A BN representation:

- Staged tree models include discrete (and context-specific) BNs as a special case.
- They are always faithful and all atomic probabilities are strictly positive.
- Tree graphs are efficient in modelling asymmetric problems.

Causal manipulations in staged trees

We can manipulate vertices to force all units to follow a certain development. E.g., manipulating $v_2$ in $T$ below forces students to live in one of the two cities.

Graphically, this is a projection onto a subtree with inherited labels. Here, the thick depicted $T(v_2) \subseteq T$.

- Situations upstream of the manipulation and counterfactuasures are not affected.
- Vertex manipulation is more general than Pearl’s atomic cause. It is more like a context-specific intervention.
- Causes and effects are events, not random variables.
- Effects are depicted downstream of a causal manipulation, so cause and effect are ordered chronologically.

Example: Advantages of an algebraic approach to causality

The intervention on $v_2$ in $T$ transforms the polynomial $c_T(\theta) = \theta_0 + \theta_0\theta_1 + \theta_0\theta_3 + \theta_2\theta_0\theta_1 + \theta_2\theta_0\theta_3 + \theta_2\theta_0\theta_5 + \theta_2\theta_0\theta_4 + \theta_2\theta_0\theta_6$ to the polynomial of the subtree $T(v_2)$, so $c_{T(v_2)}(\theta) = \theta_0\theta_2\theta_5 + \theta_0\theta_2\theta_6 + \theta_2\theta_0\theta_6 + \theta_2\theta_0\theta_4$.

The probability of a student leaving the city, assuming she was initially forced to live there, is $P_T(A | v_2) = \theta_2\theta_0\theta_6$. This is simply the probability of the subpath $v_2 \rightarrow v_3$.

There is not always a straightforward graphical way of representing a manipulation operation, e.g.,

- Forcing students to live in one of the two cities is not a vertex manipulation in the tree on the right.
- Forcing students who are renting with a grumpy landlord to move can be represented by $T(v_1) \subseteq T$ but on the right we would force a unit to go through two mutually exclusive edges simultaneously, following two different unfoldings from $v_5$.

Conclusions

Algebraic and differential methods allow us to perform causal manipulations on staged tree models without referring to the graph structure. They can be greatly generalised to models with a monomial parametrisation that do not rely on a priori problem variables.

An algebraic characterisation

We write the probability $P_T(A)$ of an event $A$ as a formal polynomial

$$c_A(T(\theta)) = \sum_{(A \in E)} \prod_{e \in E(A)} \theta(e)$$

which is a sum of atomic probabilities together with an indicator function of a unit passing through the edge $e$.

The effect of the causal manipulation on an event $A$ can be easily calculated as a differentiation on this polynomial

$$P_T(A | v) = \frac{\partial c_{T(v)\setminus A}(\theta)}{\partial \theta(v)} = c_{T(v)\setminus A}(\theta)$$

and coincides with an analogous formal polynomial defined on the subtree rooted at the intervention vertex $v \in V$.

References


UK CAUSAL INFERENCE MEETING 2016, April 12–15, London School of Hygiene and Tropical Medicine.