INTEGRATE-AND-FIRE MODELS: QUESTIONS

• Q1. Quadratic Integrate-and-Fire (QIF) Model

The quadratic integrate-and-fire model obeys

\[
\frac{dV}{dt} = q(V - V_0)^2 + \mu
\]  

which is identical to the canonical IF model, except that here the reset \( V_{re} < V_0 \) and threshold \( V_{th} > V_0 \) are finite. The model spontaneously oscillates when \( \mu > 0 \).

[Q] Show that the period \( T \) is equal to

\[
T = \frac{1}{\sqrt{q\mu}} \left( \arctan \left( \frac{(V_{th} - V_0)}{\sqrt{\frac{q}{\mu}}} \right) - \arctan \left( \frac{(V_{re} - V_0)}{\sqrt{\frac{q}{\mu}}} \right) \right). 
\]  

[A] Divide equation (1) by its RHS and integrate between \( V_{re} \) and \( V_{th} \), which are the voltages at the beginning and end of a period. It helps to rescale time and voltage.

[Q] Show that just above the critical point, when \( \mu \) is small and positive, the firing rate of the canonical Type I model is recovered which is \( r = \sqrt{q\mu}/\pi \).

[A] The argument of the first arctan approaches \(+\infty\) and the second \(-\infty\) so that the sum of the terms in the parenthesis becomes \( \pi \) - this yields the required firing rate.

• Q2. Behaviour of the Exponential IF Model near the Critical Point

The exponential integrate-and-fire model obeys the equation

\[
\frac{dV}{dt} = \frac{F(V)}{\tau_L} = \frac{E_L - V + \Delta T e^{(V-V_T)/\Delta T} + I/g_L}{\tau_L}.
\]  

With a threshold at \( V_{th} = \infty \) and \( V_{re} < V_T \) finite.

[Q] Show that the minimum of \( F(V) \) is at a voltage \( V_T \).

[A] Straightforward differentiation.

[Q] Expand \( F(V) \) around \( V_T \) to second order in voltage to show that near \( V_T \)

\[
\frac{dV}{dt} \simeq \frac{1}{2\Delta T\tau_L} (V - V_T)^2 + \frac{E_L + \Delta T - V_T + I/g_L}{\tau_L}.
\]  

[Q] Identify the critical current \( I^* \).

[A] \( I^* = g_L(V_T - \Delta T - E_L) \).

[Q] By comparison with the form for the QIF, show that for an applied current just above the critical value the firing rate of the EIF model is

\[
r = \frac{1}{\pi \tau_L} \sqrt{\frac{I - I^*}{2\Delta T g_L}}.
\]  

[A] This follows directly from \( q = 1/2\Delta T\tau_L \) and \( \mu = (I - I^*)/g_L\tau_L \).