

Optimal External Debt and Default in an Economy with a Natural Resource Endowment

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Abstract

This paper studies the optimal external debt contract for a small, natural resource endowed economy whose size in international markets implies exogenous terms of trade. I derive a normative benchmark for the optimal change in debt in response to shocks in the terms of trade. Applying the benchmark to Heavily Indebted Poor Countries whose economic features closely match those of the model, historical terms of trade data indicate that shocks since 1980 can explain, on average, 35% of actual debt relief. The findings suggest that debt contracts should be indexed to the terms of trade.

Keywords: development finance, natural resources, terms of trade, optimal debt relief

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In Malawi, tobacco accounts for over 60% of exports by value but the economy produces only 5% of world output. These export revenues are a vital source of foreign exchange used to purchase imports of capital and processed goods and repay external debt. The economy's small size on world markets implies it takes the market price of tobacco as given and its export revenues are subject to whatever whimsical movements the price takes, which can sometimes be detrimental: from 1989 to 1999, tobacco prices fell 30% in dollar terms (International Monetary Fund 2001, Poulton et al. 2007).

Malawi is not the only small country to rely heavily on natural resources for foreign exchange earnings to buy capital and processed goods from abroad. Over 50% of the annual variation in a typical developing country's terms of trade can be explained by three or fewer commodity price movements (Bidarkota & Crucini 2000). And since almost all small, poor countries are unable to influence the price of either their exports or imports on international markets they must treat their terms of trade as de facto exogenous. In combination, these features make them particularly susceptible to shocks in export and import prices that hinder their ability to import capital goods and service debt simultaneously. During the 1970s, many Sub-Saharan countries took advantage of rising commodity prices to expand import-intensive industries, thereby increasing future economic dependence on their ability to afford imports.

The downside of this exposure is the risk of default caused by a decline in export earnings. In 2001, the International Monetary Fund (IMF) identified the principal source of debt indicator worsening for fifteen small, poor countries to be a deterioration in the terms of trade.¹ In the academic literature, Catao & Sutton (2002) and Catao & Kapur (2006) find terms of trade volatility to be a key predictor of default

¹In 2001, the world price of coffee fell by 35%; cotton 19%; cashew nuts 69%; fish 21% and copper 13%, International Monetary Fund (2002). See ITFCRM (1999, Box 1, pg. 2) for a brief analysis of the importance of commodity price fluctuations on export earnings and therefore in debt indicators and countries' ability to adhere to debt sustainability targets.

in a hazard model of sovereign borrowing.²

In addition to relying heavily on natural resources, Malawi is classified by the World Bank as a Heavily Indebted Poor Country (HIPC), because of its high debt to export and debt to budget revenue ratios. Members of this group of countries have their debt sustainability analysed and, if they are found to need it, receive debt relief faster than those in traditional relief schemes. Under the original and enhanced HIPC initiatives Malawi received US\$659m of debt relief which, in combination with US\$140m from other debt relief sources, equates to 55% of the 1999 net present value debt stock. In part, this may be viewed as compensation for declining terms of trade that lowered export revenues sufficiently to place them in a position where their accumulated debt was unsustainable (Jubilee Research 2007).

Declining terms of trade are one of several factors, including rising interest rates and the decline in real net capital inflows, that the literature has suggested contribute to small, open economies' unsustainable debt positions, but comparatively little work has been done to analyse which of these factors are more important than others (Greene 1989). Exceptions are Senhadji (2003), who compares the effect of terms of trade and interest rate shocks on the steady state debt/GDP ratio, and Guimaraes (2008) who contrasts debt relief under the Brady Plan to that under an optimal contract in response to interest rate and productivity shocks.

The first contribution of this paper is to model explicitly these small, open economies' reliance on export revenues and derive the optimal debt contract that they should hold in the presence of stochastic terms of trade. The existence of projects where the marginal product of capital is higher than the risk-free world interest rate means the economy will always try to borrow as much as possible.³ Then following the literature begun by Eaton & Gersovitz (1981) I assume there is a cost to default that ensures the borrower is willing to repay up to a certain amount, and derive

²See also Hilscher & Nosbusch (2007), who find that terms of trade and its volatility has a statistically and economically significant effect on sovereign credit spreads.

³Identical results can be derived in a model of an endowment economy with an impatient government.

the optimal contract, which in these economies ensures the borrower always repays the maximum incentive compatible amount. I derive from the model a normative benchmark for the amount of debt relief that should be received in compensation for a range of adverse terms of trade shocks, and show that the greater magnitude of terms of trade shocks means that they are typically more important than shocks to total factor productivity.

In the literature on sovereign debt policy there is a debate whether the magnitude of debt relief awarded under the HIPC initiative was appropriate. Some authors argue that declining terms of trade over the period of the HIPC programme necessitates even more debt relief to sustain current debt levels (Hussain & Gunter 2005). On the other hand, Easterly (2002) shows that terms of trade growth is not significantly different between HIPCs and other low-income countries that did not accumulate such high levels of debt, and concludes that the shocks suffered by HIPCs are insufficient to justify the amount of relief they already received. To resolve this debate requires a model-based evaluation of the impact of these shocks on the economies' ability to repay and the relief required to ensure they retain sustainability.

The second contribution of this paper is to use the optimal debt contract from the model to estimate the amount of debt relief that HIPCs should have expected to receive given the terms of trade shocks they experienced. Since the debt contracts had no explicit clauses to account for exogenous movements in the terms of trade, this evaluation is essentially computing the proportion of debt relief that might be excused in the sense of Grossman & Van Huyck (1988).

In keeping with the real world, the agents are assumed to learn something about the terms of trade process of their country with each realisation. The initially high values of the 1970s cause them to misperceive the long run nature of the process, which leads to overaccumulation of debt. Successive adverse shocks during the 1980s result in debt relief for each unexpected shock and additionally lowers agents' expectation on the long run path of the terms of trade. I estimate the latter separately

and compare it to the combined estimate to show that most of the debt relief can be explained by the changes in agents' expectations about the long run path. With these reasonable expectations on the path of the terms of trade, the cumulative effect of shocks since 1980 can justify 35% of the total debt relief received by the HIPC's, and in some cases explain up to 60%, depending on the country.

This result appears to support Easterly (2002), and certainly makes it difficult to justify the call for additional relief by Hussain & Gunter (2005), however in combination with other unpredictable and adverse shocks, for example conflict, interest rate, environmental or political shocks, it remains a possibility that the accumulated optimal relief is little different from that which was received.

This exercise also emphasises the importance of terms of trade shocks to debt sustainability - the model predicts on average around 20%, and up to one-third of debt would have been relieved under the optimal contract. This result is strong justification for contingent debt contracts in countries where the terms of trade are exogenous.

The paper is set out as follows: Section 1 contains the structure of the model and Section 2 shows the optimal paths of debt in economies with deterministic or stochastic processes for the terms of trade. In Section 3, I apply the normative benchmarks to the HIPC data and discuss the results; Section 4 concludes.

1 Small, open economy with a natural resource

In this Section, I introduce a model economy that earns foreign exchange from the sale of natural resources on international markets. A central planner is given control of a small, open economy in discrete time but is without debt-repayment commitment technology. I will use this structure in both the deterministic and stochastic models of subsequent Sections.

I assume the economy is inhabited by a continuum of utility maximising agents

whose preferences may be aggregated into those of a single, infinitely-lived, representative agent with lifetime utility:

$$\sum_{t=s}^{\infty} \beta^{t-s} u(c_t) \quad (1)$$

where c_t is consumption in period t , $u(\cdot)$ is the agent's utility from consumption and $\beta \in (0, 1)$ is the discount factor. The $u(\cdot)$ function is increasing and concave.

The natural resource, hereafter termed *X-output*, is assumed to be a fixed endowment, X , in each period. Then $p_t X$ represents the sale on world markets of the period t output flow from, say, an existing diamond mine or cashew farm, where the economy's small size means that the world price, p_t , is taken as given.⁴ Should the economy default on its debt, the value of X-output on the world market permanently falls by a constant percentage, γ :

$$\text{Natural resource value on world market} = \begin{cases} p_t X & \text{No default} \\ (1 - \gamma)p_t X & \text{Default} \end{cases}$$

This assumption is empirically motivated by the observations of Borensztein & Panizza (2006) and Rose (2005) who explore the effect of sovereign default on trade. Since this paper's focus is on evaluating the normative debt contract, I take the trade decline as exogenous as in Bulow & Rogoff (1989a). For a survey of the theoretical attempts to explain the decline, see Sturzenegger & Zettelmeyer (2007).

What is crucial to the results of this paper is that there be some fall in the value of trade in the event of default. The assumed permanency of the punishment for default is more innocuous than it appears. What is required is *some* fall in the value of X-output, but the timing of that fall is not important. There exist a multitude of equivalent threats that can yield the same result depending on the timing of the

⁴Terms of trade exogeneity is also assumed in the models of Broda (2004) and Hilscher & Nosbusch (2007).

imposed cost and appropriate discounting (Mohr 1991).

In addition to its natural resource, the economy is endowed with the technology to produce another output, hereafter termed *Y-output*. This represents the non-natural resource side of the economy and takes the same linear form as in Cohen & Sachs (1986). Y-output is produced using a single input, capital, either borrowed from abroad or converted from the previous period's Y-output. Capital depreciates at rate δ . In addition to capital conversion, Y-output may be used for consumption or repayment of debt, but any remainder must be destroyed, as there is no storage technology. Denoting capital in period t by k_t , Y-output is given by:

$$Y_t = Ak_t \tag{2}$$

where A represents economic productivity. For the countries analysed in this paper, and the period concerned, it is reasonable to assume that they want to borrow as much as possible in every state of the world. This functional form is a convenient way of obtaining that assumption, and will also help to derive the closed form solutions.⁵ To that end I will assume the world interest rate $r < A - \delta$ so that the central planner wishes to borrow from abroad to invest at an infinite rate.

Explicitly modeling the natural resource, and its source as a cost of default to the economy is a key distinction from the models of Cohen & Sachs (1986), Wright (2002), Arellano (forthcoming) and others who assume a percentage decline in a country's entire economic output, in a reduced-form representation of the output costs of default.⁶ One interpretation of the above default cost is that of a loss in gains from trade and making that type of cost increasing in total output is implicitly assuming that the gains from trade must increase with total output. Whilst this may

⁵Alternatively, one could use a more general production function and bound the support of the stochastic variable, but this is less tractable and not necessary given the focus of the paper.

⁶Allowing the cost to also fall on Y-output would not change the qualitative results I derive in either the deterministic and stochastic sections, but would change the magnitude and path for debt.

be true of some economies, it is not so for those where higher income leads to less reliance on foreign trade (Eaton 1993).

It is reasonable to ask why capital is not also an input to the production process that yields the endowment of the natural resource. One interpretation is that any major capital investment was a ‘sunk cost’ decision made sometime in the past which leads to the interpretation of the endowment as a flow from an existing facility. Alternatively, the absence of flow capital may be interpreted as an economy following a diversification strategy as propounded by the OECD and the World Bank that calls for investment in other sectors of resource rich economies (Bonaglia & Fukasaku 2006). Finally, and purely from a modelling standpoint, capital is not essential to the results of this paper: an endowment economy with an impatient government will yield identical conclusions.

The financial aspects of this model begin with a large number of foreign creditors that are risk neutral and behave competitively. They seek to maximise profits and will only refuse contracts that yield strictly less than zero profits in expectation. These assumptions result in a price for debt in period t , q_t , that yields one unit of capital tomorrow given by:

$$q_t = \frac{1 - \xi_t}{1 + r} \quad (3)$$

where r is the international ‘risk-free’ interest rate, assumed constant for simplicity, and ξ_t is the time-varying, sovereign-specific probability of default. Throughout this paper, q_t is endogenous.

I assume a second cost of default to be permanent loss of access to capital markets. This cost is an important out-of-equilibrium threat that eases comparison between debt levels in default and repayment. I do not deem permanency to be more realistic than the usual assumption of temporary punishment taken by most of the literature, and which has considerable empirical support (Gelos et al. 2004, Lindert & Morton 1989). However, I view it as a tractable form of the enforcement mechanism in Wright

(2002) where financial relationships following a default are such that the economy is no better off than if it were in permanent exclusion.

The combined loss of export value and credit market access are summarised in the resource constraint that in the absence of default takes the form:

$$c_t + k_{t+1} = Ak_t + (1 - \delta)k_t + q_t d_{t+1} - d_t + p_t X \quad (4)$$

But should the economy default on its debt, it becomes:

$$c_t + k_{t+1} = Ak_t + (1 - \delta)k_t + (1 - \gamma)p_t X \quad (5)$$

By writing everything in terms of the output good, I implicitly make p_t the terms of trade in the economy.

In this paper, I am concerned with the optimal debt relief for the aggregate economy. I assume the central planner is benevolent and maximises the representative agent's utility subject to the economy's resource constraint. Then making use of the time invariant nature of the central planner's optimisation problem (denoting by a prime the future value of a variable) we obtain the value function when it has defaulted:

$$V_{def}(k, \gamma) = \max_{k'} \{u(Ak + (1 - \delta)k - k' + (1 - \gamma)pX) + \beta V_{def}(k', \gamma)\} \quad (6)$$

And when it has not defaulted:

$$V_{pay}(k, d) = \max_{k', d'} \{u(Ak + (1 - \delta)k - k' + qd' - d + pX) + \beta V(k', d', \gamma)\}$$

$$\text{where } V(k, d, \gamma) = \max \{V_{pay}(k, d); V_{def}(k, \gamma)\} \quad (7)$$

2 Optimal path of debt

2.1 *Deterministic case*

The objective in this Section is to derive the optimal path of debt in this model where the central planner is always on the boundary of the ‘no default condition’, i.e. where the value of repaying debt is weakly greater than the value of defaulting, and so the central planner never chooses to default in equilibrium:

$$V_{pay}(k, d) \geq V_{def}(k, \gamma) \quad (8)$$

We are particularly interested in the resultant optimal path when the terms of trade are unpredictable, but begin with the deterministic case to develop some intuition and suppose $p_t = \bar{p} \quad \forall t$.

First, since everything in a deterministic world is known with certainty, the outcome of the central planner’s decision to default or not will also be known. If it is known that they will default ($\xi = 1$) no lending will occur ($q = 0$), and should they subsequently default the economy loses export value. Comparing the default case of Equation 5 to Equation 4 with zero debt shows that the country would prefer not to default, since it would then avoid the export value loss. Note also that, since no default occurs in equilibrium ($\xi = 0$), the price of debt is always q^* i.e. the risk-free price from $r = r^*$; $\xi = 0$

Now, as discussed in the previous Section, because the marginal product of capital is always greater than the world interest rate, $A - \delta > r^*$, it is optimal for the central planner to borrow and invest as much as possible today. Then, as $V_{pay}(k, d)$ is decreasing in the amount of borrowing, the optimal level of debt received by the central planner will be $d' = d^{max}$ such that the no default condition binds:

$$V_{pay}(k', d^{max}) = V_{def}(k', \gamma) \quad (9)$$

Proposition 2.1.1 *Setting*

$$d' = \frac{1}{q^*}(d - \gamma\bar{p}X) \quad (10)$$

in every period yields the optimal path for debt.

Proof Since the central planner always wishes to borrow as much as possible, any borrowing constraint always binds. Substituting the equation above into the budget constraint of the central planner that chooses to repay reduces the problem to:

$$\begin{aligned} & \max_{\{c_t\}_{t=s}^{\infty}} \sum_{t=s}^{\infty} \beta^{t-s} u(c_t) \\ & \text{subject to: } c_t + k_{t+1} = Ak_t + (1 - \delta)k_t + (1 - \gamma)\bar{p}X \end{aligned} \quad (11)$$

which is identical to the problem of the central planner in a default economy. Thus their choices of consumption paths will be identical, as will the value functions corresponding to their maximisation problems, ensuring the no default condition binds.

Observe that Equation 10 is not a function of k_t so the investment decision is *not* affected by the substitution. In Guimaraes (2008), the substitution was a function of k_t which required γ be small enough that when γ were larger than zero setting the level of debt to be a function of k_t would not greatly affect the investment optimisation decision (in the previous period). Here the optimal path is independent of the magnitude of γ .

To find the optimal level of debt in this economy, I will follow Cohen & Sachs (1986) and impose a zero profit condition on the sequence of loans, so that the sum of the stream of repayments, denoted by $\{\Phi_s\}_{s=t}^{\infty}$, equals the initial debt:

$$\sum_{i=t}^{\infty} q^{*t-i} \Phi_i = d_t \quad (12)$$

Now, since $\Phi_t \equiv d_t - q^*d_{t+1}$, we may solve forwards and take limits to find a transversality condition on the level of debt, which says it grows at a rate slower than

r^* :

$$\lim_{s \rightarrow \infty} \{q^{*s} d_s\} = d_t - \sum_{i=t}^{\infty} q^{*t-i} \Phi_i = 0 \quad (13)$$

Taking Equation 10, solving forwards and imposing this transversality condition yields the optimal level of debt for a deterministic economy, which I will denote by \bar{d} :

$$\begin{aligned} d &= q^* d' + \gamma \bar{p} X \\ &= \lim_{s \rightarrow \infty} q^{*s} d_s + \gamma \bar{p} X (1 + q^* + q^{*2} + \dots) = \frac{\gamma \bar{p} X}{1 - q^*} \equiv \bar{d} \end{aligned} \quad (14)$$

Substituting this expression back into Equation 10 shows that the optimal path of debt in this economy will be constant.

$$d' = \frac{1}{q^*} \left[\frac{-\gamma \bar{p} X}{1 - q^*} - \gamma \bar{p} X \right] = \frac{\gamma \bar{p} X}{1 - q^*} = \bar{d} \quad (15)$$

Intuitively, this makes sense as the constraint will bind in every period, and it is neither unpredictable nor changing as the economy develops. Were the natural resource endowment to grow as the economy moved toward the steady state, then we would observe an increase in the incentive compatible level of debt.

Equation 15 shows that an increase in either the percentage cost of default, the terms of trade or the endowment of the natural resource all lead to higher debt as they all increase the cost to the economy from repudiating the debt. In contrast, an increase in the interest rate leads to a fall in the level of debt because it raises the cost of repayment and gives greater incentive to default.

In addition Equation 15 shows that the equilibrium level of debt is independent of the level of capital. This result is due to the simplified nature of the default cost, made for tractability, which only impacts on the value of the natural resource on international markets. The model thus predicts ‘trade neutral’ output growth: an increase in income has no effect on the level of trade and therefore no effect on the cost of default. The empirical actuality of trade neutrality/non-neutrality is economy-specific; but this is not an implausible result (Gersovitz 1983).

2.2 *Stochastic case*

The previous sub-section considered an economy without uncertainty, but to study the optimal response to terms of trade shocks to an economy that cannot affect world prices, we need that the terms of trade be a stochastic variable - as discussed in the introduction, market prices for most commodities are volatile and this leads to unpredictable changes in these economies' terms of trade. In this Section, I derive straightforward formulae to approximate the change in debt required to ensure that, following the realisation of a shock, the economy remains on the optimal path of debt.

Treating the time-series as a first order autoregressive (AR(1)) and mean-reverting process is a simplification that captures the main stochastic features observed in the data, and makes the analysis tractable. Other papers that have made a similar assumption include Mendoza (1995) and Cashin et al. (2000). An implicit assumption in this form of the process is the existence of a level for the terms of trade to which the economy tends over time. I assume that this level is that which corresponds to the deterministic path of debt derived in Section 2.1 i.e. \bar{p} , so the process may be expressed as:

$$p_{t+1}^i - \bar{p} = \phi(p_t^i - \bar{p}) + \epsilon_{t+1} \quad E_t[\epsilon_s] = 0 \quad \forall s > t \quad (16)$$

where \bar{p} is the level to which the terms of trade reverts over time. In the following proposition, the distinction between p^i and p^j is an exogenous shock to the world price of the export commodity.

Proposition 2.2.1 *When p follows the AR(1) process described above, $V_{pay}(k, d^i, p^i) = V_{def}(k, \gamma, p^i)$ and $V_{pay}(k, d^j, p^j) = V_{def}(k, \gamma, p^j)$ provided:*

$$\frac{d^i - d^j}{\bar{d}} = \frac{1 - q^*}{1 - q^* \phi} \frac{p^i - p^j}{\bar{p}} \quad (17)$$

Proof See Appendix A.

The proof follows the intuition and logic of the deterministic case, accounting in addition for the expectation that the effect of the shock will disappear over the infinite horizon.

Equation 17 is the optimal change in debt required to ensure that the debtor retains the incentive to repay, whilst allowing the creditor to extract the maximum possible repayment. Holding constant the magnitude of the shock to the terms of trade, stronger persistence leads to more debt relief due to the greater length of time that the economy is expecting to be in deviation from \bar{p} . Similarly, a larger discount factor implies greater importance of the future to the central planner and today's ideal debt relief is therefore greater. A higher steady state interest rate implies the *relative* amount of debt relief rises since, though the costs of default have not changed, a higher interest rate makes default more attractive. The magnitude of the unknown cost of default, γ , does not enter Equation 17 because the optimal change in debt is considered relative to the steady state level, and both fluctuate proportionately with γ .

To get a sense of the magnitudes of debt relief implied by Equation 17, Table 2.2 presents the values for ranges of persistence and changes in the terms of trade. The empirical literature finds that terms of trade shocks to developed and more advanced developing economies are typically large and persistent. Mendoza (1995) finds that out of seven industrialised countries and twenty-three developing countries there is just a single economy with an AR(1) coefficient of persistence below 0.21.⁷ See also Backus et al. (1992, Table 4.1) for summary statistics showing high autocorrelation coefficients and standard deviations in Japan, UK and US data.

Investigations that consider a larger set of developing countries, however, typically find that these countries do not suffer the same scale of shocks and degree of persis-

⁷The country is Argentina; the coefficient is -0.034 and the reason is unknown to me.

Table 1: Estimated debt relief for range of terms of trade shocks^{1,2}

		Persistence (ϕ)					
		0.45	0.55	0.65	0.75	0.85	0.95
$\Delta p/\bar{p}$	25%	2.2	2.6	3.3	4.3	6.5	12.8
	50%	4.4	5.2	6.5	8.7	13.0	25.6
	75%	6.6	7.9	9.8	13.0	19.5	38.5
	100%	8.7	10.5	13.1	17.4	26.0	51.3

¹ $\beta = q^* = 0.95$ (Average over period)

²All values in the table are percentage points.

tence. Cashin & Pattillo (2000) find that the persistence of shocks in sub-Saharan African countries varies more widely than that reported by Mendoza (1995).⁸ In my own analysis detailed in the next section, I find results for the magnitudes and persistence of shocks that are similar to those of Cashin & Pattillo (2000) and therefore in Table 2.2 I set the range for the change in the terms of trade to be 25-100% and the range for persistence from 0.45 to 0.95

This Table shows that, although Equation 17 is similar to that derived by Guimaraes (2008) for debt relief in the event of productivity shocks, in contrast to his conclusion that productivity shocks are of minor import, terms of trade shocks are non-trivial, because the magnitude of fluctuations in the terms of trade is typically much greater than those of productivity. It is thus unsurprising that adverse terms of trade movements are cited as justification for debt relief, such as Paris Club etc, however the extent to which historic relief may be explained by terms of trade movements remains an open question that is addressed in the next section.

⁸At one end of the spectrum: Zimbabwe is found to have a half-life for its terms of trade shocks of 0.4 years; and at the other: shocks to the terms of trade for eleven countries are found to be permanent.

3 Comparing HIPC debt relief to estimates

The HIPC initiative began in 1996 as the first programme that awarded relief for outstanding multilateral debt and considered sustainability when deciding the level of relief to award. It was adapted in 1999, and renamed enhanced HIPC, to provide ‘deeper, faster and wider relief’. From the outset, the initiative acknowledged the importance of export earnings to ensuring countries’ ability to repay by making one of the two debt sustainability criteria the ratio of present value of debt to exports of goods and non-factor services. Countries were first given relief through the Paris Club, after which their debt sustainability was analysed and a decision on the amount of debt relief they would receive under HIPC was made. This was partially awarded during an ‘interim period’ following which the remainder was awarded and debt sustainability was recalculated. In the event that debt was still found unsustainable, due to exogenous causes, further ‘topping-up’ relief could be awarded. (Gueye et al. 2007)

The goal of this section is to determine the proportion of debt relief that can be explained by shocks in the terms of trade since 1980. To this end, I compare the estimates of debt relief prescribed by the model to the actual debt relief received by HIPCs. We should expect those that rely more heavily on their export revenues to be more susceptible to movements in the terms of trade. Table 3 lists the 24 countries and their share of exports in GDP. The third column records the year in which the enhanced HIPC decision point was reached.

For the purpose of comparison, what matters is the countries’ entire relief over the period - Paris Club, HIPC relief, and other sources - but not topping-up relief which was due to exogenous changes after the decision point was reached, and which should not be affected by terms of trade movements before the decision point. I use data from Jubilee Research (2007), and the enhanced HIPC decision point when computing actual debt relief. Countries that are being considered for HIPC relief but

have not reached decision point status are not included in the analysis.

I obtained the net barter terms of trade for 1960-1996⁹ from Cashin & Pattillo (2000), Cashin et al. (2004) and updated it to 2003 using the World Development Indicators (World Bank 2006). I compute debt relief from 1980 until the HIPC decision point, as the 1980's are cited as the time when terms of trade shocks hit worst - a steady improvement in commodity prices during the 1970's led to increasingly positive terms of trade forecasts for the coming decade that encouraged sovereign borrowing. For most countries, the unexpected decline continued into the late 1990's when the magnitude of debt relief under HIPC was under discussion (Hadass & Williamson 2001).

To implement the debt relief estimation requires an estimate of what the agents thought the future terms of trade would be. I assume they forecast using the simple AR(1) in Section 2.2, and estimate the future deterministic path of the terms of trade as the mean of previous years' data. Since I begin calculating debt relief from 1980, agents have ten years of data with which to compute an initial estimate of the level around which they expect the terms of trade to fluctuate.

I view this as an agnostic procedure similar in spirit to that taken by Collier & Goderis (2007) to predict future commodity price levels. Whilst it is true that agents' expectations of a secular decline in their terms of trade cannot be ruled out, the great uncertainty in the *academic* literature in the 1980's-90's concerning the existence of a time trend, stochastic trend or structural breaks in the long-term series suggests that assuming the agents followed an agnostic approach is reasonable (Spraos 1980, Sapsford 1985, Cuddington & Urzua 1989). This assumption may even be deemed conservative compared to an estimate based on commodity price forecasts. See Figures 4 & 5 in Deaton (1999) for a dramatic demonstration of how the prices of copper and cotton were forecast to grow.

A first normative estimate of debt relief can readily be derived by considering the

⁹For most of my countries, the sample is limited to 1970-1996.

Table 2: Heavily Indebted Poor Countries' export dependence, decision point, and data sample

Countries	Exports/GDP	HIPC D.P.	Data sample
Benin	16.4%	2000	1970 - 2000
Bolivia	21.7%	2000	n/a
Burkina Faso	10.3%	2000	1970 - 2000
Cameroon	23.3%	2000	1970 - 2000
Chad	15.2%	2001	n/a
Congo, DR	22.3%	2003	1970 - 2003
Ethiopia	7.7%	2001	n/a
Gambia	50.1%	2000	n/a
Ghana	19.6%	2002	1970 - 2002
Guinea	26.6%	2000	1970 - 2000
Guinea Bissau	12.5%	2000	n/a
Guyana	81.3%	2000	n/a
Honduras	34.4%	2000	n/a
Madagascar	17.5%	2000	n/a
Mali	18.7%	2000	1970 - 2000
Malawi	24.5%	2000	1970 - 2000
Mauritania	42.5%	2000	1970 - 2000
Mozambique	10.1%	2000	1970 - 2000
Nicaragua	20.0%	2000	n/a
Niger	18.6%	2000	1970 - 2000
Rwanda	8.2%	2000	1970 - 2000
Sao Tome & Principe	n/a	2000	n/a
Senegal	27.0%	2000	1970 - 2000
Sierra Leone	20.0%	2002	n/a
Tanzania	16.4%	2000	n/a
Uganda	10.7%	2000	1970 - 2000
Zambia	33.3%	2000	1970 - 2000

Sources: World Bank WDI (\$US), 1980-2000; Jubilee Research (2008);

HIPC decision point documents

change in long run incentive compatible debt levels, caused by agents' imperception of the long run deterministic path of the terms of trade. The deceptive rise in the 1970's caused the formation of high expectations on the future path of the terms of trade. Their estimate of the long run incentive compatible level of debt is given by Equation 15:

$$E_{1980}[\bar{d}] = \frac{\gamma E_{1980}[\bar{p}]X}{1 - q} \quad (18)$$

We can compare this to agents' estimate of the long run incentive compatible level of debt following their realisation that the boom of the 1970's was a temporary phenomenon using $E_{hipc}[\bar{d}]$ where *hipc* is the decision point year, and compute debt relief as the percentage change. Intuitively, this estimate of debt relief captures the readjustment to incentive compatible debt for agents that previously contracted long term debt that was expected to be incentive compatible based on fluctuations around a mean that was higher than what actually transpired. Ceteris paribus this estimate of debt relief moves one for one with the change in agents' estimates of \bar{p} :

$$\frac{E_{hipc}[\bar{d}] - E_{1980}[\bar{d}]}{E_{1980}[\bar{d}]} = \frac{E_{hipc}[\bar{p}] - E_{1980}[\bar{p}]}{E_{1980}[\bar{p}]} \quad (19)$$

The fourth column of Table 3 shows the relief estimates for this calculation; the second column is the net present value (NPV), in 1999 terms, of HIPC relief, taken from decision point documents. The third column is the sum of NPV HIPC relief and additional relief from other sources; and the final column shows once more the ratio of exports to GDP. For the countries whose exports form a small part of their economy, the model performs poorly. Rwanda has the lowest ratio of exports to GDP, and the model predicts negative debt relief. This is not entirely surprising, given that these countries do not match the model's assumption of natural resources as an important revenue source for debt repayment. I consider these outliers, and set an arbitrary cutoff level for the ratio of exports to GDP at 10%, removing three observations from

Table 3: Debt relief from revised deterministic terms of trade path expectations

Country	HIPC Relief	Total Relief	Relief Estimates ^{1,2}	Exports/GDP
Benin	28%	51%	16%	16%
Burkina Faso	64%	65%	14%	10%
Cameroon	21%	77%	20%	23%
Congo, DR	73%	89%	23%	22%
Ghana	38%	57%	24%	20%
Guinea	24%	63%	23%	27%
Malawi	44%	55%	13%	24%
Mali	40%	50%	9%	19%
Mauritania	40%	68%	22%	43%
Mozambique	82%	85%	21%	10%
Niger	61%	78%	32%	19%
Rwanda	66%	76%	-(11)%	8%
Senegal	18%	33%	14%	27%
Uganda	51%	66%	22%	11%
Zambia	51%	83%	41%	33%

Source: Jubilee Research (2008)

¹ -() indicates an optimal increase in debt

² $\beta = q^* =$ US real interest rate over the period

Figure 3 which plots the estimate of debt relief against actual relief, where the size of the bubbles reflect the economic reliance on exports.

This estimate of debt relief ignores individual annual shocks, however, and a better one would include both long term movements in the expected deterministic path and short term shocks away from that path. In particular, one can apply Equation 17 annually and reestimate the expected value of \bar{p} with each realisation of the terms of trade. In addition, agents require an estimate of the persistence (ϕ) of shocks to the series. Unfortunately, ten observations is not enough to obtain accurate estimates of persistence¹⁰, and instead I show that whether they use actual persistence of the

¹⁰The estimates vary widely as each observation is added, and only stabilise once 20 observations are

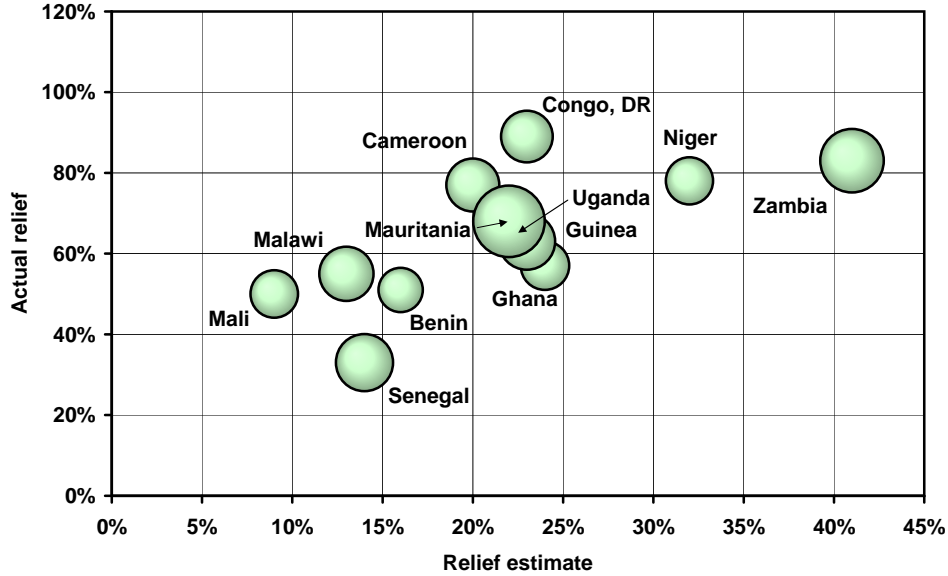


Fig. 1: Comparing estimated long run debt relief to actual relief
Countries have export/GDP>0.1

series around the mean or the average persistence for all countries ($\phi = 0.8$) results in similar estimates of relief.¹¹ I then compute the shock from the formula:

$$\epsilon_{t+1} = p_{t+1} - \hat{p}_{t+1} = p_{t+1} - \phi p_t - (1 - \phi)E[\bar{p}] \quad (20)$$

Equation 17 allows us to combine annual shocks with long run movements in the level of incentive compatible debt because the denominator can be defined as the initial estimate of \bar{p} and held constant throughout the exercise so that, by summing the annual change in debt relative to $E_{1980}[\bar{p}]$, we obtain a normative estimate of the amount of debt relief that would have been accumulated since 1980 until the HIPC

reached. This is consistent with guidelines for the number of observations required to obtain decent empirical estimates.

¹¹For some cases the individual country estimates of persistence in Cashin & Pattillo (2000) are lower than here, because they include a trend in their underlying process. As argued above, there is no reason to believe agents can predict the trend of the series.

decision point.¹²

$$\sum_{t=1981}^{hipc} \frac{1-q}{1-q\phi} \frac{\epsilon_t}{E_{1980}[\bar{p}]} \quad (21)$$

Table 3 shows the actual debt relief together with the estimates in a format similar to Table 3. The fourth and fifth columns contain the relief prescribed by the model under different expectations of the persistence of shocks. Comparing this to Table 3 shows the similarity in estimates, indicating that most of the debt relief in the combined calculation can be attributed to the fall in the expected long term deterministic path of the terms of trade. This is not altogether surprising since it affects the ability of the country to repay debt for all future periods, not just those in temporal proximity. Figure 3 plots the relief estimates against the actual relief for the same set of countries as Figure 3.

In general, the size of the bubbles is larger for points on the right hand side of the chart, indicating that as reliance on exports increases, there is an improvement in the model's ability to explain debt relief.¹³ According to the model, across the set of countries, terms of trade on average account for 35% of the total debt relief received by these countries, and virtually all of that is attributable to the adjustment of expectations on the long run path of the terms of trade. The model suggests that accurately predicting the long run trends of commodity prices is of paramount importance to these economies.

The Tables show that the level of debt relief from the sequence of terms of trade shocks since 1980 under the optimal contract would be high: around 20% on average, and about 33% in three countries. This emphasises the importance of terms of trade

¹²On a point of robustness, the average interest rate over the period is used to compute the price of debt, but using instead the actual interest rate in each period does not significantly affect the estimates.

¹³Mauritania is an exception that may be explained by its discovery of large iron deposits that improved its terms of trade, though it still had heavy debt burdens.

Table 4: Debt relief from terms of trade shocks

Country	HIPC Relief	Total Relief	Relief Estimates ^{1,2}		Exports/ GDP
			$\phi = \text{actual}$	$\phi = 0.8$	
Benin	28%	51%	7%	5%	16%
Burkina Faso	64%	65%	-3%	2%	10%
Cameroon	21%	77%	12%	8%	23%
Congo, DR	73%	89%	16%	13%	22%
Ghana	38%	57%	28%	21%	20%
Guinea	24%	63%	35%	31%	27%
Malawi	44%	55%	33%	25%	24%
Mali	40%	50%	8%	8%	19%
Mauritania	40%	68%	9%	8%	43%
Mozambique	82%	85%	47%	32%	10%
Niger	61%	78%	30%	31%	19%
Rwanda	66%	76%	-27%	-32%	8%
Senegal	18%	33%	18%	17%	27%
Uganda	51%	66%	30%	30%	11%
Zambia	51%	83%	32%	31%	33%

Source: Jubilee Research (2008)

¹ -() indicates an optimal increase in debt

² $\beta = q^*$ = average US real interest rate over period

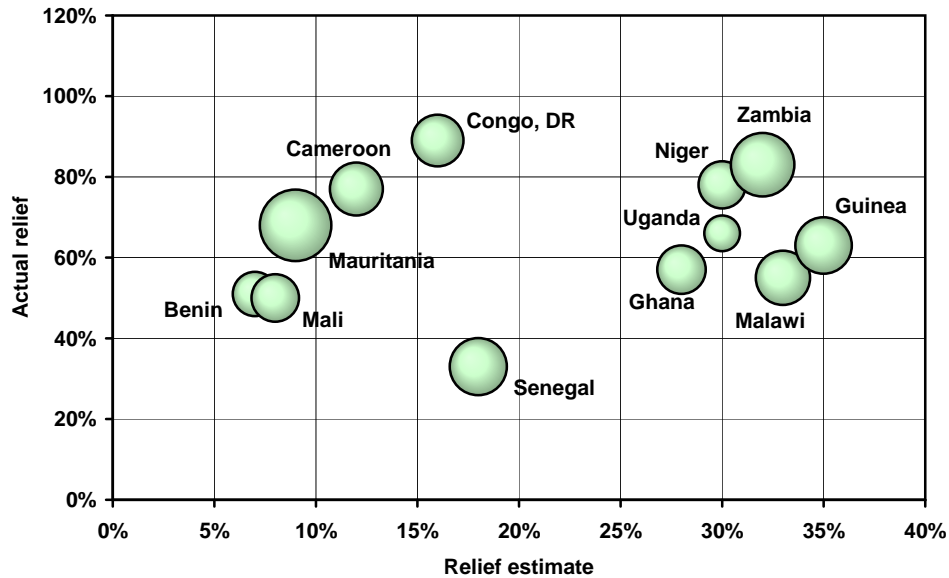


Fig. 2: Comparing estimated debt relief to actual relief
 Countries have export/GDP > 0.1

shocks to small, open economies' debt sustainability, and the care that should be taken when writing contracts in periods when commodity prices and terms of trade appear to be rising pertinaciously.

4 Conclusion

This paper provides two contributions to the literature on sovereign debt in developing countries: first it shows that, when a country that borrows as much as possible, and relies on export revenue to service the debt, suffers large and/or sustained adverse terms of trade shocks, the amount of debt relief under the optimal contract will be quantitatively significant. In general, by comparing the quantitative results to Guimaraes (2008), we see the magnitude of debt relief due for average terms of trade shocks is typically more than what is due for productivity shocks. Following his line of argument, this suggests that recent studies quantitatively matching debt

crises may be focusing on the wrong kinds of shocks by looking at technology.¹⁴

Second, this paper suggests a normative benchmark with which to evaluate the amount of debt relief awarded to the countries that benefited from the HIPC initiative. Declining terms of trade were argued to be a justification for the HIPC programme, and remain an argument for further debt relief. Famously, Easterly (2002) criticises the HIPC initiative for granting too much relief, and argues that countries would rebuild a level of unsustainable debt.

Applying the benchmark to the historical terms of trade shocks to HIPCs shows that the magnitudes of debt relief were certainly more than enough to compensate for the decline in the terms of trade since 1980, which would appear to support Easterly (2002). However, further research is needed to combine relief due to this single factor with that due for other unpredictable and adverse shocks, for example conflict, interest rate, environmental or political shocks. Until these are quantified, it remains a possibility that the accumulated optimal relief is little different from that which was received.

The exercise also shows a high level of optimal debt relief for misperceiving the long run path of the terms of trade. Acknowledging Easterly's (2002) point that debt may return to unsustainable levels, it would be sensible also to acknowledge the unpredictability and volatility of terms of trade, together with its quantitatively significant effect on the sustainable level of debt, and address it explicitly in debt contracts. Existing proposals in the area focus on commodity price shocks¹⁵ and sophisticated financial instruments.¹⁶ However, attention is due to the contribution of adverse import price shocks to debt sustainability, especially in oil-importing countries, and the fact that certain techniques require a level of sophistication in financial markets that may not exist in HIPCs. Since for these countries terms of trade are

¹⁴Cuadra & Sapriza (2006) might be seen as an exception, however, they choose an alternative view of the terms of trade as the relative price of importing primary commodities and exporting finished goods.

¹⁵Besley & Powell (1989), Gilbert & Tabova (2005)

¹⁶Kletzer et al. (1992), ITFCRM (1999), Gilbert & Tabova (2004)

de facto exogenous, explicitly writing debt clauses conditional on realisations of the terms of trade and using the simple, transparent Equation 17 actively to control debt repayments seems a better alternative.

Appendices

A Proof of Proposition 2.2.1

When p follows the $AR(1)$ process described above, $V_{pay}(k, d^i, p^i) = V_{def}(k, \gamma, p^i)$ and $V_{pay}(k, d^j, p^j) = V_{def}(k, \gamma, p^j)$ provided:

$$\frac{d^i - d^j}{\bar{d}} = \frac{1 - q^*}{1 - q^*\phi} \frac{p^i - p^j}{\bar{p}} \quad (22)$$

Proof: Since $MPK = A - \delta > r^*$ in every state of the world, the central planner will always want to borrow as much as possible. Then impose the borrowing constraint in every period:

$$d_{t+1}^i = \frac{1}{q} [d_t^i - \gamma p_t^i X] \quad (23)$$

Since the constraint always binds, the maximisation problem may be rewritten as:

$$\begin{aligned} & \max_{\{c_s\}_{s=t}^{\infty}} E_t \sum_{t=s}^{\infty} \beta^{t-s} u(c_t) \\ \text{subject to: } & c_t + k_{t+1} = Ak_t + (1 - \delta)k_t + (1 - \gamma)p_t^i X \end{aligned}$$

which is identical to that of the central planner in a default economy, and thus the value functions are identical implying the no-default condition always holds and the equilibrium price of debt is q^* .

As in Cohen & Sachs (1986), for each level of debt d_t^i , we can derive a transversality condition by imposing a zero-profit condition on the sequence of loan repayments. This is possible because the desire to borrow and the borrowing constraints ensure the no-default condition is never violated. Denoting by $\Phi_t^i \equiv d_t^i - q^* d_{t+1}^i$ the repayment due in period t

when the debt held is d_t^i , the zero-profit condition is:

$$d_t^i = \sum_{s=t}^{\infty} q^{*s-t} \Phi_s^i$$

Rearranging Equation 23 and solving forwards:

$$d_t^i = \lim_{n \rightarrow \infty} \{q^{*n} d_{t+n}^i\} + \gamma X \sum_{s=0}^{\infty} q^{*s} p_{t+s}^i$$

Now, using the AR(1) functional form for the terms of trade:

$$\begin{aligned} p_{t+s}^i - \bar{p} &= \phi(p_{t+s-1}^i - \bar{p}) + \epsilon_{t+s} \\ &\vdots \\ p_{t+s}^i &= \bar{p} + \epsilon_{t+s} + \phi\epsilon_{t+s-1} + \dots + \phi^s(p_t^i - \bar{p}) \end{aligned}$$

And substituting it into the equation for debt today:

$$d_t^i = \lim_{n \rightarrow \infty} \{q^{*n} d_{t+n}^i\} + \sum_{s=t}^{\infty} \gamma X q^{*s} \left[\bar{p} + \phi^s (p_t^i - \bar{p}) + \sum_{\tau=2}^s \phi^{s-\tau} \epsilon_{t+\tau-1} \right]$$

Then using the transversality condition and taking expectations over future shocks we find:

$$d_t^i - \bar{d} = \frac{\gamma X (p_t^i - \bar{p})}{1 - q^* \phi}$$

This is simply a rearrangement of the level of debt that ensures the value functions for the central planner in default and repayment economies are identical and thus no default occurs. Finally, we find the optimal change in the level of debt by looking at the change when the terms of trade is shocked from p_t^i to the path p_t^j :

$$\frac{d_t^i - d_t^j}{\bar{d}} = \frac{1 - q^*}{1 - q^* \phi} \frac{p_t^i - p_t^j}{\bar{p}}$$

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