SUBTHRESHOLD VOLTAGE GATED CHANNELS: QUESTIONS

• Q1. Emergence of oscillations
On the P,Q phase diagram it was seen that a system with complex eigenvalues and decaying voltage oscillations, can destabilise to give a spontaneously oscillating neuron if \( P < -1 \). However, it was noted that for the case of a neuron with only one kind of voltage-gated channel \( P = \tau_n/\tau_v \), so that \( P > 0 \). This showed that a neuron with a single voltage-gated channel cannot go into oscillation. Nevertheless, oscillating voltages are seen in neurons in vivo. How can this be explained? Consider a neuron with two voltage-gated channels. The linearised equations are

\[
\frac{\tau_v dv}{dt} = \kappa_1 y_1 + \kappa_2 y_2 - v, \quad \frac{\tau_n_1 dy_1}{dt} = v - y_1, \quad \frac{\tau_n_2 dy_2}{dt} = v - y_2
\]

[Q] Assume that the second variable equilibrates very quickly, i.e. set \( \tau_n_2 = 0 \). Reduce the three dynamic equations to two.

[Q] Rescale time \( s = t/\tau_n_1 \). Show that \( P = (1 - \kappa_2)\tau_n_1/\tau_v \).

[Q] What conditions on \( \kappa_1 \) and \( \kappa_2 \) will destabilise the neuron and cause spontaneous oscillations? What kind of currents could this correspond to? What is the physical interpretation of the actions of the currents?

• Q2. H-current effects in the response to square-current pulses
The h-current, or \( I_h \), is a common feature of neurons with large dendritic structure. It is a hyperpolarisation-activated, depolarising current, and as such can protect neurons from too-strong a hyperpolarisation. Its signature is seen in the response to square-current pulses as a non-monotonic overshoot or sag, depending on the sign of the current. The activation time constant of \( I_h \) is much longer than the voltage time constant. So \( \tau_n/\tau_v \gg 1 \) and \( \kappa < 0 \).

[Q] Using the linear forms of the voltage equation,

\[
\frac{\tau_v dv}{dt} = -v + \kappa y + RI_{app} \quad (2)
\]

\[
\frac{\tau_n dy}{dt} = v - y \quad (3)
\]

show that, in the large \( \tau_n/\tau_v \), if \( I_{app} \) is zero before \( t = 0 \) and a constant \( I_0 \) afterwards then the early-time voltage change is

\[
v \simeq RI_0 \left( 1 - e^{-t/\tau_v} \right) \quad (4)
\]

[Q] For the late-time dynamics the voltage can be assumed to be at its equilibrium, \( \tau_v dv/dt = 0 \). Use this result to express \( y \) as a function of \( v \) and solve the equation for \( y \) and show that it follows

\[
y \simeq \frac{RI_0}{1 - \kappa} \left( 1 - e^{-\frac{t}{\tau_n}(1-\kappa)} \right) \quad (5)
\]

[Q] Using the formula relating \( y \) and \( v \) from the \( \tau_v dv/dt = 0 \) approximation, show that the late-time voltage follows

\[
v \simeq \frac{RI_0}{1 - \kappa} \left( 1 - \kappa e^{-\frac{t}{\tau_n}(1-\kappa)} \right) \quad (6)
\]
Introduction to Theoretical Neuroscience

Subthreshold Voltage-Gated Channels

Week 4 Questions

1. Sketch the form of the voltage response.

2. **Q3. Eigenvalues for a neuron with the H-current**

   We now derive the eigenvalues in the same limit just considered with $\tau_n/\tau_v \gg 1$.

   [Q] In the previous question, what were the two decay time constants identified in this approximation?

   [Q] Using the forms for $P$ and $Q$ given in the lecture notes, show that in the large $\tau_n/\tau_v$ limit both $P \gg 1$ and $Q \gg 1$.

   [Q] Expand the eigenvalue equation to show that the two roots take the form

   \[
   \lambda_+ = -\frac{1}{\tau_n} \left(1 - \frac{Q}{P}\right) \quad \text{and} \quad \lambda_- = -\frac{1}{\tau_n} \left(P + \frac{Q}{P}\right).
   \]  

   [Q] Show that these two eigenvalues are consistent to leading order with the time constants derived above.