

An Investigation into the Relationship Between Counter-Terrorism Expenditure and Terrorist Attacks:

Has the threat posed by terrorism in the United States been reduced as a result of the large increases in counter-terrorism expenditure?

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Does the growth in counter-terrorism expenditure reflect an increase in the number and magnitude of terrorist attacks in the US?

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Abstract

We have used duration models to examine whether the growth in counter-terrorism expenditure has reduced the probability of a terrorist attack per unit time. We find evidence that if a perpetrator of an attack is arrested, there will be an extended period of no attacks which implies that higher expenditure could reduce attacks by increasing arrest rate. However, we fail to find the expenditure variable itself as significant and thus find no direct evidence that higher expenditure reduces attack attempts. This result is not robust, however, given the small size of our sample. Using a time series model we fail to find significant evidence that any attacks apart from 9/11 have affected the counter-terrorism expenditure growth, though our results are again not robust due to sparse data.

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I. Introduction

With an unforeseen magnitude of over 5000 casualties the September 11 attacks led to large changes in the US domestic counter-terrorism strategy. The State Department of Homeland Security (DHS) was founded to coordinate anti-terrorist efforts, and thereafter we have seen the country's domestic counter-terrorism expenditure grow from \$10.309bn in 2001 to \$34.343bn in 2011. We will investigate whether this rise in anti-terrorist spending has decreased the threat posed by terrorism in the US. We will also research if and how anti-terrorist spending responds to terrorist attacks.

Our motivation stems from the observation that despite these increases in expenditure, and while there have not been successful large-scale attacks, there have been numerous failed attempts and few smaller attacks in the US since the 9/11. The threat of terrorist attacks thus seems to remain high despite these spending increases. We are hence intrigued to test rigorously whether the threat of terrorism has decreased as a consequence of higher anti-terrorist spending. As to account for the full threat level in terms of all attempted attack², both unsuccessful and successful attacks will be considered. This is presumably also a good proxy for successful attacks and will be treated as such. Addressing this topic is imperative not just to evaluate the effectiveness of counter-terrorism expenditure but also in giving us information about the behaviour of terrorists.

Since there have been no successful large-scale attacks since 9/11, why has the expenditure been growing so much still 10 years afterwards? Have there been so many attempts which have indicated that further expenditure is necessary? Perhaps there have been so many small successful attacks in the recent years as to justify this? These questions have motivated us to find out if and how counter-terrorism responds to terrorist attacks.

² Throughout this paper “attacks” and “attempted attacks” will be used to mean the total number of attacks that include both successful and successful attempts. If we only refer to successful attacks, this will be made clear.

In this study we define the threat of terrorism to be the probability that there is a terrorist attack attempt per unit time. Data on peace durations between attacks and duration models will be used to estimate this threat and to test how it has been affected by expenditure. To study how expenditure responds to terrorist attacks, a time-series model with number of attacks and casualties as covariates will be utilised. We are not aware of any prior work that would have addressed our research questions using the same approach.

II. Literature Review

The role of economics in the study of terrorism started with Landes (1978) who showed that the introduction of metal detectors in 1973 reduced the number of skyjackings in the US. This was a seminal paper as it was the first one to assume that terrorists behave as utility maximising agents. Several authors have thereafter used this approach to study the impact of various counter-terrorism policies. For example, Enders and Sandler (1993) used a Vector Autoregression (VAR) model to show that Landes's model did not properly take into account substitution and complementary effects; while metal detectors reduced skyjackings, other modes of attack such as hostage taking increased. Even though not fully related to our research question, these papers show that terrorists respond to incentives, and can be also expected react to changes in counter-terrorism expenditure.

Barros, Passos and Gil-Alana (2006) explain the timing of ETA terrorist organisation attacks using a duration model. Their approach is similar to ours since they also to investigate how changes in variables such as season, the number of casualties in the most recent attack, political situations, and anti-terrorist activities affect the expected duration of peace period following an attack. They find that attacks are more likely to occur in the summer and occur less frequently with respect to anti-terrorist activities.

Still, the above paper differs from ours in many ways. Most importantly, the authors have not investigated the effect of counter-terrorism expenditure which is our main focus. They also include only successful attacks while we consider both successful and unsuccessful ones. In addition to the benefit of more data, this allows us to better determine the effect of covariates on the actual behaviour of terrorists in terms of their determination to attack the US, not just the outcome of attacks. Second, rather than concentrating on a single terrorist group, we take

into account all terrorist attacks. We will also use slightly different duration models. The above authors estimate five different models: Cox, Exponential hazard, Weibull hazard, and Piecewise Constant Exponential (PCE) and Heterogeneity-allowing PCE. We use only three models but they are more flexible than the ones above.

Arin *et al.* (2009) evaluate the relationship between anti-terrorist expenditure and terrorist attacks. They define four possible stages of world, consisting of the possible combinations of high and low levels of terrorist activity and anti-terrorist spending, and use Markov-switching model on UK data to estimate transition intensities between these states. They find that high defence spending is associated with reduced probability of transitioning into the state of high level of terrorism, but not the other way around. We will instead use duration and time-series models separately to study the two possible directions of causality between expenditure and terrorist attacks. This has the advantage that we are not constrained to considering only four states; rather, expenditure is a continuous variable and we can calculate its marginal effects.

Messis, Mylonidis and Paleologou (2009) use a count data model on time-series data from Greece to show that anti-terrorism spending only has a weak effect in reducing the number of terrorist attacks. The flaw in this model is that it only shows the effect of expenditure on the total number of attacks per given year. Use of duration models gives us much more detail about the behaviour of terrorists by actually allowing us to find out how different factors affect the probability of a terrorist attack per unit time and how this varies over time.

Sandler, Arce and Enders (2009) evaluate the effectiveness of the US counter-terrorism spending since the 9/11. Their main finding is that in almost all cases spending is too large; \$1 of spending only returns 30 cents on average. Much higher return could be achieved by improving highway safety which claims 30 000 lives a year. The weakness of this paper is that its conclusions rely almost entirely on this type of aggregate cost-benefit calculations.

Through the use of econometrics we can provide a rigorous analysis of the degree and direction of causality between expenditure and attacks, in particular how changes in counter-terrorism expenditure exactly affect terrorist behaviour.

III. Methodology and Data

Our study consists of two parts: we first estimate the effect of counter-terrorism expenditure on the threat of terrorist attacks. We then proceed to investigate how the number and magnitude of attacks affect anti-terrorist spending.

III. 1 – Methodology and Data for Estimating the Effect of Counter-Terrorism

Expenditure on the Threat of Terrorist Attacks

Our methodology in addressing this research topic is based on duration models. These models allow us to estimate the probability per unit time that there is a transition from a state of peace to one of a terrorist attack, conditional on that there has not been an attack for the past t days; this is called the hazard rate. We test if and how expenditure affects this rate.

Empirically this is based on estimating and testing the effect of expenditure on the duration of peace periods between consecutive attacks.

Duration models have several advantages over alternative approaches. For instance, binary dependent variable models could predict probabilities but cannot account for the relationship between peace duration and time-dependent variables and how they lead to different hazard rates at different points in time. Times-series models could only estimate an aggregate number of attacks for each time period.

Let T denote the duration of a peace period. c.d.f of peace durations is then $F(t) = P(T \leq t)$, which is called failure function since it describes the probability that an attack occurs within t days from the most recent attack. Conversely $P(T > t) = 1 - F(t) \equiv S(t)$, is the survivor function.

The function that depicts hazard rate over time, that is the hazard function, is then³:

$$\theta(t) = \frac{f(t)}{s(t)} \text{ where } f(t) = \lim_{\Delta t \rightarrow 0} \frac{P(t \leq T \leq t + \Delta t)}{\Delta t} \text{ is p.d.f of } F(t) \quad (1)$$

If hazard rate depends on covariates (which may vary over time) we have $\theta(t, X_t)$.

Hazard rate may vary in many ways over the peace duration and thus the hazard function could have many shapes. Therefore, we need to choose models that are sufficiently flexible. Barros, Passos and Gil-Alana (2006) encountered a similar problem and solved it using five different models. The Exponential and Weibull hazard models they used only allow for monotonic shapes. We replace these models with a Generalised Gamma model, which is a two parameter model that allows for much more flexible shapes.^{4 5}

Additionally we will use Piecewise-Constant Exponential (PCE) and Cox models. PCE is a flexible model as it does not specify any certain hazard function before estimation. Instead, it assumes that the hazard rate changes only between fixed intervals and within the intervals it is estimated to be some constant. For the first 30 days of peace periods we have chosen to allow hazard rate vary daily, from 30 days to 100 days between 10 day intervals, and after that it is constant for 100 day intervals up to 400 days of peace duration, after which it remains constants as data is sparse.⁶

The Cox model is the most flexible out of our models. The model's flexibility comes from using Partial Likelihood method which allows covariates to be estimated without having to specify any hazard function *a priori* and therefore the estimates are not constrained by this.⁷

³ See Appendix A.1 for how for the full derivation

⁴ The Hazard function of the General Gamma model cannot be written in closed form and is thus not displayed here. Appendix A.2 provides the necessary theoretical background for this model

⁵ We would have wanted to estimate Heterogeneous Gamma or PCE model as done in Barros *et al.* (2006) but could not estimate it given our shortage of expenditure data

⁶ See Appendix A.3 i.) and ii.) for detailed explanation of the PCE model

⁷ See Appendix A.3 i.) and iii.) for a detailed explanation of the Cox model and Partial Likelihood estimation

Its weakness is that since no hazard function is estimated, it is harder to evaluate how the hazard rate varies over time.

Hazard rates are estimated using data on peace durations between attacks and chosen covariates. To reiterate, for the Cox model partial likelihood technique is used. Maximum Likelihood method is used for PCE and Gamma models. It is chosen over OLS because OLS cannot handle time varying covariates as each single peace duration observation can have many values for the time-varying covariates such as expenditure.

RAND Database of Worldwide Terrorism Incidents (RDWTI) includes a description of all the reported terrorist attacks from 1968 until 2010 in the US, including unsuccessful ones.⁸ This dataset has been chosen due to its extensive descriptions of attacks, including the number of injuries and fatalities, perpetrator, weapon of choice and who the attack was aimed against.⁹ *RDWTI* does not provide counter-terrorism expenditure data. This has been obtained from the annual *DHS budget-in-brief* publications for 1995-2010.¹⁰ The total annual budget is multiplied by 0.61 to get the approximate amount spent domestically on counter-terrorism.¹¹ The problem is that terrorist behaviour is not likely to react to the current year's expenditure but instead to some sum of all past expenditure. We have hence created different expenditure variables, named *exp_* "XX", which are equal to the sum of all previous anti-terrorist expenditures with an annual depreciation of XX%. Variables from *exp_1* to *exp_99* are considered.¹² Using these data sources, we created a dataset of 309 peace duration

⁸ The database is available at: <http://www.rand.org/nsrd/projects/terrorism-incidents.html>

⁹ See Appendix A.4 i.) and ii.) for the full list of the descriptions given about each attack, how terrorism is defined, and the types of attacks that are included

¹⁰ Available at <http://www.dhs.gov/xabout/budget/>

¹¹ This is because part of the DHS programme is devoted to other programs such as the Federal Emergency Management Agency (FEMA). 0.61 derives from an official classification by the DHS that 61% of their budget goes to terrorism-related homeland security activities. *DHS budget-in-brief* (2004)

¹² Note that the fact that expenditure is measured annually and peace duration in days is not fundamentally a problem. Appendix A.5 explains this.

observations (dependent variable), as well as the explanatory variables we wish to use.¹³

Table 3.1.1 lists the variables we will use in our regressions and their summary statistics.¹⁴

The fact that we have counter-terrorism expenditure data only for 1995-2010 (16 observations) is a major problem. We also have only 57 peace duration observations for this period. As to minimize this problem, we will first estimate our models from 1968 to 2010 with variables as in *Table 3.1.1* except our chosen expenditure variable excluded and *prevfail* and *prevarrest* acting as expenditure proxies. We will test for their significance. After this we will include expenditure by estimating the models for the different *exp_XX* variables for 1995-2010; we consider depreciation rates from 0.01 to 0.99. The model with the most significant *exp_XX* will form the basis for our analysis. We exclude *prevfail* and *prevarrest* from these regressions as they are presumably proxies for expenditure.

Table 3.1.1 – the variables in our duration model and their summary statistics (definitions in Appendix A.6)

Name	Mean	Variance	Min. val., Max. val.
Dep. Var.:			
<i>peacedur</i>	42.882	5807.319	1, 859
Explanatory Vars.:			
<i>injprev</i>	22.420	158.407	0, 2337
<i>fatprev</i>	9.793	149.422	0, 2982
<i>forprev</i>	0.476	0.249	binary dummy
<i>prevarrest</i>	0.181	0.385	binary dummy
<i>prevfail</i>	0.283	0.203	binary dummy
<i>prevmulti</i>	0.101	0.090	binary dummy
<i>ACC</i>	0.229	0.174	binary dummy
<i>ALQ</i>	0.069	0.064	binary dummy

¹³ See Appendix A.7 for an example of an observation from our data

¹⁴ The definitions of these variables and exactly how they were calculated are in Appendix A.6

<i>explo</i>	0.703	0.209	binary dummy
<i>sum</i>	0.295	0.456	binary dummy
<i>exp</i>	18.997	91.9	5.49, 34.16
<i>exp_1</i>	n/a	n/a	n/a
⋮			
<i>exp_99</i>	n/a	n/a	n/a

The average peace duration is ~43 days. *Figure 3.1.1 a)* shows that given an attack has occurred, in vast majority of cases there will be another attack within the next few months. The variance of peace durations is large due to a few very large observations (*Figure 3.1.1 b)*), and might have large effects on our estimates. From *Figure 3.1.2* we can see that attacks occurred more frequently in the late 70s and early 80s (likely due to the politically instable situation with Cuba). Some increase in the number of attacks can be seen after 9/11, and growth in expenditure appears correlated with this. Comparing *Figure 3.1.2* with *Figures 3.1.3 a)* and *b)*, we see that arrests and failures seem to be correlated with higher expenditure (provides some justification for using them as proxies) and the number of attacks. *Figures 3.1.4* and *3.1.5* imply that failed attacks and arrests are correlated with longer peace durations.

Figure 3.1.1 a) on the left shows the distribution of peace duration for whole data, using 20 day intervals. *b)* on the right is the same but zoomed in for *peacedur > 200*

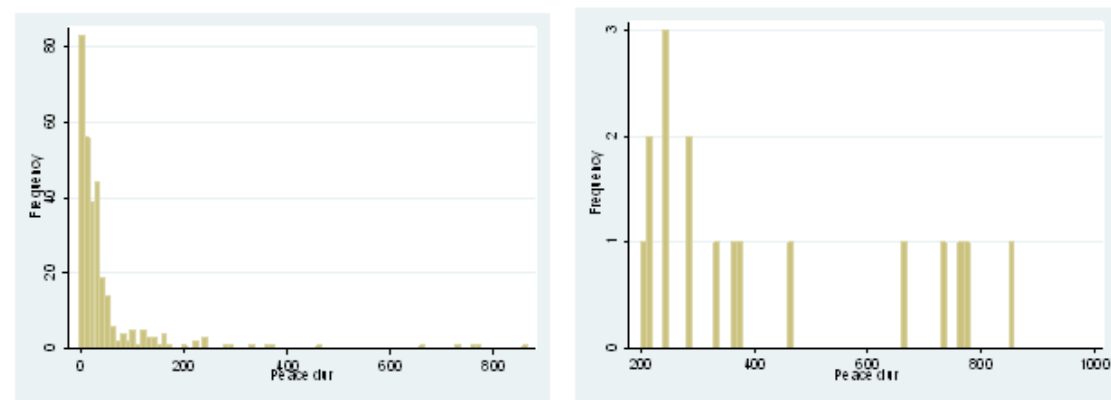


Figure 3.1.2 –Attacks (vertical lines) and counter-terrorism expenditure over time

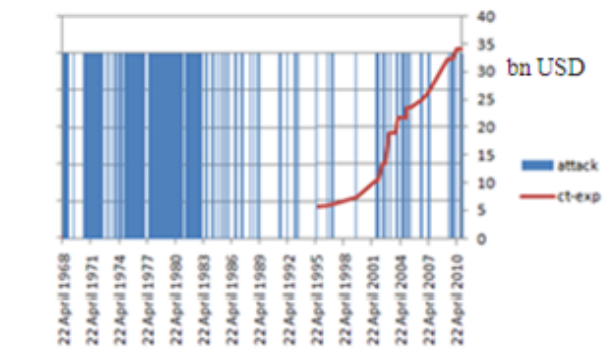


Figure 3.1.3 a) – Arrests over time

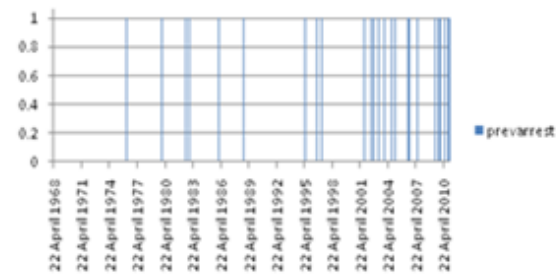


Figure 3.1.3 b) – failed attacks over time

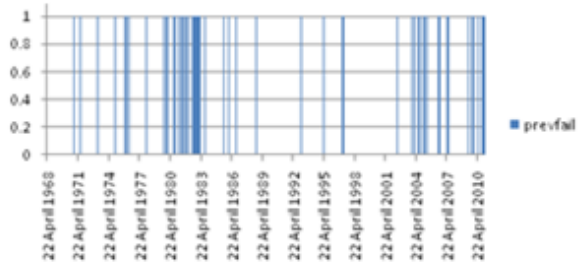


Figure 3.1.4 - Mean peace durations graphed Separately for observations with prevarrest=0 and 1

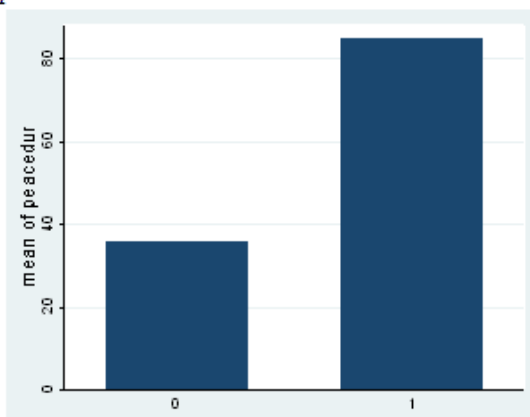
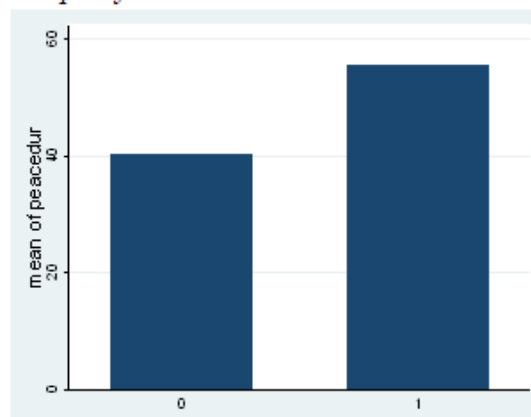


Figure 3.1.5- Mean peace durations graphed separately for observations with prevfail=0 and 1



We have created a high number of explanatory variables because our data covers four decades and thus we need to account for as much of the possible behavioural differences between the various terrorist organisations as possible. Attacks aimed at foreigners are correlated with shorter peace duration after the attacks (*Figure 3.1.6*), and since nearly half of all attacks are aimed at foreigners, including *forprev* is important. Similarly, days on which multiple attacks occur (10% of observations), ACC as the perpetrator (23% of observations), and the use of explosives (70% of observations), are all correlated with shorter after-attack peace durations. (*Figures 3.1.7, 3.1.8 and 3.1.9* respectively). Al-Qaeda attacks are correlated with longer peace durations (*Figure 3.1.10*). We have included *injprev* and *fatprev*, since a high number of casualties may, for instance, give encouragement to other terrorist groups. These variables have high proportion of zeros, and couple very high values, which could cause problems with their estimates (*Table 3.1.2*).

Figure 3.1.6 - Mean peace durations graphed separately for observations with forprev=0 and 1

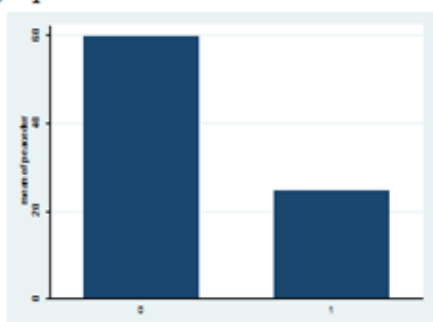


Figure 3.1.7 - Mean peace durations graphed separately for observations with prevmulti=0 and 1

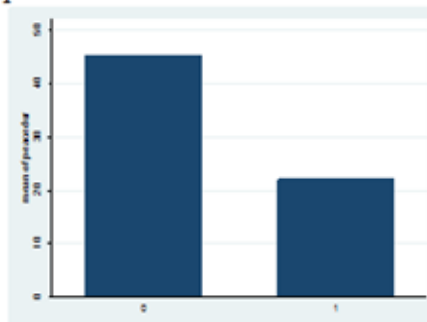


Figure 3.1.8 - Mean peace durations graphed separately for observations with ACC=0 and 1

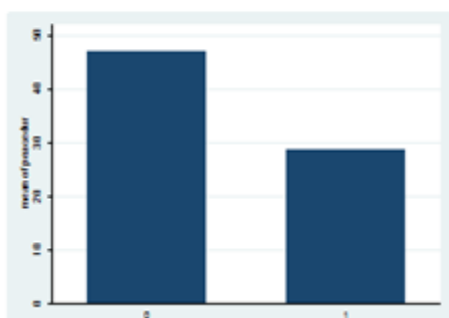


Figure 3.1.9 - Mean peace durations graphed separately for observations with explo=0 and 1

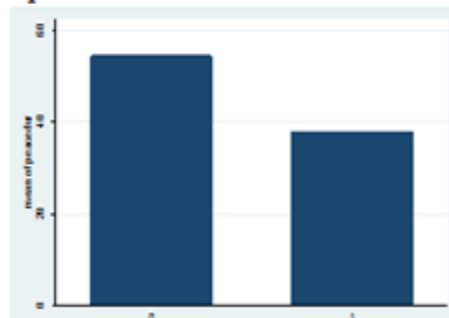


Figure 3.1.10 - Mean peace durations graphed separately for observations with $ALQ=0$ and 1

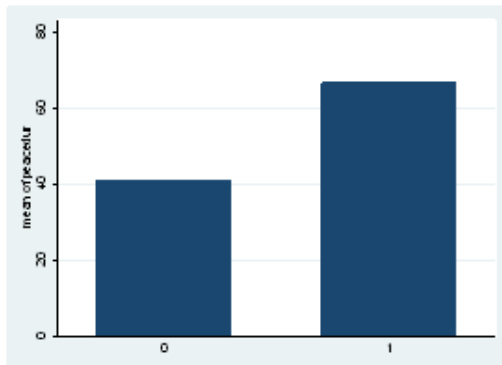


Table 3.1.2 – Distribution of values for *injprev* and *fatprev*

<i>injprev</i> value	percent	<i>fatprev</i> value	percent
0	81.29	0	87.06
1 to 10	14.03	1 to 10	2.15
10 to 20	0.21	10 to 20	0.345
20 to 100	0.308	20 to 100	0.308
100 to 1000	1.095	100 to 1000	1.26
>1000	1.06	>1000	0.25
Total	100	Total	100

III. 2 – Methodology and Data for Estimating the Effect of Terrorist Attacks on Counter-terrorism Expenditure

Since we only have 16 observations for counter-terrorism expenditure (1995 – 2010) we have developed a very simple time series model as our starting point:

$$\begin{aligned}
 exp_t = & \beta_0 + \beta_1 exp_{t-1} + \beta_2 ncas_{t-1} + \beta_3 ncas_{t-2} + \beta_4 nattacks_{t-1} + \dots \\
 & \dots + \beta_5 nattacks_{t-2} + \beta_6 GDP_{t-1} + \varepsilon_t
 \end{aligned}
 \tag{2}$$

Appendix A.8 contains all the definitions and summary statistics for these variables.

A trend variable is not included as we already account for GDP and we have assumed that rest of the expenditure is explained by terrorist behaviour which is not trending. We start with

only one-period lags for exp_t and GDP_t since we have so little data. Two lags of $nattacks_t$ and $ncas_t$ are included as they are the variables under investigation. Very insignificant lags will be removed from our final model in order to minimize problems caused by our shortage of observations. Long-run effects will be calculated. We will test for the influence of 9/11 on our estimates by approximating how much coefficient estimates would change if the observation for 9/11 was removed.

Lags of exp are included since anti-terrorism strategy can be expected to be long term. *Figure 3.2.1* shows the growth of exp overtime, and we see that exp is strongly correlated with its one-period lag. Lags of $ncas$ and $nattacks$ are chosen because the government's assessment of the necessary expenditure changes is likely to depend on the nature and number of recent attacks.

Note that while the average number of casualties annually is 367, this is pulled up by the combination of 9/11 and a large number of zero observations (*Figure 3.2.2*). *Figure 3.2.3* shows the total number of terrorist attacks attempted per year. We cannot see from *Figures 3.2.1, 3.2.2* and *3.2.3* that expenditure would be correlated with lags of $ncas$ and $nattacks$ in general, though it seems apparent that 9/11 caused a structural break in anti-terrorism expenditure. GDP is included as a control variable; as we can see from *Figure 3.2.4*, it is closely correlated with anti-terrorist spending.

Figure 3.2.1 – Counter-terrorism expenditure 1995 -2011 in billions of USD

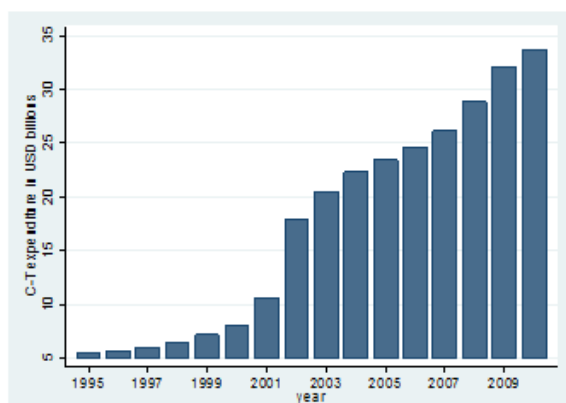


Figure 3.2.2 – Total number of casualties annually for 1995 - 2010

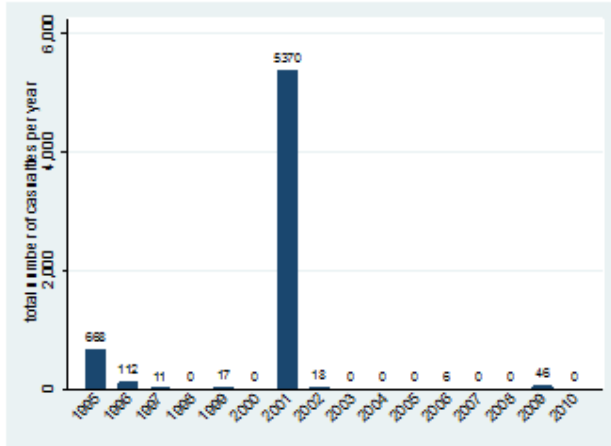


Figure 3.2.3 – Total number of attacks per year

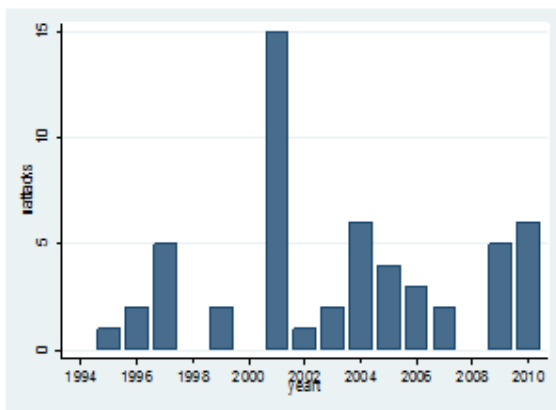
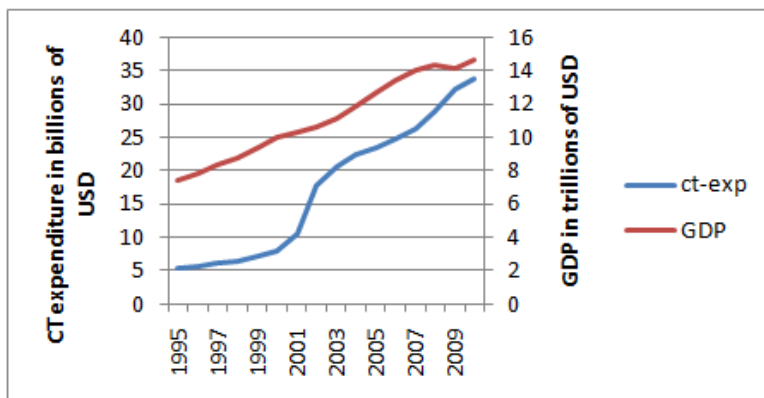


Figure 3.2.4 – GDP and counter-terrorism expenditure 1995 - 2010



IV. Results

IV. 1 – Results: the Effect of Expenditure on the threat of Terrorist Attacks

The results from regressing peace durations from 1968 to 2010 on the covariates in

Table 3.1.1, with *prevfail* and *prevarrest* as expenditure proxies are displayed in Table 4.1.1.

We have checked that all the covariates have appropriate functional forms, and that no interactions are necessary¹⁵.

Table 4.1.1 – Duration model estimates (1968-2010), with prevfail and prevarrest as proxies for counter-terrorism expenditure

<u>Dep. var:</u> <i>peacedur</i> 309 subjects	<u>Cox model</u>				<u>PCE model¹⁶</u>				<u>General Gamma model</u>			
<u>Variable</u>	<u>Coefficient</u>	<u>S.E.</u>	<u>z</u>	<u>P>z</u>	<u>Coefficient</u>	<u>S.E.</u>	<u>z</u>	<u>P>z</u>	<u>Coefficient</u>	<u>S.E.</u>	<u>z</u>	<u>P>z</u>
<i>injprev</i>	-0.001	0.001	-1.23	0.218	-0.001	0.001	-1.23	0.219	0.003	0.001	1.89	0.059
<i>fatprev</i>	0.001	0.001	1.2	0.229	0.001	0.001	1.19	0.233	-0.002	0.001	-1.58	0.115
<i>forprev</i>	0.273	0.130	2.1	0.036	0.273	0.130	2.1	0.036	-0.230	0.182	-1.27	0.205
<i>prevarrest</i>	-0.460	0.245	-1.88	0.06	-0.503	0.244	-2.06	0.04	0.834	0.348	2.4	0.016
<i>prevfail</i>	-0.196	0.173	-1.14	0.256	-0.204	0.173	-1.18	0.238	0.347	0.249	1.39	0.163
<i>prevmulti</i>	0.275	0.167	1.64	0.1	0.271	0.167	1.63	0.104	-0.434	0.234	-1.86	0.063
<i>ACC</i>	0.244	0.143	1.71	0.088	0.241	0.142	1.69	0.09	-0.253	0.197	-1.28	0.199
<i>ALQ</i>	0.199	0.150	1.33	0.184	0.192	0.149	1.29	0.198	-0.176	0.520	-0.34	0.735
<i>explo</i>	0.175	0.366	0.48	0.633	0.180	0.366	0.49	0.622	-0.227	0.208	-1.09	0.276
<i>sum</i>	0.023	0.130	0.18	0.858	0.046	0.130	0.36	0.721	0.096	0.183	0.53	0.598
constant					-3.301	0.281	-11.73	0.000	3.139	0.262	11.96	0.000
LL	-1452.551				-511.329				-545.537			

LL is the log-likelihood value of each regression. Estimates significant at 10% are bolded.

Cox and PCE coefficient estimates and standard errors are very similar. The Gamma model estimates seem very different, but this is actually not the case. The interpretations are just

¹⁵ See Appendix B.1 for how we have tested for this

¹⁶ We have not shown estimates for the dummies for different intervals since they carry no economic interpretation

different. The coefficients of Cox and PCE are interpreted as $\beta_k = \frac{\partial \log \theta(t, X)}{\partial X_k}$.¹⁷ The General

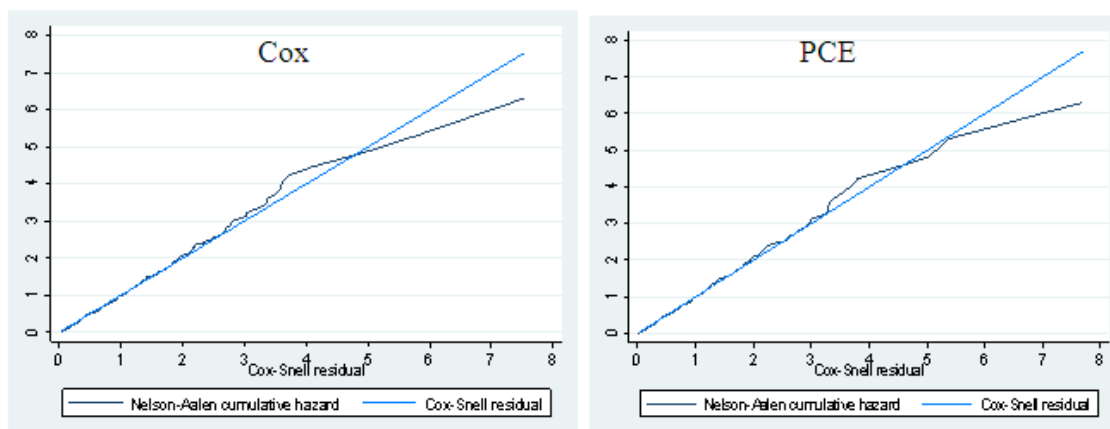
Gamma model coefficients are interpreted as $\beta_k = \frac{\partial \log T}{\partial X_k}$.¹⁸ Subsequently, Gamma model

suggest an effect in the same direction as PCE and Cox if the coefficients have the opposite sign. This is the case for all the coefficients (apart from *sum*) which indicates consistency over the models.

Testing for goodness-of-fit, we look at plots of cumulative hazard function with Cox-Snell residuals as the survival time (*Figure 4.1.1*). Line close to 45 degree indicates a good fit¹⁹.

For lower peace durations the Cox and PCE seem to provide a better fit, while the Gamma model seems to fit the data better for higher durations. In general the models provide good fit (it is normal that the fit is not good for higher durations where the data is sparse). For the Cox and PCE models we have checked that their Proportional Hazard assumptions are satisfied.²⁰

Figure 4.1.1 – Goodness of fit for the Cox, PCE and Gamma models

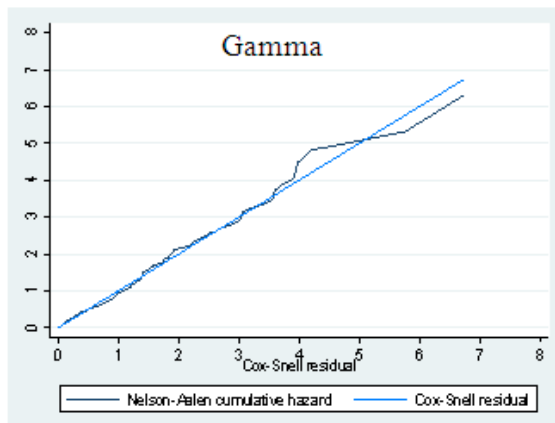


¹⁷ See Appendix A.3 i.)

¹⁸ See Appendix A.2

¹⁹ See Appendix B.2 for further details about this measure of goodness-of-fit

²⁰ See Appendix B.3



Results in *Table 4.1.1* indicate that we fail to find significant evidence that *ALQ*, *explo*, *sum* and *fatprev* have any effect on expected peace durations. The number of injuries is significant in the Gamma model but so insignificant in the other two that we cannot draw any conclusions. Same goes for ACC estimates. Cox and PCE find *forprev* to be significant, which could be due to regular bombings against foreign embassies in the 1980s which occurred in lumps, but since Gamma model finds the effect insignificant we cannot be entirely sure about its significance. All the models discover *prevmulti* to be significant, which could follow from terrorists' resources being exhausted for a longer time after multiple attacks.

Only *prevarrest* is significant out of the two expenditure proxies. This is likely to be due to the high correlation (0.457) between *prevarrest* and *preyfail*. Still, the two variables are simultaneously significant (*Table 4.1.2*). Gamma model's interpretation of *prevarrest*'s effect is the most useful; a peace period after an attack is expected to be 83.4 percentage points longer if the perpetrator is arrested. Hence, if expenditure increases lead to a higher arrest rate, it is likely to reduce terrorist attack attempts. Various possible explanations arise: the arrested person could simply be stopped from carrying out further attempts, or arrests may scare off other terrorists. If it was the latter, it could be that terrorist would spend longer

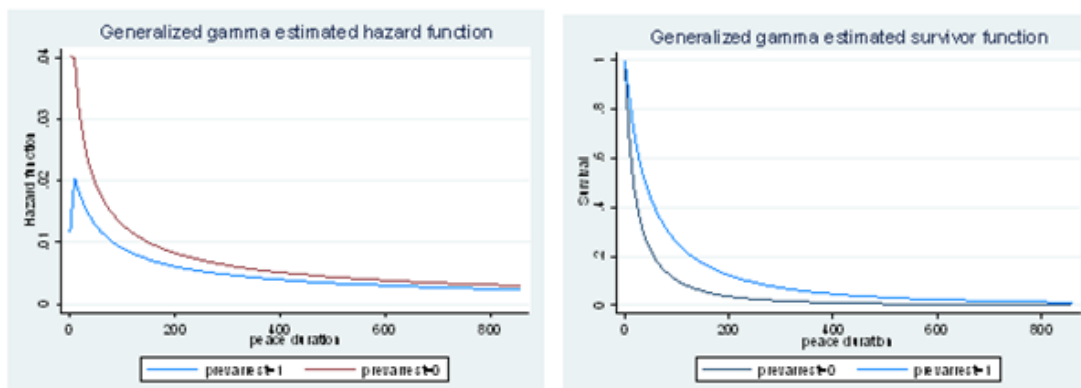
planning their attacks, in which case it is difficult to say if the total number of successful attacks would be reduced.

Table 4.1.2 – Testing $H_0: \beta_{prevarrest} = 0$ and $\beta_{prevfail} = 0$ using Likelihood-Ratio (LR) and Wald tests against 5% critical values.²¹

<u>Model</u>	<u>LR test statistic</u>	<u>Pr>chi2</u>	<u>Wald test statistic</u>	<u>Pr>chi2</u>
Cox	7.06	0.029	6.45	0.040
PCE	8.16	0.017	7.40	0.025
Gamma	11.72	0.003	11.80	0.003

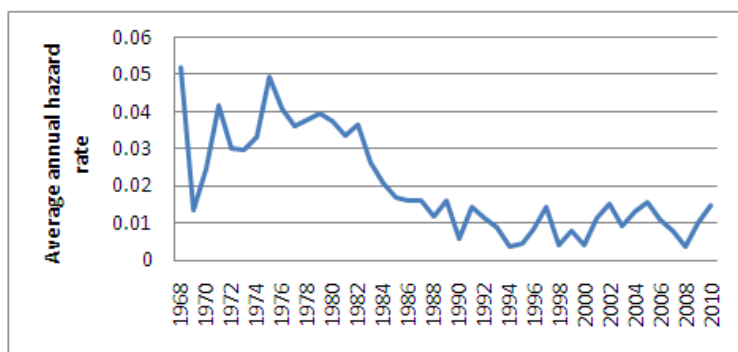
Using our estimates we can predict the shapes of the hazard and survivor functions. These are graphed in Figure 4.1.2 for $prevarrest=0$ and $prevarrest=1$. The probability of an attack per unit time is highest right after an attack, and could reflect that terrorists are encouraged by each others’ success or that they tend to plan series of attacks very near each other. The latter would explain why arrests reduce the hazard rate so much, especially right after an attack (so much that the hazard rate is actually lower immediately following an attack, than say a month afterwards). In Figure 4.1.3 we have graphed annual average hazard rates over calendar time, but fail to find clear correlation with the counter-terrorism expenditure trend (compare Figure 3.2.1).

Figure 4.1.2 – Gamma hazard and survivor function estimates for $prevarrest=0$ and 1 other vars. at means



²¹ See Appendix B.4 for an overview about the LR and Wald tests. Lagrange-Multiplier tests for duration models are unnecessarily complicated and we have therefore not used them.

Figure 4.1.3 – Average annual hazard rate (from Gamma estimates; variables at means)



After excluding the expenditure proxies and trying out the different *exp_XX* variables, *exp_85* (annual depreciation rate of 0.85) appeared as the most significant. Results of this regression are in Table 4.1.3. This is likely not to be the real depreciation rate but using the most significant expenditure variable will lead to a nice conclusion. We have ensured that the expenditure variable has the correct functional form.²² ACC is excluded as they have not performed any attacks in this period.

Table 4.1.3 – Duration model estimates (1995-2010), with *exp_85*

Dep. var: <i>peacedur</i> 57 subjects	<u>Cox model</u>				<u>PCE model</u>				<u>General Gamma model</u>			
Variable	Coefficient	S.E.	z	P>z	Coefficient	S.E.	z	P>z	Coefficients	S.E.	z	P>z
<i>Injprev</i>	0.000	0.004	0.01	0.991	0.000	0.004	0.12	0.902	0.007	0.009	0.77	0.441
<i>fatprev</i>	0.000	0.003	0.04	0.964	0.000	0.003	-0.06	0.951	-0.005	0.007	-0.73	0.466
<i>forprev</i>	0.480	0.479	1.00	0.316	0.483	0.474	1.02	0.308	0.318	0.750	0.42	0.671
<i>prevmulti</i>	0.052	0.506	0.10	0.919	0.060	0.508	0.12	0.905	-0.002	0.785	-0.00	0.998
<i>explo</i>	-0.119	0.352	0.34	0.735	0.096	0.351	0.27	0.783	-0.297	0.600	-0.50	0.619
<i>ALQ</i>	0.014	0.386	0.04	0.971	0.023	0.385	0.06	0.953	0.295	0.695	0.42	0.671
<i>sum</i>	-0.315	0.327	-0.96	0.335	-0.264	0.312	-0.85	0.397	0.907	0.548	1.65	0.098
<i>exp_85</i>	-0.027	0.118	-0.23	0.816	-0.041	0.117	-0.35	0.724	0.344	0.181	1.90	0.057
Constant					-2.89	0.646	-4.48	0.000	1.51	0.602	2.50	0.012

²² See Appendix B.1

LL	-173.703		-84.502		-108.408	
LR-chi2(1)	0.05	0.816	0.12	0.724	3.43	0.064

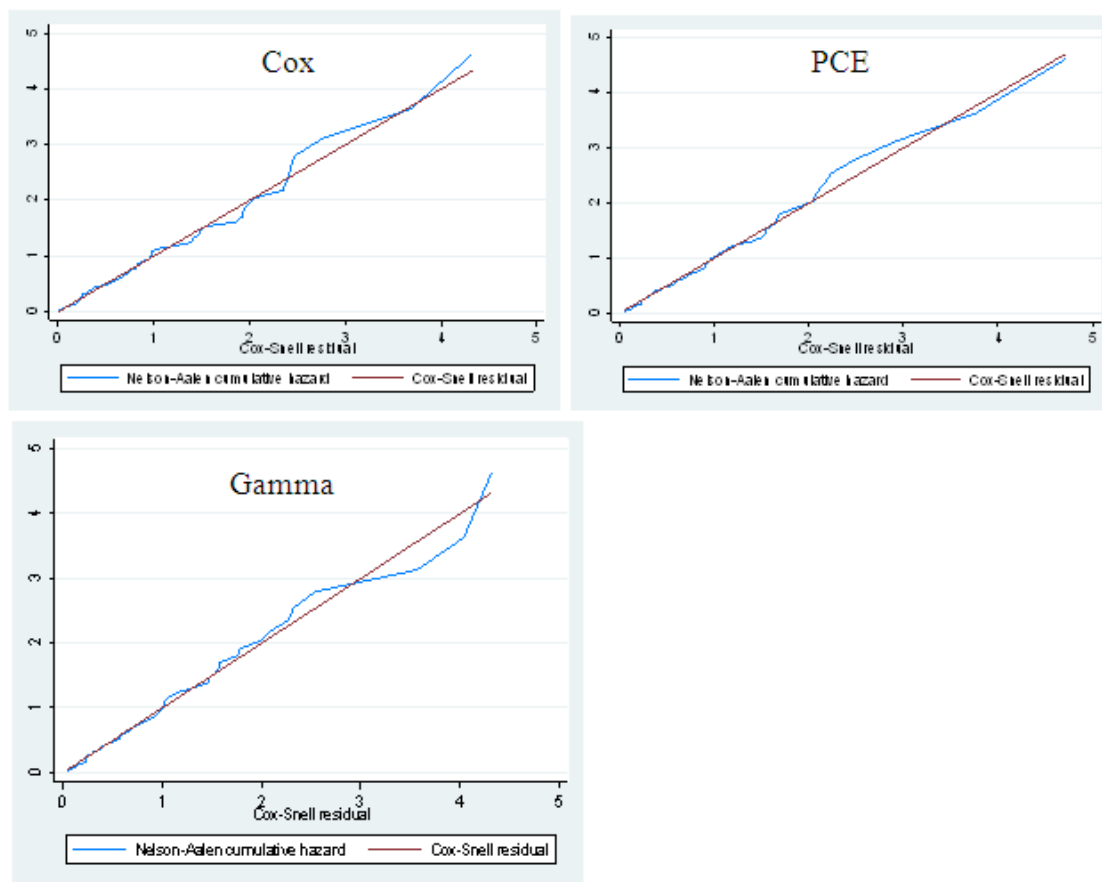
LR-chi2(1) is a likelihood-ratio test for $H_0: \beta_{\text{exp_85}} = 0$ test statistic is chi2 distributed with 1 degree of freedom. LL are log-likelihood values from each regression

The effect of *exp_85* is in the expected direction: Gamma estimates suggest that \$1bn increase in the sum of past anti-terrorism expenditure increases peace duration by 34 percentage points. The Cox and PCE models find the effect to be very insignificant. Gamma estimates show significance at %6. However, our results are shown for the depreciation rate that makes expenditure the most significant. DHS (2002) announced that counter-terrorism expenditure is intended to have long-term effects and hence the real depreciation rate can be expected to be lower than 0.85 and for those depreciation rates the expenditure estimate would have been even less significant. It follows that the even if we only relied on the Gamma estimates, the results are not sufficiently significant as to allow us to reject the hypothesis that counter-terrorism expenditure does not affect hazard rates. LR test results in *Table 4.1.3* corroborate this; we cannot reject $H_0: \beta_{\text{exp_85}} = 0$ at 5% for any of the models.

Given that we only have 16 expenditure observations, these results are not reliable. For instance, the proportional hazard assumption does not hold in the Cox and PCE models for *exp_85*²³. Another source of problems may be peace duration observations that include days from different years as they will have a large jump in the value of expenditure at the point where the year changes, which is not realistic and may cause bias. Comparing *Figures 4.1.1* and *4.1.4*, we see that our models fit the data slightly worse than before. With such a small sample, individual subjects can have large influence on estimates, and our estimates are inaccurate. All the variables are significant probably for the same reason.

²³ See Appendix B.3

Figure 4.1.4 – Goodness-of-fit for Cox, PCE and Gamma models using Cox-Snell residuals



IV. 2 – Results: the Effect of Terrorist Attacks on Expenditure

The final results from our regression with expenditure as the dependent variable are in *Table 4.2.1*. We started with the model in equation (2) of *Section III*. The Final model was obtained by deleting lags insignificant at 10% at each step since we need a very simple model as we have so little data.²⁴

Table 4.2.1 – Final model with exp_t as the dependent variable

Dependent Variable: exp_t - 14 observations

<u>Variable</u>	<u>Coefficients</u>	<u>S.E.s</u>	<u>t</u>	<u>P> t </u>
exp_{t-1}	0.829	0.065	12.69	0.000
$ncas_{t-1}$	0.001	0.0001	8.74	0.000
$ncas_{t-2}$	0.0004	0.0001	2.99	0.014
GDP_{t-1}	0.992	0.262	3.77	0.004
constant	-6.768	1.876	-3.61	0.005

All estimates are significant in the final model. The fact that *nattacks* lags were deleted implies that the government officials may only care about the number of casualties not attacks; only successful attacks influence spending decisions. We can see that \$1bn increase in counter-terrorism expenditure predicts \$0.829bn higher expenditure next year, while a \$1trillion increase in GDP prognosticates a \$0.992bn increase in anti-terrorist spending for the next year. In the long run, \$1 trillion increase in GDP causes \$5.8bn increase in counter-terrorism expenditure. Ten extra annual casualties are estimated to cause a \$10million increase in the next year's expenditure, \$4million in the year after that. The respective long run effect can be calculated as an \$82 million increase.

We have tested for this model's robustness in various ways. First, we have failed to find any evidence of serial correlation.²⁵ The main problem is that with only 16 observations the

²⁴ See Appendix B.5 for further details

²⁵ See Appendix B.6

estimates of our model are unreliable at best. It is unreasonable to assume that with such a small sample we could accurately estimate the effect of casualties since for most years the number of casualties is 0 and then for a few years the number is relatively high. We have used the DFBETA measure of influence to test for this. This predicts the change in the estimated coefficients due to deleting a specific subject, scaled by the standard error calculated with the observation deleted. The results for $ncas_{t-1}$ and $ncas_{t-2}$ are presented in *Figures 4.2.1* and *4.2.2* respectively. Indeed, the results predict that if the observation for 2002 was removed, the coefficient of $ncas_{t-1}$ would be smaller by 3.3 times the standard error. This is because for the 2002 observation, one-period lagged casualties include those of 9/11, but also because there was relatively high number of casualties in 2002 which are picked up by $ncas_{t-1}$ of 2003. Similarly, if observation 2003 was removed, it is predicted that the coefficient estimate of $ncas_{t-2}$ would be smaller by 1 times the standard error, which again reflects the influence of 9/11. Thus we cannot say from this evidence if any other attacks apart from the 9/11 would have been important determinants of anti-terrorist spending. Given the large effect of one-period lagged expenditure, it seems that the 9/11 simply set off a long-term renewal process of the US counter-terrorism measures. Due to the uniqueness of our approach we have not been able to compare these results to existing literature.

Figure 4.2.1 – DFBETA measures of influence for $ncas_{t-1}$

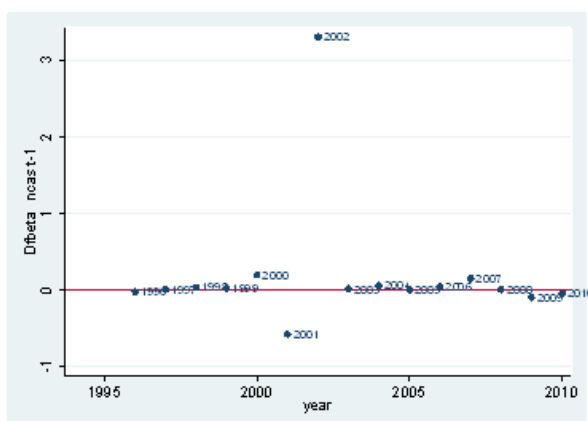
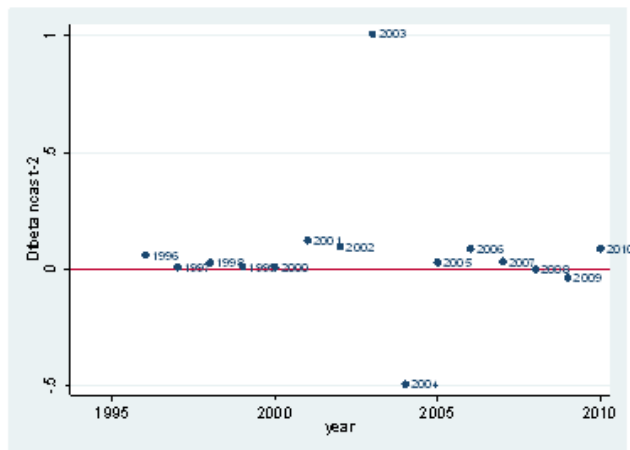


Figure 4.2.2 – *DFBETA* measures of influence for $ncast_{t-2}$



V. Conclusions

Using data from 1968 to 2010 we find that if the perpetrator of an attack is arrested, this significantly increases the expected length of the following peace period, which suggests that higher counter-terrorism spending could reduce attacks through a higher arrest rate. One reason could be that terrorists often plan to carry out many attacks within a short period of time and arrests prevent this. This could also explain our finding that the probability of a terrorist attack per unit time is highest right after an attack. We are not aware of any previous work that has provided the same results through a rigorous use of duration models. Given the robustness of the results, due to a large sample, these are our biggest contributions to the existing literature. Using the expenditure variable itself, we found some evidence from the Gamma model that expenditure could be significant; however, the evidence is not strong enough to allow us to conclude that expenditure affects the threat of terrorism. This result is far from conclusive, however. With only 16 observations we cannot expect to capture expenditure's true effect.

With expenditure as the dependent variable, we did not find significant evidence that the number of attacks in the previous two years affects the current budget. While a higher

number of casualties appeared to raise expenditure, we showed that this was almost fully due to the strong influence of the 9/11. Combined with the large and significant effect of lagged expenditure, this could imply that 9/11 alone caused changes in the US counter-terrorism strategy and the expenditure growth reflects the implementation of these long-term measures. In general, however, these results again highlight that with only 16 observations we cannot get robust estimates of the covariates. It is clear that the major weakness of our paper is thus the small amount of expenditure data.

While considering both unsuccessful and successful attacks has given us the benefit of more data as well as allowing us to consider the threat level in terms of all attempted attacks, it also has drawbacks. For example, we cannot be certain if higher arrest rate actually reduces the number of successful attacks, or if it leads to longer peace durations by providing incentive for terrorists to plan their attacks better, which could result in a higher success rate.

Additionally, we have assumed that the DHS budget decisions are affected equally by unsuccessful and successful attacks. In a more realistic extension, one would account for them separately. A further potential problem is the possibility that the data is more comprehensive for the recent years due to more public information being available. For instance, it could be that the number of failed attacks has not increased but rather that more of them are reported.

One way to extend our work would be through a better understanding of the DHS budget and what exactly constitutes anti-terrorist spending. In this paper we made an unrealistic assumption that 61% of the DHS budget goes into counter-terrorism. This is probably a good approximation, but unlikely to be accurate over years time as the structure of the DHS changes. A more detailed study of the DHS budget would also make it easier to predict the real depreciation rate rather than making conclusions based on the most significant rate. Counter-terrorism spending can take many forms and future work could account for this.

Moreover, we have not considered spending on counter-terrorism abroad, which may have large pre-emptive effects and could thus affect peace durations strongly.

Another way to extend this paper would be to build more realistic models. Our time-series model is much too simplistic and additional variables should be explored. For example, in addition to the recent attacks, the perceived future threat of terrorism is likely be an important factor. Studies in this topic are much constrained by the limited amount of publicly available national security data, however. Our duration models could also be improved. It is unrealistic to assume that terrorists' behaviour is determined fully by few variables that describe the outcome of the previous attack and are then estimated using 'off-the-rack' models. Instead, one could try to specify some decision making process for terrorists and then manually build a 'structural' duration model around this framework. One further weakness is that we consider the two possible directions of causality separately. An improvement to this would be a VAR model between attacks and expenditure. It is important to note that the usefulness of the above extensions is dependent on having a sufficient amount of counter-terrorism expenditure data and as such are currently unrealistic.

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Appendix A

1. Deriving the hazard function in continuous time (based on Lancaster 1990)

T is a continuous random variable denoting peace duration. C.d.f of these peace durations is $F(t) = P(T \leq t)$. Using laws of conditional probability we define the probability of there being an attack in a short interval dt given that there has been no attack for the past t units of time:

$$P(t \leq T < t + dt | T \geq t) = \frac{P(t \leq T < t + dt, T \geq t)}{P(T \geq t)} = \frac{P(t \leq T < t + dt)}{P(T \geq t)}$$

which can be written as $\frac{F(t+dt) - F(t)}{1 - F(t)}$ using the above definition. We divide by dt and take

limit to zero. This gives the hazard function; the instantaneous probability of there being an attack, given that there has not been one for the past t time units:

$$\theta(t) = \lim_{dt \rightarrow 0} \frac{F(t + dt) - F(t)}{dt} \frac{1}{1 - F(t)} = F'(t) \frac{1}{1 - F(t)} = \frac{f(t)}{1 - F(t)}$$

$f(t)$ is the probability density function of $F(t)$ and $1 - F(t) = S(t)$ is the survival function.

2. General Gamma model specification – based on Jenkins (2008) and Lancaster(1990)

Accelerated Failure Time models assume a linear relationship between log of survival time and covariates:

$$\ln(T) = \beta'X + u \quad (1)$$

u is an error term. Now if we let $\psi \equiv \exp(-\beta'X)$ then we will have: $\ln(T\psi) = u$ (2)

$$\text{or} \quad \exp\{u\} = T\psi \quad (3)$$

As to allow for more general models, we generalise the error term u by dividing it with a constant. So from (2) we will have: $\ln(\lambda T) = \frac{u}{\alpha}$ (4)

Now we let $\exp\{u\}$ to be gamma variate with parameter m (denoted $G(m)$). That is, if

$$\exp\{u\} = Y \text{ then it is distributed as: } f(y) = \frac{y^{m-1}e^{-y}}{\Gamma(m)} \quad (5)$$

$\Gamma(m)$ is the Gamma function which is the integral:

$$\int_0^{\infty} x^{\alpha-1} e^{-x} dx$$

From (4) we have $(\lambda T)^\alpha = \exp\{u\} = Y$. Using $\alpha\lambda^\alpha t^{\alpha-1} dt = dy$ and substituting this into (5) we can get an expression for the p.d.f of T :

$$f(t) = \frac{\alpha\lambda^{\alpha m} t^{\alpha m - 1} e^{-(\lambda t)^\alpha}}{\Gamma(m)} \quad (6)$$

This is the Generalised Gamma density function with two parameters m and α which allow for the flexible nature of the model. There is no closed form expression for the c.d.f and thus no closed form for the hazard function. From (1) we see that the coefficients for the model are

interpreted as $\beta_k = \frac{\partial \log T}{\partial X_k}$.

3. Proportional Hazard, PCE and Cox models – based on Jenkins (2008)

i.) Proportional Hazard (PH) models

For these models the hazard rate can be separated into two parts:

$$\theta(t, X(t)) = \theta_0(t) \exp(\beta' X(t)) \quad (1)$$

$\theta_0(t)$ is the baseline ‘hazard function’ which only depends on t but not on ‘personal specific’ covariates; thus it is common to all individuals. $\exp(\beta' X(t))$ takes the observation specific characteristics into account and scales the hazard rate, and consequently the peace duration, accordingly. The namesake of the model is the fact that differences in covariates for different observations will lead to proportional differences in hazard rate at each t . If we write above as $\ln \theta(t, X(t)) = \ln \theta_0(t) + \beta' X(t)$ we can see that coefficients in PH models are

interpreted as $\beta_k = \frac{\partial \log \theta(t, X)}{\partial X_k}$.

ii.) PCE model

In the PCE model hazard function is assumed to only vary between fixed intervals and within the interval it is estimated to be some constant value:

$$\theta(t, X_t) = \begin{cases} \bar{\theta}_1 \exp(\beta' X_1) & \text{for } t \in (0, t_1) \\ \bar{\theta}_2 \exp(\beta' X_2) & \text{for } t \in (t_1, t_2) \\ \vdots & \\ \bar{\theta}_K \exp(\beta' X_K) & \text{for } t \in (t_{K-1}, t_K] \end{cases}$$

Above is the PH model with the baseline hazard different for each interval. We can rewrite it as:

$$\theta(t, X_t) = \begin{cases} \exp(\log\bar{\theta}_1 + (\beta'X_1)) & \text{for } t \in (0, t_1) \\ \exp(\log\bar{\theta}_2 + (\beta'X_1)) & \text{for } t \in (t_1, t_2) \\ \vdots & \\ \exp(\log\bar{\theta}_k + (\beta'X_1)) & \text{for } t \in (t_{K-1}, t_K] \end{cases}$$

So the model is equivalent to adding dummy variables representing each interval. For instance, for the model we specified in *Section III* one would write:

$$\theta(t, X_t) = \begin{cases} \exp(\delta_t + x'\beta) & \text{for } t = 1, 2, \dots, 29 \\ \exp(\delta_j + x'\beta) & \text{if } t \in [30 + 10(j - 30), 40 + 10(j - 30)) \quad \text{for } j = 31, \dots, 36 \\ \exp(\delta_j + x'\beta) & \text{if } t \in [100 + 100(j - 37), 200 + 100(j - 37)) \quad \text{for } j = 37, \dots, 39 \\ \exp(\delta_{40} + x'\beta) & \text{if } t \in [400, \infty) \end{cases}$$

where $\exp(\delta_j + x'\beta)$ is an exponential hazard function and δ_j is a dummy variable for a specific interval. As always in estimating, we need to exclude one of the dummies.

iii.) Cox Model

The Cox model is a Proportional Hazard model; that is:

$$\theta(t, X(t)) = \theta_0(t) \exp(\beta'X(t))$$

where $\theta_0(t)$ is the baseline hazard and $\exp(\beta'X(t))$ are the observation specific effects. The special property of the Cox model is that it can estimate $\exp(\beta'X(t))$ without having to specify the baseline hazard function. This is possible through the use of Partial Likelihood (PL) estimation. First the data is ordered according to event identifiers and then in the order of survival time (peace duration), for example as in the table below:

Observation ID: i	Peace duration: t_i	Event # k :
1	2	1
2	5	2
3	6	3
4	9	4

PL is then calculated as: $PL = \prod_{k=1}^K \mathcal{L}_K$ where k is an index for events. Each \mathcal{L}_K is:

$L_k = P(\text{particular peace duration observation ends at } t_i \mid \text{one of the peace duration observations ends at } t_i).$

For example, if from the above table we define:

$$A = P(\text{event 3 is experienced by } i = 3 \text{ and not } i = 4)$$

$$B = P(\text{event 3 is experienced by } i = 4 \text{ and not } i = 3)$$

It follows that for the event #3 we therefore have $L_3 = \frac{A}{A+B}$ and in terms of hazard rates:

$$L_3 = \frac{\theta_3(6)}{\theta_3(6)+\theta_4(6)}. \text{ If we apply the PH definition (1) we get:}$$

$$L_3 = \frac{\theta_0(6)\exp(\beta'X_3(6))}{\theta_0(6)\exp(\beta'X_3(6))+\theta_0(6)\exp(\beta'X_4(6))} = \frac{\exp(\beta'X_3(6))}{\exp(\beta'X_3(6))+\exp(\beta'X_4(6))} \text{ since the}$$

baseline hazard is the same for all observations. Same logic is applied to calculate the whole PL function which can then be estimated without specifying a baseline hazard. Note that only covariate values at attack times are needed for estimation.

4. Further information about our dataset

i.) Information available about each attack: date, information source, number of casualties, if it was against US citizens or US property, suicide mission or not, perpetrator, if the attack was stopped before being carried out, if the perpetrator was arrested, location; tactic used: armed attack, hijacking, arson, kidnapping, assassination, hostage, unconventional, bombing, unknown, other; target of the attack; weapon used: remote-detonated explosive, fire or firebomb, biological agent, chemical agent, radiological agent, explosives, firearms, knives, other unknown; political motivation.

ii.) Terrorist attacks that satisfy the following definition are included: planned acts of violence that include political objective and are calculated to create fear and alarm. Counter-terrorism expenditure is presumably aimed at protecting all people residing within the US and thus we also include attacks that have been aimed at non-US citizens such as foreign embassies. Following similar logic, we include attacks by all perpetrators, even if they are US citizens. Unsuccessful attacks are included.

5. Time-varying covariates: sum and exp

Expenditure and summer month variables are problematic since they vary over time.

Fundamentally the fact that they measure monthly and annual values is not a problem despite peace duration being measured in days. For example a peace duration observation from 1st of July 2004 to 10th of July 2004, would simply have a single value for the season dummy, and one value for that year's expenditure. For observations that include non-summer and summer months and different years we have to split the observations where these variables change.

6. Variable definitions, sources and units for the duration models:

Name	Definitions & Descriptions	Source	Unit
<i>peacedur</i>	The duration of peace between consecutive terrorist attacks measured in days. Obtained by calculating the difference between consecutive attack dates	<i>RDWTI</i> 1968 - 2010	days
<i>injprev</i>	The number of people injured in the previous attack	<i>RDWTI</i> 1968 - 2010	# of people
<i>fatprev</i>	The number of people killed in the previous attack	<i>RDWTI</i> 1968 - 2010	# of people
<i>forprev</i>	A dummy variable equal to unity if the previous attack was originally aimed at foreign citizens (but also posed a risk to US citizens)	<i>RDWTI</i> 1968 - 2010	scalar
<i>prevarrest</i>	A dummy equal to one when the perpetrator was arrested right after or before an attack	<i>RDWTI</i> 1968 - 2010	scalar
<i>prevfail</i>	A dummy equal to one if the previous attack failed	<i>RDWTI</i> 1968 - 2010	scalar
<i>prevmulti</i>	A dummy equal to one if there were multiple attacks on the day of the most recent attack	<i>RDWTI</i> 1968 - 2010	scalar
<i>ACC</i>	A dummy equal to one if the perpetrator of the previous attack was a terrorist group called Anti-Castro Cubans	<i>RDWTI</i> 1968 - 2010	scalar
<i>ALQ</i>	A dummy equal to one if the perpetrator of the previous attack was Al-Qaeda	<i>RDWTI</i> 1968 - 2010	scalar
<i>explo</i>	A dummy equal to one if explosives were used in the most recent attack	<i>RDWTI</i> 1968 - 2010	scalar
<i>sum</i>	A dummy equal to one if it is currently a summer month, June, July or August – describes the current season	-	scalar
<i>exp</i>	The current year’s counter-terrorism expenditure, calculated as 61% of that year’s DHS budget. Includes spending only on domestic counter-terrorism measures	<i>DHS budget-in-brief</i> 1995 - 2010	(nominal) Billions of US dollars
<i>exp_XX</i> <i>XX</i> ∈ [0.01, 0.99]	Sum of all previous years’ counter-terrorism expenditure with an annual depreciation of XX%. One such variable has been created for each possible depreciation rate ranging from 0.01 to 0.99.	<i>exp</i> variable 1995 - 2010	Billions of US dollars

7. An example of an observation (the peace duration following 9/11) from our dataset

id	peacedur	injprev	fatprev	forprev	prevarrest	prevfail	prevmulti	ACC	explo	ALQ	exp	exp_1	...	exp_99
263	21	2337	2982	0	0	0	1	0	0	1	10.309	0.081	...	37.664

8.) Variable definitions, units and summary statistics for the time-series model in eq. (2)

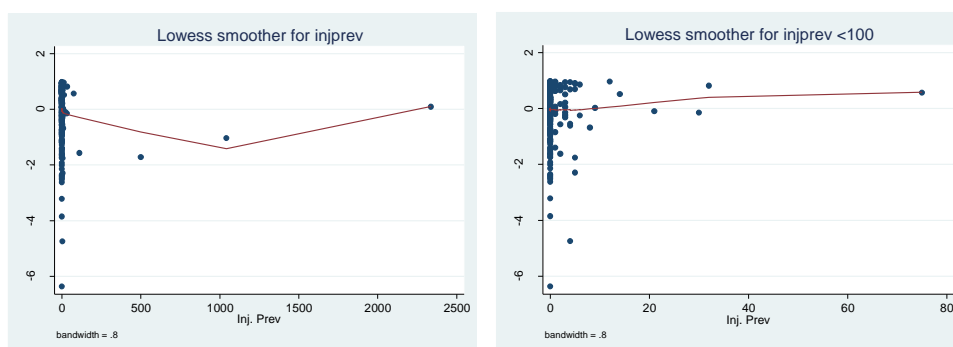
Name	Description	Unit	Mean	Variance	Min.Val., Max Val.
exp_t	The current year's counter-terrorism expenditure, calculated as 61% of that year's DHS budget. Includes spending only on domestic counter-terrorism measures	(nominal) billions of USDs	17.060	10.371	5.49, 33.733
$ncas_t$	The sum of all people injured and killed in terrorist attacks in a given year	# of people	367.529	1299.092	0, 5370
$nattacks_t$	The total number of terrorist attacks in a given year (both successful and unsuccessful)	# of attacks	4.000	24.500	0, 20
GDP_t	Annual nominal Gross Domestic Product	trillions of USDs	11.179	6.121	7.414, 14.657

Our data sources are same as before, apart from annual GDP (obtained from *IMF World Economic Outlook Database*²⁶).

Appendix B

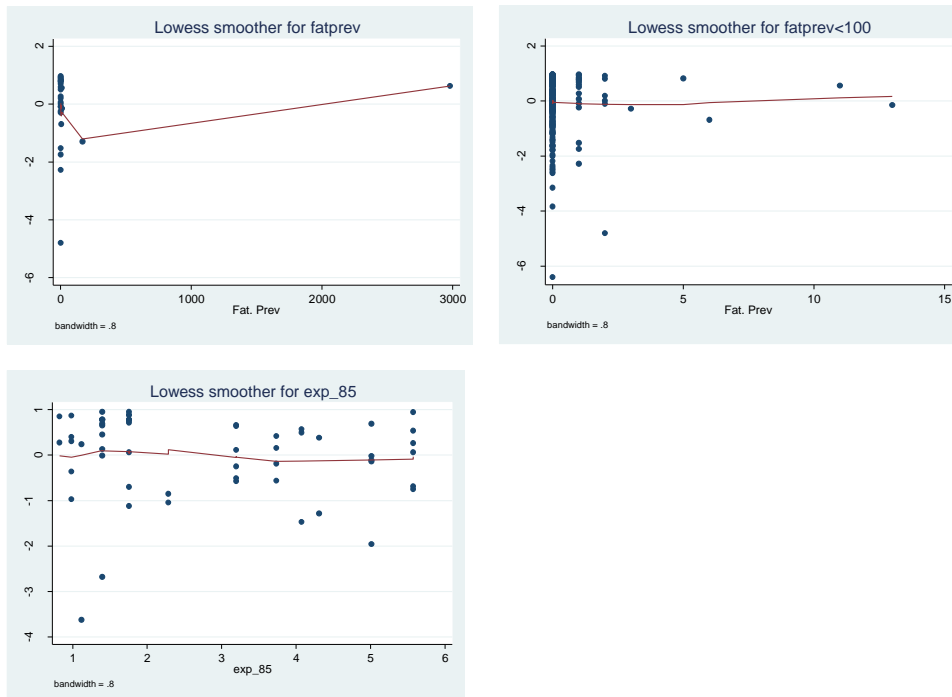
1. Checking for the functional form of covariates and interactions

All possible interaction terms were considered but as none were significant none were added. In order to check if the functional forms of our covariates are appropriate, we have run our regression without the covariate we are interested in and then saved martingale residuals. Martingale residuals are interpreted, at each survival time, as the difference between the observed and expected number of attacks given the model estimates. We plot martingale residuals against the values of the covariate we excluded to check for its functional form. Linear fit implies correct functional form²⁷ and indeed this is the case for our continuous variables $injprev$, $fatprev$ and exp_85 . However, for larger values of $injprev$ and $fatprev$ we cannot be sure about this due to sparse data. See figures below.



²⁶ IMF (2011): <http://www.imf.org/external/pubs/ft/weo/2011/01/weodata/index.aspx>

²⁷ Therneau, Grambsch and Fleming (1990)



2. Cox-Snell residual plots for goodness-of-fit – based on Lancaster (1990)

If the models are correctly specified then survivor function $\hat{S}(t_i)$ should be unit Exponentially distributed. This can be checked by plotting $-\ln \hat{S}(t_i)$ against $\hat{S}(t_i)$, which should produce close fit around 45 degree line since $-\log e^{-x} = x$. In practise this is done by getting cox-snell residuals which are defined as $-\ln \hat{S}(t_i)$. We then use these residuals as time variables and calculate the estimated survival function based on this, call it $\bar{S}(t_i)$. Next we calculate cumulative hazard function as $-\ln \bar{S}(t_i)$ in Stata using Nelson-Aalen estimator and plot it against cox-snell residuals and check for fit around 45 degree line to see if they are the same.

3. Checking for the Proportional Hazard assumption of the Cox and PCE models

Based on UCLA (2011): In Appendix C.4 we showed that PCE and Cox models are proportional hazard models such that $\theta(t, X(t)) = \theta_0(t) \exp(\beta'X(t))$, and thus have the property that hazard functions for different observations should be proportional at each point along the peace duration. We test if this holds by plotting Schoenfeld residuals against peace duration for each covariate. A Schoenfeld residual is calculated for each covariate of each observation, and is defined as $r_{ik} = x_{ik} - \hat{x}_{ik}$. Where x_{ik} is the value of the k^{th} covariate for peace duration observation i and \hat{x}_{ik} is the weighted mean of that covariate for all those peace

duration observations who are in the risk set of experiencing an attack at that point of peace duration. Testing the null-hypothesis of the proportional hazard assumption is equivalent to testing for a zero slope in a generalised linear regression of Schoenfeld residuals on functions of peace durations. If we reject this hypothesis, we reject the PH assumptions. We test this for hypothesis for all the variables in the Cox and PCE models, in both of our two regression, but only display Cox results as they are both the same. From *Table A4.1* we can see that PH assumption seems to hold for our first regression, while for second one it does not hold nearly as well. In particular, it holds very badly for *exp_85*:

Table A4.1 – Testing PH assumption for regression without exp from 1968 – 2010

<i>variable</i>	<i>rho</i>	<i>Chi2</i>	<i>Prob>Chi2</i>
<i>injprev</i>	0.068	1.74	0.188
<i>fatprev</i>	-0.048	0.77	0.382
<i>forprev</i>	0.129	5.57	0.018
<i>prevarrest</i>	0.043	0.56	0.454
<i>prevfail</i>	0.053	0.81	0.367
<i>prevmulti</i>	0.006	0.01	0.911
<i>ACC</i>	0.010	0.03	0.855
<i>explo</i>	0.030	0.28	0.596
<i>ALQ</i>	0.025	0.19	0.661
<i>sum</i>	0.049	0.73	0.393

Table A4.2 – Testing PH assumption for regression with exp from 1995-2010

<i>variable</i>	<i>rho</i>	<i>Chi2</i>	<i>Prob>Chi2</i>
<i>injprev</i>	0.198	2.55	0.110
<i>fatprev</i>	-0.182	2.09	0.148
<i>forprev</i>	0.285	4.59	0.032
<i>prevmulti</i>	-0.024	0.03	0.853
<i>explo</i>	-0.051	0.16	0.693
<i>ALQ</i>	0.175	1.51	0.218
<i>sum</i>	0.182	1.41	0.234
<i>exp_85</i>	0.348	7.28	0.007

4. Likelihood-ratio and Wald tests

Wald test: Unrestricted model is estimated, from which we get the vector of parameter estimates $\hat{\theta}$. To test some J restrictions on K parameters, that is $H_0: R\theta = q$ where R is $J \times K$ matrix and θ is a vector of the K parameters, one tests if $R\hat{\theta} = q$ are significantly far away from zero.

LR test: Both restricted and unrestricted models are estimated once in order to test if the difference in log-likelihood values between the models is significantly different from zero.

5. Deriving the final model with *exp* as the dependent variable

Dependent variable: exp_t ; 14 observations

Variable	Coefficients	S.E.s	T	P> t
exp_{t-1}	0.854	0.066	13.00	0.000
$ncas_{t-1}$	0.001	0.0003	4.92	0.001
$ncas_{t-2}$	0.0004	0.0002	1.51	0.169
$nattacks_{t-1}$	-0.119	0.081	-1.48	0.178
$nattacks_{t-2}$	0.050	0.056	0.90	0.392
GDP_{t-1}	0.922	0.255	3.62	0.007
<i>constant</i>	-6.284	1.833	-3.43	0.009

Excluding $nattacks_{t-2}$ we get:

Dependent variable: exp_t ; 14 observations

Variable	Coefficients	S.E.s	T	P> t
exp_{t-1}	0.860	0.065	13.35	0.000
$ncas_{t-1}$	0.001	0.0003	4.95	0.001
$ncas_{t-2}$	0.0005	0.0002	3.38	0.008
$nattacks_{t-1}$	-0.123	0.079	-1.55	0.155
GDP_{t-1}	0.910	0.252	3.61	0.006
<i>constant</i>	-6.12	1.81	-3.39	0.008

Deleting $nattacks_{t-1}$ brings us to our final model:

Variable	Coefficients	S.E.s	T	P> t
exp_{t-1}	0.829	0.065	12.69	0.000
$ncas_{t-1}$	0.001	0.0001	8.74	0.000
$ncas_{t-2}$	0.0004	0.0001	2.99	0.014
GDP_{t-1}	0.992	0.262	3.77	0.004
<i>constant</i>	-6.768	1.876	-3.61	0.005

This is our final model.

6. Testing for serial correlation in the final model using Breusch-Godfrey test

This test consists on regressing the model residuals on their own one-period lags and all of the explanatory variables. The test statistic is F distributed and equals 0.009 with 1 degree of freedom. The null hypothesis is that of no serial correlation and we fail to reject this since our p-value is for the above test-statistic is 0.923.