Constrained Attention and Social Coordination

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Abstract: A model of social coordination is built. The model is characterised by constrained attention on the parts of agents making them unable to distinguish between all possible types of their interacting partners. We investigate how this constraint affects players' behaviour in isolated as well as networked interactions. It is found that constrained attention in general will hamper players' ability of behavioural coordination.

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“Outside the street's on fire in a real death waltz

Between flesh and what's fantasy and the poets down here

Don't write nothing at all, they just stand back and let it all be

And in the quick of the night they reach for their moment

And try to make an honest stand but they wind up wounded, not even dead

Tonight in Jungleland”

Jungleland, Bruce Springsteen

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"Life is a collection of coordination games"

Ken Binmore, 2012

"Is there any other point to which you would wish to draw my attention?"
"To the incident of the dog in the night-time."
"The dog did nothing in the night-time."
"That was the curious incident." Remarked Sherlock Holmes.
Arthur Conan Doyle, 1901

1 Introduction

Sherlock Holmes, the protagonist of Arthur Conan Doyle's intriguing stories, had an exceptional attention to detail which was clearly contrasted with that of his companion Doctor Watson. This is a prime example of how human beings differ in their ability to make use of information. The phenomenon is of widespread importance in social sciences and an individual's processing of information in decision-making is central to the study of economic behaviour. In this work we consider individuals differing in their level of attention to their surrounding, affecting their decision-making abilities. We model this by individuals being unaware of relevant characteristics of other individuals, justified by human inability or computational cost of attention.

Social behaviour is in its essence multifaceted and difficult to characterise. Nonetheless, it is in many settings possible to view behaviour from a two-dimensional perspective scaled by various parameters. A relevant example is the study of the social contract with a vast literature on the evolution of cooperation analysing this two-dimensional perspective [3]. Similarly, we model a general situation where individuals meet and choose a cooperative or non-cooperative behaviour. The setting assumes that individuals wish to behave similarly and dislikes meeting someone behaving differently.

Our purpose is to investigate how this kind of constrained attention can affect coordination of behaviour in social interactions. We do not attempt to build a complete theory treating unawareness, but rather provide an indication to what kind of results such models can produce. We make a set of assumptions such that our model applies particularly to boundedly rational players interacting within a narrow time frame. We assume that players learn through time by observing behaviour. In particular, players attempt to reason about opponents' preferences and from this make predictions of how they will behave. The lack of generality caused by restrictive assumptions is made up for by width, in the sense that we consider individual interactions as well as networked interactions. Applications for our model are found in settings where outcomes are of low importance as this implies boundedness on individuals reasoning expressed through constrained attention, myopic behaviour and flaws in reasoning regarding formation of behavioural conjectures. When applied to multi-individual interactions, the model applies to groups with a limited amount of players as the attempted reasoning would be unreasonable given too many interactions. Individuals base beliefs on min-max regret reasoning, which provides nice intuitive properties of beliefs.

2 Literature Review

Relaxation of awareness is often a realistic assumption and has been justified in behavioural economics [17] and epistemic logic [23]. Logicians recognise that if an agent knows proposition *p,* and proposition *p´* is a logical consequence of *p*, it may not always be the case that the agent knows *p´*, especially if the consequent-relation is distant. The agent may *implicitly know p´,* but is not necessarily *aware* of this [13]. For our setting, the existence of two extreme preferences of behaviour implies by continuity the existence of a middle type, which the player may be unaware of.

Treatment of unawareness in game theory is relatively novel with recent empirical [6] and theoretical advances [7,8]. These and other treatments of unawareness include unawareness of actions, players or various types. [8] treats all these cases, whereas [7] as here focuses on types and type distributions, but from a different perspective since they seek a standard solution concept. Related to these efforts is the literature on epistemic game theory [4] which in its search for epistemic conditions in games explores relaxation of the common knowledge over type spaces [5]. These studies open exciting possibilities for exploration of epistemic variations in games, and we see our work as related although we make simplifying assumptions allowing us to neglect higher level hierarchies of beliefs.

Our model utilises the theory of min-max regret, where we are influenced by Savage [21,22]. Min-max regret theory has proved popular in decision theory and is appropriate for belief formation in games [11]. We are also influenced by attribution theory [12] from social psychology, by that individuals focuses on others' intentions rather than actual outcomes when forming beliefs [16]. The idea that attention may be scarce is also well-founded in the literature [24].

Recent years have seen intense research on social and economic networks [14]. Most relevant are [18,25] which provides studies of coordination games played within completely networked populations, providing interesting points of comparison. However, a difference is that these papers map history of play straight into strategy conjectures, implying a different reasoning. Also, we believe these papers to apply to larger populations and longer term settings where evolutionary forces have been long exerted. Our approach is also related to the theory of learning in games. There is a vast literature on this and many approaches at different levels of reasoning have been explored [9]. We see our model of learning as belonging to sophisticated learning models, see section 8 [9]. This because players attempt to derive beliefs about their opponent's preferences and map this into conjectured play, a step further in reasoning compared to adaptive models like fictitious play.

Important for interpretation of our work is the distinction of two separate views of the world, the Bayesian and the game-theoretic [1,21,22]. The Bayesian view postulates that it is always possible to put probabilities on all events, including action choices in a game. According to the game-theoretic view, action choices must be determined by an equilibrium notion and cannot be assigned prior probabilities. Our work adheres to the Bayesian view and motivated by uncertainty we assume that players always put a probability distribution over opponents' play and best-respond accordingly. We find it plausible that every setting contains a certain amount of uncertainty from the decision-makers perspective, and so here disregard the notion of pure strategy Nash equilibrium. Instead we use a setting with mixed conjectures and pure action choices, inspired by [2].

3 Individual Interactions

3.1 Model

The playing population is the set of players $I = \{1,2\}$. There exists a true type space $T = (0,1)$ and each player $i \in I$ is allocated a type $t_i \in T$, which is private information.

Players engage according to a 2-by-2 simultaneous game denoted by Γ, which is repeated infinitely but players are myopic and only care about current payoffs. When interacting, the players choose an action $a_i \in A = \{A, B\}$, which forms an action profile $a \in A \times A$. We interpret A as cooperative behaviour, B as non-cooperative. Payoffs are as follows:

$$u_i^\Gamma(A) = \begin{cases} 3 - t_i \ if \ a_j^\Gamma = A \\ -1 \ if \ a_j^\Gamma = B \end{cases}, \ u_i^\Gamma(B) = \begin{cases} 0 \ if \ a_j^\Gamma = A \\ 1 + t_i \ if \ a_j^\Gamma = B \end{cases}, \forall i, j \in \{1,2\}$$

1,2	A	B
A	3-t_1,3-t_2	-1,0
B	0,-1	1+t_1,1+t_2

To model attention, player *i* holds an awareness[1] of player *j*'s type space denoted by $T_j^i \subseteq [0,1], \forall i, j \in I$ and a probability distribution over this space denoted by t_j^i, where we assume t_j^i to initially be uniformly distributed. We denote $T_j^i = [0,1]$ as *full awareness* and $T_j^i = \{0, 0.5, 1\}$ as *limited awareness*. The probability distribution over type spaces may change through the game[2] as players update their beliefs. If an observed action makes a player exclude that the opponent can be of a certain type, the player puts probability zero on that type, which is irreversible, and equal probability on remaining types.

A *strategy conjecture* of player *i*, s_j^i, is a probability distribution over his opponent's actions. We wish these conjectures to satisfy: [11]

1. $s_j^i = s_j^i(u_j(\Gamma))$
2. $0 < s_j^i < 1$
3. $\frac{ds_j^i}{dt_j} < 0$

We seek a unique mixed strategy conjecture and a natural candidate would be mixed strategy Nash Equilibrium, but note that this would not satisfy axiom 3. Instead consider min-max regret:

[1] In general, we would have to define awareness architectures $C_i \in C = (\{0,1\}^T \times \{0,1\}^{\Delta T}) \times (\{0,1\}^T \times$ *0,1∆T×…* However, due to assumptions on belief formation to be explained, we only need first order awareness.

[2] Note that awareness does not change, but beliefs may.

Consider Γ:

i,j	A	B
A	3-t_i,3-t_j	-1,0
B	0,-1	1+t_i,1+t_j

Define a *regret* of an action choice as $\max\{0, u_i(a_i', a_j, t_i) - u_i(a_i, a_j, t_i)\}$ where a_i' maximises $u_i|a_j, t_i$. We define a regret matrix for player *j* as follows:

i,j	A	B
A	.,0	.,2+t_j
B	.,3-t_j	.,0

Supposing $s_j = \Pr(a_j = A)$, the regret matrix becomes:

i,j	A	B
B	(1-s_j)(3-t_j)	s_j(2+t_j)

The min-max principle says the player should minimise $max\{(1 - s_j)(3 - t_j), s_j(2 + t_j)\}$.

$$(1 - s_j)(3 - t_j) = s_j(2 + t_j)$$

$$s_j = \frac{3 - t_j}{5}$$

Note that this function satisfies all five axioms. We therefore define player *i*'s strategy conjecture over player *j*'s actions as $s_j^i = \frac{3-E(t_j^i)}{5}$[3]. This implies $s_j^i \in (\frac{2}{5}, \frac{3}{5})$, which is not unreasonable, although somewhat arbitrary, given our assumptions and Bayesian reasoning.

A set of actions and conjectures are considered stable if actions do not change through time, that is $a(s_j^i, s_i^j, t_i, t_j | t) = a(s_j^i, s_i^j, t_i, t_j | t + 1), \forall t > t^*, t^* \in \mathbb{Z}^+$. We denote such a situation a *stable equilibrium,* SE.

[3] Since $t_j^i \sim U(T_j^i)$, we can simply use expectation here.

We disregard cases where $t_i = t_j$ because it occurs with probability zero given continuous type spaces. In arguments note that $T = (0,1)$ while $T_j^i \subseteq [0,1]$.

3.2 Implementation

Here we state and prove Theorems 1, 2 and 3. The implications are discussed in section 5.

Theorem 1: Consider Γ with full awareness:

1. *In SE players always coordinate. The players coordinate on (AA) if $t^* < 0.5$, where $t_i = t^*$ if $|t_i - 0.5| > |t_j - 0.5|$, on (BB) if $t^* > 0.5$*
2. *If $sign(t_1 - 0.5) = sign(t_2 - 0.5)$, they coordinate in the first and all following interactions*
3. *If t_i, t_j is simultaneously higher or lower than $0.5 + \sum_{t=1}^{n} (\frac{1}{2})^{t+1} D_t$, where $D_t = \begin{cases} 1 \; if \; s_j^t = B \\ -1 \; if \; s_j^t = A \end{cases}$, they coordinate in that and all following periods. Infinite miscoordination is possible, but probability of coordination approaches one as time approaches infinity.*

In the first interaction $T_j^i = [0,1], t_j^i \sim U(0,1), \forall i \in I$. Therefore $s_j^i = \frac{3-0.5}{5}$ and from the risk-dominance properties of Γ it follows that $E(A) > E(B) \; iff \; t_i < 0.5, \forall i \in I$.

Players understand that $E(A) > E(B) \; iff \; t_i < 0.5$ in the first interaction and update their beliefs accordingly. Therefore, in the second interaction, $T_j^i = [0,0.5], t_j^i \sim U(0,0.5) \; if \; a_j = A$ and similarly $T_j^i = [0.5,1], t_j^i \sim U(0.5,1) \; if \; a_j = B$.

Consider $a_i = a_j = A$ in the first interaction. Then $T_j^i = [0,0.5], t_j^i \sim U(0,0.5), \forall i \in I$ and it follows that $s_j^i > 0.5$ must hold for any future belief refinement. Let us here calculate the critical value for t_i for which player i will play A:

$$E(A) = s_j^i(3 - t_i) - (1 - s_j^i)$$

$$E(B) = (1 - s_j^i)(1 + t_i)$$

$$E(A) > E(B) \rightarrow s_j^i(3 - t_i) - (1 - s_j^i) \; > (1 - s_j^i)(1 + t_i)$$

$$5s_j^i - 2 = t_i^c \quad (3.1)$$

Inserting the minimal possible s_j^i any player may now hold, 0.5, we get: $t_i^c = 0.5$. But since $a_i = a_j = A$ it must be that $t_i < 0.5, \forall i \in I$. Therefore both players always play A from here on. By symmetry we get similar results if $a_i = a_j = B$, both players will play B. This shows proposition 2.

Consider $a_i = A, a_j = B$, so $t_i < 0.5 < t_j$. Now we know that players beliefs are $t_j^i \sim U(0.5,1)$ and $t_i^j \sim U(0,0.5)$. This allows us to calculate strategy conjectures $s_j^i = \frac{2.25}{5}$ and $s_i^j = \frac{2.75}{5}$. Inserting into (3.1):

$$t_i^c = 0.25$$

$$t_j^c = 0.75$$

If $t_i < 0.25, t_j < 0.75$ they coordinate on A, if $t_i > 0.25, t_j > 0.75$ they coordinate on B, miscoordinate otherwise. If they coordinate, it is possible to show with an argument similar to that for proposition 2 that they always coordinate on that action. If not, players again update beliefs. If say, $a_i = B$ in the second interaction so $0.25 < t_i < 0.5$, player *j* understands this, so now $t_i^j \sim U(0.25,0.5)$ which implies $s_i^j = \frac{2.625}{5}$ which in turn implies $t_j^c = 0.625$. We see a pattern arising:

$$t_i^c = 0.5 + \sum_{t=1}^{n} (\tfrac{1}{2})^{t+1} D_t, where\ D_t = \begin{cases} -1\ if\ a_j^t = B \\ 1\ if\ a_j^t = A \end{cases} \quad (3.2)$$

Consider proposition 3. We have shown it true for $t_i^c = 0.5$. Suppose we are in time period *t*, the players have always miscoordinated, $t_i^c = X, t_j^c = Y$ and $a_i = a_j = A$ so $t_i < X, t_j < Y$. Therefore in the next interaction $t_i^c = X - (\frac{1}{2})^{t+1}, t_i^c = Y - (\frac{1}{2})^{t+1}$. We have not specified belief updating in case of coordination, so consider the minimum possible t_i^c. If we can show t_i is less than this we know that *i* will always play A.

$$t_i^c = 5s_j^i - 2 \rightarrow s_j^i = \frac{t_i^c + 2}{5}$$

$$s_j^i = \frac{3 - E(t_j^i)}{5} \rightarrow E(t_j^i) = 3 - 5s_j^i = 1 - t_i^c$$

$$max\ t_j^i = 1 - t_i^c + \frac{1}{2t}$$

$$min\ s_j^i = \frac{3 - 1 + t_i^c - \frac{1}{2t}}{5}$$

$$min\ t_i^c = t_i^c - \frac{1}{2t}$$

Recall $t_i^c = X - (\frac{1}{2})^{t+1} \rightarrow min\ t_i^c = X - (\frac{1}{2})^{t+1} - \frac{1}{2t}$

Where we have used that the width of T_j^i is $\frac{1}{t}$. Note that $(\frac{1}{2})^{t+1} > \frac{1}{2t}, \forall t > 0$, so $min\ t_i^c > X$. Since $t_i < X$ we know that $t_i < min\ t_i^c$ and so *i* will always play A, even at the minimum value of t_i^c. By symmetry we see the same if $a_i = a_j = B$.

Nothing guarantees coordination at any stage, but we see that a priori probability of coordination is 0.5 at each round, so $P(coordination) = 0.5 + (\frac{1}{2})^{t+1}$ meaning that $\lim_{t\rightarrow\infty} P(coordination) = 1$. This concludes proposition 3.

To show proposition 1, consider the fact that $|t_i^c - 0.5| = |t_j^c - 0.5|$ if players have always miscoordinated, which can be confirmed from (3.2). Without loss of generality, say $t_i < 0.5 < t_j$. Players coordinate on A if $t_i < t_i^c$ and $t_j < t_j^c$. Then also $|t_i - 0.5| > |t_i^c - 0.5|$ and $|t_j - 0.5| < |t_j^c - 0.5|$. Since $|t_i^c - 0.5| = |t_j^c - 0.5|$ it follows that $|t_j - 0.5| < |t_i - 0.5|$ if players coordinate on A. The results for coordination on B follows from symmetry.

Theorem 2: Consider Γ *with limited awareness:*

1. *If* $sign(t_1 - 0.5) = sign(t_2 - 0.5)$*, they will coordinate in the first and all following interactions.*
2. *If* $sign(t_1 - 0.5) \neq sign(t_2 - 0.5)$ *they will miscoordinate in the first interaction. Without loss of generality, say* $t_i < 0.5 < t_j$*. In SE the following holds, where* $a = (a_i a_j)$*:*
   If $t_i \in (0,0.25)$ *and* $t_j \in (0.5,0.75)$ *then a=(AA)*
   If $t_i \in (0.25,0.5)$ *and* $t_j \in (0.5,0.75)$ *then a=(AB)*

If $t_i \in (0{,}0.25)$ *and* $t_j \in (0.75{,}1)$ *then a=(BA)*

If $t_i \in (0.25{,}0.5)$ *and* $t_j \in (0.75{,}1)$ *then a=(BB)*

Proposition 1 follows from the proof for proposition 2 in Theorem 1 because $E\left(t_j^i \middle| t_j^i \sim U(0{,}0.5)\right) = E\left(t_j^i \middle| t_j^i \sim U\{0{,}0.5\}\right)$, so the first and second interaction are alike.

Suppose $t_i < 0.5 < t_j$. We know from the proof of Theorem 1 that in the second interaction $t_i^c = 0.25$ and $t_j^c = 0.75$. Therefore, if *j* observes *i* playing A in the second interaction, so $t_i < 0.25$, he reasons that *i* cannot be of type 0.5 and therefore has to be of type 0 and therefore best-respond by always playing A. Similarly if *i* played B, so $t_i > 0.25$, *j* reasons that *i* has to be of type 0.5 and so best-responds by playing B, as $t_j > 0.5$. Because we have assumed irreversibility this play continues infinitely. The remaining statements in proposition 2 follows similarly.

Theorem 3: Consider Γ *with awareness of type spaces as* $T_j^i = \{0{,}0.5{,}1\}$, $T_i^j = [0{,}1]$:

1. *If* $sign(t_1 - 0.5) = sign(t_2 - 0.5)$, *they coordinate in the first and all following interactions.*
2. *If* $sign(t_1 - 0.5) \neq sign(t_2 - 0.5)$ *they miscoordinate in the first interaction. In SE the following holds, where* $a = (a_i a_j)$*:*

   If $t_i < 0.5 < t_j$*,*then *if* $t_j \in (0.5{,}0.75)$ *then a=(AA), and if* $t_j \in (0.75{,}1)$ *then a=(BB)*

   If $t_j < 0.5 < t_i$*,*then *if* $t_j \in (0{,}0.25)$ *then a=(AA), and if* $t_j \in (0.25{,}0.5)$ *then a=(BB)*
3. *Coordination may never be achieved, but the probability of coordination approaches one as time approaches infinity.*

Proposition 1 follows from the first propositions in Theorem 1 and 2.

If they initially miscoordinate, say $t_i < 0.5 < t_j$*,* as in Theorem 2 the limited awareness player bases his play in SE entirely on the first two interactions. Therefore the characterisation of his play follows from proposition 2 of Theorem 2 and in SE $a_i = A \; if \; t_j \in (0.5{,}0.75)$ and $a_i = B \; if \; t_j \in (0.75{,}1)$. If $t_j \in (0.5{,}0.75)$ and *i* has played A in the first two interactions, so $t_i \in (0{,}0.25)$, then *j* holds beliefs $t_i^j \sim U(0{,}0.25)$ which from (3.1) implies $t_j^c = 0.875$ and since $t_j \in (0.5{,}0.75)$ he will play A. Now they always plays A, following the proof of Theorem 1. Similarly if $t_j \in (0.75,1)$ and $t_i \in (0.25,0.5)$, both players always play B from the third interaction and on. If $t_j \in (0.5,0.75)$ and *i* plays B in the second interaction, so $t_i \in (0.25,0.5)$, *j* holds beliefs such that $t_i^j \sim U(0.25,0.5)$ which from (3.2) implies $t_j^c = 0.625$ and so *j* will only play A if his type is below 0.625. Recall that regardless of *j*'s following play, *i* will always play A. From (3.2), this implies that t_j^c tends towards a limiting value, $\lim_{t\to\infty} t_j^c = 0.5 + \sum_{t=1}^{n}(\frac{1}{2})^{t+1} D_t = 0.5 + 0.25 - 0.125 + \sum_{t=3}^{\infty}(\frac{1}{2})^{t+1} = 0.75$. Since $t_j \in (0.5,0.75)$ *j* must play A in any SE. A similar argument can be made for $t_j \in (0.75,1)$ and $t_i \in (0,0.25)$ showing that this leads to that *j* must play B in any SE.

We conclude by noting that if they miscoordinate in the two first interactions, since t_j^c tends towards a limiting value that leads to coordination, the probability of coordination tends 1 as time tends to infinity.

4 Networked Interactions

4.1 Set-up

The population is now $I = \{1,2,\dots,N-1,N\}$, otherwise actions, payoffs and types are as in Γ outlined previously. The interaction structure is governed by the network *g* consisting of *N* *nodes* and a set of edges where $e_{ij} = 1$ if two nodes are linked. Define $N_i(g)$ as the set of players linked to *i* in g. If two nodes are linked they interact once each time period, all interactions occur simultaneously. All players in the game makes a single action choice valid in all interactions and the total payoff to a player is the average payoff $u_i(a_i, a_{-i}, t_i, g) = \frac{\sum_{j\in N_i(g)} e_{ij}\times u_i^{\Gamma}(a_i,a_j,t_i)}{|N_i(g)|}$.

Each player holds a belief vector $T^i = ((T_1^i, t_1^i),\dots,(T_i^i, t_i^i),\dots,(T_N^i, t_N^i))$ and form beliefs about each other player in the same way as in isolated interactions and so ignore network structure impact. We impose that (3.2) holds at all times, even after coordination, an assumption not required in isolated interactions. Players uses beliefs to form strategy conjectures s_j^i as previously and we define $\bar{s}_j^i = \frac{\sum_{j\in N_i(g)} s_j^i}{|N_i(g)|}$.

We consider two particular classes of networks; cliques and stars.

4.2 Cliques

In cliques all nodes are linked to all others, $e_{ij} = 1, \forall i, j \in I, i \neq j$. We will disregard the marginal effect of the play of an individual player[4].

Theorem 4: Consider Γ *played on a clique with limited awareness. Denote the following sets with cardinality a,b,c,d,e,f and g respectively:*

$A: i \in A \; iff \; t_i < 0.5$

$B: i \in B \; iff \; t_i > 0.5$

$C: i \in C \; iff \; t_i < \dfrac{0.75a + 0.25b}{a + b}$

$D: i \in D \; iff \; t_i > \dfrac{0.75a + 0.25b}{a + b}$

$E: i \in E \; iff \; i \in A \; and \; i \in C$

$F: i \in F \; iff \; either \; i \in A \; and \; i \in D \; or \; i \in B \; and \; i \in C$

$G: i \in G \; iff \; i \in B \; and \; i \in D$

1. *In SE,* $a_i = A$ *if* $\frac{e+0.5f}{e+f+g} > t_i$ and B otherwise.

Consider the $\bar{s}_j^i$ that a player holds in the second interaction:

$$\bar{s}_j^i = \frac{\sum_{i=1}^{N} s_j^i}{N} = \frac{\sum_{i=1}^{N} \frac{3 - E(t_j)}{5}}{N} = \frac{a \times \left(\frac{3 - 0.25}{5}\right) + b \times \left(\frac{3 - 0.75}{5}\right)}{a + b} = \frac{2.75a + 2.25b}{5a + 5b}$$

Substituting into (3.1):

$$t_i^c = 5\bar{s}_j^i - 2$$

$$t_i^c = \frac{0.75a + 0.25b}{a + b}$$

If t_i is lower than this, *i* would play A and otherwise B in the second interaction.

[4] Despite the unpleasancy, it does not alter qualitative results.

Consider players' strategy conjectures in the third interaction, recalling $T_j^i = \{0\}, \forall j \in E, T_j^i = \{0.5\}, \forall j \in F, T_j^i = \{1\}, \forall j \in G$:

$$\bar{s}_j^i = \frac{\sum_{i=1}^{N} s_j^i}{N} = \frac{e \times \left(\frac{3}{5}\right) + f \times \left(\frac{3-0.5}{5}\right) + g \times (\frac{2}{5})}{e+f+g} = \frac{3e + 2.5f + 2g}{5(e+f+g)}$$

Substituting in (3.1):

$$t_i^c = 5\bar{s}_j^i - 2$$

$$t_i^c = \frac{e + 0.5f}{e+f+g}$$

This shows proposition 1 of the theorem and note that no one will switch play so this consists a SE.

When awareness is full, network properties become harder to express analytically. Up until the third round play follows as with limited attention. But with full attention the belief updating process continues and note that the number of players playing A (likely) increases from the second to the third round. Given the effect on beliefs, the increase is bound to continue. However, in contrast to models like [25], the process will in most cases not even get close to a single action equilibrium. To see this; suppose a>b and consider a player with a type t_i close to 1. Now consider the maximum $\bar{s}_j^i$ that the player may hold which in the limit has to be $\frac{a \times \left(\frac{3}{5}\right) + b \times (\frac{3-0.5}{5})}{a+b} = \frac{3a+2.5b}{5a+5b}$. If the player should switch to A: $5\bar{s}_j^i - 2 > t_i \rightarrow \frac{3a+2.5b-2a-2b}{a+b} = \frac{1+0.5\frac{b}{a}}{1+\frac{b}{a}} > t_i$. Rearranging, we can calculate which ratio of b and a that is required for the player to switch: $t_i < \frac{1+0.5\frac{b}{a}}{1+\frac{b}{a}} \rightarrow \frac{b}{a} = \frac{1-t_i}{t_i - 0.5}$. For example, we see that the $\frac{a}{b}$ ratio required is 0.02 when $t_i = 0.99$ and 0.11 when $t_i = 0.95$. We conclude that with full awareness, we see a stronger concentration of play of the dominating action despite still not complete coordination, unless essentially all players has similar preferences and there are no players with extreme differing preferences.

When we mix full awareness and limited awareness play will occur as previously in the first three interactions as the limited awareness players play exactly the same as before. The full awareness players continue their belief updating process with a small movement towards the dominating equilibrium, but not as much as with only full awareness.

4.3 Stars

In star networks there is a node linked to all, called the *hub*, player *h*, and a set of nodes which are only linked to the hub, *spokes*. In a star, $e_{ij} = 1 \; if \; i = h, \forall i, j \in I$.

Theorem 5: Consider Γ *played on a star with limited awareness. Denote the following sets with cardinality a, b, c and d respectively:*

$A: i \in A \; iff \; t_i < 0.25, i \neq h$

$B: i \in B \; iff \; t_i \in (0.25,0.5), i \neq h$

$C: i \in C \; iff \; t_i \in (0.5,0.75), i \neq h$

$D: i \in D \; iff \; t_i > 0.75, i \neq h$

1. *If* $t_h < 0.5$ and $t_h < \frac{0.75(a+b)+0.25(c+d)}{a+b+c+d}$*, in SE all spokes play A and the hub plays A if* $t_h < \frac{a+b+0.5c}{a+b+c+d}$ *and B otherwise*
2. *If* $t_h > 0.5$ and $t_h > \frac{0.75(a+b)+0.25(c+d)}{a+b+c+d}$*, in SE all spokes play B and the hub plays B if* $t_h > \frac{a+0.5b}{a+b+c+d}$ *and A otherwise*
3. *If* $t_h < 0.5$ *and* $t_h > \frac{0.75(a+b)+0.25(c+d)}{a+b+c+d}$*, in SE all spokes play their risk-dominant choice and the hub plays A if* $t_h < \frac{a+b+0.5c}{a+b+c+d}$ *and B otherwise*
4. *If* $t_h > 0.5$ *and* $t_h < \frac{0.75(a+b)+0.25(c+d)}{a+b+c+d}$*, in SE all spokes play their risk-dominant choice and the hub plays A if* $t_h < \frac{a+0.5b}{a+b+c+d}$ *and B otherwise*

If the hub starts by repeating an action, all spokes will play this action in SE as the hub is believed to be of type 0 or 1. This occurs whenever $t_h < 0.5$ and $t_h < \frac{0.75(a+b)+0.25(c+d)}{a+b+c+d}$, or $t_h > 0.5$ and $t_h > \frac{0.75(a+b)+0.25(c+d)}{a+b+c+d}$. Now if $t_h < 0.5$ and $t_h < \frac{0.75(a+b)+0.25(c+d)}{a+b+c+d}$, consider which $\bar{s}_j^h$ and critical type value the hub holds for his third, and all following, interactions:

$$\bar{s}_j^h = \frac{\sum_{\forall j \in I} s_j^h}{N} = \frac{(a+b) \times \frac{3-0}{5} + c \times \frac{3-0.5}{5} + d \times \frac{3-1}{5}}{a+b+c+d} = \frac{3(a+b) + 2.5c + 2d}{5(a+b+c+d)}$$

$$t_h^c = 5\bar{s}_j^i - 2 = \frac{a+b+0.5c}{a+b+c+d}$$

The hub plays A in SE if his type is below this value. By symmetry we see the corresponding solution when $\mathrm{t_h} > 0.5$ and $\mathrm{t_h} > \frac{0.75(a+b)+0.25(c+d)}{a+b+c+d}$. When $t_h < 0.5$ and $t_h > \frac{0.75(a+b)+0.25(c+d)}{a+b+c+d}$ or $\mathrm{t_h} > 0.5$ and $\mathrm{t_h} < \frac{0.75(a+b)+0.25(c+d)}{a+b+c+d}$, since spokes believes that the hub is of type 0.5 they will play their risk-dominant choice. The hub form beliefs as previously, as his second action choice does not affect his SE play.

Theorem 6: Consider Γ played on a star with full awareness.

1. *If $t_h < 0.5$ and a>b, all players play A in SE.*
2. *If $t_h > 0.5$ and b>a, all players play B in SE.*
3. *If $t_h > 0.5$ and b<a or $t_h < 0.5$ and a<b, all players with sign($t_i - 0.5) = sign(t_h - 0.5)$ will play the action played by the hub in the first interaction, in all interactions. The play of the remaining players and the hub will depend on the true type distribution.*

If $t_h < 0.5$ and *a>b*, the hub and more than half of the spokes will initially play A. The hub holds $\bar{s}_j^i = \frac{0.75a+0.25b}{a+b} > 0.5$ so plays A again and the hub will now never hold a $\bar{s}_j^i$ below 0.5 as players in A will always play A, and a>b. The hub will play A infinitely which causes beliefs about his type among the spokes to approach zero and so all players plays A in SE. The case for $t_h > 0.5$ and *a<b* follows symmetrically.

If $t_h < 0.5$ and *a<b*, as before the players in A will play A in all interactions, because now their t_i^c will never go below 0.5 and $t_i < 0.5, \forall i \in A$. However, it gets complicated to formally express the play of other players. Generally, suppose without loss of generality $t_h < 0.5$. If hub's type is low it will take a large amount of spokes in B and many rounds to make him switch. Since he plays A in many rounds, the spokes belief about him approaches 0 which makes spokes start to switch from B to A. Unless there are exceptionally many spokes with high types, the hub will always play A and in this case in SE, all spokes will also play A as their belief about the hub's type is 0 in the limit. We see that if the hub never switches, in SE all players must play this action. If he does switch, due to many players in B or his type being close to 0.5, it follows that $t_h^i \in (0.25, 0.5) \forall i$ and so all spokes with $t_i > 0.75$ will play B and some spokes with $t_i \in (0.5, 0.75)$ will also play B.

For mixed limited and full awareness, consider that the hub has limited awareness and the spokes full awareness. The spokes play as previously in the first two interactions, which is the plays that the hub uses in forming final beliefs about the spokes. Therefore the hub plays as in Theorem 5. Since the hub plays the same action from the third interaction on, the belief among spokes regarding his type will approach a boundary value: $\{AAA\} \rightarrow \{0\}, \{AAB\} \rightarrow \{0.25\}, \{ABA\} \rightarrow \{0.25\}, \{ABB\} \rightarrow \{0.5\}$ and so on and the spokes best-response to this value will determine their SE play.

If the hub has full awareness and spokes limited awareness, spokes will behave as in Theorem 5, because the hub plays as before in the first two interactions and the spokes base their play only on these observations. We can therefore calculate $\lim_{t\to\infty} t_h^c$ as was done in Theorem 5.

$$If\ t_h < 0.5\ and\ t_h < \frac{0.75(a+b)+0.25(c+d)}{a+b+c+d}: \lim_{t\to\infty} t_h^c = \frac{a+b+0.5c+0.25d}{a+b+c+d}$$

$$If\ t_h < 0.5\ and\ t_h > \frac{0.75(a+b)+0.25(c+d)}{a+b+c+d}: \lim_{t\to\infty} t_h^c = \frac{a+b+0.25c}{a+b+c+d}$$

$$If\ t_h > 0.5\ and\ t_h < \frac{0.75(a+b)+0.25(c+d)}{a+b+c+d}: \lim_{t\to\infty} t_h^c = \frac{a+0.75b}{a+b+c+d}$$

$$If\ t_h > 0.5\ and\ t_h > \frac{0.75(a+b)+0.25(c+d)}{a+b+c+d}: \lim_{t\to\infty} t_h^c = \frac{0.75a+0.5b}{a+b+c+d}$$

This characterises the play by the hub in SE, A if $t_h < t_h^c$ and B otherwise.

5 Conclusions

5.1 Summary of results

Here we provide interpretations of results produced. Given that we imagine few meetings, time to coordination is important, but also characterisation of SE due to the difficulty in quantifying time in this context.

In isolated interactions, players will coordinate if they share risk-dominant behaviour regardless of attention. Players prone to cooperative behaviour will behave cooperatively versus each other, while players less averse to non-cooperative behaviour will behave non-cooperatively, which are intuitive results. More interesting is matching players with differing preferences. Theorem 1 importantly says that under high attention they will coordinate on the behaviour preferred by the player that is most extremely cooperative or non-cooperative. They eventually coordinate and stick to this coordination, but have a 50% a priori probability of miscoordinating in each meeting, until they coordinate. If these players had low attention, Theorem 2 says that players would choose a definite behaviour in the third meeting and stick to this behaviour. So if one player behaves cooperatively in the third meeting while the other acts non-cooperatively, this situation will persist as they are certain that the other player is an extremely cooperative or non-cooperative type, despite being the opposite.

Interestingly, matching players with differing attention levels, Theorem 3 says that players will eventually behave similarly, and the adopted behaviour depends entirely on how extreme the player with high attention is. The reason is that the low attention player quickly decides how to behave while the more patient player will learn more about the low attention player and eventually adopt the same behaviour.

In fully connected groups we always observe a mix of behaviour. The intuition is straightforward with limited attention, as players are “impatient” and quickly decide how to behave in all remaining interactions. Not as obvious with high attention, this arises from the fact that even though players are “patient”, they stepwise exclude possible types of their opponents which they do not reconsider. Therefore even with high attention we do not see full behavioural convergence. An intuitive reason is the level of uncertainty in conjectures assumed and the high level of connectedness making it difficult to form accurate beliefs. Summarising, we see a larger dominance of the majority-preferred behaviour with high attention, leaving mixed attention as a middle case.

In star networks we will see full convergence to the hub's preferred behaviour, unless many spokes prefer the opposite behaviour and the hub only weakly prefers this behaviour, in which case there will be a mix. With low attention there is also a small chance that if the hub is a cooperative player and there are many non-central extreme non-cooperative players, they will simultaneously "scare" each other into their non-preferred behaviour, so the hub could behave non-cooperatively while all spokes behaves cooperatively and this situation persists. If there instead is high attention, Theorem 6 says that if the hub and the majority of the spokes prefer the same behaviour, all will converge on this although it may take time until other-preferenced spokes is convinced to switch. Even if the majority of spokes and the hub does not share preferred behaviour, it is likely that all coordinate on the hubs preferred behaviour, unless many spokes prefers the opposing behaviour, leading to mixed behaviour.

If the hub has lower attention than spokes, we again see spokes "following" the hub if the hub never switches behaviour. If the hub switched behaviour but then switched back, some spokes will follow this last switch, leading to more coordination than with low attention. If the hub has high attention while spokes has low, comparing the equations on page 24 and Theorem 5 tells us that the risk of complete miscoordination of behaviour is lowered compared to low attention because the hub is more patient. The hub is also less likely to adapt his preferred behaviour if the majority of spokes behaves differently. Summarising, low attention may cause a faster convergence to a single behaviour in case the hub strongly prefers a behaviour, but also creates a risk of complete miscoordination. If the hub is weak in his preference and chooses to switch behaviour, mixed and high attention leads to more coordination than the low attention case. Also, a key insight from stars is that there exists a mechanism by which the hub forces spokes into his behaviour, which is in accordance with literature [10].

5.2 Discussion

The general view our results produce is one where high attention leads to more behavioural coordination as a result of more patient players. With higher attention we may see coordination “corrections” at late stages, whereas with low attention non-reconsiderable decisions are made early. Under mixed levels of attention we generally see a middle case between the high and low attention cases in terms of coordination. However, note that often the benefits of high attention increases with time and with only few interactions these may never be realised. Also, in cases where behaviour initially varies but will converge, low attention can lead to faster coordination convergence, an important consideration as interactions are few. On network structure we see stars exhibiting more coordination than cliques. It could therefore be desirable to organise individuals such that players with strong cooperative preferences get central positions with others more sparsely linked.

Despite analysing Γ as defined earlier, it is clear that the analysis would hold for any coordination game possessing the risk-dominance properties of Γ. The analysis is therefore mainly about coordination of behaviour in general rather than coordinating on a pareto-dominant equilibrium. The specified payoff structure of Γ is also important for the results, stated results rely heavily on the risk-dominance properties of Γ. Reservation must also be made for the very specific behavioural assumptions on beliefs made, which results may be sensitive to. On game specification, we note that if play was allowed to be sequential we could see significant increases in coordination ability. However, consideration on the plausibility of players sequentially picking behaviour in applications must then be made.

An interesting extension of the model would be to use a network-strategic element as in [15] to see whether stars, that were socially preferred to cliques, would endogenously form. Natural next steps in developing the behavioural assumptions of the model would be allowance for reconsideration of beliefs, allowing forward induction similar to [19] or allowing stochastic perturbations [18,25]. These alterations could well be successful in breaking unreasonable miscoordinations. An interesting idea close to reality which concerns stars would be to merge our model with an adaptive learning model. We could let spokes sophistically reason according to our model and let the hub play adaptively motivated by relative difficulty of computation given number of interactions. Finally, deeper consideration of computational costs is a promising extension; we could imagine explicitly modelling this cost allowing us to endogenise attention. Another similar approach would be to allow for “experts” or “automatas” [20].

Word count: 4998

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