A Behavioural Power Index

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Abstract

We propose an empirically relevant measure of voting power that uses the information about real or assumed voting patterns conveyed by a joint probability distribution on the set of voting outcomes, and apply it to the voting data of the Supreme Court of the United States.

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Key Words: a priori voting power, behavioural voting power, U.S. Supreme Court

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1 Introduction

A common criticism of the widely used Penrose (1946) or Banzhaf (1965) measure of voting power is that it fails to take account of the preferences or actual behaviour of voters. Power is assumed to arise purely from the voting rules and its measurement is confined to an examination of the possibilities that might exist for voting members to effect decisions by switching their votes given the likely voting behaviour by others. The criticism arises from the assumption that all voting outcomes that can theoretically occur are considered to be equally likely. The probability distribution of voting outcomes is not derived from evidence on actual behaviour, or assumptions about voters' preferences, but entirely *a priori*. The measure is therefore best described as a measure of constitutional voting power, power that follows from the formal rules of the voting body. Typically the rules are set at the constitutional stage of a voting body, prior to voting.

The assumption that all voting outcomes are equally probable is consistent with the binomial model of voting in which each vote has an equal probability of being for or against a motion, and all votes are independent. Felsenthal and Machover (2004) defend the binomial model on two counts. Firstly, it is a rational assumption in the absence of prior knowledge about the future issues on the ballot and how divided over these issues voters will be. It therefore suits the purpose of measuring the distribution of a priori or constitutional voting power that follows from the rules of the voting body in a purely formal sense. Second, the binomial model is attractive on normative grounds because it presumes the maximum freedom of choice for the voter. It therefore is the benchmark model to use in constitutional design.

In this paper we seek to construct an empirically relevant power measure by relaxing this very strong assumption and replacing it by the use of information about real or assumed voting patterns. Our starting point is to assume that voter preferences can be described in terms of a probability distribution over the various theoretically possible voting profiles. That is, we know the probability of occurrence of every one of the large number of possible roll-calls. This probability distribution can be estimated by observing actually occurring voting patterns over a suitable period of time.[1] We call the measure of empirically informed voting power that we derive a behavioural power index.

We apply the behavioural power index to the Supreme Court of the United States. A large amount of voting data reveal a large discrepancy between the Penrose-Banzhaf a priori measure of power and the behavioural power estimated using the relative frequencies of voting outcomes. The observed relative frequencies refute the assumption of equally probable voting outcomes and therefore also the binomial model of equally probable and independent votes. We identify positive correlation between votes as their primary cause.

In the next section we introduce a behavioural index of voting power. Section 3 describes the data and presents computations of behavioural power for the justices in the U.S. Supreme Court. Section 4 presents estimates of Coleman's measure of the overall dependence and the correlation coefficients between justices' votes. Section 5 summarizes the paper.

2 A behavioural power index

We begin with a conventional description of the Penrose power index (also known as the absolute Banzhaf index; see for example Felsenthal and Machover (1998)) to develop the analytical framework and introduce the notation.

The voting body consists of n members, labelled $i = 1, 2, \ldots, n$, $N = \{1, 2, \ldots, n\}$. Each member has voting weight w_i and must cast all her votes as a bloc either for or against a motion. Notationally, letting v_i represent the number of *Yes* votes cast by member i, we write $v_i = w_i$

[1] This obviously assumes sincere voting. We maintain this assumption throughout this paper.

if she votes *Yes* and $v_i = 0$ if she votes *No.* The decision rule is defined by a majority quota or threshold number of votes, q $(.5\sum_{i=1}^{n} w_i) < q \leq \sum_{i=1}^{n} w_i)$ such that, if the total number of votes cast by *Yes* voters is greater than or equal to q, the decision is *Yes*, and otherwise it is *No.*

We refer to any particular voting outcome, or the result of a ballot on some motion, as a *division*, represented by a set $D \subseteq N$ The total votes cast by *Yes* voters is $w(D) = \sum_{i \in D} w_i$.

It is common in the literature to refer to a division D as 'winning' if $w(D) \geq q$ and 'losing' otherwise. We will here avoid this terminology and instead refer to a *Yes* decision and a *No* decision respectively.

Now consider the power of member i. This is measured by swings. In general the power of a member is her likelihood of being able to effect a change in the decision by changing her vote, that is to be a swing voter.

A swing for i is defined in two ways:

i). An *S-swing* is a division S in which i votes *No* which would produce a *No* decision but would become a *Yes* decision if i changed her vote to *Yes.* That is, $S \subseteq N \backslash \{i\}$, such that $q - w_i \leq w(S) < q$;

ii). A *T-swing* is a division T in which i votes *Yes* which would produce a *Yes* decision unless i changed her vote to *No.* That is, $T \subseteq N$, $i \in T$, such that $q - w_i \leq w(T) - w_i < q$.

These two definitions are obviously dual because for every *S-swing* there is a *T-swing* where $T = S \cup \{i\}$. It is useful to maintain the distinction however since it is used in the definition of the behavioural power measure. More generally we will maintain the notation of S for a division in which i votes *No* and T for one where i votes *Yes.* To simplify the notation we do not index S and T or their probabilities with i but keep in mind that these are specific to each voter.

Let the number of swings, whether *S-swings* or *T-swings*, be η_i. Then the Penrose power index for voter i is defined (in the conventional notation) as

$$\beta'_i = \frac{\eta_i}{2^{n-1}}. \tag{1}$$

This is the classical *'a priori'* measure of voting power which assigns equal weight to each division. In probabilistic terms, the measure (1) assumes that the conditional probability of every set S given that i votes *No* (equivalently T given that i votes *Yes*) is equal to 2^{1-n}. That is,

$$P[q - w_i \leq w(S) < q | v_i = 0] = 2^{1-n}, \quad \text{and} \quad P[q - w_i \leq w(T) - w_i < q | v_i = w_i] = 2^{1-n}.$$

The problem now is to generalise this idea to the situation where not all swings are equally likely, but where the probability distribution is known, from empirical observation of voting behaviour, or can be assumed to be known, on some other basis. We seek a measure of the voting power of voter i in relation to decisions taken under the rules, recognising the behaviour of the other $n - 1$ voters both in relation to each other and also in relation to the behaviour of i herself. However the actual behaviour of voter i, as reflected in the voting probabilities, is not part of the measure of power.

In this we follow Braham and Holler (2005) and Morriss (2002) who argue that, because power is fundamentally defined as a dispositional concept, its possession or its magnitude does not depend on its exercise. Power exists whether or not its possessor is observed to use it. This means that the voting behaviour (or preferences) of voter i, as reflected in the probabilities of her voting *Yes* or *No* cannot have any relevance to a power measure.

Morriss (2002) makes an important distinction between 'power as ability' and 'power as ableness'. A voter's ability is defined as her power to effect decisions *whatever* the actions of the other voters. On the other hand, the term ableness refers to her power to effect decisions in the specific situation in which the voter finds herself *given specific actions by others.* The former leads to a power measure which makes no assumptions about actual or likely voting behaviour, treating all divisions that could possibly occur as equiprobable: the Penrose index. The latter means taking account of the actual or expected voting behaviour of the other $n-1$ voters besides i.

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The behavioural power index defined here is the probability of a swing for voter i. In order to achieve the greatest empirical generality we do not assume neutrality of voting. This means that the two definitions of a swing, *S-swings* and *T-swings* can no longer be treated as equivalent.

It is assumed that we know the probability distribution of voting outcomes, represented by numbers, π_D $(0 \leq \pi_D \leq 1)$, for all divisions $D \subseteq N$.

The marginal probabilities for voter i are then (using the S and T distinction for divisions):

$$P[i \text{ votes } Yes] = P[v_i = w_i] = \sum_{T:i \in T} \pi_T = \pi_i;$$
$$P[i \text{ votes } No] = P[v_i = 0] = \sum_{S:i \notin S} \pi_S = 1 - \pi_i,$$

The division probabilities π_D represent the basic information about voting behaviour which is relevant to the index. But i's marginal voting probabilities π_i and $1 - \pi_i$ are not relevant. They are used merely to find the conditional probabilities of divisions given i's vote. Since the distribution of divisions is not independent of the vote of i, we define two conditional distributions over divisions.

i). The conditional distribution of S-divisions: $P[S|v_i = 0] = \phi_S = \pi_S/(1 - \pi_i)$ for all $S \subseteq N\backslash\{i\}$. Therefore if S is an *S-swing*, ϕ_S is the conditional probability of that particular *S-swing*;

ii). The conditional distribution of T-divisions: $P[T|v_i = w_i] = \psi_T = \pi_T/\pi_i$ for all $T \subseteq N$, $i \in T$. Therefore if T is a *T-swing*, ψ_T is the conditional probability of that particular *T-swing*.

The behavioural power index is defined as the probability of a swing for voter i:

$$\alpha_i = P[i \text{ votes } No]P[S - swing|i \text{ votes } No] + P[i \text{ votes } Yes]P[T - swing|i \text{ votes } Yes].$$

We set i's voting probabilities to .5 and use the conditional probabilities defined above in this expression. The formula for the behavioural power index is:

$$\alpha_i = 0.5 \sum_{S-Swings} \phi_S + 0.5 \sum_{T-Swings} \psi_T. \tag{2}$$

This measure reflects the power of i as a capacity to influence the outcome of a vote whether or not that capacity is exercised. The fact that voter i has the a priori capacity to vote *Yes* or *No* is allowed for by using equal probabilities for both events.

Table 1 gives an example of the above calculation for a weighted voting game with $n = 3$,

$q = 3$, $w_1 = 3$, $w_2 = 2$ and $w_3 = 1$. The discrepancy between behavioural and the a priori or the constitutional power is evident. The assumed division probabilities are π_D are such that all voters have less behavioural than constitutional power. This need not be the case in general. Table 2 gives an example of a weighted voting game with $n = 5$, $q = 5$, $w_1 = w_2 = w_3 = 2$ and $w_4 = w_5 = 1$. Suppose that voters 2 and 3 never vote against one another, so that the probability of all such divisions is zero, whereas all other divisions are equally likely. By voting in unison, voters 2 and 3 effectively comprise a voting bloc with the total voting weight of 4. This increases their behavioural voting power at the expense of the other three voters, who vote independently.

A special case is where i and the other voters are independent. Then, for any S and T such that $T = S \cup \{i\}$, and $\phi_S = \psi_T$. This may be a rather restrictive assumption in practice, however, and it seems unnecessary to impose it under our assumption that we have a complete probability distribution of divisions.

Another special case is the Penrose index. Here we assume $\phi_S = \psi_T = 2^{1-n}$. Then

$$\alpha_i = 0.5 \sum_{S-Swings} 2^{1-n} + 0.5 \sum_{T-Swings} 2^{1-n} = 0.5\left(\frac{\eta_i}{2^{n-1}} + \frac{\eta_i}{2^{n-1}}\right) = \beta'_i. \tag{3}$$

3 Voting power in the U.S. Supreme Court

The U.S. Supreme Court is the highest judicial authority of the United States. The court comprises the Chief Justice and eight Associate Justices. The justices are nominated by the President and appointed with the advice and consent of the Senate to serve for life.

The court votes under simple majority rule provided at least six justices are present for the court to be in session. Justices vote in the order of their seniority, starting from the Chief

Justice. Although the Chief Justice has a number of exclusive prerogatives and responsibilities, all justices have equal a priori voting power.

It is customary to refer to the court by the name of the presiding Chief Justice, who usually is the longest-serving member of the court. We define a bench as a court in which the same group of persons sit as justices, and study three benches from the Chief Justice Warren (1953-1969), Burger (1969-1986) and Rehnquist's (1989-2005) eras.

3.1 The data

The Supreme Court Justice-Centered Judicial Databases sponsored by the University of Kentucky and maintained by Benesh and Spaeth is a comprehensive source of data on the decisions and opinions of individual justices during the three eras.

Several decisions had to be made when preparing the data. Our focus is on the relative frequencies of divisions. For a meaningful analysis we therefore confine to cases in which the same bench of justices voted. Out of 228 observed benches we chose the three which had the highest number of cases, one for each era. The high number of different benches arises from the combinations of six to nine justices possibly attending a session, and from the fact that associate justices retired and were replaced in the same era. Our selection includes 416, 1110, and 683 cases from respectively the Warren, Burger and Rehnquist eras, with the nine justices voting on each case. Since we would like the cases to be as independent from each other as possible, we exclude those with multiple legal issues or multiple legal provisions. Finally, we define a *Yes* vote as one in favour of the petitioning party.[2]

[2] In terms of the variables in the Benesh and Speath databases, we restrict $ANALU = 0$, delete the duplicate LED numbers and compute a vote variable which takes the value 1 if $MAJ\_MIN = WIN\_DUM$, and 0 otherwise.

3.2 Relative frequencies of divisions

Figure 1 shows the relative frequencies of divisions. With nine justices there will be 512 possible divisions. The abscissa scale shows the divisions ordered from a unanimous vote against the petitioning party to a unanimous vote in favour of the petitioning party. Note that:

i). only a small minority of possible divisions has actually occurred, the percentage in each bench being Warren (9.4), Burger (31.6) and Rehnquist (21.5);

ii). relative frequencies of those which are observed are very unequal;

iii). unanimity is by far the most frequent of all outcomes. The Burger bench is an exception with the 2:7 divisions against the petitioning party occurring slightly more frequently than the 0:9 divisions;

iv). a unanimous vote in favour of the petitioning party is more frequent than a unanimous vote against the petitioning party. This suggests a selection bias in the data in favour of petitions with a reasonable chance of success. This point is noteworthy because the Chief Justice ultimately decides which cases will the heard.

These observations refute the binomial model of equally probable and independent votes as an empirical description. Although comprehensive voting records such as that for the U.S. Supreme Court are scarce, data on several other voting bodies reveals that divisions typically occur with very different frequencies. This is true of the Supreme Court of Canada (Heard and Swartz 1998), the European Union Council of Ministers (Hayes-Renshaw, van Aken and Wallace 2006) and the institutions of the United Nations (Newcombe, Ross and Newcombe 1970). In the next section we show that positive correlations between votes are the primary cause of the observed pattern.

3.3 The behavioral power index

In a voting game with equal weights all voters have equal a priori powers. Under simple-majority rule with an odd number of votes the Banzhaf absolute measure equals the binomial probability of $(n-1)/2$ successes in $n-1$ trials with .5 as the probability of success. For $n = 9$ this probability equals .273.

Table 3 reports the behavioural power index α_i for each justice. It is evident that the justices' behavioural powers are not equal. We quantify this inequality by comparing the Gini coefficient of α_i. The largest inequality is observed in the Warren bench, followed by Rehnquist and Burger. The biggest losers in terms of behavioural vs. a priori voting power were justices Black, Douglas and Clark of the Warren bench. None of the justices had more behavioural power than a priori power.

4 Coleman's measure of dependence

The fact that unanimity is by far the most frequent of all divisions points at positive correlation between the justices' votes. The correlation coefficient is a pairwise measure of linear dependence. But is there a measure of the overall dependence between votes? One such measure has been proposed in Coleman (1973).

Under the binomial model the variance of the fraction of *Yes* votes is $\sigma^2 = \pi(1-\pi)/n$, where π is the probability of a *Yes* vote, here common to all voters. The sample variance s^2 will systematically differ from σ^2 if the assumptions of the binomial model are not met. This will be the case if votes are not independent, or if the true probability is not π, or both. For example, if each pair of votes correlates with the correlation coefficients c, then $s^2 \approx \sigma^2[1+(n-1)c]$, provided $(1-n)^{-1} \leq c \leq 1$. Assuming the true probability is indeed π, the ratio σ^2/s^2 is a rough measure of the overall dependence in a voting body of size n.

Coleman used this fact to compute the equivalent number of independent voters m. Setting $s^2 = \pi(1-\pi)/m$ yields $m = \pi(1-\pi)/s^2$. For independent voters $s^2 \approx \pi(1-\pi)/n$ so that $m \approx n$. Compared to $n = 9$, low values of m for the Warren (1.48), Burger (2.21) and Rehnquist (1.79) benches point at high positive correlations between votes.

Coleman's is a measure of the overall dependence. Table 4 reports estimates of the individual marginal probabilities and correlation coefficients. They show that *Yes* votes are not far from being equally probable but are highly positively correlated. For the three courts the average probabilities of a *Yes* vote are .63, .47 and .54. The spreads of the estimated probabilities range from (.44, .73) for the Warren bench to (.53, .55) for the Rehnquist bench. The average correlation coefficients between two *Yes* votes for the three benches are .67, .39 and .5.

Fisher's Exact Test[3] cannot reject the null hypothesis of conditional independence at the five percent level of significance for only 7 of 108 pairs of justices indicated by an asterisk. There is significant positive correlation in justices' decisions.

One consequence of high positive correlations is that voters often see their preferred outcome prevail. Several authors have proposed to measure voting power using this probability (Straffin's (1978) Satisfaction index, Morriss's (2002) EPW index). We believe that the probability of being on the winning side is not a valid measure of power because it does not distinguish between power and luck.[4] For example, it assigns power to a voter who cannot swing in any theoretically conceivable division of votes, a dummy. Our approach is to define power in terms of the capacity to effect a decision, based on the concept of a swing.

[3]See, Everitt (1992).

[4]This point has been extensively discussed by Dowding (1996).

5 Summary

The standard approach to measuring the voting power of a particular member of the voting body is to compute the probability of her casting a decisive vote. This analysis must encompass both the formal rules of the voting body and also the behaviour of all the voters. The member is more powerful the more frequently her vote is decisive. But this will depend in practice on circumstances created by others casting their votes so that she has opportunities to be decisive. Thus an empirical voting power measure must be based on the frequencies with which the various voting outcomes occur. As is well known, the Penrose-Banzhaf measure ignores behaviour and takes into account only power deriving from the rules construed in a purely formal, a priori sense.

We propose an empirically relevant power measure by relaxing this very strong assumption and replacing it by the use of information about real or assumed voting patterns. The behavioural power index is based on the joint probability distribution on the set of voting outcomes, which can be assumed or estimated using ballot data.

The voting behaviour of justices in the U.S. Supreme Court clearly contradicts the assumptions of equally probable and independent votes. Over a range of cases, different voting outcomes occur with very unequal relative frequencies. The observed patterns of relative frequencies point to positive correlation as their main cause, and indeed we find significant positive correlation in justices' decisions.

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Table 1: The behavioural power index for the game $\{4; 3, 2, 1\}$

v_1	v_2	v_3	π_D	$\phi_{S,1}$	$\phi_{S,2}$	$\phi_{S,3}$	$\psi_{T,1}$	$\psi_{T,2}$	$\psi_{T,3}$
$\underline{3}$	2	1	0.309	0	0	0	0.618	0.515	0.386
$\underline{3}$	$\underline{2}$	0	0.016	0	0	0.078	0.031	0.026	0
$\underline{3}$	0	$\underline{1}$	0.131	0	0.328	0	0.262	0	0.164
3	0	0	0.044	0	0.111	0.222	0.089	0	0
0	2	1	0.249	0.499	0	0	0	0.416	0.312
$\underline{0}$	2	$\underline{0}$	0.026	0.052	0	0.13	0	0.043	0
$\underline{0}$	$\underline{0}$	1	0.111	0.221	0.276	0	0	0	0.138
$\underline{0}$	0	0	0.114	0.228	0.285	0.57	0	0	0

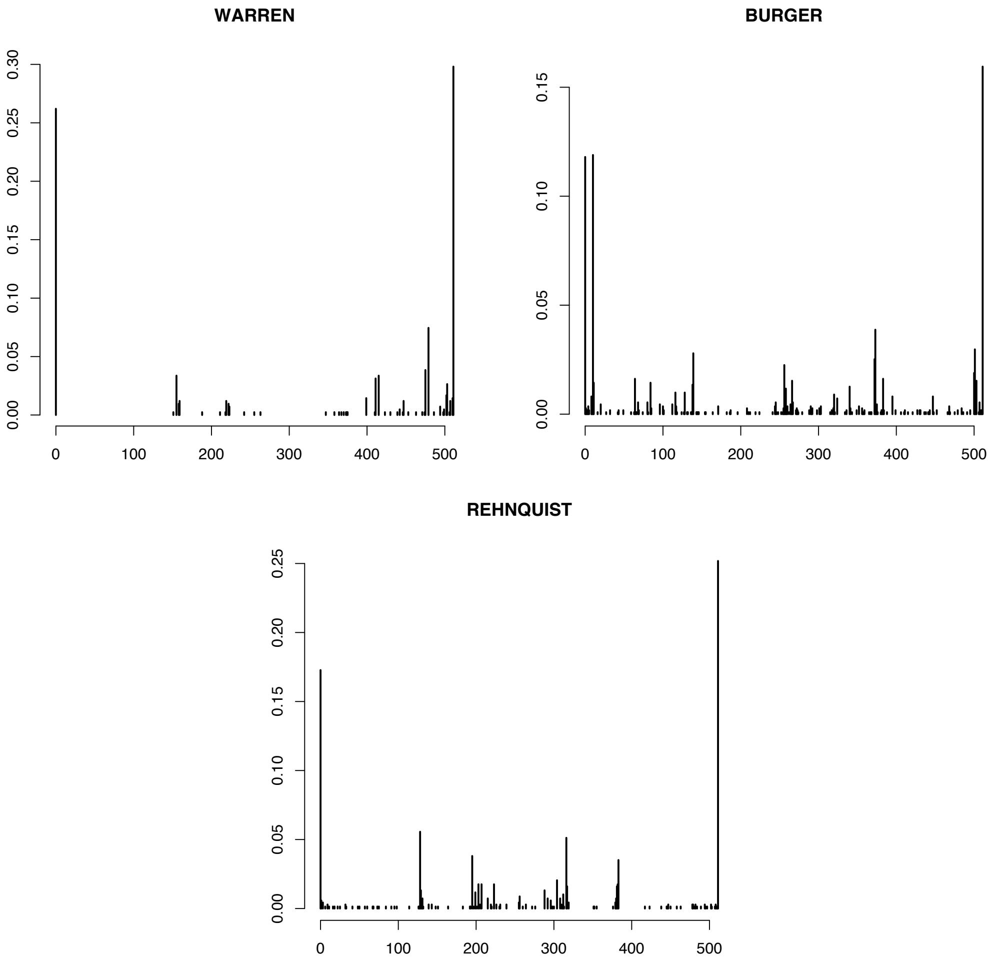
Behavioural powers equal .71, .15 and .15. A priori absolute Banzhaf powers equal .75, .25 and .25. *S-swings* and *T-swings* are indicated with an underline. The joint probability distribution π_D satisfies the following probabilities and correlation coefficients: $\pi_1 = .5$, $\pi_2 = .6$, $\pi_3 = .8$ and $c_{1,2} = .1$, $c_{1,3} = .2$, $c_{2,3} = .4$.

Table 2: The behavioural power index for the game $\{5; 2, 2, 2, 1, 1\}$

v_1	v_2	v_3	v_4	v_5	π_D	$\phi_{S,1}$	$\phi_{S,2}$	$\phi_{S,4}$	$\psi_{T,1}$	$\psi_{T,2}$	$\psi_{T,4}$
2	2	2	1	1	0.0625	0	0	0	0.125	0.125	0.125
2	2	2	1	0	0.0625	0	0	0	0.125	0.125	0.125
2	2	2	0	1	0.0625	0	0	0.125	0.125	0.125	0
$\underline{2}$	$\underline{2}$	$\underline{2}$	0	0	0.0625	0	0	0.125	0.125	0.125	0
$\underline{2}$	$\underline{2}$	0	1	1	0	0	0	0	0	0	0
$\underline{2}$	$\underline{2}$	0	$\underline{1}$	0	0	0	0	0	0	0	0
$\underline{2}$	$\underline{2}$	0	0	$\underline{1}$	0	0	0	0	0	0	0
2	2	0	0	0	0	0	0	0	0	0	0
$\underline{2}$	0	$\underline{2}$	1	1	0	0	0	0	0	0	0
$\underline{2}$	0	$\underline{2}$	$\underline{1}$	0	0	0	0	0	0	0	0
$\underline{2}$	0	$\underline{2}$	0	$\underline{1}$	0	0	0	0	0	0	0
2	0	2	0	0	0	0	0	0	0	0	0
2	0	0	1	1	0.0625	0	0.125	0	0.125	0	0.125
2	$\underline{0}$	$\underline{0}$	1	$\underline{0}$	0.0625	0	0.125	0	0.125	0	0.125
2	$\underline{0}$	$\underline{0}$	$\underline{0}$	1	0.0625	0	0.125	0.125	0.125	0	0
2	$\underline{0}$	$\underline{0}$	0	0	0.0625	0	0.125	0.125	0.125	0	0
0	$\underline{2}$	$\underline{2}$	1	1	0.0625	0.125	0	0	0	0.125	0.125
0	$\underline{2}$	$\underline{2}$	$\underline{1}$	0	0.0625	0.125	0	0	0	0.125	0.125
0	$\underline{2}$	$\underline{2}$	0	$\underline{1}$	0.0625	0.125	0	0.125	0	0.125	0
0	2	2	0	0	0.0625	0.125	0	0.125	0	0.125	0
0	2	0	1	1	0	0	0	0	0	0	0
$\underline{0}$	2	$\underline{0}$	1	$\underline{0}$	0	0	0	0	0	0	0
$\underline{0}$	2	$\underline{0}$	$\underline{0}$	1	0	0	0	0	0	0	0
$\underline{0}$	2	$\underline{0}$	0	0	0	0	0	0	0	0	0
0	0	2	1	1	0	0	0	0	0	0	0
$\underline{0}$	$\underline{0}$	2	1	$\underline{0}$	0	0	0	0	0	0	0
$\underline{0}$	$\underline{0}$	2	$\underline{0}$	1	0	0	0	0	0	0	0
$\underline{0}$	$\underline{0}$	2	0	0	0	0	0	0	0	0	0
$\underline{0}$	$\underline{0}$	$\underline{0}$	1	1	0.0625	0.125	0.125	0	0	0	0.125
0	0	0	1	0	0.0625	0.125	0.125	0	0	0	0.125
0	0	0	0	1	0.0625	0.125	0.125	0.125	0	0	0
0	0	0	0	0	0.0625	0.125	0.125	0.125	0	0	0

Behavioural powers equal, respectively, .125, .5, .5, .125 and .125. A priori absolute Banzhaf powers equal .4375, .4375, .4375, .1875 and .1875. Powers of voters 2-3 and 4-5 are equal by the equality of their weights and behaviour. *S-swings* and *T-swings* are indicated with an underline. The fact that voters 2 and 3 never vote against one another increases their voting power at the expense of the other three voters.

Figure 1: Relative frequencies of divisions



A division is represented by a binary vector $(v_1, v_2, \ldots, v_9)$, where $v_i = 1$ if i votes in favour of the petitioning party, and $v_i = 0$ otherwise. The abscissa scale shows the divisions ordered in the ascending order of the decimals their binary vectors represent, starting from the zero vector.

Table 3: The behavioural power index

WARREN		BURGER		REHNQUIST	
Black	0.009	Blackmun	0.106	Breyer	0.118
Brennan	0.046	Brennan	0.073	Ginsburg	0.072
Clark	0.017	Burger	0.089	Kennedy	0.074
Douglas	0.013	Marshall	0.106	Oconnor	0.112
Goldberg	0.047	Oconnor	0.096	Rehnquist	0.116
Harlan	0.063	Powell	0.059	Scalia	0.135
Stewart	0.019	Rehnquist	0.098	Souter	0.142
Warren	0.042	Stevens	0.060	Stevens	0.071
White	0.032	White	0.061	Thomas	0.070
Gini	0.306	Gini	0.126	Gini	0.150

Table 4: Marginal probabilities and correlation coefficients

WARREN				BURGER				REHNQUIST			
(i,j)	π_i	π_j	$c_{i,j}$	(i,j)	π_i	π_j	$c_{i,j}$	(i,j)	π_i	π_j	$c_{i,j}$
Black-Brennan	0.64	0.73	0.80	Blackmun-Brennan	0.44	0.51	0.36	Breyer-Ginsburg	0.55	0.53	0.72
Black-Clark	0.64	0.58	0.56	Blackmun-Burger	0.44	0.49	0.47	Breyer-Kennedy	0.55	0.55	0.48
Black-Douglas	0.64	0.65	0.74	Blackmun-Marshall	0.44	0.49	0.35	Breyer-O'Connor	0.55	0.55	0.54
Black-Goldberg	0.64	0.70	0.77	Blackmun-O'Connor	0.44	0.47	0.49	Breyer-Rehnquist	0.55	0.54	0.38
Black-Harlan	0.64	0.44	0.37	Blackmun-Powell	0.44	0.46	0.54	Breyer-Scalia	0.55	0.55	0.27
Black-Stewart	0.64	0.57	0.55	Blackmun-Rehnquist	0.44	0.50	0.37	Breyer-Souter	0.55	0.53	0.71
Black-Warren	0.64	0.72	0.79	Blackmun-Stevens	0.44	0.38	0.52	Breyer-Stevens	0.55	0.53	0.58
Black-White	0.64	0.64	0.69	Blackmun-White	0.44	0.54	0.46	Breyer-Thomas	0.55	0.55	0.24
Brennan-Clark	0.73	0.58	0.67	Brennan-Burger	0.51	0.49	0.00*	Ginsburg-Kennedy	0.53	0.55	0.52
Brennan-Douglas	0.73	0.65	0.82	Brennan-Marshall	0.51	0.49	0.81	Ginsburg-O'Connor	0.53	0.55	0.50
Brennan-Goldberg	0.73	0.70	0.90	Brennan-O'Connor	0.51	0.47	0.05*	Ginsburg-Rehnquist	0.53	0.54	0.41
Brennan-Harlan	0.73	0.44	0.51	Brennan-Powell	0.51	0.46	0.13	Ginsburg-Scalia	0.53	0.55	0.30
Brennan-Stewart	0.73	0.57	0.67	Brennan-Rehnquist	0.51	0.50	-0.10	Ginsburg-Souter	0.53	0.53	0.79
Brennan-Warren	0.73	0.72	0.95	Brennan-Stevens	0.51	0.38	0.36	Ginsburg-Stevens	0.53	0.53	0.62
Brennan-White	0.73	0.64	0.78	Brennan-White	0.51	0.54	0.12	Ginsburg-Thomas	0.53	0.55	0.27
Clark-Douglas	0.58	0.65	0.56	Burger-Marshall	0.49	0.49	-0.03*	Kennedy-O'Connor	0.55	0.55	0.72
Clark-Goldberg	0.58	0.70	0.59	Burger-O'Connor	0.49	0.47	0.76	Kennedy-Rehnquist	0.55	0.54	0.75
Clark-Harlan	0.58	0.44	0.62	Burger-Powell	0.49	0.46	0.73	Kennedy-Scalia	0.55	0.55	0.65
Clark-Stewart	0.58	0.57	0.62	Burger-Rehnquist	0.49	0.50	0.75	Kennedy-Souter	0.55	0.53	0.57
Clark-Warren	0.58	0.72	0.66	Burger-Stevens	0.49	0.38	0.40	Kennedy-Stevens	0.55	0.53	0.3
Clark-White	0.58	0.64	0.69	Burger-White	0.49	0.54	0.59	Kennedy-Thomas	0.55	0.55	0.63
Douglas-Goldberg	0.65	0.70	0.77	Marshall-O'Connor	0.49	0.47	0.02*	O'Connor-Rehnquist	0.55	0.54	0.71
Douglas-Harlan	0.65	0.44	0.34	Marshall-Powell	0.49	0.46	0.10	O'Connor-Scalia	0.55	0.55	0.63
Douglas-Stewart	0.65	0.57	0.52	Marshall-Rehnquist	0.49	0.50	-0.13	O'Connor-Souter	0.55	0.53	0.58
Douglas-Warren	0.65	0.72	0.83	Marshall-Stevens	0.49	0.38	0.33	O'Connor-Stevens	0.55	0.53	0.25
Douglas-White	0.65	0.64	0.63	Marshall-White	0.49	0.54	0.05*	O'Connor-Thomas	0.55	0.55	0.64
Goldberg-Harlan	0.70	0.44	0.49	O'Connor-Powell	0.47	0.46	0.72	Rehnquist-Scalia	0.54	0.55	0.76
Goldberg-Stewart	0.70	0.57	0.66	O'Connor-Rehnquist	0.47	0.50	0.75	Rehnquist-Souter	0.54	0.53	0.45
Goldberg-Warren	0.70	0.72	0.89	O'Connor-Stevens	0.47	0.38	0.46	Rehnquist-Stevens	0.54	0.53	0.12
Goldberg-White	0.70	0.64	0.69	O'Connor-White	0.47	0.54	0.53	Rehnquist-Thomas	0.54	0.55	0.75
Harlan-Stewart	0.44	0.57	0.65	Powell-Rehnquist	0.46	0.50	0.67	Scalia-Souter	0.55	0.53	0.37
Harlan-Warren	0.44	0.72	0.48	Powell-Stevens	0.46	0.38	0.47	Scalia-Stevens	0.55	0.53	0.05
Harlan-White	0.44	0.64	0.63	Powell-White	0.46	0.54	0.58	Scalia-Thomas	0.55	0.55	0.87
Stewart-Warren	0.57	0.72	0.63	Rehnquist-Stevens	0.50	0.38	0.35	Souter-Stevens	0.53	0.53	0.58
Stewart-White	0.57	0.64	0.70	Rehnquist-White	0.50	0.54	0.50	Souter-Thomas	0.53	0.55	0.35
Warren-White	0.72	0.64	0.74	Stevens-White	0.38	0.54	0.34	Stevens-Thomas	0.53	0.55	0.03