Pupils Learning Algebra with ICT in Key Stage 3 Mathematics Classrooms

A thesis submitted for the degree of Doctor of Philosophy in Education

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Author's Declaration

The ideas developed during my MSc I reported in Chapter 1 contributed directly to:

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Sections relating to research methodology in Section 3.1.5 were adapted from an assignment previously completed as part of Advanced Research Methods at the University of Warwick.

I hereby confirm that the work presented in this thesis is entirely my own. This thesis has not been submitted to any other university.

Abstract

This thesis is set in the context of learning early secondary algebra using ICT. It argues that the support offered by a collaborative interaction of elements (pupils, teachers, language and computers) in lessons, that is, the affective dimension of pupils' mathematical constructions, has not been emphasised enough in studies about the impact of ICT.

Through a classroom-based intervention study, pupils' conceptual understanding in algebra was monitored. The study examined the effect of introducing a technological tool on pupils' interest in algebra, their involvement and engagement in mathematical learning, and the role of the teacher in ICT-enhanced settings. Research was conducted during secondary mathematics lessons in England and in Kenya. This intervention used 'dialogic teaching' and *Grid Algebra* as catalysts, with traditional resources alongside new technologies.

The study sought to demonstrate that a 'blended' approach to learning can mediate the pupils' access to algebraic concepts. The study employed a mixed-method strategy including: written work, observations, interviews and pupil questionnaires. Analysis of collected data underlines the value of formative feedback, clear expectations and developing mathematical language.

The study concludes that appropriate use of computer software can have a significant impact on a whole year group. Additionally, emphasising affective aspects which reinforce ICT use in mathematics instruction can create an enabling environment for active learning. This thesis makes an original contribution to effective teacher development and delivery on the premise of a social model of learning mathematics with ICT.

Keywords: algebra; ICT-enhanced mathematics education; mathematical language; formative feedback; mathematical resilience; teacher development.

Glossary of Abbreviations

APOS -	Actions Processes Objects Schema
ASEI-PDSI -	Activity Student-centred Experiments Improvisation - Plan Do See Improve
BECTA -	British Educational Communications Agency
CEMASTEA -	Centre for Mathematics, Science and Technology Education in Africa
CPD -	Continuing Professional Development
CSMS -	Concepts in Secondary Mathematics and Science
DEAG -	Digital Education Advisory Group
epiSTEMe -	Effecting Principled Improvement in STEM Education
GCSE -	General Certificate of Secondary Education
HOD -	Head of Department
IAS -	Instrumented Activity Situations
ICCMAS -	Increasing Competence and Confidence in Algebra and Multiplicative Structures
ICT -	Information and Communications Technology
INSET -	In-service training
JICA -	Japanese International Cooperation Agency
KIE-	Kenya Institute of Education
MBRQ -	Mathematics-Related Beliefs Questionnaire
NSCT-	National Council for Science and Technology
Ofsted -	Office for Standards in Education
QASO -	Quality and Assurance Standards Officer
SMASSE -	Strengthening Mathematics and Science in Secondary School Education
SPSS -	Statistical Package for the Social Sciences
TISME -	Targeted Initiative in Science and Mathematics Education
TPACK -	Technological Pedagogical Content Knowledge
ZPD -	Zone of Proximal Development

Chapter 1: INTRODUCTION

1.0 Introduction

This chapter sets the scene for my research, to enable the reader to engage with the thesis. I first outline the aim of my research, followed by descriptive background information on the education context in Kenya with reference to the United Kingdom (UK). I link the local context to national policy and current debates in mathematics education. A description of the Information and Communication Technology (ICT) tool of choice follows explanation of my participant-researcher role in this study.

1.1 Purpose of the study

1.1.0 Research aims

I set out to examine the effect on participants' learning behaviour of Information and Communication Technology (ICT) in mathematics education in Kenya. I introduced subject-specific software, *Grid Algebra* (see Section 1.5 below), in a sequence of lessons. I intended to address inherent pupil disaffection and involvement (Skinner and Belmont, 1993) in mathematics by enabling:

- Pupil learning, when encountering algebraic concepts in 'talk'-around-ICT activity;
- Teacher 'learning', when supporting pupils' ICT-enhanced sense-making.

Learning and teaching algebra in Key Stage 3 (early secondary) mathematics classrooms has often been discussed at a cognitive level. Based on extensive research on human psychology, Bruner (1957) considered that '***thinking***' was the primary outcome of cognitive development resulting from new learning experiences of intelligent minds. Bruner (1960) favoured a spiral curriculum, whilst arguing that, as active constructors of their own knowledge, children are capable of understanding complex information. Kieran (1992) described algebra as 'difficult', with research predominantly focused on the cognitive aspects of pupil learning in literature. Hewitt (2012) questioned the attribution of 'difficulties' with learning algebra to either the nature of the topic or to pupils' characteristics instead of to the pedagogic practices.

Tall and Thomas (1991) proposed that a *'computer approach'* can mediate algebraic concepts for pupils, as argued by Noss, Healy and Hoyles (1997). According to Sutherland, Robertson and John (2009), teachers distribute responsibility between interacting sociocultural elements and technology to improve classroom learning. However, Ofsted (2008) highlighted the low

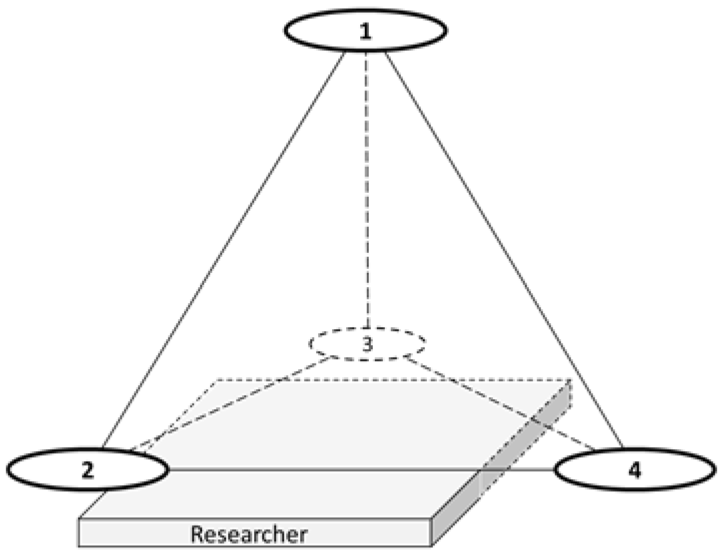
ICT uptake in early secondary mathematics lessons in the UK. Webb and Cox (2004) noted that teacher debate over the amount of control given to pupils by ICT tools featured in research; they called for teachers to be aware of pupils' beliefs on ownership of learning and to understand affordances in ICT-enhanced environments. Walshaw (2013) proposed 'hands-on' researcher support for teachers. In this thesis, I use 'computer' to refer to the computer itself, the related peripherals and software.

Using computers in mathematics has largely focused on cognition, for example Hoyles, Pozzi and Sutherland (1994). McLeod (1992) argued for integrating cognitive and affective dimensions in mathematical learning. Elder (2012) explained that, according to neuroscience, powerful positive emotional responses triggered by learning experiences can become part of the learners' most enduring memories. I envisioned 'dialogic teaching' (see Section 2.1.5), as defined by Alexander (2008), in computer-based collaborative activity creating a relaxed, enabling context for learning with pleasure. Through the enhanced social interaction that arises from 'dialogic teaching', I aimed to promote participants' learning motivation (Dweck, 2000) and their thinking about mathematics as *interesting*. Affective traits, namely the pupils' 'fun' and involvement (Nardi and Steward, 2003) and teachers' unease with ICT tools (Webb and Cox, 2004) in mathematics have been neglected. Based on Lerman (2001), I deemed both pupils and teachers to be 'learners' requiring support to adopt ICT use in lessons. In my thesis, I foreground pupil learning behaviour and assign secondary importance to teacher 'learning' in ICT-enhanced activities. This decision reflects a pedagogical approach advanced by Gattegno (1970); he placed the children at the centre of education processes.

Research by Hewitt (2009) reported Key Stage 2 pupils successfully learning formal algebra with ICT. The study findings should have challenged the on-going delay in the teaching of topics deemed too complex for children's cognitive maturity. Pupils demonstrated the capacity to learn 'difficult' concepts beyond their chronological age. They exemplified the opposition by Bruner (1960) to the Piagetian notion of readiness. Hewitt (2009) raised key pedagogical issues whilst illustrating the perspective of pupil involvement. Less attention was directed to affective aspects concerning the regular teacher's role in a classroom context. The fundamental impetus driving my research was 'dialogic teaching' and 'hands-on' ICT use (Becta, 2008) to address pupil disinterest in algebra and disengagement (Brown, Brown and Bibby, 2008) in Key Stage 3 mathematics classrooms (see Section 2.5).

This intervention stimulated the redistribution of classroom ‘talk’ through ‘dialogic teaching’ mediating ICT-enhanced activity to generate pupil interest in algebra. I incorporated ICT and non-ICT resources for learning algebraic concepts. My research design evoked the notion of ‘powers’ Gattegno (1970) argued to be the attributes of a human mind that individuals bring to learning contexts. I intended to create a ‘community of learning’ (Lave and Wenger, 1991) to harness learners’ emotions through ‘formative feedback’ (Juwah, Macfarlane-Dick, Nicol, Matthew, Ross and Smith, 2004). I evoked sociocultural concepts (Vygotsky, 1978, 1962) to emphasise social interaction and language (see Section 1.1.0) in established pedagogy. Also, I applied social cognitive theory (Bandura, 1986) in principle to focus on affective traits in interactions between pupils, teachers and available resources to redistribute learners’ agency (Gresalfi, Martin, Hand, Greeno, 2009) in classroom contexts (see Section 2.4).

My research examined the effect of intervening in a classroom ‘subculture’, mediating learners’ ICT use to address problematic pupil interest in algebra and engagement in mathematics. The support of collaborative interaction of elements in the changed learning context espoused the ‘cultural discursive psychology’ (Lerman, 2001; 1998). I ascribed catalytic roles to *pupil talk* within ‘dialogic teaching’ and to an ICT tool, *Grid Algebra*, in the research design.



Key

Element:

1 – Pupils
2 – Content: Algebra
3 – Teacher
4 – ICT: *Grid Algebra*

Process:

1
2 4 - Pupils’ instrumental genesis

1
2 3 – Teacher-mediated instruction

3
2 4 – Teacher ‘reconnaisance’

1
2 4
3 – ICT-enhanced learning

Figure 1.1: Learning processes in the mathematics classroom

In published ICT literature (Mumtaz, 2000), it is widely accepted that effective ICT use in schools is determined by the extent to which the users perceive ICT to be facilitative and relevant to the teaching intent and learning outcomes, as well as the capacity to implement ICT in pedagogically and socially constructive ways (Hennessey et al, 2005). Extensive published public literature has often decried low ICT uptake in teaching (Webb and Cox, 2004). I tried the best possible research design to achieve dual learning outcomes, as explained by Guskey (2002). I set out to modify the triadic interactions between pupils, teachers and algebra, an approach deemed in school algebra literature (Goulding and Kyriacou, 2007; Kieran, 1992) to have been of limited success. I instigated a tetrad shown in Figure 1.1, within school contexts, by integrating affordances of *Grid Algebra* to combine social, cultural and linguistic aspects in classrooms.

I envisioned disaffected pupils becoming confident and competent learners, hence enabling change from teacher-directed instruction hitherto dependent on textbooks and teacher-talk to pupil-centred learning. Such a transition relied on the software's capacity to operate in synergy with 'traditional' technologies to *supplement* existing practices through conditions, tools and outcomes in the 'learning ecology' (Luckin, 2008). Change hinges upon whether ICT use is more aligned to teacher hegemony and transmission of knowledge than it is supportive of pupil-centred approaches which recognise that learners construct their own knowledge (Bruner, 1960). Guskey (2002) showed that teachers tend to be convinced by new approaches when they see positive pupil learning behaviour and outcomes. Led by passion for pupil learning, I assumed the daunting 'facilitator' role to oversee changes, as discussed by Walshaw (2013). Increased incidence of pupil 'talk' restricted teachers to *supportive* roles in the change, thus reinforcing affective aspects of cognition and instruction in mathematics. I hoped that teacher familiarity with *Grid Algebra*'s potential (plane 234) would precede pupils exploring in plane 214 (see Fig.1.1). Enabling a learning context served to complement and bridge social and individual learning (Sfard, 2001). From an 'agentic' perspective (Bandura, 2001), many learners seemed to feel safe to realign conceptual algebraic knowledge. Following Hoyles (2001), computer mediation played a key role in regulating patterns of both classroom interaction and ICT tool exploration. 'Formative feedback' (Juwah et al, 2004) in learning contexts illuminated ICT's transformative potential.

In this thesis, I consider 'disaffection' as disengagement (Boaler, Wiliam and Brown, 2002), and 'learning' as participation with understanding (Sfard, 2001). Nardi and Steward (2003) distinguished between 'quiet' disaffection characterised by low engagement in learning from the 'visible' disaffection seen as disruptive behaviour and negative experiences of schooling. I argue that contribution of affect in secondary pupils' mathematical knowledge construction has not been discussed enough in studies about the impact of ICT, and I demonstrate what happens when affect is addressed explicitly.

1.1.1 Rationale of research

Based on the theories of Vygotsky (1894- 1934), I viewed schooling as a social experience in which learning is socially-constructed. Bruner (1961) considered the purpose of education as inspiring children's thinking which developed into transferrable problem-solving skills, and not the transmission of knowledge. In my view, teaching should be a means by which human values are experienced, shared and developed. Teachers are expected to facilitate how pupils learn by being responsible for the content and conditions: the social process. This expectation effectively makes pupil learning the reflex of teaching and accountable to the teaching intent. However, pupils should nurture behaviours, attitudes, skills and knowledge in order to make positive contributions to wider society. Gattegno (1970) envisioned learning and teaching operating in tandem for the benefit of children. He argued that pupils are responsible for their learning and effort, hence their thinking (Bruner, 1957): an individual and cognitive process.

Whereas the teacher may be largely responsible for pupils' social experiences, learning is a private process that, in the case of schooling, takes place in a public context: the classroom. Learners exercise *agency* which Bandura (1989) defined as the human capacity to control one's thought processes and motivation, and to sustain effort in the face of adversity. It was important to me to know whether teachers inspired understanding, or did work for pupils (Hewitt, 2012). Nonetheless, pupils have been known to construct their own meanings of what their teachers teach. Research evidence suggests that many pupils do not necessarily 'learn' what they are taught in classrooms (Brown, Askew, Baker, Denvir and Miller, 1998). The authors asserted that the teaching and learning processes are different and individual. Apportioning blame for many pupils' failure in learning concepts to either low 'ability' or to poor teaching, targets the individual. This essentially directs feedback to the 'self' which is limited in moving learning forward (Hattie and Timperley, 2007). Also, such practices divert attention from the affective to cognitive aspects of learning. Both pupils and teachers require support in ICT-enhanced contexts to realise intended learning outcomes. Sfard (2001) argued that 'learning' conceived as a knowledge-acquisition process divorces cognition from the context. In order to address problematic participation in mathematics lessons, acknowledgement is needed that learners may struggle with merging diverse views of desirable learning behaviour. Norris and Walker (2005) recognised that cognition and affect in mathematics cannot always be reconciled by learners. I planned to offer restructuring support for social, cultural, and linguistic processes in lessons. A *'blended'* approach to learning would provide a means of classroom instruction and organisation that made use of the available meditational tools productively. This would entail managing the complex dynamics of participants' learning behaviour within a participatory perspective in previous and altered learning contexts (see Figure 1.1).

Introduction of computer-based collaborative activities to a prevalently textbook-based ethos served to encourage more pupil 'talk' and less teacher 'talk'. Luckin, Bligh, Manches, Crook, Ainsworth and Noss (2012) argued that content and pedagogy should drive technology use. Alexander (2008) associated the concept of 'dialogic teaching' to 'assessment for learning', as advanced by Black, Harrison, Lee, Marshall and Wiliam (2003). The participant classes revisited complex algebraic concepts that they had previously encountered at a simplified level with gradually increasing difficulty (Bruner, 1960). The activities allowed pupils to solve problems by themselves. Pupils were required to demonstrate their ability to link algebraic ideas appearing in multiple representations while exploring with *Grid Algebra*. A respectful defence of their argument when challenged was expected of the pupils. Emphasis was on turn-taking as each contributor articulated their thinking to facilitate negotiation of shared understanding (Edwards and Mercer, 1987). The computer provided a 'window' (Noss and Hoyles, 1996) into contributor's reasoning. The immediate and consequential software feedback (Hewitt, 2012) provided vital constraints to regulate pupils' actions. Questioning and explaining of solutions played a vital role in promoting increased 'connectivity' (Boaler and Greeno, 2000; Askew, Brown, Rhodes, Wiliam and Johnson, 1997), and 'deep learning' (Abbot, Townsend, Johnston-Wilder and Reynolds, 2009).

For effective learning by both children and adults, Bruner (1966) proposed three modes of representation: ***enactive*** (actions-based); ***iconic*** (visually-stored images or 'mental' pictures); and ***symbolic*** (language-based in words and symbols). In this study, participation as observers in ICT-enhanced activity provided some mathematics teachers with an invaluable opportunity to 'learn' about their pupils learning algebra with enthusiasm and enjoyment. These learning experiences marked a stark departure from the established norm reported in literature (Brown et al, 2008; Nardi and Steward, 2003). In light of the feedback principles listed by Juwah et al (2004), teachers *listened* more to pupils' appropriation (Mercer, 1994) of algebraic concepts. Data collected through physical artefacts (Yin, 2009) provided culturally-invented (Lerman,

2001) evidence of the 'transfer' of learning (Lobato, 2003), which Hoyles et al (1994) reported. Data allowed teachers to reflect on actions and interactions in learning contexts. It challenged the meanings teacher participants ascribed to classroom learning processes, namely: intended curriculum objectives, perceptions of pupils, and grouping practices.

1.1.2 Context of the main study

In this section, I briefly depict a typical secondary mathematics classroom based on personal experience (see Section 1.4), and informed by literature. There are at least 42 different tribes in Kenya, each with its own ethnic language; citizens speak one national language, Kiswahili; English is the designated official language in education. Since 1989, the basic educational structure offers learners 8 years of primary schooling (Standard 1to 8), 4 years at secondary level (Form 1 to 4), and 4 years at university. Mathematics occupies a 'sacred' position of a powerful *judge* filtering those regarded as *able* from those who are not. Pupil achievement in Mathematics and English assume 'gatekeeper' roles to one's life opportunities, education and employment. This is firmly established in Kenya.

Consequently, many Kenyan parents attach great importance to education for their children. In turn, young people ascribe to the quest of achieving a good education through doing well in school. The majority of parents and pupils believe strongly in education holding the key to possible escape from manual, low-paying work and even poverty, the path to thriving careers and the ladder to success. Therefore, most teachers and pupils are under tremendous pressure to obtain good examination scores. Kanja, Iwasaki, Baba and Ueda (2001) singled out the key defect of career-oriented educational systems as certification through examinations. At school level, examination scores are given prime consideration in determining whether learners may progress to the next class. Pupils take national examinations at the end of primary education and secondary. Eligibility for admission to an excellent secondary and to university or other institutions of tertiary education is based on pupils' academic performance in these national examinations. Teachers' professional reputations and chances for promotion may be judged against their pupils' achievement.

Many learners at various levels of education are seen to rely upon 'spoon-feeding' teaching methods and rote learning. Moreover, learning conditions are hardly ideal. Large class size, on average 50 pupils, is a fact of life in Kenya. Teachers are pressured to cover all the content prescribed in national subject curricula. Additionally, they are expected to prepare candidates for national examinations amid typical teaching workloads of about 35 lessons per week. The circumstances have led many teachers to harbour beliefs that an effective teaching approach to adopt is teacher-directed in an attempt to cover large amounts of text-book driven content. Kanja et al (2001) discussed how through drill-and-skill instruction, pupils are put through their paces of practising problems before sitting their examinations. They described teachers talking to pupils incessantly instead of allowing time for pupil reasoning and thinking. Mathematical content is presented to pupils in simplified, often decontextualized and isolated chunks that encourage memorisation of rules rather than developing higher-order thinking and problem- solving skills. One pupil described the current situation in Kenyan classrooms:

We so much just sit in class, the teacher would come teach and write on the board. We are not free to talk to our desk mates and ask questions. (Pupil G044)

The remarks underlined disaffection, low pupil participation and involvement, as argued by Nardi and Steward (2003). Classroom ethos discourage many pupils from grasping content by restricting pupils' 'agency' (Pickering, 1995), as I discuss in Section 2.3.2. The learning context described tends to diminish pupils' value of the knowledge learned and its applicability to problem-solving. It reinforced my intention of promoting pupil 'talk' (Alexander, 2008) using *Grid Algebra*-based collaborative learning activity to enhance formative feedback.

Over the years, concern has grown among educators and policy-makers in government circles regarding problematic interest, participation and engagement in education. Several initiatives have been proposed to address these problems. One such approach focused on the teaching of Science and Mathematics in student-centred pedagogies over teacher-directed learning. The Kenyan Ministry of Education, Science and Technology alongside the Japanese International Cooperation Agency (JICA) sponsored an in-service training (INSET): Strengthening Mathematics and Science in Secondary School Education (SMASSE), targeting all practising subject teachers (Kanja et al, 2001). Implemented initially as a pilot in 1998, this initiative was later rolled out countrywide in 2003. SMASSE-INSET cycles were conducted annually during school holidays. The principles adopted were: **A**ctivities, **S**tudent-centred teaching, **E**xperiments, **I**mprovisation-**P**lan, **D**o, **S**ee, **I**mprove (ASEI-PDSI). Increased pupil-centred teaching was aimed at enhancing student engagement in classrooms. However, evaluative reports belied key concerns expressed by teachers: that implementing pupil-centred pedagogy demanded more teacher time, and its full application would slow down syllabi coverage. It compounded the enormous pressure on teachers to complete subjects' syllabi and preparation of pupils for examinations. Teachers have found themselves reverting back to the 'usual' practice shown in plane 312 (see Fig. 1.1). The failure of the SMASSE initiative to give direct attention to the pupils' perspectives about their own learning was a serious flaw.

A different approach to addressing problems in education systems is suggested in calls made by, for example, the Digital Education Advisory Group (DEAG) of Australia and the Joint Mathematical Council (JMC) of the United Kingdom. They advocated for the re-engineering of education by integrating digital technologies, pedagogy and curriculum. Livingstone (2012) argued that desirable educational goals to be achieved in the 21st Century are for pupils to learn critical thinking, to analyse and synthesize information from diverse sources. Hence, the Kenyan Ministry of Education, through the Centre for Mathematics, Science and Technology Education in Africa (CEMASTEA), launched an initiative to integrate ICT into education and training. This initiative advocated for the application of 21st Century methods in teaching and learning richly enhanced by ICT use. An INSET was launched in a 4-year plan (2009- 2013) to "upgrade teacher skills and competencies" (CEMASTEA, 2009, p.22); it recognised a gap between desired student-centred strategies and the prevalent pedagogy many teachers employ to enhance pupil participation and develop scientific minds. Most pedagogic skills acquired through existing initial teacher-training programmes are deemed teacher-centred. Sutherland, Robertson and John (2009) emphasised the pivotal and supportive role that teachers play in improving learning with ICT in classrooms, illustrated as 2143 (see Figure 1.1). Researchers (Sutherland et al, 2009; Askew et al, 1997) advocated against simply persuading teachers to adopt proposed pedagogies. Instead, the teachers need to develop a thorough understanding of the underlying principles upon which new strategies are based. I addressed this as my secondary research aim (see Section 1.1.0).

Nevertheless, one weakness existed within Kenya's initiative.

> *CEMASTEA has not made in-roads in the development of teaching and learning materials.*
> (CEMASTEA, 2009, p.26)

CEMASTEA attributed the lack of relevant resources to realise the vision of ICT integration in education to existing weak links between itself, the national curriculum developer, Kenya Institute of Education (KIE), and teacher-training institutions. (KIE was succeeded by Kenya Institute of Curriculum Development in January 2013.) CEMASTEA hoped that embracing the critical principles in the ASEI-PDSI approach would make Science and Mathematics less theoretical, more practical, therefore more interesting, accessible and relevant to pupils. The successful delivery of learning experience was envisioned to transform the current classroom practices through preparation, implementation and review of learning activity (CEMASTEA, 2009, p.32). CEMASTEA charged Heads of Schools and Quality Assurance and Standards Officers (QASOs) with the challenge of monitoring and evaluating ASEI-PDSI in schools. A broader goal of placing greater responsibility for learning on the students was geared towards meeting Kenya's industry requirements in line with Vision 2030 (CEMASTEA, 2009). This recognised teachers' concerns about student-centred learning restricting content coverage when emphasis is on examinations for qualifications (Kanja et al, 2001).

The CEMASTEA goal focused on teachers' application of 21st Century teaching methods to deliver effective pupil-centred learning in Mathematics and Science. I contend that this vision comes across as endorsing a transmission role for teachers; it fails to acknowledge pupils as active agents and constructors of knowledge (Bruner, 1960) in classroom contexts. I proposed to subordinate teaching to pupil learning (Gattegno, 1970) by leading and sharing innovation across countries. I sought to fill a gap by introducing mathematics-specific computer software developed in United Kingdom to Kenya through research. My study built on CEMASTEA's plans by collating participants' perspectives about pupils learning algebra in ICT-enhanced mathematics lessons. I intended to carve out a role for pupils' voices in the on-going education reforms. Whilst conducting mathematics education research in England, the teacher in me had experienced *Grid Algebra*'s potential to mediate 'difficult' concepts for pupils (Lugalia, 2009). My sharing of this resource and experience with learners culminated in research I will describe in Chapter 4. In the next section, I focus my research.

1.2 Mathematics learning

1.2.0 Scoping my research

The importance of mathematics at Key Stage 3 is stated in the UK National Curriculum.

> "*Mathematical thinking is important for all members of a modern society as a habit of mind for its use in the workplace, business and finance, and for personal decision-making. Mathematics is fundamental to national prosperity in providing tools for understanding science, engineering, technology and economics. It is essential in public decision-making and for participation in the knowledge economy. Mathematics equips pupils with uniquely powerful ways to describe, analyse and change the world*"

(Department for Education, 2007, p.1)

In this statement, mathematics is recognised as an inherently human activity. Key concepts in mathematics learning are listed:

> *"1.1 Competence*
>
> > *a. apply suitable mathematics in the classroom and beyond;*
> >
> > *b. communicate mathematics effectively;*
> >
> > *c. select appropriate tools and methods, including ICT.*
>
> *1.2 Creativity*
>
> > *a. combine understanding, experiences, imagination and reasoning to construct new knowledge;*
> >
> > *b. using existing mathematical knowledge to create solutions to unfamiliar problems;*
> >
> > *c. posing questions and developing convincing arguments.*
>
> *1.3 Application and Implications of mathematics*
>
> > *a. knowing that mathematics is a rigorous coherent discipline;*
> >
> > *b .understanding that mathematics is used as a tool in a wide range of contexts;*
> >
> > *c. recognising the rich historical and cultural roots of mathematics.*
>
> *1.4 Critical understanding*
>
> > *a. knowing that mathematics is essentially abstract and can be used to model, interpret or represent situations;*
> >
> > *b .recognize the limitations and scope of a model or representation."*

(Department for Education, 2007, p.4)

Such description succeeded in capturing the essence of the mathematical knowledge and skills that secondary school pupils should develop. This emphasised valuing the provision of variety as tools and resources that can encourage both the verbal and written communication of ideas. It implied that pupils select appropriate methods for solving problems on equivalence, algebra, proof of operations and their inverses in diverse modes of representation (Bruner, 1966). It envisioned pupils appreciating mathematics as a tool for seeking practical solutions whilst being aware of situations when mathematical models and representations may lack relevance.

Of central interest to my thesis were the social, linguistic and cultural processes expected of learners in mathematics.

> *"2.4 Communicating and reflecting*
>
> *Pupils should be able to:*
>
> *a) communicate findings effectively;*
>
> *b) engage in mathematical discussion of results;*
>
> *c) consider the elegance and efficiency of alternative solutions;*
>
> *d) look for equivalence in relation to both the different approaches to the problem and different problems with similar structure;*
>
> *e) make connections between the current situations and outcomes, and situations and outcomes they have already encountered."*
>
> (Department for Education, 2007, p.5)

In resonance with the UK National Curriculum, Kenya Institute of Education (KIE, renamed Kenya Institute for Curriculum Development in January 2013) , being the national curriculum developer, enshrined sociocultural processes in the general objectives of secondary education.

> *"By the end of this course, the learner should be able to:*
>
> - *develop a positive attitude towards learning mathematics;*
> - *perform mathematical operation and manipulation with confidence, speed and accuracy;*
> - *develop a willingness to work collaboratively;*
> - *acquire knowledge and skills for further education and training;*
> - *communicate mathematical ideas."*
>
> (KIE, 2002, p. 4)

Tall and Thomas (1991) argued that 'versatile thinking' in pupils "requires the availability of cognitive interaction between concepts represented by imagery as well as symbolically and verbally" (p.131). Despite valuing varied learning experiences (Bruner, 1966), they appeared to disregard *affective* interactions between conditions, resources and outcomes in classrooms.

1.2.1 Learning school algebra

A school-based research programme, the 'Concepts in Secondary Mathematics and Science' (CSMS), was carried out between 1974-9 to investigate the nature of pupils' understanding of various concepts in topics whilst being taught mathematics and science in the UK. In findings on algebra, the CSMS Mathematics Team reported that pupils showed considerable difficulty with accepting letters for variables and grasping the relationships between variables in algebraic expressions (Küchemann, 1981). These findings are reflected in the Kenyan context based on my experience as a secondary mathematics teacher. The UK government has launched large-scale national initiatives to address pupils' difficulties over the past 30 years. Efforts targeted improving low pupil participation in specialist mathematics beyond compulsory education. In response, Brown et al (2008) involved 1500 16 year olds in 17 UK schools in a quantitative study to establish reasons behind learner non-participation. Reported reasons ranged from: perceived dislike, lack of confidence, boredom, its perceived irrelevance and difficulty based on students' learning experiences, the teaching, older students, their parents and siblings at home. The researchers found that many secondary school pupils cannot apply algebraic concepts to problem-solving nor do they appear to demonstrate secure understanding of underlying structures. Hodgen, Küchemann, Brown and Coe (2008) wrote that the steady and substantial increment registered in attainment since the 1980s has not reflected matching improvement in student engagement. They said "independent measures of attainment suggest that these rises may be due more to "teaching to the test" rather than to increases in genuine mathematical understanding" (p. 36). The replicated science strand of the CSMS study revealed 'decline' in students' understanding of mathematical ideas. Reflecting on Hodgen et al (2008) and Stage One findings, I report in Section 4.1, led me to defining 'algebra'.

1.2.2 Working definition of school algebra

Van Amerom (2003) stated the apparent lack of universal consensus on what 'algebra' is and how it should be learned. I will highlight similar views by Mason and Sutherland in Section 2.2.5. According to Bednarz, Kieran and Lee (1996), the approach to school algebra depends largely on the emphasis placed on whichever learning activity is considered 'algebraic'. Hewitt (2011) listed 'algebraic' activity as: "the appearance of letters; working with equations with letters on both sides; working with or on an unknown; expression of generality using actions, words and gestures; seeing the particular in the general, and the general in the particular; and operations upon operations" (p. 502). The favoured approach in the UK

National Curriculum emphasises “algebra as generalised arithmetic” (Department for Education, 2007, p. 6).

In recognition of this lack of a universal approach to school algebra, I adopted the following definition for the purpose of my thesis.

> *“Algebra is a branch of mathematics that deals with symbolizing general numerical relationships and mathematical structures and with operating with those structures.”*
> (Kieran, 1992, p. 391)

To me, the definition emphasises the underlying structures in algebra expressions. Kieran (1992) argued that a ‘structural conception’ required operation on mathematical objects and working with processes as opposed to ‘procedural conception’ of algebra; the latter is associated with operations on numbers and evaluating outcomes. From the problem-solving perspective, Bell (1996) highlighted symbolic language and its manipulation in developing algebraic thinking in pupils. He argued that:

> *“I do not see algebra as an identifiable course separate from other branches of mathematics, but as appearing throughout the mathematics course in expressing generalisations, solving geometric problems, solving equations, establishing relations in trigonometry, as formulae in statistics and mensuration…”* (p. 167)

He underscored the statement made by Goulding and Kyriacou (2007) about the centrality of algebra to the vast mathematics curriculum.

1.2.3 School algebra in Kenya

My viewing ‘algebra’ as consisting of conceptual tools for operating with various areas of mathematics seemed appropriately aligned to the Kenyan context. Prescribed content in the topic ‘Algebraic Expressions’ appeared consistent with approaches that consider algebra as the study of relationships, and as a problem-solving tool (Bell, 1996; Kieran, 1992). At Stage Three, pupils were required to meet the following specific objectives.

> *“By the end of the topic, the learner should be able to:*
>
> *a) use letters to represent numbers;*
>
> *b) write statements in algebraic form;*
>
> *c) simplify algebraic expressions;*

d) factorise algebraic expressions by grouping;

e) remove brackets from algebraic expressions;

f) evaluate algebraic expressions by substituting numerical values;

g) apply algebra in real life situations.”

(KIE, 2002, p.9)

These pupils were expected to “use the idea of like and unlike terms to simplify and factorise algebraic expressions; use the term *expansion* when removing brackets, and apply brackets in multiplying expressions”. For the reader’s sake, I clarify that algebra is gradually introduced to pupils over the final four years of a ‘spiral’ primary curriculum (Bruner, 1960).

- *Standard Four: Use of letters for numbers;*
- *Standard Five: Simplify algebraic expressions; Work out simple equations in one unknown;*
- *Standard Six: Simplify algebraic expressions; Equations in one unknown; Symbols-‘ > ’ and ‘< ’; Comparison of quantities using ‘=’, ‘> ’ and ‘< ’ symbols;*
- *Standard Seven: Forming and simplifying algebraic expressions; Substitution in algebraic expressions; Forming and solving equations; Simplifying inequalities in one unknown.*

According to views I present in Section 4.4.1, it appears pedagogy and systemic dysfunction between primary and secondary schools restrict many pupils’ ability to comprehend ideas they have spent years learning. I accessed their existing understanding of algebraic concepts with a view to build on that knowledge through procedures I describe in Chapter 3.

1.3 Current trends in mathematics education

1.3.0 Introduction

In this section, I discuss briefly activity of interest launched in the UK: the Targeted Initiative in Science and Mathematics Education by the Economic Social and Research Council, with the Gatsby Charitable Foundation, The Institute of Physics, and The Association of Science Education. The TISME initiative aimed to inspire increasing pupil participation, engagement, achievement and understanding in four subject areas: Science, Technology, Engineering, and Mathematics (STEM). Two of the five programme’s projects are of relevance to my thesis.

1.3.1 The ICCAMS

A four-year (2008- 2012) project on Increasing Competence and Confidence in Algebra and Multiplicative Structures (ICCAMS) was based at Kings' College, London. ICCAMS work, underpinned by Piagetian and Vygotskian theories of teaching and learning, emphasised four pedagogic principles: *formative assessment; connectionist teaching; collaborative working; multiple representations.* In Phase 1, a large-scale survey of attainment in algebraic reasoning was conducted. The project intended to establish and provide information on 11-14 year olds' current understanding in basic algebraic ideas since the original CSMS project (Hodgen et al, 2008). The ICCAMS team related conceptual understanding and progression in attainment to pupils' attitudes, teaching and demographic factors. Their Phase 2 concentrated on formative assessment since findings by Black et al (2003) illuminated many practising teachers having difficulty with applying key principles, including quality of feedback and expectations. The ICCAMS aimed to support teachers' prudent use and interpretation of existing mathematical resources in specific areas through collaborative research. Hodgen et al (2008) hoped support would improve pupils' confidence, thus increasing participation, engagement and attainment.

1.3.2 The epiSTEMe

The Effecting Principled Improvement in STEM Education (*epiSTEMe*) project was based at the University of Cambridge. The researchers developed the pedagogic model driven by five principles: *dialogic teaching; enhanced context; domain-specific inquiry; cooperative group work; active teaching* (Ruthven, Hofmann, Howe, Luthmann, Mercer and Taber (2011). They stated that many secondary pupils in England found mathematics 'uninteresting' and difficult to learn with understanding. Sizeable gaps in pupils' learning diminished the opportunities to further education and lifelong learning. The project proposed developing principled approach grounded in research evidence to teaching and learning in the formative secondary stage. This was expected to re-engage, and guide, pupils towards understanding mathematics. Ruthven et al (2011) emphasised 'talk' and 'dialogue' as supporting subject teaching and learning; they stressed developing processes to establish collaborative small-group working and whole-class discussions. Ruthven et al (2011) identified these practices needing attention in mathematics: the posing of authentic problems; serious consideration of students' thinking; tasks' structure for cooperative group-work; training students to value group-work; linking students' interests and experiences; emphasis on higher-order reasoning. The *epiSTEMe* team proposed varying the crucial teaching role through: 'exploration' to stimulate students' thinking; 'codification' as active teaching; and 'consolidation', to check students' understanding, for effectiveness.

However, neither of the two projects made explicit the potential for use of ICT tools in their proposed pedagogic models. Hodgen, Brown, Küchemann and Coe (2010) mentioned ICT as 'wasteful resources' in mathematics learning since the 1970s.Whereas ICT use cannot be introduced in a vacuum, it can *displace* other activity, like mediating between pupils and 'difficult' concepts. This displacement hinges on *how,* not *what* ICT, is used.

The University of Warwick Institute of Education collaborated with teachers on a small-scale, action research project (2008-2009) targeting 14-19 year olds. They investigated ICT tools' potential to change the culture of educational institutions and individual classrooms. Abbot et al (2009) argued that ICT use can enable learners to experience concepts from multiple perspectives. Hewitt (2011) called for adopting alternative pedagogies to facilitate pupils' access to algebraic concepts. Healy, Pozzi and Sutherland (2001) underlined formal algebraic notation as crucial for the negotiation of meaning and problem-solving processes in pupils' experiences. Clark-Wilson, Oldknow and Sutherland (2011) claimed that developing curricula that embrace digital technologies' use can increase pupils' interest in learning mathematics, science and technology. Such moves would make learning accessible to many youths whilst contributing to securing and improving technological advancement in the UK. Clark-Wilson et al (ibid) believed that teachers and policy makers need to recognise discrepancies between pupils' personal use and the utilisation of digital technologies in formal education.

I merged a 'computer approach' (Tall and Thomas, 1991) with non-ICT resources to develop pupils' algebraic thinking, and gauge learning. This '*blended*' approach to learning algebra intended to replace problematic classroom practices (see Section 1.1.0). My own 'learning' journey through my role in this research unfolded as follows.

1.4 Researcher role

1.4.0 Personal background

This research has entailed my taking on various roles in secondary school classrooms in both England and Kenya. My PhD journey built on my awareness of the development I experienced from study on an MSc Mathematics Education course at the University of Warwick. Learning experience had challenged my professional understanding of practice. Prior to this, I taught Physics and Mathematics as a graduate teacher in secondary schools in Kenya. I worked with pupils aged 13 to 18 years, hence my preference for researching at that level. The use of ICT was not part of my initial teacher training. In Section 1.1.2, I depicted secondary mathematics learning context in Kenya as inherently teacher-centred, and based on a 'traditional' *textbook approach*, which restricted successful manoeuvres towards pupil-centred learning. Digital resources are yet to be embraced. I was confronted by the well-documented 'difficulties' of secondary pupils learning algebra in mathematics (see Section 2.2.6), and low attainment in national examinations as a teacher.

1.4.1 Encountering ICT

A 'Digital Technology and Mathematical Thinking' module within the MSc course involved my using *Grid Algebra* in a brief classroom-based research project in England. I worked with a group of 17 Year 8 pupils deemed to be at teacher-assessed UK National Curriculum attainment level 4. Pairs of pupils learned while using software on computers over four one-hour lessons. As pupils talked to each other, I realised that they learned the order of operations; the teacher pointed out to me she was yet to teach them that (Lugalia, 2009). This was what I will refer to as my first 'aha' moment; I personally experienced the power of technology to mediate abstract mathematical concepts for pupils. It captured my interest that, by engaging with the software in the learning environment where they shared freely their thoughts with each other, and with us, pupils were learning something profound, independently of teacher-directed instruction.

Given my professional background, and many experiences on the MSc course, I reflected on the project's evidence. I realised that digital technology can offer significant opportunities in terms of both how pupils learn and how I taught mathematics. I noted the Year 8 pupils' discomfort with the line notation of division in mathematical expressions; to them, $\frac{10}{2}$ was a fraction, hence hardly a supportive 'met-before' (Tall, 2004). The pupils appeared to gradually learn formal notation through acceptance of associating movements with mathematical operations (Hewitt, 2012), one software feature I highlight in Section 1.5.1. Pupils' confusion arising from line notation for division illustrated the notion of a 'procept' advanced by Gray and Tall (1994). According to Gray and Tall, pupils were demonstrating difficulty in relating the concept of 'fraction' to the process of dividing whole numbers. This episode underlined the role of symbolic language in underlying structures as pupils learn algebra in mathematics. I was further intrigued by these pupils' low level of proficiency with multiplication tables and mathematical terminology. The learning behaviour indicated quite low expectations of this group of learners. The pupils very quickly despaired rather than applying more effort to the learning tasks.

For my MSc dissertation, I investigated 'dialogic teaching' in secondary school mathematics with focus on one teacher's practice; I discuss the principles in Section 2.1.5. I continued to reflect on *how* these two elements, 'dialogic teaching' and ICT, combined to enhance pupils' conceptual understanding in mathematics. I visualised myself with pupils in an ICT-enhanced learning environment back in Kenya. I felt a moral obligation to undertake a leading role by sharing my knowledge and expertise of *Grid Algebra* designed to integrate computer teaching in mathematics education. It followed about two years of raising participants' awareness prior to conducting research at this school. Elaborate initial plans were built upon preparation and support of a proposed collaborative project with several UK software developers, Warwick Institute of Education and local Kenyan universities. This project intended to induct Kenyan teachers and pupils into using mathematics-specific software. My study was to ride on proposed project's shirt-tail as lead researcher. Unfortunately, the project did not take place. Walshaw (2013) considered researchers as 'mediators' bridging gaps between practice-based work of teachers and institutional expectations to achieve learning outcomes. Thus, I built on my own professional knowledge with new experience and understandings developed from MSc work to support effective learning of algebra in secondary mathematics classrooms.

1.4.2 Position at main research school

I conducted my main research at one boarding secondary school (see Section 3.1.0), to which I had been attached in my past role as a Physics/Mathematics teacher in Kenya. The school had the necessary hardware (computers in two laboratories), and was welcoming to the prospect of using computers in mathematics. The particular site doubled as a district training centre for SMASSE-INSETs and was earmarked by CEMASTEA for the training of primary and secondary school teachers. The Head of Department (HOD)-Mathematics is a SMASSE-INSET trainer in Mathematics with responsibility for teachers at a district level. I opted to train in an active role in Physics, and remained in close contact with developments in Kenya through my sister (a teacher of Mathematics) who became this school's departmental ICT lead. Both HOD-Mathematics and ICT lead took on the additional role of QASOs (see Section 1.1.2). Familiarity with the school's Mathematics and Science teachers in SMASSE-INSETs was crucial to my gaining access for research.

I acknowledge that my role may be considered rather ambiguous in that I am not part of the teaching staff. I was therefore an actor in my study with a flexible role to help introduce the software to both pupils and teachers. My research was intended to be a flexible intervention responding to pupils' and teachers' needs and interests (see Section 1.1.0). Without a doubt my ambiguous role did generate issues during research; I will address these in Section 3.2.4. I hasten to add here that this is not an unusual route to take in doctoral research. Hansen (2008) incorporated a design study to *'Children's geometric defining and a principled approach to task design'* at a primary school in England. Clark-Wilson (2010) conducted an exploratory study on *'How does a multi-representational mathematical ICT tool mediate teachers' mathematical and pedagogical knowledge concerning variance and invariance?"* within a secondary school in England and focused on teachers making sense of hand-held technology.

1.4.3 Epistemological stance

For the sake of the reader, this section illuminates certain assumptions that play a key role in grounding my stance to research. It includes beliefs that have influenced my research interest and interpretation of findings challenged and developed by relevant literature.

1.4.3.1 Supportive classroom interactions

I see the mathematics teacher as facilitating an enabling classroom 'subculture' (Lee, 2006; Skinner and Belmont, 1993); and recognise teachers as instrumental in managing the social ethos through establishing 'ground rules' (Alexander, 2008; Mercer and Littleton, 2007).

1.4.3.2 Articulation of mathematical ideas

I value active encouragement of pupils to articulate their mathematical ideas (Pimm, 1987) since pupils are capable of picking up and learning what is not taught (Hewitt, 2011; Lugalia, 2009). The more pupils express their mathematical ideas, the more they can develop and internalise those higher mental functions (Vygotsky, 1978, 1962) that they need to grasp and use mathematical ideas. Through talk and active participation in the learning process (Gresalfi, Martin, Hand and Greeno, 2009), learners can process new information. Pupils need more opportunities to take control of their learning (Boaler and Greeno, 2000).

1.4.3.3 Mediation of 'difficult' concepts by tools

I see the '*blending'* ICT and non-ICT resources as providing variety in pupils' mathematical learning; I recognise that ICT use, on its own, does not enhance learning (Sutherland, Barnes, Armstrong, Brawn, Breeze, Gall, Matthewman, Olivero, Taylor, Triggs, Wishart, John, 2004). The process of blending redirects focus from pedagogy and teachers' alignment of information in the context to intended learning outcomes (Ruthven, 2012) for pupils.

1.4.3.4 Value of formative feedback

I recognise the complex nature of classroom contexts as dynamic places in which a variety of resources (people, skills and knowledge) interact (Luckin, 2008); I acknowledge that human beings respond differently to stimuli in environments in which they work. Although feedback from formative assessment is crucial for promoting learning (Juwah et al, 2004), on its own it may not lead to effective learning. Pupils must take corrective action based on this feedback information (Sadler 1989) in order to learn; pupils need an enabling environment in which they feel safe to act and take managed risks (Lee and Johnston-Wilder, 2013).

1.4.3.5 Teachers as 'models'

I see the teacher's role as '*demonstrative*' (Bliss, Askew and Macrae, 1996) and *facilitative* in deciding learning tasks and allocating available resources for pupils' use within institutional contexts; pupils need induction (Claxton, 2004) to develop confident use of mathematical language (Mercer and Sams, 2006) whilst communicating verbally and in writing.

1.4.3.6 Learners responsible for their own learning

I see learning mathematics as an individual activity; I see mental construction to be subjected to assessment in a connected, social, linguistic and cultural setting (Vygotsky, 1978). Pupils need to think creatively, test predictions through critical reflection, apply their knowledge to problem situations, and communicate ideas with reason (Pickering, 1995).

1.5 *Grid Algebra* software

1.5.0 Overview

In making the choice of *Grid Algebra* for this study, I focused on mathematical processes and operations (Hewitt, 2013a) to promote a structural conception of algebra (Pierce and Stacey, 2007). *Grid Algebra* is based upon the making of 'journeys' across a multiplication grid as *supportive* 'met-befores' (McGowen and Tall, 2010). Noss and Hoyles (1996) advanced the '*Play Paradox*', suggesting that pupils attending to an experience of 'play' may fail to realise intended learning outcomes due to heightened tensions in teaching-learning processes. The arithmetic context provides a strong sense of 'purpose' (Ainley, Pratt and Hansen, 2006) for conceptual understanding as a meaningful learning outcome. Ainley et al (ibid) reiterated the belief that 'play' enables learning by transferring control over what is learned from teacher to pupils. According to Whitton (2007), ‘play’ in education can engender pupil engagement which in itself is a powerful contributing factor to effective learning. Arcavi (1994) argued that learning algebra required symbols to be readily-available sense-making tools. He defined having ‘symbol sense’ as being competent in algebra.

1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25
6	6	12	18	24	30

Figure 1.2: Software with a filled-in grid

1.5.1 Describing the software

Detailed description of features can be found at www.atm.org.uk/shop/products/sof071.html in video format. Hewitt (2013a, b) described Year 5 pupils learning formal notation, order of operations, accepting use of letters and solving linear equations with 70% success using this software. I highlight some features provided within two learning environments, the first being *Interactive Grid Algebra*, and the second being pre-set tasks within the software.

In *Interactive Grid Algebra*:

- As shown in Figure 1.2, the grid exclusively accepts integral numbers. Its size can be altered via ‘Menu’ function to allow: 5 to 24 undetermined columns; multiplication tables from -12 to 24; ‘yes/no’ option for ‘negatives’; pre-determined rows;
- A ‘Number Box’ with numerals -200 to 200;
- A ‘Letter Box’ containing the 26 small letters of the alphabet;
- An ‘Expression calculator’ for users to type expressions into a cell;
- Nature of software feedback is a consequence of *what* one drags and *where to*. The appearance of red crosses in unpermitted cells or a ‘No entry’ sign with a ‘bin’ in the right-hand corner of the screen indicated faulty movements and responses. Feedback was neither judgemental nor evaluative by explicitly stating “correct/incorrect”.

The learning environment allows users to 'play' by entering an object (letter or number) into a cell and dragging it across the grid either horizontally or vertically. Each movement (right; left; down; up) represents one of four basic mathematical operations: addition; subtraction; multiplication; division, respectively. Figure 1.3 shows examples of 'journeys' between cells. The user learns to associate particular movements with particular mathematical operations. This software represents the movements algebraically as single 'journeys' on the screen. For example, moving letter 'a' in row 2 two cells to the right is written $a + 2$; it becomes an object in its own right (Hewitt, 2013a). An expression $3\ (a + 2)$ results from dragging the new 'object' two cells down.

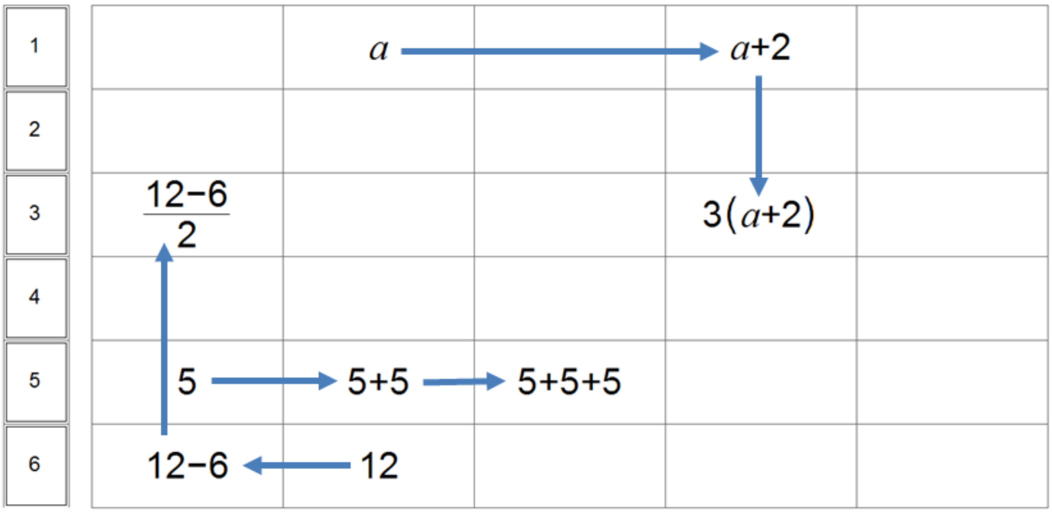


Figure 1.3: Some movements on the grid

In drawing upon pupils' knowledge of multiplication tables underpinning the grid, symbolic representation scaffolds development of structural conception of algebra within an arithmetic context. Deliberate association between physical movements and mathematical operations allows pupils to apply their existing knowledge to the new, more abstract world of algebraic notation (Hewitt, 2012). *Grid Algebra* enables pupils to link the dynamic visual representation to the formal symbolic representation to build strong a conceptual understanding of algebra.

The other learning environment of *Grid Algebra* provides *Tasks,* each consisting of questions on various concepts in algebra at varying degree of difficulty. Pupils are expected to meet the challenge of carrying out arithmetic computations since this tool does not provide answers. The disappearance of the grid after a while evoked the principle of scaffolding and 'fading' support I discuss in Section 2.1.1. Hewitt (2012) argued that non-judgemental feedback can encourage pupils to correct themselves when they are wrong, and develop confidence when they are right; they become self-regulated learners. The provision of suitable activity can encourage users to reason and make use of their existing knowledge to become aware of algebraic formal notation as something *true* (Hewitt, 2012). Pupils amend their thought processes when invited to retrial responses based on the formative feedback.

1.5.2 Factors influencing appropriate resource selection

My decision to choose *Grid Algebra* for this study was two-fold. First, classroom-based study in England (Hewitt, 2009; Healy et al, 2001) had shown that learning algebra with computers can potentially support pupils to break from arithmetic as they use arithmetic ideas in algebra. According to Arcavi (1994), technology can be harnessed such that the tool's design frees the user's mental resources to develop connections and enrich meanings necessary to undertake learning tasks. Computer software can provide rich contexts for pupils to learn *when* and *how* to apply symbolic manipulation in algebra. I had personally experienced this potential in *Grid Algebra* (see Section 1.4.1). I anticipated researching this experience in alternative settings.

The other part was largely opportunistic. My main research stage coincided with the initiative to introduce using computers in secondary mathematics in Kenya. I considered the significant levels of effort and expertise required to retrofit the computer as a tool, usually designed for tasks such as massive data storage in industry, to accomplish entirely different tasks such as mathematical learning (Kaput, 1992). Dynamic geometry software, including *Cabri-Geometre, Geometer's Sketchpad, GeoGebra* and *Autograph*, require pupils and teachers to invest significant time and effort to realise effective use in mathematics classrooms (Clausen-May, 2008). *Grid Algebra* requires users to have minimal computing skills rather than sophisticated ICT expertise. Low-level skills, such as 'click-and-drag' of a mouse, facilitate control of this software, and allow pupils to focus on learning mathematics.

1.5.3 Grid Algebra tasks

In this section, I focus on describing eight software-generated tasks which I have linked to specific curriculum objectives listed in Section 1.2.4.

1.5.3.1 Task 1: 'Calculating'

Task 1 is a number activity with tasks at ten levels of difficulty determined by the number of mathematical operations (ranging from 1 to 10) contained in each numeric puzzle.

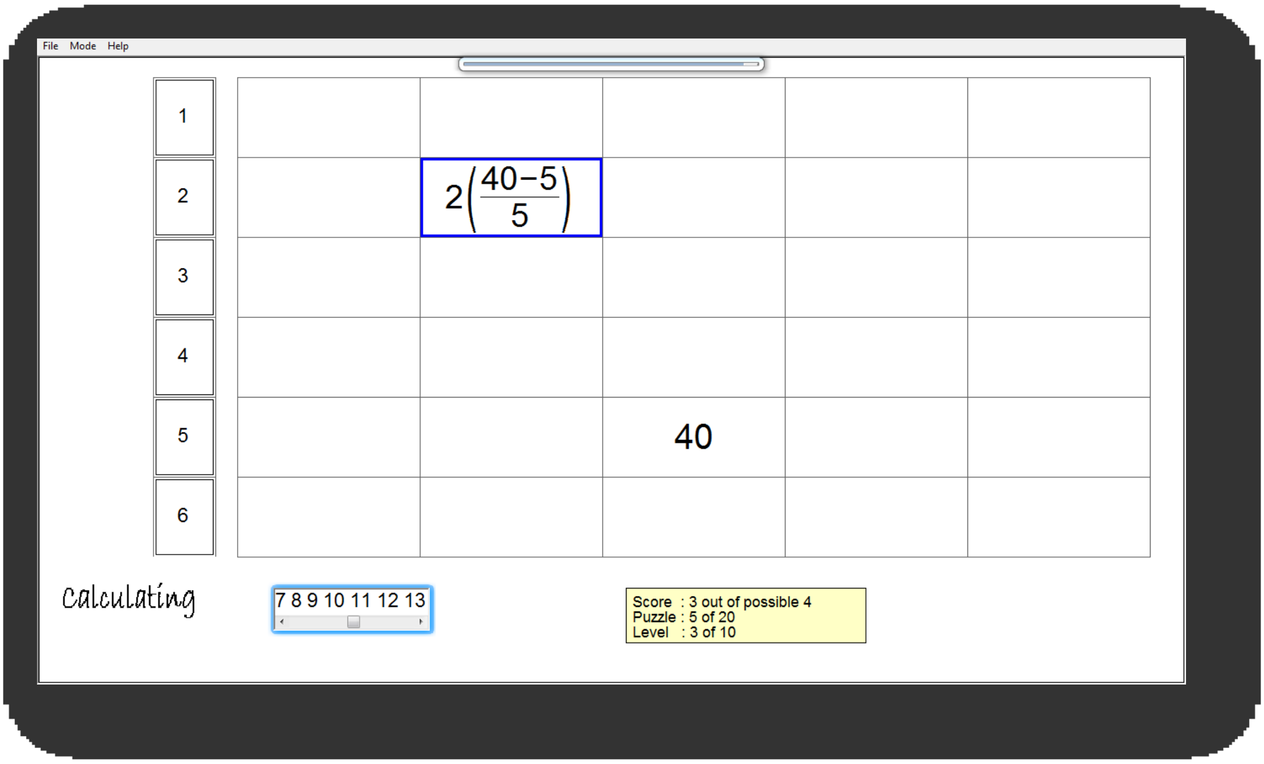


Figure 1.4: Screenshot of the software-generated task 1: 'Calculating'

Every exercise has a series of 20 questions, each involving one numeric expression in a 'blue' cell within a visible grid as shown in Figure 1.4. Questions appear as they would in printed mathematics textbooks. Pupils were expected to work out each question and then drag their answers from the 'Number Box' to the 'blue' cell. The software marked predicted responses and provided immediate feedback. Feedback appeared as a score followed by next question, or as a 'No Entry' sign in the 'blue' cell. Faulty responses hovered below the question. The feedback seemed to enable pupils to pause and reflect on reasoning underlying the responses offered. Further click of the mouse 'binned' the user's response, the score remained unaltered and the next question appeared. After the tenth question, the grid disappeared. Subsequent questions appeared in a single, enlarged cell. The software displayed final results on a score card with a brief comment, and gave the user the option of either continuing with the task by attempting puzzles at a lower, the same or next level of difficulty, or quitting the task altogether.

This task linked the learning of algebra with arithmetic (Van Amerom, 2003): it required the pupils to make connections between their current and previous mathematical knowledge (McGowen and Tall, 2010; Askew et al, 1997; UK Department for Education, 2007).

1.5.3.2 Task 7: 'Find the journey (letters)'

This activity consists of ten levels of difficulty describing 'journeys', each with between two and eleven mathematical operations, and 10 puzzles on each difficulty level.

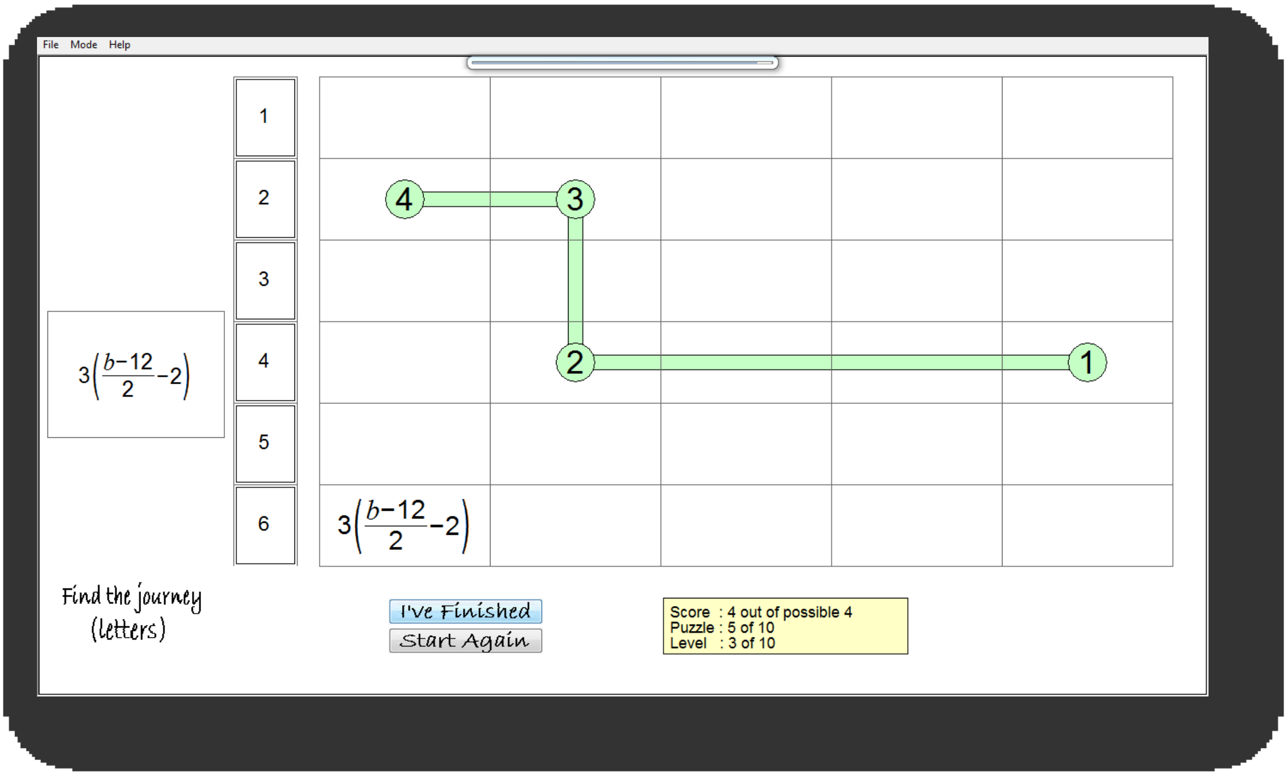


Figure 1.5: Screenshot of the software- generated task 7: 'Find the journey (letters)'

The task for the user is to create a given algebraic expression consisting of both numbers and letters by clicking on the correct cells to show the path traced by a letter across the grid. The 'journey' starts from the cell with the letter and ends at the expression. A red cross, 'X', appears briefly when the user clicks on an unpermitted cell; this encourages the user to reflect further on the mathematical operation required to recreate the given 'journey'. The software displays the path traced boldly across the grid as shown in Figure 1.5. It provides options for the user to either start the journey again or indicate they had finished the journey, whereupon instant feedback is offered and the next puzzle appears.

I hoped engagement with this task would enable the pupils to make connections between the physical movements on the grid and mathematical operations. It provided the opportunity to link the dynamic visual representation to formal symbolic algebra for pupils (Clausen-May, 2008; Noss et al, 1997) whilst they mastered using the ICT tool.

1.5.3.3 Task 25: 'What is the expression?'

A path traced by a letter across the grid from a 'blue' cell is given to the user. An expression is required to be typed into an 'Expression Calculator' as shown in Figure 1.6.

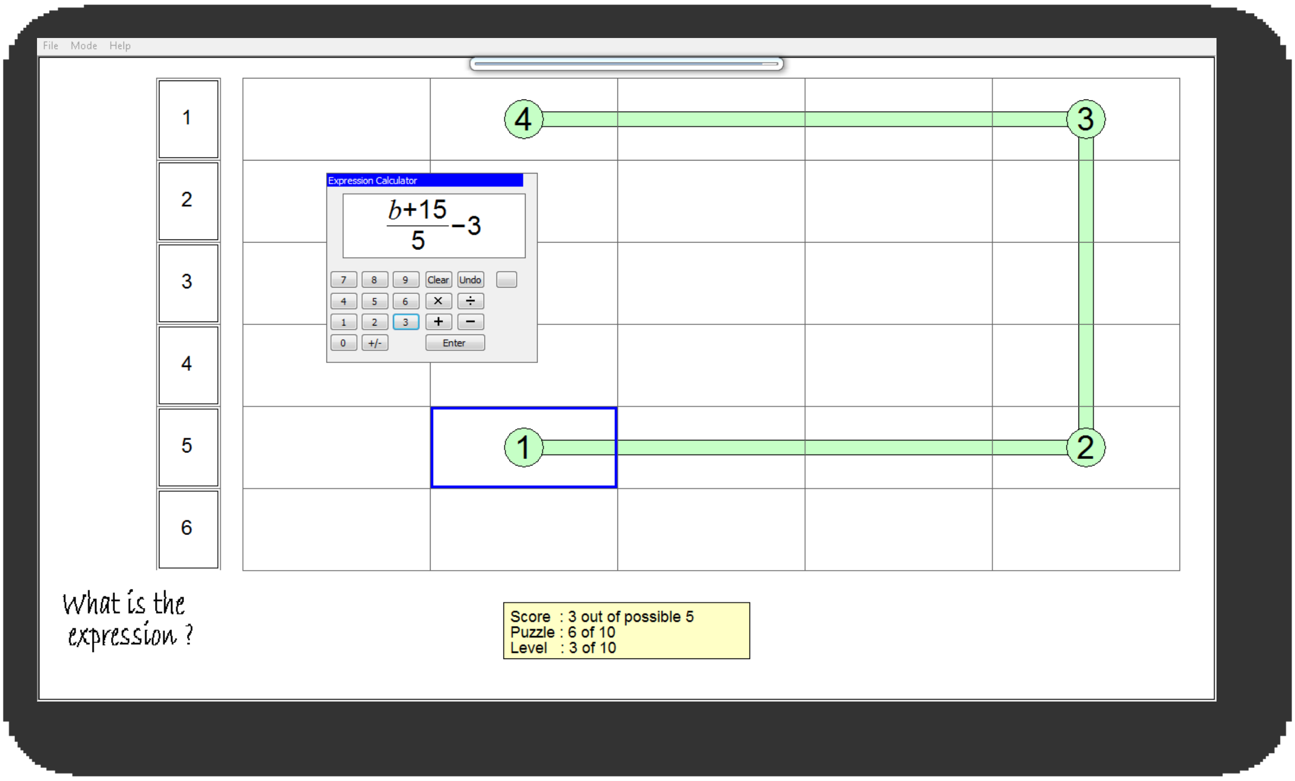


Figure 1.6: Screenshot of software-generated task 25: 'What is the expression?'

The user performs between one and ten mathematical operations on a letter represented as expressions in formal algebraic notation which the user is allowed to 'clear' or 'undo'. Each of the ten levels of difficulty consists of ten puzzles. The software provides feedback once the user presses 'Enter'; it indicates either, "Well done, Correct", or a 'No Entry' sign appears in the calculator followed by, "Incorrect, Ready for next task?", with no option to re-do the task. The user is allowed time to reflect on these responses before resuming the activity. After the tenth puzzle, the software provides results in the form of a score, a brief comment, and options to attempt the task at a lower, the same or next level of difficulty or to quit the task.

This task considered pupils' new knowledge of the software as a supportive 'met-before'. Its selection in this study was to educate pupils' awareness of the use of symbols and letters in algebraic statements, and hence to develop their 'symbol sense' (Arcavi, 1994). I hoped that engagement with this task would gauge the pupils' use of brackets for multiplication and the line notation for division as the accepted symbol convention in algebra (Bell, 1996). This is in line with the specific objective of writing statements in algebraic form (KIE, 2002).

1.5.3.4 Task 13: ‘Make the expression (letters)’

This is a timed activity.

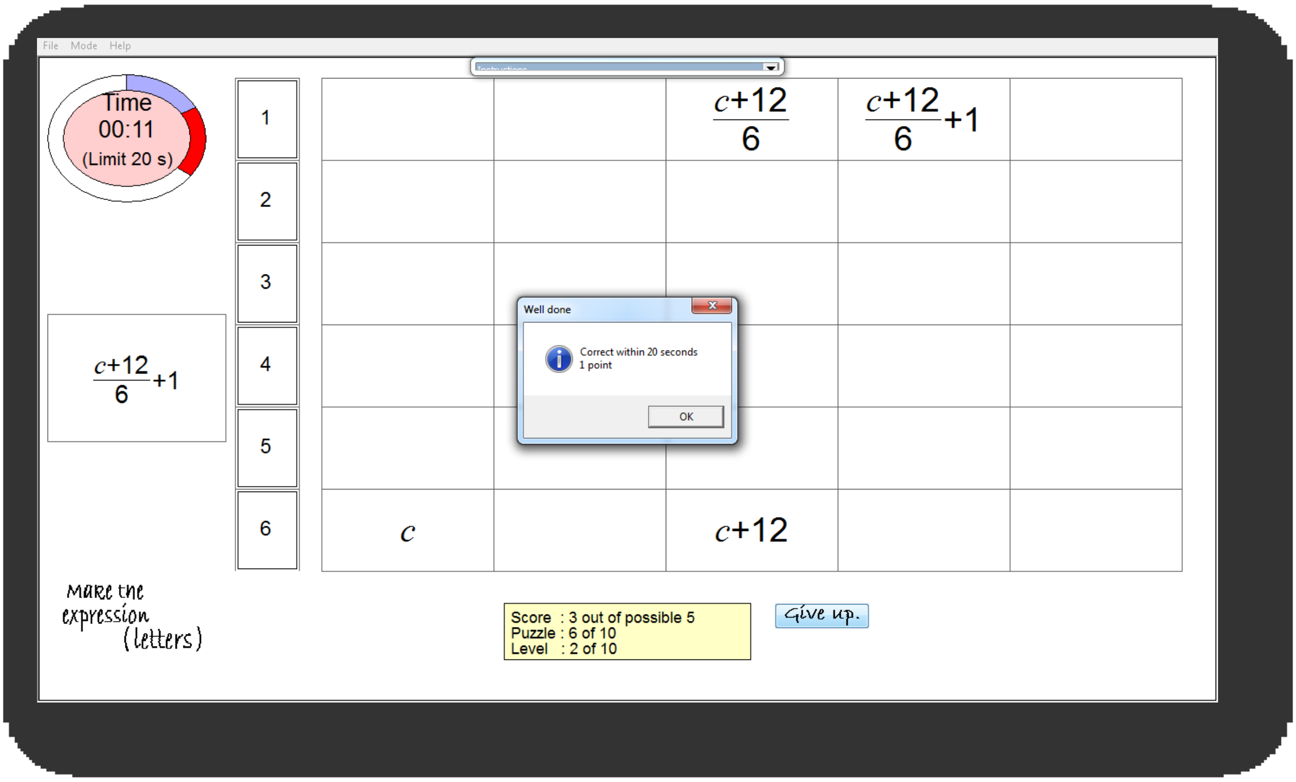


Figure 1.7: Screenshot of software-generated task 13: ‘Make the expression (letters)’

The software provides a ‘clock’ with a time limit in the top left hand corner of the computer screen, an expression below the clock, and a letter, as shown in Figure 1.7. Each difficulty level consists of ten puzzles, each with an expression created by performing between two and eleven mathematical operations on the letter. The user is required to drag the letter across the grid to create the prescribed expression within a given time frame in order to score a point, or to “give up”. They had 10 seconds for level 1, 20 seconds at level 2, 25 seconds at level 3, increasing to 60 seconds at level 10.

This task tested pupils’ knowledge of the working of the software; they read and created the given expressions whilst gauging their acceptance of the formal algebraic notation. This task was aligned with the learning objective of secondary school pupils performing mathematical operations and manipulations with confidence, speed and accuracy (KIE, 2002).

1.5.3.5 Task 12: ‘Inverse journey’

This task has three levels of difficulty: level 1 (2 to 4 operations), level 2 (3 to 5 operations) and level 3 (6 to 7 operations), with puzzles consisting of 20 questions each.

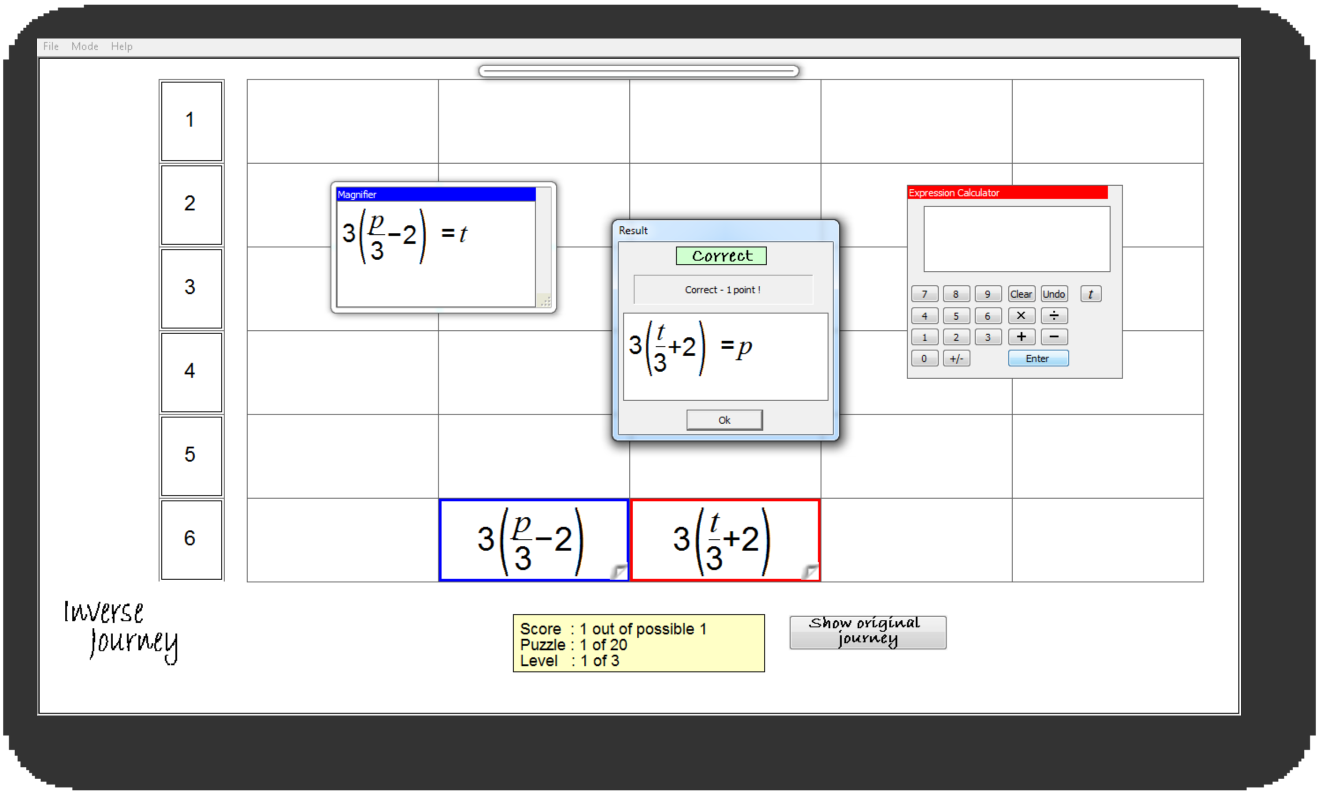


Figure 1.8: Screenshot of software-generated task 12: 'Inverse Journey'

The screen shows the grid with a letter *p* in a 'red' cell and the 'journey' traced by letter *p* is represented as an algebraic expression in the 'blue' cell. Also showing is the 'Expression Calculator' with a letter *t* on its screen and a 'Magnifier' in which the algebraic expression (the 'journey' traced by the letter *p*) is equated to the letter *t*. Each puzzle in this task required the user to type into the calculator the inverse route of the original journey shown, starting from letter *t* and ending with letter *p*. Pupils had to think of reversing each mathematical operation in order to 'undo' the expression, thereby leaving the letter *p* on its own. Once the user presses 'Enter' to indicate completion of the task, their typed response is transferred to the 'red' cell. The expected solution is displayed as shown in Figure 1.8. The software provides scaffolding for the first 10 puzzles: it allows users to see the original journey by clicking a button reading 'Show original journey'. Each correct inverse journey earns the user one point. A 'No Entry' sign indicates the entry of an error; the response remains in the calculator. In this way, the users have a chance to see and reflect upon the algebraic expressions representing the original journey, the inverse journey and the user's response.

I stated in Section 1.5.1that **the** consequential feedback provided by the software was considered crucial in enabling the pupils to make sense of algebraic meaning (see Section

2.5.4). I hoped that the focus in this task on mathematical processes would endorse a ‘structural’ conception of algebra (Kieran, 1992; Sfard, 1991). The activity stressed the argument by Bell (1996) of the role played by symbolic language and its manipulation in algebraic thinking.

1.5.3.6 Task 21: ‘Simplify’

This is an activity comprising of tasks at ten levels of difficulty, determined by between 2 and 11 mathematical operations in each puzzle, and accompanied by a set of instructions.

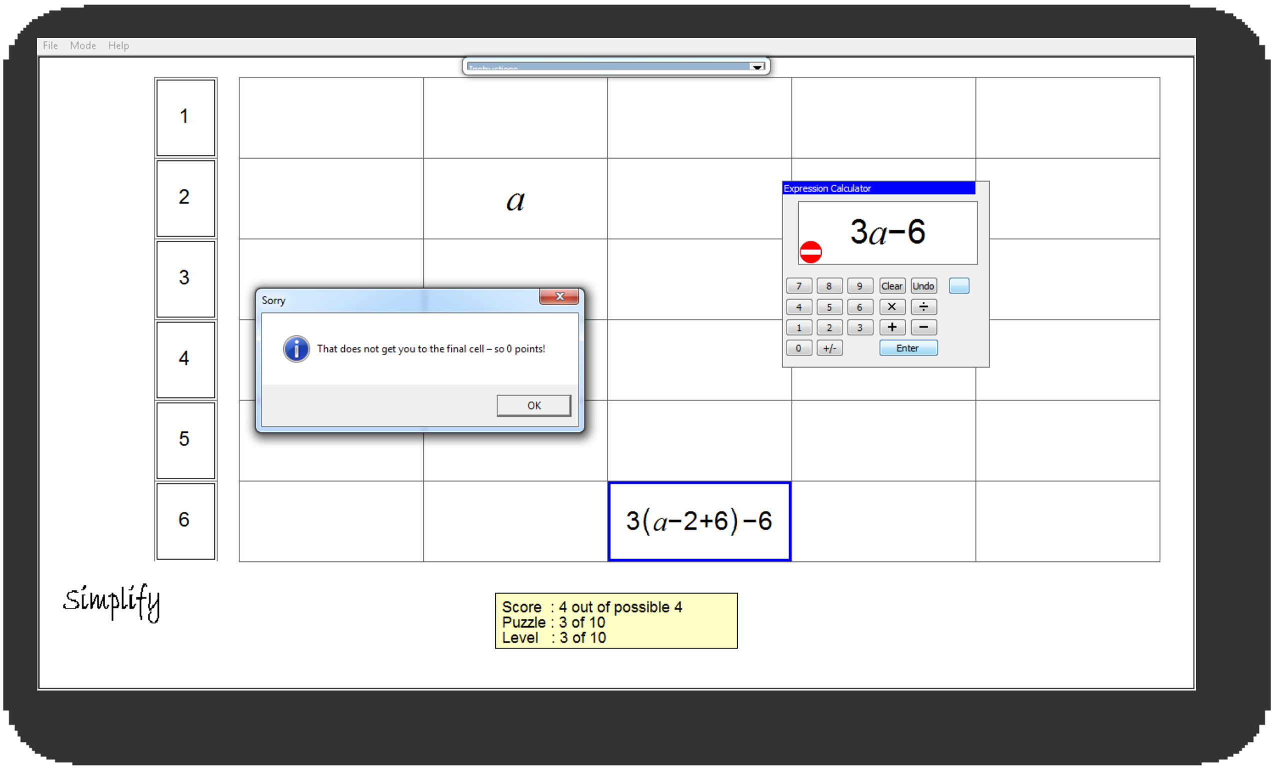


Figure 1.9: Screenshot of the software-generated task 21: ‘Simplify’

Every puzzle in a level has a series of 10 questions. Each question is an expression containing both numerals and letters, in a ‘blue’ cell with a letter in the visible grid. Questions appear as they would in the regular mathematics textbooks. The user is meant to type into the ‘Expression Calculator’ provided a simpler expression which is equivalent to the expression given. The software awards one point for a simpler expression, and two points for the simplest, as a way of encouraging the user to think of, and provide their responses in the simplest form. The grid disappears after the fourth puzzle; it leaves the letter and question lingering on the screen. In the eighth question, both the letter and ‘blue’ cell disappear: the question appears in an enlarged cell. When the user types an incorrect expression, a ‘No

Entry' sign appears in the bottom left part of the calculator screen; the software remarks, "Sorry. That does not get you to the final cell- so 0 points!" The software displays results as shown in Figure 1.9; it gives options for the user to continue with the task by solving puzzles at a lower, the same or next level of difficulty, or quitting the task altogether.

In Section 1.2.3, removal of brackets and simplifying algebraic expressions were two specific objectives to be met by pupils (KIE, 2002). The task presented an opportunity for developing a key 21st Century skill (DEAG, 2013; Luckin et al, 2012; Livingstone, 2012). The pupils were required to synthesise their grasp of various concepts within algebra and arithmetic. It evoked a connectionist belief orientation of teaching mathematics (Askew et al, 1997), which stressed making links within different mathematical concepts, a view shared by Rudduck et al (1994).

1.5.3.7 Task 22: 'Substitution'

This activity has tasks in 10 levels of difficulty; each level has 20 questions consisting of algebraic expressions with between one and ten mathematical operations. A 'red' cell with the expression, and a 'blue' cell with a letter whose value is given, are displayed as in Figure 1.10.

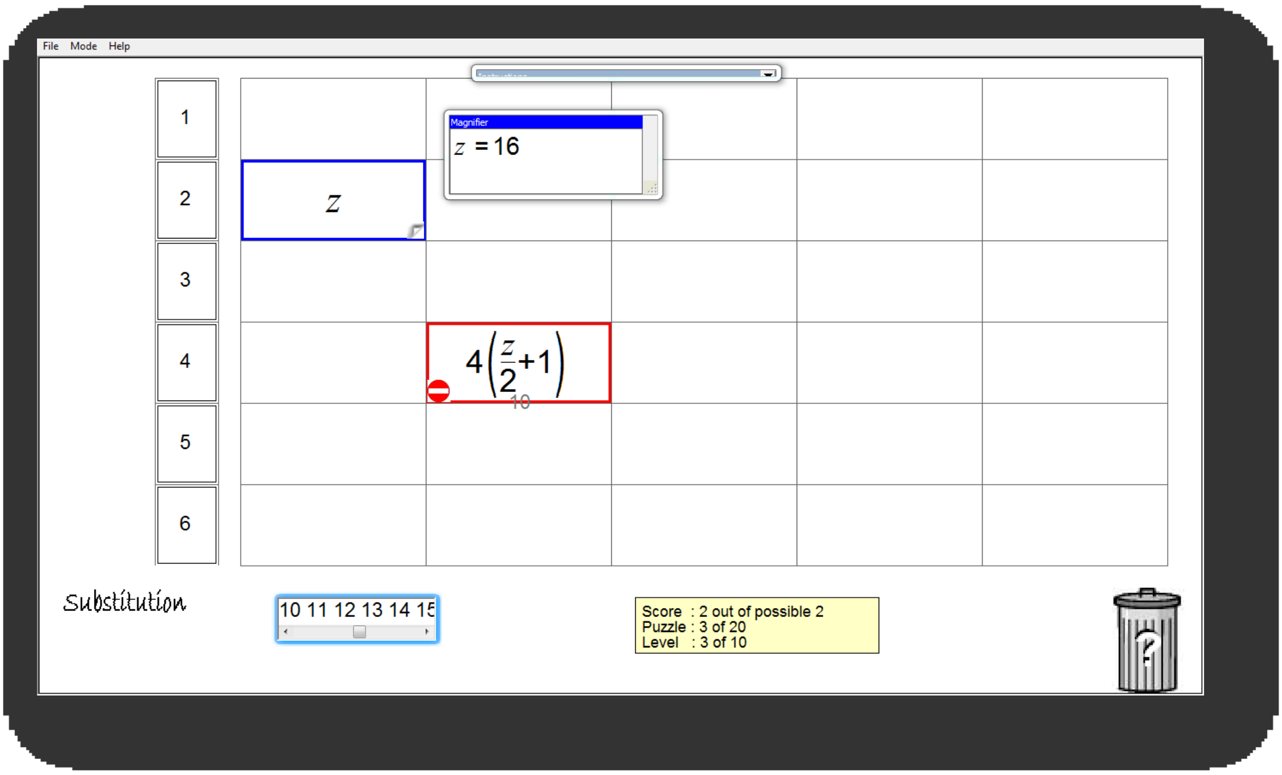


Figure 1.10: Screenshot of the software-generated task 22: 'Substitution'

The user is required to work out the value of the expression by substituting the letter in the expression, and drag their solution from the 'Number Box' into the 'red' cell. After the tenth question, the grid disappears. The software provides immediate feedback in the form of either a score and the appearance of the next question. Otherwise, a 'No Entry' appears in the 'red' cell as well as a 'bin'; the user's response lingers on the screen.

The specific learning objective to be met by the pupils was to evaluate expressions by substituting numerical values (KIE, 2002). Success depended on the pupils' knowledge of multiplication tables as a *supportive* 'met-before'. The learning activity aimed to generate reflection through 'formative feedback' provided by the software, peers and the teacher (Juwah et al, 2004).

1.5.3.8 Task 6: 'Expanding and factorising'

This activity involves the use of brackets in tasks presented in five levels of difficulty: Expanding (multiplying), Factorising (multiplying), Expanding (dividing), Factorising (dividing), and a mix of levels 1 to 4. The software displays images as shown in Figure 1.11.

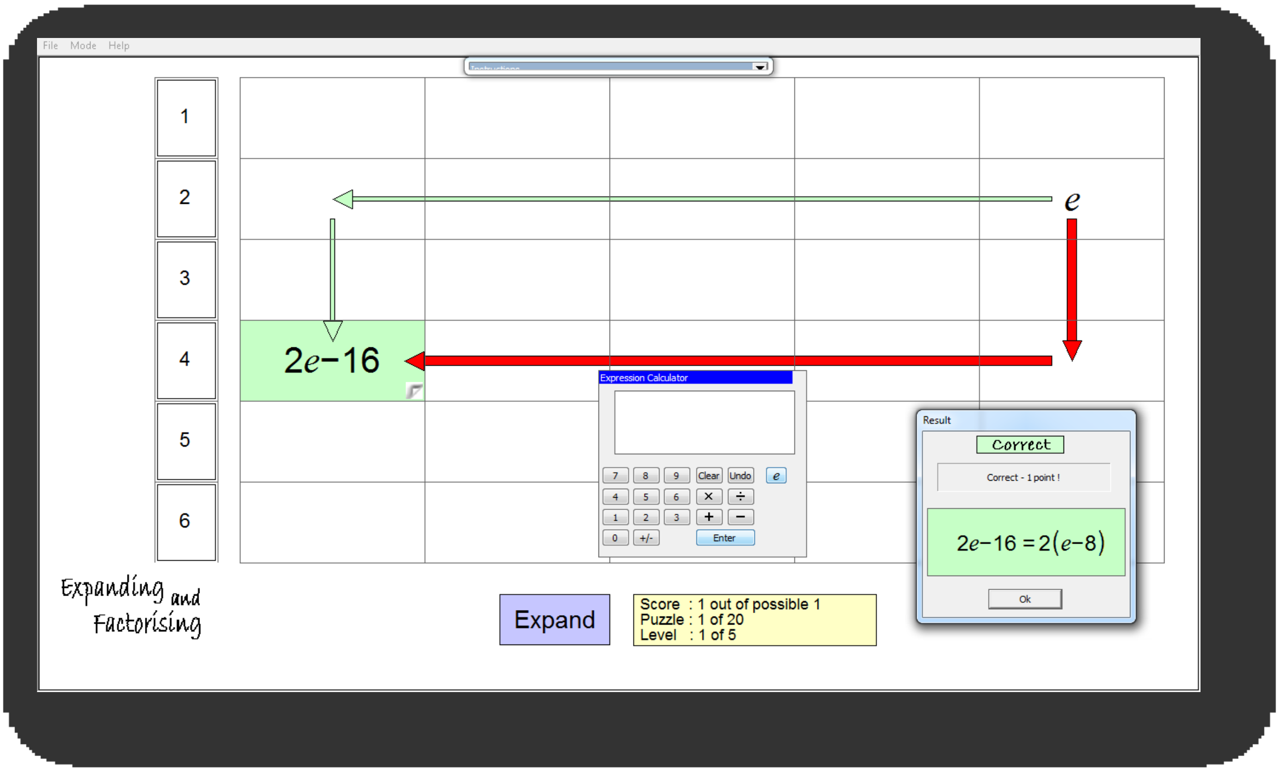


Figure 1.11: Screenshot of software-generated task 6: 'Expanding and Factorising'

A letter in a cell on the screen is the starting point of two possible routes across the grid. The user is informed that the expression in the green cell represents the 'journey' traced by the letter on the thin green route. The user's task is to generate the expression for the thick red route. There are 20 questions for each exercise. Later in the task, first the grid disappears after the sixth puzzle, leaving the letter, the expression and the two routes on the screen; after the twelfth puzzle, the routes are hidden, leaving the expression in an enlarged cell attached to the calculator. At this stage, questions appear as they would in mathematics textbooks. Once the user presses 'Enter', the software provides feedback as "Correct- 1 point!", or a 'No Entry' sign in the calculator, and "Incorrect. Have a look at the correct solution"; it then gives the result which shows the expressions from the two routes as equal to each other.

KIE (2002) emphasised the introduction and use of new mathematical terminology to pupils**.** Several researchers (Yerushalmy and Naftaliev, 2011; Clausen-May, 2008; Noss et al, 1997) argued for opportunities for increased 'connectivity' for pupils in mathematics. The dynamic visualisation and formal symbolism of the software converged with the activity-based dialogue as pupils appropriated the emerging algebraic knowledge.

I hoped that including ICT-enhanced activity in the secondary mathematics lessons would help to consolidate the pupils' conceptual understanding through the provision of alternative pathways to their learning (Abbot et al, 2009). The tasks presented learning content in ways that allowed pupils to build strong connections between the mathematical knowledge they already possessed and what they were learning.

1.6 Summary

This chapter has provided a discussion of my seemingly ambitious approach to address reported problematic pupil-centred learning in mathematics. I have described learning and teaching algebra in secondary mathematics classrooms, the *Grid Algebra* features used to realise specific learning outcomes, and I have introduced concepts for the literature review in Chapter 2.

Chapter 2: REVIEW OF LITERATURE

2.0 Introduction

This chapter provides a review of the research. I aim to identify the theoretical and empirical features that are of relevance to this study and to develop an interpretive framework for the research data. I highlight findings from several research studies where these features have been considered within the context of learning mathematics with technology. I give preference to studies using computer software. There are five sub-sections:

- the theoretical perspectives underpinning this study, highlighting the role of the computer, language and social interactions as key tools in the construction of mathematical knowledge;
- the development of knowledge, skills and understanding in mathematics as a subject discipline and meaning-making, focusing on algebra;
- learners and learning mathematics, including pupil voice, learner agency, attitudes towards learning mathematics and developing *mathematical resilience* in learners;
- the integration of ICT in education and its implications for learning and teaching in the mathematics classroom;
- the potential for use of an ICT tool in addressing the problems of learning mathematics, highlighting the importance of feedback.

2.1 Theoretical perspectives

2.1.0 Complementary theories of learning

A theory can shape our understanding by providing possible explanations for observable phenomena. According to Dubinsky and McDonald (2002), a theory in mathematics education can assist our understanding of the learning process in six ways: it can support predictions; have explanatory power; help organize our thinking about interrelated phenomena; be applicable to a broad range of phenomena; serve as a tool for analysing data; provide a language for communication of ideas about that learning.

This research was informed by the sociocultural theories of learning, principally the Zone of Proximal Development (ZPD) and internalisation, developed by Vygotsky in the 1930s. This approach emphasises that learners actively construct new knowledge whilst drawing on their beliefs and previous knowledge constructions (Vygotsky, 1978). In describing the child's potential for development, Vygotsky (1978) advanced the notion of the ZPD, which embraces the social nature of learning, and is defined as:

> *"the distance between the actual development as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers"* (p.86)

Vygotsky (1981) regarded internalisation as a process whereby social phenomena, performed between individuals on an external plane, are converted into psychological phenomena within individuals' mental planes. Vygotsky initiated the theoretical anchoring by making an explicit connection between interaction and development through the ZPD. He placed language at the heart of learning whilst underscoring the interdependence of the individual and society; social interaction was viewed as a necessary condition for learning (Vygotsky, 1978, 1962). Lerman (2001) emphasised the mediating role of linguistic (and other) tools in mathematical speaking and thinking in drawing learners forward in their ZPDs with culture preceding interaction. He viewed language and other forms of communication as 'critical' based on cultural, discursive psychology. Vygotsky (1962) suggested that we see thought and speech as two intersecting circles which coincide to produce what he referred to as 'verbal thought' in their overlapping section. Vygotsky related the development of the whole child with integrated psychological functions of perception, voluntary memory, speech and thinking whilst engaged in structured social relations with others as well as to the environment (Chaiklin; 2003; Mercer, 1994).

The ideas of Vygotsky inspire the contemporary approach to classroom education referred to as the Neo-Vygotskian theory (Mercer, 1994). Bliss et al (1996) considered both context and cultural practice in which new understandings are acquired as fundamental units within which human cognition has to be analysed. While the unit of psychological analysis is activity in the context, the principles guiding this approach are:

- Language plays a central role in learners' mental development;
- Learners construct their knowledge;
- Learning can lead to mental development (thought, language and reasoning);
- Mental development cannot be disentangled from the social context.

From the Neo-Vygotskian perspective, Mercer (1994) stressed the inextricable link between cognitive development and culture. Mercer considered learning as a communicative process through which knowledge and understanding are shared, and as social, not individualistic.

Instruction from the adult or more capable peer can support learning within the ZPD when the learner displays the will, interest and readiness to engage with maturing psychological functions; a process which Vygotsky never assumed to be enjoyable (Chaiklin, 2003). When viewing 'learning' in the light of interdependence of social and cognitive processes, I see the need for practices through which people actively influence each other's constructions. These practices may be achieved through actions and interpersonal interactions, both verbal and written, such as questioning, arguing with and elaborating on each other's ideas. According to Vygotsky (1978; 1962), social interaction increases levels of knowledge and changes in learners' thoughts and actions. Mercer (1994) considered social interaction as accounting for the increased recognition of learners as active agents who purposefully seek and construct knowledge within meaningful contexts. For this reason, Mercer and Sams (2006) argued that communication and enculturation are crucial in shaping pupils' knowledge; a view shared by Sutherland, Armstrong, Barnes, Brawn, Breeze, Gall, Matthewman, Olivero, Taylor, Triggs, Wishart and John (2004) and Morgan (2000). This has implications for our understanding of instructional practices in creating enabling learning environments. It is here that the vital role of teachers, adults, parents and peers comes to the fore in children's learning; in that they can help bring the children's knowledge to a higher level by intervening in the ZPD. This is done by providing supportive frameworks for children's thoughts, which once the learning process is complete are no longer needed by the child. Nonetheless, not all children are as 'educable' (Gattengo, 1970) in this respect: some being able to learn more in the ZPD than others.

Bandura (1986) advanced 'The Social Cognitive' theory in recognition of personal agency as operating within a causal structure determined by self-generated influences, thinking patterns, environmental factors and personal goals. Learners' engagement in classroom activities may be determined by their previous performance, present attainment and future expected goals in the context of mathematical learning. This view echoes an assertion by Bandura (1989) that self-efficacy beliefs are key determinants of human action through the motivational, affective and cognitive processes. Motivation is an integral determinant of learners' compulsion, desire and willingness to participate and be successful in learning processes. Bandura (1986) considered human agency as capacity to exercise control over one's thought processes and motivation, and to sustain effort in the face of adversity. Middleton and Spanias (1999) viewed motivation as the personal reasons for observed behaviour. They argued that motivation exists as part of pupils' goals and beliefs about what is important, and determines whether pupils ultimately engage in pursuit of goals. A belief in one's capabilities can fuel an aspiration to qualify for and pursue a mathematics-related career, and inspire hard work. Learners with high self-efficacy sustain the perseverance needed to succeed when faced with knocks, frustration and the occasional failure in mathematics. A pupil with low self-efficacy who is accustomed to experiencing failure may harbour doubts about their ability to succeed. Such beliefs can hinder productive engagement with mathematics. Bandura (1989) explained that speed of recovery of perceived self-efficacy from adversities determines the resilience of one's self-belief. Whilst one learner may recover self-assurance quickly from new set-backs, another may lose all faith in their capabilities.

2.1.1 Provision and withdrawal of support for learning

A belief that we can learn from others, of the same or higher age and developmental level, is the principle behind providing support for pupils in the ZPD. Wood, Bruner and Ross (1976) advanced the notion of **'scaffolding'** to describe the nature of crucial support offered by the adult or 'expert' as an intervention in problem-solving while assisting the less-expert child as:

> *"a kind of "scaffolding" process that enables a child or 'novice' to solve a problem, carry out a task or achieve a goal which would be beyond his unassisted efforts"*
>
> (p.90)

Sfard (2001) described this assistance as preferably entailing interpersonal and intrapersonal dialogic interactions from an affective stance. Within tutorial processes, the adult or 'expert' gradually withdraws assistance whilst the child increasingly understands the task at hand and learns to perform independently. Mercer (1994) posited that learning requires an assessment, through dialogue, of pupils' developmental levels and their progress. However, this notion of scaffolding has potential to propagate a transmission view of teaching and learning. Teachers as 'experts' instruct and demonstrate concepts while pupils imitate teachers' actions; hence a *transfer* of skills and information. Yet, development is considered a process of internalisation through which pupils make sense of their learning from encounters with tools through critical thinking and problem-solving in sociocultural contexts. This *making sense of* is referred to as 'appropriation' (Mercer, 1994), an action that is individual. For this reason, Bliss et al (1996) explained that pupils have been observed to 'travel' through the ZPD in ways which have not been anticipated, based on their own appropriation of what the teacher makes available. Also, different pupils progress at different rates and require different amounts of scaffolding. Bliss et al stressed that the quality of scaffolding needed by learners depends on the emotional and interactional conditions in home and school environments.

In contemporary sociocultural approach to learning, there has been heightened recognition of increasing pupil participation (Boaler and Greeno, 2000) in a 'community of practice' (Lave and Wenger, 1991) in which novices work supported by experts. In classroom contexts, Bliss et al (1996) proposed stimulating of the limits of children's problem-solving abilities through cognitive support, reinforcement and cultural practices targeted to groups, or communities of learners, a view shared by Mercer (1994). Luckin (2008) saw the success of that scaffolding lies in subject teachers' awareness of pupils' current understanding in offering appropriately challenging, in both quantity and quality, assistance to the learners (Sutherland et al, 2009). New learning can be influenced by experiences that the pupils have *met before* (Tall, 2004). This underscores the need for establishing the stability of learners' existing knowledge and their previous experiences in facilitating effective learning. Bliss et al (1996) argued that teachers need to have higher expectations of pupils at school. They claimed that by taking a few steps at a time, pupils can learn difficult and complex ideas (Bruner, 1960) whilst engaging in joint activity of negotiating certain specialised knowledge domain with their teacher. Mercer and Littleton (2007) considered the role of scaffolding as reducing the incidence of failure and strictly temporary whilst encouraging efforts to progress until pupils achieved the intended levels of understanding. The gradual withdrawal of this scaffolding to learning, or **'fading'** (Hewitt, 2012; Bokhove and Drijvers, 2012) is effective when it is done within the ZPD. Foster (2014) argued for minimal teacher interventions in mathematics learning. Otherwise, a failure to fade support may engender learner dependence, and 'learning' is not considered to have occurred. Alternatively, when learning support is provided outside the child's ZPD, some pupils instead learn dependency and may experience anxiety from its withdrawal.

Immediate feedback provided by ICT tools can be an essential resource, in addition to teacher expositions and textbooks, in enabling pupils to *appropriate* knowledge leading to successful mathematical learning. By imposing suitable constraints on learning processes, and provision of even-tempered, non-confrontational feedback by the computer, pupils have been seen to be capable of learning mathematical concepts without their teachers' intervention (Hewitt, 2011; Lugalia, 2009). Nonetheless, Yerushalmy and Naftaliev (2011) cautioned that visual imagery afforded by ICT tools may fail to promote effective pupil learning where it differs from static representation in the textbooks: this may require teachers to intervene to interpret and consolidate meanings for pupils. Clausen-may (2008) supported this argument. A study by Bokhove and Drijvers (2012) with grade 12 (17-18 years) Dutch secondary students investigated the design and provision of immediate feedback. They established that relevant feedback fosters algebra learning by decreasing the number of attempts needed for tasks whilst improving scores. The acquisition of algebraic expertise as fluency and symbol sense (Arcavi, 1994) were examined in terms of three aspects of feedback: 'timing and fading', 'creation of crises' and 'variation'. Consistent with the notion of scaffolding (Wood et al, 1976), this study evoked the principle of 'fading'. These pupils commenced working on tasks with a lot of structured feedback which was scaled back to less elaborated feedback as the learning tasks progressed. Subjecting pupils to 'Crises' entailed situations whereby familiar strategies would not accomplish the given tasks and no assistance was given: pupils encountered conceptual challenges after which feedback was offered to overcome the crises. On 'Feedback Variation', pupils were provided with hints for getting solutions and indications when mistakes were made. The study findings indicated that algebraic expertise was enhanced through timing, fading and feedback variation. I argue that scaffolding of pupil learning by any resource or tool should not be provided indiscriminately: learning frameworks ought to be in place before support is offered.

2.1.2 Tools and technologies in learning contexts

Sutherland et al (2004) regarded tool systems as essential for enabling people to fully express themselves, and saw education as a process of acquisition of mastery of these tools: Noss and Hoyles (1996, citing Illich) reiterated similar views. The Vygotskian perspective emphasises the idea of mediation between subject-specific knowledge and scaffolding of learning by various tools in a learner's ZPD, language being of principal importance. Mathematical language is crucial as the communicative tool between pupils and teachers in the classroom contexts. The printed textbook is an immensely powerful tool that has featured prominently in what is essentially regarded as the 'traditional approach' to teaching and learning in schools.

Digital technologies are increasingly being added to the list of tools seen to mediate between learners and knowledge. Following the advent of ICT in education, Noss and Hoyles (1996) envisioned mathematical learning to include the computer as an integral learning resource, a view shared by Kaput (1992) and Papert (1980). These authors argued for efforts to be spent on exploring relationships between technologies and education to bring out the power in each to promote learning outcomes. The computer becomes another tool, alongside pens, printed textbook and sociocultural interactions that occur in classrooms. Noss and Hoyles hoped, “the computer will open new windows on the construction of meaning forged at the intersection of pupils’ activities, teachers’ practices and the permeable boundaries of mathematical knowledge” (p.2). Therefore, focus was not on the computer itself, but what computers make possible for meaning-making in mathematics lessons: learner behaviour. Papert (1980) explained the qualities of the computer that enable users to perform actions and instil certain learning behaviour as:

> *“The computer allows or obliges the child to externalise expectations. When the intuition is translated into a program, it becomes more obtrusive and accessible to reflection”* (p. 145)

Noss and Hoyles (1996) added that, by “offering a screen on which students can express their aspirations and ideas, the computer can make explicit what is implicit” (p.5). They underlined the computer’s potential to behave as an extension of pupils’ own powers of reflection. These arguments valued the interactive relationship between pupils and tools as highly significant to the computer’s contribution to teaching and learning processes.

Clausen-May (2008) made the distinction between ‘instructional software’, as computer software designed to teach students skills and concepts, and ‘tool software’ which allow the students to ‘teach’ computers (Papert, 1980) by performing a function that enables attainment of another objective. This offers a possible justification for the proposed model of six pupil entitlements for learning mathematics with ICT by Becta (2008). Becta’s model was developed to describe what pupils may be expected to do to a greater extent with ICT use than they would without access to ICT. These entitlements included: ‘learning from feedback’; ‘observing patterns’; ‘seeing connections’; ‘working with dynamic images’; ‘exploring data’; ‘teaching computers’ through physical tool manipulation. Hewitt (2012) explained that software can be designed to provide visual imagery of structures and relationships which may allow pupils to think about the underlying processes thus gain deeper and enriched learning insights into the subject content.

Combining visual imagery with the possibility of physical interactions and manipulations can enable automating routine procedures, and provide learning experiences that are unavailable without ICT (Becta, 2008). I adopt the report by Greeno (1994) of affordances as interactive relations of agents with other agents in learning environments. Some affordances of computer use include enabling pupil independence and increased peer interaction (Hennessey, Ruthven and Brindley, 2005). Juwah et al (2004) saw these as developing self-regulation, revision of ideas, and willingness to 'have a go', thus enhancing pupil ownership of their learning. From school-based qualitative studies on ICT use involving teachers (Hennessey et al, 2005) and pupils (Deaney, Ruthven and Hennessey, 2003), researchers reported that the provision of variety and alternative teaching and learning styles can raise pupils' interest and engagement in subject learning. When computers allow pupils to control activity, provide instant feedback and more instances to reflect on results, pupils become the agents of thinking and learning: 'Computers cannot produce "good learning" but children can do "good" learning with computers' (Harel and Papert, 1991).

2.1.3 Developing mathematical language

Vygotsky (1978) stressed the importance of language as a psychological and cultural tool. He argued that inter-psychological or social activity between people, mediated through language, can trigger intra-psychological activity in individual learners. These developments constitute a learner's intellectual processes. Culture can contribute to mental development by providing tools to think with. From the resources available in learning contexts, learners can acquire the means and processes of their thinking: *means* to think, *what* to think, and *how* to think. This highlights the role of interpersonal communication in education. Several researchers (Mercer and Sams, 2006; Morgan, 2000) saw classroom discussions as a vital medium for developing effective learning. However, pupils cannot on their own, without the teachers' support, either align their everyday language use to the vocabulary in mathematical discourse or acquire the desired communication skills: they need guidance which is unfortunately lacking. In a wider context, mathematical language and vocabulary need to attain an international status and commonality of terms criss-crossing physical borders of English-speaking countries. Sharing on the world stage will always struggle with true respect unless traditional or conservative values are both recognised and adhered to in one universal framework. With Latin, Greek and Arabic words underlining the origin of mathematical language, there is no better time than now to establish them as sacred in teaching fraternities across the English-speaking world before the prevalent use reverts to parochial dialect, limiting its ability to gain wide acceptance and thereby devaluing the quality of presentation and instruction.

Evoking the notion of 'verbal thought' (Vygotsky, 1962), Pimm (1987) stated that articulation assists reflection processes by affording better access to thought itself. The power in 'talking out loud' is that it requires expressing thoughts through the use of language, including specialist mathematical terms in this context, which may easily go unnoticed when thinking to oneself. Externalising one's thoughts can help to reshape thinking: it allows clarity or difficulty in one's thinking to be exposed through clarity or difficulty in expressions. Mathematical language conveys meaning in the words used (Durkin, 1991) as the speaker draws upon the knowledge derived from previous exchanges that have lain dormant in the memory. To Pimm (1987), talking out loud helps the speaker to organize their thoughts: it externalises 'thinking', rendering it public and readily accessible to both the speaker and the listeners. Promoting mathematical talk in classrooms can enable externalisation of a speaker's thoughts or ideas. Both the speaker and the listeners weigh what is being said against their own individual thoughts or ideas. Where there is agreement between these ideas, 'internalisation' occurs (Cheyne and Tarulli, 2005; Pimm, 1987; Vygotsky, 1987). In establishing classroom cultures where the pupils are required to take turns in speaking whilst sharing their thoughts or ideas on a mathematical problem, the teacher may focus the pupils' attention on their language use.

The foundation of learning mathematics lies in solid development of mathematical language. Durkin (1991) underscored the crucial role of language in mathematical learning as:

> *"Mathematics education begins and proceeds in language, it advances and stumbles because of language, and its outcomes are often assessed in language"* (p.1)

According to Morgan (2000), there is an emerging consensus regarding the vital importance of the role of language in mathematics education. Pupils' mathematical language may be developed through active encouragement of verbal formulations of their mathematical understandings. Durkin (1991) argued that some words used in mathematics may be endowed with alternative meanings that may be more familiar to pupils. Alexander (2008) emphasised the importance of encouraging pupils to talk through their mathematics in order to consolidate their learning. Also, Cheyne and Tarulli (2005) regarded 'pupil talk' as crucial to learning processes. Pimm (1987) suggested that pupils need to be made aware of *why* they are being encouraged to talk. On further externalisation of pupils' thoughts or ideas, either through talk during discussions with ICT, with non-ICT resources, or ideas written on paper whilst they engage in pen-and-paper tasks, the learning may then be assessed (Black et al, 2003). Morgan (2000) considered whole-class discussions as teacher-guided, meaning-making experiences that offer interpersonal gateways for student to appropriate mathematical meanings for themselves. However, this can only happen if the material is suitable for discussion, and the participants are used to discussing, with each valuing the other's views. Lee (2006) explained that introducing discussion into mathematics classrooms can facilitate the 'enculturation' of pupils into the wider mathematics community; Noss and Hoyles concurred. Morgan (2000) regarded the growing acceptance of discussion in classrooms as a vital medium for enhanced mathematical learning that endorsed communication in current curricula. Alexander (2008) stressed that discussions encourage richer forms of communication, which emphasise active use of language by learners.

Mercer and Sams (2006) presented empirical evidence from a study with Year 5 (age 9-10 years) pupils working on a computer-based activity, which focused on language as the tool of choice. This evidence supported the sociocultural conception of mathematics education as successful induction of learners into a community of practice (Lave and Wenger, 1991). It demonstrated the role of talk-based activities in enhancing individual pupils' mathematical reasoning skills. Sutherland et al (2004) discussed the effect of integrating ICT within subject cultures. They stressed that, with or without ICT, the teacher's role is important in influencing the construction of new pupil knowledge. According to Vygotsky (1981, 1978), knowledge is first developed between the individual and the society through cultural tools, then shifts to the cognition of the individual; Valsiner and van der Veer (2005) and Lerman (2001) reiterated this view. The teacher's role is pivotal in creating an enabling learning environment. Teachers guide pupils' awareness and their use of mathematical language as one tool for reasoning with. Emphasis on oral proficiency may be a crucial avenue through which teachers can access the pupils' conceptual understandings. Developing pupils' mathematical ideas and language use is contingent upon mathematics educators providing suitable tools or resources, in well thought-out situations, to create environments for pupils to actively engage with mathematics.

2.1.4 The role of language in algebraic thinking

In many algebra lessons, pupils are confined to largely passive roles. The pupils listen to, and watch, teacher explanations about algebraic manipulations, some of which may make little or no sense to the pupils. Bell (1996) explained that the symbolic language in algebra plays a crucial role in developing algebraic thinking. Hence there is need to encourage pupils' proficiency in using symbolic language and to increase their awareness of the underlying structures in formal algebraic notation. Mercer and Sams (2006) demonstrated that the encouragement of pupils' language use by teachers in certain ways can lead to better learning outcomes or conceptual understanding. Therefore, it is imperative that teachers make proper, careful and consistent use of terms in the mathematics register if their pupils are to learn the universal use of mathematical language. Through teacher-led classroom interactions, pupils can gain relevant knowledge of mathematical terms, concepts or procedures. Teachers model the correct use of mathematical language and vocabulary whilst providing pupils with opportunities to develop communicative competence. However, the teacher cannot make the pupils learn mathematical vocabulary: pupils have to be willing to engage with the learning process if they are to grow in confidence and join the wider mathematics community (Mercer and Sams, 2006; Lee, 2006).

2.1.5 'Dialogic teaching' in mathematics classrooms

From a neo-Vygotskian perspective, talk and teachers are two central and significant features of interest for research on learning in educational contexts. To Mercer (1994), through talking and listening in the classroom, alternative perspectives may be considered as information and understandings are shared, explanations or justifications are offered, and ideas are exchanged. *Dialogue* with pupils in focused discussions is paramount for a teacher with a 'connectionist' belief orientation (Askew et al, 1997). This allows for consideration of varied understandings or interpreted meanings, and exploration of mathematical problem-solving strategies (Ruthven et al, 2011). Following ideas grounded in counselling, Foster (2014) called for the development of listening skills in mathematics teachers to encourage pupils' autonomy in decision-making whilst problem-solving. Lee and Johnston-Wilder (2013) stressed that although a lot of 'talk' ensues in mathematics classrooms, experience indicates that most of it is by teachers and there is very little *pupil talk* (Kanja et al, 2001). To Alexander (2008), 'teacher talk' may take the form of:

- *rote* through drilling facts and routines via constant repetition
- *recitation* in cuing pupils to work out answers through stimulated recall
- *exposition,* imparting information, explaining principles and procedures
- *discussion,* in which ideas are exchanged with a view to solving problems
- *dialogue* with common understandings achieved through exchange of structured reasoning and discussion

The key characteristics of dialogic talk are that it is: *collective* in involving all participants in tasks; *reciprocal* through listening to each other whilst considering alternative viewpoints; *supportive* of free contribution to discussions with mistakes valued; *cumulative* in negotiating a consensus in participants' thinking; and *purposeful* in meeting the intended learning outcomes. Alexander (2008) argued that while reorganisation of classroom settings and interactions can lead to teaching that is collective, reciprocal and supportive, being cumulative and purposeful can present pedagogic challenges for teachers. He observed that:

> *"Discussion and scaffolded dialogue have by far the greatest cognitive potential. But they also, without doubt, demand most of teachers' skill and subject knowledge. Rote, recitation and exposition give us security. They enable us to remain in firm control not just of classroom events but also of the ideas with which a lesson deals".* (p.31)

Through '***dialogic teaching***', the teacher invites pupils to share their thinking in sequences of reflective talk. Wrongly-phrased talk is used as 'learning points', to ask pupils more questions, thus keeping the lines of enquiry open, extending classroom discussions, and testing both the teacher's and the pupils' understandings. Alexander (2008) listed characteristics of 'dialogic teaching' as:

1. *questions structured to provide thoughtful answers*
2. *answers provoke further questions and are seen as building blocks of dialogue rather than terminal points*
3. *individual teacher-pupil and pupil-pupil exchanges are chained into coherent lines of enquiry rather than left stranded and disconnected*

(p.42)

According to Alexander (2008), the types of 'talk' within a teacher's repertoire will determine the quality of the opportunities to learn that will be made available to pupils who have to voice their own thinking. The classroom 'subculture', which depends on the learning environment created by the teacher, can determine the level of pupil participation and the kinds of interactions in lessons. Alexander stressed the need for teachers appropriately to balance the social and cognitive uses of *talk* in order to facilitate pupil participation and extended dialogues. He considered 'cumulation', in particular of the five 'dialogic teaching' principles, as posing the greatest challenge because it places greater demands on the teachers':

- Mastery of content knowledge;
- Professional skills;
- Ability to gain insight into their pupils' existing understanding and capacities;

- Ability to listen to and to review their pupils' contributions, and then to judge the best way to scaffold the pupils' thinking to achieve the intended understanding.

Alexander (2008) cautioned that 'dialogic teaching' and scaffolded discussions were likely to curtail the information flow from teachers to pupils through awkward questions.

Pimm (1987) stressed the need for classroom dialogues to be task-focused with an emphasis on the style and level of explicitness of talk. He argued that the amount to 'pupil talk' should not be seen as an end in itself: the quality rather than the quantity of talk is what is important. Extending the Vygotskian idea of 'thinking aloud', Mercer (1994) said that to encourage talk and communicating with others through speech is to engage in a social mode of thinking. By laying appropriate 'ground rules' (Edwards and Mercer, 1987) in lessons, and asking questions that provoke reflection and thought, teachers can create the opportunity to assess whether learning may be taking place as it occurs (Hattie and Timperley, 2007; Black et al, 2003). Externalised reasoning in 'dialogic teaching' simultaneously evokes the internalisation, appropriation and scaffolding of pupils' learning within their ZPDs. In this thesis, I contend that both pupils and teachers become aware of the content intended, content taught and content learned by observing the 'ground rules'. Collectiveness can provide a supportive atmosphere within which pupils may feel safe when invited to express their reasoned mathematical arguments: mistakes are valued as 'learning points'. Mercer and Littleton (2007) stated that pupils trained to work in this way progress more successfully in joint problem-solving and improve their scores on written tests.

Whilst offering criticism of the establishment of 'ground rules' governing classroom talk, Lambirth (2006) asserted that the practice can reinforce social inequalities and disempower pupils from less privileged backgrounds rather than enhancing all pupils' communicative and cognitive activity. Nevertheless, the effect of 'ground rules' in the classroom may be seen as enabling the development of resilience in activity systems. Bottrell (2009) viewed 'resilience' as the coping and competence strategies adapted by individuals despite adversity occasioned by the interaction of social and cultural factors. Reporting on a study of marginalized youths in Sydney, Australia, Bottrell emphasised the significance of individuals' social identities and collective experiences to their resilience by underscoring the voices of 'others' in narratives of pupils' experiences. Herein lies the recognition that the management of pupils, teachers, and technologies as classroom resources (Luckin, 2008) is important for enabling resilience. Bottrell (2009) underscored the dynamic nature of resilience, whilst pointing to the dialectical relationship between individuals and their context, in stating that adversity requires resistance. Hernandez-Martinez and Williams (2013) considered 'resilience' to be the dynamic interplay between the sociocultural context and the agency of learners in transition. In this thesis, I see collaborative, computer-based mathematics classrooms as activity systems requiring explicit expectations. Hence, the criticism of 'ground rules' falls short of acknowledging the pupils' resilience arising from the developing interaction between learners and the classroom context.

2.1.6 The importance of collaborative learning

Collaborative learning refers to pupils working *together* on a shared activity with a common goal, usually in small groups. Following Barnes (2000), social processes involving the pupils negotiating shared understandings play a central role for meaning-making. Noss and Hoyles (1996) stated that collaboration offers an opportunity to make mathematics inclusive, and hence more human. Lee and Johnston-Wilder (2013) concurred; they saw collaboration as developing 'mathematical resilience' in pupils through more involvement and motivation of the pupils in their learning of mathematics. They explained that encouraging pupils to work together, whilst articulating individually-held ideas or beliefs, can promote learner agency. It can enable some pupils to help themselves and others, and to learn to express things confidently as argued by Boaler and Greeno (2000). Since one teacher cannot access all the pupils in a classroom at once, it is crucial that pupils are encouraged to take on more responsibility for their own learning processes within small groups (Hoyles, Healy and Pozzi, 1994). 'Dialogic' interactions (Alexander, 2008) offer the pupils an opportunity to articulate their views, and to appropriate new meanings for themselves as they learn, whilst allowing teachers to assess the pupils' learning. Encouraging 'exploratory talk' (Mercer and Littleton, 2007) - indicated by terms such as 'if', 'I think', 'because', 'agree', 'so' - in discussions can scaffold the pupils' mathematical understandings. Gresalfi et al (2009) advanced that mathematical competence grows when learners are accountable *for* their contributions *to* their peers: the learners weigh alternative reasoning against their own constructions in the quest for shared understandings. Sutherland et al (2004) considered group discussions to be critical in allowing the pupils' deeper insights and understandings of content; this deeper understanding is referred to as 'connected knowing' (Boaler and Greeno, 2000). Taylor, Harlow and Forret (2010) discussed that, while collaboration may not automatically foster subject-specific knowledge, it can enable 21st Century skills to develop in the pupils, such as the demonstration of leadership qualities, articulation and listening.

Hoyles et al (1994) concluded that computer-based activities foster collaboration within the learners' ZPDs, where pupils work with their peers on mathematical learning tasks in small groups with equal-status partnerships. The ensuing classroom interactions, and feedback from computer software and from each other, can facilitate access to conceptual learning by acting as triggers to the construction of mathematical knowledge. This learning may occur at various levels: individual, paired or whole-class. Sutherland et al (2004) underlined the importance of group-working in orchestrating the provision of peer feedback for pupils. This practice facilitates the shift of emphasis from individual knowledge to shared knowledge. These groups present pupils with an invaluable opportunity to own their learning; the pupils are free to direct their comments to each other as they negotiate the construction of personal insights into intended learning. Where an ICT tool is integrated into a subject discipline, both teachers and pupils need to personally acquaint themselves with the tool's potential from learning and teaching perspectives. Yet, it is possible that teachers may not be endowed with as much knowledge as the pupils about the new technologies as learning tools. Hence, Luckin (2008) saw this situation as presenting opportunities for 'reciprocal' teaching and learning: a collaborative relationship with respect to expertise. She explained, "learners need to know how they can use these tools to learn more about particular subject or skills, and teachers need to know enough about these tools to scaffold learning" (p. 341). Herein lies the suggestion that teachers consider learning about the potential of the ICT tool from their pupils.

2.1.7 Factors hindering effective collaborative learning

Certain beliefs and misconceptions, which both pupils and teachers bring to the classroom, about the mathematics being 'abstract', about pupils' 'mind-sets' (Dweck, 2000) and about teachers' hegemony of mathematical knowledge, do very little to dispel these subject-related myths. According to Barnes (2000), as pupils collaborate, and struggle to *appropriate* the curriculum content, they are at the same time confronting perceptions of themselves as learners of mathematics. These perceptions may be fixed, or may shift and change, depending on the interactions with other people and the classroom context. Hence, there is a need to facilitate a relaxed learning environment and more classroom interactions as pupils learn. Mercer and Littleton (2007) observed that pupils can work *in* groups, but still working individually or locked in disagreements, yet not *as* groups and thinking together. Barnes (2000) viewed some pupils as dominant in mathematics classrooms: these are usually males in co-educational settings, and may be the 'more able' pupils in same-sex classrooms.

However, this dominance is not unique to whole-class settings; it may still be evident in small-group settings. Some pupils can eclipse their peers, believing that they know more, and hence doing and talking more; the others spectate and remain quiet, believing they have nothing to contribute. Traditionally, a teacher is considered the 'expert', and is expected to possess mastery of subject content knowledge. Yet, this belief consolidates the teacher's hegemony over content knowledge. Pupils with such beliefs tend to disregard contributions made by their peers; they consider their peers' views as invalid. However, these beliefs serve to propagate a transmission role for the teacher as the conveyer of all legitimate mathematical information in the classroom. Nevertheless, Alexander (2008) expected teachers, as the 'expert' knowers in subject disciplines, to play the crucial role of addressing any misconceptions and invalid constructions that may arise.

Following a 'sociocultural perspective', a belief in the transmission of knowledge from teacher to pupils fails to take into account individual learners' constructions of their own meanings as they make sense of the information presented. Small-group collaborative learning in mathematics classrooms can afford more pupils the opportunity to articulate their ideas than whole-class discussions, which tend to be monopolised by the more dominant personalities. However, many pupils require induction in becoming active speakers and listeners, and need to learn to value pupil-pupil discussions, in order for group-working to offer effective ways of learning (Claxton, 2004). Taylor et al (2010) described the teacher's role in a collaborative learning environment to be one of encouraging a culture of listening to, and respecting others' views, in order to learn from each other. The teacher models for pupils how to work together in developing task-based discussions through enquiring, reasoning and making joint decisions. Mercer and Sams (2006) suggested that providing clear instructions to pupils on what they are expected to do can help to address those peer group interactions that are unproductive, uncooperative and inequitable. The support provided by such explicit rules of engagement can assist in improving the quality of 'talk', and in keeping the pupils on-task.

2.1.8 Visual imagery contributing to developing mathematical skills

In this section, I discuss *how* the cultural context in the wider society can interact with the tools available for use in schools to influence the development of certain skills in pupils. In a study involving 5th graders (10-11 year olds) in Taiwan, Stigler, Chalip and Miller (1986) reported the enormous value placed on pupils developing rapid mental arithmetic skills. The skills had resulted from abacus skills training: they had positive implications for many pupils' cognitive development and attainment in school. Stigler et al (1986) reported both quantitative benefits and a qualitative impact on attitudes of children towards reading, science and mathematics.

An abacus is a wood-framed tool composed of movable beads in columns used for arithmetic calculation. Users can perform addition, subtraction, multiplication, division, even square and cube roots, to various levels of expertise. It is widely used in Asia. Abacus skills' training is conducted primarily for national and international competitions in Japan, China, Taiwan and India (Frank and Barner, 2012; Stigler et al, 1986). The cultural emphasis on training is such that, while it forms a part of Taiwan's mathematics curriculum, it is not meant for application in problem-solving. Consistent with results of an earlier study conducted with Chinese pupils, Stigler et al (1986) observed about 15% of pupils, who opted for additional, paid training for greater expertise, carrying out complex mental computation, a feat attributed to the additional training. By first forming a "mental abacus", a visual image of an abacus, then moving beads on the "mental abacus" in the same way as they would an actual tool, pupils were observed to be capable of extremely rapid and precise mental arithmetic. Such skills contrasted sharply to American and Western cultures: less emphasis is placed on mental arithmetic skills which are regarded as instilled in 'rote learning'. Visual imagery relating to abacus use is hypothesised to accurately, albeit non-verbally, represent one's number sense.

Stigler et al (1986) stressed that practice with any skill is bound to realise some improvement in other domains. According to Frank and Barner (2012), intense practice can reinforce visual imagery effects in abacus users competently executing mental arithmetic. They conducted an experimental study on children in India, aged between 5 and 16 years, to investigate the role of language in mental abacus computations. The results indicated images of detailed, column-based models are stored in users' visual working memory independent of verbal interference: they are utilised as visual resources in arithmetic calculations. It may be worth adding that the visual image is formed through practice by touching and manipulating actual tools in abacus training sessions. I suggest that cues from visual processing, similar to the cues provided through abacus use, can facilitate consolidation of pupils' number sense; they may enhance cognitive development. Since information processing varies with types of learners, individual strengths and weaknesses are subject to change and development, especially when the learner is young.

Papert (1980) proposed the idea of mathematics that is not separate from the body; instead, it depends on concrete experiences as well as abstraction. Learners develop new strengths when presented with opportunities in a variety of ways (Bruner, 1966). Some pupils respond well to visual representations of mathematical concepts; others look for kinaesthetic stimulus; others may seek auditory information (Fleming, 1995). Rohrer and Pashler (2012) refuted the importance of such differences, stating that there is little empirical evidence to support the existence of different 'learning styles'. To Clausen-May (2008), ICT tools possess the potential to provide dynamic images for pupils to work with. She argued that these can supplement textbooks, or possibly challenge the dominance of print, as the legitimate learning medium in mathematics classrooms. Visual representations and dynamic affordances within appropriately-designed software can support the externalisation of pupils' mental constructions onto computer screens. Using ICT tools, pupils can access difficult mathematical concepts in ways which would otherwise be extremely difficult.

I have underscored the values of linguistic, social interaction and cultural aspects in enlisting active pupil participation. I have emphasised the provision of language and ICT tools alongside textbooks in mathematics classrooms. Next, I consider mathematics as a subject.

2.2 Mathematics and Learning

2.2.0 Introduction

I embrace the stance that, mathematics is "a human activity, a social phenomenon, a set of methods used to illuminate the world, and it is part of our culture" (Boaler, 2009, p. 17). This position may be behind the reason why mathematics is a core subject in most countries, and hence it influences my consideration of mathematical activity from the learners' perspective.

2.2.1 'Three Worlds' of mathematics

Tall (2004) theorised the development of an individual's cognitive growth in mathematics to advance through three distinct 'worlds':

- ***Embodied world***, that develops out of objects and concepts that can be sensed and perceived both physically and mentally, hence a 'conceptual-embodied world';
- ***Proceptual world***, one used for calculation and manipulation in arithmetic, algebra and calculus, beginning with actions as concepts, through the use of symbols that

facilitate a smooth transition from processes to carry out to concepts to think about, hence a 'proceptual-symbolic world';

- ***Formal world***, one based on axioms, unfamiliar objects of experience, which are formulated to define mathematical structures in terms of specified properties, hence a 'formal-axiomatic world.'

Dubinsky and McDonald (2002) developed the APOS theory of mathematics education from Piaget's work on reflective abstraction. They argued that individuals tend to construct mental **a**ctions, **p**rocesses, and **o**bjects that they organize into **s**chemas in problem-solving situations. These four elements combine to form an individual learner's mathematical knowledge. According to Dubinsky and McDonald, an 'action' is a step-by-step transformation of an object through operations that are perceived as 'external', while a 'process' is an internal mental construction arising from reflections on repeated actions with which the learner can think. They advanced that an 'object' is constructed once the learner becomes aware of the process as a stand-alone entity and the transformations that can be performed on it. To them, a 'schema' for a mathematical concept consists of collection of actions, processes, objects and related schemas linked to that concept in the learner's mind. Dubinsky and McDonald (2002) emphasised that each of these four components of APOS must be constructed before the next step is possible. They argued that observed success or failure of learners' handling mathematical tasks may be attributed to the difference in learners' mental constructions regarding specific concepts. However, Hewitt (2012) proposed the possibility of ICT tools enabling objects to precede processes, and become concepts for pupils to think, and work, with.

Tall (2004) noted that individuals develop at different rates, and in ways influenced by their reorganising of previously-constructed ideas, through the three 'worlds' of mathematics. Some learners are observed to linger in the number concept-formation phase, challenged by actions such as counting, which fail to develop into thinkable concepts. Arithmetic becomes simple for 'more able' learners, whilst becoming increasingly difficult for the 'less able'. According to Gray and Tall (1994), success in learning mathematics is facilitated when a pupil is able to recognise that the same notation represents both a process and an object (Sfard, 1991). Successful pupils are able to switch between these two ways of 'seeing'. Gray and Tall (1994) proposed the notion of '**procept**' as particularly vital in developing algebraic thinking; successful pupils possess the ability to think of and algebraic expression $m + 3$ as a single object to be operated on, and yet remain aware of the expression as representing a process: the arithmetic operation of addition. An inability to regard such an expression as a 'concept' poses great difficulty for one to whom the notation represents only a 'process'. Tall, Thomas, Davis, Gray, Simpson (1999) suggested that the main focus for such pupils is on the procedure of evaluating an answer rather than on the expression itself as a stand-alone 'procept' that can be manipulated. This difficulty implies that such pupils learn mathematics procedurally; they cope through sticking to familiar routines. Familiarity can help to organize such pupils' thinking and, to some extent, clarify their understanding of mathematical relationships. However, this is not in the least a suggestion for making pupil learning *simpler* by reducing mathematics to techniques to be followed. Such practice neither consolidates firm conceptual understanding nor provides pupils with intellectual challenge in lessons. Instead, it can enhance disinterest and disengagement in mathematics (Brown et al, 2008; Nardi and Steward, 2003).

2.2.2 Constructing new mathematical knowledge

In response to the importance of familiarity in developing new learning, Tall (2004) advanced the theoretical construct of **met-before**: McGowen and Tall (2010) defined this as "mental structure that we have now as a result of what we have met before" (p.172). This construct provides a mathematics education through which greater insight into pupil learning can be considered by mathematicians, the teachers, mathematics educators and cognitive scientists. Citing Ausubel (1968), and building on Piaget, McGowen and Tall (2010) asserted:

> *"new experiences that build on old experiences are much better understood and what does not fit into prior experience is either not learned or learned temporarily and easily forgotten"* (p.2)

McGowen and Tall (2010) considered 'met-befores' to be supportive for pupils when existing knowledge, created in previous positive learning experiences, makes sense in new contexts and can be related to new experiences. The feelings of pleasure and success generated in pupils can build confidence. Otherwise, when pupils' 'met-befores' are seen not to work in new learning contexts, they are deemed to be *problematic*: they impede new learning, and generate feelings of anxiety and failure. Teachers of mathematics and mathematics educators need to gauge the security and role of pupils' 'met-befores' in different learning situations. Askew et al (1997) aligned the practice to a connectionist belief orientation, a view shared by Hodgen et al (2008).

Yerushalmy and Naftaliev (2011) reported 13-14 year old pupils studying algebra in graphic, numeric and symbolic modes in an interactive environment prior to traditional instruction in Israel. They claimed that animations are carriers of meaning. Hence, the profound difference between representations in traditional printed media, and visuals in interactive textbooks should be considered in evaluating all the affordances on offer in effecting technological shifts in educational materials. Noss et al (1997) argued that the construction of knowledge depends on the manner in which connections between visual and symbolic representations are built, as well as the kinds of discourse around mathematical activity, rather than on the activity itself. Yerushalmy and Naftaliev (2011) proposed that the design of resources needs to incorporate opportunities for actions that fit with pupils' supportive 'met-befores'. Designers ought to consider the type of knowledge to be developed when pupils encounter unfamiliar tasks. This resonated with the argument by Hattie and Timperley (2007) that, if educators gave thoughtful consideration to, and were more aware of the effect of 'met-befores' on their own thinking, they would be in a better position to perceive learners' difficulties.

2.2.3 Developing mathematical skills

As a core subject in schools, mathematics poses difficulties for many learners the world over due to its abstract nature. Mathematics seems to be riddled with a litany of inconsistencies; Chinn (2004) listed the fact that multiplying does not always make something bigger as an example. Kilpatrick, Swafford and Findell (2001) proposed the 'mathematical proficiency' concept as identified in five distinct yet inexorably interdependent skills to be developed in pupils for their success in learning school mathematics. These skills are listed as: *conceptual understanding, procedural fluency, strategic competence, adaptive reasoning* and *productive disposition.* Kilpatrick et al (2001) emphasised 'conceptual understanding' as "comprehension of mathematical concepts, operations and relations", and 'procedural fluency' as "skill in carrying out procedures flexibly, accurately and appropriately" (p.116). Procedural fluency appears to imply automation in the execution of techniques, informed by a developed understanding of the structures underlying the given mathematical relationships. Unfortunately, not many mathematics learners are so endowed.

Gresalfi et al (2009) argued against viewing these skills as attributes of individual pupils, and for shifting more focus to the opportunities available in mathematics classrooms for pupils to participate **competently**, and the ways in which pupils take up these opportunities. Claxton (2004) saw mathematics classrooms as 'mind gyms' providing pupils with mental, emotional, and social exercise. Provision of variety resonated with the modes of representation proposed by Bruner (1966). Skinner and Belmont (1993) wrote that pupils who are highly motivated to engage in school are seen to select tasks at the edge of their academic competencies, to initiate actions when given opportunities, and to exert intense effort and concentration whilst executing learning tasks. Such pupils try hard, persist and actively cope with challenges and setbacks. A value of automaticity is exemplified in the case of abacus skills training in Asian learners and its facilitative role in mental arithmetic (see Section 2.1.8) as vital to success in mathematics. This feat can be accomplished by perfecting 'procedural fluency' through practice to enhance 'conceptual understanding' in learners. Claxton (2004) concurred, adding that pupils' minds require training and sustained practice.

2.2.4 Types of 'understanding' in mathematics

Success in handling the mathematics curriculum, which spans arithmetic, geometry, algebra, measurement and statistics, requires a firm foundation comprising of a sound number concept, adequate mental arithmetic skills and knowledge of estimation. Kieran (1992) observed that algebra, despite being central in the school mathematics curriculum, is a topic from which many pupils disengage, and regard as 'difficult'. Goulding and Kyriacou (2007) and Chinn (2004) shared these views. Bokhove and Drijvers (2012) highlighted an on-going debate in mathematics education about whether focus should be on pupils' procedural fluency or conceptual understanding in school algebra. Development of these two key skills in pupil learning is of particular interest to my thesis. One argument is that an emphasis on knowing 'how' can lead to the reproduction of procedures to acquire correct answers; it favours 'instrumental understanding' (Skemp, 1976) and a superficial form of learning that overlooks the conceptual structures underlying mathematical relationships. Tall and Thomas (1991) attributed to 'traditional' pedagogy the failure of many pupils to recognise algebraic expressions as processes that may be regarded as 'objects'. Such teaching depicts algebra as a manipulative activity, which is laden with rules to be memorised; Tall and Thomas (1991) listed some as:

- "do multiplication before addition";
- "calculate expression in brackets first";
- "collect together like terms";
- "of means multiply";
- "add the same thing to both sides";

- “change sides, change signs”;
- “to divide, turn upside down and multiply”. (p.126- 127)

Pupils in many mathematics classrooms are relegated to passive roles, and with their thinking devalued (Ruthven et al, 2011). Kieran (1992) explained that a majority of pupils tend to adopt coping mechanisms to arrive at ‘acceptable answers’, and inevitably develop an ‘instrumental understanding’, which they eventually deem to be the key essence of algebra. Boaler and Greeno (2000) labelled such classroom practice as ‘received knowing’. They argued that such an approach to learning undermines pupils’ agency; it dents their identity as ‘mathematicians’ and it cultivates a belief that mathematics is a “closed, rule-bound subject” (p. 180). Some secondary students handling the task ‘Factor $(2x+1)^2 - 3x\ (2x+1)$’ exemplified the difficulty as reported by Tall and Thomas (1991). While appearing to interpret the ‘factorisation’ concept as a process, the students proceeded to expand the question to obtain a product or answer. They ended up with a quadratic expression instead of immediately spotting the common factors. Noss et al (1997) argued that such actions result when the learning attention is focused on the number attributes of the outputs, in which algebra is considered to be an endpoint rather than a tool for problem-solving. They lamented that school mathematics in the UK was seemingly reduced to a ‘pattern-spotting’ activity, which tends to deprive pupils of appreciation and understanding of the crucial structures underpinning learning, a view shared by Hewitt (1992).

Conversely, an emphasis on learning and understanding ‘why’ underlying relationships work is argued to engender ‘relational understanding’ (Skemp, 1976). According to Tall (2013), both ‘instrumental’ and ‘relational’ understanding are pleasurable; a suggestion of affective traits. While ‘instrumental’ understanding promotes one’s success in procedural fluency, ‘relational understanding’ enables more connectivity of mathematical ideas. This view is consistent with Noss and Hoyles (1996) conception of ‘learning’ as:

> *“the construction of a web of connections between classes of problems, mathematical objects and relationships, ‘real’ entities and personal situation-specific experiences”.*
>
> (p.105)

The more connections pupils make, the deeper the learning of the subject content. Whilst advocating for ICT use within subject cultures, Abbot et al (2009) defined ‘deep learning’ as an approach to knowledge and learning that related previous knowledge to new knowledge.

They explained that understanding existing knowledge levels, and linking it to new material, combined with ICT use, potentially leads to ‘deep learning’ in pupils; this is in contrast to ‘surface learning’. Wilensky (1997) believed that a pupil’s yearning for ‘deep learning’ may lead them to develop “epistemological anxiety”, which often lurks in the background. The pupil experiences a sense of loss from their failure to comprehend the legitimacy of understandings, meanings and purposes in the mathematics they are engaged in. Tall (2010c) explained that such ‘anxiety’ indicates a lack mathematical understanding despite the pupils’ ability to execute the necessary procedures, citing the following statement from Wilensky (1993):

> *“I was good at maths, but I didn’t really like it. I felt like I was getting away with something, like I was cheating. I did well in tests but really did not know what was going on”.*

These feelings clearly suggest that pupils need more than rules or techniques for working out mathematics questions; instead they value understanding the *‘why’*. According to Tall (2013), ‘epistemological anxiety’ may indicate a pupil’s inability to achieve ‘relational understanding’. He added that such pupils may resort to rigid procedures and ‘instrumental understanding’ to achieve ‘acceptable’ success levels, and to ease their sense of frustration. The danger here is that such pupils may eventually come to believe that mathematics is ‘boring’, and that learning it cannot be ‘enjoyable’. Such pupils may disengage from mathematics. Boaler and Greeno (2000) argued that, through actively participating in discussions, and contributing to a shared understanding of mathematical ideas, pupils are seen to develop ‘connected knowing’.

Although mathematics educators acknowledge the need to develop an increased fluency and understanding in pupils, there is a persistent lack of consensus about the important relationship between the two skills, and how to prioritise both (Bokhove and Drijvers, 2012). Skemp (1976) wrote about a possible ‘mismatch’ existing between pupils and teachers in mathematics classrooms. He considered the situation grave when a teacher values ‘instrumental understanding’ whilst some pupils yearn for ‘relational understanding’. Taking individual needs into account, the promotion of classroom practices that are consistent with providing pupils with the necessary tools to construct and develop shared mathematical understandings is desirable.

2.2.5 Approaches to school algebra

Bednarz, Kieran and Lee (1996) listed different approaches for introducing pupils to school algebra as:

- the rules for transforming and solving equations;
- the solution of specific classes of problems;
- the generalisation of laws governing number;
- the concept of variable and function;
- the study of algebraic structures.

Bednarz et al (1996) suggested that the range of approaches taken to school algebra determines greatly the access that pupils have to algebra. The favoured approaches to school algebra may also be blamed for problematic algebraic conceptions that pupils develop throughout their schooling and beyond, including:

- the meanings pupils ascribe to algebraic symbols and notation (Van Amerom, 2003);
- poor strategic decisions pupils make in problem solving (Tall and Thomas, 1991);
- a 'difficult' relationship with mathematics and/or algebra beyond schooling (Boaler, 2009; Brown, Brown and Bibby, 2008; Nardi and Steward, 2003; Boaler, Wiliam, Brown, 2002; Quilter and Harper, 1988).

Bell (1996) stressed the importance of pupils experiencing various approaches: *generalising, problem-solving, modelling,* and *functions*. All pupils need to develop essential processes of representing, manipulating and interpreting algebraic relationships. Within these various experiences, a pupil's attention may be drawn to structures underlying algebraic relationships. Mason and Sutherland (2002) also advocated that none of these approaches be valued over others. They suggested merging approaches in order to make the teaching of algebra more effective, since each approach, used on its own, is inadequate. They further underlined stark differences in approach to school algebra adopted by various countries as follows:

- Hungary, France, Italy emphasising algebra as a study of systems of equations;
- England and Ontario in Canada laying emphasis on algebra as a means of capturing number patterns as formulae;

- Victoria and Queensland in Australia also laying emphasis on algebra as a means of capturing number patterns as formulae, as well as expressing generality and giving evidence for conjectures obtained from mathematical investigations;
- United States of America seeing algebra as generalised arithmetic and a systematic way of expressing generality and abstraction.

Healy et al (2001, citing Hewitt, 1992) argued that the emphasis on school algebra in the UK as generalising from patterns undermines algebraic activity, since teachers often fail to draw pupils' attention to the underlying structure of the situations from which numeric data is derived. In agreement, Noss et al (1997) proposed a view of mathematics learning in which pupils are encouraged to link visual, symbolic and verbal mathematics through providing various modes of expressions. Van Amerom (2003) underlined the dialectic relationship between arithmetic and algebra as: "arithmetic involves straightforward calculations with known numbers, while algebra requires reasoning with unknown quantities and recognizing the difference between specific and general situations" (p.64).

2.2.6 Understanding formal algebraic notation

Research has highlighted various difficulties experienced in the learning of algebra, including the: "lack of closure" obstacle (Tall and Thomas, 1991, p.126); meanings associated with use of letters (Küchemann, 1981); failures to see 'process' and 'object' in expressions (Sfard, 1991); and expressing operations and relations in formal algebraic notation (Goulding and Kyriacou, 2007). Kieran (1992) illustrated certain approaches to tasks in textbooks that placed an emphasis on procedural techniques. She posited that algebraic content itself contributes to the difficulties that pupils have in algebra. Chinn (2004) explained that several 'inconsistencies' in algebra are a source of 'difficulty' for many pupils, including:

- The signs for addition and subtraction are retained, while those for multiplication and division 'disappear' and are replaced by brackets and the line notation respectively;
- A group of terms can be combined and regarded as a single object on which an operation may be performed; for example, $2a + 4$ as a single entity which learners should be encouraged to write as $3(2a + 4)$ when multiplied by 3, and $\frac{2a+4}{4}$ when divided by 4;

- The instruction ‘Remove the brackets $(2m + 3)(m - 1)$ does not mean literally doing so.

Van Amerom (2003) involved grade 5 and 6 pupils (10-12 years) in primary, and grade 7 pupils (12-13 years) in secondary school in a study in the Netherlands. She reported pupils’ difficulties in adopting formal symbolism despite their solving linear equations. However, in contrast to algebra literature, Hewitt (2009) underlined the role of teachers in drawing pupils’ attention to particular details. His study involved Year 5 (10-11years) pupils assessed to be at levels 2 to 5 in the UK National Curriculum: they had received no prior instruction in algebra. After 4 hours using computer-based and pen-and-paper tasks on three consecutive days, these pupils were able to learn formal notation and to solve complex linear equations with confidence.

Anyone learning algebra needs to develop their symbol awareness. Formal notation involves applying four basic number operations on both numerals and letters. These numerals, letters and other mathematical signs can be treated as symbols, and they combine to form algebraic terms and expressions that can be rearranged or simplified. Arcavi (1994) claimed this awareness, ‘symbol sense’, should:

> “*include the intuitive feel for when to call on symbols in the process of solving a problem, and conversely, when to abandon symbolic treatment for better tools”* (p.25)

Arcavi (1994) seemed to endorse a focus on developing of algebraic conceptual understanding. He identified eight behaviours embodying ‘symbol sense’ as: making friends with symbols; reading through symbols; creating symbolic expressions; generating equivalent expressions; choice of symbols; flexible manipulation skills; symbols in retrospect; symbols in context. He stated the connectedness and interdependence between ‘procedural fluency’ and ‘conceptual understanding’. One skill reinforces the other to develop effective learning of algebra. Bokhove and Drijvers (2012) underscored the existence of this symbiotic relationship.

> *“Basic skills involve procedural work with a local focus and emphasis on algebraic calculation while symbol sense involves strategic work with a global focus and emphasis on algebraic reasoning”* (p. 44)

Arcavi (1994) suggested that ‘symbol sense’ may not be fully developed in pupils who know how to perform algebraic manipulation but fail to consider the relevance of symbols, which is revealed in the structure underlying an algebraic problem. Failure to develop 'symbol sense' while in school results in a negative impact on pupils' affective domain. It can lead them into feeling inadequate in mathematics. Boaler (2009) argued that this lack of understanding becomes a source of anxiety for many learners: this anxiety may turn into a 'hatred' for mathematics that the learners nurse well into their adulthood, leading them to become potentially less productive members of society, avoiding mathematics at all costs. Quilter and Harper (1988) wrote about graduates in a professional group, who cited teaching methods, language and symbolism, and the discontinuity between arithmetic and algebra as key reasons for their dislike of mathematics despite being academically-qualified. According to Hewitt (2011), the adoption of alternative pedagogic approaches may address the persistence of pupils' difficulties at secondary school in order to make algebra more accessible to pupils. Kieran (1992) argued that teaching in ways that make little or no sense to pupils may render the content incomprehensible and encourage pupils to overlook the underlying structures such as formal symbolism. Hewitt (2012) questioned whether the learning difficulties are due to pedagogic practice rather than algebra as an inherently 'difficult' topic. While the abstract nature of algebra and mathematics can undeniably pose problems for many pupils, questions may be raised about the role of subject educators as their learners make sense of their mathematical knowledge. I now consider the pupils' role in their own learning.

2.3 Pupils Learning Mathematics

2.3.0 Introduction

From a Vygotskian perspective, the essence of pupils' views of their learning within targeted school-based initiatives recognises pupils as active constructors of knowledge (Bruner, 1960).

2.3.1 Upholding pupil voice

There is increasing emphasis on metacognition and self-regulation as desirable 21st Century skills (Livingstone, 2012) and as foundations for effective life-long learning. However, pupils' views of pupil learning are seldom-considered resources in most educational reforms. Wall, Higgins and Smith (2005) underlined the need to actively seek pupils' views about their own ICT-enhanced learning. Researchers, including Lee and Johnston-Wilder (2013); Flutter and Rudduck (2000), have argued for policy makers to value the pupil perspective on what would make learning more effective. According to Rudduck, Harris and Wallace (1994), pupils have a rich, *observant*, usually untapped understanding of the factors affecting learning processes, and need to be offered greater agency as responsibility for their own learning. Yet, a mathematics classroom in England is often characterised as a quiet environment where pupils are expected to work individually on textbook exercises, pausing when they get 'stuck' to ask their teacher for assistance and explanation (Ofsted, 2008). Kanja et al (2001) reported this practice as the norm in many secondary mathematics classrooms in Kenya (see Section 1.1.2). Discussion amongst pupils is unusual: pupils work individually through textbook exercises after teacher-led examples or explanations. Learning mathematics ends up being regarded as an individual activity: pupils engage in silence and isolated from peers. Classroom practices can propagate beliefs about mathematics as a threatening and isolating activity (Noss and Hoyles, 1996).

Although such predictable routine in mathematics classrooms, to an extent, guarantees order and security, it may instil a sense of isolation in pupils and lack of variety that renders lessons 'boring'. Pupils' agency in learning is undermined through limited opportunities to be active as they develop mathematical understanding. Rudduck et al (1994) explained that the 'truth' about any learning in mathematics classrooms can only be inferred since 'understanding' is not observable and learning may be said to be 'going on well'. They argued that while pupils may neither be aware of nor talk about their 'understanding' in lessons, pupils are clear about 'not understanding' when they are asked to express perceptions of their learning experiences. Hannula (2002) warned that a lack of understanding generates negative experiences, low expectations, and hence leads to anxiety and frustration. Negative emotions may lead to pupils avoiding and even disliking mathematics as reported by Brown et al (2008). Boaler et al (2002) reported pupils' views on imposed classroom grouping and structural boundaries in curricula systems. Bandura (1989) argued that such practices may lead pupils to develop low self-efficacy and a fixed 'mindset' (Dweck, 2000) as beliefs that each individual's capacity to learn has a 'ceiling'. These factors combine into attitudes towards mathematics that influence pupil participation and progress.

2.3.2 Learner agency in mathematics

Viewing mathematics classrooms as activity systems requires an understanding of how pupils, teacher, curriculum resources including ICT tools, and physical environment, interact in the construction of competence. Gresalfi et al (2009) described 'mathematical competence' as the system of expectations established around an acceptable classroom 'subculture'. It defines what pupils need to *know* or to *do* when "correct", sharing misconceptions and mistakes in order to be considered 'successful' by their peers and their teacher. Thus, pupils play a role in shaping *what*, if any, learning occurs in classrooms. They construct with their teacher a system of competence by negotiating the distribution of responsibility and accountability, *for what* and *to whom*, in their learning. The role is determined by pupils' levels of conscious or unconscious participation in mathematical activity. According to Gresalfi et al (2009), each individual can always exercise 'agency' by complying or resisting: it is the *ways* of exercising agency, and the *consequences* of doing so, that change in learning contexts. A pupil's participation in a group or a class activity may form a pattern and develop into a set of expectations regarding the agency of that pupil's actions. These actions include: articulating ideas; agreeing or disagreeing with explanations; refraining from responding to contributions; becoming 'helpless'; and whether a pupil's own actions are in turn accepted, challenged or ignored.

Various definitions have been offered to describe 'agency' in acknowledgement of learning actions (Gresalfi et al, 2009; Blair, 2009; Boaler and Greeno, 2000). The classroom 'subculture' dictates the pupil participation and accountability. The nature of the learning tasks will determine the *kind* of agency that pupils have an opportunity to exercise as they complete a task. Skinner and Belmont (1993) considered 'autonomy support' as the amount of freedom that pupils are given to determine their own behaviour. They suggested that, when pupils experience more 'autonomy support', their independence in learning can strengthen. Increased conceptual understanding in mathematics requires the application of thought by pupils. The prevalent practices in many mathematics classrooms need to be addressed if pupils are to take responsibility for their own thinking (Bruner, 1957) and learning. Boaler and Greeno (2000) lamented that:

> "*Traditional pedagogies and procedural views of mathematics combine to produce environments in which most students surrender agency and thought in order to follow pre-determined routines. Many students are capable of such practices but reject them as they run counter to their developing identification as responsible, thinking agents*" (p. 171)

Blair (2009) described 'agency' as the ways in which people act and interact in sociocultural settings. She viewed individual agency as driven by one's desire to fulfil basic needs and take control of one's own mental activity, competence and freedom. Autonomy and relatedness can determine pupil engagement in learning activities (Skinner and Belmont, 1993). Gresalfi et al (2009) concurred; they stated that individual pupils' actions contribute to the joint activity of the group in which they are participating: 'agency' refers to the ways in which pupils act or refrain from action. According to Hernandez-Martinez and Williams (2013), reflexivity was vital for cultivating the pupils' resilience when in sociocultural contexts plagued with adversity. They claimed that reflexivity can foster the development and exercising of learner agency and can lead pupils to negotiate successfully through their learning. Emirbayer and Mische (1998) viewed 'agency' as a capacity for action determined by a dynamic interplay of 'routine', 'purpose' and 'judgement': these three elements contextualize the habits and future projects within situational contexts, a view shared by Hannula (2002). The level of pupil engagement in a classroom activity may be determined by their previous performance, present attainment and expected goals in mathematics achievement. Thus, it is desirable to promote conditions that may enhance active pupil participation despite unsuccessful initial attempts at solving tasks. Developing learner agency in classrooms creates a crucial avenue for learning-with-understanding (Sfard, 2001).

In this thesis, I adopt the definition of 'agency' offered by Pickering (1995) for actions within sociocultural contexts. According to Pickering (1995), the pupils exercise ***conceptual agency*** when they engage in reasoning and in joint decision-making whilst choosing methods, and when they explore strategies to negotiate meaning for relationships in mathematical concepts. Otherwise, pupils exercise ***disciplinary agency*** when they adhere to correctly executing established procedures; hence they cede agency to mathematics. In a classroom where pupils articulate their ideas, they are accountable *for* demonstrating understanding and also accountable *to* both their peers and teacher. Pupils are thus required to put more thought and effort into justifying the reasoning in their solutions. Blair (2009) viewed learner agency as pupils moving from powerlessness to a sense of control over their own future when they realise the potential for growth in transformative classroom learning experiences. Lee and Johnston-Wilder (2013) reported pupils' yearning to grow as learners and increased ownership of learning process. They described pupils' desire for the provision of opportunities that enable enhanced understanding and discovering things for themselves. When given a chance, pupils are willing to participate in collaborative tasks, relate to each other, and be respected and valued in a community of mathematical learning.

2.3.3 Affective traits in mathematics

The following discussion illuminates the factors that may influence learning and achievement in mathematics separate from cognition. Kilpatrick et al (2001) listed 'productive disposition' as a key skill that promotes success in mathematical learning when developed in pupils. Several researchers (Diego-Mantecón, Andrews and Op't Eynde, 2007; Brown, 1996) reported that the learners' self-concept, teaching, curriculum and classroom contexts determine both 'affect' and mathematical thinking. Writing on 'Lesson Study' in the Far East, where teachers collectively review content delivery, Tall (2010c) stated the need to tune into the feelings elicited in pupils as they engage with understanding mathematics. He underlined the effect of emotional reactions on the quality of learning. According to McLeod (1992), emotions are states of consciousness or feelings distinguished from cognition, whereas attitudes are "affective responses that involve positive or negative feelings of moderate intensity" (p. 581). Therefore, attitudes are manners of acting and feeling that show one's disposition, while emotions are considered to be less cognitive and change more rapidly than attitudes. Hannula (2002) advised that we acknowledge that pupils have awareness of their emotions and that they may reflect on and control them. He considered cognition to be a neural activity determined by pattern recognition, categorisation and association; emotions combine with cognition to produce attitudes towards mathematics observed in:

1. emotions pupils experience during mathematical tasks;
2. automated association of emotions with the concept of 'mathematics';
3. pupils' expectations as a consequence of doing mathematics;
4. pupils' value of mathematics-related goals in the global structure.

The reconceptualization illustrated the dynamic influence of past experiences and expectations of the future upon pupils' present engagement in mathematics learning. McLeod (1992) wrote that pupils experience negative emotions (panic and frustration) upon encountering a difficulty; the expressions of 'aha' indicate positive emotions that signal satisfaction with delightful insights into problem-solving. Diego-Mantecón et al (2007) conducted a comparative study, involving 625 12 and 15 year olds in England and Spain, to investigate the impact of mathematics-related beliefs on students' engagement and learning. They reported that pupil beliefs are related to mathematics attainment. According to Hannula (2002), attitude towards mathematics can improve within a short time when raised interest supports learning: anxiety can hinder learning. He reported the case of 'Rita' from a study involving pupils in grades 7 to 9 (13-16 years) in a Finnish secondary school. This pupil did not like learning mathematics due to some problems experienced since primary school in understanding mathematics. She valued working hard and had a positive 'attitude', despite having low expectations. Following more positive experiences in lessons, she actively participated, became more confident, and looked forward to learning. According to Hannula (2002), understanding mathematics was a new learning experience that engendered in her positive expectations that served as personal goals for success. The more 'Rita' understood mathematics, the more she liked it. This case illustrates the need for making mathematics classrooms places in which pupils nurture a sense of pride in their work. It highlights the teacher's role in inspiring and motivating their pupils to learn, through making reasoned judgements by themselves (Gattegno, 1970; Bruner; 1961), a view shared by Tall (2013) (see Section 2.2.4). By sharing ideas within an enabling learning experience, pupils can develop their confidence and competence in mathematics.

2.3.4 Developing 'mathematical resilience'

Researchers have, over the years, identified the 'enjoyment' of learning mathematics as crucial to the levels of pupil engagement and pupil participation in mathematics lessons (Tall, 2010c; Brown et al, 2008; Hannula, 2002). Nardi and Steward (2003) listed negative perceptions of mathematics as **"tedious, isolated, rote-learning, elitist, de-personalised"** (p.345), following a qualitative study involving Key Stage 3 (11-14 years) pupils in the UK. This study illuminated the distinct lack of due consideration for the affective dimension in mathematics learning. The failure to feel successful in mathematics had led to these pupils warding off any emotional involvement with mathematics. Many pupils considered mathematics to be 'boring', lacking in challenge, 'fun', and opportunity for one to be creative due to their loss of control over learning activities and tasks. According to McLeod (1992), attitudes towards mathematics tend to be more negative as pupils progress from primary to secondary school. He attributed pupils' feelings of greater frustration as they faced barriers to the increased difficulty of the mathematical concepts learned. It is undesirable, if not impossible, to divorce challenge from mathematics. Research findings (Brown et al, 2008; Nardi and Steward, 2003) suggested that attempts to protect pupils from challenges in mathematics are likely to nurture 'helplessness' and future disappointment.

This increased difficulty in concepts necessitates the development in all pupils of a positive adaptive stance that will enable the pupils to persevere with learning. Johnston-Wilder and Lee (2010b) have named this construct as 'mathematical resilience'. It requires pupils to have a 'growth mind-set' (Dweck, 2000); to value learning from mistakes; to be willing to reflect upon problem-solving processes, starting again or trying new strategies; to regard asking questions as clever; and to be purposeful in seeking meaning in their own learning. My own view is that pupils need to face mathematical barriers in order to progress with learning at secondary school with greater confidence and self-efficacy, so that they can take on more positive identities as mathematicians. In order to encourage more pupils to develop a resilient approach to learning in mathematics classrooms, Lee and Johnston-Wilder (2013) listed:

- more pupil talk, less teacher talk, and the development of mathematical language;
- more expectation of effort;
- provision of challenging and hard work;
- collaborative learning, working in groups and supporting each other.

Where pupils willingly work with others, they are seen to ask questions when unsure, to share with one another their insights into ideas, to take responsibility for their own learning and to check their own working. Hence, pupils increasingly exercise ‘conceptual agency’. Hattie and Timperley (2007) explained the importance of classroom ‘subcultures’ that foster peer and self-assessment and allow learning from mistakes. In this way, learning is viewed in terms of ‘growth’ in accordance with the Theory of Malleable Intelligence advanced by Dweck (2000). She proposed the idea of a ‘growth mind-set’ to suggest that one’s intelligence is not fixed: it can be cultivated and increased through effort. With guidance and raised expectations of more effort to be put into understanding mathematics, every pupil can increase their intellectual abilities. Lee and Johnston-Wilder (2013) proposed effective ways of learning mathematics that can inspire pupils’ confidence and motivate them to persevere in their learning. They listed: an emphasis on understanding; inclusion; valuing hard work and the introduction of variety with ICT tools providing dynamic and involving tasks. Luckin et al (2012) advocated for digital technology use within a ‘learning ecology’ (Luckin, 2008): the conditions, resources, practices and outcomes. They argued that it may be naïve to assume that ICT tools will change teaching and learning when used on their own. Provision of variety in classroom contexts offers positive learning experiences. Pupils can express what conditions stimulate effective learning for them, and this can allow them to take ownership of their learning process. Thus, it is contingent upon educators to listen to pupils’ views whilst considering education reforms. The following discussion is about how this may achieved using ICT.

2.4 ICT use in education

2.4.0 Perceived effects of ICT integration

The early use of new technologies in education was based on their potential to improve the basic skills of reading, writing, mathematics and science. Livingstone (2012) wrote about the promises that ICT would enhance pupil attainment in standardised examinations whilst reducing some disadvantage inherent in traditional assessment regimes. Researchers (Crisan, Lerman &Winbourne, 2007; Hennessey et al, 2005) reported that unfortunately ICTs have historically been regarded as an ‘add-on’, something extra, since prevailing assessment practices are still considered to be more aligned to teacher-centred pedagogies. Such views can be contested from a sociocultural perspective. Crisan et al (2007) stated that the resistance which met the use of calculators and computers was located in a belief that technologies trivialised mathematics rather than engaging pupils in learning. Hence, digital technologies remain unavailable to pupils in examinations. Kaput (1992) decried this restricted integration of technologies into mathematics education, stating:

> *“major limitations of computer use in the coming decades are likely to be less a result of technological limitations than a result of limited human imagination and the constraints of old habits and structures”* (p. 515).

Kaput stressed that, when used appropriately, the computer can make possible fundamental changes in the conditions for pupils’ learning of mathematics. He believed that it can shift the focus from memorising techniques to understanding concepts through pupils interpreting, analysing and assessing data. Galbraith and Haines (1998) argued that attitudes to mathematics can improve when pupils learn with computer tools in addition to the affordances detailed in Section 2.1.2. From a neo-Vygotskian perspective, the plethora of digital technologies available in this day is increasingly being seen as able to support the development of desirable skills in learners.

2.4.1 The development of ‘new’ skills

The current educational system in the UK is a far cry from the *basic skills-oriented* education, whose emphasis was the acquisition of the 3 R’s: reading, writing and arithmetic, as proposed by Sir William Curtis in 1795. Nonetheless, the challenge facing educators all over the world today is in applying the principles that define ‘Vygotskian’ theories alongside these emerging technologies, whilst ensuring the provision of productive learning and relevant environments. Livingstone (2012) proposed a reconceptualization of the skills to be learnt by pupils for the 21^{st} Century as “play, improvisation, experimentation, simulation, multimodal navigation and remixing, multitasking, networking, negotiation, and ability to judge diverse information sources” (p.17). Claxton (2004) proposed mathematics as nurturing versatile learners through developing their emotional, attention, reflective, social and cognitive skills.

In a collaborative initiative drawing on expertise from key stakeholders: educators, academics government and industry, the Australian Government commissioned the Digital Education Advisory Group (DEAG) to look into all aspects of education. It scrutinised teacher training, schools' infrastructure, curriculum design, assessment, and community participation. DEAG (2013) listed 21st Century skills as: creativity, innovation; critical thinking; problem solving; decision making; life-long learning; collaboration and communication; ICT literacy; personal and social responsibility; consciousness of being local and global citizens. Taylor et al (2010) reported that the Ministry of Education in New Zealand recognised the contribution of ICT in developing new ways of learning in the 21st Century: problem-solving, collaborative working as well as critical and creative representation, negotiation and communication of ideas. Pupils are expected to develop subject-specific matter alongside the key competencies identified by this Ministry, including: "*thinking, using language symbols and text, relating to others, managing self, participating and contributing*" (Taylor et al, 2010, p.561). These researchers noted that teachers, and pupils, are challenged by this blending of mathematics, nurturing competencies and ICT use. This resonated with a view that teachers need to "walk the talk" whilst teaching pupils (Claxton, 2004, p.31) as part of their continuing professional development (CPD).

Integration of ICT in England has been largely imposed on teachers through a requirement to use it within subject teaching in the National Curriculum (Hennessey et al, 2005). The same can be said of New Zealand (Taylor et al, 2010). Luckin et al (2012) regarded this externally imposed pressure to include ICT in schemes of work, regardless of whether its use supported a particular subject's aspect, as valuing technologies over pedagogies. Hennessey et al (2005) reported that a greater emphasis is placed on hardware rather than on how ICT is used. Crisan et al (2007) argued that this situation largely endorses the subordination of existing theories that underpin classroom teaching and learning in mathematics, and the curriculum to ICT, a view shared by Sutherland et al (2004). Hennessey et al (2005) wrote about concern that ICTs' use in subject teaching and learning ought to be appropriate, discriminating, and geared towards the specific learning outcomes. This argument was supported by Luckin et al (2012) who identified eight 'learning themes' of promising innovation with technology. These researchers stressed that it is not using the technology on its own or for technology's sake, but rather *how* the technology is used that has been shown to determine its impact on pupils' learning and to realise innovative examples of technology use (Livingstone, 2012). Ruthven and Hennessey listed the benefits of ICT use as enhancing participation, motivation, pace and productivity, and progression in pupil learning. However, Crisan et al (2007) and Hennessey et al (2005) insisted that an emphasis on the acquisition of new skills and knowledge that has been reported to develop in learners through more ICT uptake in schools would best be encouraged through the establishment of new assessment regimes.

2.4.2 ICT integration and teachers as 'learners'

Despite numerous, well-documented challenges hindering ICT uptake, Tabach (2011) stated that emerging digital technologies are steadily creeping into education systems. Livingstone (2012) attributed this change to the efforts of innovative teachers who embrace meaningful use of technology into everyday classroom practice. Luckin et al (2012) emphasised that educational contexts determine the way in which ICT is integrated in schools, with the learner(s) at the centre, and resources available in terms of the '*environment*, *tools*, *people*, *skills and knowledge levels*', dedicated towards supporting pupil learning. This view was shared by Rodriguez, Nussbaum and Dombrovskaia (2012) who emphasised the 'sustainability' of that integration in their ICT for education model in Chile. Research continues to highlight the crucial role of teachers in determining the impact of using ICTs in classrooms (Sutherland et al, 2009). However, this is not by any means an easy task. Integration necessitates a fundamental shift at a personal level, including teachers' beliefs systems (Webb and Cox, 2004), skills (Foster, 2014) and knowledge levels (Alexander, 2008) in order to harness these ICTs' affordances to benefit pupil learning. The Technological, Pedagogical and Content Knowledge (TPACK) framework is a model for developing teachers' knowledge for ICT integration in education (Mishra and Koehler, 2006) as shown in Figure 2.1.

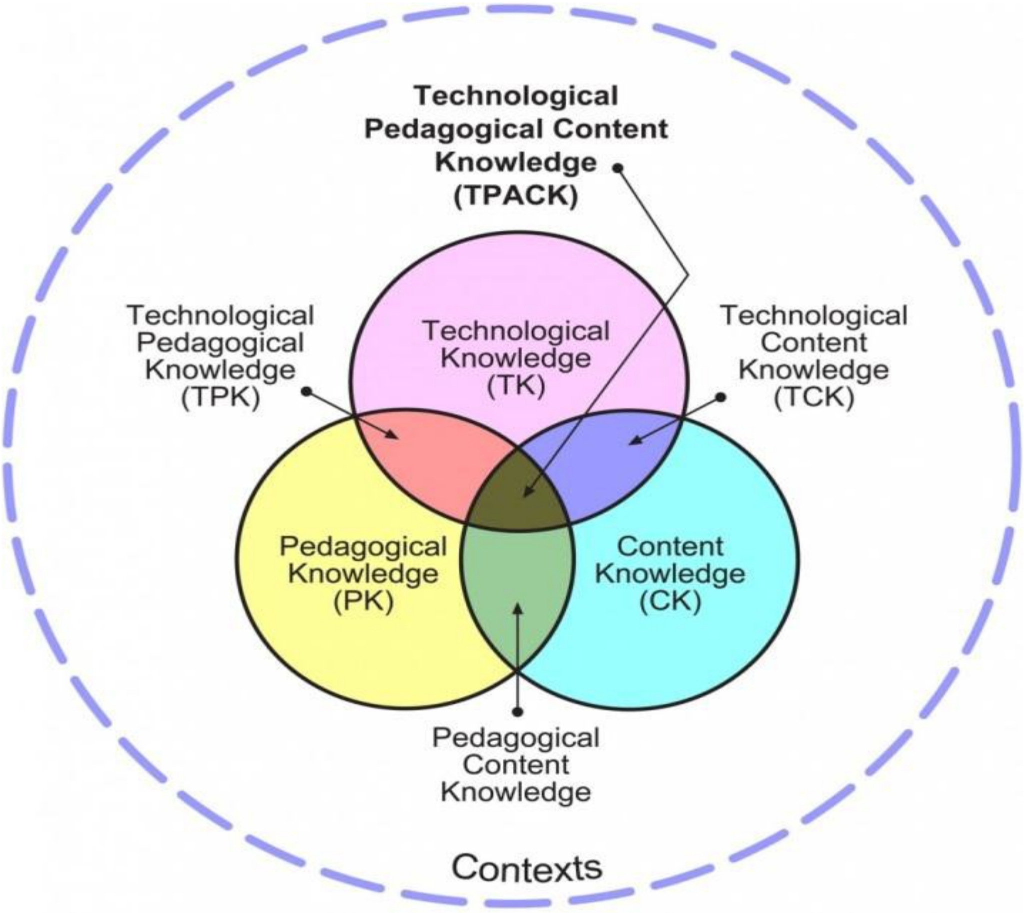


Figure 2.1 The TPACK Model ("Reproduced by permission of the publisher© 2012 by tpack.org")

'*Content Knowledge*' (CK) consists of the teachers' conceptions of the subject matter to be taught, including their knowledge of concepts, theories, evidence and proof as well as the established practices and approaches towards developing of that knowledge (Koehler and Mishra, 2009). '*Pedagogical Knowledge*' (PK) encompasses the teachers' pedagogical content conceptions including theories of teaching and learning, lesson planning and practices about classroom management skills (Koehler and Mishra, 2009). '*Technological Knowledge*' (TK) essentially stems from teachers' ICT content and curricular conceptions (Crisan et al, 2007). Tabach (2011) saw TK as entailing knowledge about connecting, installing and using hardware and software, as well as an ability to learn about the emerging digital technologies. However, it is considered an unstable knowledge that is difficult to define due to the constant influx of new technologies (Koehler and Mishra, 2009). While Crisan et al (2007) advocated for upgrading some teachers' knowledge of technical aspects of ICT, Tabach (2011) underscored the ability to adapt from one version of an application to a new one. An integrated form of knowledge is created at the point of intersection of any of the three different forms of knowledge.

Pedagogical Content Knowledge (PCK) consists of an understanding of what makes any subject difficult or easy to learn, an awareness of common misconceptions, how to address them, and how concepts are organised, adapted and presented in such a way as to target different learners' interests and abilities.

Technological Pedagogical Knowledge (TPK) implies an appreciation of the influence of technology use on a teacher's repertoire and pupil learning that requires acquaintance with a tool in order to adopt the appropriate pedagogic strategy to suit a specific learning objective. Yerushalmy and Naftaliev (2011) espoused the construction of TPK in teachers by stating:

> *"Teaching with an interactive textbook should be considered more than a technological change; indeed it is an attempt to create new paths for the construction of mathematical meaning"* (p. 243)

They acknowledged that technology can facilitate new ways of making sense of knowledge.

Technological Content Knowledge (TCK) involves knowing how subject content and ICTs mutually influence and constrain each other (Tabach, 2011). According to Koehler and Mishra (2009), teachers need to understand which technologies are best suited to represent concepts, and need an awareness that content may dictate or change the technology, and vice versa.

While TPACK is the knowledge generated where all three forms of knowledge overlap and integrate (see Figure. 2.1), it entails an understanding of:

- learners' prior knowledge and use of technologies to strengthen and build on existing knowledge and to develop new ways of knowing;
- representation of concepts using pedagogic techniques that use technology constructively;
- how technology can help address some misconceptions and problems that learners face.

Effective integration of ICT in education is determined by the interaction of and relationship between teachers' and pupils' actions, the intended learning outcomes and the conditions for teaching. The emphasis ought to be on promoting shifts towards pupils being more responsible for their own learning; pedagogies shifting from teacher-centred to pupil-centred; and shifting from procedural fluency to the development of communication skills and conceptual understanding in subject learning. I contend that policy makers have often focused on cognition while largely ignoring the affective aspects of learning, which I explore next.

2.4.3 Enablers and inhibitors to ICT integration

Several studies have identified numerous influencing factors, of both a contextual and a personal nature, as teachers learn about the potential and the limitations of ICT and about how to integrate ICTs into teaching (Crisan et al, 2007; Hennessey et al, 2005; Mumtaz, 2000) (see Figure. 2.2).

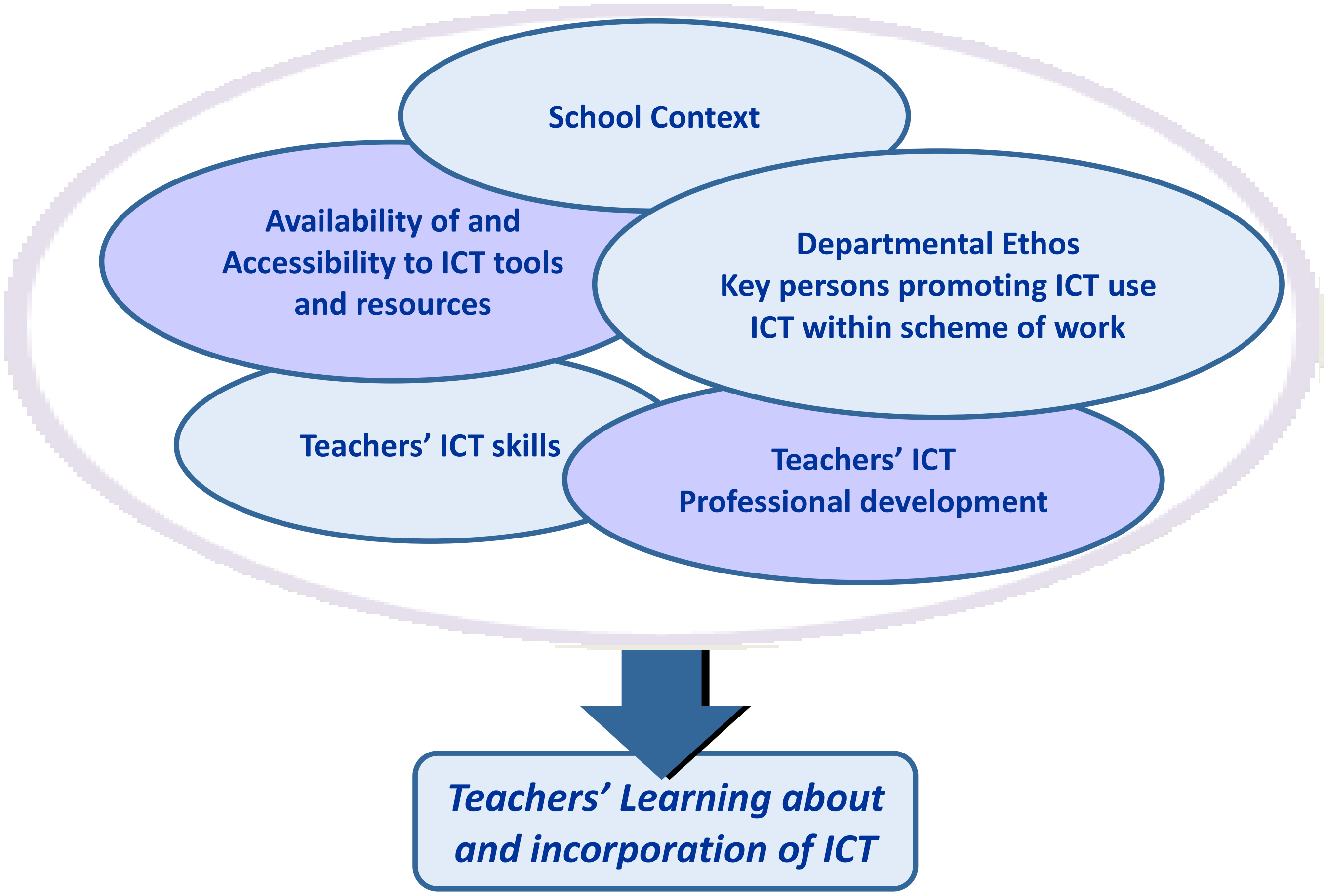


Figure 2.2 Factors affecting teachers' learning about and integration of ICT into their classroom practices (Crisan et al, 2007)

In a review of pedagogy related to ICT, Webb and Cox (2004) discussed the role played by beliefs and values in dictating the extent to which teachers fully integrate ICT and student-centred approaches in education. Mishra and Kohler (2006) explained that the teachers own learning experiences with ICT contribute to the development of a personal ICT pedagogical construct, the TPACK, a view shared by Crisan et al (2007). Rodriguez et al (2012) argued that while ICT was never conceived for educational purposes, its integration may thrive in a model defined by concepts, discourses and practices, dependent on factors such as:

- pedagogical approaches and beliefs;
- teacher confidence, attitudes and skills relating to ICT;
- time spent by teachers on meetings, training, exercises and lesson planning;
- school's ICT infrastructure, supervision and technical support;
- involvement and leadership of school's management.

(Rodriguez et al, 2012, p. 292)

The TPACK framework assists in thinking about how teachers' technological knowledge might complement their existing pedagogy and subject content knowledge so as to foster innovative classroom practices and improve pupil learning outcomes. However, effective integration of ICT within a subject may be demonstrated when the teachers embrace learning for themselves by using the ICT tools to transform their subject content knowledge and adjust their teaching repertoire accordingly.

Crisan et al (2007) highlighted the important role of individual teachers charged with the responsibility of promoting ICT within subject departments. This is achieved by modelling ICT's effective integration into the school's context, enabled and constrained by institutional factors. This may convince colleagues to adopt relational pedagogies, such as the potential of linking arithmetic to algebra through ICT use. Sutherland et al (2004) admitted to hindrances to knowing how to use ICT tools to transform learning in schools, stating:

> *"This is because new ICT tools often challenge an existing practice of teaching and threaten a well-established knowledge domain"* (p. 424)

Koehler and Mishra (2009) emphasised that "an approach is needed that treats teaching as an interaction between what teachers know and how they apply what they know in unique circumstances and contexts within their classrooms" (p. 62). Hennessey et al (2005) reported the many challenges that new technologies pose to teachers such that there is no 'best' way of integrating ICT into subject teaching. Koehler and Mishra (2009) agreed that ICT use has implications for the reconceptualization of teacher education and professional development.

Niess, Ronau, Shafer, Driskell, Harper, Johnston, Browning, ÖzgÜn-Koca and Kersaint, (2009) advanced that teachers need to develop a specific knowledge with which to teach mathematics with ICT, by undergoing a process of '*recognising*', '*accepting*', '*adapting*', '*exploring*', '*advancing*': the Mathematics TPACK shown in Figure 2.3.

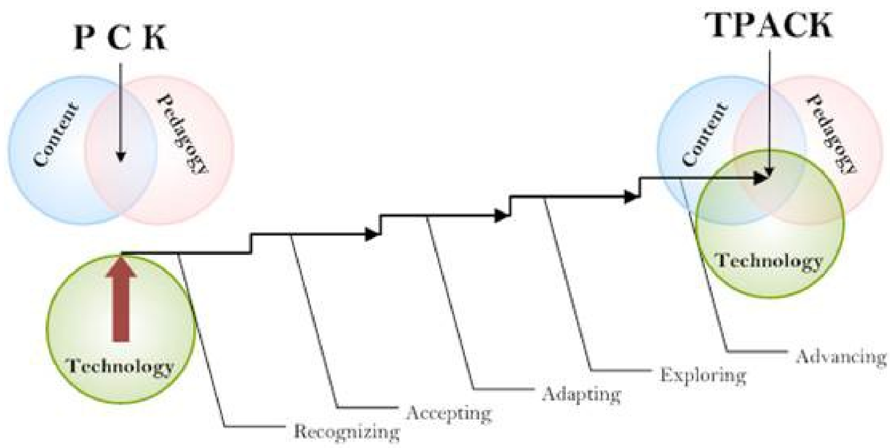


Figure 2.3 Visual description of mathematics teacher levels as their thinking and understanding merge towards the interconnected and integrated manner identified by TPACK (Niess et al, 2009)

As mathematics teachers acquaint themselves with the ICT tool, they may:

- recognize the alignment of the tool with the curriculum content, yet fail to embrace it;
- form an attitude towards teaching and learning with the ICT tool;
- engage in activities that lead to a choice to adopt or reject the ICT tool;
- actively integrate the ICT tool within the teaching and learning of mathematics;
- confirm their decision with regard to the integration of ICT tool in their repertoire.

According to Tabach (2011), this integrated knowledge is strongly related to mathematics and needs to be elaborated further within mathematics. Research (Crisan et al, 2007; Hennessey et al, 2005; Ruthven and Hennessey, 2002; Mumtaz, 2000) has shown factors that encourage teachers to use computers in mathematics teaching include: access to computers; teachers' own skills; beliefs that computers help pupils learn; availability of software relevant to the curriculum; policy decisions of the school; technical support; and 'fun' for pupils. The discouraging factors include: difficulties accessing computer rooms; lack of technical support; inadequate lead-in time to learn software; inadequate resources; lack of confidence. The use of ICT in education is viewed with mixed perceptions which largely depend on users' beliefs, knowledge and values. I now consider the convergence of the issues I have so far highlighted as they affect my research.

2.5 ICT-enhanced learning contexts and mathematical thinking

2.5.0 Introduction

This discussion revolves around the potential of ICT tools to address difficulties of learning algebra in the mathematics classroom.

2.5.1 Using ICT in mathematics learning

Several researchers, including Noss et al (1997), Askew et al (1997), Wilensky (1997), Noss and Hoyles (1996), have argued for learners increasing connections within their mathematical knowledge. Rudduck et al (1994) labelled the 'connectedness' as 'coherence', which implies pupils learning mathematics with 'relational understanding' (Skemp, 1976) according to Tall (2013), leading to their 'deep learning' (Abbot et al, 2009). Central to the achievement of this goal is the emphasis on the role of pupils moving from passive recipients (Boaler and Greeno, 2000) to active constructors of knowledge (Bruner, 1960). Pupils need to make connections between their previous knowledge and the material being learned. Abbot et al (2009) argued that 'deep learning' requires a firm grasp of previously-learned concepts that can be recalled with ease, and applied creatively to new content in subject learning by pupils, with the help of ICT. The authors wrote that using ICT tools can provide pupils with learning experiences of concepts from multiple experiences, a view elaborated by Becta (2008) (see Section 2.1.2). It is important that the pupils' attention is focused on enhancement of conceptual understanding as opposed to tool mastery. According to Reed, Drijvers and Kirschner (2010), a construction of new understanding requires reflection by pupils on knowledge that is a result of linking tool techniques to learning tasks and the underlying structures of the mathematical concept. It is imperative that ICT use is purposeful (Ainley et al, 2006) in pupils' learning as they seek to achieve intended learning outcomes, and not just for the sake of using technology.

2.5.2 Effect of using ICT on meaning-making in mathematics

Galbraith and Haines (1998) reported findings following an experimental study on a group of first year UK students on engineering, mathematics and actuarial science degree courses. The implication of interaction of technology on student confidence, motivation and engagement is such that computer attitudes were found to be more influential in facilitating active student engagement in computer-related activities in mathematical learning. These results established the influence of computer use on learning environments in determining attitudes to computer-mathematics interaction. Galbraith and Haines (1998) stated that students with high computer motivation find computer learning more enjoyable since they are more sure of their answers when supported by immediate feedback; mistakes are readily resolved, thus, mathematical learning is enhanced. Despite the fact that some students had minimal prior experience with computers and keyboard skills, hence they were low on computer confidence, this study showed that the computer-based activity allowed individual subject development. These findings implied that computers have a positive effect on the learning environment by providing students with the freedom to experiment, test new ideas whilst expressing their thoughts; hence computer use has implication for pedagogic practices in mathematics classrooms. However, few studies have explored the motivational effect of ICT translating into learning gains. Claxton (2004) listed curiosity, critical thinking, sociability, ability to tolerate frustration and self-regulation among the skills students can acquire through practice and experimentation in mathematics classrooms. Reed et al (2010) cautioned against indiscriminate use of ICT. They argued that instructional practices seen to suit less able pupils can lose their effectiveness when used with less able pupils. Hence the vital role played by the teacher in assessing the appropriateness of activities and resources in meeting the learning needs of their pupils.

Healy et al (2001) described the positive effect on pupils' mathematical constructions in the interactive computer environment (see Section 1.3.2). This view resonated with the 'versatile thinking' developed in pupils by a 'computer approach' to teaching and learning algebra (Tall and Thomas, 1991). They believed this approach enables pupils' moving into the 'proceptual-symbolic' world of mathematics' (Tall, 2004) by developing 'proceptual thinking' (Gray and Tall, 1994). It encouraged pupils to switch effortlessly between seeing symbols in algebraic expressions as mathematical processes to do and concepts to think about. For Hewitt (2012), students received a meaningful introduction to algebra and became more successful at overcoming well-documented difficulties: the use of letters for variables, formal notation of algebraic expressions and embodied objects. From their classroom-based research in England involving teachers and students in which activities were dictated and constrained by curriculum demands, Healy et al (2001) reported that:

"*Through interaction with particular computational environments, students could:*

- *accept that a letter represented a general number*

- *accept and work with 'unclosed' algebraic expressions*
- *express general mathematics relationships expressed in a computer language*"

(p. 237)

Healy et al (2001) stressed the development of pupils' conceptual understanding of algebra in mathematics classrooms:

> "*We noted that in paper and pencil settings a similar set of structured activities often fail to lead students towards algebraic approaches as the 'pre-algebraic' activities do not engage many students in algebra related activity, but can be successfully negotiated through arithmetic problem-solving methods. The students are unlikely to spontaneously construct algebraic concepts and are unlikely to experience some of the power of the algebraic representation system. There needs to be a cognitive pay-off which motivates students to incorporate algebraic methods into their mathematical practices.*" (p. 238)

Healy et al (2001) decried approaches through which pupils in the UK have traditionally been introduced to algebra in school mathematics, and certain initiatives that under-emphasise the crucial role played by language in the developing of algebraic thinking. Bell (1996) suggested that the practice of generalising focuses learners' attention on the techniques of extracting formulae, leading to an approach which effectively circumvents the essence of algebraic processes, resulting in procedural learning. Healy et al (2001) reported that students using a '*computer approach*' (Tall and Thomas, 1991) progress to make use of ideas they obtain in computer environments to pen-and-paper algebra activities, which Bell (1996) identified as:

- being able and willing to operate with symbolic (algebraic) expressions;
- learning the linguistic aspects of algebra (write and read notation correctly and meaningfully);
- learning to manipulate (algebraic notation) correctly and fluently;
- acquiring the strategic know-how needed to deploy algebraic language in activities.

Healy et al (2001) stated that research with computer environments poses a challenge to the notion of 'readiness' on which the UK curriculum is based, despite research evidence, seen in Hewitt (2012), showing that pupils are ready to take on algebraic ideas much earlier than expected. These sentiments resonated with observations I made during my recent pilot study in a classroom in England (see Section 3.2.1), given that ICT has featured in UK classrooms for almost forty years.

2.5.3 The importance of feedback

In an extensive research review, Black and Wiliam (1998) distinguished between *formative assessment* as assessment *for* learning and *summative assessment* as assessment *of* learning. Formative assessment aids learning by generating *feedback* to inform both pupils and teachers. Hattie and Timperley (2007) viewed feedback as information when provided by an agent regarding aspects of one's performance rather than praise, rewards or punishment. The agents providing this information range from teachers' comments on learners' written work and verbalisations, and peers in collaborative learning environments, to ICT tools in technology-enhanced contexts. According to Black and Wiliam (1998), feedback realised positive benefits for learning and attainment across all content areas and levels of education when the information is acted upon; only then can assessment be said to be 'formative'. From an instructional view, formative assessment aims to empower students as self-regulated learners; thus Sadler (1989) suggested the following sequence as a formative assessment activity:

- learner receives task-related information on their performance and the desired level;
- learner perceives a gap between their existing understanding and the desired level;
- learner takes appropriate action which leads to some closure of that gap.

Hence, it is incumbent on learners actively to take corrective action in order to complete the feedback loop for learning to occur and learners to exercise conceptual agency (Pickering, 1995). Yet as Sadler (1989) observed, feedback provided to learners is often deficient in informing learners what is necessary to close the gap. Sadler (1998) believes the quality of feedback is important in terms of its accessibility to the learner as communication, coaching value and ability to inspire hope and confidence. Black et al (2003) stressed that offering only a correct/incorrect response may not be very effective in facilitating learning; it may be more useful to provide information about what is 'wrong' with an answer, leaving the pupil with the initiative to act.

Whilst emphasising the benefits to both pupils and teachers, Juwah et al (2004) proposed the following principles of good formative feedback:

1. *Facilitates the development of self-assessment (reflection) in learning;*

2. *Encourages teacher and peer dialogue around learning;*
3. *Helps clarify what good performance is (goals, criteria, standards expected);*
4. *Provides opportunities to close the gap between current and desired performance;*
5. *Delivers high-quality information to students about their learning;*
6. *Encourages positive motivational beliefs and self-esteem;*
7. *Provides information to teachers that can be used to help shape the teaching.*

(Juwah et al, 2004, p.2)

These authors argued that, while feedback on performance can enable students to restructure their current understandings, to consider alternative strategies and to construct more powerful capabilities, teachers receive information regarding students' difficulties and where to target their teaching efforts. Juwah et al (2004) saw students as active constructors of knowledge, whose level of engagement is determined by their 'met-befores', motivational beliefs, personal interpretation of the requirements and propriety of the task, and quality of feedback . Affective processes which can influence learners to close the gap include increased effort, motivation or engagement in the learning process. Otherwise, cognitive processes that involve restructuring understandings and the confirmation of responses indicate that more information, or the adoption of alternative strategies, is required to achieve the desired understanding. Hattie and Timperley (2007) state that feedback is preceded by performance of a task, and the power of feedback is greatest when it builds on changes from previous trials and addresses misconceptions; feedback that identifies a total lack of understanding can become threatening to the learner. In order for pupils to assume responsibility for learning, information offered as feedback should address the goal of the task, the progress being made, and corrective measures towards achieving that goal. Feedback information needs to be varied during the execution of the task and may be targeted at:

- task level: how well tasks are understood and performed (FT);
- process level: main process needed to understand and perform tasks (FP);
- self-regulation level: self-monitoring, directing, regulating actions (FR);
- self-level: personal evaluations and affect (usually positive) on learner (FS).

Different combinations of feedback are reported to achieve different levels of effectiveness. According to Black and Wiliam (1998), the provision of written comments about a pupil's performance (FT) is more effective than providing grades and can promote attainment in learning. FP is specific to processes underlying tasks, hence is more suited to 'deep learning' (Abbot et al, 2009) where construction of meaning is related more to relationships and transfers to unfamiliar and more challenging tasks: a 'surface learner' would benefit more from FT since their understanding involves the acquisition, storing and recollection of procedures. FR requires commitment, control and confidence for a learner to monitor, direct, and regulate their actions toward achieving a learning goal. Hattie and Timperley (2007) argued that corrective FP and FR is most powerful when perceived to lead to future learning, although the evaluative aspects of mathematics lessons introduce the risk of failure, which may constrain student engagement in learning. Hence, there is a need to establish a classroom subculture where articulated views are respected and valued by all, and for pupils to consider mistakes as learning points. On the *timing* of feedback, Hattie and Timperley (2007) advised that it is important to consider both the difficulty of the task and the level, namely 'task', 'process', 'regulation', 'self', at which feedback is targeted. While immediate feedback may be more powerful for FT, and delayed feedback more powerful for FP, pupils would benefit more through delayed feedback when involved in difficult tasks. Bokhove and Drivjers (2012) listed the factors affecting the effectiveness of feedback as: "*elaboration*, *student achievement levels*, *depth of understanding*, *attitude towards feedback*, *learner control*, *timing* and *response certitude"* (p.45). Immediate and elaborate feedback is suited to less able students; more able students may gain more from feedback that allowed for active processing.

2.5.4 Formative feedback from ICT tools

The instant feedback provided by ICT tools can be an essential resource, in addition to the teacher's explanations and textbooks, in enabling pupils to *appropriate* knowledge leading to successful mathematical learning. By imposing suitable constraints on that learning process, and provision of even-tempered, non-confrontational feedback offered by the computer, pupils have been seen to be capable of learning mathematical concepts without their teachers' intervention (Hewitt, 2011; Lugalia, 2009). However, Yerushalmy and Naftaliev (2011) cautioned that visual imagery afforded by ICT tools may fail to promote effective learning where it differs from the static representation in textbooks, and may require teacher intervention to interpret and consolidate meanings for pupils. This view was supported by Clausen-May (2008). A study by Bokhove and Drijvers (2012) with grade 12 (17- 18year old) Dutch secondary students investigated the design and provision of automatic feedback; the study demonstrated that relevant feedback fosters algebra learning by decreasing the number of attempts needed for a task while improving scores. The acquisition of algebraic expertise, in terms of 'procedural fluency' and 'symbol sense', were investigated in the intervention along three aspects of feedback: 'timing and fading', 'creation of crises' and 'feedback variation'. From these findings, Bokhove and Drivjers (2012) concluded that algebraic expertise was facilitated through timing and fading as well as through feedback variation targeted to FT and FR used formatively; their findings are consistent with Hattie and Timperley (2007).

2.5.5 Instrumental genesis

Consideration needs to be given to the theoretical approach of 'instrumental genesis', based on work developed by French Mathematics Educators in the 1990s. Knowledge construction may occur in the course of practical activity which involves interaction with artefacts. The ***Instrumented Activity Situations*** **(IAS)** model is proposed by Verillon and Rabardel (1995), through which the user associates a tool with an action in order to perform a task. This model consisted of a subject (user or operator), an artefact (technological tool) and an object on which an action using the artefact is performed (task). Within the complete interacting system, in a complex process, the technological tool is transformed into an *instrument* for performing the task. The instrument intervenes as a mediator between the subject and the object by modifying the action and presenting features which link the subject's operation with the content of the task. This transformation process, referred to as *instrumental genesis*, consists of two elements: *instrumentalisation* and *instrumentation*. *Instrumentalisation* is determined by the tool's design, its potentialities and constraints, and the results of users' conjectures and explorations that progress through stages of discovery and selection, personalisation and transformation. Reed et al (2010) and Bretscher (2008) stated that the user is then able to interpret and reflect on their engagement in the activity once *instrumentalisation* is achieved. Guin and Trouche (2002) noted that *instrumentation* related the organisation of actions to the way the ICT tool conditioned the user's behaviour in the activity; when pupils are introduced to an ICT tool, they begin to form a mental scheme of what the tool can do. Tabach (2011) explained that:

> *"a tool together with the schemes of the tool the user has developed (and continues to develop) become an instrument in the hand of the user. Thus, instruments are psychological constructs actively constructed over time as they use a tool and become more acquainted with it"* (p. 251)

In my research study, as represented in the IAS model triad, the *instrument* was *Grid Algebra* software loaded onto computers, the *subject* was the secondary mathematics pupils, and the *object* was the learning of algebra as shown in Figure 2.4.

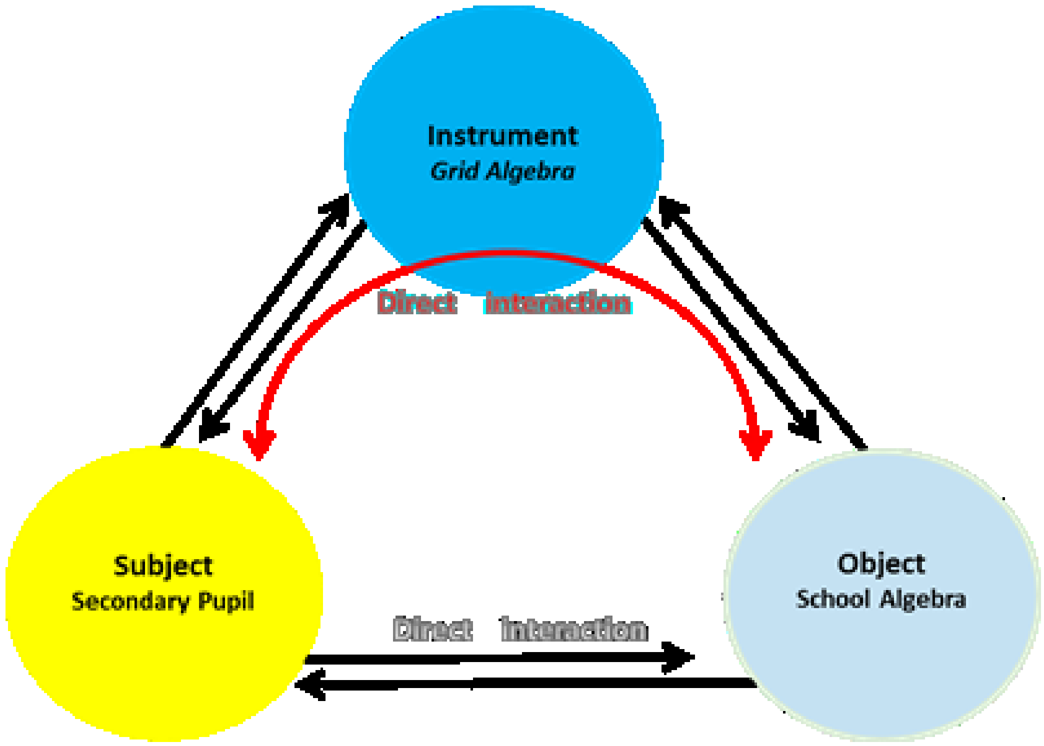


Figure 2.4 Adapted IAS Model: the triad characteristic of Instrumental Activity Situations (Verillon and Rabardel, 1995, p. 85)

I considered the role of the mathematics teacher in the ICT-enhanced learning environment because the teacher is influenced by the instrumentalisation therein. Collaborative, ICT-enhanced sessions evoke a ‘didactical tetrahedron’ (Ruthven, 2012) since the software adds the technological dimension to the interaction of pupils, content and teacher. Hence, when technology plays a role in mathematics lessons, the computer *complements* rather than *displaces* the older tools and perceptions of teaching and learning. The teacher usually takes on the mediating role between concepts and pupils in the non-ICT mathematics lessons; this is generally referred to as ‘teaching’. Guin and Trouche (2002) argued that instrumental genesis has individual and social aspects whose balance depends on affordances of the ICT tool and its access within the school context. According to Ruthven (2002), technological tools can reorganise senses if not rebalance user’s senses. In this case, using the software appeared to assist some pupils to distinguish between the left and right side as they learned to associate physical movements with mathematical operations from the software feedback presented on computer screens. This ICT-enhanced learning environment facilitated pupils working with dynamic images, reflection and learning from feedback (Becta, 2008) as they 'taught' the computer. Reed et al (2010) and Bretscher (2008) considered the teacher's role crucial in supporting pupils' instrumental genesis by consolidating individual constructions. Several researchers considered adding of computers to the learning 'ecology' (Luckin, 2008) inevitably leads to a shift in the teaching and learning relations, and required the teacher to assume various roles (Monaghan, 2004; Sutherland et al, 2004; Healy et al, 2001). This may be quite disturbing for teachers who, on realising that pupils are apparently learning mathematics without the teacher's input, feel threatened about the computer taking over their jobs, and rendering them redundant. To the contrary, the teacher is freed up to concentrate on several roles. Monaghan (2004) listed these as: *task-setter, facilitator, collaborator, assessor, evaluator,* in the ICT-enriched classroom setting for the desired effect of enhancing pupils' learning of algebra to be achieved. Sutherland et al (2004) identified a 'complex shifting of perspectives, from "more knowledgeable other' to a *co-constructor* of knowledge to the vicarious participant" (p. 420). In this way, the teacher has the opportunity to learn about ICT as well as the learning of algebra from the growing pupils' confidence with formal notation through to solving linear equations by revealing the understandings and misconceptions they articulated. Willing teachers' Mathematics TPACK (Niess et al, 2009) and 'listening skills' (Foster, 2014) can develop, including pedagogical decisions. While pupils learn more about algebra supported by the classroom interactions in ICT-enhanced learning, their teacher also 'learns' more about the teaching and learning of mathematics, supported by an ICT tool use.

From a sociocultural perspective (see Section 2.1.0), these were my initial research questions:

a) What effect will such intervention have on pupils learning algebra at Key Stage 3 in terms of:
    i. classroom interactions;
    ii. competence, and confidence, with mathematical language use;
    iii. ownership of learning?
b) To what extent do pupils benefit from the use of mathematics-specific ICT resources in addition to non-ICT resources, in mathematics lessons?

2.6 Summary

This chapter has detailed an overview of theoretical ideas shaping research which aimed to demonstrate that, within carefully-designed settings, with the benefits of appropriate tools and interactions, pupils can access 'difficult-to-grasp' mathematical concepts. Chapter 3 describes the plans I invoked in the study in order to view the participants' actions through a sociocultural lens in processes reported in Chapter 4.

Chapter 3: RESEARCH DESIGN

"The most important single factor influencing learning is what the learner already knows.

Ascertain this and teach them accordingly."

(Ausubel, 1968)

3.0 Introduction

This chapter outlines the development of research strategy for my entire project. I discuss my approach to methodology which I link to theories shaping this research, and justify research methods in Section 3.1. In Section 3.2, I detail implications of two pilot stages and how they informed my final research questions and strategy.

3.1 Selecting research strategy

3.1.0 Overview

The core research theme was examining the potential of mathematics classroom 'subculture' (Bauersfeld, 1998) to mediate ICT tool's use in early secondary algebra. Papert (1980) linked mathematical thinking to kinaesthetic computer-based activity. I placed the mathematics-specific ICT tool, *Grid Algebra* on computers, into pupils' hands at Key Stage 3. I envisioned aspects of 'dialogic teaching' around *Grid Algebra* tasks stimulating formative feedback (Juwah et al, 2004) in a collaborative learning context. I hoped to observe ensuing social interactions and monitor what pupils were able to do when using ICT that they could not do to such an extent without ICT. My prime research interest was seeking the secondary pupils' perceptions about learning algebra in ICT-enhanced mathematics lessons. My project evolved into three distinct stages: Stages One, Two and Three. This section details my thinking about the processes that informed research strategy at each stage which culminated into the main study. I embarked on Stage One between May and July 2011, at a mixed11-18 community college in the West Midlands, England. I conducted subsequent stages, in January and summer 2012 respectively, at a girls-only, state-run, fee-paying, boarding school in Nairobi, Kenya.

3.1.1 Target participants

There is considerably less curricular and societal pressure on learners at early secondary level, compared with older pupils, in both England and Kenya. Stage One had 34 pupils (16 boys and 18 girls), out of the 250 Year 8 (age 12- 13) pupils in this school, and two specialist teachers I will refer to as ‘Teacher 1E’ and ‘Teacher 2E’ in Section 4.1. The pupils had achieved teacher-assessed UK National Curriculum attainment levels 4 or 5, and working towards level 6. The class had “handled the basics in algebra”, including ‘Use of letters’ and ‘Substitution’. Both teachers described Year 8 as “heavy with algebra”, and welcomed more work on the topic.

The participants at Stage Two were 231girls in Form 2 (second year of secondary education) aged on average 15 years (actual 14.66, ranging from 13 to 17 years) in five classes, and the regular teachers. Pupils participated in varying degrees I describe briefly in Section 4.2. Pupil achievement in examinations varied from low to high, and they followed the spiral Kenyan mathematics curriculum (KIE, 2002). Thus, the pupils had encountered some formal algebra at primary school and also at Form 1.

Stage Three participants were pupils aged on average 14 years (actual 14.22, ranging from 12 to 16 years), and five specialist teachers of mathematics I will refer to as: ‘T1’; ‘T2’; ‘T3’; ‘T4’; ‘T5’ in Section 4.3. 270 pupils learned in five mixed-ability classes of between 53 and 56 each. According to Hernandez-Martinez and Williams (2013), all pupils in transition need sensitive induction to develop emotional intelligence and social capital to negotiate secondary education successfully. I regarded the Form 1 cohort as ‘pupils in transition’ since they were merely three-months old at secondary level, with the majority being first-time boarders.

3.1.2 Sampling

As indicated in Section 3.1.0, I conceived my research as an intervention study. One possible approach to the issue under study, *learner behaviour*, may be located within the experimental designs discussed by Hartas (2010). I was not keen on conducting an ‘experiment’ on human beings for ethical reasons. I could not justify creating randomly-assigned groups of pupils in the same class, one *experimental* the other the *control*, and my subjecting equally-deserving groups to different learning conditions for investigating the role of classroom ‘subculture’ to mediate ICT use in mathematics lessons. An experimental design would entail instigating collaborative learning by combining ‘dialogic teaching’ and *Grid Algebra* to address interest and participation as ‘new treatment’ for an *experimental* group. ‘Old treatment’ for the *control* group would consist of textbook-based teacher exposition but neither collaborative learning nor *Grid Algebra* use.

According to Ercikan and Roth (2006), it is not possible to draw random samples of pupils to participate in intervention programs in schools, a view reiterated by Hoyles et al (1994). This stance questioned the feasibility of conducting experimental designs in schools. Arguing for ethical balance, Dane (1990) highlighted researchers' obligation to offer the 'new treatment' to participants in the control group. Given my limited time in each research setting, I decided against using experimental designs. I involved several whole classes of pupils and teachers at a time. All participants received the same treatment: 'dialogic teaching' with *Grid-Algebra*-based collaborative learning during mathematics lessons. I judged the feasibility of available resources at each research stage based on my personal experience that I detailed in Section 1.4.0.

I selected participants on the basis of purposive sampling as defined by Denscombe, (2010). Teddlie and Yu (2007) wrote that this sampling technique provides greater depth and less breadth than probability sampling. I valued depth over breadth in this trade-off because I was aware of the chosen ICT tool's availability at both research sites. I qualified classes with prior encounter of algebraic concepts as appropriate for participation in this classroom-based study. This pragmatic approach (Denscombe, 2010) of opting for exploratory samples (Dane, 1990) rather than using samples statistically representative of the whole population yielded good quality research findings.

3.1.3 Starting out

My research journey commenced in a student-tutor role supporting pupils and teachers across low, middle and high-attaining sets during Years 7 to 10 mathematics lessons in England. I gained invaluable insights into the UK National curriculum. While I appreciate the hard work of the UK mathematics teachers, I was baffled by their interpretation of 'levels' (Brown et al, 1998). There was no clear consensus whether 'levels' indicated content, understanding or skills the pupils were able to demonstrate. I encountered many pupils in different schools unable to grasp the concept of 'perimeter', persistent reference to mathematical constant π as "squiggle", nor the instrument 'protractor' as an angle measurer. Of poignant interest to this thesis was 'difficulty' with using letters in algebra (Küchemann, 1981). It was commonplace to hear many secondary pupils grumbling: "why use letters in maths? That's just dumb! Maths should only have figures", reported by Brown et al (2008). Such examples crystallised my interest in developing mathematical language as stated in the curriculum for Key Stage 3:

"The language of mathematics is international. The subject transcends cultural boundaries and its importance is universally recognised."(Department for Education, 2007, p. 2)

However, robust modelling of language use for pupils through revising of previous lesson and homework activity was lacking, for reasons beyond the scope of my study. I suggest that impoverished language lay at the heart of what ails the UK secondary mathematics learning. I ask when timing is deemed *right* for pupils' mathematical language. Based on Boaler (2009), delayed skills can lead to pupils 'hitting a brick wall' at all levels of schooling. Confident language use may nurture a strong basis for understanding mathematics (Boaler and Greeno, 2000) and boost pupils' readiness to cope with university mathematics. Teachers possibly lay greater value on generating 'feel-good' factors to the detriment of rich learning experiences.

Webb and Cox (2004) highlighted the minimal impact had on established pedagogy by laptops and interactive whiteboards intended to enable student-centred learning. ICT should neither be used primarily to entertain nor distract pupils as reported by Crisan et al (2007). Despite the availability of a wide array of ICT resources in England, I observed the above practices and teachers often handling technology during mathematics lessons. The prevalent practice reflected the lament in Ofsted (2008) report about the relative decline in contribution of ICT to pupil mathematical learning, technological advancements notwithstanding. Many pupils' kinaesthetic ICT use was restricted mainly to homework activity and as occasional reward. Teachers subscribed more to managing classroom behaviour than to harnessing affordances attributed to ICT-based collaborative learning discussed by Hennessey et al (2005). However, pupils and teachers need different skills for this type of working to be effective. It can allow pupils to benefit from *social* learning in ways to develop conceptual understanding, such as:

1. the articulation of one's ideas which opens them to assessment by self and others;
2. the evolution of communicative competence, constructive argument and reasoning;
3. to gain formative feedback vital for refining and developing one's ideas;
4. the development of strategic competence in handling challenging problems;
5. learning to work *with* and learn *from* others.

Several researchers (Black et al, 2003; Juwah et al, 2004) endorsed these affordances. Rarity of the practices described above in mathematics lessons led to classrooms I experienced in England resembling those in Kenya described in Section 1.1.2. My dilemma was *where* to conduct research based on my own professional experience (see Section 1.4) and wider literature: in England, where mathematics-specific ICT resources had existed well over three decades, yet issues of formal mathematical language use and access to participants plagued my mind? Or in Kenya, where there was little in terms of 'dialogic teaching' (Kanja et al, 2001), no ICT in education, yet a wide-spread boarding set-up at secondary level promised enhanced access to participants. I considered myself as being on a 'learning journey', to discover what either worked or did not work, within real-life settings in processes I will discuss in Section 3.2.

Primary focus on pupil learning located my initial interest in 'exploratory' research (Dane, 1990). I was curious about effect of classroom interactions triggered by combining 'dialogic teaching' and *Grid Algebra* on mathematical learning. I wished to investigate whether such intervention was more likely to succeed in raising interest in algebra and pupil engagement than the prevalent textbook-based instruction. I envisioned collaborative activity enabling pupils to learn in a 'fun', relaxed manner. I hoped to improve pupils' mathematical reasoning and decision-making skills whilst freeing teachers to observe pupils learning, and then target teaching accordingly. I contend that 'formative feedback' (Juwah et al, 2004) can develop the pupils' symbolic language and algebraic thinking (Bell, 1996). My observations, and wider literature, suggested scarcity of such interactions at secondary school. I trialled, then *modelled,* an intervention to emphasise affective traits in mathematics. This study was approved by the Research Ethics Committee, Warwick Institute of Education and complied with University of Warwick Ethical Guidelines (see sample in Appendix 1). I obtained prior informed consent from participants in accordance with the British Educational Research Association (BERA) Guidelines for Educational Research (2004). I frequently consulted my supervisors (Lindsay, 2010). I will clarify ethical issues relating to research in Kenya in Section 3.2.6.

3.1.4 Research methodology

According to Denscombe (2010), a research strategy, as a *plan of action* aimed to realise clear goals, ought to be appropriate, ethical and feasible. I underscored in Section 2.1.0 that Mercer (1994) deemed sociocultural theory of learning as better suited to 'naturalistic' methodology. For Lincoln and Guba (1985), naturalistic study involved studying phenomena in the natural setting of activity. Norris and Walker (2005) emphasised the researcher directly experiencing people *in situ* and staying with them in some role acceptable to those being studied. Denzin (1971) described naturalistic inquiry as entailing one's commitment to empirical research by active immersion in others' worlds to investigate human behaviours, languages, attitudes and feelings. Researcher's conduct in interaction during the study effectively reflects their unique epistemological stance and becomes part of the interpretive frame (Norris and Walker, 2005). Effective research implementation entailed my becoming embedded in classrooms. I assumed a 'facilitator' role and introduced all participants to the ICT tool.

Ruthven and Hennessey (2002) advanced the need for naturalistic inquiry to provide analyses and perceptions of ICT tools use in mathematics education by participants across different settings. The researchers' laudable study conducted in England focused on teachers' views of computer-based tools. One example of naturalistic study is the *Thinking Together* programme involving Year 5 (aged 9-10 years) children by Mercer and Sams (2006). Pupils were taught language use as a reasoning tool for solving mathematical problems. Another example is the investigation of Year 5 and 6 pupils' views concerning the effect of introducing interactive whiteboards into some primary schools in England by Wall et al (2005). My interest to merge the cognitive and affective aspects of mathematics influenced selective thinking from various research traditions: 'ethnography'; 'design study'; 'action research'; 'case study'; 'surveys'.

Norris and Walker (2005) regarded commitment to detailed description of behaviour arising from direct contact with participants as 'ethnography'. According to Goldbart and Hustler (2005), ethnographers understood by living in the worlds constructed and utilised by people whilst maintaining cultural meanings which inform actions. They defined 'culture' as what is *enacted*; an 'ethnographic' focus provided holistic approach for investigators to engage with cultural meanings and processes within the research strategy. For Jones and Somekh (2005), duties assigned to researchers by the group posed a possible danger of distraction. However, becoming absorbed in mathematics classroom *subcultures* afforded unique insights into real-time social interactions. Direct contact allowed me to obtain data about what the participants were thinking and doing as discussed by Eisenhart (1988). As the human research instrument (Lincoln and Guba, 1985), I studied actively multiple realities constructed in the lessons. My intervention departed from 'ethnography' by altering existing *subcultural* norms. I instigated elements I deemed crucial in contributing to successful mathematical learning:

- Offering pupils opportunities to articulate their mathematical ideas;
- Confidence in using mathematical language while expressing their ideas;
- Social interactions within a supportive classroom 'subculture';
- Blending non-ICT and mathematics-specific ICT resources during the lessons.

Following Alexander (2008), *new* classroom 'subculture' (Bauersfeld, 1998) was realised in increased *pupil talk* as something pupils considered valuable in learning activity. While I may have subscribed to thinking that informs naturalistic inquiry, I underscore specific limitations to my research being considered 'naturalistic'. Learners were at various points relocated from classroom settings to computer laboratories within their schools. My 'naturalistic' claim lies partly in the fact that most research activity occurred during time-tabled mathematics lessons.

Lerman (2001) described learning as the outcome of focus on work-related practices. Cobb, Confrey, diSessa, Lehrer and Schauble (2003) advanced ***design experiments*** geared towards:

"both 'engineering' particular forms of learning and systematically studying those forms of learning within the context defined by the means supporting them, with the designed context subject to test and revisions" (p.9).

A 'design study' tradition conducted research to develop theories targeting domain-specific learning processes. Cobb and Steffe (1983) argued that the discussions between teachers and researchers complimented knowledge of 'what' pupils learned. Steffe and Thompson (2000) resonated by stating that researchers obtained first-hand experience of pupils actually doing mathematics through teaching. The video-recording of social interactions served as source of data. Play-back of critical episodes stimulated the reflective discussions with the teachers for retrospective analyses. Multiple observers' views of learning behaviour fed into iterations of activity. These discussions reinforced a view by Norris and Walker (2005), that any behaviour conceived as socially constructed may not always be what it seems. Cobb et al (2003) valued the situated nature of retrospective analyses for linking observed learning to its organisational and support structures. Nevertheless, Engeström (2011) critiqued 'design study' researchers for planning interventions in social settings whilst failing to take into account the participants developing resistance. Sutherland et al (2009) described video data as crucial to professional development. Yet, they noted grudging teacher acceptance of camera use in lessons. Based on Hartas (2010), I adopted 'design study' thinking as far as being: ***multi-levelled***, by combining cognitive and affective aspects in learning and teaching of algebra; ***process-focused*,** through monitoring the participants' learning by examining patterns of learning behaviour and effects of instructional artefacts on learners' thinking and reasoning; ***utility-oriented***, with supporting effective ICT use to alter classroom learning. Following Gorard with Taylor (2004), research intervention created an enabling learning environment in lessons. To evoke data triangulation, argued by Cohen et al (2011) and

Denscombe (2010), I held retrospective discussions about significant pupils' mathematical behaviour with teachers. I valued weighing my observations and interpretations against teachers' perceptions whilst being acutely aware of pressing demands on teachers by huge workloads. I conceived the participants' 'resistance' as *learner agency* and individuals exercising their right to participate or not in this research. I equated 'disciplinary' (Pickering, 1995) to 'proxy' agency, defined by Bandura (2001) as participants relying on other *agents* in contexts to secure desired outcomes. Unlike 'design studies', I instigated change in learning processes across settings by involving different participants described in Section 3.1.1. I shelved video-recording based on my negative experience of participants' inhibitions due to such intrusion (Denscombe, 2010). I acknowledge a weakness of relying on human memory albeit backed by physical artefacts (written work and my notes) to stimulate recall of events.

Denscombe (2010) described 'hands-on', small-scale and applied projects conducted to solve social problems as 'action research' (Lewin, 1946). This study was not classic action research (Johnston-Wilder, 2010) undertaken by a practitioner in their usual work context. My role in this research focused on issues of classroom practice labelled by Noffke and Somekh (2005) as "first order" and clearly distinguished from "second order" concerned with improving my practice as 'facilitator' (p. 90). Following Lewin (1988), the impetus for research was change in mathematics classrooms through deepening the participants' understanding of ICT use and developing strategies to achieve improved learning behaviour. I linked theory to practice (Noffke and Somekh, 2005) and conducted this study from within lessons; research became an integral part of participants' practice. Denscombe (2010) emphasised that while research may instigate practitioners' thinking about their established practice, mere reflection does not qualify as 'action research'. Although certain teachers reflected on pedagogical practice, this argument restricted my identifying with the 'action research' tradition.

The importance of the context and observing learning in real-life may have suggested a 'case study' research strategy (Norris and Walker, 2005). Cohen et al (2011) considered context a powerful determinant of both cause and effect. According to Stark and Torrance (2005), 'case study' researchers lay more emphasis on 'objective' investigation of 'the case'. Yet, I placed considerable value on the respondents' views, especially pupils' perceptions of their learning behaviour. I enlisted thinking that informs a 'case study' tradition: using direct observations, interviews and analyses of physical artefacts (Yin, 2009). I limited 'case study' thinking to preferring *depth* to *coverage* and more *face-to-face* to remote research methods. Descriptive accounts from research settings intended to allow the reader to identify with aspects of their personal experience and judge the quality of evidence as discussed by Stake (1994). I failed to actualise teachers' 'reconnaissance' with *Grid Algebra* as a predetermined event proposed by Lewin (1988). Valuing formative feedback in research design enhanced *learner agency*. I adapted to flow of unravelling events as the study progressed to allow crucial insights into the dialectic learning processes at work. Pupils' learning resulted in some teacher 'learning', something I believed to be achievable within the scope of this research.

Hoyles et al (1994) employed a multisite case-study design to investigate pupils aged 9-12 years learning mathematics in groups on computers without their teachers. Each group had three boys and three girls of mixed teacher-assessed attainment levels. The research sessions, each roughly 150 minutes, involved both computer-based and pen-and paper tasks in three areas: *Logo* programming, geometry and data handling. The data collection methods included pupils' written work, rigorous observations and video-recordings of task-based interactions, talking to teachers, group interviews and two pupil questionnaires. Constructivism informed this study as the theoretical perspective. It directed researchers' focus on cognitive aspects of mathematics. Researchers noted that other 'learning' seemed to occur in the collaborative context which determined pupils' progress as evidenced by data from both pupils and teachers. Their observations reiterated a view by Lerman (2001) about social constructivism not accounting for social interaction yet valuing individuals' development. Hoyles et al (1994) concluded by underlining task design and group work with computers as facilitative factors for *talking* thus developing *language* and *actions* to construct shared understanding. Their report emphasised diminished benefit from either form of involvement when taken singularly. As one researcher recorded systematic observations of task-based interactions about the learning outcomes, the second researcher took ethnographic notes of pupils' motivation and involvement. I suggest that Hoyles et al (1994) overlooked information concerning affective dimension to account for pupils' emotional, motivational and involvement in learning. In Section 3.1.5.6, I will detail using questionnaires as 'survey' research (Dane, 1990) to gather such information.

My studying computer-based learning of algebra required a strategy that would capture the complexity of interactions between elements in the learning context. In seeking to establish a balanced position from which to conduct my investigation, I finally settled on using a 'family approach' discussed by Bryman (2001). I drew on several research strategies that I regarded as 'fit for purpose' and explained by several researchers (Denscombe, 2010; Creswell, 2009; Morse, 1991). Holloway and Todres (2003) emphasised the notion of 'appropriateness' of method. They suggested that mixing approaches can lead to greater clarity about the problem under investigation and questions posed. I planned to devise an optimal typology (Leech and Onwuegbuzie, 2009) to address my research aims (see Section 1.1.0), provoking sustainable and transformative pupil learning and teacher 'learning' processes in ICT-enhanced contexts. It involved my assessing conditions in each site. I employed an eclectic research strategy for comprehensive approach to investigate pupils learning algebra with ICT at secondary school within an interpretive framework (Creswell, 2009). It allowed me to explore a wider scope of social, cultural and linguistic dimensions to the research. Different strategies applied in this complex research design (Bryman, 2001) limited interaction between datasets during data collection (Morse, 1991). I mixed study findings at the interpretation stage (Bryman, 2006). Eisenhart (1988) discussed how interpretivist research seeks to allow us to make sense of the mathematical activity from the participants' perspectives through first-hand involvement as an 'insider' yet reflecting upon it as an 'outsider'. Denscombe (2002) and Norris and Walker (2005) explained how these studies highlight ways people shape the society, with researchers an integral part of the world they study.

3.1.5 Data collection methods

3.1.5.1 Pupils' written work

According to Yin (2009), worksheets, as physical or cultural artefacts, provided evidence of work done in addition to ascertaining the nature of pupils' actual use of ICT. Lerman (2001) argued for the role of language and other cultural tools in mediating human consciousness. Hoyles et al (1994) stated that written tests provided best evidence of pupil learning. Lobato (2003) explained that 'transfer' of learning was an active process distributed across mental, material, social and cultural planes, including language use and focusing phenomena. Yet the weakness of physical artefacts as stand-alone instruments was underlined by Yin (2009). He advocated for their prudent use in tandem with corroborative research methods. Sutherland et al (2009) developed diagnostic instruments to assess subject-based learning outcomes in the Interactive Education Project. The researchers employed questionnaires, document analyses, focus group interviews, selected individual interviews, case studies, observations and video-records of lessons as data collection techniques. Hoyles et al (1994) endorsed pupils' written work in tests for pragmatic reasons, including project costs and demands on the participants' time. They noted the inherent weakness with written work's focus on cognition in learning.

The UK National curriculum listed ability to communicate effectively as 'competence'. Also, 'communicating mathematically' was a crucial secondary education objective in Kenya (KIE, 2002). Both curricula endorsed developing pupils' ability to convey mathematical meanings when writing and speaking. Usual assessment of pupil cognition in mathematics is based on written rather than spoken work. I deemed it necessary to direct my attention to written tasks. I focused on the pupils' conceptual algebraic understanding indicated by mathematical language use. I will present in Chapter 4 the role symbolic language had in algebraic thinking, revealed by pupils' awareness of underlying structures in formal notation.

I planned for my research study to commence with pupils taking a diagnostic exercise at each stage. This exercise aimed to establish pupils' understanding of algebraic concepts and target the intervention accordingly. Owing to my failure to locate a suitable standardised test for use at Stage One, the research design incorporated the following:

I) reliance on teacher assessment of the pupils;

II) one short, teacher-approved diagnostic exercise derived from the Mathematics Project of 'Concepts in Secondary Mathematics and Science' (CSMS) research programme (1974-79) conducted in UK schools.

In the course of this study, I developed and administered a series of worksheets selected from *Grid Algebra*-generated tasks and from past-GCSE paper questions. I merged the pre-study diagnostic exercise with items in worksheets and two puzzles from software-generated tasks in Stage One. The merged material, which I refer to as 'Baseline' (see Appendix 10), became the pre-study diagnostic instrument administered to pupils in Kenya at Stages Two and Three. This instrument gauged the pupils' existing algebraic understanding and strategic competence (Kilpatrick et al, 2001). I designed some worksheets related to eight software-generated tasks (see Section 1.5.3); I considered their alignment to specific learning objectives (KIE, 2002). I employed written tasks to monitor developing mathematical language revealed as acceptance of formal algebraic notation as I report in Section 4.4.4. I collected and marked pupils' scripts at the three stages, thus reducing the load on teachers. My having no prior pupil acquaintance maintained objectivity in marking. Data from written work provided solid evidence of pupil learning. Pupils and teachers at Stage Three highlighted remarkable pupil progress in teacher-assessed examinations; I will discuss this in Section 4.4.3. Based on the pupils' 'met-befores'

(Tall, 2004) in the 'Baseline' assessment, I determined the appropriate level at which to pitch my introduction of software-generated tasks, and monitored pupils' symbolic language use in written work alongside observations, interviews and questionnaires as shown in Table 3.1.

3.1.5.2 Participant observation

Denscombe (2010) described participant observation as the observer participating in the daily life of people under study, listening to what is said, and questioning this observed behaviour, over extended periods of time. Norris and Walker (2005) associated this method to 'ethnography'. Cohen et al (2011) argued that it facilitates the generation of 'thick descriptions' and data that is 'strong in reality'. Its suitability for real-time studying of enacted learning behaviour rather than given accounts influenced my using it to examine classroom 'subculture' in mathematics lessons. Eisenhart (1988) saw the enabling of access to vital insights on non-verbal behaviour through first-hand experience of what pupils and teachers were actually doing and thinking as an advantage of using the method in mathematics classrooms. Denscombe (2010) argued that preserving the naturalness of the setting, with participants oblivious to the researcher, gained insights, and the context-enhanced ecological validity of data contributed to accurate explanations and interpretation of events. While a researcher gathers data on enacted actions as field notes, this method enables the switching of researcher roles on a continuum shown in Figure. 3.1.

OUTSIDER				INSIDER
Detached as observer	Observer	Observer as participant	Participant	Complete participant

Figure 3.1 Roles for Observers (Adapted from Cohen, Manion and Morrison, 2011, p. 233)

Yin (2009) and Denscombe (2010) listed potential dominance of researcher participation over observer role, bias and immersion in group advocacy as factors that might influence findings, thus potentially reduced data reliability. Yin (2009) underlined negotiating access to research sites and researcher's ability to perceive reality as an insider as one weakness of this method. Cohen et al (2011) proposed using participant observation with methods such as interviews to elicit participants' views of events and accounting for actions. While participant observation targets *behaviour* which participants can explain, interviews address *forms of talk* as actions narrated and performed within social life (Goldbart and Hustler, 2005).

I intended to support the redistribution of accountability and responsibility in lessons, linking research to practice (Walshaw, 2013) by demonstrating productive use of tools, linking pupil learning to teacher development (Guskey, 2002). An enabling learning context would allow pupils to narrate algebraic reasoning particularly to their peers in ICT-enhanced sessions. At all three research stages, I introduced participants to *Grid Algebra*, and provided assistance when requested in mathematical learning activity. I designed pen-and-paper worksheets; the teachers commented on task suitability prior to their distribution to pupils. I recorded my observations of events and reflections in a research diary as the study progressed.

Pupil actions allowed me, and teachers at Stages One and Three, to attend to pupils' learning behaviour. Full participation allowed me to share the groups' learning experiences, and led to building trust and acceptance. I switched from being a completely passive observer to being a completely active participant as the situation demanded (Swain, 2006). Stage Two presented an opportunity for me to interact with pupils more than Stage One. Thus, I gradually became aware of the need for more structure indicated by 'ground rules' (Edwards and Mercer, 1987), thus enhancing the pupils' interest, involvement and engagement in mathematics as was argued by Ruthven et al (2011); I will discuss this in Section 3.2.2. As 'facilitator', I established rules of engagement in ICT-enhanced activity (see Section 3.2.3), and revised research questions (see Section 3.2.5) at Stage Three. Some teacher actions and accounts of pupil learning at Stages One and Two necessitated my paying increased attention to teacher 'learning' behaviour.

At Stage One, I attended all mathematics lessons for the Year8 group, and became a familiar fixture in their learning context. I established informal relationships with participants to help easing possible reactions effected by my being non-Western. Participant observation in the Kenyan phase augmented in line with unfolding developments in the country's education sector (see Sections 1.1.2 and 1.4.0). In the trade-off, I was open to proposals of alternating teaching and observation roles with regular teachers, hence my role as a teacher-researcher (T-R). Consequently, I experienced enhanced difficulty of balancing participant and observer roles. The more I participated, the less I observed. The more I veered towards 'teaching', the less I attended to learning behaviour. The tension between research and teaching hindered my taking sufficient notes and raising appropriate questions about events occurring in learning activity. To combat this weakness, argued by Yin (2009), I wrote as many notes as possible at the end of each lesson as Jones and Somekh (2005) advised. It may raise some doubts about accuracy of notes as I searched deeply to authenticate them. In relying on memory, I incurred reduced reliability, hence needed data triangulation (Denscombe, 2010).

3.1.5.3 Direct observation

The natural setting of time-tabled mathematics lessons enabled direct observation of relevant learning behaviour and contextual conditions as Yin (2009) explained. He considered direct observations as useful for providing additional information about phenomena being studied. According to Jones and Somekh (2005), observers notice 'new' behaviour patterns by being present in situations and gathering sensory data. They stressed the importance of recording one's impressions of events for subsequent scrutiny and analysis. They listed complexity of human behaviour, compiling full records of actions and researcher's subjectivity as weaknesses with observations. Goldbart and Hustler (2005) underlined the tension arising from actively engaging in data collection whilst interpreting meanings of observed behaviour. Flick (1998) remarked that research may disrupt routine without offering real benefit to systems such as classroom 'subculture'. Cohen et al (2011) described structured observation as the systematic generation of numerical data which may overlook context-specificity of behaviour. Jones and Somekh (2005) highlighted inevitable effects observers have on the observed. They argued that while audio and video-recording provide a permanent record, partial representations of events lack 'reflexivity' (Denscombe, 2002) afforded by one's "self". Intrusiveness of video-recording in research settings and time-consuming transcription of audio and video data prior to analysis are weaknesses, as argued by Denscombe (2010).

I examined classroom 'subculture' mediated by use of *Grid Algebra* in mathematics lessons. 'Ethnographic' thinking in research influenced my decision to use unstructured observations (Cohen et al, 2011), which are considered highly participatory by Jones and Somekh (2005). Of key importance to me was compiling a complete record: the learning resources available, number of participants present, the norms structuring learning activity, and my own conduct. I was careful to strike a delicate balance between being intrusive and maintaining a degree of detachment. I hoped to identify pupils whose opinions of the intervention would supplement my notes and teachers' views on learning behaviour. Using unstructured observations made fewer demands on teachers who were attending to pupils learning in ICT-enhanced sessions.

Through practice, I had learned to position myself, usually at the back of the room, where my presence would be least obtrusive. My attire and note-taking drew little attention. I avoided making eye-contact with participants, and I moved around only when I believed movement would least distract pupils and teachers. I collected a great deal of data *in situ* on: actual *Grid Algebra* use; verbal, enacted or written learning behaviour; and the problems encountered by participants. From this record, I composed rich descriptions of events as they occurred; I will discuss this in Section 4.4.5. Working as a single researcher, direct observations enabled me to interpret mathematical meanings in social interactions inherent in classroom 'subculture' (Yin, 2009). I hoped teachers would embrace supportive roles and encourage pupils to justify their reasoning to either peers or adults present. Computers provided 'windows' into pupils' thinking (Noss and Hoyles, 1996) by displaying mathematical ideas on screens. I intended to provoke sustainable transformative learning processes. I hardly expected the participants' absolute acceptance of this intervention. Since building on Stage One findings hinged on enlisting teacher support at Stage Three, I became wary of direct observations as follows.

Hoyles et al (1994) argued that computers supported collaborative mathematical learning; the classroom practices became increasingly 'dialogic' (Ruthven et al, 2011). I paid attention to the extent to which classroom interactions were *dialogic*: reciprocal, collective, supportive, cumulative and purposeful (Alexander, 2008). Crotty (1998) argued, "only through dialogue can one be aware of the perceptions, feelings and attitudes of others and interpret meanings and intent" (p.75). I will report in Chapter 4 some invaluable cognitive and affective insights gained in both non-ICT and ICT-enhanced sessions using unstructured observations at Stage One. Teachers' lack of enthusiasm for pupil-centred computer-based collaborative learning was apparent to me. I observed five non-ICT lessons at Stage Two which afforded insights into prevalent verbal and non-verbal learning behaviour inherent in mathematics classroom 'subculture'. Barnes (2000) suggested that some learners may become excitable while others become restrained owing to researcher's presence in classroom context. Unfortunately, I was dismayed by some teachers' reaction to direct observations. They appeared conscious of my presence and may have been inhibited. Negative teacher reaction, overlapping lessons, large number of participants and my augmented role (see Section 1.4.0) altered the research design at Stage Three. I limited research activity to time-tabled and make-up ICT-enhanced sessions for pragmatic reasons which I elaborate in Section 3.2.3. I refrained from observing non-ICT lessons to manage, and prevent possible distortion to, research. I witnessed explicit distortion caused by teacher's reaction to observers in an ICT-enhanced session (see Section 4.4.6.5).

Some aspects of teacher behaviour were exaggerated and others downplayed. Lesson delivery seemed executed for the observers' benefit rather than focused on pupils' learning. Teachers appeared not to act interact with their pupils in productive ways. Barnes (2000) explained that such teachers may have felt constrained in their actions driven by a desire to *assist* research. She underscored the likelihood that observers' presence may have altered the fundamental nature of the teacher-pupil interactions.

3.1.5.4 Retrospective discussions

Eisenhart (1988) regarded 'ethnographic interviews' as useful complements to observational data. Hoyles et al (1994) conducted retrospective discussions to inform about pupils as group members and individual learners, to complement observed behaviour. Sutherland et al (2009) employed the technique when reviewing video data with teachers.

My intention to elicit participants' perspectives on the potential for classroom 'subculture' to mediate *Grid Algebra* use in mathematics lessons informed my engagement in learning-based discussions. I was able to use my full description of participants' learning behaviour in ICT-enhanced context to access corroborating evidence in subsequent discussions with learners. In this way, I was able to obtain unique personal insights into learning behaviour, and additional crucial information, from participants affected by the phenomena under study; I will discuss this in Section 4.4.3. I was aware that discussions would provide the best source of evidence concerning positive and negative emotions, motivation and involvement as affective traits of learning behaviour. Informal conversations developed as appropriate when events of interest occurred. 'Ethnographic interviews' were particularly useful for guiding my discussions with teachers based on pupils' written work. I shared my interpretation of pupil learning behaviour with teachers and reflected on these discussions. I presented marked scripts of pupils' work to teachers. We deliberated on specific learning behaviour of concern, shared our interpretation of pupils' algebraic constructions and used this information to inform subsequent study iterations. The discussions generated cooperative relations with participants. During Stage Three, I insisted on holding these discussions to keep teachers abreast with pupils' progress whenever opportunities arose. I used ethnographic interviews with pupils as follows.

From an affective stance of mathematics learning, computer confidence can influence pupil motivation and engagement as argued by Galbraith and Haines (1998). Following Boaler and Greeno (2000), I arranged 'extra' ICT-enhanced sessions for a considerable number of pupils who were unable to participate fully in computer-based tasks at Stage Three. The mathematics teachers and I had noticed pupils exhibiting phobic behaviour during ICT-enhanced sessions owing to their low computer confidence. The pupils lacked computing skills which distracted attention from learning algebra. Some of them were categorical about never having a chance to operate computers. Mathematics teachers were hard pressed for time since their ‘teaching’ focused on cognition in pupils’ mathematical *understanding*, as was reported by Hennessey et al (2005). Thus, I organised access to the computer laboratory for those pupils possessing resolve and willingness to learn basic computing skills. The ‘extra’ sessions took place over lunch breaks, during evenings after school’s time-tabled lessons, and occasionally at weekends. I attended to varied groups of pupils keen on spending time to upgrade skills. The pupils concentrated on individual learning needs, computing and written, they had identified. Additional access to computers was taken up by some 15 to 20 pupils from across the classes at any given session. I intervened as and when required with the HOD-Mathematics usually in the vicinity. Pupils were free to choose learning tasks to practice further; most worked in pairs. They indicated developing ‘resilient’ behaviour (Hernandez-Martinez and Williams, 2013).

The ‘extra’ sessions facilitated my learning more from the pupils. Some confided in me how their pleas for assistance went unheeded during Computer Studies lessons despite requesting for assistance. Teachers in this department apparently assumed every pupil in the cohort had the basic computing skills. Yet, it was patently clear many of them never had prior access to such facilities as I will elaborate in Section 3.2.3 and Chapter 4. Unfortunately, the perception that some teachers would regard disadvantaged pupils in their classes as not fitting the image of the school was felt by these pupils. Descriptions of some teachers given by Stage Three pupils echoed a study by Bottrell (2009) involving 13- 24 year old girls in Sydney, Australia. Bottrell explained that pupils are clear about teachers who do not care or give more attention to higher achievers. She observed such pupils develop behaviours to reject the ‘labels’ given to them by teachers and work hard to prove their worth. Some Stage Three pupils voluntarily concentrated on those areas and skills that they recognised needed more effort on their part. They were neither put off from learning algebra nor in despair at their lack of success. These pupils showed intent to acquire relevant educational and social ‘capital’ (Hernandez-Martinez and Williams, 2013) to negotiate transition to secondary school with surprising success. They displayed developing ‘mathematical resilience’ (Johnston-Wilder and Lee, 2010b) through conscious reflection. Pupils felt able to assume control of their learning and persevered when faced with adversity, easing pupil participation and inclusion in 'learning community' as was argued by Sfard (2001) and (Boaler and Greeno, 2000).

3.1.5.5 Interviews

Yin (2009) described interviews as guided conversations with key participants. This research method allows researchers simultaneously to attend to their critical lines of inquiry and to pose related questions in a friendly albeit unbiased manner. Participants provide personal opinions about events, facts or issues under investigation. An interview can afford access to privileged and sensitive information. Denscombe (2010) argued that direct contact during interviewing increased validity of data; data can be checked for accuracy and relevance with appropriate prompts and probes guiding the conversations. However, Yin (2009) stated that interpersonal influence affected reliability of data and advocated cautious use of interviews alongside other methods. Dane (1990) viewed the flexibility of focused interviews as both an advantage and a weakness. Denscombe (2010) described interviews as time-consuming due to the additional transcribing and coding tasks which render data analysis considerably difficult.

I intended to elicit the extent of participants' awareness of classroom 'subculture' mediating *Grid Algebra* use in mathematics lessons. My plan was to conduct pupil interviews in groups of up to four within the learning context of mathematical activity to ensure pupils felt at ease (Denscombe, 2010). I planned on conducting individual teacher interviews at venues chosen by respondents. Topics of conversations aimed to tease out teachers' perceptions about policy and management of ICT in mathematics education, pupils' learning with *Grid Algebra* and teachers' own 'learning'. Pupil and teacher interviews each had a schedule (see Appendix 9) with main questions backed by sub-questions in pursuit of specific lines of inquiry of interest. Respondents viewed the preliminary agenda in advance. I requested the individual's permission to audio-record interviews for these conversations to flow naturally; this enabled permanent records. I employed focused interviews (Dane, 1990) with some participants for short periods of time (Yin, 2009) at the end of each of the three stages. Each interview lasted between 10 and 30 minutes. I transcribed audio recordings fully to retain the extent of dialogue as much as possible. At Stage One, I obtained no information from a pair of pupils who degenerated into a fit of giggles, a targeted pupil failing to turn up for school, and another shying away, saying very little on interview day. I conducted one successful individual pupil interview. I conducted two individual and three group interviews to further probe pupil views collected in questionnaires at Stage Two. I requested pupils to speak in turns whilst I noted the identity of respondents to facilitate my accurate transcriptions of group interviews (Denscombe, 2010). At Stage Three, I shelved focused individual pupil interviews and relied on the wealth of participants' insights from ethnographic interviews, pupil questionnaires, and focused individual teacher interviews, due to time constraints (see Section 3.2.4). I conducted one teacher interview at Stage One, none at Stage Two, and three at Stage Three on the basis of teacher availability.

3.1.5.6 Questionnaires

Denscombe (2010) described questionnaires as an economical method for collecting facts and opinions directly from large groups of people based on researcher's knowledge of contexts. He argued for group-administration with 30 or more respondents writing their information in response to questions posed. Dane (1990) argued how self-reporting by respondents can limit one's ability to verify the data. Between 230 and 270 pupil participants in the Kenyan phase, and Stage One results (see Section 4.1), were factors influencing my use of questionnaires to collect pupil background information and satisfaction with research intervention (see Section 4.4.5). Classroom 'subculture', seen in unity of body and mind (Bauersfeld, 1998) defined by both 'talk' and context, mediated *Grid Algebra* use. In light of factors (teaching and learning, classroom context and curriculum content) addressed in International Mathematics Surveys (Brown, 1996), 'affect' was deemed more likely to influence pupil learning and achievement.

The mathematics-related beliefs questionnaire (MRBQ) was developed at the University of Leuven in Belgium and used with Flemish students. An adaptation of the MRBQ instrument was used more recently by Diego-Mantecón et al (2007) with 12 and 15 year olds in England and Spain. This instrument contained 73 items, asked on a six-point Likert scale that ranged from 'Agree Strongly' to 'Disagree Strongly'; they included:

a) perceptions of the classroom context, especially the teacher's role;
b) ability to succeed in mathematics;
c) the global significance and relevance of mathematics to each pupil at a personal level;
d) personal access to mathematical concepts;
e) the nature of mathematics as a body of knowledge accessible only to some.

I hoped the responses data would help contextualise the participating pupils' pre-study stance towards learning mathematics. However, the following two items, related to learning context, raised an issue of MBRQ's ecological validity:

‘My teacher puts mathematics posters or problems on our classroom walls’

‘My teacher puts examples of our work on our classroom walls’

Such practices are rare in the secondary school classroom context in Kenya. Although I was apprehensive about the relevance of these items for respondents, I administered the MRBQ pre-study to pupils in its entirety at Stages Two and Three. I chose not to exclude these items from the instrument because I was uncertain of the effect such exclusion would have on the MBRQ’s fidelity; I will detail addressing my concerns in Section 3.1.6. Respondent tendency to place questionnaire items into personal context raises issues about the pupils’ beliefs that I elicited. I acknowledge difficulty of knowing precisely what interpretation pupils assigned to words in MRBQ items given the learning context at Stage Three. I question meanings pupils attached to group working, class discussions, and relevance of mathematics. Pupils possibly saw mathematics as important for the sake of qualifications and future aspirations.

I designed a second questionnaire which revisited the ethical principle of informed consent. I assured confidentiality and anonymity on its face sheet (see Appendix 13). I requested the absence of the regular teachers during data collection to offer pupils some privacy. Following Denscombe (2010), I group-administered the pupil questionnaire post-study in regular mathematics rooms rather than in computer laboratory to differentiate learning activity from research. Every item contained clear instructions for completion. Items 1 and 2 were closed items concerning age and prior primary schooling and computing background. Items 3 and 4 collected the pupils’ perspectives about their learning behaviour in *Grid Algebra*-enhanced mathematics lessons. Each began with a closed question followed by space for an open-ended response. The questionnaire served to gather succinct information from 270 pupils for describing the learning context. Also, written textual information required no subsequent transcription.

Table 3.1 details for the reader data collection methods I have included in Section 3.1.5 to illuminate decisions at each research stage as an ‘audit trail’ (Lincoln and Guba, 1985).

Research Method	Research Stage		
	One: Pilot Study-UK	Two: Pilot Study-Kenya	Three: Main Study-Kenya
Pupils' written work			
- Pre-study diagnostic	May 2011	January 2012	May 2012
- Worksheets	✓	✓	✓
- Examination results	July 2011		March-July 2012
Direct observation of classroom 'subculture'	✓	✓	
Participant observation in learning activity	✓	✓	✓
Ethnographic interviews with pupils	✓	✓	✓
Retrospective discussions with teachers	✓	✓	✓
Individual interviews with targeted pupils	✓	✓	
Focused group interviews with pupils		✓	
Individual interviews with teachers	✓		✓
Questionnaire to pupils on mathematics-related beliefs		✓	✓
Questionnaire to pupils on classroom 'subculture' mediating *Grid Algebra* use		✓	✓

Table 3.1: Summary of data collection methods used within stages

3.1.6 Data analysis

Pre-coded responses to MBRQ were quantitatively analysed using the Statistical Package for the Social Sciences (SPSS) software version 20. I collected 230 responses at Stage Two and 270 responses at Stage Three for analysis. In addressing the ecological validity of the MRBQ I raised in Section 3.1.5.6, 98% of Stage Three pupils (267) responded with either “Disagree Strongly” (69%; 185) or “Disagree” (30%; 82): three pupils did not provide responses to the two items. Overall, this result offered some measure of confidence in the degree of honesty in pupils’ responses to the MRBQ items. Further analysis to measure of reliability of the pupils’ data revealed the Cronbach Alpha (α) level of 0.775 for all 73 items, and 0.778 when the two items were removed. A level between 0.8 and 1 (perfect reliability) is deemed acceptable: the α-level appeared to vindicate my decision not to exclude these items from the instrument. The increment to the reliability measure caused by removal of the two items in analysis is hardly significant. 230 responses collected at Stage Two yielded a level of 0.797 using all 73 items, and 0.800 with the removal of these two items. Despite the reliability analysis excluding 58 respondents (due to non-response of some items) it still met an acceptable α-level. Descriptive analysis provided some insight into the pupils' mathematics-related beliefs; I will discuss these in Chapters 4 and 5. I selected items containing specific words directly linked to certain aspects of interest to my thesis, including: *understand*, *discussion*, *mistakes*, and *quit*.. Consistency of responses enabled me to contextualize pupils' general perceptions of learning mathematics.

I formulated a sorting process for the wide array of qualitative data obtained from the open-ended questions. I put these together with transcriptions of three individual teacher interviews conducted at Stage Three. Following Wengraf (2001), I deconstructed written texts into units using similar words. I counted these units which I tabulated. I explored the units for similar meanings to identify categories of concepts on the basis of school algebra and ICT literature reviewed in Chapter **2**. I classified the categories into themes I will present in Chapter **4**.

3.1.7 Reliability and validity

To ensure the internal validity of research findings, I upheld triangulation by collecting data using various methods and from different sources. Cohen et al (2011) defined triangulation as the use of two or more data collection methods to study an aspect of human behaviour from alternative viewpoints. Complementary data aimed at increasing confidence in study findings and providing holistic insight into learning behaviour, as argued by Denscombe (2010). Study findings are triangulated to the extent that results are consistent across different data sources and perspectives, a view shared by Morse (1991). A mixed-method strategy has the potential to capture unique features instrumental to understanding a given situation that may well be lost in large-scale research. In this report, I present a description of events in research settings followed by analysis of the findings grounded in literature. Stake (1994) argued for valuing 'naturalistic generalisation' on the basis of sound evidence as generalising from 'the case' rather than from a statistically representative sample of the whole population. I acknowledge the sampling technique limits the generalisability of any claims I make from my findings.

I unilaterally selected the topic of study and the inclusion or omission of material in the final report. For this reason, the results may not be generalisable, and are difficult to cross-check since they are biased, selective, and personal, as Cohen et al (2011) explained. In resonance, Denscombe (2010) described subjectivity influenced by the personal nature of new learning experiences for participants or self-interest fuelling tendency to give favourable accounts as threats to reliability. Distorted impressions of observed events based on selective recall may contribute to modified accounts in interviews to ward off emotional aspects. Nevertheless, I checked my interpretation of the observed learning behaviour against participants' meanings of events and physical artefacts. Kvale (1996) noted that prolonged researcher engagement in the settings with keen and persistent observation and note-taking can improve the validity of findings, hence increase generalisability of results. According to Kvale, it is usual in this kind of work to talk about 'trustworthiness' and 'credibility' of research findings.

3.2 Effecting change in mathematics classroom 'subculture'

3.2.0 Coordinating research intervention

Digital technologies are becoming established learning tools in education. The mere presence of ICT should not be mistaken as the panacea guaranteed to cure all educational problems. I reflected on thet statement 'computers cannot produce "good learning" but children can do "good" learning with computers' (Harel and Papert, 1991). I facilitated adding *Grid Algebra* on computers to secondary mathematics classroom contexts. This research advocated for change in classroom 'subculture' to enable pupils as active constructors of mathematical knowledge to engage in realigning conceptual understanding in algebra by providing them with tools to think with:

- language, spoken and written;
- social interaction, with peers and adults;
- content, usually in printed textbooks;
- computers, linking alternative representations.

I recognised that such change required the restructuring of classroom practice that involved more responsibility and accountability in learning activity for pupils and supportive roles for teachers. My study evolved into three research stages I explain as follows.

3.2.1 Implications of Stage One: Pilot Study-UK

Stage One illuminated for me the severity of the problem for those pupils disaffected with learning algebra in secondary mathematics classrooms, discussed by Ruthven et al (2011);

see Section 1.3.2. I initiated a formal introductory meeting with both teachers in the school's staffroom. Neither had seen nor used *Grid Algebra* software. Its withdrawal from the school's system suggested that the resource may have been considered *inappropriate* for learning algebra. The Mathematics department had relinquished a computer suite within three years of use with a previous Year 8 group (see Section 1.4.1). This development suggested under-use of the facilities. However, the software was located in the departmental cupboard and re-loaded onto computers prior to this study. Teachers 1E and 2E described participating pupils as "cooperative, ready to share, willing to step up and try, working well together". I attended all mathematics lessons for the class over 12 weeks punctuated by: a half-term break, a HOD-Science interview, two Bank Holidays, a national teachers' strike, and end-year examinations. I will discuss in Section 4.1 my use of written work, observations, discussions and focused interviews with participants. I witnessed apparent undermined accessibility of algebra I outlined in Sections 1.2.2 and 3.1.3. Pupils had very limited opportunities to value collaborative learning and experience serious consideration for their thinking (Ruthven et al, 2011).

The lack of universal consensus on what learning activity is considered 'algebraic' (Hewitt, 2011; Van Amerom, 2003) caught my attention based on strategic decisions of pupils solving word problems. I was puzzled by specialist teachers disputing the relevance of *Grid Algebra* in algebra; it stimulated my reading more about approaches to school algebra. Pupils seemed willing to discover new ways of working while teachers hesitated. Minimal ICT use seemed aligned to teacher-directed instruction; teachers reinforced individual pupil learning. Pupils' silence when required to offer immediate responses in whole-class discussions suggested they exercised agency to deflect any labelling and judgment of their learning abilities. A change in learning strategy had revealed startling insecurity of several pupils' number sense. Higher expectations of effort can motivate engagement in mathematics, as pupils demonstrated when presented with GCSE-level questions. Agency, affect and mathematical thinking took on new dimensions. I attributed the apparent disinterest in pupils' kinaesthetic *Grid Algebra* use to stimulate learning-based talk to teachers accustomed to controlling pupil thoughts or actions in mathematics. Teacher hegemony can restrict positive learning experiences; conditions, opportunities and resources may render lessons dreary and isolating (Nardi and Steward, 2003). Pupils' hatred of self-checking work indicated 'learned helplessness' (Dweck, 2000) or possibly low expectations and limitations imposed by curriculum or examination boards. Teachers have been known to set bars of attainment at the minimum. I have often heard a teacher telling a pupil something to the effect that "Grade B in mathematics is brilliant! You only need to get C in GCSEs!" It is hardly surprising that some researchers (Brown et al, 2008; Nardi and Steward, 2003; Boaler et al, 2002) described secondary mathematics classrooms as landscapes riddled with quiet disaffection. Some researchers (Healy et al, 2001; Bednaz et al, 1996) argued that favoured curriculum options inform pupils' strategic competence. Thus, diminished access to algebraic concepts can be attributed to pedagogic decisions to a greater extent than to pupils' mathematical abilities or the nature of algebra. I propose further investigation into the defining of algebra as 'generalised arithmetic' in the UK National Curriculum; it seems many teachers construe the emphasis as evaluating numerical solutions and finding patterns (Hewitt, 1992). Bell (1996) argued that 'generalising' may direct pupils' attention to procedural learning.

To a large extent, this 'exploratory' stage reaffirmed the heightened need to acknowledge that pupils actively construct knowledge despite being 'taught' by teachers. Bruner (1961) argued for teachers to inspire pupil learning rather than transmitting knowledge. The UK pilot study highlighted vital benefits to learning afforded by software increasing classroom interactions, namely peer collaboration and teacher intervention. It strengthened my conviction concerning schooling as an inherently social process overseen by the teaching fraternity that provides and manages variety for pupils in mathematics lessons. Combined ICT and non-ICT resources in classroom environments can be an enabling factor for formative assessment and 'transfer' of learning that may occur when pupils encounter 'difficult' concepts. Classroom contexts can encourage pupils to either be helpless or to learn mathematics with understanding. Positive learning behaviour reinforced my thinking that affective aspects can influence mathematical learning. Pupils can welcome appropriate and challenging targets; they engage and work well when encouraged, and take responsibility for their learning when provided with opportunities to *act*. Otherwise, pupils are less likely to 'author' high self-expectations; many see no need to increase effort in learning activity, hence low pupil participation. I contend that teachers need not burden pupils with their own fears and inadequacies; neither should teachers bow to external pressures to the detriment of positive learning experiences. These findings convinced me to amend my research design and questions to take into account the effect of using ICT on the teachers and on teaching in lessons. I reflected on my own role supporting teachers making sense of ICT-enhanced pupils' algebraic learning and developing TPACK as teacher 'learning'. I set out to try this intervention in a different setting in the next stage as follows.

3.2.2 Implications of Stage Two: Pilot Study-Kenya

Dane (1990) argued for *pretesting* aspects of research prior to full-scale implementation. This research stage was essentially a four-week feasibility study. I was able to assess: functionality of ICT resources and facilities; how to implement research with minimal disruption to class routines; the extent to which I would access insider's perspective (Flick, 1998) as 'teacher-researcher'; and the pupils' comprehension of instructions and length of time required to complete instruments. Site conditions enabled me to examine *new* mathematics classroom 'subculture' mediating pupils' *Grid Algebra* use. Stage Two marked the subtle genesis in research nature from 'exploratory' to 'descriptive' (Dane, 1990). I developed my aim to embrace the learning context described by Luckin (2008) in the people, conditions, tools, and knowledge and skill levels. I concentrated on affective dimensions concerning pupils (interest, involvement and engagement) and teachers (sense of *unease* with ICT). I acknowledged the effect of ICT-enhanced learning on mathematics teachers' thinking, discussed by Tabach (2011). I modified my research questions (see Section 3.2.5) to reflect the entire learning context and teachers' decision-making on social interaction and resources in classroom lessons (Sutherland et al, 2009). Following Walshaw (2013) and Hodgen et al (2008), teachers required support.

I involved Form 2 pupils in trialling data collection instruments to be employed during Stage Three; the Form 1 cohort was yet to join secondary school. Schemes of work for the school term had already been planned. Some teachers expressed concerns about ICT-enhanced activity disrupting lessons. They instead invited me to observe classroom interactions within non-ICT lessons. The teachers' concerns echoed some listed in Section 1.1.2 and voiced in SMASSE-INSETs I attended (see Section 1.4.2). Instead of negative pupil behaviour reported by Barnes (2000), I observed some teachers apparently unsettled by my presence in classrooms. I considered the negative teacher reaction to being studied (Denscombe, 2002) a concern, given my proposed intervention. Of importance to me was apparent teacher inertia and opposition to doing things differently for the sake of mathematical learning. I became convinced that most teachers and teaching posed the immediate threat to positive learning experiences. I enlisted the pupils as *agents* of change to differentiate *pupil-centred* learning from *teacher-directed* instruction for participants. I adapted to teacher concerns by negotiating with HOD-Computer Studies whose subject had sole access to computer laboratories. I facilitated ICT-enhanced sessions on two evenings between 7pm and 10pm. Computer Studies teachers agreed to be on hand to solve any technical or behavioural issues.

I oversaw the loading of software onto 3 computers in a SMASSE room for teachers' and 13 computers in one laboratory for pupils' use. The lack of projection facilities necessitated my sourcing a projector for whole-class demonstration from outside the school. I presented the 'Baseline' to teachers for prior scrutiny and approval. One teacher pointed out that algebraic concepts in this instrument were "Standard Five" (primary school) level. In this remark was a subtle suggestion the 'Baseline' was very easy, and that the pupils would get everything correct. Another teacher signalled approval for the content. He hoped the exercise would test the pupils' grasp of *basic* algebra, remarking that although pupils may have been previously taught the concepts, they probably would have 'forgotten'. This teacher indicated willingness to court risk-taking. I will describe maximising productive use of limited lesson time in transformative learning processes in Section 3.2.3. I will report using pupils' written work, two questionnaires, observations and interviews in Section 4.2. I presented preliminary findings to HOD-Mathematics prior to dissemination.

My direct observations at Stages One and Two suggested teacher behaviour can affect pupils' learning (Skinner and Belmont, 1993). Some researchers (Reed et al, 2010; Bretscher, 2008) argued that effective ICT use required teacher acquaintance or awareness of tools' potential to mediate mathematical concepts. Such knowledge enabled the teachers' support of pupils' appropriation of learning. Participating teachers were initially not familiar with using *Grid Algebra* resource for learning algebra due to institutional constraints within schools. Some 'legitimate' reasons given as explanations, which varied from time-tabling, design and size of computer rooms, appeared to mask teachers' *sense of unease* concerning ICT use. None of the teachers relished developing mathematics TPACK alongside pupils. Hence, I planned to hold a 'reconnaissance' *Grid Algebra*-based session for teachers at Stage Three.

3.2.3 Implementing Stage Three: Main Study-Kenya

This stage marked actualising increased pupil interaction, involvement and engagement when learning mathematics, following reports by Brown et al (2008) and Nardi and Steward (2003). According to Australian DEAG (2013), quality learning environments can improve learning outcomes based on three principles:

- *There is a direct relationship between how pupils learn and what they learn;*
- *Making it possible for every pupil to learn by developing personalised learning;*
- *Emphasis on pupil-centred learning.*

These principles added to calls for shifting responsibility for learning to pupils discussed by several researchers for example Lee and Johnston-Wilder (2013), Gresalfi et al (2009) and Boaler and Greeno (2000). I envisioned harnessing classroom 'subculture' to mediate *Grid Algebra* use and distribute responsibility and accountability in learning. The timing of Stage Three coincided with the term during which Form 1 pupils learn 'Algebraic Expressions' according to the secondary mathematics curriculum (see Section 1.2.3); every class has six 40 minute time-tabled mathematics lessons per week. It was agreed in a meeting with the Principal and HOD-Mathematics, following Stage Two findings, to allocate one lesson per week per participating class, hence 40 lessons, to ICT-enhanced algebra learning. One largely underused computer laboratory with 25 pristine computers was dedicated to this project. On the premise of ethical arguments in Section 3.1.2, the entire cohort of 270 had access to *Grid Algebra* software. I group-administered the MRBQ to 270 pupils whose responses I discuss in Chapter 4 in light of pupils' strategies in the 'Baseline' (see Appendix 10). I perceived data in Table 4.2 crucial in informing about pupil cognition as 'met-befores' (Tall, 2004) instead of relying on teacher assessment of pupil learning. It revealed difficulties pupils had with writing mathematics.

Each class had a total of 8 ICT-enhanced mathematics lessons. Pupils invested greater lesson time learning collaboratively on 8 of the 26 software-generated tasks discussed in Section 1.5.3. However, this study highlighted the plight of pupils who lacked basic computing skills. The pupil questionnaire collected quantitative data shown in Table 3.2. The SPSS output displays descriptive information on the pupils' primary schooling and computing background.

		Class of respondent					Total
		1	3	4	5	2	
Did you have any computer lessons before you were admitted to this school?	Yes	32	35	40	37	38	182
	No	22	18	14	17	17	88
Total		54	53	54	54	55	270

Table 3.2: Distribution of the pupils' prior computing experience per class

Approximately 67% of Stage Three pupils had prior experience with computers upon joining secondary school as shown in Table 3.2. Between 30 and 40% of pupils in a particular class had to acquire extra computing skills whilst grappling with learning algebra. Their initial low computer confidence caused by a lack of ‘click-and-drag’ skills threatened the pupils’ active or full participation in ICT-enhanced learning. They faced being excluded from experiencing positive feelings engendered by computer-based activities that have been found to influence pupil engagement in mathematical learning. By offering ‘extra’ sessions (see Section 3.1.5.4), I supported in full the plan to provide 270 pupils with access to uniform learning experiences. Pupils subsequently encountered related written tasks based on *Grid Algebra* and textbooks. Two Bank (Public) holidays, Sports’ Day, Speech/Prize-giving Day, five-day half-term, and an impromptu school inspection disrupted some sessions. I arranged make-up sessions with affected pupils outside time-tabled hours in these instances. I agreed to lead ICT-enhanced sessions for the first few weeks. I introduced *Grid Algebra* to the participants whilst teachers observed. This plan was intended to build teacher confidence to take over and allow me to resume participant observer role. My assuming the lead in ICT-enhanced sessions provided ‘lead-in’ time and the support (Crisan et al, 2007) the participants required to use ICT in practical ways.

The weekly sessions adopted a similar pattern to the one I describe shortly. Following Pimm (1987), in the first session with each class, I explained my expectations of the *why*, *how*, *when, where* then *what* of the ‘ground rules’ to facilitate productive working in ICT-enhanced activity. Each subsequent session usually began with brief whole-class revision of written work focusing on areas of difficulty followed by introducing new tasks, after which pupils worked collaboratively on a software-generated task in much of lesson time such that:

1. *Groups of at most three pupils formed per computer;*
2. *Pupils took turns to speak and listen respectfully to each other;*
3. *The contributor operated software whilst explaining their reasoning;*
4. *Other pupils offered justification for alternative views.*

When a class entered the computer laboratory, *Grid Algebra* was already lit up on the screens of working desktops on tables for pupils’ use. The software beamed from my laptop onto the whiteboard ready for use in whole-class demonstration. Pupils complied with instructions by gathering in groups of at most three around desktops. Two pupils stood on either side of one seated peer in order for each to have clear vantage point of the screen and whiteboard in front of them. Each had their marked ‘Baseline’ script for reference. With pupils’ eyes fixed on the projected grid, they became quiet and waited for the *why* that shaped the learning experience.

Having studied strategies pupils readily used in the 'Baseline' (see Table 4.2), I initiated ICT-enhanced sessions by introducing the main software features (see Section 1.5.1). A whole-class discussion ensued. By using question-and-answer technique to revise 'Baseline' questions 5, 8 and 9, I aimed to consolidate pupils' understanding of *Grid Algebra*. I intended the pupils to *see* formal notation for themselves: that addition and subtraction signs were preserved whereas multiplication and division signs 'disappeared', and replaced by brackets and line notation respectively. This activity served to link the *why* to *when* and *where.*

To consolidate the *how* of learning, I selected a software-generated task for projection. Pupils collectively read the accompanying instructions made as explicit as possible. I hoped to ensure every pupil understood what was expected of them. Upon selecting the lowest level of difficulty, I invited the whole class to answer the projected puzzle. Low murmurs erupted; pupils solved the question, some individually while others discussed softly. The pupils raised hands to indicate their willingness to contribute. The 'ground rules' governing the activity required that we all listen to one contribution at a time. A randomly selected pupil offered their response which I entered into the program. Everyone else listened to the speaker whilst considering the software feedback. When an answer was deemed to be 'wrong', a 'No Entry' sign and a bin at the right-hand corner appeared on the screen. I paused to request the contributor to explain *how* they obtained their answer. The rest of the class listened, assessing the reasoning behind the solution. A whole-class discussion ensued; I invited the other pupils to point out the flaw in reasoning, and possibly correct it. In this way, *why*, *when*, *where*, and *how* of learning fed into each other. The class worked together through a level of difficulty in order to make pupils aware of what to expect at the end of each level. The regular teacher and I watched and listened closely. Many pupils seemed to revel in this learning experience; they interacted freely and participated in debates about *why* solutions worked or not.

I then asked pupils to turn to their desktops, and work as expected through a selected task: the *what.* The software provided immediate feedback on solutions. As instructed, the pupils took turns working through puzzles whilst operating computers: each talked through their answers. Pupils seemed more confident in their comprehension of *what* was expected of them as they explored, negotiated and discussed their answers. The regular teacher and I offered assistance when and where it was asked for. We at times drew whole-class attention to the salient points in summaries. Some related written tasks were then administered and done individually.

I held retrospective discussions with regular teachers, and later with some pupils, as detailed in Section 3.1.5.4. Through the discussions, I learned that pupils apparently transported 'new' learning behaviour into non-ICT mathematics lessons. Surprisingly high levels of pupil interest, involvement and engagement were reported by participants in interviews and questionnaires.

3.2.4 Enabling and inhibiting factors

Certain factors affected implementation at Stage Three; they included: participants' computer confidence, technological issues, harnessing learners' enthusiasm, and '*mismatched*' learning processes. To enable full pupil participation in ICT-enhanced sessions, basic computing skills were essential for one to operate the software. Table 3.2 shows about 33% (88) of pupils were adversely affected by the low computer confidence I discussed in Section 3.1.5.4. Mathematics teachers lamented the obvious discrepancy in pupils' skills levels, which teachers attributed to a newly-implemented admissions policy affecting the Form 1 2012 cohort. The Kenyan Ministry of Education resolved to promote national cohesion through education. Changes resulted in this research school enrolling pupils of wide-ranging schooling and socio-economic backgrounds. However, these teachers were accustomed to handling mainly urban high-achieving pupils. Analysis of some data collected in the pupil questionnaire attributed diversity in computing experience to factors determining access to digital resources: private versus public, and urban versus rural settings. I noted that not all pupils with urban schooling automatically had prior basic computing skills, contrary to popular belief. The issue raised questions about the role of the school's Computer Studies department, which I addressed through 'extra' ICT-enhanced sessions (see Section 3.1.5.4). The persistence of resilient pupils in acquiring basic computing skills in addition to learning algebra seemed to challenge teachers who lacked enthusiasm for ICT use in mathematics due to their own inadequate skills. It helped to remind teachers that they were hardly required to be technologically-adept in order to embrace ICT integration. Clausen-May (2008) explained that such prerequisite may depend on the ICT tool's design. This study afforded mathematics teachers an opportunity for 'reciprocal learning' explained by Luckin (2008). She argued that teachers can learn *with* and *from* pupils. Teachers gave due consideration to alternative ways of achieving pupil learning. Rather than outright dismissal of ICT integration in education, pupils' enthusiasm about *Grid Algebra* use helped teachers to feel more positive about ICT in mathematics lessons. It made teachers interested to realise that there might be something in the software worth considering.

Throughout this project, I noted many pupils' difficulty in comprehending instructions within the software. As detailed in Section 3.2.3, each session began with whole-class demonstration to explain related instructions prior to pupils exploring with the tool on their own. The lack of technical support and projection facilities were sticky issues. Luckin et al (2012) proposed we "make better use of what we've got" for the sake of pupil learning. Yet, the Computer Studies department seemed unhappy with the school's decision to allow my research project access to one computer laboratory on premise that the facilities lay unused. The decision was argued to effectively sanction reallocation of ICT facilities to Mathematics. As a 'pseudo-technician', I checked and confirmed the number or working desktops every morning. A personal pocket projector served adequate use for whole-class demonstration.

In what I considered well-thought out plans, I anticipated holding a 'reconnaissance' session for mathematics teachers as indicated in Section 3.2.2 following Lewin (1988). I envisioned teachers exploring *Grid Algebra*'s potential preceding pupils' use. However, getting all five teachers to convene outside normal school activities was impossible. It became clear that my plan needed tweaking to facilitate a decent measure of success. I intended to switch lead and observer roles with teachers when they were ready. I had one rare opportunity to engage three teachers in an exploratory session on one morning: two for Mathematics (T1 and T2) and one Computer Studies (TCS). Teachers highlighted factors they felt were barriers to my project in light of the relatively large number of pupils per class. The following exchange ensued.

***TCS**: What if the pupil is unable to do the question?*

***T2**: Then they ask the teacher for assistance.*

***TCS**: What if the teacher is unable to do the work?*

Teachers T1 and T2 stated they would provide support being specialist subject teachers. They selected Task 21: 'Simplify' to solve the puzzles; they reacted to the consequential feedback.

***TCS**: This is great, because pupils have to do the mathematics by themselves!*

***T1**: Yes! And the computer marks the work for them as they give their answers.*

***T2**: This is good! I can see it does well to combine 'Integers' and 'Expressions'. These pupils really have problems with 'Integers'.*

The session, albeit brief, was of great importance to my research; some teachers had explored the ICT tool's potential. They considered its appropriateness for Form 1 Mathematics and its inclusion in the learning context. A key aspect for these teachers was the fact that pupils had to solve questions by themselves whilst the software provided feedback; it seemed to capture teachers' interest and imagination in learning activity. The departmental ICT lead recognised *Grid Algebra*'s potential; it linked 'Integers' to algebra. Teacher T4 reiterated these views in our retrospective discussion on the following day.

My taking action to organise 'extra' sessions became a point of discussion in an unscheduled meeting with the Principal and HOD-Mathematics. I raised my concern about observed pupil running at speed to the sessions, in terms of their safety, and their reluctance to leave the laboratory, in terms of their need to get to the next lesson. Although safety issues fuelled my concerns, I had doubts at that time whether pupils were indeed learning algebra or simply were eager to get away from their usual learning environment. My doubt resonated with educators' concerns relating to pupils feeling good about being in school yet fail to learn anything, as reported by Skinner and Belmont (1993). These views illuminated an argument that enjoyment may not necessarily indicate engagement with learning. However, the Principal begged to differ; she preferred to interpret observed behaviour as an indication of "pupils loving mathematics". She looked forward to receiving reports on the research project's impact on pupils' progress. I acknowledge with thanks the enabling support for change that I from this school's leadership team (the Principal; HOD-Mathematics and departmental ICT lead as QASOs; Senior Mistress) in assessing facilities (Guskey, 2002). Luckin et al (2012) and Crisan et al (2007) considered such support vital for ICT integration. Despite congestion, technological resources were subordinated to some 270 pupils' learning needs. I subordinated CEMASTEA's goal of upgrading teachers' skills to pupils' learning in research (see Section 1.1.0). These factors magnified pupil engagement in mathematics and spread enthusiasm across this school. Mathematics teachers talked about the pupils' change from *disaffected* to highly *participatory* learners with colleagues. Pupils ran to ICT-enhanced sessions and performed tasks with focused animation and increasing confidence. Although transformation was evident, pupils' learning outstripped teacher 'learning' behaviour within this study.

I prioritised pupils' learning through supportive pedagogic and organisational practices in this research. I observed some pupils willing to support their peers when motivated at Stage One. I gambled on considerable pupil enthusiasm to engage with *Grid Algebra* and embrace *new* mathematical learning behaviour acting as triggers for reflection and awareness in teachers: teacher 'learning'. Two teachers embraced observation roles; they attended all ICT-enhanced sessions and interacted with groups of the pupils working on software-generated tasks. Three teachers made some *technical* appearances or missed sessions altogether. Only two teachers felt confident enough to lead some ICT-enhanced sessions. This had overall impact on the research design; I participated more as 'facilitator' in the sessions than intended. I jeopardised researching to 'facilitate' ICT-enhanced learning. Therefore, participants' words may be less accurate in my recollections. I balanced participation in diverse roles: teacher-researcher, trainer, confidant, 'technician', sister. This entire research experience was time-consuming and emotionally-draining. It entailed my spending very long days, planning and writing up field notes in the evenings when free, at the school. I restored my researcher role to some extent by holding retrospective discussions with the teachers.

3.2.5 Final research questions

This research upheld small-group collaborative working, articulation of ideas, inclusion, and learner agency mediating 'hands-on' *Grid Algebra* use. My final research questions were:

1. Did the intervention address levels of interest in algebra and pupil concerns about lack of involvement and engagement in mathematics lessons?
2. What effect did intervention have on the competence, and confidence, of these pupils' mathematical language use in classroom interactions?
3. What consequences did the intervention have for the role of teachers in ICT-enhanced learning contexts?
4. In what ways did the intervention differ from the participants' usual classroom practices in terms of distribution of responsibility and accountability in learning?

3.2.6 Ethical considerations

To conduct research in Kenya, it is imperative for the researcher to obtain clearance from the National Council for Science and Technology (NSCT) upon affiliation to a designated local university. I requested and obtained ethical approval from Warwick Institute of Education in accordance with University's and BERA (2004) guidelines (see sample Appendix 1). I duly affiliated myself to the Faculty of Education and Community Studies, Egerton University, to obtain NSCT research permit (see Appendix 2). I obtained prior informed consent from both Principal and teaching staff that stand in *loco parentis*, the research site being a boarding school where pupils are drawn from far flung provinces with limited access and communication. The mathematics teachers signed written consent forms (see Appendix 3) on behalf of each class. Pupils gave their verbal consent, as described in the next paragraph, endorsed by enthusiastic participation. This is acknowledged as a deviation from BERA (2004) guidelines, upheld in the Kenyan context by *loco parentis* (Denscombe, 2002). *Consent* is defined by explicit acts (verbal or written); *assent* is indicated by one's willingness to participate (Gallagher, 2008).

I gave on-going thoughtful consideration to each research process. My ethical conduct was not restricted to form-filling and seeking clearance from research ethics committees. I upheld long-term teacher-pupils relations. I adhered to behaving ethically towards teachers as adults facilitating my access to pupils. To ascertain voluntary and informed participation, I drew up a contract of participation (see Appendix 4). I read it out during my initial contact with all participants in their mathematics rooms at each stage. I introduced myself and underlined my status as a University of Warwick research student. I outlined research procedures for pupils' benefit as a way of seeking their informed consent for classroom-based study. I emphasised participants' right to withdraw from the study whenever they chose to. I use pseudonyms for research participants in the report to preserve their confidentiality and anonymity. I revisited consent and voluntary participation principles whilst distributing physical artefacts to pupils. I clarified that worksheets and questionnaires were not pieces of school work.

My role as teacher-researcher became blurred when I assumed lead in the lessons despite my being wary of imposing either role on the participants. At Stage One, I was addressed by my first name by the participants. At first I struggled with this norm which was alien to my own culture where failure to use titles when addressing others may be deemed *disrespectful*. With time I recognised its effect: it distanced me from the teachers. In the Kenyan phase, I became more of the *teacher* than *researcher* in most ICT-enhanced sessions. Pupils often referred to me as 'Teacher' at Stage Three. This role-blurring possibly limits extent to which I can claim pupil participation was voluntary given cultural deference to authority figures. Their reference was hardly surprising since my presence provided additional structure and involvement for pupil learning whilst supporting teacher 'learning'. I found it quite difficult to distance myself from the activity with the objectivity required of a researcher. I struggled to remember which role to assume, to what extent and to suppress the *teacher* in me from getting 'carried away' in the passion of moments as events unfolded.

3.3 Summary

This research examined whether classroom 'subculture' mediating *Grid Algebra* use had the potential to promote conceptual understanding in algebra, and hence to include disaffected pupils in secondary school algebra. In Section 3.1 of this chapter, I discussed data collection and analysis tools that I selected to use to address the research questions, and my rationale for doing so. In Section 3.2, I discussed developing my research methodology, and questions, through two pilot stages, one in UK and one in Kenya. I report on the results in Chapter 4.

Chapter 4: RESULTS

4.0 Introduction

In this chapter, I present research findings as provided by different groups of participants and different individuals. Sections 4.1 and 4.2 describe in brief Stages One and Two respectively; Section 4.3 provides a full description of Stage Three. I concentrate on *what* I found in the field using various research methods (see Section 3.1.5) rather than what the data mean, which will be discussed in Chapter 5.

4.1 Stage One: Pilot Study-UK

The pre-diagnostic exercise consisted of five algebra questions drawn from the CSMS project (see Appendix 5). I distributed 34 worksheets and discreetly watched the pupils complete the scripts. Many pupils compared their answers with peers, and some then made alterations. I collected 24 scripts; 10 pupils opted not to hand in their scripts. Only15 scripts were complete. 23 pupils demonstrated a good understanding of 'perimeter'. 12 pupils gave responses to a word problem; only one pupil elaborated their reasoning. This data revealed the pupils' strategic competence; the majority apparently attended to obtaining numerical solutions and valued arithmetic methods over algebraic strategies.

Teacher 1E expressed concern to the class about previously-learned concepts. She suggested weekly strategies with more practice intended to improve performance. The pupils' objection to the proposal had me thinking about establishing whether mathematics-related beliefs were held by the pupils. Therefore, I sourced the MBRQ (see Section 3.1.5.6) used in later stages.

The group had four one-hour ICT-enhanced sessions in which I took the lead as explained in Section 3.1.5.2. The first session was conducted in the group's mathematics classroom using a whole-class format with the software projected on the whiteboard at the front. After a brief introduction, I asked pupils to predict numbers in specified cells within defined grids. This activity proved difficult for those pupils who appeared to struggle with their knowledge of multiplication tables. Others demonstrated ease with describing journeys traced by letters dragged across the grid. Next, I selected Task 1: 'Calculating- small grids'; I invited pupils to provide responses to puzzles. The pupils worked at different paces; some struggled with calculations while others grumbled about the questions being "too easy". The class became noisy. Quite a few pupils seemed disengaged; they stopping working on the task. It appeared to me that while some pupils had difficulty with their number sense, others had no difficulties. The evident differences in the positions of these pupils' cognitive development stimulated my reflection on the 'ground rules' to govern collective activity with an emphasis on mutual respect. I viewed the teacher's role as facilitating and enforcing a supportive classroom 'subculture'.

The second *Grid Algebra* session was held in a computer room that proved to be rather small for the class. A number of computers failed to start up; some pupils waited for mini-laptops. The room had no provision for projection, thus no opportunity for whole-class demonstration. Pupils worked in pairs or in threes. I requested them to select Task 13: 'Make the expression-small grids'. I observed pupils working with high engagement to beat the clock in the timed-task. Pupils discussed among themselves; some demonstrated fairly good mastery of how the software works. They made links between the grid movements and mathematical operations. Such pupil behaviour suggested their willingness to participate actively in the learning activity.

Halfway through the lesson, I requested pupils to select Task 25: 'Write the expression'. The majority remained on-task while some stopped working soon after commencement. Towards the end of the lesson, I administered a worksheet with four questions (see Appendix 6) which required pupils to provide algebraic expressions when given a starting letter and paths traced across grids. Some pupils' responses resembled description of journeys rather than algebraic expressions; they wrote, $c + 1 \times 2 + 6$, instead of $2\ (c + 1) + 6.$ Through demonstration with software, I drew these pupils' attention to the fact that multiplication and division signs were replaced by 'brackets' and 'line notation' respectively. 7 of the 34 pupils reattempted the work before the lesson ended, and Teacher 2E gave the following feedback.

- The software was too "open-ended"; I needed to set time limits for each difficulty level, say every pupil should finish a given level in five minutes;
- Pupils had too much control over what they were doing;
- I should write 'BODMAS' on the board as a reminder for the pupils;
- It was hard to see how *Grid Algebra* related to what the pupils did in class exercises.

The remarks revealed Teacher 2E's perceived need for *speed* when working in mathematics, control of learning activity, and linking resources to curriculum objectives. It stimulated my reading more about approaches to school algebra. I resolved to modify and re-administer this task following Teacher 2E's feedback as part of consolidating the computer mediation of formal notation supported by pupil-pupil interactions in lessons.

In the third ICT-enhanced session, the class was accommodated more comfortably in a larger computer suite with projection facilities. Following a brief whole-class demonstration, the pupils worked well in pairs through Task 25: 'What is the expression?' Teacher 1E and I observed whilst moving round the room. I then requested the pupils to repeat the previous written task with the additional information in the instructions (see Appendix 7). I collected 24 scripts; only one of the pupils was successful at using formal notation. Pupils then selected Task 6: 'Expanding and factorising'. After a whole-class demonstration with explicit instructions, pupils settled down to work. When one pair requested Teacher 1E for assistance, she replied, "I do not know how this software works." She asked them to explain as she watched closely. One pupil became a 'more-knowing other'; Teacher 1E became the 'learner' to exemplify 'reciprocal teaching'. Using animated hand gestures, the pupil argued *why* his solution worked. A second pupil worked steadily, before stopping short when the grid disappeared; he could not proceed. This exemplified the need for appropriate 'fading' of support. This pupil may have become dependent on the grid's scaffolding; he felt 'lost' without it. Another pair of pupils requested clarification of software instructions after which they collaborated with high engagement. Despite the grid's disappearance, the pair continued working with considerable mastery of *Grid Algebra*. Teacher 1E remarked on pupils' learning after the session.

Teacher 1E*: This was much better this time! But I am honestly struggling to see how they relate what they are doing with the software with what they do in class.*

Both teachers had made no attempt to engage with *Grid Algebra* or to assess its alignment with learning algebra. However, I was encouraged by the pupil engagement and involvement. We agreed on the class taking a breaking from ICT use. I observed non-ICT lessons. Teacher 2E led a non-ICT lesson involving 'Substitution'. She wrote questions on the blackboard; she imposed time limits. Pupils appeared not to comprehend how to proceed: they kept silent and offered no responses. Teacher 2E initiated whole-class discussions of questions before asking pupils to solve similar questions in textbooks. I observed four pupils working individually *in* a group; they did not reason through questions *as* a group. Instead, pupils focused on answers and demonstrated low confidence in their own solutions. They persisted in obtaining answers from peers. The teacher explained this learning behaviour as:

Teacher 2E*: They simply hate checking their work.*

I viewed this as a missed opportunity to encourage self-assessment in the pupils. Self-assessment can help pupils develop into independent and confident learners in mathematics (Black et al., 2003).

Teacher 1E revisited the concepts of 'Substitution' and 'Changing the subject' in a non-ICT lesson the following day. The pupils appeared totally bewildered by the 'Substitution' questions, and seemed unsure of what was expected of them, which led to their teacher remarks.

Teacher 1E*: Always do what the formula tells you to do. This is level 4 work, it's too easy!*

The pupils refrained from asking any questions or offering even partial or 'wrong' responses; they instead remained silent. The behaviour made it difficult for Teacher 1E to intervene; she said:

Teacher 1E*: Seriously I have doubts whether you will be able to do higher GCSEs.*

Teacher 1E seemed to vent frustration by directing feedback at the pupils' 'self' rather than at their task performance. She appeared to ignore feedback information, and instead continued posing more questions that were met with silence, before switching to 'Change of subject' questions. She reiterated her concerns about 'ability'.

Teacher 1E*: I am seriously surprised and disappointed about the kids not coping with work involving 'Substitution', but okay with 'Change the subject', which to me is much harder. I really wonder whether they really will be able to cope with higher-tier GCSEs. I found that lesson utterly draining!*

At this point, I suggested revisiting the 'Substitution' concept using *Grid Algebra* to scaffold the pupils' learning. Teacher 1E welcomed my changed strategy, from non-ICT to ICT in a whole-class format with the software projected at the front, such that:

- I explained the task;
- Teacher 1E nominated the pupils at random;
- The pupils suggested answers to puzzles which I entered into the program.

Some pupils grumbled about the selected level being *too easy*; others had difficulty obtaining answers. Once the grid disappeared, the pupils became attentive; they watched and listened to each other's contributions. Pupil participation improved through Teacher 1E probing the pupils' reasoning, which they explained. One pupil complained about this format exerting *too much pressure*: he could not think that fast. Teacher 1E altered the activity to emphasise individual working and solutions.

***Teacher 1E**: Now I want everyone to write down the answer to each question, then show it to your desk-mate when I say so.*

Whilst acceptable to Teacher 1E, such ICT use appeared to reinforce individualistic, teacher-directed learning which is not what I had planned for. The pupils neither had 'hands-on' use of *Grid Algebra* nor shared and extended each other's ideas. The activity revealed many pupils' weak grasp of multiplication tables as a mental resource in problem-solving that rendered the use of 'met-befores' hardly *supportive*. I observed several pupils who took to finger-counting, a strategy that appeared out-of-step with working at speed when solving problems. One pupil stated their difficulty in knowing which operation to perform: "What I do not get at times is how I will know what to do first."Teacher 1E replied: *"You use 'BODMAS'."*

For such pupils, the order of operations was problematic despite previously being taught to use the technique 'BODMAS'. This was consistent with pupils failing to *learn* what they have been 'taught'. The evidence suggested that reminding pupils to use 'BODMAS' was not helpful since it was not sufficiently understood. Such use can encourage the pupils' reliance on procedures instead of assuming responsibility for their own thinking and learning.

On one morning, a non-ICT lesson started with Teacher 1E delayed in traffic by an accident. I initiated paired-working of pupils on a worksheet I had designed to make the pupils aware of the nature of algebra questions in GCSE examinations (see Appendix 8). The pupils responded to the task with high levels of engagement and participation evidenced in animated dialogues. They exhibited difficulty with 'Substitution' questions; their confidence was jolted when solutions turned out to be negative. Nevertheless, most pupils applied great effort in the paired discussions. When Teacher 1E finally joined the class, she initiated a whole-class discussion in solving the following questions:

1. $8p + 4$
2. $12a + 4a^2$
3. $32ab^2 + 18b$

4. $8y^3 + 12y^2$

The pupils applied commendable effort in ‘Factorising’, a concept apparently beyond curriculum level 5; they proved able and willing to handle ‘difficult’ ideas when motivated and trusted.

I administered worksheets on ‘Substitution’ and ‘Expanding and factorising’; each had five questions selected from software-generated tasks. The pupils solved questions with considerable ease. Teacher 1E and I went round the room marking this work; the pupils retained their scripts. A teacher-led revision of the questions in a whole-class discussion followed.

I interviewed only one pupil for about 10 minutes. He described the ICT-enhanced mathematics lessons as *enjoyable*; his ‘surprise’ was realising an improvement from attainment level 4 to level 6. I interviewed Teacher 1E for about twenty minutes in the mathematics staffroom. She expressed ‘surprise’ at the pupils comfortably handling level 7 work despite assigning levels 4-6 to the majority. She raised again the relevance of *Grid Algebra* to pupils learning algebra.

Teacher 1E*: To be honest, I just could not see how pupils could relate it with what they were learning in class.*

She underscored marked differences revealed in the pupils’ mathematical abilities.

Teacher 1E*: The other lesson, when we had them working as a whole class on the tasks, they clearly struggled with questions produced by the software. Differences amongst them were so very clear, much as they are in the same set! And oh! They struggled with substituting values.*

The teacher puzzled over the setting of the pupils, and the class’ prior encounter with the concept.

Teacher 1E*: I really wonder, because we’ve tackled those kinds of problems so many times!*

However, she was pleased with the pupils’ handling what she referred to as ‘level 7’ work.

Teacher 1E*: I am very impressed with their effort in coping with the work on Expanding and Factorising, very impressed.*

She seemed to suggest that using *Grid Algebra* had enabled the pupils to handle concepts beyond their attainment level. To me, Stage One results stressed the importance of teachers assessing relevance of ICT tools through personal acquaintance. Software on its own does not improve learning. It is the way ICT is used in the classroom as integral to a learning context and *Grid Algebra*'s design that can inspire pupils successfully engaging with algebra.

4.2 Stage Two: Pilot Study-Kenya

As the school in Kenya was a boarding school, I was able to group-administer the MBRQ to the pupils on a Saturday evening. They completed the MBRQ in roughly 30 minutes. I collected 230 responses for analysis with SPSS. I administered the 'Baseline' assessment on a Sunday afternoon. 231 pupils took approximately 25 minutes to complete the instrument. I collected and marked the scripts. From a cognitive stance, none of the Stage Two pupils got everything *correct*. Thus the 'Baseline' proved adequate for gauging existing algebraic understanding, hence it was appropriate for deciding how to pitch the ICT-enhanced sessions. I attended one lesson in each of the five classes. High pupil disengagement was widespread; many pupils dozed openly in teacher-directed instruction.

Subsequently, I gave a class of 48 two one-hour *Grid Algebra*-based sessions on consecutive evenings. In Session 1, I introduced the pupils to software features: the 'Tools Menu' and the nature of the software's feedback. I used a projected grid to demonstrate the 'Baseline' items 5, 8 and 9 in a whole-class discussion. I evoked a revision activity to direct the pupils' attention to structures in algebraic expressions. The pupils then collaborated in small-group working on two software-generated tasks 1 ('Calculating') and 21('Simplify'). The pupils adhered to instructions by taking turns to operate the computers, talking and listening to each other respectfully. Their talk was highly task-focused and punctuated with pupils giving squeals of delight upon receiving software feedback. After a brief whole-class introduction in Session 2, the 45 pupils present worked on two software-generated tasks, 13 ('Make the expression') and 25('What is the expression?'). I noted high engagement indicated by the pupils using hand gestures as they negotiated responses. Whilst some groups worked successfully through to higher levels of task-difficulty, others requested my assistance. I then distributed the worksheets used at Stage One (see Appendix 6) and collected 45 scripts. I administered a pupil questionnaire (see Appendix 13) and collected 45 responses in 25 minutes. I subsequently conducted two individual focused interviews, and three group interviews, each involving four pupils. In one interview, I probed a pupil's views of the intervention.

Doing work manually on a working out paper helps me to exercise my brain and how fast I am in maths.

Although she said: “it is really exciting to use a computer when learning maths”, she also stated that she had experienced some *unease* with ICT-enhanced learning, preferring the *textbook* approach.

You feel like it’s your own work, that you can actually do the sum; you have your own speed without the help of the computer.

This pupil further revealed apparent constraints of the ‘learning community’.

It’s okay if we discuss after attempting the work on our own, and not when we do it together because everyone gets their own answers, everyone has their own method. Once you get your answer, you can discuss to see who is correct.

This response illuminated the pupil’s view on the role of classroom ‘subculture’ in mediating *Grid Algebra* use. I recognised that pupils may not welcome exercising ‘collective agency’ in a collaborative learning effort; they exercised ‘disciplinary agency’ owing to being accustomed to ceding their responsibility to either teachers or textbooks. In general, the pupils’ consensus that the ICT-enhanced activity enabled exciting albeit challenging high engagement, participation and more accessibility to algebra in effective pupil learning behaviour was described as:

Grid Algebra makes learning algebra fun, interesting and easy. You get to understand better as you do it practically and you also enjoy. It helped me realise that expressions are not as hard as everyone thinks. The books make mathematics look hard a lot! I got to understand deeply the basics of expressions, how they come to be in a more real manner.

I used this experience to formulate ‘ground rules’ for Stage Three (see Section 3.2.3).

For the school’s Speech Day, a group of six pupils displayed a mathematics-with-ICT exhibit of the *Grid Algebra* software projected on a white screen. They explained Task 6: ‘Expanding and factorising’ to parents, teachers and curious pupils from higher classes: Form 2, 3 and 4. The 6 pupils acted as ‘teachers’ as they engaged their audience. One of the older pupils said:

This has really made me do more maths than I have ever had to do in class!

When she was asked who was doing the work, the computer or herself, the pupil stated that she definitely made the mathematical decisions, and thought for herself; she only used the computer to express her own ideas. This pupil was adamant that the computer was neither displacing nor replacing the teacher's input; the tool played a complementary role. She underscored the need for her to have heard the teacher describing how the tool works. She summarised her views as follows.

This makes mathematics very enjoyable, involves critical thinking, and is a more relaxed way of learning.

I will draw on this pupil's brief experience of learning with *Grid Algebra* in Section 4.4.1, plus Teacher T3's description of her role in the ICT-enhanced learning context in Section 4.4.6.3.

It became clearer to me that technology needs to be used in tandem with 'active teaching'. A final report based on the interviews and the questionnaire data was published in the 7th International Technology, Education and Development conference proceedings (Lugalia, Johnston-Wilder and Goodall, 2013). Following the preliminary results, I decided to continue with supporting participants in actual classroom contexts (see Section 3.2) as follows.

4.3 Stage Three: Main Study-Kenya

At this research stage, I was able to implement the intervention as I had planned by working within institutional constraints (see Section 3.2.3). I group-administered the MBRQ to pupils on a Saturday evening; Table 4.1 shows some analysis of the 270 complete responses collected in roughly 30 minutes. I then administered the 'Baseline' assessment on a Sunday afternoon. The 270 pupils took about 25 minutes to complete the instrument. I collected and marked the scripts; Table 4.2 shows the analysis of the Stage Three pupils' responses. I attended a total of 40 ICT-enhanced sessions, with 40 time-tabled minutes per task, in the five classes of over 50 pupils each. I abandoned both observing the non-ICT lessons and the focused pupil interviews in favour of retrospective discussions with the participants, based on their written work (see Section 3.1.5.4).

The management of over 50 pupils in the five classrooms underscored the teacher's role in interpersonal interactions during the teaching and learning of mathematics. As displayed in Table 4.1, 92% of 270 pupils (248) agreed with the statement that working in groups helped them to learn, while 99% (262) considered discussions a good way of learning. However, only 27% (72) indicated that group work, and 22% (59) talking to peers, was allowed in their classrooms. Some talking seems to have occurred, mostly when pupils were required to mark their own work. One teacher shed some light on this classroom norm whilst explaining the frequency of marking pupils' books by saying:

T1*: Our system is: we give them an exercise, they do the work, and then you mark the following day as a whole class; they have to participate. I involve all the (pupils).*

It appears that teacher-centred pedagogy was the favoured norm in the lessons since 87% (246) agreed with 'my teacher always shows us, step-by-step, how to solve a mathematical problem, before giving us exercises'. However, 91% (245) indicated their willingness to be given time to think on their own, an apparent disaffection with the established practice. It would appear that the mathematics teachers emphasised the value of mistakes in learning. 91% (245) were in agreement that 'my teacher thinks mistakes are okay as long as you learn from them'. This was reinforced by 94% (253) agreeing with the statement that 'making mistakes is part of learning mathematics'. The beliefs related to mistakes resonated with the 97% of 230 Stage Two pupils (223) in agreement with both statements.

Most of the 270 pupils indicated that they had high self-expectations in mathematics since 95 % (256) agreed with 'I think I will do well this year'; this matched 97% of 230 (223) indicating the same at Stage Two. It seems that 61% of the Stage Three pupils (164) believed in their ability to understand even the most difficult topics; the other 39% (106) reported feeling unable to engage with mathematics they deemed difficult. Only 42% (113) agreed with 'I understand everything we have done in mathematics'; therefore 58% (157) indicated 'not understanding' some concepts taught so far. Around 19% (51) admitted a tendency to quit trying when faced with adversity. One teacher explained that a considerable number of the pupils clearly needed support to learn, saying:

T1*: I can say that this class, at least three quarters are above average. There is a quarter there that requires to be really tutored in mathematics properly.*

The pupils' response to the statement that 'everyone can learn mathematics' was 97% (259) in agreement; compared to 98% of 230 (225 pupils) at Stage Two. This belief was emphasised with only 3% (6) of the Stage Three pupils agreeing with the statement that 'only very intelligent students can understand mathematics'. Another 13% (35 pupils) preferred to memorise the rules they learn; the response provided a measure of the pupils' value of procedural understanding. This suggested that 87% (235 pupils) would probably welcome gaining a more holistic picture when learning mathematics, hence ‘relational understanding’. Also, only 6% (16 pupils) considered ‘getting the right answer in mathematics is more important than understanding why the answer works’. According to the results, the majority of the pupils would welcome an approach to learning that emphasised conceptual understanding.

Item No.	Statement	270		Number of Pupils			
		Agree Strongly		Agree		Agree Slightly	
		%	No.	%	No.	%	No.
1.	Making mistakes is part of learning mathematics	58.1	157	30.7	83	4.8	13
2.	Working in groups helps me to learn mathematics	48.9	132	30.4	82	12.6	34
4.	It's a waste of time when our teacher makes us think on our own	4.8	13	2.2	6	2.2	6
5.	Everyone can learn mathematics	75.6	204	16.3	44	4.1	11
10.	Mathematics is used all the time in people's daily life	68.9	186	23.0	62	4.8	13
14.	I think I will do well in mathematics this year	77.4	209	15.2	41	2.2	6
16.	I like doing mathematics	68.5	185	21.5	58	6.7	18
19.	I understand everything we have done in mathematics this year	5.2	14	15.6	42	21.1	57
20.	I think mathematics is an important subject	89.6	242	8.9	24	1.1	3
22.	I can understand even the most difficult topics taught me in mathematics	14.8	40	27.8	75	18.1	49
27.	If I cannot solve a mathematics problem quickly, I quit trying	3.7	10	5.6	15	9.6	26
28.	Only very intelligent students can understand mathematics	0.7	2	1.1	3	0.4	1
29.	Ordinary students cannot understand mathematics, but only memorise the rules they learn	2.6	7	5.9	16	4.4	12
35.	Getting the right answer in mathematics is more important than understanding why the answer works	1.5	4	1.1	3	3.3	9
36.	Discussing different solutions to a mathematics problem is a good way of learning mathematics	74.4	201	19.6	53	3.0	8
40.	The mathematics we learn in school has little or nothing to do with the real world	2.7	2	1.5	4	8.1	6
42.	Knowing mathematics will help me earn a living	67.4	182	21.9	59	7.0	19
44.	Mathematics has no relevance to my life	1.1	3	0.7	2	0.7	2
54.	My teacher thinks mistakes are okay as long as we are learning from them	48.5	131	30.4	82	11.9	32
56.	My teacher always shows, step by step, how to solve a mathematics problem before giving us exercises	58.1	157	18.5	50	10.7	29
60.	We do a lot of group work in this mathematics class	4.1	11	8.5	23	14.1	38
68.	We are not allowed to ask our neighbours for help during class work	7.8	21	7.0	19	7.0	19

Table 4.1: Distribution of some pupils' beliefs in 'Attitudes to Mathematics' questionnaire (MBRQ)

The personal value of mathematics was indicated when 97% (260 pupils) agreed with ‘knowing mathematics will help me earn a living’. Only 3% (7 pupils) agreed with ‘mathematics has no relevance to my life’. These responses mirrored the claims I made in Section 1.1.2 regarding mathematics as a ‘gatekeeper’ in Kenya. A positive belief system was hinted at by 97% (261) of pupils agreeing with ‘I like doing mathematics’. Also, a possibly great value of the subject was suggested by 99.6% (269 pupils) agreeing with ‘I think mathematics is an important subject’. Emphasis on mathematics as a ‘human activity’ was upheld by 97% (261 pupils) agreeing with ‘mathematics is used all the time in people’s daily life’. It matched the 4% (12 pupils) agreeing with ‘mathematics we learn in school has little or nothing to do with the real world’. These pupils indicated their awareness of the societal importance attached to learning the subject.

In general, the responses indicated that the majority of the pupils valued mathematics as an important subject. The results suggested that the pupils would probably welcome increased opportunities to think on their own and discuss as they learn mathematics. Although quite a number of the pupils were aware of ‘not understanding’ the mathematics that they had been taught, many indicated their willingness to learn. The administration of the next instrument was therefore important in establishing the knowledge these pupils possessed in algebra.

In Question 1, about 60% interpreted correctly the meaning of ‘perimeter’ as distance around a figure; the other 40% seemed unable to simplify algebraic expressions. This indicated some limited understanding of symbolic language. Three pupils indicated an unwillingness to work with letters by measuring each side; they added the measurements to obtain their response.

In handling a word problem, 81% (218 pupils) formed algebraic equations which they solved successfully in Question 2. The data indicated the pupils’ ability and willingness to work with symbolic expressions. This compared to 95% of 230 (219 pupils) at Stage Two. The result was in sharp contrast to some of the strategies employed for the same question at Stage One. None of the 34 pupils used algebraic strategies: only one explained their arithmetic reasoning, most pupils simply copied answers from peers. At Stage Three, 9% (25) formed $\mathbf{4x + x = 10}$ as their equation; this indicated their interpretation of the question. Other pupils wrote $\frac{\mathbf{10}}{\mathbf{2}}$ as their interpretation of ‘two children sharing 10 sweets’, although this question stated that one had 4 sweets more than the other.

In Question 3, 67% (182 pupils) were able to simplify the expression; the other 33% (88 pupils) exhibited uncertainty when handling negative numbers. This ‘difficulty’ was exhibited by 12% (28 pupils) at Stage Two. In Question 4, between 10% and 12 % of the pupils were unable to evaluate letters given in an algebraic expression.

In Question 5, 76% (206 pupils) did not observe the symbol convention in algebra by using ‘×’ and ‘÷’ when writing: some offered $\left(\frac{b}{4} + 2\right) 5$ as the final expression. Others appeared to treat the ‘expression’ as an ‘equation’; they performed manipulations in an attempt to ‘solve’ it. The pupils’ difficulty pointed to a need to focus on formal algebraic notation.

When solving Question 6, 18% (38 pupils) offered incomplete work. While some pupils multiplied out the brackets correctly without simplifying, others erred in carrying out the process of expanding. Hence, there was a need to focus on simplifying expressions.

Notable pupils’ responses in Question 7 revealed 89% (240 pupils) as lacking understanding of the concept of factorising. Some pupils were seen to incorporate ‘prime factorisation’ which may be considered a ‘problematic’ ‘met-before’, and something to be watched out for when this concept was taught. This underlined ‘Expanding and Factorising’ as a major area of focus in mathematics lessons. 23% (61 pupils) and 17% (45 pupils) incomplete responses were offered, including $\frac{0}{4}$ and $\frac{0}{3}$ to Questions 8 and 9 respectively. The data seemed to indicate that some pupils had no mathematical meaning for when ‘zero’ was divided by a number. Question 10 underlined the ability to communicate mathematically when writing. Although 56% (152) of pupils had correct responses, this question highlighted solving linear equations as an area that required attention.

In summary, the ‘Baseline’ results enabled me to identify some key algebraic concepts to be considered, in conjunction with the teachers, when addressing the pupils’ learning needs. The written exercise served a diagnostic function in assessing the pupils’ knowledge of algebra, which could be built upon as the class learned ‘Algebraic Expressions’ in *Grid Algebra*. As was the case at Stage Two, none of the 270 Stage Three pupils got everything correct in the ‘Baseline’, despite some mathematics teachers showing initial misgivings that the content would be too ‘easy’. Although I allayed the pupils’ fears that the ‘Baseline’ was not a ‘test’, I noted with concern one pupil who failed to answer a single question. The ‘Baseline’ results highlighted the need to focus the pupils’ attention on structures in algebra using *Grid Algebra* in the ICT-enhanced sessions, and on communicating mathematically within written tasks.

I administered a pupil questionnaire followed by three focused interviews with teachers, thus collected the following views about learning algebra using *Grid Algebra* in mathematics.

Q.	Content	Response
1	Write expression for perimeter of shapes a) equilateral triangle, side '$\boldsymbol{e}$' b) quadrilateral with 4 sides labelled 'h' and one labelled '$\boldsymbol{t}$' c) pentagon with 2 sides labelled '$\boldsymbol{u}$', and the other 3 givens as 5, 6 and 5	Correct expressions in simplest form for (a), (b) and (c) by 167 (62%), 163 (60%) and 159 (59%) pupils respectively. Some wrote: a) $\boldsymbol{e^3}$ (11pupils); $\boldsymbol{e+e+e}$ (87 pupils) b) $\boldsymbol{h^4+t}$ (10 pupils c) $\boldsymbol{u^2+16}$ (11 pupils) 3 pupils measured the sides then added them to give a numerical result
2	Word problem	218 pupils (81%) correct algebraic strategy 27 pupils (10%)correct arithmetic methods 25 pupils (9%) formed equation as $\boldsymbol{4x+x=10}$
3	'Simplify' $\boldsymbol{7x+2y-x-3y}$	182 pupils (67%) correct; 88 pupils exhibited uncertainty with handling Integers, including $\boldsymbol{6x+-y}$ (39 pupils);$\boldsymbol{6x+y}$ (17 pupils); $\boldsymbol{6x-5y}$ (15 pupils);$\boldsymbol{7x+y}$ (3 pupils); $\boldsymbol{8x-y}$ (6 pupils); $\boldsymbol{8x-3y}$ (2 pupils); $\boldsymbol{8x+5y}$ (2 pupils)
4	'Substitution' a) $\boldsymbol{p=4k-10}$ Work out value of k given $\boldsymbol{p=50}$ b) $\boldsymbol{y=4n-d}$ Work out value of y given $\boldsymbol{n=2, d=5}$	a)242 pupils (90%) correct; majority of the rest found $\boldsymbol{k=10}$ (23 pupils) b)238 pupils (88%) correct; the rest erred in the substitution such that $\boldsymbol{y=7}$ (11 pupils); $\boldsymbol{y=8-15}$, responses of 23, -23 (12 pupils)
5	Write down an algebraic expression from a given description	206 pupils (76%) gave incorrect responses: -Did not observe symbol convention -Tried to manipulate the algebraic expression
6	Expand and simplify $\boldsymbol{3(x+5)+2(5x-6)}$	232 pupils (86%) correct. 38 pupils either give correct expansion, unable to simplify, or both incorrect; most common response $\boldsymbol{13x+9}$ (10 pupils)
7	'Factorise' a) $\boldsymbol{5x+10}$ b) $\boldsymbol{12a+4a^2}$ c) $\boldsymbol{32ab^2}$**+18b**	240 pupils (89%) indicated no grasp of concept of factorising; confused it with 'prime factorisation'
8	Find the value of $\boldsymbol{\frac{x-4}{4}-1}$ when $\boldsymbol{x}$=16	61(23%) incorrect; gave response as $\frac{0}{4}$
9	Find the value of $\boldsymbol{\frac{c+6}{3}-6}$ when $\boldsymbol{c}$=12	45(17%) incorrect; gave response as $\frac{0}{3}$
10	'Solve' $\boldsymbol{2x+3=10}$	152 pupils (56%) correct working Communicating mathematically

Table 4.2: Stage Three pupils' responses in the 'Baseline'

4.4 Thematic Breakdown of Participants' Perspectives

4.4.0 Introduction

I broke down the text in each participant's data into conceptual units whose frequencies I recorded, and show below in figures 4.1, 4.2, 4.3, 4.6 and 4.8. I present six themes that emerged: changed learning environment; learner agency; changed motivation; accessible learning; affect and enjoyment; variable teacher 'learning' behaviour. I illustrate each theme with raw data from responses to the pupil questionnaire, the interviews and the observations. I collate the pupils' views with relevant quotations from Teachers T1, T3 and T4.

4.4.1 Changed Learning Environment

This theme arose from comments related to the provision and development of opportunities for positive learning experiences attributed to the introduction of ICT in mathematics lessons. The ICT-enhanced sessions provided: variety, change in venue, alternative ways of working, and the acquisition of new skills as shown in Figure 4.1.

R236: *You get to have fun in learning mathematics outside the normal class lessons. Normal class lessons tend to be boring and the computers help to explore more on maths.*

U065: *It makes algebra seem simpler and fun than when it was taught in the classroom. It made algebra more fun and exciting.*

G022: *I got to do algebra practically thus increasing my rate of understanding the topic and dozing became less often as it was more fun.*

These responses compare to the view of using *Grid Algebra* as "an enjoyable, more relaxed way of learning that involves critical thinking" expressed by the older pupil in Section 4.2. While some pupils hinted at valuing their autonomy in the learning process, for others, the 'fun' lay in the change of venue for mathematics lessons from the usual rooms.

R229: *It broke the monotony of staying in class and gave me ideas of fun and educative ways of using a computer.*

P175: *It was fun to get to do algebra using the computer and it also broke the monotony of being in class all the time and this was easier to understand and more fun than if the teacher would come and teach the topic in class.*

Several pupils described valuing the combined change in the learning zone and conditions. Emphasis was placed on increased opportunities for discussion with peers and/or the teacher.

P182*: I got a chance to be out, away from the classroom and learn more while interacting with classmates around the computer.*

G028*: With Grid Algebra we work together and also it gives you a chance to use a computer.*

K151*: In class you don't get the chance to discuss freely and say what you don't understand but in the lab you can pour out your views/issues to the teacher, others in the group.*

Teacher T1 acknowledged the potential of collaborative learning.

T1: *They (pupils) feel relaxed, even those that do not know, strive to know from the others because they are working as a group.*

This was an apparent endorsement of the 'community of learning' provided by ICT-enhanced sessions. 40 % of the pupils (107) underlined the variety of learning resources.

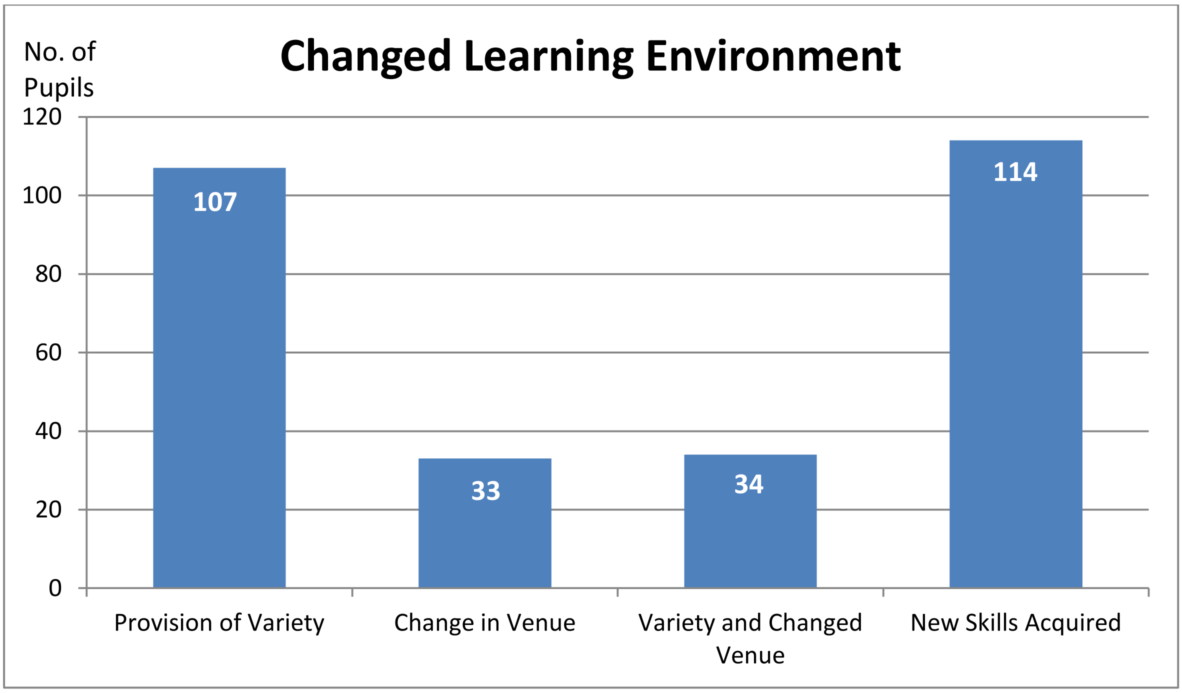


Figure 4.1: Pupils' views on the changed learning environment in pupil questionnaire

G028*: Algebra gets boring when doing it in your book daily.*

K146: *I think it is very interesting to do maths another way other than the usual work we do in class.*

P204*: We got to learn more about algebra beside the one in our books and also had fun. We got to use computers in our mathematics lessons.*

As Teacher T4 observed, the format of the ICT-enhanced sessions enabled pupil involvement in the mathematics lessons.

T4*: When using the chalkboard, rarely do you invite all of pupils to come and* ***participate.*** *But when using the computers, every (pupil) has access to it, and they can take part in the activity. Around each computer, we had at least about three of them so that in a lesson of 40 minutes, each pupil can easily be able to have at least 10 minutes doing the work, trying to practice, has the opportunity to interact with the content.*

To the teachers' surprise, the change to increased pupil involvement may have accounted for some pupils running to sessions (see Section 3.2.4). T1 pointed out (see Section 4.4.6.1) some pupils who rarely spoke in non-ICT lessons yet were engrossed in the ICT-enhanced discussions; 42% (114 pupils) attached importance to their acquisition of a variety of new 'skills'.

G040*: Aside from learning how to do algebra with ICT, I was also learning some computer skills, having not used it before. Most of the pupils like anything to do with technology so Grid Algebra gave me enthusiasm to do maths even more.*

P171*: Because I had never used a computer before, I get this chance to know some skills on operating the computer. It has given me another way of learning.*

R229*: I discovered easier and faster ways to solve algebraic questions in a totally exciting and fun way.*

K111*: For one it was fun and interesting thus making me understand better. Not only did we learn mathematics but also computers.*

The organisation of ICT-enhanced sessions was designed to encourage the development of other skills including externalising thoughts. The pupils were required to communicate mathematically whilst listening and speaking in turns in the small-group working format.

G035*: We were discussing, doing the work in groups with each other.*

K111*: We learnt together in groups at our pace since the tasks were at different levels.*

R270*: One is able to understand what the teacher says and you can do it practically as a group helping each other as you learn more.*

U091*: Since algebra was my weakness, I managed to expand my knowledge since it may be hard to understand a teacher. Working in groups I learn more from the others.*

These pupils appeared to feel safe when given the opportunity to share with, and learn from, their peers. My interpretation reflected that of Teacher T1, who observed:

T1*: They (pupils) feel relaxed, even those that do not know, strive to know from the others because they are working as a group.*

It seems that working out a question displayed on a computer screen while articulating their reasoning to other members of the group, or to the teacher, challenged some of the pupils.

K135*: All because it makes you challenged mentally in some way since you have to do the calculations with your mind without a pen or a pencil but the computer and reasoning with others in the group.*

G023*: I learnt to calculate sums fast and I also liked it because you learn how to use the software and get to listen to each other as you work together on sums and figure out expressions.*

The clear expectations that governed the ICT-enhanced sessions (see Section 3.2.3) required each pupil to talk through their reasoning to the group as they negotiated their solutions. All pupils took turns to listen and speak, a challenge that every pupil appeared to relish rather than back down from. The format of the ICT-enhanced sessions was regarded as useful in facilitating equal opportunities to the resources available, including increased teacher-pupil contact.

R227*: It is exciting and one gets to have a one-on-one talk with the teacher who deals with each of us at a time.*

P203*: Because it (Grid Algebra) makes algebra much more interesting than it seems in books. The lesson was much more interesting than before because everybody has an opportunity to try on the computer as they learn.*

Nevertheless, some pupils viewed the sharing of resources in these sessions as a limitation.

U086*: It made me even to learn how to use a computer though I have not known very well. Working with others in groups did not give me a chance to learn it but with extra time working on my own, I am getting it.*

R237*: It is beneficial for those who did not know how to use a computer and enjoyed doing maths in groups with those who are good in using it.*

Given the high pupil-to-computer ratio, the pupils had to share the resources and took turns performing learning tasks within their small groups. It is possible that those pupils who were opposed to sharing may have preferred an individual learning style. Alternatively, it may be that others with inadequate skills did not wish to be exposed in their groups. Several pupils viewed the study as an opportunity to acquire computing skills.

R234*: Because one who does not know how to use a computer will have more time to use it and in a short space of time, they will know as they learn maths on the computer. The Grid Algebra lessons changed the way I learn because I did not know how to use a computer but at least I am trying though I have not known fully.*

One teacher described their supportive role for pupils in the ICT-enhanced environment.

T3*: At times I would ask some pupils, 'Are you sure you are doing the right thing?' because I did not want to interfere....*

Some pupils indicated an awareness of the learning in other schools whilst expressing their appreciation of using *Grid Algebra* in their mathematics lessons.

R263*: Because I think it is privilege learning with ICT, with Grid Algebra, unlike other schools.*

R232*: Beneficial since some of the high schools may not be learning it, but to us we have an extra advantage than them.*

One teacher may have propagated this view when discussing the experience with the pupils.

T3*: I told the pupils, 'You are privileged, more than those in the higher classes, learning on computers! It could be the same for other topics'.*

In summary, it was difficult to isolate the particular aspects of the sessions which led to them being described by some pupils as being fun, enjoyable, or exciting. As Figure 4.1 illustrates, several permutations of the factors in the learning environment may have combined to affect the pupils differently. The ICT-enhanced activity provided variety, and a learning community with increased opportunities for learner agency.

4.4.2 Learner Agency

This theme revolves around verbalisation when using ICT-enhanced opportunities for active learner participation in mathematical activities. Many pupils indicated an awareness of the role of the computer in motivating them to take control of their own learning.

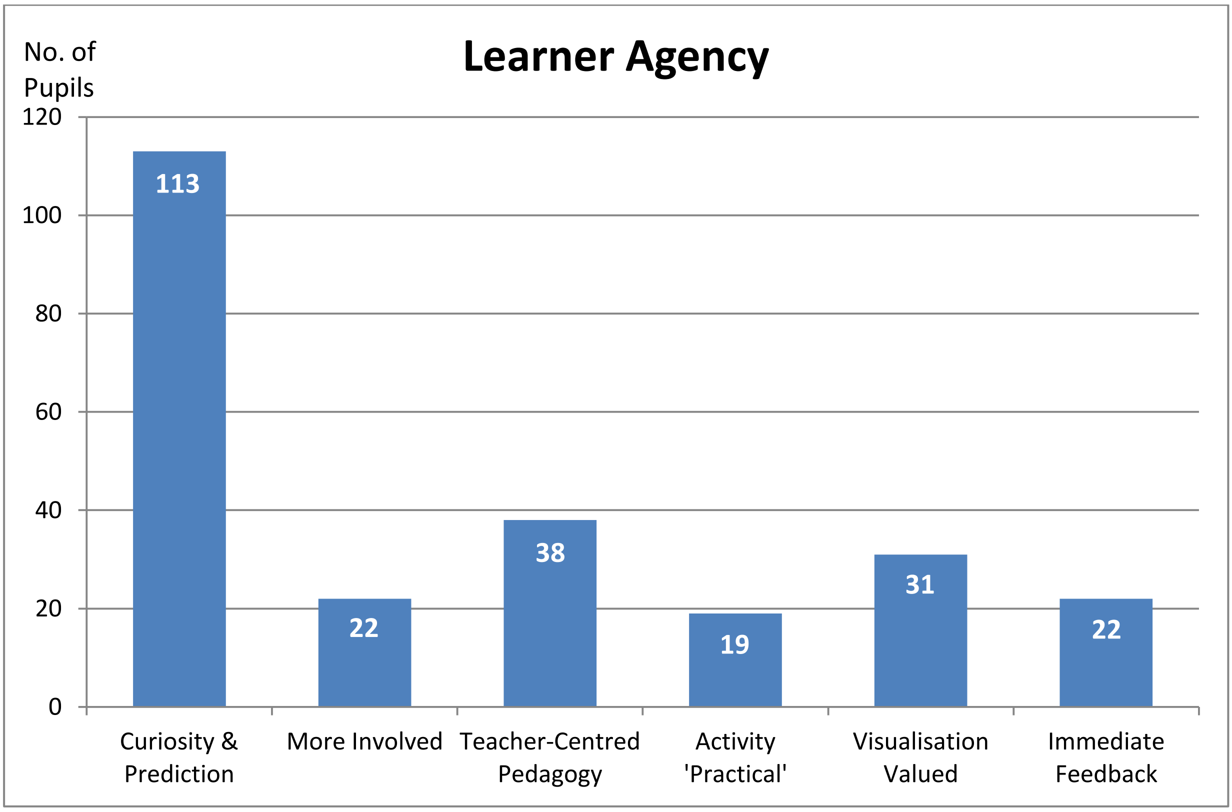


Figure 4.2: Distribution of pupils' views on learner participation in the computer-based activity

This alluded to increased learner agency in the mathematics-with-ICT environment. These pupils stated feeling more engaged.

P190*: I get to understand more when using Grid Algebra and I could also apply that even in my book. It is fun to use the computer to study. You get to study and became active. Some people do not get active in class but there (in the lab) one is totally active so that you may understand the steps one needs to take.*

G053*: I prefer to learn algebra through Grid Algebra. It takes me step by step to learn a certain expression. And it also involves my participation as a student.*

K146*: We are used to teachers in class every day and writing. It is good to explore technology. It is easier for me when I master the concepts used in the computer then I apply them on paper. It opens up the mind.*

G022*: It is also easy to operate the computer. I got to do algebra practically thus increasing the rate of my understanding the topic.*

P177*: It is important to move with technology. It has boost* (sic) *my thinking by making me find solutions for myself.*

These views resonated with those expressed by the HOD-Mathematics (see Section 4.4.6.5); they illuminated pupils' inactivity in teacher-directed instruction. The views echoed claims that I made about the Kenyan context where teacher-talk usually dominated in mathematics lessons (see Section 1.1.2). Other pupils apparently welcomed the increased opportunities for classroom dialogues around learning and involvement.

G044*: We so much just sit in class, the teacher would come teach and write on the board. We are not free to talk to our desk mates and ask questions but at least after this, we changed our learning zone and were able to discuss.*

P193*: It opens up one's mind so as to think and know how to operate different equations. It changed the way I learnt because in class, sometimes I cannot understand how the teacher came up with different solutions, but Grid Algebra explains and sometimes shows arrows on how to operate different equations.*

R254*: It was easier to work with a computer than by hand. Learning changed since I understood well than when a teacher explains it on the board.*

These statements pointed to pupils' valuing of the emphasis on articulation, variety and group discussions as learners negotiated their mathematical understandings. Some pupils expressed feeling excluded and demotivated in their regular mathematics lessons.

R223*: Having these lessons has helped me a lot in my class work, encouraged a lot to put effort in my work since I was previously very frustrated. We got to explore learning using computers and it made mathematics fun.*

K117*: Before Grid Algebra was introduced, I hated the topic in mathematics because I understood nothing! But when it was introduced, I like the topic because I enjoy doing it and also I can understand what I am taught.*

It is arguable that the passive role, to which these pupils were usually relegated, engendered feelings of frustration, lack of understanding or even dislike for learning mathematics. Figure 4.2 illustrates that 42% of pupils (113) valued trialling and re-trialling to satisfy their curiosity and to test their predictions as they explored and experimented on the computers.

R264*: The way we learned has changed in that Grid Algebra has made us understand more about algebra since you experiment by yourself making you get the concept well.*

U101*: This has helped us know how algebra is worked out. You are able to see and get to know how algebraic expressions are formed.*

P181*: It has helped solve my many algebraic puzzles. At least now I know how to manipulate different expressions. Algebra used to be a problem for me. I've been able to now understand how and why the final answer comes to be.*

U060*: It was beneficial since it helped me know how to calculate using computers. It helped me to be able to believe in the answers I give during calculations.*

This suggested that kinaesthetic activity was contributing to the pupils' cognitive development. Placing the ICT tool in the pupils' hands appears to have facilitated individual construction of mathematical connections; and hence 'deep learning' and 'relational understanding'.

P211*: I got to understand algebra in a different way, as journeys on a grid and solving equations as inverse journeys, which was much easier and more fun.*

R256*: Grid Algebra helped me understand deeper and deeper since algebra was a problem from primary but now it isn't a problem again. It helped me to understand algebra and the steps of solving any question from algebra.*

Others described the learning activities as 'practical'; they learnt more by 'doing'.

P183*: It helps us understand more fast* (sic) *since we do it practically. Before we learnt in class where most people didn't understand but Grid Algebra is easy to understand.*

G049*: Because the Grid Algebra helps you to work practically and helps you to be active unlike in just oral which we do not understand or is even boring. It has instructions and examples which made me understand and brought reality to the ways of working.*

P165*: We got to understand algebra better since it was practical than when taught in class.*

U095*: It has enabled me to see the real sense of mathematics in calculations.*

Many pupils valued understanding 'why' in mathematics over getting 'correct' answers.

P166*: Grid Algebra gives us chance to understand the concepts of doing algebra before tackling the actual question. Before, I did not know how I came up with the answers I got in algebra but now because of the program I am able to understand fully.*

K117*: I like the topic because I enjoy it and also I can understand what I am taught. Before Grid Algebra was introduced, I passed but I didn't know what I was doing since I practised the spoon-feeding method from teachers.*

These pupils' responses depicted some form of 'anxiety' with the focus on procedures rather than understanding in their usual textbook-based and teacher-directed instructional practice.

T3*: My happiness is when I bring a concept, in my own way, I give them the steps, and then I see the (pupils) working step-by-step, and they produce, that is my achievement, what I love!*

It seems that Teacher T3 valued transmission of knowledge through exposition. *Grid Algebra* use emphasised the structural relationships, hence consolidated the symbol convention.

T4*: It has been so difficult for them (pupils) to know 'how do I introduce this bracket or why or where?' But when using this* (pointing to the *Grid Algebra* program on the screen)*, they see the way the bracket is being introduced for them, they understand why it is so, so that even when it comes to written work, it becomes very easy, and the (pupils) can do it just like they saw on the computer!*

This indicated teacher awareness of pupils' problematic access to formal notation. Renewed focus on the symbolic language provided a more meaningful introduction in algebra.

T3*: The use of brackets was easier; it taught the pupils that when you see brackets, whatever is inside, when you open, you multiply with what is outside. We are not having an issue, no hitch on that.*

Other pupils underscored the scaffolding offered to their conceptual understanding by various features of the ICT tool as they learned.

G043*: It shows us the step by step of how to get to an answer rather than having some examples with teachers and you wouldn't know why.*

P179*; It's a new way and easier to understand compared to when you are only taught on paper. Grid Algebra provides answers, clues e.g. routes which help one understand better compared to when you are doing it without any clues.*

I encouraged all pupils to pause and think about the questions and feedback whilst discussing with each other. In each of the five classes, increased requests for assistance indicated pupils' confusion particularly with vertical movements and deciding what to multiply with or divide by in Task 7: 'Find the journey (letters)' during ICT-enhanced session 2. Apparently, they had forgotten how the software works. I attributed the difficulty to a week-long hiatus of sessions. I was prompted to recapitulate the association between grid movements and mathematical operations, directing the pupils' attention to the fact that the rows were pre-determined. Whole-class discussions appeared to refresh the pupils' mastery of the software. Most pupils worked successfully through levels 1, 2 and 3: some managed level 4 with increasing confidence. The pupils considered their peers' actions and listened to contributions; they squealed with delight at the interactivity. Towards the end of each session, I drew the pupils' attention to Task 25: 'What is the expression?' which they received very well; the activity gave pupils something to anticipate in one week's time.

R222*: It is kind of practical and easy to understand for example inverse journeys. In Grid Algebra, you are shown the arrows when reading and writing expressions and how to move operate, unlike the normal maths lesson.*

U101*: This has helped us know how algebra is worked out. You are able to see and get to know how algebraic expressions are formed.*

About one fifth of all pupils described a sense of apparent 'ease' in software-based tasks.

G048*: Understanding algebra became easier. Learning by using grid algebra is easy and methods for calculating sums are easier, like inverse journeys for equations.*

U100*: Easy to learn. It has helped me find another interesting way of learning math thus making me love it.*

R228*: Using Grid Algebra is easier to understand and it is easy to learn with. It helped me to improve in algebra which would most of the time, prove difficult to me.*

The views indicated the pupils' appreciation of *Grid Algebra* as a tool 'to learn' as well as to 'learn with'. Visual imagery provided by the software feedback was explicitly mentioned as supportive by 31 pupils.

P171*: It has given me another way of learning. It has made me form a mental picture when calculating algebraic expressions, e.g. when I remember how the arrows move in the computer.*

U108*: When taking my exams, I recall the Grid Algebra lessons that were so scintillating and interesting. The layout is so real and unforgettable thus sticks in my mind, so I imagine a grid in my exam. It helps a lot.*

R239*: This is because it has really assisted me in carrying out various algebraic questions and improved my understanding of algebra. It improved my learning skills while working algebraic questions since while answering the questions, I had an idea of how the question would look like in Grid Algebra.*

These pupils described forming and using mental images of *Grid Algebra*. The views suggest that dynamic visual imagery can support some pupils' accessing of mathematical concepts in both classroom learning and examinations. 'Fading' of visual support was described by other pupils who felt the scaffolding offered by the software had its limitations.

P207*: It helps us visualise the work in algebra but does not help when it comes to exams. The grid algebra lessons helped to understand the steps followed in an equation.*

However, this view was not shared by all pupils.

R217*: It shows the exact steps to take without taking shortcuts and can be used in exams.*

Nevertheless, these views highlight the effect of visual imagery on pupils' constructions of mathematical meanings, which I will discuss in Chapter 5. The immediate feedback provided by the software was regarded as vital to pupils' learning; importance was attached to its timing.

G019*: The questions are marked and you are corrected on the spot.*

G032: *It marked the work we did on the spot giving us scores and encouraged us to work some more. In class, it is not usual for the teacher to mark our work on the spot.*

R257: *Since learning algebra with ICT is interesting, one does not forget since they had a nice time. We get to learn mathematics in a more relaxed way than being in class. Sometimes the teacher bores or criticises you whenever you wrong a question* (sic). *But in the computer, you'll be told 'Incorrect' and learn afterwards.*

It seems that the temporary nature of the software feedback affected these pupils' self-efficacy when learning.

U078: *When we are wrong, the book will just be filled up with red crosses! And every time you open your book, it puts you off maths! With the computer, okay the bins and red crosses appear when you are wrong then disappear, no matter how many sums you get wrong, it does not stay with you. That motivates you!*

K133: *It made me stop and wonder, I thought I knew this! Then I was like Okay, let me try and get a better result in the next one.*

These responses illustrate the pupils' ability to respond to set-backs: their self-efficacy. They welcomed the instant and non-judgemental nature of the software feedback. It implied pupils' willingness to learn from their mistakes, as indicated in the MBRQ (see Section 4.3).

Many pupils' willingness to assume more control of their own mathematical learning through active participation in ICT-enhanced activity was clearly indicated in Figure 4.2. They valued making and seeing connections for themselves rather than being told facts in lessons by their teachers; they laid emphasis on 'hands-on' *Grid Algebra* use, and valued knowing *why*. Thus, these pupils prescribed a supportive role for mathematics teachers in pupils' learning.

4.4.3 Changed Motivation

This theme surrounds the change in perceptions of learning mathematics that the pupils associated with their use of *Grid Algebra* in mathematics lessons. Some pupils considered their non-ICT mathematics lessons in regular classrooms to be 'boring'; many described being quite disengaged with learning, a common report by their teachers.

P205: *It offers better understanding of concepts in algebra from the normal class lessons as one can actually see how the sums come about. They offer mathematical gain from the usual boring class lessons.*

G019: *It encourages me that maths is enjoyable and can be worked even on computers, and also makes me attentive in class since I can't doze off yet there is some fun programme in front of me.*

As Figure 4.3 indicates, several pupils noted changes in themselves. Some positive emotions (curiosity, enthusiasm, and interest shown in activity) are considered an indication of pupils' high self-efficacy and engagement with their learning. The ICT-enhanced activity seemed to engender raised levels of engagement and concentration in learning.

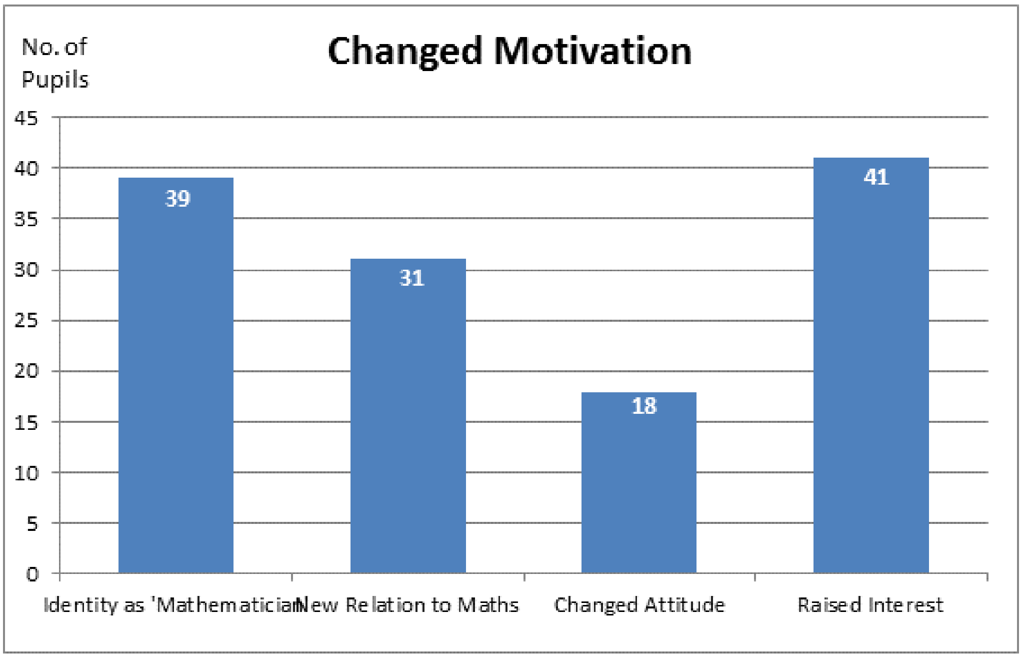


Figure 4.3: Distribution of some affective aspects in pupils' views of ICT-enhanced learning

I explained the need for every group to select software-generated tasks at lower difficulty levels and gradually work their way to higher levels. The majority of pupils complied with this advice to build self-confidence; they worked, tentatively at first in animated discussions, then with increasing concentration as they received, reflected on and responded to feedback.

K115: *It is more fun and easier to understand. It has improved my concentration level a great deal.*

G054: *I get to be alert every time a question is runned* (sic) *on the computer.*

One teacher described many pupils' evident positive disposition to learning.

T4: *I have never seen these pupils so absorbed in the work they are doing like they were today! The fact that the Grid Algebra was marking as the pupils worked and even giving them* *words of encouragement such as 'Excellent!' was really giving the pupils the impetus to do more and more work! That was great! It has ensured good classroom management. I have especially liked the way the software encouraged, motivated them to try more work.*

Other pupils (11%; 31) stated fresh perspectives in their approach to mathematics attributed to the weekly ICT-enhanced sessions.

***P187**: With Grid Algebra, I understand algebra a bit more than before. It is also fun to work with a computer while doing maths. Most of the time I do not 'anticipate' to learn maths. With Grid Algebra, I usually 'anticipate'; it made me understand maths especially algebra.*

***K122**: It helped me to like algebra because I never used to like it. But now I do like it and understand it. It made mathematics very fun and a lesson to look forward to every time.*

A marked change in pupils' attitude to learning was attributed to the new learning resource.

***K129**: In the past, I never used to get algebra so I had a negative attitude towards it but it changed when I started doing Grid Algebra and I understood algebra more.*

***R251**: It makes mathematics fun thus helps to develop a positive attitude towards mathematics.*

***U070**: This is because working with the computer changed my attitude in maths and made me see maths as easy and enjoyable.*

***R242**: It makes algebraic equations seem easier and makes one develop a positive attitude towards it. Using Grid Algebra is much more fun and exciting, helps one learn computer as well as mathematics, and it is much faster.*

Some pupils reported shifting relations to mathematics as a subject and algebra as a topic due to intrinsic values. The pupils signalled changes in their identity as 'mathematicians'.

***P194**: It's because it makes you feel assured of yourself and believe you can do algebra questions given confidently.*

***U059**: This shows us that mathematics is not only about books and mathematics is something you can enjoy doing.*

***P198**: It creates an attitude toward math and especially algebra since it is seen as a difficult topic. It opens up minds to show that there is always a way out of what seems to be difficult* *questions. Algebra could not have been well understood without using grid algebra. We know and understand, while in normal learning we try even if knowing is impossible.*

***P178**: Learning has really become interesting. The lessons give one encouragement to learn more and more about maths.*

The pupils showed increased confidence and competence in their ability as mathematical learners. The introduction of *Grid Algebra* was reported by the regular teachers to have raised pupil interest in their learning in other non-ICT mathematics lessons.

***T1**: Very good participation in class now because, everybody is striving to answer the teacher! You find them enjoying, and the interest is high now.*

***T3**: Now I go to class with pieces of chalk, and they work on the board! Substitution, they are doing it very well! And when you have Integers like -2, the bracket comes in automatically.*

These teachers' remarks showed awareness of pupils applying what they learned in ICT-enhanced sessions. Some pupils claimed that software use may have resulted in increased progress.

***G015**: It was beneficial to me because it helped me improve in algebra by making mathematics (algebra in specific) more interesting.*

***R267**: Using Grid Algebra changed learning in maths lessons because it is more interesting. It has helped me improve in mathematics and I did enjoy it.*

***P209**: It became more interesting learning while using the computer so it gives me (psyche) to continue.*

The pupils' increased motivation to learn algebra was associated with the overall contribution of ICT-enhanced activity. As shown in Figure 4.3, this led to the raised levels of interest, self-concept, and changed attitudes to learning. It appeared to have influenced the pupils' progress in mathematics, such that it warrants discussion. Measurement of the impact of *Grid Algebra*'s use on overall pupil attainment in mathematics was beyond the scope of my study. Nevertheless, both teachers and pupils drew my attention to pupils' progress in three examinations taken in the course of the school term. These examinations were set at a departmental level and marked by the respective subject teachers. Pupil performance is as shown in Figure 4.4; it includes results for the last examination taken prior to this research. The percentage mean scores per class in a given examination are indicated.

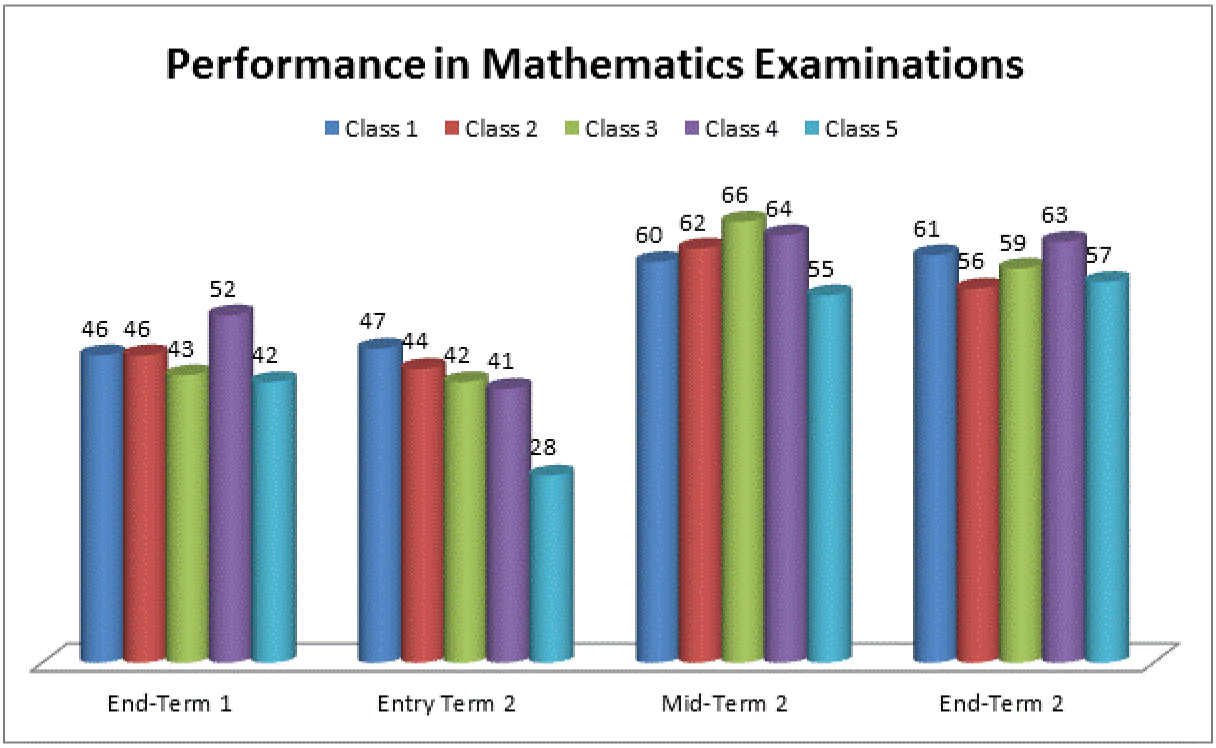


Figure 4.4: Pupil performance in mathematics examinations (Percentage means per class)

Questions on algebra featured in the end-of-term 2 examinations that were underway as I left the field (see Appendix 14). 'Surprising' feedback revealed pupils' handling of questions. Stage Three pupils demonstrated a firmer grasp of algebraic concepts compared with previous years. The mathematics teachers attributed this change to pupils' engagement with *Grid Algebra*. Their reports amplified a comparison of two cohorts taught by the same teacher.

***T3**: When I check their (entry) marks, unless something 'fluky', they are able; they are able! I cannot compare them to Form Three class (who would) brainstorm you, but these ones…*

This teacher observed that, although the Form 1 2010 pupils were considerably a more 'able' cohort (based on their end-of-primary education scores) compared to Form 1 2012 cohort, the latter handled 'difficult' algebraic concepts better. The mathematics attainment for these two cohorts is as shown in Figure 4.5.

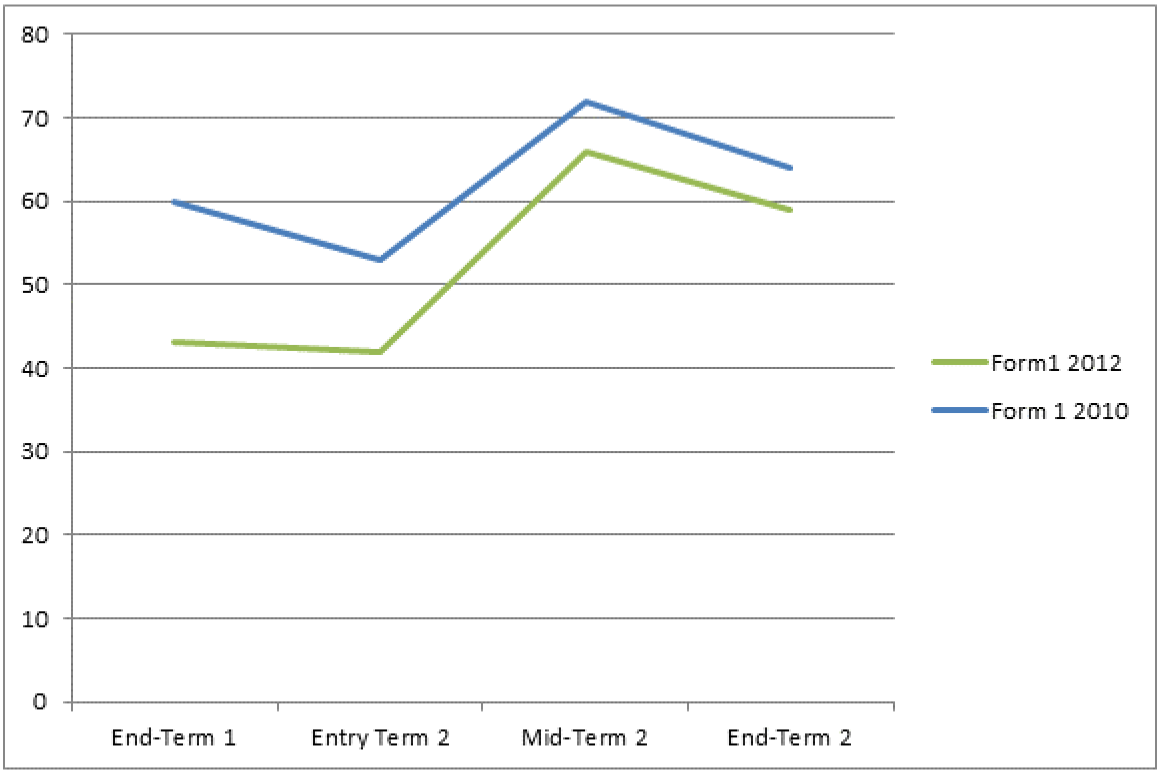


Figure 4.5: Performance of Form 1 2010 and Form 1 2012 (mathematics-with-ICT) in examinations

These results indicate that the 2012 cohort registered a higher improvement in mathematics than the 2010 cohort without the ICT-enhanced collaborative learning over the same academic period. Their teacher remarked on the notable differences in conceptual understanding.

T3*: When we tell them* $a\ (c + d) - b\ (c + d)$ *and ask them, 'What is that?' They say, 'It is the same!' They can see* $c + d$ *as… (Speech is barely audible), and that way, they are able to factorise, because it is in both terms. They know terms which I have to teach the Form Three now since it seems they have not gotten the concept. Sometimes when you write* $-8\ (-6)$*, they write* $-8 + \ -6$*, which means the concept of a term is not there! I am very grateful because the (Form 1s) have gotten it.*

Teacher T3 attributed the difference, discussed in Section 2.2.4, to the class' interaction with *Grid Algebra* defining the pupils' learning.

T3*: You have removed us from a stupor! What came out was, so mathematics can also be this interesting!*

In summary, the comparison data in Figure 4.5 helped me to make sense of the participants' appreciation of the intervention. The results in Figures 4.4 and 4.5 seemed to corroborate all participants' attributing increased pupil attainment in mathematics to the changes introduced in the learning context. Albeit pleasing, the impact of learning gains revealed in the mathematics examinations was incidental to my overall research aim: transformative learning processes.

4.4.4 Accessible Learning

This theme focuses on what Stage Three pupils had to say about ICT tool use affecting their *engagement* with learning 'Algebraic Expressions' and the extent to which it reshaped that learning. Figure 4.6 indicates that the majority of pupils (57%; 155) clearly reported that *Grid Algebra* enhanced their conceptual understanding and application in algebra.

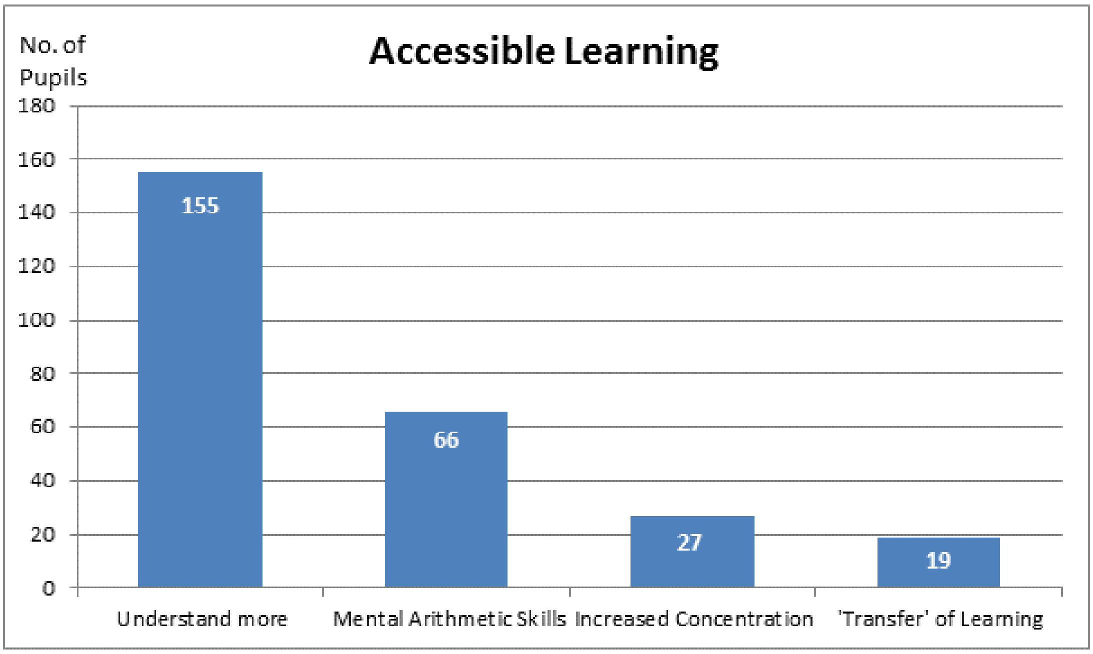


Figure 4.6: Distribution of pupils' views on accessibility of algebra

Pupils described understanding 'more', or 'better' than on their previous encounters.

K157: *It made me know how to work with algebraic questions better than before.*

One teacher attributed the pupils' difficulties in grasping algebra to its inherent nature.

T4: *I highly recommend for the software to be used in high school because algebra is quite a bit abstract because of the fact that we move from the numbers and are now using letters to represent, and* (inaudible) *the pupils are not able to see it clearly. You know algebra in itself* *also has other topics like Integers, plus Expansion, Quadratic Expressions...all these are developed from algebra. The software makes the learning of algebra much more interesting.*

The participants stated awareness of the software's role in enabling the pupils' understanding. By the end of the study, 83 pupils felt the topic was 'easier'.

P208: *Since I started learning Grid Algebra in the computer, I understand algebra better than learning it in class. Algebraic expressions have now become simpler and easy.*

G036: *Through Grid Algebra, I was able to understand algebra more and see how easy it is. The mathematics was still the same as that we learnt in class only this time we did it on computer.*

K116: *It has helped me especially the work concerning the Inverse Journeys. On my part, it changed my perspective on that topic and showed me ways on how to go about certain equations and now I can do the algebraic equations with utmost ease.*

The pupils' sense of 'ease' of understanding algebra seems to have been attributed to the idea of associating expressions with 'journeys'. Engagement with Task 12: 'Inverse Journeys' was singled out by 38 pupils as quite useful. It assisted many pupils to grasp various algebraic concepts including formal notation, writing expressions (33), and expanding/factorising (11).

P048: *It benefitted me since understanding algebra became easier to learn. Learning by using Grid Algebra is easy and the methods for calculating sums are easier, like Inverse Journeys for Equations.*

G030: *It help(s) in knowing the way of using the Inverse Journeys in Factorising and Simplifying expressions. It made algebra a bit easier and understandable.*

U059: *It shows us the steps in solving and made us understand more about the symbols we have in maths (e.g. use of multiplication and division signs).*

K162: *We learnt about Inverse Journeys which encouraged us to reverse the operations which we were not learning in algebra lesson.*

I monitored the pupils' developing 'symbol awareness' using written exercises in this study. Following the small-group pupil engagement with Task 25: 'What is the expression?' in each of the five classes' ICT-enhanced session 3, the pupils worked individually on a related worksheet (see Appendix 6). This written exercise had four questions drawn from level 3

(Expression with 3 operations) of the software-generated task because I had observed that very few pupils managed to work to difficulty level 4. Figure 4.7 displays results from the 263 scripts that I collected and marked to describe the pupils' grasp of formal algebraic notation.

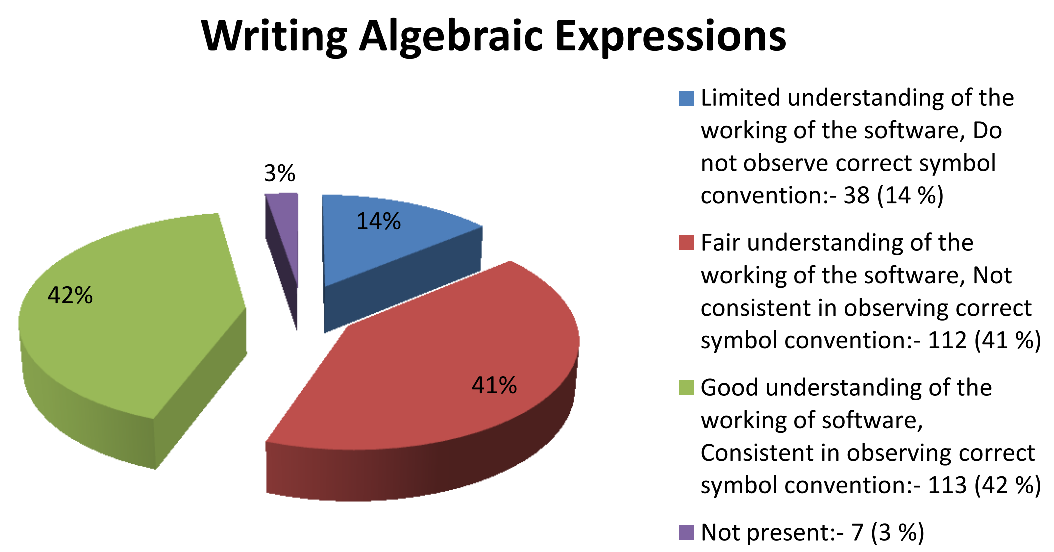


Figure 4.7: Pupils' results in pen-and-paper task on 'Reading and Writing Algebraic Expressions'

As illustrated in Figure 4.7, I considered that 42% (113) of the pupils had been successful in the task. This was an improvement from 24% of pupils in the 'Baseline' (see Section 4.3). More effort was required to draw pupils' attention to formal algebraic notation. Seven pupils were absent at the time of administration of the written task; these pupils were involved in co-curricular activity, Music Festivals, outside their school. When I administered this instrument at Stage One, only one of 34 pupils (3%) was successful after two attempts. 58% (26 out of 45 pupils) were successful at observing the formal symbol convention when writing expressions at Stage Two. I used the *Grid Algebra* software to revise the worksheet questions so that pupils could see the expected results. The purpose of the revision activity was to enhance the pupils' awareness of formal algebraic notation in ICT-enhanced mathematical learning.

Following engagement in groups with the software-generated Task 12: 'Inverse journeys' in ICT-enhanced session 6, pupils worked individually on a related worksheet consisting of 14 questions (see Appendix 11). Table 4.3 displays results of all the 55 pupils present in Class 2 at the time of its use (one pupil was absent due to ill health).

Content	No. of Pupils
Work completed; All responses correct Symbol convention observed; Adherence to instructions	24
Work completed; Good attempt, mostly correct Indication of lapse of concentration with questions on division	21
Incomplete work All correct	4
Incomplete work Wrong 'Inverse Journeys'	6

Table 4.3: Results of pen-and-paper task on 'Inverse Journeys' for Class 2

From the results shown, I considered that 51% (n = 28) of the pupils present had been successful; 38% (n = 21) demonstrated they had 'accepted' the use of brackets; line notation required more effort on the pupils' part. Although I distributed this worksheet, collected and marked scripts in the other four classes, its uniform administration was hampered by the various activities (see Section 3.2.3) in which pupils were actively involved.

Teachers T1 and T2 expressed appreciation of the pupil engagement with this particular task; they considered this task invaluable in preparing the pupils for future learning of concepts in mathematics. I also observed very good pupil-pupil interactions as they worked on Task 22: 'Substitution' in ICT-enhanced session 7. The pupils pinpointed errors in peers' reasoning when supported by software feedback as they assessed each other's views. Some pupils struggled with this task; they demonstrated low confidence with mental arithmetic and multiplication tables. I paired up and worked with one struggling pupil. Her confidence in solving algebraic puzzles appeared to grow gradually with increased practice. She attended the 'extra' sessions (see Section 3.1.5.4) regularly and worked on *Grid Algebra* tasks. Pupils worked individually on a worksheet with 6 questions based on difficulty level 3 (see Appendix 12). Based on the results of Class 2, 55%; 30 of the 55 pupils observed the symbol convention and presented all their working well. The gradual improvement from 24% to 42% and from 51% to 55% indicated *acceptance* of formal algebraic notation. T4 noted their developing 'awareness' of underlying structures, and the 'add-on' value of dynamic visual imagery to pupils' accessing algebra.

***T4**: They are very much comfortable with the concept (Expanding and factorising). I like the way it was presented in the software, showing the two different routes of arriving at an expression; that one really helped them.*

Teacher T4 summarised many participants' acknowledgement that using *Grid Algebra* had consolidated pupils' grasp of algebraic concepts as well as other areas of mathematics.

In all the five classes, I observed that, once all pupils settled in their small groups, they coped very well with questions in Task 21: 'Simplify' at levels 1 and 2. Several pupils appeared to struggle with solving questions at level 3 (expressions with 4 mathematical operations). These puzzles required pupils to combine ideas including: 'Integers', use of brackets and collecting like terms. Various groups requested assistance with reasoning through their solutions to the questions posed. This prompted whole-class discussions through which some teachers and I used explanations and question-and-answer techniques to stimulate the pupils' thinking. Such discussions seemed to enable these pupils to proceed successfully with working on the task.

The combination of multiple algebraic concepts in questions at difficulty level 3 appeared to pose difficulties for most Stage Three pupils. I observed the pupils who were challenged by simplifying puzzles such as $\boldsymbol{\frac{a-6}{3}+3-6}$. I attribute pupils' difficulties to applying concepts that the five classes were yet to encounter in their regular mathematics lessons. Their teachers and I resorted to 'active teaching' by stimulating pupils' thinking and supporting their engagement with the software-generated task. The pupils were observed to continue working after receiving assistance.

U099*: I learnt lots of things I didn't know in algebra through the computer, like writing expressions with brackets, the operations and more on Integers. The Grid Algebra helps us to develop new ways and skills of answering the questions and helps us to understand better.*

G007*: It made learning more understandable like now I am using Inverse Journeys in topics I was not really understanding like Linear Equations.*

Nineteen pupils made references to the notion of 'transfer' of learning.

K145*: Using ICT is beneficial because it is easier to understand and has all the steps laid out unlike that I would get from word of mouth. I have understood algebra and it has become easy for me to apply even when it comes in a written test.*

G011*: It made it much easier for me to do algebra on paper (theoretically). I now apply what I learnt on the computer in solving linear equations and even simultaneous equations.*

K151*: This is because as I use the computer, it prepares my mind on how to do the sums practically in class. Actually it expanded my knowledge.*

G026*: It was beneficial since the steps can be used when tackling questions in written form and is much simpler.*

Defining 'transfer' as the application of knowledge learned in one situation to another, the pupils reported applying skills learnt in the computer learning environment to pen-and-paper work both in class exercises and tests. This ability was underlined by their teacher.

T3*: Previously, we did algebra just the way it is then we went on to (Linear) Equations. Do you know what this (Grid Algebra) did? It combined the two topics! Now when I am giving them a question, 'John is three times as old as his daughter. In five years' time, he will be... five or ten years ago he was... Get his present age.' They could not perceive that, they could not conceptualize it! But now, I am telling you, it is just automatic!*

Several pupils also reported that the new learning medium effected a change in their execution of tasks in terms of the amount of writing required. This allowed pupils to concentrate more on learning and understanding algebra.

G042*: It has really helped me understand algebra more and have an interest in it. It made everything seem easier due to the less or no calculations, just understanding.*

U056*: They were beneficial because I learnt to do algebra using a computer. It has made me use my brain in most of the calculations instead of paper and pen.*

K136*: I got to discover an easier way of inverse journey for example and it helped in saving time while calculating. It gave you time to think mentally without putting it down on paper.*

The pupils described valuing the activity for saving time spent on writing work. This allowed them to concentrate on solving the mathematical puzzles on the screens. Figure 4.6 indicates that 66 pupils stated the ICT-enhanced learning experiences enhanced their mental arithmetic skills, making them think 'more' or 'faster'.

K137*: For me mental sums are not easy but with the computer I have at least speeded up even though it is not much. Solving the equations needs the brain without a pen. It helps me in doing mental sums.*

U076: *Grid Algebra has helped me to sharpen my maths skills by being able to think quickly and accurately on the spot.*

K142: *It was beneficial because you got to jog your mind and is also quick, requires training. In this way it helped us feel relaxed working out mathematics questions unlike in class.*

U077: *It made me think in a faster way than I did.*

P176: *It also makes a person work at the same pace of computer. It increases thinking capacity and speeds our thinking by how we see the working and also discuss.*

The pupils appeared to acknowledge the software's role in enabling them to apply themselves mentally as they handled learning tasks. Teacher T1 made a similar observation.

T1: *I have assessed their performance: they are good! Especially when you started the Algebraic Expressions, I saw them solve sums off-head, and they were doing it! I carried the process further to class, and then like in the Linear equations, I started asking them to, solve* $\mathbf{2x+4=8}$ *, and I realised that they are even able to think faster!*

Teacher T1 admitted to taking advantage of many pupils' enthusiasm and interest in learning algebra to cover other related topics in the schemes of work for the next term.

T1: *I was trying to link Linear Equations to that (Co-ordinates and Graphs) because I do not want them ... I do not want to go back to that! I want them to work so that once we complete Algebra, Linear Equations, Simultaneous (Linear) equations, Graphical, then they are done!*

Nevertheless, the increased mental activity may have contributed to 80 pupils feeling more competent and confident in mathematics as another 34 pupils claimed to have made progress in mathematics.

U109: *It exercises your mind and makes one think faster. It helped me be quick in tackling questions.*

G006: *It has helped me improve my maths, mental calculations and computer skills.*

P199: *I was very confident when answering questions on algebra.*

K143: *I easily answer the questions when they are brought in exams.*

R263: *It has made me work more efficiently, making my work accurate.*

G031: *I have learnt a lot which has greatly improved my grade in mathematics.*

Some pupils found themselves able to use visual memory when the software was not available (U108; Section 4.4.2).

The evidence of progress in mathematics attainment alluded to in these claims is presented in Section 4.4.3 (see Figure 4.4). It shows a marked improvement in these pupils' performance in mathematics examinations in the term described by their teachers.

In summary, as shown in Figure 4.6, many pupils valued the contribution made by their using *Grid Algebra* in facilitating more access to algebraic conceptual understanding. The majority underlined the requirement to apply themselves mentally in ICT-enhanced learning. Several pupils saw the change in execution of tasks as 'less calculations, just understanding'. Pupils welcomed the increased connectivity within mathematics: this may have contributed to their progress in attainment in algebra and generally in mathematics.

4.4.5 Affect and Enjoyment

U076: *It has made me discover that maths is fun when you understand what you are doing.*

This view defined for me involvement in activity based on pupils' perception of learning as 'pleasurable'. This relates to the emotional dimension of pupils in classrooms; yet it is often overlooked that the pupils' emotional state may affect the quality of mathematical learning. Using computers can allow pupils to explore ideas, exercise responsibility for their reasoning, hence facilitate enhanced understanding in the changed classroom environment and generate enjoyment of their learning. Figure 4.8 shows that 67 % of all the pupils (181) reported 'enjoying' the change in their learning due to the presence of computers in the mathematics lessons. The pupils clearly attributed their enjoyment to the use of the *Grid Algebra* software on computers.

K150: *Grid Algebra is easier to understand than oral hearing. It makes you enjoy maths and you want to do more. It has made me quicker in thinking. It has made me love mathematics more and enjoy the lessons more.*

R235: *I love computers very much and it is easy to understand when I am using it to learn mathematics. I learn by doing something I love.*

R225: *It's enjoyable and enables one to understand algebra easily. It has made learning algebra easier and more enjoyable.*

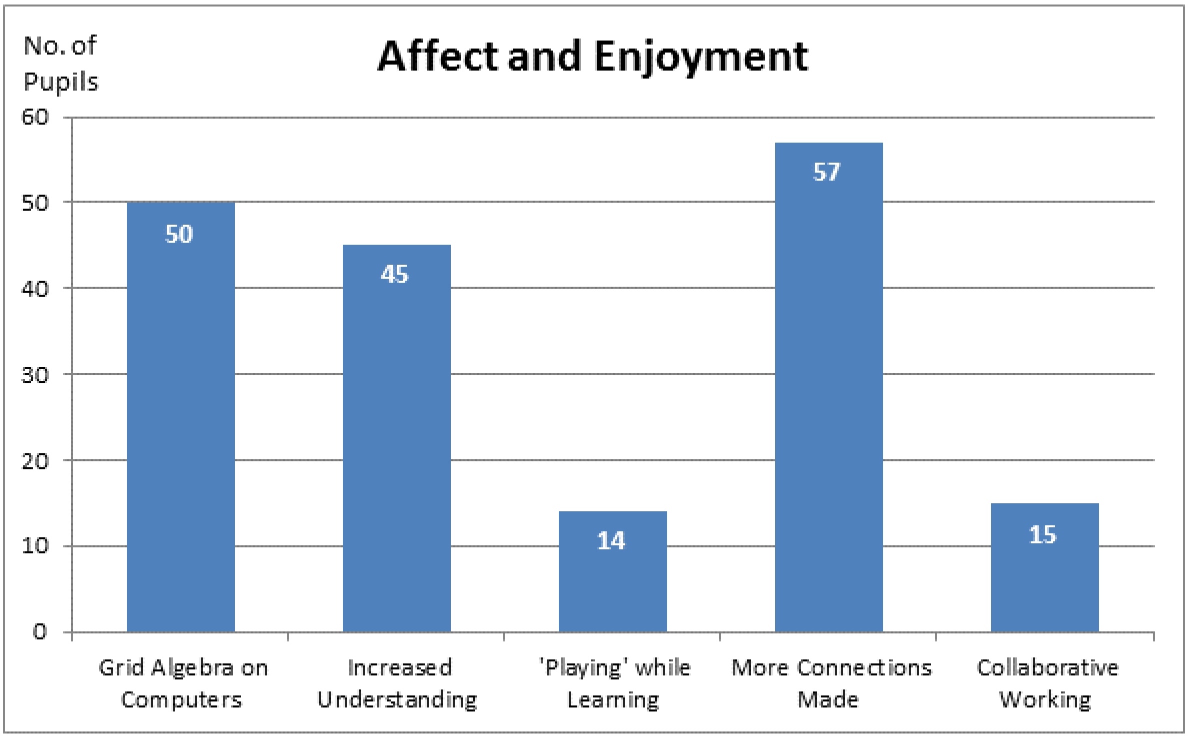


Figure 4.8: Distribution of pupils' views on the experiences in ICT-enhanced sessions

For other pupils, their 'fun' seemed to be due to their increased understanding of algebra.

***U104**: I never knew how to carry out algebraic expressions. Now I really enjoy calculating.*

***P184**: It makes a topic that felt so difficult, understandable. It made me enjoy algebra while doing it.*

***K154**: I got to understand a huge bit of algebra that was giving me a problem. This includes how to simplify, writing expressions and especially the use of signs as I was never able to tackle questions with positive and negative numbers. It has made me feel that mathematics is easier and even more fun as you learn more.*

'Understanding more' of algebra was something the pupils were clearly aware of (see Section 4.4.4). Figure 4.8 indicates that many attributed their 'enjoying' to increased connectivity of algebraic concepts.

***R220**: Using computers is very fun and when something is enjoyable, you understand easily. It is a more enjoyable way of learning and one actually sees the process of doing the sum.*

***K129**: Because for one it was fun and interesting thus making me understand better. Not only did we learn mathematics but also computers.*

For some pupils, mastery of the software was a gradually transformative process.

U075: *It was difficult at first but with more practice I was able to understand and work more easily, even on my own. It helped me learn faster with fun in mathematics.*

P206: *At first one may not understand well therefore just follows the arrows without knowing why you are adding, subtracting, dividing or multiplying. With more practice and knowledge of how (Grid Algebra) works you get to understand algebra much better.*

These pupils appeared to suggest that using the software enabled them to move from one form of learning to another; from *procedural*, where they simply made the movements across the grid without understanding why, to *relational,* once they had mastered using the software. It seems that pupils' mastery of *Grid Algebra* may have facilitated more effective pupil learning of algebra. This idea was emphasised by one teacher.

T3: *Conceptualizing in algebra has actually been taken to some levels higher, like 2 or 3, or more, more than that! They know the terms: that a term in algebra is differentiated by a sign, either positive or negative; these pupils know that!*

This teacher's view was amplified in several pupils' descriptions of their developing formal notation and use of mathematical terminology. I responded to some questions on 'variables', 'coefficients', 'terms' and 'constants' from textbooks the pupils had attempted. This seemed to help the pupils use the new vocabulary when they engaged in small-group discussions and writing.

K128: *It has benefitted me since it has helped me understand from the smallest bit. It helped me understand how mathematical terms are written in algebra e.g. instead of* $\mathbf{3} \div \boldsymbol{x}$, *we use* $\frac{3}{x}$.

P182: *I have a chance to at least understand mathematical signs (brackets, division line) and learn new terms like inverse, factorise.*

U089: *It has helped me learn a lot of what I did not really know before like writing expressions, use of brackets through Inverse Journeys.*

K152: *Doing algebra using a computer is much easy to understand... It helped me to understand how to multiply the signs in and out of brackets.*

For some pupils, it was the emphasis on communicating mathematical ideas when writing.

P185*: It helped me in learning how to arrange my calculations.*

G002*: I've known how to arrange my work neatly and present meaningful work.*

G043*: I can now write my mathematics work in order without jumping some steps, making my work tidy.*

U089*: It has helped me learn a lot of what I did not really know before like writing expressions, use of brackets through Inverse Journeys. It has really helped me improve in my grade. It has changed the way I learn maths what I did not know of I know now like how to arrange my work when writing.*

A sense of 'learning while playing' evoked in ICT-enhanced sessions was alluded to.

P165*: For a person like me, I had a lot of problems with algebra but now I have just found it easy and exciting because it is all about playing with numbers.*

All pupils were highly absorbed; some showed positive emotions and concentration on tasks. They worked collaboratively through difficulty levels. When the sessions ended, many pupils were reluctant to leave the room: pupils asked to be allowed to continue with the activity, as one said:

R233*: I love it!* (Dancing) *It is quite addictive; hard to let go…*

Whilst most pupils seemed very absorbed in solving the *Grid Algebra*-based tasks, one pupil inquired about acquiring the software for personal use.

P205*: It would be great to have it at home instead of just having computer games! I can play but at the same time I'll revise my work and build confidence in maths even as I relax.*

This pupil described the ICT-enhanced mathematics sessions as "different from the usual boring class lessons". Other pupils working in the adjacent group asked me whether a mobile phone 'app' for *Grid Algebra* was available. I honestly admitted to having no such information. The pupils' enjoyment of ICT-enhanced sessions was echoed in teachers' observations such as:

T1*: One thing we can say is that there is a lot of fun in these mathematics lessons compared to the classroom teaching.*

What emerged as clear was that the pupils appeared to develop a sense of ‘love’ and ‘enjoyment’ in their learning during the ICT-enhanced mathematics lessons.

K141*: Because it assist(s) one to understand algebra in a fun and exciting way.*

U082*: It has made me love mathematics and I have enjoyed a lot. It has enabled me have more love in mathematics than before.*

P200*: Because you have fun as you are learning at the same time. The way I understood Integers is different from the computer as in the side of arithmetic.*

K156*: It’s enjoyable and during that period one actually learns a lot. It’s really fun calculating maths there! It enabled me understand better.*

U065*: It makes algebra seem simpler than when it is taught in the classroom. It made algebra more fun and exciting.*

Some powerful emotions and effect on the pupils’ concentration were evident during the timed Task 13: ‘Make the Expression (letters)’. This particular task elicited reactions of increased chatter from pupils in all the five classes; observed norms of classroom protocol and social inhibition fell away whilst the pupils worked with great excitement in battling to ‘beat the clock’. So great was the noise generated in learning activity that several members of the senior management team rushed to the laboratory only to find pupils engrossed in their learning, totally oblivious of attracting undue attention. Every pupil remained very much focused on brainstorming in their small groups the possible movements to make across the grid. I saw pupils making physical actions as they spoke, and trying out different routes, to create prescribed expressions within time limits imposed by the software. I found it difficult to concentrate on or hear what some members in any one group were telling each other since my attention focused on ensuring the classes did not disrupt the rest of the school: these pupils were that noisy!

P180*: It made me pay a lot more attention since once the questions passed, it brought more questions and somehow harder.*

K148*: They were very enjoyable since they were in game so you had to be on your toes so as to get any sum correct.*

The pupils stated that albeit being ‘fun’, the software-generated tasks were in no way simple. With the variation in prior computing expertise attributed largely to the diversity in schooling background (see Table 3.2 in Section 3.2.3), some pupils seemed to value working and discussing in groups.

R251: *We got involved in group discussions and help each other with the work given which most of the time does not happen in the classrooms*

K160: *We get to learn to do mathematics using a computer and together, which makes it easier to discuss and understand.*

Great value was placed on the ‘community of learning’ facilitated by collaborative working since many pupils developed their confidence and skills of communicating mathematically as they articulated their spoken ideas.

G023: *I also liked it because you learn how to use the software and get to listen to each other as you work together on sums and figure out expressions.*

U077: *Grid Algebra has helped me understand algebra because I had problems in understanding and reasoning.*

In summary, many pupils clearly appreciated using the *Grid Algebra* software on computers in realising enhanced *enjoyment* of learning, conceptual understanding and connectivity. The task of distinguishing specific factors that engendered pupils’ *fun* was not easy. It is possible the pupils’ developing belief that computers make learning more ‘enjoyable’ may have been responsible for increased accessibility of ‘difficult’ algebra.

4.4.6 Variable teacher ‘learning’ behaviour

Teacher positioning shown in Figure 4.9 is based on ‘learning’ behaviours I observed in the ICT-enhanced sessions.

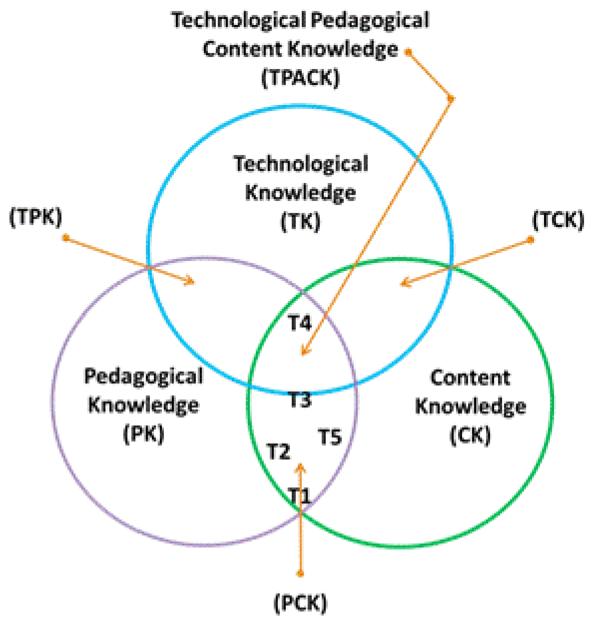


Figure 4.9 Mathematics teachers TPACK in the ICT-enhanced learning environment: Stage Three

4.4.6.1 Teacher T1

This teacher had been at this school for 10 years, with a proven track record of competence in Physics and Mathematics for 26 years. T1 was ambivalent about this research project despite professing to personal ICT use. He remarked severally about a potential threat to the pace of work posed by mathematics-with-ICT.

At this rate we will never cover the syllabus! Our (pupils) are lazy: they need to be pushed! The method is a bit too slow for our (pupils), and you see we are rushing to complete syllabus and appearing at the top! We may not do a lot…

These remarks implied that Teacher T1 was more concerned about covering the curriculum at speed, and being in league rankings, than pupils' learning. His reference to "pushing" pupils and his emphasis on content coverage hinted at leaning towards the transmission view of teaching. He commented about the organisation of ICT-enhanced sessions.

Why don't you divide the class into two; have one in the computer lab, and the other in their classroom in one week, then swap the groups over the next week?

This suggestion seemed to disregard the fact that there were five different classes, including his Class 1 of 54 pupils, involved in the study. T1 felt that the ICT-enhanced classroom learning arrangement was "not good". He argued in to favour of pupils working individually.

With this number, you may find you cannot control the (pupils). A whole class may not move with you unless they have the interest because you have at least three on a computer and only one can manipulate it. The others are left to watch what is going on. If we had computers for all, you will find lessons flowing and even move faster; but now, you slow down because you have to give the others a chance.

T1 appeared to be dismissive of enforcing either equitable 'hands-on' use of available resources among pupils or collaborative small-group working. He nevertheless acknowledged that some pupils were intimidated by the prospect of working on computers. On a contradictory note, T1 noted the prohibitive cost required to equip the school with adequate facilities; he cited the lack of a smart board as evidence disputing the myth that this school had well-endowed infrastructure. T1 voiced insights into some beliefs about ICT-enhanced learning contexts.

You can also learn from computers, give (pupils) programs to do by themselves even without the teachers' presence…they can be able to produce the work you expect.

This statement illuminated the teacher's sense of displacement by a computer's introduction in the learning ecology. T1 hinted at a discovery view of teaching by emphasising the equipment and pupils working things out for themselves. Pupil enthusiasm for ICT-enhanced learning drew T1 to observe and interact with pupils during most of session four after his initial non-participation. In session 4, I led a whole-class discussion on 'Inverse Journeys' which I described as performing the opposite mathematical operations to those given. It involved an oral exercise which emphasised use of mathematical language, formal notation and checking of working. I wrote the following series of examples on the blackboard.

$$6 + 2 = 8; 9 - 3 = 6; 4 \times 2 = 8; \frac{14}{2} = 7; 2(4 + 1) = 10;\ 2x + 5 = 7$$

The pupils identified 'Inverse Journeys' when given linear equations. These oral exercises elicited a very good level of pupil participation and responses; they answered the questions very well. I followed the activity with a whole-class demonstration of the software-generated task. I invited the pupils to work through level 1 as a class. Small-group working followed; the pupils selected this task on their computers. Most groups reached difficulty level 4 after which all pupils worked individually on a related written task as Teacher T1 observed.

What have you given the (pupils)? They really seem to love mathematics in your lesson! Did you see how absorbed they were? I never knew some of them could even talk yet here…

He noted the high levels of animation and pupil involvement after ICT-enhanced session 7.

Just look at how seriously engaged in learning pupils are! They really love coming to the lab. What are you giving them? They even come running to the lab for mathematics! Look at that one; she hardly ever speaks in class, and now here she is very much alive…

He remarked on the pupils' willingness to learn, and was hopeful that the evident enthusiasm for ICT-enhanced sessions would translate into high achievement in mathematics examinations. I elicited T1's perception of the affordances in the changed learning environment.

There is a lot of fun in these lessons compared to classroom teaching, and the interest is high.

Teacher T1 underlined the choice of algebra for integration of ICT as very appropriate seeing that algebra appeared in almost all other topics in mathematics. He argued that, once learners were disheartened by failure to grasp algebra, they were bound to remain "lost"; he added:

I love the idea of the 'Inverse Journey'! It prepares the pupils for solving equations very well. I also love the way the software is making pupils apply themselves mentally to mathematical tasks. This is laying firm foundations for 'Change the Subject' and Solving Linear Equations.

The pupils' attachment to ICT-enhanced sessions did not escape T1's notice. He claimed to have 'pushed' his class to cover two additional topics, and initiated a third outside the departmental schemes of work. T1 attributed this feat to the pupils' engagement with *Grid Algebra.* He described a transformation in the pupils' learning from disaffection to high participation in non-ICT lessons.

Oh! Very good, because everyone is striving to answer…you find them enjoying, even the one who is asleep wakes up!

T1 referred to an apparent 'transfer' of learning by the pupils.

They are using the skills learnt to go systematically, the way the computer has trained them.

Teacher T1 appeared to review his perception of pupil learning, their participation in lessons, and he considered alignment of the ICT tool with the mathematics curriculum. He conceded to having taken advantage of the raised levels of pupil interest in 'Algebraic Expressions', while it was fresh in their minds, to cover related topics: Linear Equations, Simultaneous Equations, Coordinates and Graphs. This indicated that T1 valued that use of *Grid Algebra* had in fact enabled syllabus coverage at speed, despite his earlier prediction.

4.4.6.2 Teacher T2

Teacher T2 had taught Physics and Mathematics for 19 years. He recognised *Grid Algebra*'s potential for pupils' learning of algebra in a brief reconnaissance session (see Section 3.2.4). 56 pupils in Class 2 had their first ICT session in the presence of the Mathematics departmental ICT lead, with whom he had exchanged teaching duties. T2 observed briefly a group of pupils negotiating with each other and evaluating formative feedback during session 2. However, he missed subsequent time-tabled sessions for various reasons including a "late breakfast". This behaviour seemed to indicate an affective trait of 'avoidance'. I kept T2 informed of the pupils' progress as described in Section 3.1.5.4. Despite his absence during the ICT-enhanced sessions, T2 commented on the corrected scripts of pupils' work on 'Inverse Journeys'.

This is very impressive work that is preparing the pupils for 'Change of Subject' of formulae, and they are doing it in one step!

These remarks focused on the pupils' cognition that I had highlighted as the weakness of written work in Section 3.1.5.1. While my research aimed to address disengagement in mathematics learning, T2 exercised his individual right to selective participation in this study.

4.4.6.3 Teacher T3

Of her 29 years' experience teaching Accounting and Mathematics at secondary school level, T3 had taught Mathematics exclusively for 3 years. She had minimal experience and contact with computers. Before ICT-enhanced session 1 involving 53 pupils in Class 3 began, T3 asked, "Must I be there?" I reiterated the agreed plan for teachers to first observe for some weeks. She struck me as very apprehensive at the thought of taking over the lead from me. T3 complied with the plan and attended all 8 ICT-enhanced sessions. She remarked on the pupils' engagement as the first session concluded.

They are very excited! They seem to have found that interesting and have enjoyed themselves

The pupils' reactions in the ICT-enhanced activity had drawn T3's attention to what the pupils were doing on the computers. She observed and listened to pupil-pupil dialogues without intervening. This was important training for both participants. The pupils had opportunities to *talk* whilst they solved questions; the teacher watched and listened to the pupils' reasoning. T3 and I observed 53 pupils highly engrossed in negotiating grid movements to create prescribed expressions in the second session. T3 noted that some pupils were unwilling to exit the program and proceed to the next lesson.

The pupils are very absorbed.

During Class 3's fourth ICT-enhanced session, I was pleased to observe Teacher T3 engaging with several groups of pupils as they worked on software-generated tasks. She assisted these pupils to make sense of the mathematics on computer screens. This was a key turning point in my research: Teacher T3's gradual inclination towards the 'new' learning context. T3 seemed very happy with the 'inverse journeys' idea; she remarked on its importance to pupil learning. She considered it very useful in the teaching and learning of algebra, with solving linear equations and algebraic fractions. There was a subtle shift in T3's stance towards computers in mathematics that was likely triggered by the pupils' enthusiasm in the ICT-enhanced sessions. She noted the changed engagement in pupils' learning. I realised from retrospective discussions that T3 was paying very keen attention to the pupils' talk and the information presented on the computer screens. During the interview, Teacher T3 expressed her preference for placing emphasis on 'exposition' (see Section 4.4.2). She offered the pupils what she regarded as clear explanations of concepts, detailing steps to be followed in non-ICT mathematics lessons. This suggested a transmission belief orientation to teaching. She expressed her dismay that many pupils failed to learn from her intensive efforts and from practising the textbook exercises.

However, while observing pupils being very absorbed and involved in the ICT-enhanced activity, she was more interested in *what the* pupils were learning. These pupils' reactions in the new learning experiences directed T3's attention to changes in the pupils' attitudes to learning mathematics; they enjoyed it and found it interesting. T3's presence in the learning context provided a crucial structure and involvement to stem poor pupil-pupil interactions in one ICT-enhanced session. Some pupils shouted at each other and refused to work together in groups. The negative pupil behaviour forced T3 to intervene; she encouraged the members to work in harmony and listen to each other respectfully. After the awkward start, the pupils eventually settled down to participate in the whole-class discussion and small-group working. It underlined for me the difficulty of effective collaborative learning in classrooms.

Such involvement led to invaluable observations of the teacher 'learning' about pupils learning mathematics with ICT. I noted that T3 never explored *Grid Algebra* personally. Nevertheless, she clearly appreciated the contribution of the dynamic visual imagery to the classroom 'subculture' in mediating 'difficult' algebraic concepts. She became intent on aligning what these pupils appeared to learn from the software feedback with specific learning objectives. She underlined her supportive role in ICT-enhanced sessions as follows:

It made it possible for me to see what the pupils were doing, which area has not been covered, and how do I approach that area in the classroom lessons. That one helped me especially when tackling Algebraic Fractions. We did it so well, now it is okay.

T3 applied her confidence in mathematical knowledge; she reflected on the conceptual algebraic understanding of two cohorts she taught (see Section 4.4.3). She underscored *Grid Algebra*'s success in making algebra as explicit as possible: it showed pupils the underlying structures and process. T3 professed to a change of heart regarding computer teaching in mathematics. She described kinaesthetic activity as beneficial for the pupils' learning.

It is hands-on. It challenges the pupils because the computer tells them, 'You are incorrect', so they go thinking, they were fully involved! They would all be alert, working! Nobody would waste time dozing. And for every lesson in the lab, they would come running…when they are in class, they switch off! But here, they could not! It was hard to tell them to go for the next lesson; that is what we need in maths! That encouragement, that spirit!

As the pupils interacted, Teacher T3 pointed out the salient features and misconceptions. To me, T3 corroborated some of the views expressed by the older pupil in Section 4.2. What appeared powerful and exciting about this was T3's recognition of the role she carved out in the pupils' learning in that she aligned pupils' developing algebraic constructions with conventional mathematics. In Teacher T3, I witnessed the greatest transformation representing a shift in her attitudes and beliefs towards ICT-enhanced learning brought about by the pupils' instrumental genesis.

4.4.6.4 Teacher T4

T4 had 5 years' experience of teaching Chemistry and Mathematics at secondary school, and was confident about content knowledge. T4 described his technological experience as "just average, and not so much in terms of ICT". He expressed preference for 'dialogic teaching'; he invited and considered pupils' views before building on them. I observed that T4 was very empathetic and highly committed to pupil learning. He paid special attention to pupils' skills levels and backgrounds; he considered how these factors affected new learning. T4 interacted with 54 pupils in Class 4, and attended all 8 ICT-enhanced sessions with keen interest. He underscored pupils' difficulties after the first session.

Some of these pupils claim they have never operated a computer before in their lives! And this software is so good! The pupils have to deal with questions on Integers which many of them have problems with! This offers great practice. The pupils really enjoyed themselves, the experience, and are looking forward to the next. I am also very keen to know more about it so that I can help them. I look forward to teaching with it! The only problem is there is a lot of work...and getting in here is a problem.

I informed T4 that teachers could access the software in the SMASSE room, plus I welcomed requests for exploratory sessions from willing individuals. T4 remarked on the pupils' high levels of engagement in the ICT-enhanced learning activity.

... since it was marking for everyone on the spot; that is more efficient than when they are working in their books and I am unable to get to each and every one of them during the lesson! Did you see they were very happy and willing to try more? For me this represents an excellent, relevant example of what the Ministry is trying to represent!

Teacher T4 echoed the ICT lead, T1 and T2, by observing that *Grid Algebra* linked 'Integers' to algebra. It provided pupils with vital practice. He listed additional knowledge and skills as:

They need to be computer literate, and know their mathematical tables, very well!

T4 underlined the effect of feedback from the software upon pupils' motivation to learn; T4 emphasised that some pupils lacked the requisite basic computing skills to participate fully in the activities. After ICT-enhanced session 2, T4 decided to personally explore with the tool. He responded to the software's feedback, and appeared quite pleased to see the result displayed as:

$$2\,(f-4)=2f-8$$

Whilst acknowledging the potential of the tool to mediate algebraic concepts to pupils, T4 voiced concerns about CEMASTEA's initiative to integrate ICT in education.

I would urge those who are pushing for it to be very careful, and ensure that the teachers have the know-how of how to incorporate it because it is not just a matter of integrating it. They also have to make sure it is effective!

T4 made time for a two-hour 'hands-on' session. He explored *Grid Algebra* personally while we discussed using ASEI-PDSI format. He focused on Task 6: 'Expanding and factorising' as he prepared to assume the lead an in an ICT-enhanced session, and teach pupils 'Expanding' as removal of brackets and 'Factorising' as inserting brackets in expressions.

The incorporation of ICT in the mathematics lessons was observed by CEMASTEA officials. Teacher T4 introduced the 38 pupils present to the task in a whole-class discussion. About 11 observers crammed into the computer laboratory for this session. This restricted movement in the tiny room despite a relatively lower number of pupils. Pupils were free to engage with the software-generated task after a whole-class demonstration. T4 kept his intervention to a minimum: he offered occasional, succinct explanations to the class. He drew attention to areas the pupils found challenging as they worked through difficulty levels 1 and 2. The demeanour of the pupils and the teacher was impeccable in the face of such intense scrutiny. The lesson flowed smoothly as the learners appeared completely relaxed and unfazed by the presence of the observers. I answered questions from the curious 'visitors' who wondered where pupils were getting the numbers to divide, multiply, add and subtract from. Observers whispered to each other, "amazing!", "one of a kind." At the end of the session, the observers left, only to be replaced by an even bigger group who were ready to observe the next ICT-enhanced session. Unfortunately, the 'visitors' left the school without offering any feedback to the teachers whose lessons had been observed, a lack of 'review' of ASEI-PDSI (discussed in Section 1.1.2), which was lamented by T4.

It is important that these officials have come around today and seen for themselves that we are steps ahead of them in this area of ICT integration, that we are using a resource that cannot be found anywhere in Kenya!

This was reference to a lack of locally-produced resources for ICT integration in education in Kenya (see Section 1.1.2). T4 played a crucial role in supporting the pupils' instrumental genesis, whilst developing mathematics TPACK.

4.4.6.5 Teacher T5

In 5 years as a qualified teacher of Chemistry and Mathematics, Teacher T5 was in the unique position of teaching mathematics for the first time at this school; he mostly taught Chemistry. T5 was an experienced and confident user of computers. The ICT-enhanced sessions for Class 5 of 54 pupils were the most affected by the disruptions to school routine that I listed Section 3.2.3. I held make-up sessions with the class. Thus, he observed only one session, without interacting with any of the pupils. By that time, T5 had started teaching 'Algebraic Expressions' in non-ICT mathematics lessons. The class was looking at the introduction of brackets, the effect brackets had on signs of numbers ('Integers') and factorising. Following a brief observation of the class, he remarked that, although the pupils appeared to be doing algebra competently on the computers, they were unable to tackle the exercises in textbooks. T5 added that he "told the class off for being carried away by all this technology business!" This statement may have indicated an unfavourable attitude towards computer teaching in mathematics.

T5 had made no attempt to explore the ICT tool even after observing the pupils, albeit briefly in session 2. After session 4, every teacher of Science and Mathematics received notice of an impending visit to the school by Ministry of Education, Kenya officials and a CEMASTEA team. Some lessons were to be observed. T5 then requested a 'tutorial' in using *Grid Algebra.*

Please take me through this software so that I can have a better hang of how the program works as I plan to prepare and present the lesson around 'Substitution' since the next topic is 'not ASEI-PDSI friendly'.

This initiated my discussing with T5 and the HOD-Mathematics about computer teaching in mathematics. The teachers described using computers in mathematics lessons as synonymous with empowering pupils' ownership of learning as advanced by student-centred approaches.

HOD: *If we fully adapted this approach, teaching will be less strenuous, relaxed and thoroughly enjoyable, with the pupils taking on the bulk of the learning process.*

Both HOD-Mathematics and Teacher T5 considered the ICT-enhanced sessions as extremely compatible with ASEI-PDSI principles (see Section 1.1.2) as they were 'activity-based' and 'student-centred'. The consensus was that by making use of the under-used school facilities, computer-based activity espoused the principles of 'experiment' and 'improvisation'. This left the teacher free to move around and observe what pupils were doing because the software promptly marked the pupils' work. The teachers remarked that, in this way, the teacher's role changed from 'instructor' to 'facilitator' in support of pupil learning while the pupils took on more responsibility for their own learning.

A team of about 50 Ministry of Education inspectors and CEMASTEA trainers arrived in the school. This session was one among many lessons observed in Science and Mathematics. I refer to the school inspectors and trainers as 'observers' since I had no way of distinguishing between them. 15 to 20 'observers' attended the ICT-enhanced session in Class 5; all the 54 pupils were present and shared 20 computers. In a bid to control the learning activity using both computer-based and pen-and-paper methods, T5 continually disrupted the pupils' tempo of working. Rather than allowing the pupils to engage freely with the software-generated task, Teacher T5 read out questions for pupils to work out: he insisted on individuals moving in the cramped room to have their books marked. This behaviour suggested the teacher's adherence to the *textbook approach.* It is likely that his actions on this occasion were for the benefit of education officials observing the lessons. Nevertheless, the events illuminated for me the argument that teachers, as much as the pupils, require feedback to learn effectively. I underline the lack of formative feedback I noted from policy makers for the teachers 'learning' to incorporate ICT into subject teaching. Despite upholding ASEI-PDSI principles (see Section 1.1.2), the CEMASTEA trainers failed to *review* the ICT-enhanced sessions on the aspects of teaching and learning that they had observed with mathematics teachers. It seemed to devalue the commendable efforts invested by these teachers in the preparation and execution of lessons. When some form of feedback was finally made available, the report was: "they were very impressed". The emphasis was on a spatial perspective of the lessons: the report contained no information to move either teaching or learning forward. I therefore raise questions about the effectiveness of ICT integration into education on the premise that local resources were yet to be developed.

4.5 Summary

The findings in this chapter strongly indicate the transformation of the pupils learning of algebra at secondary school with an emphasis on collaboration, articulation, agency and variety mediating the use of an ICT tool. Using *Grid Algebra* led to many pupils' engagement, enjoyment, new confidence, and eagerness to participate in mathematics. I found that six major themes emerged: changed learning environment, learner agency, changed motivation, accessible learning, affect and enjoyment, and variable teacher 'learning' behaviour. In the next chapter, I relate these findings to the literature in Chapter 2.

Chapter 5: ANALYSIS

5.0 Introduction

In this chapter, I interpret the findings in Chapter 4. I base my understanding on sociocultural theories of learning, complemented by Social Cognitive theory, as detailed in Chapter 2. This analysis involves: the changed learning environment; learner agency; changed motivation; accessible learning; affect and enjoyment; variable teacher 'learning' behaviour.

5.1 Changed learning environment

Of crucial interest to me was the nature of classroom interactions, notably the teacher's role in supporting pupil learning. The prevalent mathematics classroom practice in Kenya, depicted by pupil G044 in Section 1.1.2, appeared to me to be skewed towards a teacher-centred pedagogy. This appears to have severely diminished pupil agency (Boaler and Greeno, 2000). In Section 4.4.1, pupil R251 listed group discussions, and pupils helping each other, as rare; pupil K151 described pupils as unable to verbalise their lack of understanding; Teacher T4 lamented the constraints on participation that are due to large pupil numbers in a 40-minute lesson activity. Knowledge was conveyed to the pupils through teacher expositions (Alexander, 2008). In Section 4.4.2, Teacher T3 defined succinct explanations for pupils to reproduce which pupil K117 described as "spoon-feeding". The practice echoed the report by Nardi and Steward (2003) that teachers, albeit being driven by good intentions, tended to present over-simplified chunks of information that carried little meaning to many pupils in mathematics classrooms.

Over 90% of pupils valued discussions, believing that collaborative work in mathematics would help them to learn. The meaning they attached to these practices is not clear from the MBRQ. Morgan (2000) saw discussions as interpersonal ways for pupils to appropriate mathematical meanings for themselves. The MBRQ data suggest that a possible 'mismatch' exists between pupils who value 'connected' knowing and their teachers' emphasis on 'received' knowing (Boaler and Greeno, 2000), explained by pupil P193 and Teacher T3 in Section 4.4.2. Teachers were apparently unaware that most of their pupils preferred to be given time to think on their own. Having over 50 pupils per class implied that collaborative learning (Barnes, 2000) would be necessary since it is difficult for one teacher to reach every pupil, unless as a lecturer; but, lectures need careful strategies to be effective at communicating. In Section 4.4.1, pupil P175 described valuing *social* learning (see Section

3.1.3); pupil U091alluded to ‘difficulty’ due to problematic teaching (Brown et al, 2008; Nardi and Steward, 2003), while pupil R227 valued a teacher’s supportive role, described as crucial (see Section 4.2) for pupils learning in ICT-enhanced contexts (Sutherland et al, 2009).

In Section 4.4.1, pupil R263 described valuing the ICT-enhanced experience in the study through ‘hands-on’ use of *Grid Algebra* in mathematics lessons for pupil-centred learning. Following Neo-Vygotskian theory (Mercer, 1994), my emphasis in the changed pupil learning was on ‘talk’ with justification, turn-taking and small-group working (P182) instead of the learning behaviour described by pupils G028 and P204. Based on Vygotsky (1978), I considered articulation to be important for enabling more formative feedback on pupils’ algebraic ideas, thus promoting processes of self-reflection and assessment by others described by pupil U091. Externalising ideas in alternative ways of working (pupil K146) allowed the clarity and difficulties in the pupils’ thinking to be exposed, and shaped in expression (Pimm, 1987) as described by pupil R270. Both pupils and teachers required different induction for positive learning experiences to be effective (Elder, 2012). Pupil P182 described valuing the peer interactions; Teacher T3 described listening to pupils’ algebraic *talk* (Foster, 2014) and stimulating pupils’ thinking; pupil P203 and Teacher T4 claimed that expectations as ‘ground rules’ (Edwards and Mercer, 1987) encouraged the pupils to ‘have a go’, leading to their reasoning being valued (Ruthven et al, 2011; Mercer and Littleton, 2007).

In Section 4.4.1, the participants described valuing the provision of variety in ICT-enhanced activity, the changed venue and the collaborative working; Teacher T1 recognised the role of capable peers and *Grid Algebra* in scaffolding pupils’ learning; pupil K111 described positively working in small groups at their own pace, since the software tool provided tasks at different difficulty levels; pupils K135 and G023 described the relish of the mental challenge in learning and exercising ‘conceptual agency’ (Pickering, 1995); others valued controlling their activity (G022), honing new skills in mathematics (R229), communication (G035) and computing (P171). The pupils indicated their awareness that this had been made possible, on this occasion, by the use of *Grid Algebra* in their lessons alongside textbooks (P204), and not just the software.

5.2 Learner agency

My focus is on whether pupils were enabled to participate competently (Gresalfi et al, 2009) in algebraic activity in keeping with a sociocultural approach (Lerman, 2001). It was important for me to establish what stance pupils adopted towards mathematics. In Section 4.4.2, pupil G053 described their being active and involved, an indication that they were willing (Bell, 1996) to engage with learning algebra. Mercer and Sams (2006) and Hoyles et al (1994) emphasised inducting pupils in assuming ownership of their learning by expressing ideas for themselves. Many pupils stated their preference for having an active role in meaning-making (pupils P177 and P190) over being passive learners and being 'told' by teachers (pupil G043); Teachers T3 and T4 described kinaesthetic activity aiding pupils' in understanding formal algebraic notation; pupil R264 valued self-exploration with *Grid Algebra*; pupil P166 upheld knowing the *why* in solutions (Tall, 2013; Wilensky, 1997).

In the MBRQ, 97% of pupils (259) embraced the view that 'everyone can learn mathematics'. This result suggested that many pupils possibly held an incremental view of intelligence (Dweck, 2000). This view was corroborated when only 2% (6 pupils) agreed that 'only very intelligent students can understand mathematics' and 13% (35 pupils) considered 'ordinary students' as those who valued memorising *rules* over understanding. However, care is vital when interpreting this bearing in mind the view by Barnes (2000) that it is not easy to know what is meant by 'understanding'. Pupils may claim to comprehend concepts taught quickly, and yet be seen as more interested in procedural learning (Kieran, 1992) than fitting ideas together (Tall, 2013) between various aspects of mathematics. It was important for me to establish whether pupils focused more on knowing *how* (the techniques) than on *why* (connections) or valuing deeper understanding of mathematics they learned. Pupils are clear about 'not understanding' when given opportunity to speak (Rudduck et al, 1994). The results from the MBRQ indicated that this was the case for 58% (157 pupils) concerning the mathematics they had been taught so far; only 6% (16 pupils) valued getting 'right' answers more than understanding 'why answers work', a hint at procedural and surface learning for this minority. In Section 4.4.2, pupil P181 described valuing 'relational understanding' (Skemp, 1976); pupil U101 stated that seeing dynamic images in *Grid Algebra* enabled access to diverse reasoning (R257) and 'connected knowing' (Boaler and Greeno, 2000) which made mathematics "real" (U095; G049). I consider such pupils to be 'deep learners' (Abbot et al, 2009) who valued more conceptual understanding in algebra.

I determined the need for directing pupils' attention (Hewitt, 2011) to structural relationships (Kieran, 1992) if pupils were to make sense of 'Algebraic Expressions' following 'Baseline' results (see Table 4.2). The presentation of written work can reveal a learner's reasoning through their use of symbols. It signifies the linguistic aspects of mathematics, symbolic thinking (Bruner, 1961) in algebra, and communication to include speaking and writing one's ideas. Therefore, I monitored the pupils' ability to convey mathematical meanings as they spoke, as well as when they engaged in writing. I invited the pupils to challenge the meanings conveyed in examples of work in revision activity. For the sake of algebraic reasoning (Bokhove and Drijvers, 2012), I focused on developing mathematical language revealed by response to 'Baseline' Question 5, which required pupils to link reading to writing an algebraic expression. Some pupils, who seemed to 'manipulate' final expressions, may have been seeking a numerical solution in their quest for "closure" (Tall and Thomas, 1991). Such pupils seemed unable to work with letters in algebra unless they knew what the letters represented in their expressions, in itself an unwillingness to operate on specific unknowns (Küchemann, 1981). Using *Grid Algebra* to revise Question 5 allowed pupils to encounter formal algebraic notation (Hewitt, 2012), and hence to develop their 'symbol sense' (Arcavi, 1994). While my intention was to familiarise the project participants with operating the software, revision activity consolidated symbol convention and enabled drawing attention to:

- the retention of the signs for addition and subtraction whilst those for multiplication and division 'disappear', being replaced by brackets and division respectively;
- the use of mathematical terminology, including 'variable', 'coefficient', 'term' and 'constant';
- the combination of groups of numbers and letters to form terms that are regarded as a single object upon which an operation may be performed.

These pupils appeared to appreciate seeing that the structure of the final algebraic expression given as solution to this 'Baseline' question was $5\left(\frac{b}{4}+2\right)$ and not$\left(\frac{b}{4}+2\right)5$.

In Section 4.4.2, pupils R222 and P211 described 'transfer' (Lobato, 2003) attributed to a focus on mathematical processes (Sfard, 1991) in Task 12: 'Inverse journeys' evoking a structural conception of algebra (Kieran, 1992). This illuminated the important role played by symbolic language for pupils' algebraic thinking from a problem-solving stance (Bell, 1996) according to Teacher T3. Thus, it reflected the development of symbolic thinking and problem-solving in pupils (Bruner, 1961). Emphasis was on pupils exercising 'conceptual agency' since every pupil justified their reasoning in connected knowing (pupil U101) while paying attention to mathematical meanings conveyed in their written and spoken contributions (Pickering, 1995). In Section 4.4.2, pupil R239 described gaining *symbol awareness* and problem-solving skills since the software assisted pupils to develop proficiency in symbolic language use in algebra.

5.3 Changed motivation

Since the 1980s, advances in technology have witnessed the development of educational software whose design can mediate pupil learning of the 'difficult to teach' concepts (Hewitt, 2012). Kaput (1992) asked whether using digital technology in classrooms can "help us do better what we have been trying to do" (p. 548) in terms of: hastening pupil progress; increasing the percentage of pupils succeeding; and raising test scores on previous assessment. Kaput listed "perceived human and monetary resources and decision-makers' vision and expectations" (p. 548) as potential constraints on the contribution of new technologies' to achieving the mathematics education objectives. Few research studies, like Galbraith and Haines (1998), have managed to translate the motivating effect of computer use on pupil engagement to 'better' learning gains.

Nearly all 270 pupils seemed aware of, and possibly accepted the significant role accorded to mathematics in Kenya (see Section 1.1.2). I acknowledge that the pupils' interpretation of the MBRQ items limits my ability to distinguish between the relevance of qualification and the relevance of pupils' mathematics learning experiences (see Section 3.1.5.6). Ninety-five percent (256 pupils) indicated a high self-expectation to succeed in mathematics. To Hannula (2002), expectations can be a crucial motivating factor in pupils' learning. Based on the theory of Bandura (1986), I considered the mathematics-related beliefs would affect pupil learning behaviour. As shown in Table 4.1, 19% (51 pupils) indicated their tendency to quit when faced with adversity; 61% (164 pupils) indicated having a positive conviction in their ability to access 'difficult' topics taught. This stance suggested to me that most Stage Three pupils would welcome strategies intended to enhance 'resilient' approaches (Lee and Johnston-Wilder, 2013) to mathematical learning.

In Section 3.1.1, I regarded pupils' knowledge of multiplication tables and acquaintance of algebraic concepts as 'met-befores' (Tall, 2004). Following Ausubel (1968), researchers have argued for establishing, and building on, pupils' prior understandings that are 'supportive'

(McGowen and Tall, 2010). It is fundamental within a constructivist framework that a learner's existing knowledge must influence the extent to which their new learning is organised and related to their existing knowledge (Bruner, 1960), hence enabling possible misconceptions to be identified and addressed. This argument informed my need to establish those pupils' mathematical 'met-befores' that were either 'supportive' or 'problematic' in order to help shape the subsequent teaching of algebra (Juwah et al, 2004). Claims in the MBRQ by 58% (157 pupils) that they had "not understood" concepts previously taught (see Section 4.3), and had 'difficulty' accessing algebra (pupils K129 and P187 in Section 4.4.3) vindicated my plan.

I considered 60% (163) of pupils to be successful in interpreting 'perimeter' and letters used as 'objects' (Küchemann, 1981); the rest indicated their 'symbol sense' (Arcavi, 1994) by using indices in final expressions. During the whole-class revision activities, I invited pupils to express the 'meanings' that such answers conveyed to them, and to offer alternatives; in this way any misconceived symbolic thinking (K128; Section 4.4.5) was exposed and addressed. Three pupils indicated their focus on obtaining numerical solutions (Tall et al, 1999); they measured then added the sides of given figures. This strategy indicated that the pupils were possibly attached to procedural learning; they may be lingering in what Tall (2004) described as the 'first world' of mathematics. Many pupils' vulnerable number sense was exposed when they attempted Task 1:'Calculating'; the task challenged these pupils' security of 'met-befores' in arithmetic, especially with negative numbers, described by their teachers in Section 3.2.4.

Throughout this study, many pupils were observed to struggle with 'Simplifying expressions' despite having had previous encounters with the concept: this qualified as one *problematic* 'met-before' (McGowen and Tall, 2010). In the 'Baseline', Question 3 required pupils to ignore letters and to collect like terms without ascribing any values to letters (Küchemann, 1981); only 67% (182 pupils) managed to simplify the expression. The data indicated that a majority were able and willing to tolerate 'uncertainty' as argued by Bell (1996) and Arcavi (1994). Several pupils appeared to forget parts of Question 6: either 'expanding' or 'simplifying'. This underlined a need to develop 'reflection' (Juwah et al, 2004) or 'self-assessment' (Black et al, 2003) in pupils with a firm emphasis on them checking their own work prior to submission for marking.

I attributed the evident struggling of numerous pupils when engaging with Task 21:‘Simplify’ (see Section 4.4.4) to increased demands made on the pupils’ cognitive abilities to synthesise concepts in given questions. I established through the ‘Baseline’ data and confirmed from the primary mathematics syllabus (KIE, 2002) that ‘Factorising’ was indeed new at Stage Three; ‘Factorising’ and algebraic fractions concepts could hardly be considered ‘met-befores’ (see Section 1.2.3). The teachers and I, as mathematics ‘experts’ (Bliss et al, 1996; Wood et al, 1976) in some ICT-enhanced sessions, were prompted to use an ‘enhanced context’ (Ruthven et al, 2011) in keeping with providing scaffolding and ‘fading’ learning support for pupils. Thus, we assumed ‘support’ roles (Monaghan, 2004) and built on pupils’ mastery of *Grid Algebra* and the expressions to stimulate pupils’ thinking. We refrained from solving the puzzles for pupils within either whole-class or small-group discussions. Using questions and explanations, we assisted the pupils to link their current knowledge of ‘fractions’ to new concepts. In Section 4.4.3, pupil P194 described feeling self-confident whilst solving algebra questions; Teachers T3 and T4 supported pupils’ ‘instrumental genesis’ (Guin and Trouche, 2002), and developed their own mathematics TPACK (Niess et al, 2009).

Written responses to a word problem informed me about ‘strategic competence’ (Kilpatrick et al, 2001). In this study, pupils’ proficiency with formal notation indicated to me their ability and willingness (see Table 4.1) to engage with the linguistic aspects of algebra (Bell, 1996) needed to link visual, verbal and symbolic ideas (Noss et al, 1997), as described by G023 (in Section 4.4.1). Küchemann (1981) explained that pupils’ forming of equations hinges on their interpretation of the language in questions; such interpretation may determine the accuracy of mathematical statements derived, and may lead to incorrect answers. To a large extent, the emphasis favoured in the teaching of algebra can direct the strategies that pupils employ. The ‘Baseline’ data indicated that pupils chose arithmetic methods at Stage One (see Section 4.1); a majority of pupils at Stages Two and Three (90%; 243) chose algebraic strategies in Question 10. These results underlined the argument by Bednarz et al (1996) that favoured curriculum options will inform pupils’ ‘strategic competence’. The results illuminated for me the stark differences in approaches to school algebra used in various countries. Researchers (Healy et al, 2001; Bednarz et al, 1996) have argued that pedagogic approaches can reduce ‘connectivity’ (Askew et al, 1997) in pupils’ mathematical learning. It is imperative that we enhance pupils’ appropriation of ideas by combining pedagogic approaches in algebra (Mason and Sutherland, 2002).

In Section 4.4.2, pupil U078 described working with accuracy, speed and a tendency to avoid collaboration to obtain good results where 'correct' work in mathematics was highly valued; pupil P183 described their lacking comprehension in 'procedural' learning. Nearly all pupils (94%; 254) indicated their dissatisfaction with the prevalent emphasis on 'performance goals' (Middleton and Spanias, 1999) in the MBRQ, and instead valued knowing 'why' whilst they learned. In Section 4.4.3, pupil P205 described valuing the 'structure' (Skinner and Belmont, 1993) in ICT-enhanced sessions for changing current perceptions of learning algebra (P198); many pupils were observed to be more persistent when they found the work challenging (P178), were willing to spend more time on challenging learning activities (G019), and claimed developing self-belief (P242); Stage Three pupils' enthusiasm, curiosity, interest and optimism indicated their changed motivation in mathematics lessons. Such qualities signalled pupils valuing a crucial shift towards 'learning with understanding' (Sfard, 2001) from 'performance goals' argued by several researchers (Middleton and Spanias, 1999; Skinner and Belmont, 1993).

Bandura (1989) asserted that self-efficacy contributes to motivation by determining: the goals pupils set for themselves; the effort they apply in learning tasks; how long they persevere in the face of adversity; and their 'resilience' to failure. Most pupils acknowledged greater effort on their part to engage with learning algebra in mathematics. In Section 4.4.2, pupil K133 described feeling "okay" when unsuccessful, then re-trialling tasks. Many of these pupils may be considered to have developed high self-efficacy and 'mathematical resilience' (Lee and Johnston-Wilder, 2013); they regarded their 'challenge' of learning algebra as something they were capable of mastering. They were seen to develop rapt interest in the ICT-enhanced sessions; they appeared to recover quickly from failure and set-backs. In Section 4.4.3, pupils P194 and U070 described developing a 'can-do' stance. The achievement in mathematics examinations (see Figure 4.4) supported these pupils' claims of improved attainment (G015) attributed to 'understanding more', and underscored the centrality of algebra in mathematics (Teacher T3) argued by Kieran (1992); the data strongly suggests that pupils may have been driven by their progress. Middleton and Spanias (1999) advanced that success can be a powerful motivating factor to achieve, such that learners' effort and motivation in mathematics may be considered interchangeable: pupils who apply greater effort are considered to be motivated. Many pupils associated their increased motivation to learn with the overall contribution of using *Grid Algebra* including: scaffolded access to algebra (R267); more 'conceptual agency' (U059); enjoyment (K122); more 'connectivity' (P209) due to increased structure, autonomy and involvement in mathematics (Gresalfi et al, 2009; Skinner and Belmont, 1993). High pupil engagement and confidence engendered in the ICT-enhanced sessions appeared to influence pupils' raised levels of interest and their changed attitude (Hannula, 2002) to learning; Teachers T1 and T3 described the pupils exporting this change to non-ICT lessons, indicating 'transfer' (Lobato, 2003) of learning and pupils developing a 'productive disposition' (Kilpatrick et al, 2001) in mathematics.

5.4 Accessible learning

This was the most profound theme for me within participants' views. In Section 4.4.4, 57% (155 pupils) indicated greater access to algebra in the pupil questionnaire; I consider this 'learning with understanding' (Sfard, 2001); pupil K145 described linking visual (Teacher T4) and symbolic (U059) to verbal meanings in mathematical, advanced as increased 'connectivity' (Askew et al, 1997); pupil U099 described developing 'skills' and algebraic understandings owing to *Grid Algebra* use providing different modes of representations (Bruner,1966) and ICT enabling pupils to experience concepts from multiple perspectives (Abbot et al, 2009); pupil G007 described valuing 'relational understanding' (Skemp, 1976).

Van Amerom (2003) noted the strong dialectical relationship between arithmetic and algebra, a view shared by Healy et al (2001). In Section 3.2.4, I described the regular teachers' encounters with *Grid Algebra* in rare exploratory sessions; they acknowledged the ICT tool's potential to link 'Algebraic Expressions' with 'Integers', a problem area for which many pupils required practice, and described by pupil K154 (see Section 4.4.5). In Task 1: 'Calculating', the pupils were expected to solve a numeric puzzle and select predicted answers from a box. Therefore, the main learning theme was *learning through inquiry*, supported by *assessment* and *learning through practising* (Luckin et al, 2012). In Section 4.4.4, pupil U076 described *Grid Algebra* linking technology to algebra in a way that enabled pupils to feel safe practising (K142) their mental arithmetic skills (Claxton, 2004), as noted by Teacher T1. It built solid foundations for the pupils' mathematical knowledge through more connections, developing their 'procedural fluency' (Kilpatrick et al, 2001) described by pupil K137; Teacher T3 described how the tool facilitated 'webbing' (Noss and Hoyles, 1996) of concepts within mathematics for the pupils.

The main learning theme evoked by Task 7: 'Find the journey (letters)' was *learning through exploring*, supported by *assessment* and *learning with others* (Luckin et al, 2012). The pupils' attention was not explicitly drawn to their 'symbol sense' (Arcavi, 1994) despite the requirement for written work implying a clear need for developing formal algebraic notation. I shelved my plan momentarily after I observed some pupils struggling with operating *Grid Algebra* following week-long hiatuses. I focused on developing pupils' mastery of the software through their engagement with this task immediately after the introduction to *Grid Algebra*. Task 7's structure evoked the third principle of 'effective formative feedback' (Juwah et al, 2004): this helped to clarify goals and "good" performance for the pupils. As users created prescribed expressions, the software represented the 'journey' traced across the grid in bold lines: a 'cross' appeared to indicate unpermitted cells. In Section 4.4.2, the tool's scaffolding was described as "clues" by pupil P179 or "arrows" by pupil R222; pupil K136 described increased efficiency in working, and "just understanding" (G042). The pupils negotiated appropriate grid movements for particular operations; they *saw* the results of their actions (Hewitt, 2012). Thus, the pupils were observed to form stronger links between physical movements and mathematical operations (Papert, 1980) as 'consequential' software feedback aided the pupils' decisions about what they multiplied with, or divided by.

Pupils' ability to 'read and write' 'Algebraic Expressions' was an identified learning outcome (KIE, 2002), hence a focus in my study. Engagement with Task 25: 'What is the expression?' required pupils to possess an understanding of the software's working. I monitored pupils' developing formal algebraic notation through a written task after ICT-enhanced activities. All pupils were required to put their thinking into action and write down expressions representing given 'journeys' across the grid. The main learning theme was *learning by making*, supported by *others* and *assessment* (Luckin et al, 2012). I expected pupils to make use of the visual imagery provided by the software and peer feedback, to demonstrate their understanding of the structures and processes underlying algebraic expressions. This written task was performed with average success (see Figure 4.7 in Section 4.4.4). Revision activities of a written task using *Grid Algebra* took on an open-ended approach; these pupils were *learning through exploring* (Luckin et al, 2012) whilst teaching computers (Becta, 2008; Papert, 1980) to create expressions for them. In Section 4.4.4, pupil U059 described her symbol awareness.

The main learning theme for Task 13:'Make the expression (letters)' was learning by making; pupils were required to create prescribed algebraic expressions within the given time frames. The theme was supported by *learning through inquiry* and *with others* because exploring of grids was geared to specific ends; all pupils worked together, questioned each other and tested their reasoning with structured actions (Luckin et al, 2012). In Section 4.4.5, pupil

P180 and K148 described the increased levels of concentration that were engendered in pupils by the learning activity.

The Task 12: ‘Inverse journeys’ encouraged *active thinking* in order to reverse one’s actions (Hewitt, 2012). Pupils were expected to ‘undo’ mathematical operations: addition became the reverse of subtraction, multiplication the reverse of division. The task of reversing operations appeared to gradually replace ‘rules’ to be memorised (Tall and Thomas, 1991), especially:

- “add the same thing to both sides”;
- “change sides, change signs”;
- “to divide, turn upside down and multiply”.

Owing to Task 12, pupils seemed to accept thinking about linear equations as ‘journeys’ that required ‘undoing’, as described by pupil K162 in Section 4.4.4. The requirement to put their ‘thinking into action’ motivated many pupils to construct and share their understanding. Thus *learning by making* became the main learning theme, supported by *assessment* and *learning through practising* (Luckin et al, 2012); pupil P176 valued the feedback offered in response to reversing action as pupils undid algebraic expressions, and this represented a *critical voice* as internal feedback (Juwah et al, 2004) that triggered the self-awareness (U109); pupil U077 described how their mental skills (Bruner, 1957) developed their mathematics learning (G006).

The main learning theme for Task 21: ‘Simplify’ was *learning through inquiry*, supported by *learning through practising* and *assessment* (Luckin et al, 2012). Pupils were required to give simplified versions of puzzles. Pupils examined their predictions whilst trying to discover the simplest expressions possible; *Grid Algebra* provided consequential feedback (Hewitt, 2012) valued by Teacher T4 in Section 4.4.4; adults, as the ‘more knowing others’ (Vygotsky, 1978), provided some support for pupils’ ICT-enhanced learning (see Section 5.3), and thus pupils were *learning from experts*. The ‘scaffolding’ (Wood et al, 1976) assisted many pupils to make sense of new information on their screens. The experts played a crucial consolidating role in the pupils’ mathematical constructions, as argued by Tabach (2011). This emerged as one task that several pupils favoured in the ‘extra’ sessions (see Section 3.1.5.4); some pupils requested independent practice and assistance to secure their algebraic knowledge. This indicated their developing ‘mathematical resilience’ (Johnston-Wilder and Lee, 2010b).

In Task 22: ‘Substitution’, the main learning theme was *learning through inquiry*, supported by *learning through practising* and *assessment* (Luckin et al, 2012). The pupils solved given puzzles and received feedback on the numerical answers they offered. More classroom dialogues (Alexander, 2008) indicated the incidence of enhanced formative feedback (Juwah et al, 2004) as pupils negotiated their responses in small groups (Elder, 2012). This task appeared to also test the security of pupils' existing arithmetic constructions (see Teacher T4 in Section 4.4.4).

In Task 6: 'Expanding and factorising', the main learning theme of *learning through making* was supported by *learning through practising* and *assessment* (Luckin et al, 2012). This task required pupils' mastery of *Grid Algebra* to create expressions from 'journeys' given. These pupils relied on *learning from experts* for the same reasons as Task 21 above; several pupils chose the software-generated task for further practice in 'extra' sessions (see Section 3.1.5.4).

This data appeared strongly to suggest that pupils in secondary mathematics classrooms can engage with increased 'connectivity', and gain a deeper understanding of, 'difficult' algebraic concepts (see pupil R256, Section 4.4.2) when provided with appropriate tools and conditions in enabling and safe learning environments as discussed by DEAG (2013).

5.5 Affect and enjoyment

Following Bandura (1986), I considered the computer to be an integral tool in pupils' algebra lessons. Several researchers, for example Tall (2010c) and Hannula (2002) linked emotional traits with the quality of learning. Bandura (1989) viewed human behaviour as determined by what one feels, thinks and believes. McLeod (1992) suggested that either positive or negative feelings elicited in pupils determine their levels of engagement as they learn and understand mathematics. In Section 4.4.2, pupil R256 described their low self-belief due to problematic accessibility of primary school algebra, as did P165 (Section 4.4.5); pupils described loving mathematics (K150), and finding algebra easy and 'fun' (K154), once they understood. Such views echoed reports about emotions, expectations and values as influential factors of attitude in learners (Hannula, 2002). The ICT-enhanced learning seemed to affect pupil engagement with 'Algebraic Expressions'. This intervention incorporated *Grid Algebra*'s potential to mediate between concepts and learners through the six 'pupil entitlements' (Becta, 2008):

Learning from feedback, since the instant, notational *Grid Algebra* feedback (Hewitt, 2012) appeared to trigger self-assessment (Black et al, 2003) in the pupils; they paused to reflect on their reasoning. Formative feedback was targeted at task performance rather the 'self' (Hattie and Timperley, 2007). Many pupils regarded mistakes as crucial to their learning (Alexander, 2008): they indicated this belief consistently in the MBRQ (see Section 4.3). In Section 4.4.2, *Grid Algebra*'s feedback was described as non-judgemental (R257) and immediate (G019) by pupils. The software design incorporated the principle of 'fading' (Hewitt, 2012) by reducing the amount of support provided to pupils in tasks (Bokhove and Drijvers, 2012): this diminished the risks of developing dependency in learning (Mercer and Littleton, 2007). 'Faded' scaffolding appeared to trigger phases of reflection in most pupils (G023, Section 4.4.5); they paused to *think* and *talk* through their reasoning processes when their responses did not work.

Observing patterns, as pupils explored their *Interactive Grid Algebra* learning environment with numbers and letters. Algebraic expressions that formed on pupils' screens culminated in an understanding of abstract concepts, including formal notation (Healy et al, 2001). Askew et al (1997) advanced the contribution of mental imagery in enabling pupils to become 'numerate' rather than have understanding through procedural learning (Kieran, 1992). In Section 4.4.4, pupils described developing an ability to think 'more' (P176) and 'fast' (U109), and developing mental skills (U076). In Section 4.4.2, pupil U108 described using mental grids in examinations, a finding that resonated with research conducted in Asia which showed that abacus training developed rapid mental arithmetic skills and apparently consolidated the learners' number sense (Frank and Barner, 2012; Stigler et al, 1986).

Seeing connections, as hands-on use of *Grid Algebra* in mathematics lessons allowed pupils to visualize processes and appropriate (Mercer, 1994) algebraic concepts, as argued by Papert (1980). The visual and verbal elements in the ICT-enhanced activity appeared to complement each other and promote effective mathematical learning (Wall et al, 2005). In Section 4.4.2, pupils P181 and P171 described how seeing the formation of the expressions led them to clearer understanding, and hence to more 'connectivity' in ICT-enhanced mathematics lessons (Noss et al, 1997).

Working with dynamic images, as control of the activity, through manipulating the mouse, enabled dragging of numbers and letters across grids. All pupils saw the structure of algebraic representation on the screens. Software-based activity linked the dynamic images with the formal symbolic representation (Noss et al, 1997). This connection enhanced pupils' appropriation (Mercer, 1994) of formal algebraic notation (see Section 4.4.4). In Section 4.4.2, Teacher T4 described how the dynamic imagery of 'Expanding and factorising' concepts as 'journeys' by *Grid Algebra* had consolidated pupils' understanding of these ‘difficult’ algebraic ideas. Many pupils appeared to appreciate the visualised differences between two mathematical processes (Sfard, 1991). In the example $2\ (f-4)=2f-8$, the software presented these processes as different, albeit equivalent, ‘journeys’ using different routes, and ending in the same cell (Hewitt, 2012).

Exploring data, as *Grid Algebra* generated formal algebraic expressions as data for pupils to think with. In Section 4.4.2, pupil K146 described how the kinaesthetic learning involved in the ICT-enhanced activity enabled her engagement with written tasks. Figure 4.2 shows that many pupils claimed that the consequential nature of software’s feedback (Hewitt, 2012) encouraged them to experiment with the tool; 42% (113 pupils) described making predictions, which they tested, and which subsequently modified their thinking.

Teaching the computer, since the pupils made reasoned decisions on ‘journeys’ across each grid as externalised ideas. Pupils increasingly processed information by linking their physiological experiences to their abstraction of mathematical ideas (Clausen-May, 2008). Pupil P177 described how her own ‘thinking’ was valued following opportunities to find solutions for herself in Section 4.4.2.

In Section 4.4.5, these pupils described enjoying themselves and having fun in the ICT-enhanced sessions; many appeared to develop beliefs that computers made learning algebra “enjoyable” (pupil K146); pupil R235 described “loving” computers for the enhanced understanding they experienced; exciting learning experience and positive emotions apparently generated some ‘pleasure’ (Tall. 2013) and “concentration” (pupil K115); pupil P205 valued learning through “playing” as they attended to the intended curriculum objectives (Noss and Hoyles, 1996). The ICT-enhanced activities evoked ‘purpose’ and ‘utility’ (Ainley et al, 2006) since pupils mastered *Grid Algebra* use to create algebraic expressions; pupils K128 and K182 described accepting formal algebraic notation and mathematical language. In Section 4.4.2, pupil U060 believed that their computer use developed their self-belief; pupil K117 said that lacking understanding bred negative feelings; however, the more she understood and liked algebra, the more she enjoyed learning. The findings suggested that computers can influence confidence, motivation and active pupil engagement in mathematics, as argued by Galbraith and Haines (1998).

5.6 Variable teacher 'learning' behaviour

The secondary research aim illuminated the role of regular teachers in the ICT-enhanced learning context. My study presented opportunities for teachers to develop their mathematics TPACK (Niess et al, 2009). According to Lee and Johnston-Wilder (2013), variety and inclusion are important for pupils' positive learning experiences. Following Bruner (1961), the teacher's role should be to align pupils' constructions to universally-accepted conventions (Ruthven, 2012) and provide necessary structure and support (Belmont and Skinner, 1993). Teachers need to be acquainted with the potential of their learning resources in order to be able to welcome their availability in learning contexts. It is crucial that teachers should actively interweave ICT and non-ICT elements in their classrooms for effective pupil mathematical learning.

The organisation in ICT-enhanced sessions enabled more teacher-pupil interaction, as described by pupil R227 in Section 4.4.1, and valued by Teacher T3 in Section 4.4.6.3. The teachers were freed to attend to pupils either in small groups or individually. It required teachers to hone their listening skills (Foster, 2014); they had to be well-prepared to handle unforeseen questions and challenges arising from increased pupil-pupil dialogues, given the wide variety of abilities and talents. However, this has implications for one's ability to cope with such a complex setting (Norris and Walker, 2005). Each teacher has to put more thought and effort into lesson preparation whilst exploring various teaching approaches in order to effectively cater for diverse pupil learning needs. This argues for recognition that each pupil, with self-effort and support, is capable of higher achievement.

I encountered teacher concern about the extent to which the tasks should be 'open-ended' in line with reports by Webb and Cox (2004). They argued that teachers require awareness of their pupils' beliefs concerning the degree of control they should have of their own learning, and an understanding of the affordances of ICT-enhanced learning contexts. Teacher 2E's insistence that pupils should work at speed apparently disregarded the obvious range in the pupils' learning abilities and hinted at some possible exertion of control over pupils' thoughts and actions. Evidence suggested that reminding pupils to apply 'BODMAS' was hardly helpful since it was not sufficiently understood in Section 4.1. Such rule-following practices can limit many pupils from assuming responsibility for their own learning (Boaler and Greeno, 2000). Several pupils illuminated the value of formative feedback in the learning context. The consequential software feedback (Hewitt, 2012) mediated pupil-pupil dialogues and triggered reflection. Teachers seemed to shun computer mediation of algebraic concepts by expressing unawareness of *Grid Algebra*'s potential. Observed learning episodes illuminated that neither ICT nor pen-and-paper resources may realise intended curriculum goals on their own (Luckin et al, 2012). The invaluable data in Section 4.4.3 implied that dialogic interactions can support pupils' mathematical constructions (Tabach, 2011). While I observed Teachers 1E and 2E instructing pupils to "pay attention to the structure", they each invested little effort in developing 'symbol awareness' in their pupils or distinguishing between algebraic 'expressions' and 'equations'. According to Arcavi (1994), developing 'symbol sense' can enable pupils to perform manipulations and to consider the relevance of underlying algebraic structure. Brown et al (2008) reported that diminished 'symbol sense' while in school impacted negatively on pupils' affective domain: pupils felt inadequate in mathematics. There is value in encouraging *pupil talk* around learning activity. My findings suggest the importance of pupils articulating ideas when solving mathematical problems to reveal their 'difficulties'.

At Stage Three, I observed that two teachers, Teachers T4 and T3, were led to discover 'new' teaching skills. In Section 4.4.6.3, Teacher T3 described gaining new insights into the teaching of 'Algebraic expressions'; she reconsidered the content to be covered, the appropriate resources and methods to address the pupils' learning needs. In Section 4.4.6.4, Teacher T4 described pupil learning behaviour in ICT-enhanced sessions. I observed Teachers T3 and T4 listening silently to pupils' dialogues in small-group interactions, before intervening when the occasion called for it (Foster, 2014). Therefore, T3 and T4 became increasingly 'dialogic' (Alexander, 2008). Whereas Teacher T1 embodied the effect of teachers' beliefs on learning processes, Teacher T2 embodied the apparent disconnect between cognition and affect in mathematics (McLeod, 1992); T2's absence from the ICT-enhanced sessions alienated him from developing emotional and motivational involvement (Skinner and Belmont, 1993) in pupil learning. I refer to Teacher T3's behaviour in this study as the clearest embodiment of the *didactical tetrahedron* (Ruthven, 2012) shown in Figure 1.1. Niess et al (2009) and Mumtaz (2000) reported that teacher attitudes towards teaching with technology may depend on how they perceive that such teaching will impact pupil learning and their beliefs about what it means to *understand* mathematics. Such involvement also led to invaluable observations of a teacher 'learning' about pupils learning mathematics with ICT (Tabach, 2011). Despite avoiding any personal exploration with *Grid Algebra*, Teacher T3 gradually started interacting with pupils in the ICT-enhanced sessions. In Section

4.4.6.3, Teacher T3 described substantial shifts in her perception of the value of ICT-enhanced learning (Hennessey et al, 2005). Her initial attitudes, distinct lack of computing skills and unwillingness to engage with digital technology would often have acted as barriers. Instead, there was remarkable transformation as a result of her experiences in the sessions. Algebraic concepts were no longer just in T3's head and in her explanations (Ruthven, 2012) or passively recorded in textbooks (Yerushalmy and Naftaliev, 2011). Teachers' own learning experiences with ICT contribute to development of TPACK (Koehler and Mishra, 2009). It is possible for a teacher observing pupils learning with ICT to develop the integrated knowledge as exemplified by T3. She drew on her pupils' reactions and the software's feedback to inform and shape her teaching in the non-ICT lessons; she evoked the seventh principle of formative feedback (Juwah et al, 2004), and acted as an 'external organising agent' (Ruthven, 2012) for pupils. Some researchers (Tabach, 2011; Reed et al, 2010) have advanced that this supportive role is crucial for pupils' learning.

Teacher T4 finally evaluated *Grid Algebra*'s appropriateness to indicate that he went through mathematics TPACK development processes Niess et al (2009) described as:

- ***Recognising***, the 'knowledge' phase; he appreciated *Grid Algebra*'s alignment with the curriculum;
- ***Accepting***, the 'persuasion' phase; he formed a favourable attitude towards ICT use in mathematics;
- ***Adapting***, the 'decision' phase; he explored the software and planned an ASEI-PDSI lesson;
- ***Exploring***, the 'implementation' phase; he led some *Grid Algebra* sessions;
- ***Advancing***, the 'confirmation' phase; he evaluated the results of that decision.

In my view, Teacher T4 embodied the contribution of teacher professional development to effective pupil learning through 'connectionist teaching' (Askew et al, 1997). Teacher T5's learning behaviour reinforced my thinking about the teacher's role changing from 'instructor' to 'facilitator' in support of pupil learning (Bruner, 1957), enabling the pupils to assume more responsibility for their own learning.

5.7 Summary

The introduction of *Grid Algebra* enabled new learning experiences to develop in the ICT-enhanced secondary mathematics classrooms. My analysis has covered: changed learning environment; learner agency; changed motivation; accessible learning; affect and enjoyment; and variable teacher 'learning' behaviour observed at Stage Three. I regarded the participants as 'learners', with instrumental genesis for pupils, and a platform for teachers developing their TPACK. The discussion I take forward to Chapter 6 will link the six emergent themes detailed in Chapters 4 and 5 to the final research questions that I listed in Section 3.2.5.

Chapter 6: DISCUSSION

6.0 Introduction

This is not a study of one single factor realising change using ICT; ICT use can be effective, or equally ineffective. It is the integration of affective aspects into cognition that strengthens ICT use in learning and instruction of algebra in ICT-enhanced Key Stage Three mathematics classrooms. The findings suggest shifts in pedagogy, and the enhancement of theory from a sociocultural perspective. I assess in Sections 6.1 to 6.4, whether my final research questions, listed in Section 3.2.5, were addressed by the study.

6.1 Did the intervention address levels of interest in algebra and pupil concerns about lack of involvement and engagement in mathematics lessons?

The questions that were being asked in this study were: whether the use of *Grid Algebra* had the potential to enhance conceptual understanding of algebra in secondary school pupils; and also whether it facilitated the inclusion of disaffected Kenyan pupils in mathematics learning. I concurred with the viewing of disaffection as disengagement by several researchers (Nardi and Steward, 2003; Boaler et al, 2002). Participant observation, focus group interviews and pupil questionnaires provided invaluable data which crystallised my thinking at Stage Two; I found evidence of disengagement as pupils dozed in predominantly teacher-centred lessons (see Section 4.2). My mission at Stage Three was to demonstrate ways of blending 'old' and 'new' technologies in mathematics education to seek inclusion. I considered computer use as *supplementing*, rather than replacing, existing classroom 'subcultures' hitherto dependent on 'traditional' resources, including teacher-'talk' (G044, Section 4.4.2). To me, *Grid Algebra* use in actual classroom learning of 'Algebraic Expressions' seemed consistent with the Ministry of Education's vision to modify mathematics teaching and learning in Kenya. The pedagogic goals set out by CEMASTEA (2009) emphasised teacher-centred, theoretical classroom instruction being replaced by practical, more relevant and interesting activities for pupils (R270, Section 4.4.1). I evoked the first principle of 'quality learning environments' (DEAG, 2013) in research design: by relating *how* pupils learn to *what* they learn. In Section 4.4.1, participants described valuing the learning activity through 'dialogic teaching' (K151, Section 4.4.1) mediating the participants' ICT use; pupil R227 described this study as enabling ways of empowering the 'pupil voice' (Flutter and Rudduck, 2000), and stimulating pupils' interest (K146), involvement (G035) and engagement (P204); Teacher T4 described how the pupils' kinaesthetic use of appropriate software, *Grid Algebra,* on computers enabled high engagement (T1) from what would have occurred without the *new* tool (P198, Section 4.4.3). In turn, the activity challenged the teachers' conceptions of 'teaching' for engagement in a way that they could manage. The availability of digital technologies has the potential to change significantly what learners experience as 'learning' (Hoyles, 2001).

However, it is unavoidable that, whilst they learn, many pupils become aware of the gap between their current knowledge level and what they are expected to know. Pedagogic practices which only value correct, and immediate, solutions to given problems can obstruct mathematical learning. In such classrooms, pupils face very real prospects of humiliation and judgement of their learning abilities; they may resort to indecision, confusion and isolation. According to McLeod (1992), intense feelings of anxiety, frustration and disaffection develop and can have severe implications on pupil participation in mathematics, as described by pupil P198 who indicated struggling with understanding algebra in teacher-directed instruction (see Section 4.4.3). For disaffected pupils, patterns of disengagement can be coping mechanisms by which they can survive adversity in the learning context. Such pupils attempt to forge the identity, relationships and learning behaviour that they deem effective in deflecting the emotional upheaval inflicted either by judgement or labelling of mathematical ability. Pupils can exercise agency through participating minimally or not at all, as *self-protection* when the classroom ethos fails them (Gresalfi et al, 2009). They can refuse to articulate their own ideas or to improve on and build on peers' contributions.

Teachers can inspire active pupil participation through intelligent risk-taking (Elder, 2012) in mathematics. 'Dialogic teaching' stimulates interpersonal dialogues. In Section 4.4.1, pupil R270 described dialogues allowing both pupils and teachers to build on their own and each other's ideas and arguments and to modify their thinking where necessary (Mercer and Littleton, 2007); Teacher T3 described her minimal intervention in task-based pupil 'talk' as she nurtured and gave due consideration to pupils' reasoning. The classroom 'subculture' can foster pupils' confidence in their learning. Participants in dialogic lessons consider 'mistakes' as learning points rather than as sources of embarrassment (U078, Section 4.4.2); pupil K133 described valuing her re-trialling activity when *wrong*. It is important for *dialogic* teachers to provide increased opportunities for task-based pupil 'talk', described as 'transactional, interrogatory, expressive, exploratory and evaluative' (Alexander, 2008), while they interact. Teachers should manage the ensuing extended dialogues for shared understandings (K135, Section 4.4.1) since any learning progress hinges on the joint acceptance (G023). Teachers need to believe that dialogue makes teaching easier because as pupils talk about the work they are doing, they let the teachers know the quality of assistance, if any, they require (T3, Section 4.4.6.3); I saw Teacher T3 attending to pupils' 'talk' (Foster, 2014) and learning behaviour in the ICT-enhanced sessions (Sutherland et al, 2009). In Section 4.4.5, pupil R251 described a 'new' classroom 'subculture' enabling communication skills and growing confidence (U077) in the learning processes by making better use of dialogue.

A key feature of my research design was illuminating and harnessing pupil attitudes, values and emotions to influence cognition in mathematics. McLeod (1992) described how the affective domain, indicated as *emotions*, *moods* and *feelings*, exceeded cognition. He advocated the merging of affect and cognition in mathematics learning. The pupils deemed 'understanding algebra' as significantly difficult. In Section 4.4.2, pupil R223 described feeling "frustrated"; pupil P181 described "anxiety" attributed to a lack of understanding; pupil R256 described harbouring problems since primary school. In Section 4.4.1, pupil U091 described "learning more" in group discussions with peers, alongside the teacher, in the ICT-enhanced interaction. In Section 4.4.5, 67% of 270 Stage Three pupils (181) indicated in a pupil questionnaire that their use of *Grid Algebra* made learning 'Algebraic Expressions' and mathematics "fun" or "easier", thus consolidating their *symbol sense* and use of mathematical language (K154); the 'play' factor was described by 6% (14 pupils) as facilitating more conceptual understanding in algebra, as argued by Whitton (2007).

In Section 4.4.4, pupils talked about their developing 'thinking' (30%, 80 pupils), mental arithmetic skills (24%, 66 pupils) and attainment (13%, 34 pupils). Stigler et al (1986) argued that mental activity has both quantitative and qualitative benefits for pupils' cognition. Stage Three pupils' mental arithmetic skills may have contributed to their increased mathematical attainment in examinations (see Figure 4.4 in Section 4.4.3). This finding strongly supports providing pupils with mental exercise for practice (Claxton, 2004) in mathematics lessons.

Of major importance to pupil learning was the potential for the software's immediate feedback to encourage self-regulation (Wall et al, 2005); the 'consequential' feedback triggered the user to reflect upon and assess their actions, as described by pupil U078 (see Section 4.4.2). Teacher T4 valued dynamic visualisation of formative *Grid Algebra* feedback for enhancing the pupils algebraic understanding (see Section 4.4.6.4); the scaffolding and 'fading' support depended on pupils' actions in the grid. Several researchers have emphasised that simply indicating that a response is correct or incorrect may not be very effective in facilitating learning (Black et al, 2003; Black and Wiliam, 1998). Sadler (1989) argued that it may be more useful to provide learners with information about what is ‘wrong’ with their answers, and leave learners with initiatives for corrective action. The findings in the present study strongly imply that feedback is more powerful (Hattie and Timperley, 2007) when it is ‘consequential’ (Hewitt, 2012) rather than evaluative. In Section 4.4.1, 107 pupils (40%) indicated valuing the variety in lessons enabling positive experiences since the changed learning context seemed to encourage pupils to act by amending their predictions, as described by pupil K133, Section 4.4.2, who felt safe to make mistakes and try again. Nevertheless, the quality and timing of feedback from peers, teachers and computer-based programs, are crucial features in determining how well pupils learn. The software increased pupils’ concentration (K115, Section 4.4.3) in the ICT-enhanced sessions. In my thinking, *Task 7: ‘Find the journey’* (see Section 1.5.3.2) was effective in enabling pupils to relate specific movements across the grid to particular mathematical operations, creating given algebraic expressions. I observed pupils discussing, and reflecting on, the decisions about making movements to create the expressions (p. 175, Section 4.4.5). They had to think ‘If I do this, this will happen’ (K133, Section 4.4.2). The actions required pupils to apply arithmetic in algebra (P200, Section 4.4.5), challenging the security of pupils’ ‘met-befores’ (Tall, 2004); pupils handled such tasks with perseverance, indicating their self-efficacy (Bandura, 1989).

In the small groups, mediated by the computer and supported by the specialist mathematics teachers, all pupils had equal chances to prove themselves and excel at learning algebra. The same ‘ground rules’ for talk governed the sharing of resources (P203 and U086, Section 4.4.1). Thus, collaborative activity transformed the mathematics lessons into domains of enhanced classroom participation. The ICT-enhanced sessions offered the pupils opportunities to adopt improved learning behaviours, contrary to their previous ascriptions of either intelligence or mathematical failure. The setting provided benefits for all (R234, Section 4.4.1) since the more able pupils were challenged by the questions asked by the group members; the less able provided encouragement and support for their peers. The higher achievers were expected to respectfully answer questions from their less able peers; they had the chance to test their algebraic thinking on computers through exercising ‘conceptual agency’ (Pickering, 1995). The low achievers also had turns to express their contributions, and discovered that they too had something to say. The observed pupil enthusiasm (Galbraith and Haines, 1998) in most ICT-enhanced sessions led to the pupils' productive disposition (Kilpatrick et al, 2001). In Section 4.4.3, pupil P194 described feeling assured of her ability to solve algebra questions. Some 39 pupils (14%) described their 'new' confidence as mathematical thinkers. Boaler and Greeno (2000) regarded 'identity' as a crucial factor in determining participation in learning activities. The pupils' self-identity served to increase the levels of motivation to learn; it improved task engagement and attainment levels in terms of both quality and quantity of work. McLeod (1992) argued that attitudes towards learning mathematics tend to be more negative for pupils moving from primary to secondary school as they encounter harder concepts. However, the ICT-enhanced sessions seemed to ease Stage Three pupils' transition into secondary school; pupil P198 described how she avoided experiencing the dreadful sense of anxiety or failure occasioned by isolated, individual working on textbook exercises which many pupils deemed inaccessible. It is hard for one to be disruptive in an environment where everyone else seems to be interested and working hard (T1, Section 4.4.6.1). The support offered by the positive nature of the small-group collaboration using *Grid Algebra* served to dispel any feelings about *difficult* algebra that the pupils might have brought to the sessions, as described by P184, Section 4.4.5. I observed high pupil involvement and engagement in ICT-enhanced mathematics lessons.

Some teachers' perceptions seemed to change in the light of the pupils' engagement with the software. Teacher 1E described the pupils' ability to factorise as evidence of 'higher-level' algebra by the end of Stage One (see Section 4.1). At the end of Stage Three, Teacher T1 admitted that more work had been covered from related topics than expected; Teachers T3 and T4 underscored the pupils' raised levels of conceptual understanding in Section 4.4.2.

6.2 What effect did the intervention have on the competence, and confidence, of these pupils' mathematical language use in classroom interactions?

The data collected from written work provided significant evidence with respect to cognition, of pupils' initial competence and confidence in communicating mathematically, illustrated in Table 4.3. Development of mathematical language is not limited to using mathematical words in speech ; writing mathematically and using algebraic symbols appropriately (K128, Section 4.4.5) also pose 'difficulties' for many secondary school pupils (U099, Section 4.4.4). Papert (1980) described instruction in mathematics as an enterprise of extracting content knowledge from the meeting of body with context. Teaching school algebra in 'traditional' classrooms appears to have become a process by which we ask pupils to forget their natural mathematical experiences and to learn new sets of 'rules' (K146, and P179, Section 4.4.3; T4, Section 4.4.4). Using participant observation and retrospective discussions provided ample evidence of pupils' confident use of mathematical language (T3, Section 4.4.3). I considered forms of *talk* as examples of social action. Dominance of teacher *talk*, including 'exposition' (Alexander, 2008), arguably contributes to pupils' disengagement in classroom learning. This intervention encouraged talk around a small-group learning activity enabling pupils to appropriate algebraic concepts using *Grid Algebra* (G030; Section 4.4.1) in mathematics lessons. Many pupils described applying their speaking and listening skills to perform social actions as externalised reasoning (K151; Section 4.4.1). Each pupil justified to their peers and teachers their 'verbal thoughts' (Vygotsky, 1962) that were displayed in the software feedback on their screens. By associating physical movement and mathematical operations, the pupils enacted algebraic learning behaviour within socially-shared frames of reference (see Section 3.2.3). Enacted social actions were clarified by data from worksheets and examinations, in ethnographic interviews; these actions were accounted for when pupils described their own (U077) and peers' deeds in the pupil questionnaire (G023, Section 4.4.5). This evidence was reinforced by data from the teachers (T1, Section 4.4.1) in ethnographic and focused interviews.

The mathematics register has borrowed a lot of words from the 'natural' language. Pupils can learn mathematical vocabulary (P182, Section 4.4.5) following consistent and appropriate use by their teachers (Durkin, 1991). Thus, teachers become the primary models in the classroom as part of the pupils' enculturation into mathematical discourse and the community at large (Lee, 2006). Whilst discussing modelling as a mechanism for cognitively supporting pupils through the ZPD, Bliss et al (1996) asserted that teachers should be wary of assuming that their pupils have understood what is being demonstrated. They argued that 'formative feedback' (Juwah et al, 2004) which allows pupils to consider alternative perspectives whilst comparing their reasoning against established standards (U089, Section 4.4.5) is extremely powerful. In this study, I incorporated the role of attention (Hewitt, 2011) in enabling pupils' to access algebraic concepts in talk-based, ICT-enhanced mathematics lessons (see Section 5.2).

The pupils' use of *Grid Algebra* in mathematics lessons seemed to challenge their teachers to think differently about their own mathematical thinking and teaching of algebra, particularly linking algebra to arithmetic. On this score, the teachers in Kenya were similar to teachers in

England. Both teachers at Stage One repeatedly expressed their failure to see the relevance of *Grid Algebra* to the content pupils learned in lessons (see Section 4.1). I reported the teachers apparent indifference to algebraic structure (Sfard, 1992) using an 'enhanced context' (Ruthven et al, 2011) in Section 3.2.1, and the observed pupils 'strategic competence' (Kilpatrick et al, 2001) in Section 5.3. At Stage Three, four teachers 'recognised' (Niess et al, 2009) the tool's appropriateness; it linked algebra to arithmetic (see Section 3.2.4). The *Grid Algebra* software is based upon 'journeys' and 'inverse journeys' across a grid. The activity tested the security of these pupils' existing mathematical knowledge, including their grasp of multiplication tables and 'Integers'. I monitored the participating pupils' acceptance of formal algebraic notation using written work in this study. The pupils demonstrated a gradual improvement from 24% to 42% to 51% to 55% in written tasks (see Section 4.4.4); both pupils and teachers reported that the ICT-enhanced activity enabled more understanding in 'Algebraic Expressions'. For many pupils, their developing 'symbol sense' (Arcavi, 1994) seemed contribute to greater competence and confidence (Hodgen et al, 2008) when they learnt 'Linear Equations' and 'Simultaneous Linear Equations' (T1, Section 4.4.4). More pupils appeared to have made firmer connections (Noss et al, 1997) in algebra, as described by T3 (see Section 4.4.3), and with other related topics (K154, Section 4.4.5). Using the *Grid Algebra* software generated new possibilities for mathematical expression for these pupils.

In Section 5.4, I illustrated a case for secondary pupils' entitlements of learning mathematics with ICT (Becta, 2008) through linking the effective learning themes proposed by Luckin et al (2012) to the eight software-generated tasks that pupils engaged with in this study. Thirty-eight pupils in the pupil questionnaire, and four teachers, singled out task 12 ('Inverse journey') as having an impact on pupils' thinking (T1, Section 4.4.4); the task required the user's reversing action on mathematical operations to *undo* expressions. The externalising processes involved seemed to affect the pupils' thoughts and actions (K116, Section 4.4.2) while applying this reversing process to questions (K162, Section 4.4.2) in written tasks. Pupils' written work had revealed 'difficulty' with symbol conventions (Figure 4.7 and Table 4.3 in Section 4.4.4).

The computer environment can facilitate collaborative learning; it allows for less teacher talk whilst reducing the incidence of pupils distracting or disturbing each other. In this study, the small-group working promoted more pupil talk (G023, Section 4.4.1) and opened 'windows' (Noss and Hoyles, 1996) into the pupils' thinking for the teachers (T3, Section 4.4.6.3). The collaborative learning and 'hands-on' use of computers with the *Grid Algebra* software led to the pupils valuing new 'social' skills (G023) in discussions (G044, Section 4.4.2). This enabled more 'connectivity' (Noss et al, 1997), as illustrated in problem-solving (T3, Section 4.4.4); pupils developed 'procedural fluency' (Kilpatrick et al, 2001) to support their increased attainment in mathematics (see Figure 4.4 in Section 4.4.3). The teachers were freed to give assistance to those pupils who needed extra support to learn. Subject 'experts' in the context became the 'external organising agents' (Ruthven, 2012) for the pupils. They helped to link the ideas in the ICT-enhanced learning environment to non-ICT resources in line with the curriculum for the pupils. The data in Section 4.4.4 strongly suggest that more pupils learned with *understanding* (Sfard, 2001) as claimed by 57% (155) pupils (see Figure 4.6). At Stage Three, I relied upon the dynamic imagery of consequential software feedback to direct pupils' attention to the salient features in formal algebraic notation (see Section 3.2.3) and mathematical speaking. Subsequently, I observed the pupils using formal terminology: 'coefficient', 'variable', 'term' and 'constant' (P182, Section 4.4.5) in their small-group discussions.

The visual imagery afforded by *Grid Algebra*, together with the possibility to interact with it physically and manipulate it, provided learning experiences that are unavailable without ICT. The computer provided dynamic images of formal expressions in powerful ways (T4, Section 4.4.4); it enabled pupils to access a firm and clear understanding of algebraic concepts and of the appropriate use of algebraic symbols. The scaffolding facilitated by the visualisation of the software feedback was essential in directing the pupils' attention to the underlying structures in algebraic expressions. In Section 4.4.2, Teachers T3 and T4 claimed that the dynamic nature of the images made a difference to the pupils' use of brackets. The images inevitably promoted more connectivity and understanding of algebraic ideas (P205, Section 4.4.3). Making *journeys* across a multiplication grid in the *Grid Algebra* software design required the application of previously-met knowledge enabling many pupils to move safely outside their comfort zone (P198, Section 4.4.3). The findings in Chapter 4 illustrated that software can be designed to provide the pupils with experiences that are not available without ICT (Clausen-May, 2008). In Section 4.4.3, Teacher T3 stated that what Stage Three pupils had learned using *Grid Algebra* was new and different from, and perhaps more than, what the previous cohorts had learned without ICT. The dynamic visual imagery of the pupils' reasoning that was represented in formal symbolic notation by *Grid Algebra* allowed the pupils to 'think more' and to reflect upon their use of algebraic language (57 pupils, 21%, in Section 4.4.5). I attribute the developing pupils' *symbol awareness* described as "understanding more" (G036; Section 4.4.4) to focusing their attention to written and spoken mathematical language.

Some teachers' perceptions seemed to change in the light of the pupils' increased understanding of algebraic language while using the software. Teacher 1E described the pupils' ability to factorise, which might be seen as an advanced manipulation of algebraic language, as evidence of 'higher-level' algebra by the end of Stage One (see Section 4.1). At the end of Stage Three, Teacher T1 admitted that more work had been covered than expected from related topics; Teachers T3 and T4 underscored the pupils' raised levels of conceptual understanding in Section 4.4.2; T1 and T3 reported more pupil participation in non-ICT lessons (see Section 4.4.3), indicating some 'transfer' of learning (Lobato, 2003); many pupils readily contributed in the whole-class discussions that unravelled at some length until the class had reached a consensus and everyone was satisfied (T1, Section 4.4.6.1). The pupils' competence and confidence in mathematical language use, which were revealed and further developed in non-ICT lessons, were also attributed to this intervention (T3, Section 4.4.3 and Figure 4.4).

6.3 What consequences did the intervention have for the role of teachers in ICT-enhanced learning contexts?

According to CEMASTEA (2009), 'traditional' textbook-based, teacher-directed instruction in mathematics classrooms is prevalent in Kenya (see Section 6.1); ICT in education is yet to be embraced. I emphasised 'dialogic teaching' to inspire more learning-based talk using *Grid Algebra* on computers as a fourth element (see Figure 1.1) stimulating 'formative feedback' (Juwah et al, 2004) for improved pupil learning (Sutherland et al, 2009). My challenge was harnessing the ensuing classroom interactions for the participants' benefit. Data collected from participant observation, retrospective discussions with participants, three focused teacher interviews and two pupil questionnaires provided some invaluable evidence regarding the role of teachers in the pupils' ICT-enhanced learning of algebra as follows.

The successful practice of 'dialogic teaching' hinges squarely on the teacher. It is important that, for more secondary mathematics classrooms to become *dialogic*, subject teachers' awareness must first be drawn to the ways in which different kinds of *talk* can be used in classroom learning. In Section 4.4.1, T3 described being fully engaged, listening to the small-group pupil 'talk', whilst actively avoiding "interruption" of ideas, thus provoking pupils' reflection in the ICT-enhanced sessions (Foster, 2014). She subsequently used these insights when planning for non-ICT lessons (see Section 4.4.6.3); this indicated that T3 was giving serious consideration to pupils' mathematical reasoning (Ruthven et al, 2011). Alexander (2008) stressed the need for teachers appropriately to balance social and cognitive uses of *talk* in order to facilitate pupil participation and extended dialogues (R227, Section 4.4.1).

Sometimes 'dialogic teaching' and scaffolded discussions may curtail the information flow from teachers to pupils through awkward questions (Alexander, 2008); this happened to pupil G043, Section 4.4.2. This experience can be unsettling for ill-prepared teachers, hence, the importance of each teacher having a secure conceptual map of content knowledge (Kohler and Mishra, 2009) in order to allow their pupils freedom to explore the teacher's understanding. Teachers need to think about the logical sequencing of subject matter and the appropriate questions to guide pupils towards the desired learning outcomes; teachers have to consider the pace of learning, how much 'thinking time' (Black et al, 2003) to allow, and also effective ways of handling the pupils' responses. It helps when teachers view themselves as joint participants in the classroom activity (T1, Section 4.4.6.1). Occasionally, teachers may request time to consider a pupil's contribution; this signals to the pupils that sometimes answers require careful thought.

Through the acquisition of new techniques and knowledge, teachers are expected to adapt their teaching strategy whilst being mindful of meeting the individual learning needs of the pupils they teach. Askew et al (1997) argued that research demonstrates the difficulty that teachers may experience in adopting new professional practices without developing an appreciation of, and core beliefs in, the underlying principles; this was indicated by T5 valuing teacher-directed instruction for pupils' *Grid Algebra* use (see Section 4.4.6.5) and may have accounted for T2's learning behaviour (see Section 4.4.6.2). Askew et al (ibid) suggested that teachers may appear to adopt some 'desirable' practice in teaching without altering their actual practices. Lack of thorough teacher understanding of proposed pedagogies, however effective, is likely to persuade teachers to maintain familiar, albeit less effective, strategies in their teaching repertoire (T3 and T4, Section 4.4.2), as argued by Guskey (2002). It is likely that teachers' beliefs influence pupils' learning to greater extents than realised; beliefs can take time to change (T1, Section 4.4.6.1).

I confined my examination of the teachers' beliefs to ICT-related aspects, with respect to pedagogical reasoning, in the 'learning' behaviour that I observed. The teachers' ICT beliefs appeared to confirm published research. According to Webb and Cox (2004), secondary mathematics teachers are widely reported to be relatively strongly aligned with the transmission belief orientation of teaching (Askew et al, 1997). Such teachers use ICT to support a ‘subculture’ consistent with their beliefs, including an emphasis on pupils working at speed. At Stage One, Teacher 2E questioned the “open-ended” activities; she wished to see pupils working within imposed time limits (see Section 4.2). In Section 4.4.6.1, Teacher T1 expressed concerns that the overt inclination towards pupil-centred learning that was adopted in my study threatened syllabus coverage; however, pupil engagement in the ICT-enhanced sessions facilitated more work (T1, Section 4.4.4). While appearing to value pupils working at speed, T1’s belief that using computers was ‘slow’ remained unchanged. T5 insisted on pupils solving the sums that he read out before presenting their books for marking instead of allowing their engagement with a software task. Despite investing effort in leading an ASEI-PDSI lesson, T5 appeared to be reluctant to change his beliefs; perhaps this indicated a lack of experience (see Section 4.4.6.5).

My discussion in Section 5.6 added to the debate about whether ICT use is more aligned with the traditional, teacher-centred pedagogies seen to endorse the transmission of knowledge, or with pupil-centred approaches which recognise that learners construct their own knowledge. This study leaned towards the latter. It incorporated an aspect of ‘dialogic teaching’ (Alexander, 2008) through increasing the incidence of ‘pupil talk’. The design facilitated an increased focus on pedagogy; the study presented teachers with the opportunity for ‘reciprocal learning’ (Luckin, 2008). The teachers became ‘learners’: they learned about pupils learning algebraic concepts in the ICT-enhanced sessions. The computer-based mathematical activity resulted in the pupils’ ‘instrumental genesis’ (Guin and Trouche, 2002). The teachers assumed varied roles (Monaghan, 2004) in support of pupil learning. Teachers T3 and T4 provided ‘involvement’ in the ICT-enhanced context by compensating for the struggling pupils’ learning behaviour (Skinner and Belmont, 1993). Both teachers interacted with their pupils in the small groups and in whole-class discussions; they helped to reinforce a structural conception of algebra (Sfard, 1992) by directing pupils’ attention to the algebraic expressions. They prompted the pupils to reflect on formative feedback provided by peers and software. These teachers’ role in the ICT-enhanced context became more facilitative for pupils’ learning. They modelled the conventional use of specialist vocabulary and mathematical language as a tool for reasoning (Mercer and Sams, 2006). They consolidated the pupils’ appropriation of the information on the screens, and aligned it to specific objectives. I suggest that this was possible only where the teacher was willing to embrace a supportive role for pupils' learning and to recognise the pupils as actively constructing (Bruner, 1960) their knowledge. As discussed, Teacher T3 seemed to develop her pedagogic skills through observing the pupils' instrumental genesis, listening to pupil-pupil dialogues, and the whole-class discussions I facilitated in the ICT-enhanced sessions. This teacher made the learning content more relevant and accessible to the pupils. T3 became an 'external organising agent' in the didactical tetrahedron (Ruthven, 2012); hence the pupils' attainment increased (see Section 4.4.3). Such teacher 'learning' behaviour requires a belief system which places greater value on pupils making meaningful connections between the information displayed on computer screens and their existing knowledge of the intended learning (G011, Section 4.4.4). Mathematics provides mental, emotional and social 'exercise' in learning experiences (Claxton, 2004); *variety* in lessons enabled pupils to reflect upon which conditions work best for them (P165, Section 4.4.2; K145, Section 4.4.4). Pupils require sustained practice using their mental resources to build more connections with mathematical concepts they encounter.

According to Crisan el (2007), the integration of ICT into subject teaching can be limited by a lack of teacher enthusiasm for ICT, since many teachers yield to external pressure for examination qualifications (T1, Section 4.4.6.1). Using ICT can challenge, and may even appear scary for, teachers attuned to a traditional *textbook approach*, which emphasises resources that explain procedures or techniques which pupils then reproduce. The lack of teacher enthusiasm for the efforts required to change practice can threaten pupils' developing curiosity and imagination (Claxton, 2004) as they explore learning with ICT tools. Not every teacher's 'learning' behaviour was supportive in certain instances. T1 expressed low perceptions of pupils' ability to 'appropriate' mathematical knowledge; he stated that pupils needed "pushing" (see Section 4.4.6.1). T5 admitted offering negative remarks to the class regarding their involvement in the ICT-enhanced sessions (see Section 4.4.6.5). Aspects of teacher behaviour can demotivate pupils (R257, Section 4.4.2), as these pupils compared feedback from their teachers with that provided by the software for the pupils' learning. While pupil R257 valued the non-judgemental software feedback, pupil U078 underscored *Grid Algebra*'s temporary formative feedback; she described it as "motivating"; Teachers T3 and T4 suggested that the nature of software feedback encouraged pupils exploring of algebra and hence their appropriation of formal algebraic notation, especially use of brackets.

The existing initial teacher training programmes may endow specialist teachers with adequate pedagogic skills to supplement their content knowledge (Niess et al, 2009). Teachers need to be aware of and clear about how to teach subject-matter as domain-specific inquiry (Ruthven et al, 2011). Teachers T3 and T4 assumed multiple roles in the ICT-enhanced sessions, including: monitoring the relevance of conceptual knowledge in pupils' communication and reasoning, managing social interaction in interpersonal discussions, and asking stimulating questions to create a relaxed atmosphere for learning with pleasure (Tall, 2013). Unfortunately, it appears that training programmes barely equip teachers to handle emotions or values in the classroom (Norris and Walker, 2005) as affective aspects needed for emotional 'intelligence risk-taking' (Elder, 2012) in mathematics. Interweaving mathematical thinking, discourse and classroom practice (Lerman, 2001) can merge Vygotskian and constructivist conceptions of knowledge. More focused attention shifts to the affective dimension of cognition. Some teachers appeared to find 'learning' in ICT-enhanced contexts easier than others in this study. Teacher 1E seemed to accept 'learning' more readily than Teacher 2E at Stage One. At Stage Three, T2 and T5 seemed to experience more *difficulty* in coming to terms with their 'learning' process than T3, T4 and, to limited extent, T1. I considered that T2's absence from the ICT-enhanced sessions (see Section 4.4.6.2) was within his right to exercise 'proxy agency', which Bandura (2001) defined as "relying on others to act on one's behest to secure desired outcomes" (p.13). However, T2's avoidance allowed me to 'facilitate' the sessions for 56 pupils in line with my primary research aim: to promote pupil learning. I recognised that teachers with 'good' pedagogic skills and content knowledge may struggle with handling emotions and feelings in classrooms.

It is crucial to identify the appropriate resources for mathematics teaching from a pedagogic and cultural stance, a view underscored by Lerman (2001). Webb and Cox (2004) underlined the value of teachers making pedagogic decisions about how best to use ICT alongside the traditional tools. In Kenya, the lack of development of relevant, locally-produced materials by KIE rendered the teachers unable to either envision or implement the ministerial policy in their pedagogy. The linking of content in a new tool to subject syllabi can be challenging. It requires teachers to come to grips with digital technology while simultaneously covering the curriculum. This was expected of teachers within the context of the practical conditions of their working environments, including the huge teaching workloads and pupil numbers that I described in Section 1.1.2. In truth, practising teachers lack time to identify and explore ICTs appropriate for their use (T4, Section 4.4.6.4); they require support. Walshaw (2013) called for linking theory and practice; she endorsed the practice of researchers supporting teachers in bridging the gap between actual practice and policy within institutional constraints, a view shared by Guskey (2002). The argument implied contextualised professional development. Some teachers may seem to appreciate the strategies advanced in CPD training only to revert back to their preferred practice in classroom settings. I acknowledged my supportive role for pupils and teachers in the ICT-enhanced classroom contexts in Section 1.4.1. This influenced my empathetic stance towards teachers in Kenya. It was the transformative potential for pupils learning algebra with *Grid Algebra* in secondary mathematics classrooms in England that set me on the course I adopted. Invaluable insights into ‘dialogic teaching’ which I had gained on an MSc course led to my emphasis on providing pupils with opportunities to discuss while they learn. My perceptions of mathematics teaching and learning had developed; they existed in my own mind as a set of interconnected conceptual structures. I ‘facilitated’ the ICT-enhanced sessions in this research in that my knowledge of the ‘dialogic teaching’ concept mediated the participants’ ICT use in classroom lessons. This study offered contextualised CPD to the mathematics teachers. Embracing ICT in mathematics facilitated the technology transforming the pedagogy from teacher-directed towards positive pupil-centred *social* learning (Luckin et al, 2012), as valued by T4 in Section 4.4.3. The pupils’ successful ICT-enhanced learning enabled some teachers to acquire some TPACK, both directly (T4, Section 4.4.6.4) and indirectly (T3, Section 4.4.6.3). Valuing pedagogy over technology requires ‘good’ quality teaching and that teachers facilitate effective use of ICT in achieving the intended learning outcomes.

6.4 In what ways did the intervention differ from the participants’ usual classroom practices in terms of distribution of responsibility and accountability in learning?

Data from participant and direct observations, pupil questionnaires, ethnographic and focused teacher interviews provided ample evidence regarding shifts in the participants’ responsibility and accountability realised by the classroom ‘subculture’. The pupils had access to multiple tools to think with: language, printed books, social interaction, and computers. Classroom learning activity in mathematics ICT-enhanced sessions was as shown in Figure 6.1.

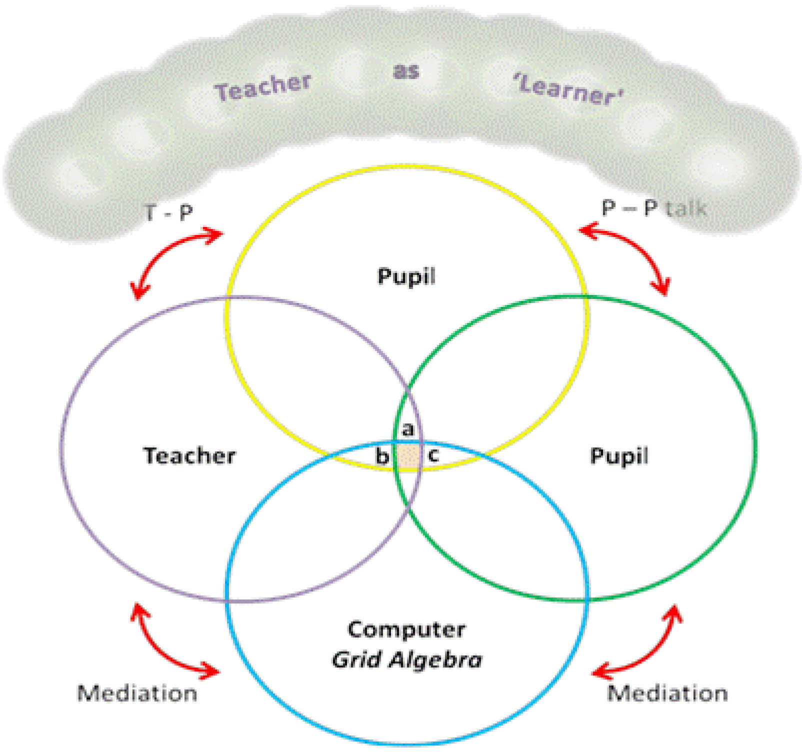


Figure 6.1 Collaborative computer-based mathematical activities (Mercer and Littleton, 2007, p. 82)

The *Grid Algebra* software took on the mediating role between the pupils and the algebraic concepts by providing the means of making the mathematics as explicit as possible. Tabach (2011) was categorical about the need for every participant to contribute in making sense of the mathematics at hand. The ensuing task-based discussions (G035, Section 4.4.1) focused on the algebraic expressions created by the software on computer screens (R270, Section 4.4.1). The ICT-enhanced activity enabled pupils to become informed sources of mathematical ideas (G023, Section 4.4.1). The *Grid Algebra* software became an alternative authority, independent of the teacher's control of knowledge (R222, Section 4.4.2); the 'legitimate' mathematics was no longer statically recorded in textbooks Ruthven (2012) or contained in the teacher's head and explanations (T3, Section, 4.4.2). I observed positive pupil-teacher interactions in small-group and whole-class discussion (see Section 4.4.6.4); Teachers T3 and T4 listened to the pupils' mathematical contributions during the eight ICT-enhanced sessions for their classes; they attended to the realignment of difficult concepts and misconceptions for their pupils while occasionally providing assistance in supportive roles to individuals or groups of pupils where it was required (see Section 5.3). T3 described reflecting on pupils' actions in the ICT-enhanced sessions when planning for non-ICT lessons, as argued by Ruthven (2012). Therefore, this intervention enabled positive changes in the ‘learning’ behaviour of T3 and T4 following the pupils’ instrumental genesis and the nature of the mathematics learned.

The mediating tools between the pupils and algebraic concepts were mainly the *Grid Algebra* software in the ICT-enhanced sessions, and the teacher in the non-ICT lessons. The tools enabled the pupils: to use the resources in the learning context pragmatically, to link the contrasting representations, and to derive meaning from that use. Whilst taking on a supportive role in the ICT-enhanced sessions, some teachers assisted in consolidating their pupils’ understanding. I contend that the ideal relationship between teachers and pupils is one where participants ‘relax’ and hold meaningful dialogues (T1, Section 4.4.1). A ‘subculture’ of task-based pupil ‘talk’ enabled the teachers to gain access to the learning process and to see the ‘difficulty’ in reasoning experienced by their pupils in understanding algebra (U077, Section 4.4.5). It is imperative that teachers expect ‘disciplined behaviour’ from pupils because mathematics, with its extensive register, requires a pupil to be attentive to what they say, do and hear. This calls for both discipline and self-discipline, as facilitated by the ‘ground rules’ (Edwards and Mercer, 1987) that operated in the ICT-enhanced sessions (P203, Section 4.4.1). Every pupil had the responsibility to engage with, check and complete their work (K162, Section 4.4.4) in written or oral tasks, as they responded to ‘formative feedback’ (Juwah et al, 2004).

This intervention has identified a move beyond simplistic pupil-tool interaction towards: the complex process of instrumental genesis (Guin and Trouche, 2002); the role of teachers; and combining ‘talk’ in ICT-enhanced sessions with ‘traditional’ pedagogies to realise a *blended learning* in secondary mathematics for pupils. The *Grid Algebra* software provided a variety of algebraic activity for the pupils:

- generational activity, as they created algebraic expressions and equations;
- transformational, rule-based activity, through manipulating and simplifying algebraic expressions, ‘inverse journeys’ for solving linear equations;
- meta-level activity, as they visualised algebraic structure and problem-solving.

In the meta-level activity, using *Grid Algebra* involved these pupils in learning processes for creating algebraic expressions through the associating physical movement with mathematical operations. Hence, the pupils became aware of *what* the process was (R220, Section 4.4.5). In this way, the grid allowed pupils to perform the process of forming the algebraic expressions, and viewing the process itself as an object that could be manipulated (P181, Section 4.4.5). This indicated the development of 'proceptual thinking' (Gray and Tall, 1994) in the pupils. The ICT-enhanced activity encouraged the generation of second-level awareness in pupils (U099, Section 4.4.4). This was achieved through the establishment of 'purpose' (Ainley et al, 2006) that transformed the learning of algebraic content in a way that allowed the pupils to question and to reflect on whether or not the things they did were mathematical. In Section 4.4.2, pupil U101 described how *Grid Algebra* enabled the pupils to know how algebraic expressions are formed; pupil U095 described gaining "the real sense" of mathematics. The computer-based activities supported the pupils' awareness of what they did when working on pen-and-paper exercises in textbooks and examinations. The pupils' task in the ICT-enhanced sessions changed to becoming aware of *what* they were doing, and at the same time *doing it*: to think about the questions and explain their reasoning, not just doing questions (G026, Section 4.4.4).

My focus on learning behaviour recognised the mathematics classroom 'subculture' (Bauersfeld, 1998) as something that participants enacted. My overall research purpose was to provoke sustainable transformative learning processes that would be owned by the participants to address the problematic pupil interest in algebra and to increase pupils' involvement and engagement in mathematics lessons. Several pupils talked about their ICT-enhanced sessions as learning experiences that were *memorable* for articulated reasoning with their peers (G023, Section 4.4.1). The benefits that the pupils gained from 'dialogic teaching' included increased confidence, engagement with algebra and an ability to crystallize what they were thinking. This resonated with claims made by Ruthven et al (2011), Alexander (2008) and Mercer and Littleton (2007). What was remarkable, both to me and to Teachers T1, T3 and T4, was that the pupils transported their re-engagement to non-ICT lessons. The teachers, who previously despaired of preventing the 50-plus pupils from dozing during their 40-minute lessons, had to contend with managing the raised pupil participation (T3, Section 4.4.6.3). The pupils' mathematical competence is constructed when they are inspired to engage with the algebraic learning process and to assess the sensibility of their mathematical ideas (U077, Section, 4.4.5); the pupils discussed meanings of algebraic representations with group members, and learned from their 'mistakes'(K133, Section 4.4.2) in the ICT-enhanced sessions. This signalled clear intentions to *teach* pupils self-regulatory learning behaviour; there was less emphasis on pupils' ability to finish the work given, and instead for pupils to:

- note when a peer's contribution did not make sense;
- make that confusion clear to the contributor;
- offer suggestions to improve the idea.

The format of these sessions embraced the incremental theory of intelligence (Dweck, 2000): that everyone can become better at handling and using algebraic ideas by facing the difficulties they come across whilst they learn. Lee (2006) argued that emphasis should be placed in teaching an incremental view of mathematics learning: that every pupil can improve with perseverance from self and with supportive help from others. Hence, I stressed certain key aspects: *small-group collaborative learning*, *articulation*, *inclusion*, and *learner agency*. These were notable since they marked a departure from the participants' usual 'subculture'. I discuss them each in turn.

6.4.1 Small-group collaborative learning

Skinner and Belmont (1993) explained that a learning environment, in which pupils are free to discuss, and to support and challenge each other, requires the support of explicit expectations. Pimm (1987) proposed that articulation of one's ideas helps in the process of reflection by providing better access to one's mental reasoning. Talking enables thoughts to be 'externalised' making them accessible to the speaker's own and to others' scrutiny, in keeping with linking thoughts to speech (Vygotsky, 1981). Once a pupil utters their contribution, it becomes a concrete fixed expression as well as being an idea that all pupils can internalise and reconsider against their inner thoughts. I contend in Section 3.1.3 that both the pupils and the teachers need to develop speaking and listening skills for this type of classroom learning to be effective, as stressed by Alexander (2008) and Foster (2014). In Section 4.4.1, pupil R236 described their regular mathematics lessons as "boring"; to pupil R229 they were "monotonous"; in contrast, T1 described the learning atmosphere, when pupils worked in small groups using *Grid Algebra*, as "relaxed". The new and enabling classroom 'subculture' was reported to engender greater pupil participation in whole-class discussions.

Pupils had freedom to identify the peers they chose to work with on the *Grid Algebra* tasks in the ICT-enhanced sessions. They appeared to relish working in these units since the small groups generally operated well and effectively. These pupils took the opportunity to develop interpersonal and communication skills. They shared understandings irrespective of their varied mathematical ability rather than forming 'gangs' based on their differing sociocultural backgrounds, which were the result of an admissions policy (see Section 3.2.4). It was possible for me to distinguish groups of pupils with similar, usually 'urban' characteristics, including being active and speaking readily and confidently in whole-class discussions. The peers in the Stage Three classes mixed very well. The pupils were free to choose where to sit and work from. I failed to discern patterns in their choice of group members. The observed pupil behaviour negated strongly the assertion by Lambirth (2006) about the setting of 'ground rules'. Lambirth (2006) argued that such rules reinforce social inequalities and disempower pupils from less privileged backgrounds without enhancing communicative and cognitive activity. The support provided by expectations in the ICT-enhanced sessions generated a sense of pride, commitment and 'ownership' of learning, leading to academic progress in many pupils from diverse backgrounds, notwithstanding the huge number of pupils in a class and the design of the room. It seemed that what mattered most for these pupils was to stay on task for the duration of the sessions. Pupils shared their ideas with fellow group members, and reflected on the results displayed on the screens. My interpretation of the effect of these pupils' thoughts, beliefs and feelings on their behaviour is based on the theory of Bandura (1986). The ICT-enhanced sessions were seen to enable coping strategies or protective mechanisms in the Stage Three pupils. The pupils developed increasingly high mathematical self-efficacy (Bandura, 1989); they proceeded to solve written algebraic tasks with new confidence (P194, Section 4.4.3) and competence. The pupils registered significant progress in the examinations set and marked by their teachers (see Figure 4.4 in Section 4.4.3).

6.4.2 Articulation

Pimm (1987) explained the practice of verbalising mathematical talk as focusing attention on arguments and convictions behind pupils' reasoning; it allows for more precise and succinct expression of ideas which may be readily verified if necessary. The requirement to verbalise thoughts and ideas out loud is based on Vygotsky's (1978, 1962) theories. It allows both the contributor and the listening pupils to reflect on the mathematical problem and the reasoning in solutions offered. When individual pupils were encouraged to articulate their explanations, their peers listened attentively and considered each contribution, whether to add to it or improve on it (G023, Section 4.4.1). This 'subculture' had the cumulative effect of checking and improving pupils' problem-solving strategies as well as enabling pupils to develop content knowledge, and to use mathematical language. Therefore, as they collaborate on mathematical problems, pupils can develop expertise (Claxton, 2004) in using language as a tool for reasoning (Mercer and Sams, 2006). Task-based 'talk' enabled the pupils' co-construction of 'meaning' for algebraic concepts (G023).

The provision of feedback on one's performance by peers, the teacher and ICT tools plays a vital role in both triggering reflection on one's thought process, and encouraging self-assessment. Black and Wiliam (1998) emphasised the value of pupils taking corrective action in order for learning to take place. I observed that many pupils seemed comfortable about externalising ideas following supportive feedback from the software and their peers (K135, Section 4.4.1). The ensuing dialogues were beneficial to pupils' learning because:

1. the pupils who had just mastered the working of the software were better able to explain it in a language and a way that was more accessible to their peers than teachers;
2. dialogues exposed all pupils to alternative views and strategies for problem-solving;
3. the pupils could develop objectivity in judging work through commenting on their peers' contributions against standards which they then could apply to their own work;
4. dialogues motivated the pupils by encouraging them to persist in achieving the learning goals;
5. the teacher's assessment may be harder for the pupils to accept than peer assessment expressed in dialogue (R257, Section 4.4.2).

The Stage One pupils' silence when they were required to solve questions on 'Substitution' was a poignant example of the importance of *talk* in the learning process. The pupils' silence offered the teachers no information about the underlying conceptual difficulties. The learning behaviour highlighted the importance of pupil talk as a conduit of a pupil's difficulty in understanding the concepts. The pupils' behaviour may have caused Teacher 1E's frustration that led to her raising doubts about the pupils' ability to cope with higher-level GCSEs. The teacher's remarks targeted pupils' self-level (Hattie and Timperley, 2007) rather than the task, process or self-regulation levels; they did little that I could see to move the learning forward. Insecure 'number sense' (Tall, 2004) was revealed in many pupils through the change in learning strategy (see Section 3.2). In as much as formative feedback provides pupils with information about their learning, it also helps teachers to target their teaching to meet pupils' learning needs. Gresalfi et al (2009) argued that mathematics educators need to be concerned with whether or not learners act, and more so with how learners act, when they are provided with opportunities.

6.4.3 Inclusion

The introduction of computer-based activities advanced the notion of 'play' (Whitton, 2007) with emphasis on the affective aspects of classroom learning in mathematics lessons. The pupils in this study experimented with curiosity and discussed their predictions. Pupil G023 described the group dynamics whilst exploring algebra using *Grid Algebra* in Section 4.4.1. The activity harnessed the potential for the technology to mediate the mathematical concepts. This mediation combined opportunities for pupils to engage in discussion with their peers devoid of power relations. Lee and Johnston-Wilder (2013) stated that collaborative working can foster the creation of a supportive 'community' that provides a safe learning zone for pupils sharing their partial understandings as they work on mathematical tasks (R251, Section 4.4.5).

The ensuing formative feedback that pupils received in the ICT-enhanced learning environment, from peers, adults, books and *Grid Algebra*, encouraged reflection on their intuitions. Mistakes were valued as 'learning points'. Verbalisation of partial understandings was considered acceptable and an invitation for further contributions to improve on incomplete answers. Small-group working enabled pupils developing their mental skills (K135, Section 4.4.1). At Stage Three, some pupils were willing to explore algebra further with *Grid Algebra* (U086, Section 4.4.1). Nevertheless, 88 pupils (33%) pupils lacked the requisite low-level computing experience (see Table 3.2 in Section 3.2.3). I observed the resolute intent of some determined pupils to voluntarily take up 'extra sessions'. I described in Section 3.1.5.4 pupils working on software-generated and associated written tasks (R234, Section 4.4.1); they indicated their developing 'mathematical resilience' (Lee and Johnston-Wilder, 2013).

6.4.4 Learner agency

Claxton (2004) suggested that mathematics in educational systems should focus on teaching pupils self-regulating self-efficacy. The prevalent practice in Kenyan mathematics classrooms is such that the pupils are expected to reproduce techniques (K117, Section 4.4.2); the pupils exercise 'disciplinary agency' (Pickering, 1995). Pupils are accountable for demonstrating their mathematical understandings to their teachers in class exercises, tests and examinations (Kanja et al, 2001). Thus, the pupils are unconsciously encouraged to cede responsibility for their thinking either to the teacher (T3, Section 4.4.2) or to class textbooks. Many pupils stick to the step-by-step explanations without question (P198, Section 4.4.3).

This study incorporated secondary pupils' use of an ICT tool as they learned algebra, since it has been argued that technology can give the user actions for which they can be responsible, and of which they have ownership. The behaviour I hoped would be elicited in the Stage Three pupils by observing the 'ground rules' included:

- knowing what to do;
- believing they can take action while learning;
- not feeling helpless and dependent;
- welcoming the change in ownership of responsibility for their learning;
- not abdicating responsibility for their thinking to the teacher.

This study evoked the various stimulating features of the computer to incorporate the aspect of 'play' while pupils learned algebra in ICT-enhanced sessions. These features included the pupils controlling the software using a mouse in small-group collaborative working as they engaged in solving puzzles. The learning activity involved working with the visual display of the contributor's input, described as "clues or routes" (P179) and "arrows" (P193), which allowed the pupils to reflect on their thinking as they shared their ideas. The pupils' description of the apparent 'ease' with which they mastered using *Grid Algebra* as a learning tool alluded to the skill enabling the software to develop into, not a mere tool, but a resource with which to learn (R228, Section 4.4.2). The 'hands-on' sessions in the computer laboratory provided screens onto which pupils expressed algebraic ideas, something 181 pupils (67%) claimed to enjoy (R225, Section 4.4.5). In Section 3.2.4, mathematics teachers remarked about pupils solving puzzles in *Tasks* by themselves. The *Interactive Grid Algebra* mode allowed options for the pupils to explore the tool through testing their curiosity and predictions in positive learning experiences as they learned. The software provided the pupils with opportunities to enhance their 'adaptive reasoning' (Kilpatrick et al, 2001). It opened a 'window' (Noss and Hoyles, 1996) into their mathematical thoughts. The visual processing enabled the pupils to link algebraic representation with 'proceptual thinking' (Gray and Tall, 2004). In time, many pupils became able to switch between two contrasting ways of seeing an algebraic expression: as a process, and an object encapsulated in single expressions (Sfard, 1991). Teachers T3 and T4 described the pupils' symbol awareness; algebraic learning was reshaped (Deaney et al, 2003) in the ICT-enhanced sessions. Algebraic activity evoked the 'Play Paradox' (Ainley et al, 2006; Noss and Hoyles, 1996) since the pupils learned whilst they ‘played’ (P165, Section 4.4.5). This aspect stressed the value of games in education as an effective influence in supporting and maintaining learning processes (Whitton, 2007).

The introduction of collaborative learning, with an emphasis on *pupil talk*, in the ICT-enhanced sessions appeared to change the ‘subculture’ of the mathematics lessons. The activity created more opportunity for pupils to act and test their own thinking (R233, Section 4.4.2). Small-group working on software-generated tasks helped in facilitating active and productive engagement in the classroom learning activity (Gresalfi et al, 2009). Each pupil was accountable to the other group members through a requirement to convince the others of the mathematical sense in solutions, to field questions from their peers, and to discuss strategies to solve given tasks (G023, Section 4.4.1). Hence, the pupils exercised ‘conceptual agency’ (Pickering, 1995). This made each pupil work harder than before (K135, Section 4.4.1). The expectations enforced by the ‘ground rules’ in the ICT-enhanced sessions reportedly placed demands on the mental arithmetic skills of 66 pupils (24%). The pupils described the shift in the concentration of ‘conceptual agency’ in mathematics from the teachers to the pupils; they described how they had to ‘think more’ (T1and T3, Section 4.4.4). This signalled a return of the responsibility for learning to the pupils.

In a worrying development, I observed some teachers taking it upon themselves to make subject content ‘simpler to understand’ in a bid to keep the pupils ‘on-task’. This was illustrated in the suggestion that I write ‘BODMAS’ on the board for every ICT-enhanced session to serve as a reminder for the pupils. It is of note that many pupils seemed unable to apply BODMAS (see Section 4.1). This was described as a “spoon-feeding method from teachers” (K112, Section 4.4.2); it expressed their sense of *unease* with ‘instrumental understanding’ (Skemp, 1976) labelled as ‘epistemological anxiety’ (Wilensky, 1997). The practice essentially shifts the responsibility for thinking about the mathematical processes from the pupils (Pickering, 1995); this implies doing the work on behalf of learners. However, the data have strongly suggested that, through collaborative learning, articulation, inclusion and agency, the pupils were capable of absorbing and developing this knowledge for themselves given the opportunity. Hence, mathematics teachers at Key Stage 3 need to believe in their pupils’ ability to take responsibility for learning content (HOD, Section 4.4.6.5) in classrooms. This can best be achieved by the teachers strengthening their role of facilitating that learning. Learners need support when appropriate, then the gradual ‘fading’ of that support in order to discourage dependency which can hinder effective pupil learning from taking place.

6.5 Summary

In this chapter, I have shown that, when careful consideration is invested in providing quality learning environments, several theories put together can work to promote 'social learning' in mathematics classrooms. The findings provide fresh insights into a classroom 'subculture' with 'dialogic teaching' in conjunction with the use of ICT tools adding to the theory of thought and speech (Vygotsky, 1978, 1962) and the social cognitive theory (Bandura, 1986). The use of 'resilience' thinking can lead to effective change in pupils learning of algebra at Key Stage 3. In the next and final chapter, I summarise the key findings from this study and the implications they have for pedagogy and research in mathematics education.

Chapter 7: CONCLUSION

7.0 Introduction

This chapter is a summary of the lessons learned from my study into pupils learning algebra with *Grid Algebra* at Stage Three. The fundamental impetus for this research evolved from a proposed reform to education in Kenya. The stated desire was for active pupil engagement with learning rather than quiet acquiescence to teacher authority in the classrooms (Kanja et al, 2001). The initiative for this change was envisioned as increasing interest and questioning minds in the pupils to develop 21st Century skills in readiness for their future life challenges (CEMASTEA, 2009). My study has illuminated some crucial issues concerning pupil engagement and participation in secondary mathematics classrooms: the value of harnessing 'pupil voice' in educational reforms; the pedagogic significance of ICT use in mathematics teaching; and how successfully to involve teachers in change. In the discussion that follows, I emphasise the transformative potential of effective integration of interacting elements in the key findings, the limitations experienced in this study, and the implications for further research.

7.1 Key findings

7.1.0 Introduction

My thematic discussion in Chapter 5 entailed: changed learning environment; learner agency; changed motivation; accessible learning; affect and enjoyment; variable teacher 'learning' behaviour. In this section, I underscore the following three meta-developments: increasing pupil engagement and interaction; ICT-enhanced pupils' learning about algebra; pupils' success affecting teachers' attitudes to ICT use.

7.1.1 Increasing pupil engagement and interaction through talk

Firstly, my study established the importance of stimulating 'pupil talk' and of learning from that talk in lessons, and its impact upon pupils' motivation and attainment. The verbal dimension of mathematical learning was used explicitly to support pupils' cognition. The contributory features in transforming mathematics classrooms to create an enabling learning environment (DEAG, 2013) included: articulation; collaboration; inclusion; and learner agency in computer-based activity (Figure 6.1, Section 6.4). The traditional *textbook* approach in Kenya placed an emphasis on teacher exposition; it appears that many of the pupils did not have control of their own learning. Following Pickering (1995), pupils basically exercised disciplinary agency by reproducing procedures in classroom exercises and examinations. It appears that whenever the pupils got 'stuck', without their teacher, they felt that they were unable to act (P198, Section 4.4.3). The pupils' stance was one of dependency and 'learned helplessness' (Dweck, 2000). The ICT-enhanced context encouraged pupil-centred dialogic talk (see Section 6.2) around the learning tasks, and a classroom 'subculture' unfamiliar to the participants (see Section 1.1.2).

Requiring the pupils to speak as they solved puzzles served to externalise their thinking (Pimm, 1987); the pupils' algebraic decisions were displayed as consequences on their computer screens. Linking of thought and speech (Vygotsky, 1962) in this way made the pupils' thinking available for assessment by 'self', their peers and teachers. This articulation enabled ideas to be shared, and built upon, and allowed difficulties to be addressed (Alexander, 2008). The incidence of interpersonal dialogue centred on algebraic ideas was increased significantly. Invaluable data collected from participant and direct observations, two pupil questionnaires, ethnographic and focused teacher interviews provided ample evidence of meaningful pupil interactions and engagement in mathematics lessons as discussed in Section 6.1. In keeping with the Vygotskian concepts of 'internalisation' and 'zone of proximal development' in the social, linguistic and cultural aspects of mathematics education (Lerman, 2001), I focused on the affective dimensions of learning. I facilitated both 'dialogic teaching' and *Grid Algebra* use in lessons after seeing them in action generating increased pupil algebraic 'talk' (see Section 6.3).

This study focused on how the enrichment of classroom learning experiences stimulated changes in the pupils' thinking. It stressed developing an array of skills, including: considering, probing, and exploring as the pupils engaged in mathematics. Elder (2012) argued that teachers demand 'good' use of language as a tool for reasoning by pupils to ensure progress is realised. In agreement, Foster (2014) proposed minimal teacher intervention when moments of 'struggle' surrounded the pupils as effectively placing ownership of learning in pupils' hands (U060, Section 4.4.2).

Classroom 'subcultures' demanding complete solutions tend to discourage many pupils from sharing their partially-formed ideas; such 'subcultures' can promote an apparent sense that everyone else is clear about what is being taught. Teachers should acknowledge that pupils deemed to be in the same set may indeed be in different 'worlds' of mathematics (Tall, 2004). Some concepts may be easy for one pupil yet pose great difficulties for another, depending on their position in actual and potential developmental levels (Bliss et al, 1996). Pupils placed in the same set or deemed to be at a certain 'level' can reveal startling differences in attainment. This raises concerns about pupils learning in set classes as though of homogeneous 'ability'. From a sociocultural perspective (Vygotsky, 1978), it is imperative for pupils to externalise their complete, partial or faulty thinking. When the classroom ethos values 'mistakes' as learning points, pupils share their understandings which may then be assessed and built upon without fear of shame and embarrassment (R270, Section 4.4.1). Given the positive outcome of such an approach in my study in Kenya, the practice should be vigorously embraced and enforced by teachers, as argued by several researchers (Alexander, 2008; Mercer and Littleton, 2007; Lee, 2006). The pupils' task-based algebraic 'talk' provided insights into their thinking and conceptual beliefs. This information guided teachers' reflections on pupil learning (Guskey, 2002). In Section 4.4.1, Teacher T3 described holding back and drawing pupils' attention to their actions; she instigated their reflection on the information displayed by the computers. Positive teacher 'learning' behaviour realised a match between the teaching aims and the learning outcomes.

7.1.2 ICT-enhanced pupils' learning about algebra

Secondly, pupils engaged successfully with *Grid Algebra* and learning algebra in the Kenyan context. This was despite some inhibiting organisational challenges described by Teacher T1 (see Section 4.4.6.1) and that 88 (33%) pupils had no prior computing experience, and hence had low computer confidence (U086 and R234, Section 4.4.1). Given the ratio of pupils in a class to computers available, it was necessary for the pupils to share the ICT resources. This naturally led to collaborative learning and more social interaction in the ICT-enhanced mathematics lessons. Collaborative small-group working facilitated more opportunity for many pupils to develop interpersonal skills, including speaking (R251) and listening (G023); such skills are deemed to be essential 21st Century skills (DEAG, 2013). In Section 4.4.2, pupils P190, G053 and K146 described their enabled active participation; they discussed freely and shared their ideas in critical discussions; pupil G022 described how kinaesthetic activity enhanced her algebraic understanding, a view expressed by Teacher T3 in Section 4.4.6.3. In Section 4.4.6, Teacher T1 described the pupils as "relaxed"; Teacher T3 stated that the pupils were "absorbed and enjoying themselves". Teacher T4 remarked about the pupils being "happy, willing to have a go; and looking forward to the next lesson".

The 'ground rules' of engagement in the ICT-enhanced sessions (see Section 3.2.3) provided the pupils with structure and support for autonomy (Skinner and Belmont, 1993) in their learning (P203, Section 4.4.1). The learning environment enabled the pupils to feel safe and to take action in the following ways: to express their partially-formed ideas (Lee and Johnston-Wilder, 2013), by allowing an exchange of ideas and alternative viewpoints (K151), and to make decisions on movements across the grid, creating expressions (P171). The use of trialling, re-trialling and amending of thought processes allowed the pupils to experiment; they explored what 'works' when solving puzzles. The pupils saw, and understood the 'why' in algebraic relationships (U091). Thus, they exercised conceptual agency (Pickering, 1995).

Wall et al (2005) underlined the scarcity of educational research about pupils' understanding of their own learning; they considered meta-cognition to be an essential basis for effective life-long learning, itself a 21st Century skill (DEAG, 2013). The raised expectations of pupils to think about and articulate their reasoning in the ICT-enhanced sessions, instead of just solving puzzles in algebraic tasks, generated meta-cognition in the pupils (P177, Section 4.4.2). The Stage Three pupils made reasoned judgements about mathematical decisions; they indicated an awareness of their actions; some 155 (57%) were clear about understanding algebra 'more', and showed that they were aware they had *learned* something. In Section 4.4.2, pupil U060 described how she developed a belief in her own solutions, an awareness of her own learning; many pupils stated that the practical aspect of the activities had enhanced their understanding (R264). In Section 4.4.1, pupil P198 reinforced this claim by explaining that algebra could not have been well understood without *Grid Algebra*. In Section 4.4.4, pupil G011 described their ability to use knowledge learned in the computer environment in the solution of written tasks with reasonable success, as argued by Healy et al (2001) and Hoyles et al (1994). My findings show the evolution of a second-level of awareness in pupils: awareness of their improved understanding of algebra, evidenced in better than expected outcomes compared to previous cohorts (Figure 4.5, Section 4.4.3).

As an alternative resource in the learning context, the *Grid Algebra* software provided the pupils with tasks, and an invaluable opportunity to practice and apply their understanding of mathematics. In Section 4.4.2, pupils P171 and U108 described using mental images of the software in class exercises and examinations, akin to the case of the 'mental abacus' (Frank and Barner, 2012). The formation of mental images may have assisted in consolidating the pupils' number sense, and may have influenced their cognitive development. The claims suggested that an emphasis on developing pupils' mental arithmetic skills can enhance problem-solving. Stigler et al (1986) wrote about skills practice contributing to Asian pupils' cognition; they decried the tendency in Western cultures to regard mental arithmetic skills as synonymous with rote learning. I argue that, rather than devaluing the potential of rapid and precise computation, encouraging such skills can develop internal resources such that pupils can recall at speed. Automaticity can free pupils to attend to making connections for more conceptual understanding in mathematics.

The explicit blending of the traditional with 'new' technologies in this study appears to have created alternative pathways to learning algebraic concepts for the pupils. Many pupils (67%; 181 pupils) were clear about 'enjoying' the opportunities for self-regulated learning on computers in the ICT-enhanced activity, which they considered to be "easier", satisfying and motivating (see Section 4.4.5). Healy et al (2001) and Bell (1996) emphasised the vital role of symbolic language in pupils' algebraic thinking. Tall and Thomas (1991) advanced the development of 'versatile thinking' in pupils using the 'computer approach' to teaching and learning algebra; the strategy enables pupils to move into the 'proceptual-symbolic world of mathematics' (Tall, 2004). In Section 4.4.4, Teacher T4 outlined pupils' difficulties with accessing algebraic ideas. I described how pupils 'learn with understanding' (Sfard, 2001) by engaging with the various software-generated tasks in Section 5.4; the tasks introduced the pupils to new learning experiences of algebraic concepts. *Grid Algebra*'s potential to link dynamic visualisation with formal symbolism led to increased connectivity in the pupils. The arithmetic context provided by the software's grid allowed the development of a structural conception (Kieran, 1992); the users' actions were displayed algebraically. The dynamic images of the 'journeys' showed the mathematical processes represented by algebraic expressions. The dynamic visualisation of images enabled pupils' attention to be directed at underlying structures in 'Algebraic Expressions'. This allowed Stage Three pupils to engage successfully with *Grid Algebra*: 'connected knowing' (Boaler and Greeno, 2000) and 'relational understanding' (Skemp, 1976) leading to 'deep learning' (Abbot et al, 2009) of algebra were all described by pupil U099 in Section 4.4.4.

7.1.3 Pupils' success affecting teachers' attitudes to ICT use

The third key finding was the consequence of the increased pupil engagement and attainment on the teachers' interest in the use of ICT. Although *Grid Algebra* deals with only a section of the mathematics curriculum, it breaks down algebra into easier-to-master steps for pupils; the software encouraged the re-trialling of thoughts (K133, Section 4.4.2). The 'journeys' across a multiplication grid seemed to make the complex algebraic concepts simpler to explain. The 'consequential' nature of the software feedback (Hewitt, 2012) required the pupils to make reasoned decisions about physical actions and the related mathematical knowledge. The design expected pupils to rely on their 'met-befores' (Tall, 2004); it made demands on the pupils' number sense, and their knowledge of multiplication tables and 'Integers'. This revealed the security of pupils' prior knowledge. The grid linked arithmetic to algebra and engendered 'coherence' (Rudduck et al, 1994), defined as fitting ideas in meaningful ways (Tall, 2013). I discussed the software building capacity of pupils taking action and modifying their thinking (see Section 6.4). *Grid Algebra* linked visual imagery with formal symbolic notation, enabling pupils to develop their 'symbol sense' (Arcavi, 1994) and to increase their 'connectivity' (Noss et al, 1997).

I viewed the mathematics classroom 'subculture' through a sociocultural lens as an enabling environment for pupils learning algebra with ICT. The laying of 'ground rules' (Mercer and Littleton, 2007) created a specific classroom ethos. This encouraged pupils to 'have a go', to articulate and share their understandings, and to ask and answer questions whilst working collaboratively on the *Grid Algebra* tasks; they exercised 'conceptual agency' (Pickering, 1995). There is little doubt from the findings that the pupils found the use of the new technological tool in their mathematics lessons quite engaging (G019, Section 4.4.3); their faces lit up when the weekly ICT-enhanced session was imminent; they raced at speed to the session, ready to engage with algebra (T1, Section 4.4.6.1); and they were extremely loath to exit the laboratory as their sessions ended, as observed by the teachers (T3, Section 4.4.6.3; T4, Section 4.4.3).

My study findings strongly suggest that using *Grid Algebra* led to increased connectivity and attainment. This claim is evidenced in data collected from pupils' written work in worksheets (see Section 4.4.4) and examinations (see Section 4.4.3), as argued by Guskey (2002). I saw pupils gradually accepting and using formal algebraic notation (see Section 6.2). The design based on pupils' mathematical 'met-befores' (Tall, 2004) made the software successful at presenting algebraic ideas in creative and non-threatening ways. Pupils were observed to synthesise mathematical concepts in solving unfamiliar problems (T3, Section 4.4.3). Pupils' progress in mathematics achievement indicated a 'transfer' of learning (Lobato, 2003). The teachers described how valuing *Grid Algebra* use provided the pupils with invaluable practice and changed motivation (T4, Section 4.4.6.4) and connectivity within mathematics (T1, Section 4.4.6.1).

In Section 6.3, I highlighted some differences in teacher 'learning' behaviour in support of pupils' learning in ICT-enhanced contexts. It is imperative that we give consideration to the teachers' affective domain as influencing the successful uptake of ICT in subject teaching. At Stage Three, I observed: Teachers T3 and T4 overcoming their *unease* and being supportive following *new* pupil learning behaviour and skills; T1's ambivalence since he seemed to emphasise covering work more than attending to pupils' learning; and T2 and T5's notably non-supportive behaviour (see Section 4.4.6). When used appropriately, ICT can stimulate the fundamental shift from a teacher-hegemony pedagogic model as favoured by T5 (see Section 4.4.6.5), to a more pupil-centred model of learning. The change in pedagogy enabled a classroom 'subculture' facilitating pupil interest and engagement in mathematics (K142, Section 4.4.4). Teacher T3 realised the greatest transformation of 'learning' behaviour; while relying on my support in the sessions, T3 formed favourable attitudes to computer teaching in mathematics (see Section 4.4.6.3). I also noted that the pupils embraced their learning more readily than teachers (see Section 3.2.2). Successful pupil engagement and attainment at Stage Three inspired the continued use of the ICT tool by some teachers after I left the field.

7.1.4 Overview of research questions

The engagement factor played a key role in keeping secondary pupils on task. In the process of working collaboratively using the *Grid Algebra* software whilst verbalising their ideas, many pupils seemed to develop considerable insights into algebra. They eventually appreciated learning algebra, which the literature suggests they might not have achieved if they were working by themselves after teacher-directed instruction. I suggest that secondary pupils learned and developed sociability and experimentation as invaluable skills (Claxton, 2004). The ICT-enhanced activity made algebraic learning experiences less isolating and more inclusive for pupils. The final research questions that guided Stage Three were answered adequately in this intervention. My aims to increase pupils' interest in algebra and engagement, leading to some of the teachers 'learning' about pupils learning algebra with ICT at secondary school, were achieved, and showed the significant impact of the research.

7.2 Limitations of this study

I acknowledge that I do not fully address the role of teachers in ICT-enhanced contexts. I became increasingly aware of the enormous influence of affective traits in teacher 'learning' behaviour as the study progressed. With hindsight, teachers' ICT-related motivational beliefs, on their own, could have been examined further using a teacher-specific instrument, similar to the MBRQ (see Section 3.1.5.6) for firmer conclusions.

I recognise that participants emerging from fresh personal experience may provide distorted accounts; others may describe themselves favourably when driven by self-protection, and hence may compromise the reliability of the data. Collecting more data might have been useful, but was not possible for me as a single researcher with limited resources and time in the field.

7.3 Implications of my research

7.3.0 Introduction

The findings of my study suggest that appropriate and effective use of ICT can make 'hard to teach' concepts accessible to pupils in mathematics education. The use of ICT can engender pupil engagement and effective pedagogy in the secondary mathematics classroom learning.

7.3.1 Pedagogy

The results of my research add to the calls for greater consideration to be given to increasing learners' engagement in mathematics lessons in Kenya (Kanja et al, 2001). In Section 6.4, I described the emphasis on collaboration, articulation, inclusion and agency in the study's design. This emphasis paid attention to the power talk-based use of *Grid Algebra* enabling a *social* model of learning algebra with formative feedback. The variety provided new learning experiences described by Stage Three pupils in Section 4.4 as developing their mental, social and emotional skills (Claxton, 2004). In Section 4.4.2, pupil P206 described her gradual mastery of *Grid Algebra* enabling her learning algebra with the tool. However, educational research on successful group-work seems less focused on classroom-based interaction with computers, and more on distance and collaborative learning between adults (Webb and Cox, 2004). My study has shown that Kenyan pupils can be encouraged to assume more responsibility for their learning. In Section 6.3, I suggested there may be value in modelling for practising teachers the effective use of ICT in real-life classroom conditions. Teacher commitment needs to be geared fully towards integrating digital technology (Hennessey et al, 2005) into mathematics classrooms. I described the effect of pupils' learning on teacher 'learning' behaviour in my study. Teacher T3 managed to reflect on her classroom practice; she embraced an 'external organising agent' role (Ruthven, 2012) in the ICT-enhanced context. This case illustrates that increasing pupil engagement can challenge a teacher's knowledge, skills and beliefs. The findings have shown that the social model of learning mathematics with ICT is worthy of further research in terms of effective CPD and delivery (Guskey, 2002).

7.3.2 Future research

I am pleased that the teachers in the Kenyan school have continued to use *Grid Algebra* with subsequent cohorts. The managing of resources available in this school, namely the: people, conditions, knowledge and skills, at Stage Three, reflected the emphasis on sustainability in the Chile model (Rodriguez et al, 2012). In Section 3.2.4, I suggested that the effect of ICT use in mathematics lessons was magnified through involving an entire cohort. In Section 4.4, I presented participants' perceptions verbatim of the 270 pupils' behaviour in ICT-enhanced sessions; the participants described the general enthusiasm generated by this research study. The transformation of the pupils and some of the teachers, including at Stage Two (Lugalia et al, 2013), may have influenced the continued use of *Grid Algebra*. These findings strongly indicate the value of involving a large population. Further research may be conducted at this school to measure the quantitative impact of continued ICT use on pupils' attitudes and to assess the predictive potential for long-term growth in learning mathematics at Key Stage 3.

7.3.3 Educational technology

Several pupils inquired about acquiring the ICT tool for personal practice (see Section 4.4.5). Rapid advancements in digital technology appear to be registered within mobile technologies. I propose the development of a mobile phone application for the *Grid Algebra* software; it may enhance the software's accessibility. I base my suggestion on the prevalence of mobile digital technologies in Kenya.

Notwithstanding the lack of local, digital curriculum resources, the policy for ICT integration in education exists in Kenya. KIE should hasten developing appropriate ICT tools for use in subject teaching to facilitate pedagogic decisions at the classroom level.

7.4 Summary

My study created safe environments for spontaneous learning through cycles of collaboration, reflection, culture and agency. I instigated transformative learning processes in mathematics to challenge conceptions of 'teaching' in ways that teachers felt able to manage, and that would be owned by the participants. Actualising this vision required an increased consideration to affective traits through appropriate design of the learning activity and use of available resources, resulting in active pupil participation. Despite representing a small scale change, I set out to demonstrate this objective as *achievable* and one that has the potential to be rolled out into a much wider ICT agenda in education. My study illuminates how computer mediation can make algebraic concepts more accessible to pupils, alongside traditional tools, within supportive learning contexts. If, by this work, pupils actively engage with a *blended learning* of algebra with ICT whilst teachers are offered a ‘social’ CPD in secondary mathematics classrooms, then this research was worth it.

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Appendices 1 – 14

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2- Ethical approval, NCST

3- Informed consent form

4- Contract of participation

5- Pre-study diagnostic exercise

6- Reading and Writing expressions: Attempt 1

7- Reading and Writing expressions: Attempt 2

8- GSCE algebra questions

9- Interview schedules

10- ‘Baseline’

11- Inverse journeys

12- Substitution

13- Pupil questionnaire

14- Algebra in End-Term 2 Mathematics Examinations

Appendix 1: Ethical approval, University of Warwick

THE UNIVERSITY OF WARWICK
Institute of
EDUCATION

Application for Ethical Approval for Research Degrees
(MA by research, MPHIL/PhD, EdD)

Name of student: Mary A. Lugalia | | | **PhD**

Project title: Pupils Learning Algebra with ICT in KS 3 Mathematics Classrooms

Supervisor: Sue Johnston-Wilder, Associate Professor, Mathematics Education
Dr. Janet Goodall, Research Fellow, Parental belief and parental engagement research

Funding Body (if relevant)

Please ensure you have read the Guidance for the Ethical Conduct of Research available in the handbook.

Methodology
Please outline the methodology e.g. observation, individual interviews, focus groups, group testing etc.

In this study, I intend to work closely with pupils and teachers at a secondary school in Kenya, as they use *Grid Algebra*, mathematics-specific software, in their mathematics lessons while I investigate the nature of the pupils' development of algebraic concepts.

This mixed-method research will be conducted in two distinct phases, beginning with a four-week pilot study at the start of the first term of the school's academic year in January 2012, involving the Form 2 class (in the second year of secondary school education). The Pilot Study will commence with the administration of an 'Attitudes to Mathematics' survey instrument and a written test I will refer to as the 'Baseline' to the Form 2 pupils. This will be run concurrently with a period of reconnaissance with the teachers in the school's mathematics department for them to personally experience and identify learning opportunities with the ICT tool. I will briefly interview the teachers to obtain a sense of the pupils' current attainment levels based on the Kenyan secondary school mathematics curriculum. The teachers' perceptions of the pupils will be considered alongside the pupils' performance in the 'Baseline'. This will be followed by intervention with *Grid* Algebra in the mathematics lessons with Form 2 pupils using the ICT tool as I observe the ensuing classroom interactions while conducting ethnographic interviews with all participants over a period of two weeks as I join them for their normal mathematics lessons. I will then administer a pupil questionnaire, to be followed by semi-structured interviews with groups of pupils and individual teachers, discussing their perceptions of the *Grid Algebra* lessons and the intervention in general.

After a period of reflection and review, the second phase of the research, the Main Study, will be undertaken, involving Form 1 pupils, who will have just joined secondary school, and their subject teachers. The timing of this phase will be designed to coincide with the teaching of the topic 'Algebraic Expressions' in the second term of school year (May to July 2012) in fourteen lessons according to the Secondary

Education Syllabus (Kenya Institute of Education, 2002). This phase will also commence with the administration of the 'Attitudes to Mathematics' survey and the 'Baseline' test to Form 1 pupils to report on their entry behaviour. This will be followed with intervention with *Grid Algebra* in the mathematics lessons with ethnographic interviews conducted with the participants. A pupil questionnaire will then be administered, followed by semi-structured interviews with groups of pupils and individual subject teachers, to collect the participants' perceptions of their mathematics-with-ICT lessons.

Participants

Please specify all participants in the research including ages of children and young people where appropriate. Also specify if any participants are vulnerable e.g. children; as a result of learning disability.

Participants will be pupils in Form 1 (13- 14 years) and Form 2 (aged 14- 15 years), equivalent to the UK National Curriculum Year 7 and 8 respectively, and three to five teachers of mathematics at a secondary school in Nairobi, Kenya which is welcoming to the idea of exploring the integration of computer teaching in secondary school mathematics, In addition to having the hardware and having acquired the *Grid Algebra* software, this particular school has been earmarked by the Centre for Mathematics, Science and Technology Education in Africa (CEMASTEA) as a training centre for integration of ICT in education for both primary and secondary schools.

Respect for participants' rights and dignity

How will the fundamental rights and dignity of participants be respected, e.g. confidentiality, respect of cultural and religious values?

In strict adherence to Guidelines 8, 9 and 13 of the Revised Ethical Guidelines for Educational Research (BERA, 2004), I will request, in writing, for the pupils' and teachers' permission to conduct the investigation, and subsequent intervention, with the two classes by preparing a contract of participation. In the contract, I will invite both pupils and teachers to be active participants in the research, and assure them of their right to withdraw from the study at any time they may choose to do so, without any penalty. I will also assure the participants that any information relating to them shall not be used in the reporting of the study should they choose to withdraw from the study. This contract will be read out to all participants at the beginning of the study as I seek to establish familiarity with all participants.

Privacy and confidentiality

How will confidentiality be assured? Please address all aspects of research including protection of data records, thesis, reports/papers that might arise from the study.

In accordance to BERA (2004) Guidelines 23, 24, 25, 26, 27 and 29, I will anonymize the research by not disclosing either the names of any of the participants or the school. I will keep all data from field notes, questionnaires, audio-recordings and interview transcripts under strict restricted access, using a password-protected computer, and the raw data locked in a cabinet only accessible to me. In writing my report, I will use codes for the participants which will be kept very separate from the raw data to minimize risk of identification.

Consent - will prior informed consent be obtained? **Yes**

- from participants? **Yes** from others? **Yes**

- explain how this will be obtained. If prior informed consent is not to be obtained, give reason:

In order to conduct research in Kenya, I will endeavour to secure a research permit in accordance with the requirements of the National Council for Science and Technology, Ministry of Higher Education, Science and Technology, Republic of Kenya.

In line with Guideline 10 (BERA, 2004), I will state the purpose, content and clearly spell out the procedures of the research in the contract of participation to be read out to all participants at the start of the study. I will obtain, in writing, voluntary informed consent from the participants, prior to the commencement of the research, to be signed by the teachers and the Principal of the school, who stand in *loco parentis* on behalf of the pupils as this is a boarding school. I will seek prior permission to audio-record the interviews from the each interviewee.

- will participants be explicitly informed of the student's status?

Yes, the participants will be explicitly informed of my status as a research student at the University of Warwick, United Kingdom.

Competence

How will you ensure that all methods used are undertaken with the necessary competence?

I will strictly adhere to the BERA Guidelines for Educational Research and will be in frequent consultation with my supervisors.

Protection of participants

How will participants' safety and well-being be safeguarded?

As the research involves pupils regarded as children and vulnerable young people, in compliance with Guidelines 14, 15, 16 and 17 (BERA 2004), I have been CRB-checked, for the position of Education/Schools Volunteer by Warwick Volunteers, and cleared to work with young people in schools.

The study, including the intervention and group discussions, will be conducted in the natural setting of the participants' mathematics lessons. This is intended to maximise the participants' sense of ease during the entire study. I will observe Guideline 18 (BERA, 2004), with a teacher being present at all times in case any of the pupil participants experience any distress or discomfort in the research process.

The teachers will be interviewed in the mathematics department room for a maximum of 30 minutes but in two distinct sessions; once before the intervention and the other at the end of the entire study period. The pupils will be interviewed in their classroom in groups of at most four, for 10 minutes. The tests and questionnaires will be administered in the mathematics classrooms.

Child protection

Will a DBS (Disclosure and Barring Service formerly CRB) check be needed?

Yes (If yes, please attach a copy.)

Addressing dilemmas

Even well planned research can produce ethical dilemmas. How will you address any ethical dilemmas that may arise in your research?

Any dilemmas that may arise in the course of the research shall be dealt with the sensitivity they deserve, in immediate consultation with the class teachers and my supervisors.

Misuse of research

How will you seek to ensure that the research and the evidence resulting from it are not misused?

I will ensure strict protection of all data records, reports and papers that will arise from the study in strict adherence to Guidelines 44, 45 and 46 (BERA, 2004).

Support for research participants

What action is proposed if sensitive issues are raised or a participant becomes upset?

A teacher will be present at all times during the study and the pupils will know where they are. Any upset participant will be reminded they can leave the room or study with no penalty.

Integrity

How will you ensure that your research and its reporting are honest, fair and respectful to others?

In compliance with Guidelines 41 and 43 (BERA, 2004), I will invite the teachers to peruse a rough draft of the report, in addition to working closely with my supervisors to ensure that the research, and its reporting, is strictly honest, fair and respectful to all participants and that all identifying details are anonymized.

What agreement has been made for the attribution of authorship by yourself and your supervisor(s) of any reports or publications?

Joint authorship by contribution, with myself as the lead author, in line with Guidelines 47 and 48 (BERA, 2004)

Other issues?

Please specify other issues not discussed above, if any, and how you will address them.

Signed Ayuma. 09/12/2011

Research student Mary A. Lugalia Date 05/02/2013

Supervisor Sue Jackson Wilde Date 5/2/2013.

Action

Please submit to the Research Office (Louisa Hopkins, room WE132)

Supervisor [signature]

Date 13/12/2011

<u>Action</u>

Please submit to the Research Office (Louisa Hopkins, room WE132)

<u>Action taken</u>

- [x] Approved 13.12.2011 SJW
- [ ] Approved with modification or conditions – see below
- [ ] Action deferred. Please supply additional information or clarification – see below

Name [illegible] Date 17/12/12

Signature [signature]

signature omitted in error on 13.12.2011 SJW

<u>Stamped</u>

<u>Notes of Action</u>

The case of study is not specified. The proposal is satisfactory but rather limited + brief

...and for the Stage 1 study, just the last page as evidence:

Action

Please submit to the Research Office (Louisa Hopkins, room WE132)

Action taken

- [x] Approved
- [ ] Approved with modification or conditions – see below
- [ ] Action deferred. Please supply additional information or clarification – see below

Name G. LINDSAY Date 3/5/11

Signature

Stamped

Notes of Action

Appendix 2: Ethical approval, NCST

REPUBLIC OF KENYA

NATIONAL COUNCIL FOR SCIENCE AND TECHNOLOGY

Telephone: 254-020-2213471, 2241349
254-020-310571, 2213123, 2219420
Fax: 254-020-318245, 318249
When replying please quote
secretary@ncst.go.ke

P.O. Box 30623-00100
NAIROBI-KENYA
Website: www.ncst.go.ke

Our Ref: **NCST/RCD/14/012/09**

Date: **1st March 2012**

Mary Ayuma Lugalia
University of Warwick
21a Cyril Street
Northampton
NN1 5EL

RE: RESEARCH AUTHORIZATION

Following your application for authority to carry out research on "***Pupils learning algebra with ICT in KS3 Mathematics classrooms,***" I am pleased to inform you that you have been authorized to undertake research in **Nairobi Proince** for a period ending ***30th September 2013.***

You are advised to report to **The Provincial Commissioner and the Provincial Director of Education, Nairobi Province** before embarking on the research project.

On completion of the research, you are expected to submit **two hard copies and one soft copy in pdf** of the research report/thesis to our office.

DR. M. K. RUGUTT, PhD, HSC.
DEPUTY COUNCIL SECRETARY

Copy to:

The Provincial Commissioner
The Provincial Director of Education
Nairobi Province.

"The National Council for Science and Technology is Committed to the Promotion of Science and Technology for National Development."

Appendix 3: Informed consent form

Mary A. Lugalia

The University of Warwick

Coventry

CV 4 7AL

Dear Teacher,

RE: INFORMED CONSENT

I am a research student at the University of Warwick. In this study, as partial fulfilment of the requirement for the award of a doctorate in Mathematics Education, I am investigating the nature of pupil learning as they integrate the use of *Grid Algebra* in their mathematics lessons. I would be very grateful if you would sign below to indicate that you are happy for the information you and the group of pupils in your class provide to be used in the write-up of the research study.

As a participant in this study, you have a right to answer as many or as few of the questions asked as you wish, in as much or as little depth as you deem fit. You may request the removal of some or all of the data from the study at any time during, or after, up to 10 days following the administration of a particular research instrument.

I would like to assure you that any details collected that may link you to the data, including the identity of yourself, the pupils and the school, will be kept in the strictest of confidence, and will not be used in the reporting of the outcomes of this research. Any information you provide will not be used or shared without your prior consent.

Should you have any concerns or queries with regard to any aspect of this study, please do not hesitate to contact me at M.A.Lugalia@warwick.ac.uk, and I will be happy to assist in any way possible.

Thank you for your participation and note that all your help is very gratefully received.

Please write **'I agree'** to confirm your agreement with the terms of participation that have just been described.

Sign: [signature] **Date:** 18/05/11

....

I agree to the information I and the pupils provide being used in the write-up of this study.

Sign: [signature] **Date:** 18/05/11

Appendix 4: Contract of participation (presented orally to the participants)

Good Morning. I am Mary Lugalia from the Institute of Education, University of Warwick. I am a research student interested in investigating the nature of pupil mathematical learning with ICT.

Today, I'd like to invite you to participate in a study that will involve you, your teachers and I in a research project. Firstly, I will attend your mathematics lessons and make observations in whole class and small group settings. Secondly, I will introduce you to work with *Grid Algebra* software on computers in the computer laboratory for a short series lessons, and in conjunction with your teachers, assist you to learn more about algebra.

We may ask you questions about why certain things happen the way they do. As part of this study, I will study what you produce in class exercises given so that I can see what effect the software may have to your learning of algebra and interactions in the mathematics lessons. I will ask you to fill out a questionnaire to answer a few questions about working on the computers. I may invite you to be interviewed.

You can decide at any time not to be involved in the study. To indicate your decision to withdraw, you may let either your teachers or me know; and at that time, I will not use any information regarding you in the research.

Title of Study: Pupils learning mathematics with ICT in a secondary school classroom

Purpose: This study seeks to explore how pupil learning of algebra at secondary school level is affected by the use of ICT, in particular *Grid Algebra* software, and the ways in which pupils and teachers interact in technology-rich mathematics classrooms

Procedures: During this study, pupils will be asked to answer questions verbally or in writing, fill out questionnaires and may be asked to take part in interviews which will be audio-taped. In addition, test scores and school records may be accessed. Selected excerpts will be used in dissemination of what is learned from this study

Risks: There are no potentially harmful risks related to participating in this study

Benefits: As a result of participation, the pupils' grasp of mathematics in general and algebra in particular may be increased. The study provides pupils with valuable insights into understanding algebraic concepts and an opportunity to look into a different approach and practice in the teaching and learning of mathematics

Withdrawal: Participation in the study is voluntary. You may withdraw without penalty at any time.

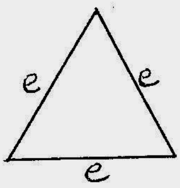
Confidentiality: All information collected in this study will be kept private and you, your class and school shall not be identified by name. All information you provide in this study shall be locked in a filing cabinet and in a password-protected computer.

Appendix 5: Pre-study diagnostic exercise

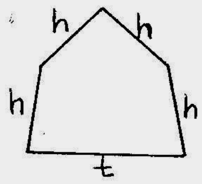
WORKSHEET 1

Name ______________________ Date ____________

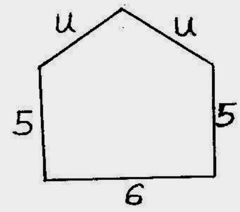
1. Write down an expression for the perimeter of each of the following shapes.



Perimeter = ____________



Perimeter = ____________



Perimeter = ____________

2. What can you say about a if $a+5=8$?

$a=$ ________

3. What can you say about m if $m=3n+1$ and $n=7$?

$m=$ ________

4. What can you say about u if $u=v+w$ and $u+v+w=30$?

$u=$ ________

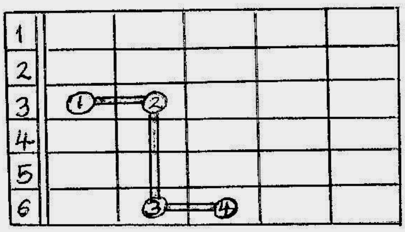
5. 10 sweets are to be shared between Sophie and Ali so that Sophie has 4 more than Ali. How many sweets does each of them get ?

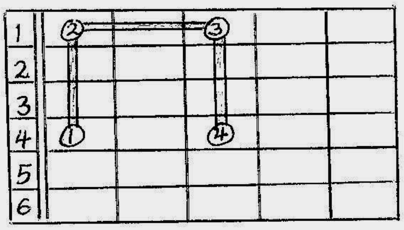
Appendix 6: Reading and Writing expressions: Attempt 1

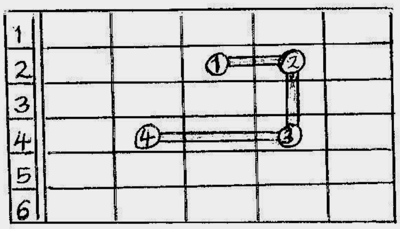
Name ______________________ Date ____________

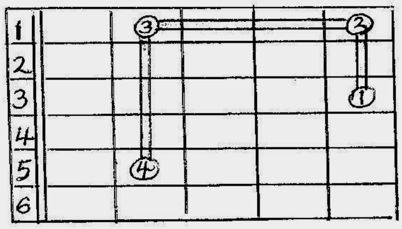
Reading and Writing Expressions

I. Starting with the letter *a* in the cell labelled 1, and tracing the journey shown in each question, write down the expression you would expect in the cell labelled 4.

1. 

2. 

3. 

4. 

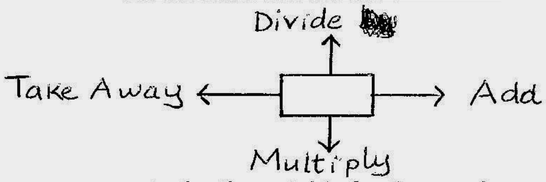
Appendix 7: Reading and Writing expressions: Attempt 2

GRID ALGEBRA

Name ______________________________ **Date** ______________

Reading and Writing Expressions

Remember

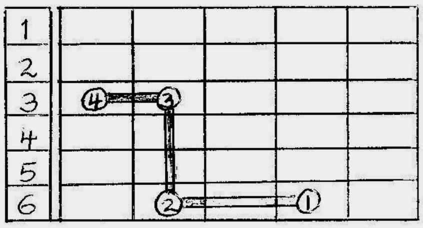


- **Each row represents the times table for the number on the left**
- **Numbers and letters entered into a cell may be dragged around the grid to create expressions**
- **The journey traced as a cell is dragged across the grid determines the final expression written in the destination cell**

Instruction

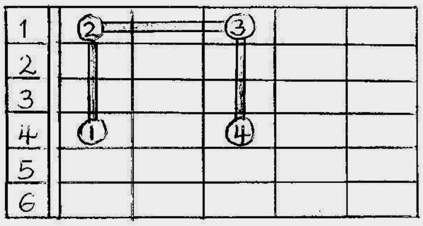
Write down the expression for the journey shown, starting with the letter 'b' in the cell labelled 1 and ending in the cell labelled 4, for each of the following:

1.



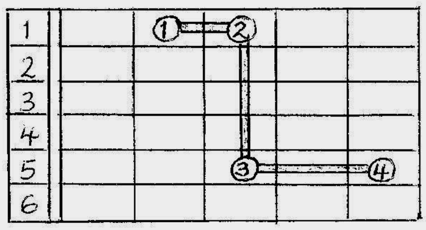
____

2.



____

3.



____

Appendix 8: GSCE algebra questions

Name ______________________ Date __________

1\. a) $P = 4K - 10$

$P = 50$

Work out the value of K. (2 marks)

b) $y = 4n - 3d$

$n = 2$

$d = 5$

Work out the value of y (2 marks)

2\. Simplify

a) $4x + 3y - 2x + 5y$ (2 marks)

b) $7x + 2y - x + 3y$ (2 marks)

3\. Expand and simplify

a) $3(x+1) + 2(x-1)$ (2 marks)

b) $3(x+5) + 2(5x-6)$ (2 marks)

4\. Factorise $5x + 10$ (1 mark)

5\. Solve $2x + 3 = 10$ (2 marks)

Appendix 9: Interview schedules

Pupil Interview

1. Do you think that having computers available in your mathematics lessons has changed the way you learn and/ or were taught mathematics? If so, give examples.
2. What is your view about using *Grid Algebra* in your mathematics lessons?
3. Did you have any 'surprises' in the lessons? If yes, what were they?
   Did anything in particular capture your interest?
4. Talk about your experiences in each of the sub-topics covered
5. If you were to use one word to describe your lessons with *Grid Algebra*, what would it be?

Teacher Interview

1. Background Information

- How many years' experience do you have in teaching?
- How many years have you taught at this school?
- What do you see as your strengths in teaching?
- Have you had any ICT training?
- Describe your own level of ICT knowledge and use

2. Use of *Grid Algebra* in mathematics lessons

In your own view,

- In what ways were your focus of teaching and the pupils' focus on understanding affected by the use of technology?
- What additional knowledge and skills did the pupils need?
- Do you feel the pupils benefitted in any way from engaging with the software in the mathematics lessons? If so, explain

3. Lesson Organisation

- What is your view about computer teaching in mathematics?
- Was the focus of your teaching on developing skills or understanding in pupils?
- What worked, and did not work, in the mathematics lessons?
- What would you do differently next time?

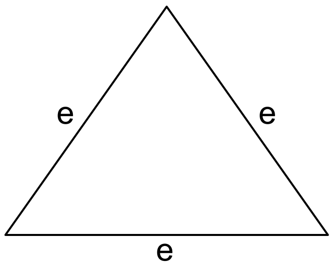
4. Analysis of pupils' work in the lessons

- Reflection and feedback from collected work and written tests
- Reflection and feedback from classroom observations, questionnaires and interviews

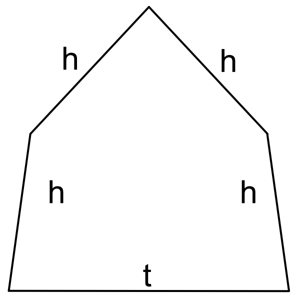
Appendix 10: ‘Baseline’

<u>**Instructions**</u>: Answer the following questions in the spaces provided. Show your working as clearly as possible

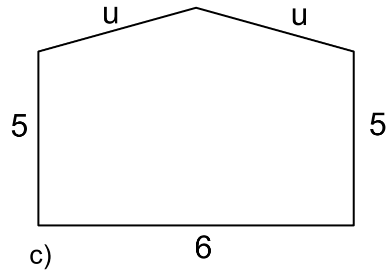
1. Write down an expression for the perimeter of each of the following shapes (3marks)



a)



b)



c)

Perimeter=____________ Perimeter = ______________ Perimeter = _____________

2. 10 sweets are to be shared between Sophie and Ali so that Sophie has 4 more than Ali. How many sweets does each of them get? (3 marks)
3. Simplify $7x + 2y - x - 3y$ (2 marks)
4. a) $p = 4k - 10$
   p = 50
   Work out the value of k (2 marks)
   b) $y = 4n - 3d$
   n = 2
   d = 5
   Work out the value of y (2 marks)
5. Starting with the letter **b**, divide by 4, then add 2, and then multiply the result by 5. Write down the final algebraic expression. (2 marks)
6. Expand and simplify (2 marks)
   $3(x + 5) + 2(5x - 6)$
7. Factorise each of the following:
   a) $5x + 10$ (1 mark)
   b) $12a + 4a^2$ (1 mark)
   c) $32ab^2 + 18b$ (1 mark)
8. A pupil in England was asked the following question: ‘Find the value of $\frac{x-4}{4} - 1$ when x = 16’, and gave the answer as 11. What would your own solution to the question be? (2 marks)
9. Find the value of $\frac{c+6}{3} - 6$ when c = 12 (2 marks)
10. Solve $2x + 3 = 10$ (2 marks)

Appendix 11: Inverse journeys

Write down the inverse journey for each of the following expressions.

1. $\frac{34-8}{2} + 1 = 14$
2. $\frac{31 \times 2 - 6}{2} = 28$
3. $4\,(\frac{24}{3} + 17) - 32 = 68$
4. $4\,(x + 3) - 16 = k$
5. $3\left(\frac{m}{5} + 7\right) - 18 = r$
6. $2\left(\frac{s - 16}{4} + 27\right) - 14 = g$

For each question, write down the inverse journey, starting at the final number, to end up back at the letter. Then find the value of the letter in each case.

1. $2\,(t + 3) = 46$
2. $3V + 12 = 132$
3. $\frac{k - 8}{4} = 17$
4. $\frac{2t - 8}{4} = 41$
5. $\frac{3d - 18}{6} + 1 = 37$
6. $4\left(\frac{z}{2} + 3\right) = 68$
7. $\frac{2\,(w+6) - 18}{3} = 86$
8. $\frac{2j + 12}{3} - 6 = 26$

Appendix 12: Substitution

1. Find the value of $\frac{x+\ 6}{3} - \ 4$ when $x = 48$

2. What is the value of $2\left(\frac{z}{3} + 2\right)$ when $z = 48$?

3. Find the value of $3\left(\frac{c+6}{3}\right)$ when $c = 12$

4. Find the value of $2x - 12 \ - 6$ when $x = 9$

5. Find the value of $2\left(\frac{b}{2}\right) + \ 12$ when $b = 6$

6. What is the value of $3\left(\frac{d}{6} + \ 3\right)$ when $d = 42$?

Appendix 13: Pupil questionnaire

As part of my doctorate research course in Mathematics Education at The University of Warwick, United Kingdom, I would like to investigate how pupils learn algebra with ICT in secondary mathematics classrooms.

Confidentiality: The name of the school, the pupils and teacher will not be used in reporting the outcomes of this research. Any information you provide will not be shared with any other member of the school without your consent. If you have any concerns about how the information I will collect will be used, or would like to know more about this research study, please ask me or email me.

1. Age (in years) ……………………………………..

2. (a) In which province did you sit your KCPE? ……………………………………………………………

Please tick only one of the boxes.

(b) What type of primary school did you attend? Public ☐ Private ☐

(c) Did you have any computer lessons before you were admitted to this school?

Yes ☐ No ☐

(d) Do you ever use a computer at your home?

Yes ☐ No ☐

3. How did you find the mathematics lessons with *Grid Algebra*?

Please tick only one of the boxes. Beneficial ☐ Waste of time ☐

Explain your answer.

………

………

………

4. Do you think having *Grid Algebra* in your mathematics lessons changed the way you learn?

Please tick only one of the boxes. Yes ☐ No ☐

Yes/ No

If Yes, in what way?

………

………

………

Thank you for your responses. All your help is highly appreciated.

M. A. Lugalia

Appendix 14: Algebra in End-Term 2 Mathematics Examinations

1. Simplify the expression $\frac{2a-4}{5} - \frac{a-b}{3}$ (3 marks)

2. If a : b = 3 : 2, find the ratio of (a + 3b) : (8a – 3b) (3 marks)

3. In fourteen years' time, a mother will be twice as old as her son. Four years ago, the sum of their ages was 30 years. Find out how old the mother was when her son was born (4 marks)

4. When we double a number and add 17, the result is 59. What is the number? (2 marks)

5. In a Form One class, there are 5 more boys than girls. On a certain day, one quarter of the boys and one fifth of the girls went to the Music Festival. Find the number of students in this class. (4 marks)

6. In a by-election, $\frac{1}{10}$ of the registered voters in Mathare constituency did not vote. $\frac{1}{5}$ voted for Mr. Masinde, and the rest voted for Mrs. Wafula. If Mr. Masinde received 3 500 votes, how many registered voters were in that constituency?