

Expectations and Fluctuations : The Role of Monetary Policy

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Abstract

How does the economy respond to shocks to expectations? This paper addresses this question within a cashless, monetary economy. A competitive economy features producers and consumers/workers with *asymmetric* information. Only workers observe current productivity and hence they perfectly anticipate prices, whereas all agents observe a noisy signal about long-run productivity. Information asymmetries imply that monetary policy and consumers' expectations have real effects. Non-fundamental, purely expectational shocks are conventionally thought of as demand shocks. While this remains a possibility, expectational shocks can also have the characteristics of supply shocks: if positive, they increase output and employment, and lower inflation. Whether expectational shocks manifest themselves as demand or supply shocks depends on the monetary policy pursued. Forward-looking policies generate multiple equilibria in which the role of consumers' expectations is arbitrary. Optimal policies restore the complete information equilibrium. They do so by manipulating prices so that producers correctly anticipate their revenue despite their uncertainty about current productivity. I design targets for forward-looking interest-rate rules which restore the complete information equilibrium for any policy parameters. Inflation stabilization per se is typically suboptimal as it can at best eliminate uncertainty arising through prices. This offers a motivation for the Dual Mandate of central banks.

JEL Classification: E32, E52, D82, D83, D84

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1 Introduction

Recent empirical work suggests that shocks to expectations contribute significantly to economic fluctuations.¹ But how so? This is a recurrent question for academics, practitioners, and op-ed columnists. There is a growing consensus that if, for instance, consumers overstate the economy's fundamentals, the economy booms at the cost of inflation. A recent literature has formalized this idea:² non-fundamental, purely expectational shocks behave like demand shocks. When positive, they increase output and employment, and are inflationary. Stabilizing inflation emerges then as a natural policy recommendation.³

Nevertheless, Figures 1 - 4 show that the US economy was characterized by high cyclical employment and relatively low inflation in the mid-80s and the second half of the 90s, which are recalled as periods of exuberant optimism. Notably, Figures 3 and 4 reveal that consumer sentiment and inflation are negatively correlated.⁴ An interpretation of expectational shocks as demands shocks does not seem to fit.

This paper reconsiders the nature of purely expectational shocks within a competitive, monetary, cashless economy where producers and consumers/workers have asymmetric information about fundamentals and inflation (prices). I show that expectational shocks can have implications for the business cycle associated with supply shocks: when positive, they increase output and employment, and they lower inflation, which is incompatible with the Phillips curve.⁵ Nonetheless, the possibility that expectational shocks manifest themselves as demand shocks remains. The underlying forces are producers' expectations which push toward a supply-shock interpretation and consumers' expectations which push toward a demand-shock interpretation; which one (demand or supply) prevails depends on the monetary policy pursued.

A natural question that emerges concerns the role of the monetary authority and its optimal response to shocks. With flexible prices, producers' incomplete information is the only source of inefficiency. Asymmetric, as opposed to incomplete but symmetric, information about inflation (prices) implies that monetary policy has real effects. Optimal policies restore the complete infor-

¹ Empirical studies on the contribution of changes in expectations to business cycle fluctuations include Beaudry and Portier (2006), Schmitt-Grohe and Uribe (2008), Blanchard et al. (2009)), Beaudry and Lucke (2010) and Barsky and Sims (2011a,b).

² See for example Blanchard (2009), Angeletos and La’O (2009), and especially Lorenzoni (2009, 2011).

³ See the baseline case in Lorenzoni (2009).

⁴ At a quarterly basis (Figure 3), the correlation of consumer sentiment and inflation is -0.53 . Data are described in Appendix B.

⁵ Gali (1992) considers the textbook IS-LM model coupled with a Phillips curve and explores the effects of demand and supply shocks on the US business cycle. A discussion of the Phillips curve can be found in Mankiw (2001).

mation equilibrium. Inflation stabilization per se is typically suboptimal as it at best eliminates uncertainty arising through inflation without removing producers' incomplete information. Optimal policies manipulate inflation so that producers correctly anticipate their revenue despite their uncertainty about productivity. Bearing this in mind, I design targets for forward-looking policies which restore the complete information equilibrium for any chosen policy parameters.

A competitive (neoclassical) economy features two representative agents, a consumer/worker and a producer, and a monetary authority. The worker supplies labor to a firm, managed by the producer, which produces a single commodity. Productivity consists of a permanent and a temporary component. There is asymmetric information about its current realization: it is specific and known to the worker, while the producer faces uncertainty about it. The monetary authority sets the riskless short-term nominal interest rate. I consider two interest-rate rules: a “contemporaneous” one and a forward-looking one.

Each period is split into two stages: In the first stage, the worker realizes his current productivity -not its individual components-, both agents observe a noisy public signal about the permanent (equivalently, long-run) productivity component, and the labor market opens (and closes). In the second stage, with production predetermined from stage 1, the commodity and the nominal bond markets open (and close) and all payments materialize. Prices are flexible in all markets and agents are price-takers.

The nominal wage, announced in stage 1, reflects the producer's expectations about productivity as well as stage 2 inflation (or prices). With constant returns to scale, the scale of production is pinned down by labor supply. The worker has complete information, so his labor decision and, consequently, production depend on the nominal wage and the inflation he knows will prevail in stage 2.

Inflation, in turn, depends on current productivity, on the producer's expectations about it, and the consumer's expectations about long-run productivity in a way decided by monetary policy. Asymmetric information about current productivity leads agents to form heterogeneous expectations about the inflation to prevail; this opens the door to monetary policy. Further, to the extent that inflation depends on the consumer's expectations about long-run productivity, the producer needs to second-guess the consumer. Then the consumer's expectations also have real effects, indirectly, through inflation. Therefore, that inflation is realized after the labor market has cleared not only prevents productivity from being revealed, but, in combination with asymmetric information, it implies that monetary policy and the consumer's expectations have real effects.

Purely expectational shocks affect both agents' expectations. The consumer's expectations about long-run productivity push toward a demand shock interpretation. A consumption smoothing motive is behind this. Consider, for instance, positive purely expectational shocks. A consumer overly optimistic about the long-run prospects of the economy raises his current demand. If the producer had complete information about current productivity, flexible prices would increase and wages would proportionally adjust leaving the real wage intact. However, under incomplete information, the producer overestimates the inflationary pressure caused due to the consumer's expectations. As a result, the nominal wage increases more than proportionally and a higher real wage prevails. This induces the worker to increase his labor supply and production to expand.

The producer's expectations about current productivity per se point toward a supply shock interpretation. A higher real wage reflects the producer's overly optimistic expectations; employment increases, production expands and, for a certain demand level, prices need to fall for the commodity market to clear.

It should not perhaps come as a surprise that the producer's incomplete information manifests itself as a distortion in the labor wedge originating from the labor demand side. The labor wedge is defined as the ratio of the marginal product of labor to the marginal rate of substitution of leisure for consumption.⁶ Chari et al. (2007) find that it is countercyclical and accounts for more than half of the US output variance. When the real wage exceeds the marginal product of labor, the labor wedge falls. Positive expectational shocks, then, induce a countercyclical labor wedge.⁷

Whether expectational shocks cause an inflationary or a deflationary pressure depends on the monetary policy pursued. Taking into account that employment and output both increase (positive co-movement), it follows that it is up to the monetary authority whether a demand or a supply shock interpretation best fits expectational shocks.

In particular, the policy weight on the current output gap is central to which interpretation prevails. To see this, fix the real interest rate and note that, for a "contemporaneous" rule, expected inflation is zero, which implies that the real interest rate coincides with the nominal one. The nominal interest rate targets inflation and the output gap. A positive expectational shock results in a positive output gap. A higher weight on the output gap implies less inflationary pressure which, in fact, may turn to a deflationary one.

⁶See for example Hall (1997), Chari et al. (2007) and Shimer (2009).

⁷ Related papers generating a countercyclical labor wedge in response to expectational shocks include Angeletos and La'O (2009), La'O (2010) and Venkateswaran (2011). Unlike these papers, the present paper emphasizes the connection of monetary policy with the labor wedge.

Turning to productivity shocks, agents' expectations underreact in response to positive productivity shocks. As a result, a lower real wage prevails which induces employment to fall,⁸ whereas output increases, however by less than under complete information. Following the same line of thought as above, the policy weight on current output gap determines whether productivity shocks are inflationary or disinflationary. Of course, agents learn over time and their expectations eventually converge to the underlying productivity level.

Considering forward-looking policies, the main difference with “contemporaneous” ones is that forward-looking policies generate a continuum of equilibria for any choice of policy parameters.⁹ Importantly, what distinguishes equilibria is the role of the consumer's expectations which is arbitrarily specified. Furthermore, the short-run volatility of output due to expectational shocks is considerably higher under forward-looking policies than under “contemporaneous” ones. These results can contribute to the discussion about the desirability of forward-looking policies.¹⁰

The nominal implications for forward-looking rules also differ, even after controlling for the consumer's expectations. This is because “contemporaneous” interest-rate rules pin down inflation, whereas forward-looking ones pin down price levels. To see this, consider a positive purely expectational shock and let prices depend positively on the producer's expectations, which is true for “active” policies, i.e. policies in which the monetary authority responds to inflation more than one-to-one. Price levels exhibit a non-monotonic pattern in response to expectational shocks: they increase on impact, however as agents update their beliefs over time, they gradually return to their long-run level. Thus, positive expectational shocks cause an inflationary pressure on impact and a deflationary one from the following period onwards. By the same logic, positive permanent productivity shocks are inflationary, until prices reach their higher steady-state level.

The producer's incomplete information is the only source of inefficiency. Optimal monetary policies restore then the complete information equilibrium. To do so they manipulate inflation (prices) so that the producer correctly anticipates his stage-2 revenue, even though still uncertain about current productivity. Inflation stabilization per se is typically suboptimal as it at best eliminates

⁸ In the business cycle literature, Gali (1999) and Basu et al. (2006) also argue that positive technology shocks cause a temporary fall in employment.

⁹ As my focus is on the effects of purely expectational shocks as well as those of productivity shocks, I do not discuss determinacy in the sense, for example, of Clarida et al. (2000) or Bullard and Mitra (2002) (although that discussion has been recently revived with Cochrane (2011)). Nevertheless, it is important to mention that there is no real indeterminacy here, since possible “sunspot” shocks lie outside the information sets of both agents thereby having no real effects, a point on which I elaborate below. I resume this discussion in fn. 52.

¹⁰ Clarida et al. (1999, 2000) and Giannoni and Woodford (2003) also consider forward-looking policies, however in different settings. The Bank of England is suggested to follow a forward-looking policy (Nelson (2000) provides an account of the period 1992 - 1997).

the indirect, inflation, channel of expectations without removing the producer's uncertainty about current fundamentals. I design forward-looking interest-rate rules which restore the complete information equilibrium. The rules "punish" deviations of expected inflation and expected growth from targets which adjust to their complete information levels.

In an extension, I consider a forward-looking monetary authority with superior information and let it communicate its information with noise. The noise could be thought of as a measurement error or a monetary policy shock. The nominal interest rate serves then as an endogenous public signal. To the extent that prices depend positively on productivity, I show that positive measurement errors and monetary policy shocks raise the producer's expectations about the following period's productivity which results in higher prices and output.

Related literature. The idea that changes in expectations affect the business cycle has its origins at least in Pigou (1926) and has recently been revived by Beaudry and Portier (2004).¹¹ Christiano et al. (2010) show that expectational shocks are disinflationary in a New-Keynesian framework.¹² However, this strand of literature distinguishes between shocks to current and future productivity, whereas I emphasize the distinction between fundamental and non-fundamental shocks to expectations.

This paper lies in the literature following Phelps (1970) and Lucas (1972) which has formalized the idea that incomplete information can open the door to non-neutralities of non-fundamental factors.¹³ The closest paper is Lorenzoni (2009). Lorenzoni (2009) restricts attention to the consumer side within a New-Keynesian framework and suggests that purely expectational shocks cause effects associated with demand shocks. Instead, I consider both the producer and the consumer side in a competitive economy with flexible prices¹⁴ and suggest that purely expectational shocks can behave like supply or demand shocks depending on the monetary policy pursued. To the best of my knowledge, this paper is the first to suggest so.

This paper shares with Weiss (1980), King (1982) and Lorenzoni (2010) the idea that monetary

¹¹See also Beaudry and Portier (2006, 2007) and Jaimovich and Rebelo (2009).

¹²It has also been suggested in the empirical work of Barsky and Sims (2011b).

¹³ Polemarchakis and Weiss (1977), Weiss (1980), King (1982), Bulow and Polemarchakis (1983) and, especially, Grossman and Weiss (1982) are related papers of the early literature. The literature has been revived with Woodford (2001), Morris and Shin (2002), Mankiw and Reis (2002) and Sims (2003). Hellwig (2008), Mankiw and Reis (2010), Lorenzoni (2011) and Chapter 9 in Veldkamp (2011) offer excellent surveys of the literature.

¹⁴ A strand of literature, which for instance includes Angeletos and La'O (2009), Angeletos and La'O (2011a) and La'O (2010), also considers both sides however within non-monetary "Lucas-islands" frameworks featuring Dixit-Stiglitz monopolistic competition. This strand of literature emphasizes the link between dispersed information and strategic complementarities across islands which I abstract from.

policy is non-neutral when there is asymmetric information about variables the monetary authority will respond to.¹⁵ Crucially, it is asymmetric, rather than incomplete but symmetric, information that breaks the policy irrelevance, proposed in Sargent and Wallace (1975, 1976). Furthermore, the proposed optimal policies here differ from the one in Weiss (1980). In Weiss (1980), prices perfectly reveal the unknown fundamentals, while here prices are observed with a delay, so, by construction, this possibility is non-existent.

The structure of the paper is as follows. Sections 2 and 3 present the model. Section 4 considers a “contemporaneous” interest-rate rule and shows that purely expectational shocks can have the features of demand or supply shocks for different policy specifications. Section 5 presents and analyzes the equilibria when a forward-looking interest-rate rule is followed and, in an extension, endows the monetary authority with superior information. Section 6 discusses the role of monetary policy and proposes optimal policies. Section 7 concludes.

2 Environment

The competitive economy features two agents: a representative consumer/worker supplying labor to a representative firm he owns and a producer managing the firm. The firm produces a non-storable commodity. The economy is cashless and the only relevant financial market is a nominal bond market; a monetary authority sets the price of a riskless short-term nominal bond according to a “Taylor-type” rule.¹⁶ Agents are price-takers in all markets. Time is discrete and infinite commencing in period 0. Each period comprises two stages: in stage 1 only the labor market opens, whereas in stage 2 the commodity and the nominal bond markets open.

The consumer’s preferences are given by

$$E_0^c \sum_{t=0}^{\infty} \beta^t U(C_t, N_t), \quad (1)$$

with period- t utility

$$U(C_t, N_t) = \log C_t - \frac{1}{1+\zeta} N_t^{1+\zeta}. \quad (2)$$

C_t and N_t denote consumption and employment in period t , respectively, and $\zeta > 0$ denotes the inverse of the constant marginal utility of wealth (“Frisch”) elasticity of labor supply. The consumer’s time preference is parametrized by $\beta \in (0, 1)$.

¹⁵ Recent papers studying monetary policy in environments with informational frictions include Adam (2007), Paciello and Wiederholt (2011) and Angeletos and La’O (2011b).

¹⁶ Chapter in Woodford (2003) provides a treatment of cashless monetary economies.

The consumer faces a sequence of budget constraints given by

$$P_t C_t + Q_t B_{t+1} = B_t + W_t N_t + \Pi_t, \quad (3)$$

where Q_t and B_{t+1} denote the price and holdings of nominal bonds maturing in $t+1$, respectively, P_t and W_t the commodity price and the nominal wage in t , respectively, and Π_t the firm's profits that accrue to the consumer.

The firm's technology is

$$Y_t = A_t N_t, \quad (4)$$

where A_t denotes the worker's productivity.

Productivity consists of a permanent and a temporary component (henceforth lowercase letters will denote natural logarithms),

$$a_t = x_t + u_t, \quad (5)$$

where x and u denote the permanent and temporary components, respectively. Productivity - not its components - is specific and known to the worker, whereas the producer faces uncertainty about it.¹⁷

The permanent component x_t follows a random walk stochastic process

$$x_t = x_{t-1} + \epsilon_t, \quad (6)$$

where ϵ_t is an i.i.d shock and $\epsilon \sim N(0, \sigma_\epsilon^2)$. The temporary component u_t is i.i.d. and $u \sim N(0, \sigma_u^2)$.

All agents have costless access to a public signal about the permanent productivity component

$$s_t = x_t + e_t, \quad (7)$$

where e_t is i.i.d. and $e \sim N(0, \sigma_e^2)$. Shocks u , ϵ , and e are mutually independent. Hereafter, I will call e a purely expectational shock.

The distinction between permanent and temporary productivity introduces persistence in the shock effects.

¹⁷ It may be argued that it is in the worker's best interest to reveal his type as he is the firm's owner. This is only an abstraction. Although I have not explored this possibility, an economy with many islands and complete financial markets which preserves the asymmetry of information *within* an island would presumably generate similar implications.

2.1 Timing

Each period is divided into two stages. In stage 1, the consumer realizes his temporal productivity a_t , agents and the monetary authority observe the public signal s_t about the permanent productivity component, and the labor market opens (and closes).

Let me note at this point that I split stage 1 into two sub-stages, although I will not make a distinction between these hereafter: in the first sub-stage of stage 1, after the new information is realized, the nominal wage is announced which, due to constant returns to scale, is independent of the labor submitted. The producer decides whether to “accept” it if it solves his problem, in which case he commits to accommodate any labor supply, or not. With the equilibrium nominal wage announced in sub-stage 1, in sub-stage 2 of stage 1 the worker decides on his labor supply and, since -as I argue below- he has complete information, his consumption and bond holdings.

In stage 2, with production predetermined from stage 1 (sub-stage 2), the commodity and the nominal-bond markets open (and close). I specify the role of the monetary authority in the following section. All payments materialize in stage 2 and are perfectly enforceable.

3 Towards Equilibria

The producer’s labor demand in stage 1 maximizes the firm’s expected profits, $E_t^p[\lambda_t \Pi_t | I_{t,1}^p]$, conditional on the producer’s information set in stage 1, $I_{t,1}^p$.¹⁸ Taking into account the firm’s technology (4), period-t profits are given by $\Pi_t = (P_t A_t - W_t) N_t$. Profits are evaluated according to the consumer/owner’s period-t Lagrange multiplier, λ_t . Constant returns to scale imply that the producer accommodates any labor supply at the following wage:¹⁹

$$W_t = \frac{E_t^p [\lambda_t P_t A_t]}{E_t^p [\lambda_t]} . \quad (8)$$

The consumer has complete information about the state of the economy and, as a result, makes all decisions in stage 1. He chooses consumption, labor supply, and bond holdings to maximize his expected utility (1)-(2) subject to his sequence of budget constraints (3) and a usual no-Ponzi-scheme constraint. Nominal bonds are in zero net supply, hence market clearing in the nominal

¹⁸Henceforth, the producer’s expectations will always refer to his expectations as of stage 1 unless otherwise stated.

¹⁹ It is central in the paper that the nominal wage prevailing in stage 1 be such that the producer’s *expected* evaluated profits are zero. Given the linear technology (4), at that nominal wage the producer is willing to hire any labor supplied which will typically result in a production level not desirable *ex-post*: once the state of the economy is realized, the real wage will typically be higher or lower than productivity, yielding losses or profits, respectively, with profits (losses) added (subtracted) in a lump-sum fashion to (from) the consumer/owner’s income. Even though, the nominal wage can be set flexibly, this specification could be roughly interpreted as a form of nominal wage stickiness.

bond market requires $B_{t+1} = 0$ for all t . As such, I suppress bond holdings from the state of the economy. The consumer's optimality conditions are²⁰

$$N_t^\zeta = \frac{W_t}{P_t C_t} \quad (9)$$

$$C_t = \frac{Q_t}{\beta P_t} E_t^c [P_{t+1} C_{t+1}] , \quad (10)$$

where $E_t^c [\cdot]$ refers to the consumer's expectations conditional on his information set I_t^c .

3.1 Linear equilibria

I focus on linear equilibria.²¹ All equilibria are rational expectations equilibria. In log-linear form the optimality equations are²²

$$w_t = E_t^p [a_t] + E_t^p [p_t] \quad (11)$$

$$\zeta n_t = w_t - p_t - c_t \quad (12)$$

$$c_t = -\log \beta + \log Q_t + E_t^c [c_{t+1} + \pi_{t+1}] . \quad (13)$$

Combining (11) and (12) results in

$$\zeta n_t = E_t^p [a_t] + E_t^p [p_t] - p_t - c_t . \quad (14)$$

I use the optimality conditions (13) and (14) in the rest of the analysis.

The existence of a monetary policy rule can get round the equilibrium indeterminacy, nominal or real depending on whether agents have complete information or not, that would have prevailed in its absence. However, as Section 5 illustrates, the presence of a monetary authority per se need not be enough.

²⁰Appendix A.1 provides the equilibrium definition and offers an analytical demonstration of the agents' problems.

²¹I ignore whether non-linear equilibria exist.

²²Where applicable, approximations are first-order around the stochastic steady state to be characterized in Section 4.4.

Monetary authority. The monetary authority sets the gross nominal interest rate (equivalently, the inverse of the logarithm of the nominal bond price), $i_t = -\log Q_t$, according to an interest-rate rule. Two commonly used rules will be considered in sequence, a contemporaneously-looking one (henceforth, rule 1) and a forward-looking one (henceforth, rule 2):²³

$$i_t = -\log \beta + \phi_\pi \pi_t + \phi_y (y_t - a_t) \quad (\text{Rule 1})$$

$$i_t = -\log \beta + \phi_\pi E_t^m [\pi_{t+1}]. \quad (\text{Rule 2})$$

where i_t denotes the nominal interest rate and π_t denotes inflation in period t , defined as $\pi_t := p_t - p_{t-1}$. In the case of rule 1, the monetary authority targets the output gap defined as the deviation of output from its complete information counterpart a_t . I restrict attention to positive values of the policy weights, ϕ_π and ϕ_y .

The monetary authority's information is solely based on the sequence of public signals as well as information extraction from prices and quantities. In Section 5.5, I let it be endowed with superior information when it follows rule 2 and subsequently study the information extraction problem of the agents. I consider more rules in Section 6 which explicitly studies the optimal monetary policies in the current framework.

3.2 Expectations and the state of the economy

The state of the economy as of period t coincides with the entire history $\Psi_t = \{(a_\tau)_{\tau=0}^t, (s_\tau)_{\tau=0}^t\}$. Past realizations of productivity and the public signal are part of the current state due to the agents' formation of expectations. In particular, the evolution of the agents' expectations about permanent productivity is given by the Kalman filter algorithm. This is because inflation (prices) and/or quantities perfectly reveal productivity in stage 2 of each period. For this reason, the monetary authority's information set when it steps in, I_t^m , coincides with the state. It follows then that $I_t^m = I_{t,2}^p = I_t^c = \Psi_t$. The producer's expectation about current productivity as of stage 1 coincides with his expectation about its permanent component which follows from (5) and the fact that his information set in stage 1, which I show in the next section, is $I_{t,1}^p = \{(a_\tau)_{\tau=0}^{t-1}, (s_\tau)_{\tau=0}^t\}$. More

²³ Rule 1 has been suggested by Taylor (1993, 1999) to capture adequately the Fed's policy during the period 1987-1992. Among other papers, rule 2 is considered in Clarida et al. (1999, 2000). Nelson (2000) proposes that a forward-looking rule fits well the Bank of England's policy in the period 1992-1997.

analytically and bearing (5) in mind, agents and the monetary authority's expectations evolve as

$$E_t^p[a_t] = E_{t,1}^p[x_t] = (1 - \mu) E_{t-1,2}^p[x_{t-1}] + \mu s_t \quad (15)$$

$$E_{t,2}^p[x_t] = E_t^c[x_t] = E_t^m[x_t] = (1 - k) E_{t-1}^c[x_{t-1}] + k [\theta s_t + (1 - \theta) a_t], \quad (16)$$

where μ, k, θ depend on the variances $\sigma_\epsilon^2, \sigma_e^2, \sigma_u^2$ and are in $(0, 1)$. Appendix A.2 offers an explicit treatment of the formation of expectations.

4 Equilibrium under Rule 1: Demand or Supply?

4.1 Complete information benchmark

Consider the case in which the state of the economy is common knowledge. Then, the real side of the economy is determined irrespectively of the public signal and the pursued monetary policy; we can confirm that $n_t^* = 0$ and $y_t^* = a_t$. On the nominal side, conjecture that $\pi_t = \vartheta_1 E_t^c[x_t] + \vartheta_2 a_t$ and then confirm that $\pi_t^* = \frac{1}{\phi_\pi} (E_t^c[x_t] - a_t)$. The consumer's expectations about permanent (long-run) productivity have only nominal effects: a consumption smoothing motive leads to changes in the consumer's current demand depending on his expectations about permanent productivity; however, flexible prices appropriately adjust in stage 2 and the nominal wage proportionally adjusts in stage 1 leaving the real wage intact and preventing the consumer's expectations from having real effects.

4.2 Incomplete information

Conjecture that

$$c_t = \xi_1 E_t^p[a_t] + \xi_2 a_t \quad (C1)$$

$$\pi_t = \kappa_1 E_t^p[a_t] + \kappa_2 E_t^c[x_t] + \kappa_3 a_t. \quad (C2)$$

Conjectures (C1) and (C2) imply the state of the economy can be summarized as $\Psi_t = \{E_t^p[a_t], E_t^c[x_t], a_t\}$. This is a direct consequence of the way agents form their expectations, described in Section 3.2, which disciplines the treatment of public signals and productivities within the state. The monetary authority can fully extract the current state by observing the public signal in stage 1 and inflation in stage 2 (alternatively, production or employment) which by conjecture (C2)

(respectively, (C1)) perfectly reveals productivity a_t . In other words, when the monetary authority steps in at the beginning of stage 2, it shares the same information set with the consumer. This applies to the producer in stage 2 as well; that is $I_t^m = I_{t,2}^p = I_t^c = \Psi_t$.

Adding and subtracting p_{t-1} in the labor market optimality condition (14) and combining the Euler equation (13) with rule 1 implies

$$\zeta n_t = E_t^p[a_t] + E_t^p[\pi_t] - \pi_t - c_t \quad (17)$$

$$c_t = -[\phi_\pi \pi_t + \phi_y (y_t - a_t)] + E_t^c[c_{t+1} + \pi_{t+1}], \quad (18)$$

respectively.

Combining conjectures (C1) and (C2) with the optimality conditions, (17) and (18), and market clearing (Appendix A.3 collects the derivations) yields

$$y_t = \xi_1 E_t^p[a_t] + (1 - \xi_1) a_t \quad (19)$$

$$\pi_t = \frac{1}{\phi_\pi} [-(1 + \phi_y) \xi_1 E_t^p[a_t] + E_t^c[x_t] + [(1 + \phi_y) \xi_1 - 1] a_t] \quad (20)$$

$$\xi_1 = \frac{\phi_\pi - 1 + k(1 - \theta)}{\phi_\pi(1 + \zeta) - (1 + \phi_y)}, \quad (21)$$

where k, θ are parameters associated with the consumer's learning problem introduced previously and derived in Appendix A.2.²⁴

Equation (19) shows that output is a weighted average²⁵ of productivity and the producer's expectations about it. The respective weights depend on the Frisch elasticity of labor supply, parametrized by ζ , and the monetary policy parameters ϕ_π, ϕ_y .

The presence of ϕ_π, ϕ_y in (19) leads to the *first* key remark: monetary policy is non-neutral. This is attributed to the *heterogeneity* of the agents' expectations in stage 1 about inflation in stage 2 as we can see from (17). Of course, heterogenous expectations are attributed to the agents' asymmetric information about current productivity. Crucially, incomplete yet symmetric information would imply a neutral monetary policy.

²⁴ Output is non-stationary. Stationarity can be restored by normalizing it with the permanent productivity component. For instance, in the case of output we could instead use $Y_t^s = \frac{Y_t}{e^{x_t}}$ ($y_t^s = y_t - x_t$ in logs). However, throughout the paper I use the non-normalized variables.

²⁵This is a direct consequence of preferences logarithmic in consumption.

A *second* key remark is that the consumer's expectations have real effects despite prices being flexible. Once again, this is a direct consequence of asymmetric information. To the extent that inflation depends on the consumer's expectations, the producer needs to second-guess the consumer when forming expectations about inflation.²⁶ In particular, as (90) in Appendix A.3 shows,

$$E_t^p [E_t^c [x_t]] = E_t^c [x_t] + k(1 - \theta)(E_t^p [a_t] - a_t).$$

What matters for the labor decision and hence production -through the inflation channel- is the *wedge* between the producer's and the consumer's expectations about inflation. Given conjecture (C2) and the fact that $E_t^c [\pi_t] = \pi_t$, it follows that

$$E_t^p [\pi_t] - E_t^c [\pi_t] = E_t^p [\pi_t] - \pi_t = [\kappa_2 k(1 - \theta) + \kappa_3](E_t^p [a_t] - a_t). \quad (22)$$

The presence of the parameter κ_2 in (22) attests that the consumer's expectations have real effects.

Importantly, what lies in the common information of the agents (for example, the producer's expectations) and what lies outside both agents' information sets (possibly, non-fundamental shocks - see fn. 52) has *no* real effects through the inflation channel.

I will first discuss purely expectational shocks, which operate only through agents' expectations. Insulating the analysis from productivity shocks will allow me to focus solely on the "mechanics" of agents' expectations. Subsequently I discuss productivity shocks which operate both directly and through agents' expectations. Before continuing, let me point out that

$$\kappa_1 + \kappa_2 + \kappa_3 = 0 \quad (23)$$

$$\kappa_1 + \kappa_3 = -\frac{1}{\phi_\pi}. \quad (24)$$

Combining (23) and (24) implies $\kappa_2 = \frac{1}{\phi_\pi} > 0$, which we can see in (20); the consumer's expectations are positively related to inflation, and, consequently, indirectly through inflation positively related to output. The logic underlying this is a permanent income hypothesis one: if, for instance, a purely expectational shock leads the consumer to overstate the long-run prospects of the economy, consumption smoothing results in an increase in current demand which in turn causes an inflationary pressure. If the producer had complete information, prices would fully absorb the

²⁶One could conjecture that consumption in (C1) also depends directly on the consumer's expectations only to verify that, in fact, the consumer's expectations do not enter equilibrium output directly. This happens because what matters for the labor decision in stage 1, and hence the real side of the economy, is productivity and the producer's -not the consumer's- expectations about it as well as about inflation, as (17) attests.

increased demand in stage 2 and nominal wages would proportionally adjust in stage 1; both would imply an unaffected real wage and, as a result, the absence of real effects. However, this is not the case under incomplete information: an overly optimistic producer -the public signal coordinates agents- overestimates the inflationary pressure. This implies the nominal wage increases more than proportionally compared to inflation, which results in a higher real wage. The latter causes labor to increase and production to expand, therefore partly accommodating the increased demand. Purely expectational shocks via the consumer's expectations push then toward a demand shock interpretation.

Turning to the producer, we can see from (19) - (21) that his expectations cause output and inflation to move in opposite directions. In other words, they point toward a supply-shock interpretation.²⁷ A sufficient condition for the producer's expectations to be positively related to output and negatively related to inflation is $\phi_\pi > \max\{\frac{1+\phi_y}{1+\zeta}, 1\}$.²⁸ That is for sufficiently “active” policies, expectational shocks via the producer's expectations push toward a co-monotone supply shock interpretation. As I have already implied, the inefficiency caused due to the producer's incomplete information manifests itself as a distortion in the labor optimality condition. In particular, it causes a shift in labor demand: the overly optimistic, for instance, expectations of the producer will result in a higher real wage. This induces the worker to increase his labor supply and, as a result, production to expand. For a given demand level, this causes a deflationary pressure; prices need to fall for the commodity market to clear.

Will a demand or a supply shock interpretation prevail for purely expectational shocks? Suppose that the expectational shock affects the agents' expectations in the same way.²⁹ Then it follows that a positive expectational shock lowers inflation as long as $\kappa_1 + \kappa_2 < 0$. By (23) and (24), this is equivalent to requiring $\kappa_3 > 0$. Inspecting (20), we can see that the term κ_2 does not respond to changes in the monetary policy weight on the output gap, ϕ_y , whereas κ_3 increases

²⁷ As already argued, since the producer second-guesses the consumer when forming expectations about inflation, the consumer's expectations matter indirectly as the term $\kappa_2 k(1-\theta)$ in (22) shows. In fact, since $\kappa_2 > 0$, this term only accentuates the supply shock interpretation as (92) in Appendix A.3 shows.

²⁸ For $\min\{\frac{1+\phi_y}{1+\zeta}, 1 - k(1-\theta)\} < \phi_\pi < \max\{\frac{1+\phi_y}{1+\zeta}, 1 - k(1-\theta)\}$, positive expectational shocks behave like negative supply shocks: they lower output and raise inflation. For appropriate policy parameters (ϕ_π, ϕ_y) , inflation depends negatively on productivity. Then the indirect, inflation, channel of expectations lowers the total effect of the producer's expectations: an overly optimistic producer expects the worker to be more productive than he is, while inflation lower than it will actually be. For the suggested parameter values, the negative indirect effect outweighs the positive direct one. As a result, the nominal wage increases by less than inflation, hence, the real wage falls compared to its complete information counterpart, which induces the worker to decrease his labor supply.

²⁹ That is I assume $k\theta = \mu$ in the learning problems of the agents. This is a good approximation if the temporary productivity shock has a high variance relative to the expectational shock, which is consistent with the parametrization in Table 1 below.

in it;³⁰ the sign of κ_3 depends on how the policy weight on the output gap relates to the Frisch elasticity of labor supply $1/\zeta$. In particular, a value of ϕ_y greater than or equal to the inverse Frisch elasticity of labor supply ζ is a sufficient condition for κ_3 to be positive and, consequently, expectational shocks to be negatively related to inflation.³¹

The picture that emerges is that expectational shocks can exhibit features associated with supply or demand shocks depending on the monetary policy pursued. The policy weight on the current output gap is central to how expectational shocks manifest themselves. In particular, as long as $\phi_\pi > \max\{\frac{1+\phi_y}{1+\zeta}, 1\}$, a higher weight on the output gap pushes toward a supply shock interpretation. To provide an intuition for this, first note that (20) implies expected inflation is zero, that is the nominal and the real interest rate coincide:

$$r_t = i_t = \phi_\pi \pi_t + \phi_y (y_t - a_t). \quad (25)$$

Fix for a moment the real interest rate and consider the case of overly optimistic expectations which implies a positive output gap. Controlling for general equilibrium effects, the higher the policy weight on the output gap, the lower the inflationary pressure has to be for the real rate to remain constant.³²

However, the real -and, hence, the nominal- interest rate increases in response to a positive purely expectational shock. This is a consequence of the overreaction of expectations: expected future output increases by more than current output since the latter is in part disciplined by current productivity, whose long-run component agents overstate. To what extent or whether this increase will be translated into higher inflation depends on the weight put on the (positive) output gap.

Turning to productivity shocks, $\phi_\pi > (1 + \phi_y) \max\{\frac{1}{1+\zeta}, \frac{k(1-\theta)}{\zeta - \phi_y}\}$ is a sufficient condition for them to be positively related to output. On the nominal side, maintaining the assumption that expectational shocks affect the agents' expectations in the same way, a direct implication of (23) and (24) is that productivity and expectational shocks cannot both increase or lower inflation. To connect the results with the previous analysis, consider a positive productivity shock. Under complete information, inflation would depend positively on the wedge between the consumer's expectations and productivity. Following a positive productivity shock the consumer's expectations underreact; as a result, demand underreacts as well which implies that prices must fall for the market to clear.

³⁰A sufficient condition for this is that $\phi_\pi > 1$.

³¹The term κ_3 exhibits discontinuity at $\phi_\pi(1 + \zeta) - 1$. As a result, this is true as long as $\phi_y < \phi_\pi(1 + \zeta) - 1$.

³²One may wonder what happens when the policy weight on inflation, ϕ_π , changes. In fact, general equilibrium effects complicate things considerably as both κ_2 and κ_3 (alternatively, κ_1) depend on ϕ_π . As a result, a similar reasoning applies only locally and it becomes hard to generalize. Hence, I will abstract from this consideration.

However, under incomplete information, prices will not fall as much as they would under complete information, whereas they can even increase. The reason is that the producer's expectations also underreact, hence supply underreacts as well.

Along the lines of the above analysis, the weight on the output gap proves to be key as to how supply responds. Revisiting the real side, the underreaction of the producer's expectations implies that the increase in output falls short of the increase in productivity, therefore the output gap is negative and employment falls. Holding the real and, since they coincide, the nominal interest rate constant, the higher the weight on the output gap, the lower the nominal interest rate will be, hence the less the required fall in prices (see also (25)). However, both the nominal and the real interest rate fall after a positive productivity shock, a consequence of the underreaction of expectations.

Last, note that for $E_t^p[a_t] = a_t$ the complete information equilibrium prevails.

4.3 Labor wedge

Formalizing the intuition above, the producer's incomplete information has an impact on his labor demand and, consequently, distorts the labor optimality condition. This causes fluctuations in the labor wedge. Following Chari et al. (2007) and Shimer (2009), the labor wedge is defined as the ratio of the marginal product of labor to the marginal rate of substitution of leisure for consumption by construction equal to $\frac{1}{1-\tau_{n,t}}$ in the expression below:

$$-\frac{U_{n,t}}{U_{c,t}} = (1 - \tau_{n,t}) MP_{n,t}.$$

$U_{n,t}$ and $U_{c,t}$ denote the marginal disutility of labor and marginal utility of consumption, respectively, and $MP_{n,t}$ denotes the marginal product of labor in period t . The above expression becomes in this case

$$N_t^{-(1+\zeta)} = \frac{1}{1 - \tau_{n,t}}.$$

Under complete information, $N_t^* = 1$ and the labor wedge is equal to 1. Under incomplete information this will generally not be the case; switching to logs and using $n_t = y_t - a_t$ from the firm's technology implies

$$n_t = \frac{\phi_\pi - 1 + k(1 - \theta)}{\phi_\pi(1 + \zeta) - (1 + \phi_y)} (E_t^p[a_t] - a_t). \quad (26)$$

For $\phi_\pi > \max\{\frac{1+\phi_y}{1+\zeta}, 1\}$, employment depends positively on the distance of the producer's expectations from the underlying productivity. The labor wedge in logs is given by the LHS below:

$$-\log(1 - \tau_{n,t}) = -\frac{[\phi_\pi - 1 + k(1 - \theta)](1 + \zeta)}{\phi_\pi(1 + \zeta) - (1 + \phi_y)} (E_t^p[a_t] - a_t). \quad (27)$$

Maintaining that $\phi_\pi > \max\{\frac{1+\phi_y}{1+\zeta}, 1\}$, in case $E_t^p[a_t] > a_t$, the log-labor wedge is negative, and positive, otherwise. Then purely expectational shocks induce a countercyclical labor wedge. This is easy to see: a positive, for instance, purely expectational shock raises output whereas it lowers the labor wedge. This is in line with the documented countercyclicality of the labor wedge (see for example Chari et al. (2007) and Shimer (2009)) and suggests that purely expectational shocks can possibly account for it. Interestingly, fluctuations in the labor wedge depend on the monetary policy pursued. Section 6 elaborates on this.

4.4 Equilibrium dynamics

I deal with this case numerically, even though a closed-form representation of the dynamics can be obtained along the lines of Section 5.3.4 below. The baseline parametrization is in Table 1. In that I follow Lorenzoni (2009) and one may check the references therein. The parametrization implies the Kalman gain terms, μ and k , are 0.22 and 0.23, respectively, whereas the relative weight the consumer places on the public signal, θ , is 0.96. In addition, I initially set the response to the output gap $\phi_y = 0.5$.³³

Before continuing with the impulse response functions, let me point out that the *stochastic* steady state is pinned down by the permanent productivity component x_t , which by (6) evolves as a random walk (see also fn. 24). The steady state is typically different from the efficient, complete information level of the economy which is pinned down by aggregate productivity a_t . In the analysis of the impulse response functions below, the economy has already reached its steady-state which, I assume, coincides with its complete information counterpart before any shocks hit. As such, the two will remain coincidental after a permanent productivity or an expectational shock and they will only differ on impact following a temporary productivity shock. In particular, the steady state is given by $a = x$ and $E^p[a] = E^c[x] = x$. With no loss of generality, I set $x = 0$.³⁴ In all figures, impulse response functions are for one standard deviation shocks. Periods, appearing on the horizontal axis of the figures, are interpreted as quarters.

Figure 5 shows the impulse responses to positive purely expectational shocks. As already argued, as expectations increase, output and employment increase, the labor wedge falls and the interest rates increase. For the considered parametrization, inflation falls. With no change in the underlying productivity, all effects die out in the long run and variables return to their steady-state values.

³³The monetary policy parameters are based on Taylor (1993).

³⁴These imply $y = x$, $n = 0$, $\pi = 0$, and $r = i = -\log \beta$. For ease of exposition, I have suppressed constants, hence in all figures the nominal and the real interest rate are zero at the steady state.

Table 1: Calibrated parameters

Inverse Frisch elasticity of labor supply	ζ	0.5
Monetary policy weight on inflation	ϕ_π	1.5
Standard deviation of permanent productivity shock	σ_e	0.0077
Standard deviation of temporary productivity shock	σ_u	0.15
Standard deviation of expectational shock	σ_e	0.03

Figure 6 shows that after a positive permanent productivity shock agents expectations underreact. This causes an increase in output, however by less than under complete information, which in turn causes a fall in employment. As employment falls, the labor wedge increases and is therefore procyclical. For the considered parametrization, inflation increases, whereas, since expectations underreact, the nominal and the real interest rate fall. As expectations converge to the underlying higher productivity level, all variables converge to their steady-state levels.

The impulse responses to a temporary productivity shock (Figure 7) are initially similar to the ones of a permanent productivity shock and, subsequently, to the ones of an expectational shock. As argued above, this is because they affect productivity only on impact, whereas from the following period onwards they serve as purely expectational shocks.

As I have already pointed out, the impulse responses when rule 1 is followed are generally sensitive to the specification of the monetary policy rule and, in particular, to the policy weight on the output gap, ϕ_y . Consider the case in which the authority does not respond to the output gap, that is $\phi_y = 0$, with all other parameters as in Table 1. Figures 8 - 10 show the impulse responses to one standard-deviation positive permanent productivity, expectational, and temporary productivity shocks, respectively. While everything else remains unchanged, the implications for inflation are reversed. In particular, positive permanent productivity shocks lower inflation whereas positive expectational shocks increase inflation. The last results are in line with Lorenzoni (2009). Notably, unlike in Lorenzoni (2009), they are generated in a perfectly competitive environment where prices are flexible and the real interest rate can freely adjust.

Juxtaposing figures 5 and 8 illustrates the first main result of the paper: purely expectational shocks can behave like supply or demand shocks. A natural question is why the current framework can accommodate both cases. The reasons are, first, the explicit role assigned to the producer's ex-

pectations and, second, the presence of asymmetric information between consumers and producers. Crucially, the latter pushes monetary policy and the consumer's expectations through the door.³⁵ The producer's expectations point toward a supply-shock interpretation, whereas the consumer's expectations, as in Lorenzoni (2009), point toward a demand-shock interpretation. The monetary authority decides which one will prevail.

5 Equilibria under Rule 2: Beyond Demand and Supply

5.1 Complete information benchmark

Like before, under incomplete information $y_t^* = a_t$ and $n_t^* = 0$. Conjecture for prices that $p_t = \vartheta_3 E_t^c[x_t] + \vartheta_4 a_t$. The Euler equation (13) becomes

$$E_t^c[a_{t+1}] - a_t = (\phi_\pi - 1)(E_t^c[p_{t+1}] - p_t). \quad (28)$$

A family of solutions is given by $p_t^* = \frac{1}{\phi_\pi - 1} a_t + \vartheta_3 E_t^c[x_t]$; price levels depend arbitrarily on the consumer's expectations.

5.2 Incomplete information

Consider the conjectures:³⁶

$$c_t = \xi_3 E_t^p[a_t] + \xi_4 a_t \quad (C3)$$

$$p_t = \kappa_4 E_t^p[a_t] + \kappa_5 E_t^c[x_t] + \kappa_6 a_t. \quad (C4)$$

Conjectures (C3) and (C4) imply the state can sufficiently be described by $\Psi_t = \{E_t^p[a_t], E_t^c[x_t], a_t\}$. The information sets of the agents and the monetary authority are like before. Since $I_t^m = I_t^c$, the Euler equation (13) becomes

$$E_t^c[c_{t+1}] - c_t = (\phi_\pi - 1)(E_t^c[p_{t+1}] - p_t). \quad (29)$$

³⁵It is key that the consumer has complete information about the current state; this enables me to abstract from wealth effects which are the subject of Beaudry and Portier (2004) and Jaimovich and Rebelo (2009) among other papers.

³⁶See also fn. 26 on why I do not include $E_t^c[x_t]$ in (C3).

Taking familiar steps (see Appendix A.4) yields

$$\xi_3 = \frac{1 + \kappa_6 + k(1 - \theta)\kappa_5}{1 + \zeta} \quad (30)$$

$$\xi_4 = 1 - \xi_3 \quad (31)$$

$$\xi_3 = (\phi_\pi - 1)\kappa_4 \quad (32)$$

$$\kappa_4 + \kappa_6 = \frac{1}{\phi_\pi - 1}. \quad (33)$$

There are 4 equations and 5 unknowns. Equations (30) - (31) follow from the labor market optimality condition (14), whereas (32) - (33) follow from the Euler equation (29). When matching coefficients, an equation is missing from the latter because the real interest rate is determined irrespectively of the consumer's expectations.

Combining (30) - (33) yields

$$\xi_3 = \frac{\phi_\pi}{\phi_\pi + \zeta(\phi_\pi - 1)} + \frac{(\phi_\pi - 1)k(1 - \theta)}{\phi_\pi + \zeta(\phi_\pi - 1)}\kappa_5. \quad (34)$$

Prices can be expressed as

$$p_t = \frac{1}{\phi_\pi - 1}y_t + \kappa_5 E_t^c[x_t]. \quad (35)$$

As in the case of rule 1, asymmetric information implies monetary policy and the consumer's expectations have real effects as the presence of ϕ_π and κ_5 , respectively, in (34) attests.

However, a crucial difference with the case of rule 1 is the existence of multiple equilibria each corresponding to a different value of κ_5 . An immediate monetary policy implication is that targeting expected inflation invites multiple (linear) equilibria, notably for any value ϕ_π in the interest-rate rule. Interestingly, the role of the consumer's expectations is arbitrarily specified across equilibria, which is the second main finding of the paper. As already pointed out, this is because the real interest rate is independent of the consumer's expectations. Additionally, rule 2 specifies price levels as opposed to inflation in the case of rule 1.

As expected, depending on κ_5 , expectational and productivity shocks can raise or lower employment and price levels. Further, equation (34) suggests that short-run output volatility caused by expectational shocks increases in the absolute value of κ_5 .

5.3 A baseline equilibrium

To explore the dynamics of the producer's expectations in the equilibrium under rule 2, I will suppress the role of the consumer's expectations. This corresponds to setting $\kappa_5 = 0$ in (34) and (35). It is straightforward to extend the results to equilibria with $\kappa_5 \neq 0$.

5.3.1 Complete information benchmark

Like before, on the real side complete information implies $y_t^* = a_t$ and $n_t^* = 0$. A solution for price levels is $p_t^* = \frac{1}{\phi_\pi - 1} a_t$.³⁷

5.3.2 Incomplete information

Setting $\kappa_5 = 0$ in (34) and (35) pins down the equilibrium given by equations (36) - (37) below:³⁸

$$y_t = \frac{1}{\phi_\pi + \zeta(\phi_\pi - 1)} (\phi_\pi E_t^p [a_t] + \zeta(\phi_\pi - 1) a_t) \quad (36)$$

$$p_t = \frac{1}{\phi_\pi - 1} y_t. \quad (37)$$

Like before, equation (36) shows that output is a weighted average (also fn. 25) of productivity and the producer's expectations about it. The respective weights depend on the Frisch elasticity of labor supply, parametrized by ζ , and the monetary policy parameter ϕ_π . By (37), prices are a monotone transformation of output.³⁹ For an “active” monetary policy ($\phi_\pi > 1$), output and prices depend positively on the producer's expectations about productivity and productivity itself.⁴⁰

It follows from (36) and (37) that

³⁷ Since $E_t^c [a_{t+1}] = E_t^c [x_t] \neq a_t$ (see also Section 3.2), the possibility of prices being fixed in equilibrium appears only as a limit case for $\phi_\pi \rightarrow \infty$. It is also a possibility in the special case where $\sigma_e^2 = \sigma_u^2 = 0$. Constant prices could also have prevailed (as a unique non-explosive path) if either productivity a_t evolved as a random walk, or if the economy was a static one.

³⁸ Conjectures (C3) and (C4) for $\kappa_5 = 0$ combined with (11) imply $w_t = (1 + \kappa_4 + \kappa_6) E_t^p [a_t]$; the nominal wage perfectly reveals $E_t^p [a_t]$ to the consumer and the monetary authority in stage 1. Hence, if the signal s_t , instead of publicly observed, was privately observed by the producer, the nominal wage would generally perfectly communicate it to the consumer and the monetary authority.

³⁹ Output and prices are non-stationary. See also fn. 24.

⁴⁰ For $\frac{1}{1+\zeta} < \phi_\pi < 1$ output depends positively on the producer's expectations about productivity and negatively on productivity, whereas for $0 < \phi_\pi < \frac{1}{1+\zeta}$ it depends negatively on the producer's expectations and positively on productivity. The opposite relations are true for price levels. Employment has the same sign as the weight of expectations in output as (39) below shows.

$$\pi_t = \frac{1}{(\phi_\pi - 1)[\phi_\pi + \zeta(\phi_\pi - 1)]} (\phi_\pi (E_t^p[a_t] - E_{t-1}^p[a_{t-1}]) + \zeta(\phi_\pi - 1)(a_t - a_{t-1})) . \quad (38)$$

Inflation, by equation (38), is a weighted average of the change in producer's expectations and the change in productivity in the last two periods.

Let me make some remarks. First, each value of ϕ_π is associated with a unique equilibrium; the equilibrium with constant prices is obtained in the limit as $\phi_\pi \rightarrow \infty$. Second, observe that the optimal monetary policy in the baseline equilibrium under rule 2 is a zero-response to expected inflation policy, $\phi_\pi = 0$. In this case, all variables are at their complete information (efficient) level. I elaborate on this in Section 6 where I further consider an enriched version of rule 2. Last, note that for $E_t^p[a_t] = a_t$ the complete information equilibrium prevails.

5.3.3 Labor wedge

It follows from (36) and the firm's technology (4) that

$$n_t = \frac{\phi_\pi}{\phi_\pi + \zeta(\phi_\pi - 1)} (E_t^p[a_t] - a_t) . \quad (39)$$

Equation (39) shows that employment depends proportionally on the wedge between the producer's expectations about productivity and productivity itself. Taking the same steps as in the case of rule 1, the labor wedge in logs is given by

$$-\log(1 - \tau_{n,t}) = -\frac{\phi_\pi(1 + \zeta)}{\phi_\pi + \zeta(\phi_\pi - 1)} (E_t^p[a_t] - a_t) . \quad (40)$$

For $\phi_\pi > \frac{1}{1+\zeta}$, in case $E_t^p[a_t] > a_t$, the log-labor wedge is negative, and positive, otherwise. In addition, it is decreasing in the monetary policy parameter, ϕ_π ⁴¹ and becomes zero for $\phi_\pi = 0$. We can once again observe that purely expectational shocks induce a countercyclical labor wedge, which is in line with the documented countercyclicality of the labor wedge.

5.3.4 Equilibrium dynamics

Turning to the impulse response functions, the signs I report below refer to $\phi_\pi > 1$; that is the monetary authority follows an “active” policy, along the lines of Taylor (1999).⁴² Figures 11-16 show the impulse response functions to one-standard deviation shocks for the parametrization in Table 1. Periods are interpreted as quarters.

⁴¹Note that there is a discontinuity for $\phi_\pi = \frac{1}{1+\zeta}$.

⁴²See fn. 40 for the dynamics when $\phi_\pi < 1$.

If a unit shock expectational shock, e_t , arises, the consumer's expectations in period $t+s$ increase by $(1-k)^s k \theta$. The producer's expectations increase on impact by μ and in period $t+s$ for $s \geq 1$ by $(1-k)^{s-1} (1-\mu) k \theta$. The impulse response functions are

$$\frac{dy_t}{de_t} = \frac{dn_t}{de_t} = \frac{\phi_\pi \mu}{\phi_\pi + \zeta(\phi_\pi - 1)} > 0 \quad (41)$$

$$\frac{dy_{t+s}}{de_t} = \frac{dn_{t+s}}{de_t} = (1-k)^{s-1} \frac{\phi_\pi (1-\mu) k \theta}{\phi_\pi + \zeta(\phi_\pi - 1)} > 0, \quad \text{for } s \geq 1 \quad (42)$$

$$\frac{d\pi_t}{de_t} = \frac{\phi_\pi \mu}{(\phi_\pi - 1)[\phi_\pi + \zeta(\phi_\pi - 1)]} > 0 \quad (43)$$

$$\frac{d\pi_{t+1}}{de_t} = - \frac{\phi_\pi [\mu - (1-\mu) k \theta]}{(\phi_\pi - 1)[\phi + \zeta(\phi_\pi - 1)]} \quad (44)$$

$$\frac{d\pi_{t+s}}{de_t} = - (1-k)^{s-2} \frac{\phi_\pi (1-\mu) k^2 \theta}{(\phi_\pi - 1)[\phi + \zeta(\phi_\pi - 1)]} < 0, \quad \text{for } s \geq 2. \quad (45)$$

Equations (41) and (42) (also Figure 11) demonstrate the positive co-movement result, already discussed: output and employment increase in response to a positive expectational shock. The result is due to the producer overstating the worker's productivity. In the limit as $s \rightarrow \infty$, expectations converge to the true level of productivity implying both output and employment return to their steady-state levels.

A key difference between the equilibrium under rule 1 and the equilibrium under rule 2 is that the former specifies inflation whereas the latter price levels. In the baseline case considered here, prices are positively related to the producer's expectations. Hence, a positive expectational shock causes an increase in price levels (Figure 12). However, as agents update their beliefs over time, their expectations become more aligned with fundamentals and, hence prices return monotonically to their steady-state level, $p = \frac{1}{\phi_\pi - 1} x$, generating thereby a deflationary pressure as (45) shows from the following period onwards.⁴³ Put differently, price levels respond non-monotonically to positive expectational shocks. They are higher compared to their complete information level, yet inflation,

⁴³ Whether there is inflation or disinflation in period $t+1$ depends on the variances of the shocks. The parametrization here implies the latter.

by definition, measures changes in price levels between periods.⁴⁴ All effects vanish as $s \rightarrow \infty$.

To pin down the impulse responses of the nominal and the real interest rate (Figure 12) I need to specify the impulse response of expected inflation:

$$\frac{dE_t^c[\pi_{t+1}]}{det} = \frac{\zeta(\phi_\pi - 1)k\theta - \phi_\pi(\mu - k\theta)}{(\phi_\pi - 1)[\phi_\pi + \zeta(\phi_\pi - 1)]} \quad (46)$$

$$\frac{dE_{t+s}^c[\pi_{t+s+1}]}{det} = (1-k)^{s-1} \frac{\phi_\pi(\mu - k) + [\zeta(\phi_\pi - 1)(1-k)]k\theta}{(\phi_\pi - 1)[\phi_\pi + \zeta(\phi_\pi - 1)]}, \quad \text{for } s \geq 1. \quad (47)$$

The nominal interest rate is $i_{t+s} = \phi_\pi E_{t+s}^c[\pi_{t+s+1}]$ and the real interest rate is $r_{t+s} = (\phi_\pi - 1) E_{t+s}^c[\pi_{t+s+1}]$.⁴⁵

Inflation expectations increase,⁴⁶ given the parametrization, resulting in higher nominal and real interest rates. In the limit $s \rightarrow \infty$, inflation expectations, the nominal, and the real interest rate all return to their steady-state values.

If a shock to the permanent productivity component $\epsilon_t = 1$ arises, the consumer's expectations about productivity in period $t+s$ increase by $1 - (1-k)^{s+1}$ as (16) implies, whereas the producer's expectations increase by $1 - (1-\mu)[1 - (1-k)^s]$ as (15) implies. The impulse response functions are

$$\frac{dy_{t+s}}{d\epsilon_t} = 1 - (1-k)^s \frac{\phi_\pi(1-\mu)}{\phi_\pi + \zeta(\phi_\pi - 1)} \in (0, 1) \quad (48)$$

$$\frac{dn_{t+s}}{d\epsilon_t} = -(1-k)^s \frac{\phi_\pi(1-\mu)}{\phi_\pi + \zeta(\phi_\pi - 1)} < 0 \quad (49)$$

$$\frac{d\pi_t}{d\epsilon_t} = \frac{1}{\phi_\pi - 1} \left(1 - \frac{\phi_\pi(1-\mu)}{\phi_\pi + \zeta(\phi_\pi - 1)} \right) > 0 \quad (50)$$

$$\frac{d\pi_{t+s}}{d\epsilon_t} = (1-k)^{s-1} \frac{\phi_\pi(1-\mu)k}{(\phi_\pi - 1)[\phi_\pi + \zeta(\phi_\pi - 1)]} > 0, \quad \text{for } s \geq 1. \quad (51)$$

⁴⁴A similar result is obtained in Lorenzoni (2005), though not associated with disinflation. The increase in prices can become less severe and prices can even fall for reasonable values of ϕ_π under an extended forward-looking rule targeting expected growth in addition to expected inflation. The logic is similar to the rule 1 case: for a given real interest rate, the greater the weight placed on expected growth, the lower expected inflation will be, controlling for general equilibrium effects.

⁴⁵In addition, notice that $E_t^c[y_{t+s}] = E_t^c[x_t]$ and $E_t^c[\pi_{t+s}] = 0$ for $s \geq 1$. These results follow from (36) and (38) combined with (16).

⁴⁶This is unlike the case of rule 1. The difference between the two lies in that rule 1 specifies inflation rather than price levels.

A unit increase in the permanent productivity shock causes an equivalent change in steady-state output and no change in steady-state employment. We can see from (48) and (49) (see also Figure 13) that a positive permanent productivity shock causes output to increase by less than one and employment to fall temporarily. By (40), the labor wedge increases temporarily. This happens because expectations underreact after a positive permanent productivity shock. As a result, labor demand shifts inwards and the real wage falls relative to its efficient level. Equation (51) suggests productivity shocks are inflationary (see also Figure 14). The positive dependence of prices on expectations for $\phi_\pi > 1$, as (36) and (37) imply, underlies this result. Hence, as expectations converge to the new permanent productivity level, prices get closer to their steady-state level, implying inflation along the way.

The impulse response of the consumer's inflation expectations (also Figure 14) is

$$\frac{dE_{t+s}^c [\pi_{t+s+1}]}{d\epsilon_t} = (1 - k)^s \frac{\phi_\pi (k - \mu) - \zeta(\phi_\pi - 1)(1 - k)}{(\phi_\pi - 1)[\phi_\pi + \zeta(\phi_\pi - 1)]}. \quad (52)$$

Figure 14 shows that following a permanent productivity shock inflation expectations fall and so are the nominal and the real interest rate. In the limit as $s \rightarrow \infty$, expectations converge to the new productivity level, output converges to its new steady state, whereas the remaining variables return to their pre-shock levels.

A temporary productivity shock causes on impact responses similar to those in the permanent productivity shock case; from the following period onwards, it only affects the agents' expectations, hence the responses resemble the ones in the expectational shock case. The consumer's expectations in period $t + s$ increase by $(1 - k)^s k (1 - \theta)$, whereas the producer's expectations are unchanged on impact, as changes in the temporary productivity component affect their expectations with one-period lag, and increase by $(1 - k)^{s-1} (1 - \mu) k (1 - \theta)$ in period $t + s$ for $s \geq 1$. In particular, in period t the responses are

$$\frac{dy_t}{du_t} = \frac{\zeta(\phi_\pi - 1)}{\phi_\pi + \zeta(\phi_\pi - 1)} \in (0, 1) \quad (53)$$

$$\frac{dn_t}{du_t} = - \frac{\phi_\pi}{\phi_\pi + \zeta(\phi_\pi - 1)} < 0 \quad (54)$$

$$\frac{d\pi_t}{du_t} = \frac{\zeta}{\phi_\pi + \zeta(\phi_\pi - 1)} > 0. \quad (55)$$

In the subsequent periods the responses are

$$\frac{dy_{t+s}}{du_t} = \frac{dn_{t+s}}{du_t} = (1-k)^{s-1} \frac{\phi_\pi (1-\mu) k (1-\theta)}{\phi_\pi + \zeta(\phi_\pi - 1)} > 0, \quad \text{for } s \geq 1 \quad (56)$$

$$\frac{d\pi_{t+1}}{du_t} = - \frac{\zeta(\phi_\pi - 1) - \phi_\pi (1-\mu) k (1-\theta)}{(\phi_\pi - 1) [\phi_\pi + \zeta(\phi_\pi - 1)]} \quad (57)$$

$$\frac{d\pi_{t+s}}{du_t} = -(1-k)^{s-2} \frac{\phi_\pi (1-\mu) k^2 (1-\theta)}{(\phi_\pi - 1) [\phi_\pi + \zeta(\phi_\pi - 1)]} < 0, \quad \text{for } s \geq 2. \quad (58)$$

The response of inflation expectations is given by

$$\frac{dE_t^c[\pi_{t+1}]}{du_t} = \frac{\phi_\pi k (1-\theta) - \zeta(\phi_\pi - 1) [1 - k (1-\theta)]}{(\phi_\pi - 1) [\phi_\pi + \zeta(\phi_\pi - 1)]} \quad (59)$$

$$\frac{dE_{t+s}^c[\pi_{t+s+1}]}{du_t} = (1-k)^{s-1} \frac{[\phi_\pi (\mu - k) + \zeta(\phi - 1) (1-k)] k (1-\theta)}{(\phi_\pi - 1) [\phi_\pi + \zeta(\phi_\pi - 1)]}, \quad \text{for } s \geq 1. \quad (60)$$

Figures 15 and 16 display the impulse response functions.

5.4 Short-run volatility

In what is a separate exercise, I compare the short-run (one-period) output volatility caused by purely expectational shocks, e_t , among the equilibria for the considered interest-rate rules.⁴⁷ The parametrization is the one in Table 1. I normalize to one the short-run output volatility generated by rule 1 for $\phi_y = 0$ to make comparisons easier. Table 2 reports the results.

We can see that the baseline case of rule 2 generates considerably higher short-run output volatility than the considered cases of rule 1. This can be further increased by assigning a role to the consumer's expectations (see also (34)). Considering the analyzed equilibria for rule 1, “supply” shocks ($\phi_y = 0.5$) generate considerably higher volatility than “demand” shocks ($\phi_y = 0$). Indeed, for ϕ_π high enough so that $\phi_\pi (1 + \zeta) - (1 + \phi_y) > 0$, the short-run output volatility due to expectational shocks increases in ϕ_y .

⁴⁷Short-run output volatility in the cases of rule 1 and 2, respectively, is

$$\frac{\phi_\pi - 1 + k (1-\theta)}{\phi_\pi (1 + \zeta) - (1 + \phi_y)} \mu \sigma_e^2$$

$$\frac{\phi_\pi}{\phi_\pi + \zeta (\phi_\pi - 1)} \mu \sigma_e^2.$$

Table 2: Short-run volatility

Rule 1 ($\phi_y = 0$)	1
Rule 1 ($\phi_y = 0.5$)	2.78
Rule 2 (baseline)	4.43

5.5 Monetary authority with superior information

In this section I lift the assumption that the monetary authority has no superior information compared to the agents. Instead, I assume that the monetary authority has information about the following period's state. To prevent the forward-looking⁴⁸ nominal interest rate from being fully revealing about the following period's state, I require that the monetary authority either reports the following period's price with a measurement error or transmits "surprise" monetary policy shocks. In both cases the nominal interest rate serves as a public signal about the following period's productivity. However, in the former case the monetary authority misreports the following period's prices unintentionally, as opposed to intentionally in the latter. The aim of this section is twofold: first, to analyze the informational implications per se when the monetary authority communicates its superior information with noise; second, to equip the monetary authority with an additional monetary policy tool, the monetary policy shocks, and pin down its equilibrium effects. I further explore monetary policy shocks in Section 6. The focus throughout this section will be on the baseline case of rule 2, which corresponds to setting $\kappa_5 = 0$ in (34) and (35). Extending the results to the other equilibria is straightforward.

When the monetary authority reports the following period's price with a measurement error, the prevailing nominal interest rate in $t - 1$ is

$$i_{t-1} = \phi_\pi \tilde{\pi}_t, \quad (61)$$

where $\tilde{\pi}_t \equiv \tilde{p}_t - p_{t-1}$, with

$$\tilde{p}_t = p_t + w_t. \quad (62)$$

The error term is i.i.d with $w_t \sim N(0, \sigma_w^2)$ and is independent of the shocks ϵ_t , e_t , and u_t .

⁴⁸ Since agents have complete information about the current state when the monetary authority steps in, there can only be information extraction if the monetary authority is forward-looking. Therefore, I restrict attention only to rule 2.

In terms of observables as of stage 2 in period $t - 1$, this can be expressed as

$$\tilde{p}_t = \frac{1}{\phi_\pi} (i_{t-1} + \phi_\pi p_{t-1}).$$

In the case of monetary policy shocks the nominal interest rate is

$$i_{t-1} = \phi_\pi \pi_t + \omega_t, \quad (63)$$

where ω is i.i.d. with $\omega_t \sim N(0, \sigma_\omega^2)$ and is independent of the shocks ϵ_t , e_t , u_t , and w_t .

Agents now extract

$$\hat{p}_t = \phi_\pi p_t + \omega_t, \quad (64)$$

which in terms of observables in stage 2 of period $t - 1$ can be expressed as

$$\hat{p}_t = i_{t-1} + \phi_\pi p_{t-1}.$$

5.6 Linear equilibria

Equilibrium is given by equations (36) - (38). The state of the economy is now augmented by the public signal about period t 's productivity which the monetary authority transmits. I denote this by z_t in the case of a measurement error and \hat{z}_t in the case of a monetary policy shock. The state can sufficiently be described then by $\Omega_t = (\{a_\tau\}_{\tau=0}^t, \{s_\tau\}_{\tau=0}^t, z_t)$, replacing Ω_t with $\hat{\Omega}_t$ and z_t with \hat{z}_t in the case of a monetary policy shock. What distinguishes the two cases is the information set of the monetary authority; in the case of measurement errors it is $I_t^m = \Omega_t \setminus \{z_t\}$, whereas in the case of monetary policy shocks it is $I_t^m = \hat{\Omega}_t$. That is, in the latter case, the monetary authority takes into account the effects of the signal it transmits. I assume it is common knowledge what the case is each time a shock hits. As I show in Appendix A.4.1, the endogenous public signals associated with the two cases are

$$z_t = a_t + \frac{\phi_\pi + \zeta(\phi_\pi - 1)}{\zeta} w_t \quad (65)$$

$$\hat{z}_t = a_t + \frac{\phi_\pi + \zeta(\phi_\pi - 1)}{\phi_\pi \zeta} \omega_t. \quad (66)$$

Agents (perfectly) disentangle the endogenous public signals upon the realization of the public signal s_t in stage 1 of period t .⁴⁹ The producer's information set then becomes $I_{t,1}^p = \Omega_t \setminus \{a_t\}$,

⁴⁹ This happens because they know the stochastic process of prices given by (37).

whereas the consumer's $I_t^c = \Omega_t$. As I show in Appendix A.4.1, the producer's expectations about productivity are

$$E_t^p [a_t | I_{t,1}^p] = \delta E_t^p \left[x_t | I_{t,1}^p \setminus \{z_t\} \right] + (1 - \delta) z_t, \quad (67)$$

where δ is a coefficient in $(0, 1)$ (respectively use $\hat{\delta}$, \hat{z}_t and $\hat{\Omega}_t$ in the case of a monetary policy shock). Importantly, δ and $\hat{\delta}$ depend on the monetary policy parameter ϕ_π .⁵⁰

It is apparent from (65) and (66) that the economy's response to measurement errors and monetary policy shocks is very similar. In particular, for $\phi_\pi > 1$ positive interest rate shocks raise the producer's expectations about productivity in the following period. This happens because for $\phi_\pi > 1$ prices are positively related to productivity. Therefore, a higher nominal interest rate overstates the following period's price and leads the producer to partially attribute it to an increase in productivity.⁵¹

5.7 Equilibrium dynamics

The dynamics when shocks ϵ_t , e_t , and u_t are realized are very similar to the ones in Section 5.3.4.

Unlike there, the effects of a measurement error or a monetary policy shock last only one period. This is because it generates a signal about a_t , which consumers learn and producers realize once the labor decision is made. If a shock $w_t = 1$ arises, the impact responses are

$$\frac{dy_t}{dw_t} = \frac{dn_t}{dw_t} = \frac{\phi_\pi (1 - \delta)}{\zeta} > 0 \quad (68)$$

$$\frac{dp_t}{dw_t} = \frac{d\pi_t}{dw_t} = -\frac{d\pi_{t+1}}{dw_t} = \frac{\phi_\pi (1 - \delta)}{\zeta (\phi_\pi - 1)} > 0. \quad (69)$$

It can be seen from (68) and (69) that interest rate shocks boost output and prices. These responses are in the same direction as the ones after a shock to the public signal s_t . This is because measurement errors and monetary policy shocks serve as purely expectational shocks: when positive,

⁵⁰ The case analyzed in Section 3 corresponds to $\delta = 1$ which would prevail if the conditional variance of the endogenous signals was infinite.

⁵¹ In case the monetary authority has no superior information and this is common knowledge, monetary policy shocks have no real effects because they are unanticipated by both agents, hence they have no effect on the labor decision in stage 1. They can immediately be extracted by the agents which implies they have no effect on the consumer's inflation and output expectations for the following period. As a result, they only affect the current price, in a co-monotone way for $\phi_\pi > 1$. On the contrary, in the superior information case agents extract monetary policy shocks with one-period lag, hence their nominal effects are realized in the following period. In addition, monetary policy shocks have real effects since they are not simultaneously fully extracted by both agents.

they increase the producer's expectations about productivity without any change in the underlying fundamentals.

The impact responses to a policy shock $\omega_t = 1$ are scaled down by ϕ_π as (66) suggests:

$$\frac{dy_t}{d\omega_t} = \frac{dn_t}{d\omega_t} = \frac{(1 - \hat{\delta})}{\zeta} > 0 \quad (70)$$

$$\frac{dp_t}{d\omega_t} = \frac{d\pi_t}{d\omega_t} = -\frac{d\pi_{t+1}}{d\omega_t} = \frac{(1 - \hat{\delta})}{\zeta(\phi_\pi - 1)} > 0. \quad (71)$$

The previous comments apply. However, in the next section I show that the two cases generate partly different monetary policy implications.

6 Monetary Policy

The equilibrium nominal wage in stage 1 is given by

$$w_t = E_t^p[a_t] + E_t^p[p_t].$$

Consequently, through the nominal wage, the real side of the economy reflects the producer's expectations about productivity. The producer's expectations enter the nominal wage both directly and indirectly through inflation in the case of rule 1 and prices in the case of rule 2. Monetary policy can have real effects through the indirect inflation (price) channel. To see this, observe that the labor market optimality condition (17) can more generally be written

$$\zeta n_t = E_t^p[a_t] + E_t^p[\pi_t] - E_t^c[\pi_t] - E_t^c[c_t]. \quad (72)$$

Taking the producer's uncertainty as given, monetary policy has real effects as long as agents form *heterogeneous* expectations about the inflation to prevail in stage 2, that is $E_t^p[\pi_t] \neq E_t^c[\pi_t]$. By construction, this is the case here. Crucially, what matters for labor decision and, hence the real side, is the wedge in the agents's expectations about inflation, $E_t^p[\pi_t] - E_t^c[\pi_t]$. Anything common in the agents' information sets and anything lying outside both agents' information sets (for instance, non-fundamental shocks - see fn. 52 below) has no real effects through the inflation channel. Then, it should not perhaps come as a surprise that incomplete yet symmetric information about current productivity would imply a neutral monetary policy.

The producer's incomplete information is the only source of inefficiency. Optimal monetary policy restores then the complete information equilibrium. An infinitely aggressive policy on inflation

policy implies $\pi_t = 0$ and only removes the indirect, inflation (price) channel of expectations. As a result, it is typically suboptimal.

By direct implication of (14), the complete information equilibrium is restored if and only if

$$E_t^p[a_t] + E_t^p[\pi_t] - (\pi_t + a_t) = 0 \quad (73)$$

$$E_t^p[a_t] + E_t^p[p_t] - (p_t + a_t) = 0, \quad (74)$$

for rules 1 and 2, respectively.

It follows that monetary policy succeeds, not by removing the producer's uncertainty, but rather by making it irrelevant. To see this, note that inflation (prices) depends on productivity and agents' expectations in a way decided by monetary policy. Optimal monetary policy manipulates inflation in such a way that the producer correctly anticipates his stage-2 revenue, which is all he is interested in.

One would argue that the inefficiency here arises exactly because of agents's asymmetric information; if agents had incomplete yet symmetric information, then the complete information equilibrium would prevail. However, this is true only because of logarithmic preferences in consumption; in more general environments, the producer's incomplete information would suffice. Nevertheless, it is asymmetric, rather than incomplete but symmetric, information in combination with the existence of a nominal bond market that enables the monetary authority to drive the economy closer to the complete information equilibrium. If a real bond market was in the place of the nominal bond market, then the inflation (price) channel would be absent, and there would be no way to drive the economy to the first best.

Optimal policy here has different implications from the one in Weiss (1980) which implies that prices perfectly communicate fundamentals. By construction, this is a nonexistent possibility here. However, this paper shares with Weiss (1980), King (1982) and Lorenzoni (2010) the insight that, at the time the labor decision is made, it is asymmetric, as opposed to incomplete but symmetric, information about variables the monetary policy will be based on that breaks the policy irrelevance proposed in Sargent and Wallace (1975, 1976). Implicit in this is that the monetary authority is more informed when it steps in than the least informed agent (here, the producer) at the time the labor decision is made. This is true here since the time advantage of the monetary authority is essentially an informational advantage; in fact, the monetary authority perfectly observes or extracts the variables in question (inflation, output, current productivity) when it steps in.

Crucially, that inflation stabilization is suboptimal is in contrast with the baseline case in Lorenzoni (2009) in which the limit $\phi_\pi \rightarrow \infty$ restores the efficient equilibrium. In Lorenzoni (2009), producers have complete information, however nominal rigidities prevent prices from fully absorbing the consumer's expectations about long-run productivity. Stabilizing inflation resolves this. In contrast, here prices flexibly adjust, however producers have incomplete information about productivity and, consequently, their (stage-2) revenue; stabilizing inflation only eliminates their uncertainty about inflation but not about their revenue.

Below I consider both interest-rate rules and explore how the monetary authority can mitigate the effects of incomplete information and drive the economy closer to its complete information counterpart in each case. In the context of rule 2, I design policy targets which restore the complete information equilibrium for any choice of policy parameters.⁵²

6.1 Rule 1

If interest-rate rule 1 is followed, setting $\phi_\pi = 1 - k(1 - \theta)$ is optimal; however, this policy is unrealistic as it requires the monetary authority to be fully aware of the agents' learning problems which is hardly realistic. Crucially, in the limit as $\phi_\pi \rightarrow \infty$, $\pi_t \rightarrow 0$;⁵³ inflation is constant and the indirect (inflation/price) channel of expectations, through which the consumer's expectations also operate, is muted. However, even in this limit case, the producer's expectations continue to matter via the direct channel. Hence, inflation stabilization can at best eliminate the uncertainty arising through the inflation channel and, as such, is suboptimal.

An implication of Section 5.4 is that, for moderate values of ϕ_y , short-run output volatility due to purely expectational shocks increases in ϕ_y . However, perhaps not surprisingly, in the limit $\phi_y \rightarrow \infty$, the economy is at its complete information counterpart; a policy infinitely responsive to deviations of output from its complete information level is therefore optimal.

⁵²My focus so far has been on the effects of purely expectational shocks as well on those of productivity shocks. However, a fair question, especially when it comes to monetary policy, is about the possible presence of indeterminacies. The answer is that here there is no *real* indeterminacy. To see this, first note that the labor decision is intratemporal and is anyway made before inflation prevails. Turning to inflation, even if it is indeterminate, i.e. susceptible to possibly non-fundamental ("sunspot") shocks, since the "sunspot" shocks lie outside the information sets of both agents, they cannot affect the labor decision in stage 1, i.e. they cannot have real effects (see (72) and the analysis that comes with it). Nevertheless, there may well be nominal indeterminacy. To rule out "sunspot" shocks in the case, for instance, of rule 1, on which the positive part of this paper is based, we would need $\phi_\pi > 1$. But I should repeat here, that I abstract from such considerations.

⁵³As ϕ_π increases, the sign of the change in the weight of the producer's expectations, ξ_1 , is given by the sign of $-(1 + \phi_y) + [1 - k(1 - \theta)](1 + \zeta)$ (see also (19) and (21)). It is negative for a high enough ϕ_y relative to the inverse Frisch elasticity ζ , while there is a discontinuity at $\frac{1+\phi_y}{1+\zeta}$.

6.2 Rule 2

A first policy implication generated by the equilibrium analysis (see Section 5) is that a forward-looking rule, like rule 2, invites multiple equilibria in which the consumer's expectations is arbitrarily specified, which is not the case when a contemporaneously-looking rule is followed.⁵⁴ Second, as I showed in Section 5.4, the short-run volatility of output due to expectational shocks is substantially higher for forward-looking rules than for contemporaneously-looking ones, for the parametrization in Table 1.

I initially consider the baseline equilibrium in which the consumer's expectations have no role.⁵⁵ This corresponds to setting $\kappa_5 = 0$ in (34). Observe in (36) that the weight of output placed on producer's expectations is $\kappa_4 = \frac{\phi_\pi}{\phi_\pi + \zeta(\phi_\pi - 1)}$, whereas the weight placed on productivity is $1 - \kappa_4$. The former decreases in ϕ_π (see also fn. 41); the greater ϕ_π , the weaker the indirect (price) channel of expectations will be. In the limit as $\phi_\pi \rightarrow \infty$, prices are constant. As in the previous case, only the indirect channel of expectations is muted, therefore inflation stabilization is suboptimal.

The focus so far has been on active policies, which correspond to the monetary authority setting $\phi_\pi > 1$. However, setting $\phi_\pi = 0$ in (36) and (37) returns $y_t = a_t$ and $p_t = -a_t$; a Friedman-rule policy completely eliminates the role of expectations and keeps the economy at its complete information level. To provide an intuition for this, observe that for $\phi_\pi < \frac{1}{1+\zeta}$ the price effect becomes negative, which implies the indirect channel effect mitigates the direct one. For $\phi_\pi = 0$, the two effects precisely offset each other, rendering, therefore, incomplete information irrelevant in equilibrium. Notably, such a policy implies that the producer's revenue is constant across states: high prices prevail for low productivities and vice versa. Summarizing the above, the Friedman rule, for different from the usual reasons, emerges as an optimal policy; however, if an active policy is to be pursued, then it should be as aggressive on inflation as possible.

Next, I analyze monetary policy when the monetary authority has superior information. As we saw earlier, the monetary authority can either, unintentionally, report prices with a measurement error or, intentionally, fuel the economy with "surprise" monetary policy shocks. A straightforward option for a "benevolent" monetary authority in the latter case is to use monetary policy shocks to eliminate the producer's expectational errors. However, I will focus on the monetary policy parameters that can insulate the economy against measurement errors and can serve as a commitment device against monetary policy shocks.

⁵⁴At least, I have failed to find other linear equilibria for rule 1.

⁵⁵See fn. 58 below for the general case.

One can see from (68) and (70) and Appendix A.4.1 that the monetary policy parameter ϕ_π affects the equilibrium not only directly, but also indirectly by affecting the precision of the endogenous public signal, z_t or \hat{z}_t , it generates. The precision of the public signal is inversely related to δ ($\hat{\delta}$ for the monetary policy shock).

Considering the case where the authority reports prices with a measurement error, in the limit as $\phi_\pi \rightarrow \infty$, the precision of the endogenous public signal z_t becomes zero and $\delta \rightarrow 1$; hence, agents ignore the public signal which then has no real effects. Alternatively, a Friedman-rule policy ensures immunity to measurement errors as well, for the reasons outlined above. Hence, both extreme policies imply measurement errors have no real effects.

In the case of monetary policy shocks, ϕ_π matters only through the parameter $\hat{\delta}$ as we can see from (70). Appendix A.4.1 shows that the variance of the signal \hat{z}_t tends to infinity only when a Friedman-rule policy is pursued, which is the unique optimal policy in this case allowing the monetary authority to commit against “surprise” shocks. Even though, for $\phi_\pi > 1$ (a sufficient condition), the variance of the signal increases in ϕ_π , in the limit $\phi_\pi \rightarrow \infty$ the public signal’s variance is still finite, hence $\hat{\delta} \neq 1$. This implies that a policy infinitely aggressive on inflation cannot serve as a commitment device against monetary policy shocks.

6.2.1 Optimal monetary policies

In this section I design targets for forward-looking interest-rate rules which restore the complete information equilibrium for any choice of policy parameters (ϕ_π, ϕ_y) . I start with the baseline case of rule 2 and subsequently deal with the general form that equilibria can have when a forward-looking policy is followed, given by (30) - (33).

Baseline equilibrium. I will follow a reverse engineering process. The optimal policy suggested above requires setting $\phi_\pi = 0$. It is straightforward to check that this implies $y_t = a_t$ and $p_t = -a_t$ (see also fn. 58 below).

Consider the rule

$$i_t = -\log \beta + \phi_\pi E_t^m [\pi_{t+1} - \hat{\pi}_{t+1}] + \phi_y E_t^m \Delta[y_{t+1} - \hat{y}_{t+1}], \quad (75)$$

where the target levels of prices and output are set equal to their above specified levels: $\hat{\pi}_{t+1} = -E_t^m [\Delta a_{t+1}]$ and $\hat{y}_t = a_t$. This rule involves the monetary authority “punishing” deviations from

the efficient inflation and growth rates.^{56,57}

Taking the same steps as in the derivations of (36)-(37) shows that *any* chosen coefficients (ϕ_π, ϕ_y) can drive the economy to its efficient level. The Friedman rule is a special case obtained for $\phi_\pi = \phi_y = 0$.

Multiple equilibria. Consider the interest-rate rule given by (75). Make the following modification:

$$\hat{\pi}_{t+1} = \kappa_7 E_t^m[\Delta a_{t+1}] \quad (76)$$

$$\kappa_7 = -1 + \frac{\phi_\pi - 1}{\phi_\pi} k(1-\theta)\kappa_5. \quad (77)$$

This rule drives the economy to its complete information counterpart. Once again, observe that the inflation and growth targets are related to their complete information levels.⁵⁸ The proposed rule is invariant to changes in ϕ_y , whereas, as (77) shows, it adjusts to the chosen value of ϕ_π .⁵⁹ To get an intuition for the latter, first use the former result and set $\phi_y = 0$ in order to bring the equilibrium closer to the equilibrium given by (34)-(35). Then observe in (34) that the real effects of the consumer's expectations depend on the monetary authority's response to expected inflation, ϕ_π . Hence, the targets in the suggested policy (75)-(77) also need to adjust accordingly to changes in ϕ_π . All derivations are collected in Appendix A.5.

The monetary authority can extract the role of the consumer's expectations, parametrized by κ_5 , and productivity a_t by observing output and prices (see also (35)) when it steps in; subsequently, it

⁵⁶ As already emphasized, the authority has complete information when it sets the nominal interest rate.

⁵⁷Orphanides (2003) discusses the benefits of targeting output growth.

⁵⁸ To get an intuition for this, recall that the efficient equilibrium requires $\xi_3 = 0$; this implies $\kappa_4 = 0$ by (32) and $\kappa_6 = \frac{1}{\phi_\pi - 1}$ by (33). Given these, we can see in (34) that $\xi_3 = 0$ prevails for ϕ_π such that $\kappa_5 = -\frac{1}{k(1-\theta)} \frac{\phi_\pi}{\phi_\pi - 1}$. Then, the equilibrium is

$$y_t = a_t \quad (78)$$

$$p_t = \frac{1}{\phi_\pi - 1} a_t - \frac{1}{k(1-\theta)} \frac{\phi_\pi}{\phi_\pi - 1} E_t^c[x_t]. \quad (79)$$

The rule given by (75)-(77) and $\hat{y}_t = a_t$ yields

$$y_t = a_t \quad (80)$$

$$p_t = -[1 + k(1-\theta)\kappa_5] a_t + \kappa_5 E_t^c[x_t]. \quad (81)$$

Setting $\kappa_5 = -\frac{1}{k(1-\theta)} \frac{\phi_\pi}{\phi_\pi - 1}$ returns (78)-(79).

⁵⁹The rule will not adjust to changes in ϕ_π for $\kappa_5 = 0$, as already shown.

can invoke the rule given by (75) - (77) and $\hat{y}_t = a_t$ and restore the complete information equilibrium for any choice of policy parameters (ϕ_π, ϕ_y) .

Last, observe that setting $\kappa_5 = 0$ returns $\kappa_6 = -1$ and $\kappa_7 = -1$, which correspond to the baseline rule (75).

7 Conclusion

This paper has reconsidered the nature of purely expectational shocks within a competitive, cashless, monetary economy. Asymmetric information about current fundamentals is the driving force in the model. Informational asymmetries lead agents to form heterogeneous expectations about inflation; as a result, monetary policy and consumers' expectations have real effects through inflation. Traditionally, expectational shocks are viewed as Keynesian demand shocks: when positive, they increase output, employment and inflation. I have shown that this interpretation remains a possibility but is not the only one; expectational shocks can cause business cycle patterns associated with supply shocks: when positive, they increase output and employment and they lower inflation. Such an interpretation seems in line with the low inflation and the high cyclical employment in the mid-80s and the second half of the 90s, which are recalled as periods of exuberant optimism. Whether expectational shocks manifest themselves as demand or supply shocks reflects the monetary policy pursued.

I have considered different interest-rate rules and shown that forward-looking rules generate multiple equilibria in which consumers' expectations have an arbitrary role. Further to this, expectational shocks cause substantially higher short-run output volatility under forward-looking policies than under "contemporaneous" ones. Inflation stabilization per se is typically suboptimal, as it can at best eliminate uncertainty arising through prices. Optimal monetary policies manipulate inflation so that the producer correctly anticipates his revenue. In this way, producers' incomplete information about productivity becomes irrelevant. I have designed targets for forward-looking interest-rate rules which restore the complete information equilibrium for any chosen policy parameters.

Recovering purely expectational shocks from the data will shed light on their seemingly shifting nature. Of course, the literature on the identification of expectational shocks remains far from settled (for example, Beaudry and Portier (2006), Blanchard et al. (2009) and Barsky and Sims (2011a,b)). On the policy front, introducing capital, investment and credit constraints is a rather natural extension with potentially promising monetary policy implications.

A Omitted derivations

A.1 Equilibrium definition and agents' problems

Equilibrium definition. A rational expectations equilibrium *under an interest-rate rule* $Q(\Psi_t)$ consists of prices $\{P_t(\Psi_t), W_t(\Psi_t \setminus \{a_t\})\}$, an allocation $\{N_t^d(\Psi_t \setminus \{a_t\}), Y_t(\Psi_t)\}$ for the producer and an allocation $\{C_t(\Psi_t), N_t^s(\Psi_t), B_{t+1}(\Psi_t)\}$ for the consumer such that:

1. $\{N_t^d(\Psi_t \setminus \{a_t\}), Y_t(\Psi_t)\}$ solves the producer's problem, laid out below, at prices $\{P_t(\Psi_t), W_t(\Psi_t \setminus \{a_t\}), Q(\Psi_t)\}$.
2. $\{C_t(\Psi_t), N_t^s(\Psi_t), B_{t+1}(\Psi_t)\}$ solves the consumer's problem, laid out below, at prices $\{P_t(\Psi_t), W_t(\Psi_t \setminus \{a_t\}), Q(\Psi_t)\}$.
3. All markets clear: $Y_t = C_t, N_t^d = N_t^s, B_{t+1} = 0$ for all t with $B_0 = 0$.

Producer's problem. Stage 2 profits of the consumer-owned firm are given by $\Pi_t = P_t Y_t - W_t N_t$, where $Y_t = A_t N_t$.

In stage 1 of each period t , the producer chooses $N_t \geq 0$ to maximize the firm's expected profits evaluated according to the consumer's Lagrange multiplier denoted λ_t :

$$E_t^p[\lambda_t \Pi_t].$$

Expectations are with respect to the information set of the producer in stage 1 which, as specified in the main text, is $I_{t,1}^p = \Psi_t \setminus \{a_t\}$. The maximization problem does not yield a solution for $W_t < \frac{E_t^p[\lambda_t P_t A_t]}{E_t^p[\lambda_t]}$.

The producer accommodates any labor supply for

$$W_t = \frac{E_t^p[\lambda_t P_t A_t]}{E_t^p[\lambda_t]}. \quad (82)$$

Since, technology is linear in the worker's productivity, the scale of production is pinned down by the labor supply side. Put differently, the producer commits to accommodate any labor supplied which will pin down the output produced as long as the nominal wage is the one given by (82).

The case in which the LHS of (82) is greater than the RHS implies $N_t = 0$ which in turn implies $Y_t = 0$, which is not possible in equilibrium as the consumer's problem requires $C_t > 0$.

On another note, since production takes place after the nominal wage is announced and depends on the consumer/worker's productivity, Y_t in the definition of equilibrium above is a function of the state Ψ_t rather than the producer's information set in stage 1, $\Psi_t \setminus \{a_t\}$.

Consumer's problem. Given $B_0 = 0$, the consumer solves the following problem:

$$\max_{\{C_t, N_t, B_{t+1}\}_{t=0}^{\infty}} E_0^c \sum_{t=0}^{\infty} \left(\log C_t - \frac{N_t^{1+\zeta}}{1+\zeta} \right)$$

subject to the sequence of budget constraints

$$P_t C_t + Q_t B_{t+1} = B_t + W_t N_t + \Pi_t,$$

and a no-Ponzi-scheme constraint (requiring that $B_{t+1} > -\Gamma$ for any $\Gamma > 0$ at all t would do since in equilibrium $B_{t+1} = 0$ for all t). The FOCs with respect to C_t , N_t , and B_{t+1} respectively are:

$$\frac{1}{C_t} = \lambda_t P_t \tag{83}$$

$$N_t^\zeta = \lambda_t W_t \tag{84}$$

$$Q_t = \beta \frac{E_t^c [\lambda_{t+1}]}{\lambda_t}, \tag{85}$$

where, as noted above, λ_t is the current-value Lagrange multiplier associated with the period- t budget constraint. Expectations are with respect to the information set of the consumer which coincides with the state Ψ_t (see also the analysis in the main text). I have made no distinction between stages 1 and 2 for the consumer as he has the same information in both stages. Combining (83) with (84) and (83) with (85) yields, respectively,

$$N_t^\zeta = \frac{W_t}{P_t C_t} \tag{86}$$

$$Q_t = \beta E_t^c \left[\frac{P_t}{P_{t+1}} \frac{C_t}{C_{t+1}} \right]. \tag{87}$$

In addition, the no-Ponzi-scheme condition and the fact that nominal bonds are in zero net-supply imply $B_{t+1} = 0$ in equilibrium.

A.2 Kalman filter

Let me start with the consumer's case which is easier to handle. The consumer's information set is $I_t^c = I_{t-1}^c \cup \{s_t, a_t\}$. Suppose the consumer's prior in period t is $x_t | I_{t-1}^c \sim N(0, \tilde{\sigma}_x^2)$, where

$\tilde{\sigma}_x^2 \equiv \text{Var}_{t-1}^c[x_t]$. Before applying Bayes' Law, recall that all shocks are serially uncorrelated, mutually independent, and normally distributed. The consumer's posterior is

$$x_t | I_{t-1}^c \sim N\left(0, \left(\frac{1}{\tilde{\sigma}_x^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_u^2}\right)^{-1}\right).$$

This implies the following period's prior is

$$x_{t+1} | I_t^c \sim N\left(0, \left(\frac{1}{\tilde{\sigma}_x^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_u^2}\right)^{-1} + \sigma_\epsilon^2\right).$$

Using $\hat{\sigma}_x^2$ to denote $\text{Var}_t^c[x_{t+1}]$, it follows that

$$\hat{\sigma}_x^2 = \left(\frac{1}{\tilde{\sigma}_x^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_u^2}\right)^{-1} + \sigma_\epsilon^2. \quad (88)$$

I will assume the consumer's prior in period 0 is $x_{0|-1} \sim N(0, \sigma_x^2)$, where I let σ_x^2 denote the solution (a fixed point) to the Riccati equation (88). This implies the learning problem of the agents is at its steady state when time commences. Turning to the coefficients in (16), let

$$k \equiv \frac{\frac{1}{\sigma_e^2} + \frac{1}{\sigma_u^2}}{\frac{1}{\tilde{\sigma}_x^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_u^2}},$$

denote the Kalman gain term, that is the precision of new information relative to the prior's precision. This is time-invariant due to the consumer's learning problem being at its steady state. In addition, let $\theta \equiv \frac{\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_e^2} + \frac{1}{\sigma_u^2}}$, denote the relative precision of the signal s_t within the new information $\{s_t, a_t\}$.

Turning to the producer's learning problem, recall from the analysis in the main text that agents have the same information set at the end of each period, that is $I_{t-1,2}^p = I_{t-1}^c$. As a result, they have the same prior in the following period. However, their information sets differ in stage 1. In particular, the producer's information set is $I_{t,1}^p = I_{t-1,2}^p \cup \{s_t\}$. Letting $\mu \equiv \frac{\frac{1}{\sigma_e^2}}{\frac{1}{\tilde{\sigma}_x^2} + \frac{1}{\sigma_e^2}}$ yields the coefficient in (15).

A thorough demonstration of the Kalman filter can be found in Anderson and Moore (1979), Harvey (1989) and Technical Appendix *B* in Ljungqvist and Sargent (2004).

A.3 Equilibrium under Rule 1

Let me elaborate first on the filtering problems of the agents. The producer's and the consumer's expectations, respectively, are (see also (15) and (16)):

$$E_t^p[a_t] = E_{t,1}^p[x_t] = (1 - \mu) E_{t-1,2}^p[x_{t-1}] + \mu s_t$$

$$E_{t,2}^p[x_t] = E_t^c[x_t] = (1 - k) E_{t-1}^c[x_{t-1}] + k[\theta s_t + (1 - \theta)a_t].$$

Then, the consumer's expectations in period t of the producer's expectations in $t + 1$ are given by

$$E_t^c[E_{t+1}^p[a_{t+1}]] = E_t^c[x_t], \quad (89)$$

and the producer's expectations in period t of the consumer's expectations in t are given by

$$E_t^p[E_t^c[x_t]] = (1 - k) E_{t-1}^c[x_{t-1}] + k[\theta s_t + (1 - \theta) E_t^p[a_t]] = E_t^c[x_t] + k(1 - \theta)(E_t^p[a_t] - a_t). \quad (90)$$

Substituting conjectures (C1), (C2), and (90) in (17) implies

$$\zeta n_t = (1 + \kappa_3 - \xi_1) E_t^p[a_t] + \kappa_2 k(1 - \theta)(E_t^p[a_t] - a_t) - (\kappa_3 + \xi_2)a_t. \quad (91)$$

Substituting (91) in the firm's technology, $y_t = a_t + n_t$, using market clearing, $y_t = c_t$, and, subsequently, matching coefficients with conjecture (C1) yields

$$\xi_1 = \frac{1 + \kappa_3 + \kappa_2 k(1 - \theta)}{1 + \zeta} \quad (92)$$

$$\xi_2 = \frac{\zeta - \kappa_3 - \kappa_2 k(1 - \theta)}{1 + \zeta}. \quad (93)$$

Observe that

$$\xi_1 + \xi_2 = 1, \quad (94)$$

a direct consequence of preferences logarithmic in consumption.

Turning to the Euler equation (18), conjectures (C1) and (C2) combined with (89) imply

$$E_t^c[c_{t+1}] - c_t = -\xi_1 E_t^p[a_t] + (\xi_1 + \xi_2) E_t^c[x_t] - \xi_2 a_t \quad (95)$$

$$i_t - E_t^c[\pi_{t+1}] = (\phi_y \xi_1 + \phi_\pi \kappa_1) E_t^p[a_t] + [-\kappa_1 + (\phi_\pi - 1)\kappa_2 - \kappa_3] E_t^c[x_t] + [\phi_y \xi_2 + \phi_\pi \kappa_3 - \phi_y] a_t. \quad (96)$$

Matching coefficients in (95) and (96) yields

$$-\xi_1 = \phi_y \xi_1 + \phi_\pi \kappa_1 \quad (97)$$

$$\xi_1 + \xi_2 = -\kappa_1 + (\phi_\pi - 1) \kappa_2 - \kappa_3 \quad (98)$$

$$-\xi_2 = \phi_y \xi_2 + \phi_\pi \kappa_3 - \phi_y. \quad (99)$$

Summing (97) - (99) across sides and using (94) yields

$$\kappa_1 + \kappa_2 + \kappa_3 = 0, \quad (100)$$

whereas summing across (97) and (99) and again using (94) yields

$$\kappa_1 + \kappa_3 = -\frac{1}{\phi_\pi}, \quad (101)$$

which are equations (23) and (24), respectively, in the main text.

Solving (93), (94) and (97) - (99) for $\xi_1, \xi_2, \kappa_1, \kappa_2, \kappa_3$ returns (19) - (21) in the main text.

A.4 Equilibria under Rule 2

Equations (30) - (31) can be obtained by combining the equilibrium labor decision (14) with the firm's technology and market clearing. They coincide with (92) - (94), derived in Appendix A.3 above (one only needs to replace ξ_1 with ξ_3 , ξ_2 with ξ_4 , κ_2 with κ_5 , and κ_3 with κ_6).

Turning to the Euler equation, conjectures (C3) - (C4) imply $E_t^c [c_{t+1}] = (\xi_3 + \xi_4) E_t^c [x_t]$ and $E_t^c [p_{t+1}] = (\kappa_4 + \kappa_5 + \kappa_6) E_t^c [x_t]$. Then the LHS and the RHS of the Euler equation (13) after taking into account that $I_t^m = I_t^c$ (equation (29) in the main text) become, respectively,

$$E_t^c [c_{t+1}] - c_t = (\xi_3 + \xi_4) E_t^c [x_t] - \xi_3 E_t^p [a_t] - \xi_4 a_t \quad (102)$$

$$i_t - E_t^c [\pi_{t+1}] = (\phi_\pi - 1) E_t^c [\pi_{t+1}] = (\phi_\pi - 1) [(\kappa_4 + \kappa_6) E_t^c [x_t] - \kappa_4 E_t^p [a_t] - \kappa_6 a_t]. \quad (103)$$

Matching coefficients in (102) - (103) and using (31) yields (32) - (33).

A.4.1 Omitted derivations in Section 5.5

First, I deal with the case in which the monetary authority reports the following period's prices with a measurement error. Next, I follow the same process in the case of a monetary policy shock. Recall that what distinguishes the two cases is the information set of the monetary authority.

Measurement error. Suppose at the end of period $t - 1$ the nominal interest rate serves as a noisy signal about the price in t , as in (61). Agents extract

$$\tilde{p}_t = p_t + w_t, \quad (104)$$

where $\tilde{p}_t \equiv \frac{i_{t-1} + \phi_\pi p_{t-1}}{\phi_\pi}$. The monetary authority's information set is $I_{t-1}^m = \Omega_t \setminus \{z_t\}$, where z_t is the public signal about period- t productivity which I derive below and Ω_t denotes the state of the economy in t . The latter is $\Omega_t = (\{a_\tau\}_{\tau=0}^t, \{s_\tau\}_{\tau=0}^t, z_t)$. Using (37), (104) becomes

$$\frac{\tilde{p}_t - \kappa_4 E_t^p [a_t | I_{t,1}^p \setminus \{z_t\}]}{\kappa_6} = a_t + \frac{1}{\kappa_6} w_t, \quad (105)$$

where κ_4, κ_6 are coefficients given by (30)-(33) for $\kappa_5 = 0$. The producer's information set in stage 1 is $I_{t,1}^p = \Omega_t \setminus \{a_t\}$. The LHS in (105) is the endogenous public signal in stage 1 of t denoted by z_t . It follows then that

$$z_t \equiv \frac{[\phi_\pi + \zeta(\phi_\pi - 1)] \tilde{p}_t - \frac{\phi_\pi}{\phi_\pi - 1} E_t^p [a_t | I_{t,1}^p \setminus \{z_t\}]}{\zeta} = a_t + \frac{\phi_\pi + \zeta(\phi_\pi - 1)}{\zeta} w_t. \quad (106)$$

The conditional variance of productivity is then $\sigma_z^2 \equiv \text{Var}[a_t | z_t] = \left(\frac{\phi_\pi + \zeta(\phi_\pi - 1)}{\zeta}\right)^2 \sigma_w^2$.

Turning back to the producer, suppose for a moment that z_t is not part of his information set. Then, the producer's posterior distribution of a_t is

$$a_t | I_t^p \setminus \{z_t\} \sim N\left(E_t^p [x_t | I_{t,1}^p \setminus \{z_t\}], \sigma_x^2 + \sigma_u^2\right),$$

where $E_t^p [x_t | I_{t,1}^p \setminus \{z_t\}]$ is given by (15) and σ_x^2 is the fixed point in (88). Taking z_t into account, the producer's posterior becomes

$$a_t | I_{t,1}^p \sim N\left(\delta E_t^p [x_t | I_{t,1}^p \setminus \{z_t\}] + (1 - \delta) z_t, \sigma_a^2\right), \quad (107)$$

where $\delta = \left(\frac{1}{\sigma_x^2 + \sigma_u^2}\right) / \left(\frac{1}{\sigma_x^2 + \sigma_u^2} + \frac{1}{\sigma_z^2}\right)$ and $\sigma_a^2 = \left(\frac{1}{\sigma_x^2 + \sigma_u^2} + \frac{1}{\sigma_z^2}\right)^{-1}$.

Monetary policy shock. In case the monetary authority transmits monetary policy shocks, its information set additionally includes \hat{z}_t , that is $I_{t-1}^m = \hat{\Omega}_t$. Taking the same steps as before, agents observe $\hat{p}_t = \phi_\pi p_t + \omega_t$, where $\hat{p}_t \equiv i_{t-1} + \phi_\pi p_{t-1}$. The monetary authority transmits the public signal

$$\hat{z}_t = a_t + \frac{\phi_\pi + \zeta(\phi_\pi - 1)}{\phi_\pi \zeta} \omega_t,$$

where

$$\hat{z}_t \equiv \frac{[\phi_\pi + \zeta(\phi_\pi - 1)] \hat{p}_t - \frac{\phi_\pi^2}{\phi_\pi - 1} E_t^p[a_t | I_{t,1}^p]}{\phi_\pi \zeta}. \quad (108)$$

The conditional variance of productivity is $\sigma_{\hat{z}}^2 \equiv \text{Var}[a_t | \hat{z}_t] = \left(\frac{\phi_\pi + \zeta(\phi_\pi - 1)}{\phi_\pi \zeta} \right)^2 \sigma_\omega^2$. The producer's posterior is

$$a_t | I_{t,1}^p \sim N \left(\hat{\delta} E_t^p \left[x_t | I_{t,1}^p \setminus \{\hat{z}_t\} \right] + (1 - \hat{\delta}) \hat{z}_t, \sigma_{\hat{a}}^2 \right), \quad (109)$$

where $\hat{\delta} = \left(\frac{1}{\sigma_x^2 + \sigma_u^2} \right) / \left(\frac{1}{\sigma_x^2 + \sigma_u^2} + \frac{1}{\sigma_{\hat{z}}^2} \right)$ and $\sigma_{\hat{a}}^2 = \left(\frac{1}{\sigma_x^2 + \sigma_u^2} + \frac{1}{\sigma_{\hat{z}}^2} \right)^{-1}$.

Observe that unlike in (106), the producer's expectations in (108) are conditional on the entire information set of the producer. To fully extract \hat{z}_t use (109) to get

$$\hat{z}_t \equiv \frac{\phi_\pi - 1}{\phi_\pi} \hat{p}_t - \frac{\phi_\pi}{\phi_\pi + \zeta(\phi_\pi - 1)} \hat{\delta} E_t^p \left[x_t | I_{t,1}^p \setminus \{\hat{z}_t\} \right].$$

A.5 Derivations in Section 6.2

Consider the interest-rate rule

$$i_t = -\log \beta + \phi_\pi E_t^m [\pi_{t+1} - \hat{\pi}_{t+1}] + \phi_y E_t^m \Delta[y_{t+1} - \hat{y}_{t+1}],$$

where $\hat{\pi}_{t+1} = \kappa_7 (a_{t+1} - a_t)$ and $\hat{y}_t = a_t$. A reverse engineering process will pin down κ_7 .

The labor market optimality condition implies

$$\xi_3 = \frac{1 + \kappa_6 + k(1 - \theta)\kappa_5}{1 + \zeta}$$

$$\xi_4 = 1 - \xi_3,$$

which correspond to equations (30) - (31) in the main text. Taking familiar steps, the Euler equation implies

$$1 - \phi_y = (\phi_\pi - 1)(\kappa_4 + \kappa_6) - \phi_\pi \kappa_7 - \phi_y \quad (110)$$

$$(1 - \phi_y) \xi_3 = (\phi_\pi - 1) \kappa_4 \quad (111)$$

$$(1 - \phi_y) \xi_4 = (\phi_\pi - 1) \kappa_6 - \phi_\pi \kappa_7 - \phi_y. \quad (112)$$

Setting

$$\kappa_7 = \frac{(\phi_\pi - 1) \kappa_6 - 1}{\phi_\pi} \quad (113)$$

implies $\xi_3 = 0$ and $\xi_4 = 1$ as required, $\kappa_4 = 0$ and $\kappa_6 = -[1 + k(1 - \theta)\kappa_5]$. Combining the latter with (113) yields (77) in text.

B Data

Data in Figures 1-4 are collected from the St. Louis Fed and refer to the US economy for the period 1965:1 - 2010:1. Data in Figures 1 and 3 are quarterly, whereas in Figures 2 and 4 they are annual. Employment refers to “All Employees: Total Nonfarm Employees (Thousands of Persons)” (series PAYEMS) and is seasonally adjusted. It is logged and HP-filtered with penalty 1600 for quarterly and 100 for annual data, respectively. Figures 1-4 show its cyclical component scaled up by 50 for expositional clarity. Inflation in Figures 1 and 3 refers to percent changes in the “Gross Domestic Product: Implicit Price Deflator” (series GDPDEF) and is seasonally adjusted. Inflation in Figures 2 and 4 refers to percent changes in the “Consumer Price Index for All Urban Consumers: All Items” (series CPIAUCSL) and is seasonally adjusted. Consumer Sentiment refers to “University of Michigan: Consumer Sentiment” (series UMCSENT1, UMCSENT) and is not seasonally adjusted. It is scaled down by 25 in Figure 3 and by 10 in Figure 4.

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Figure 1: Changes in GDP Deflator and Cyclical Employment: 1965 - 2010 (quarterly)

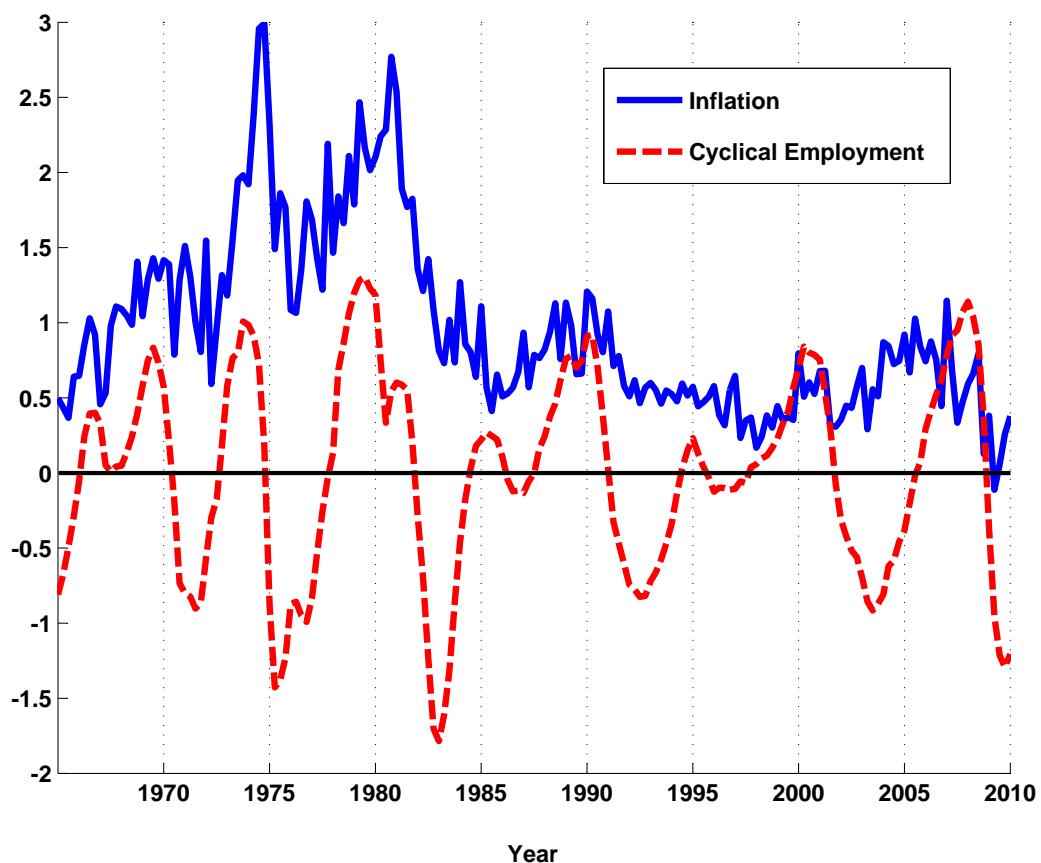


Figure 2: CPI Inflation and Cyclical Employment: 1965 - 2010 (annual)

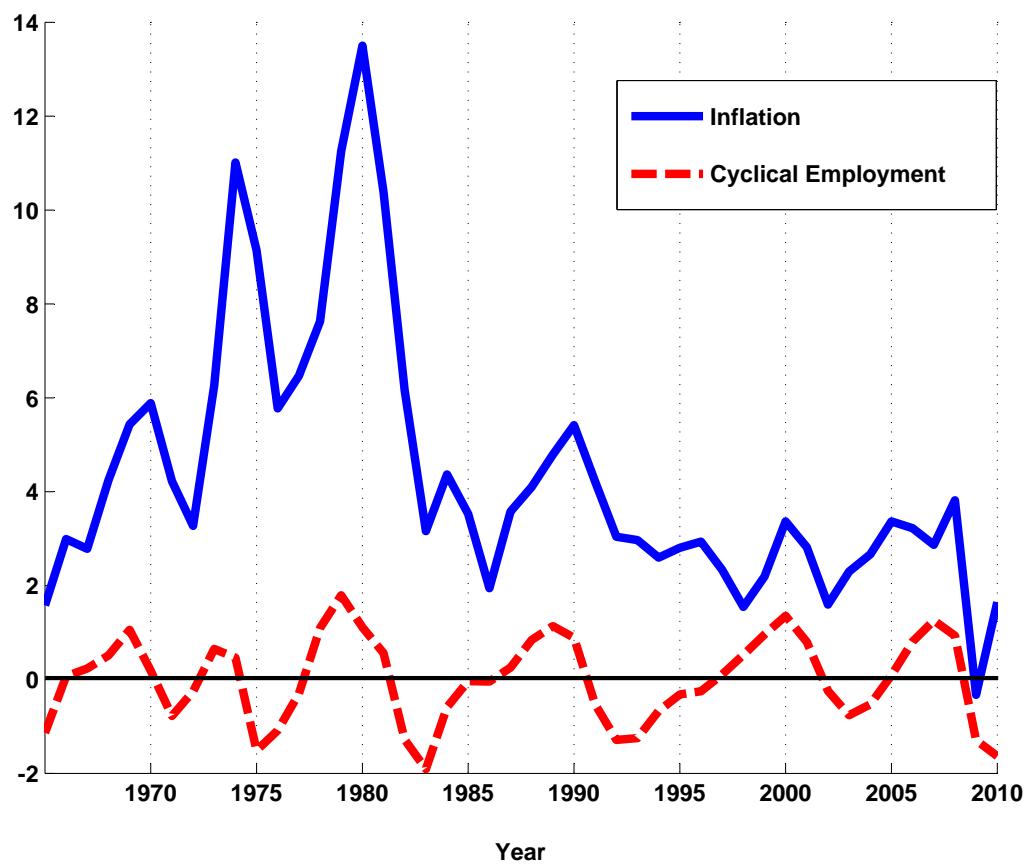


Figure 3: Changes in GDP Deflator, Cyclical Employment and Consumer Sentiment: 1965 - 2010 (quarterly)

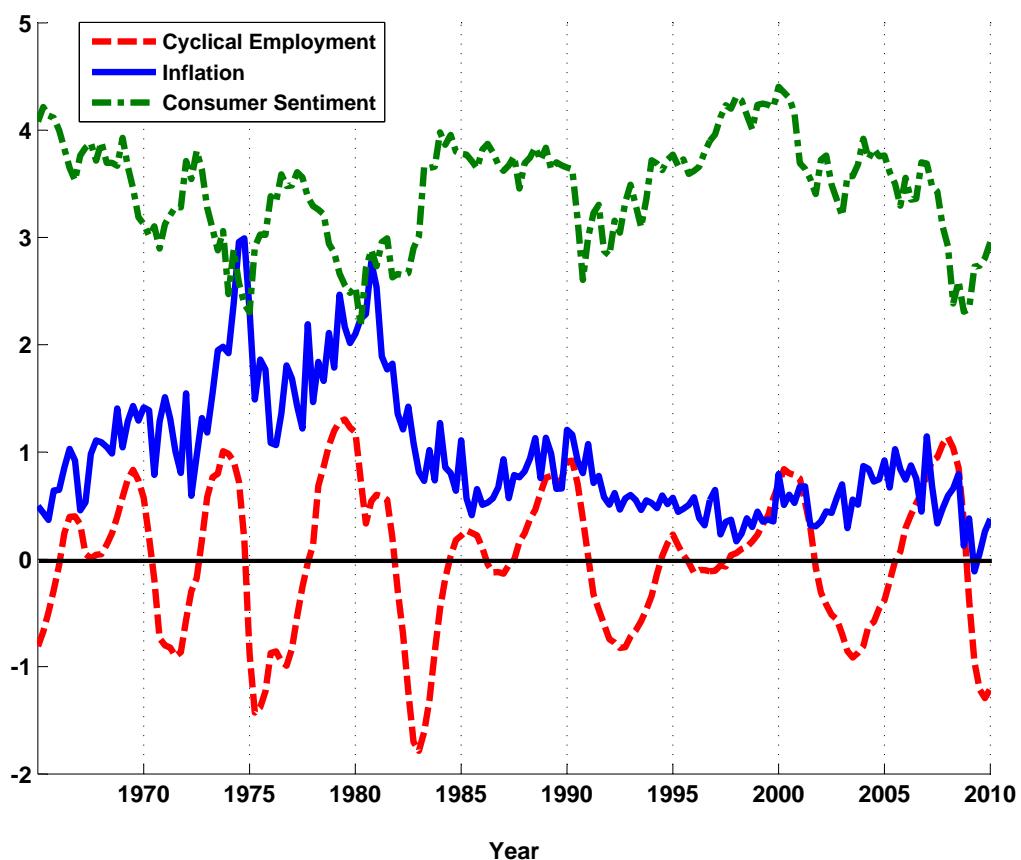


Figure 4: CPI Inflation, Cyclical Employment and Consumer Sentiment: 1965 - 2010 (annual)

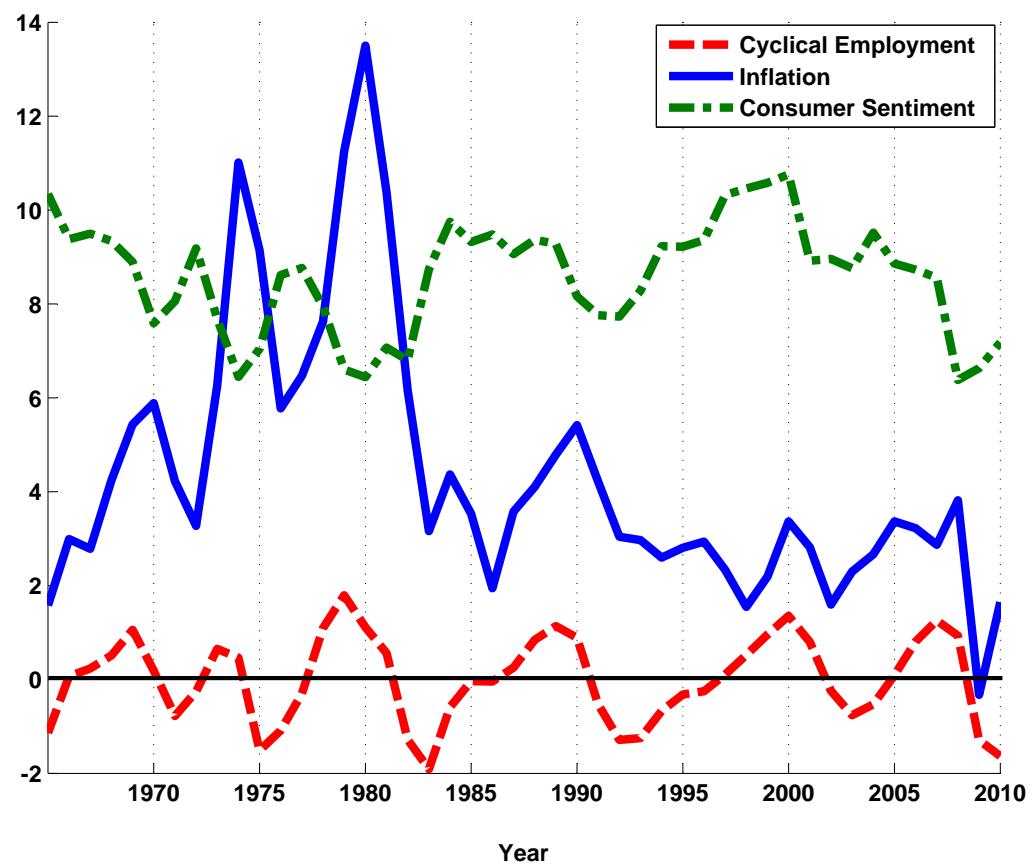


Figure 5: Impulse responses to a positive expectational shock for rule 1 with $\phi_y = 0.5$

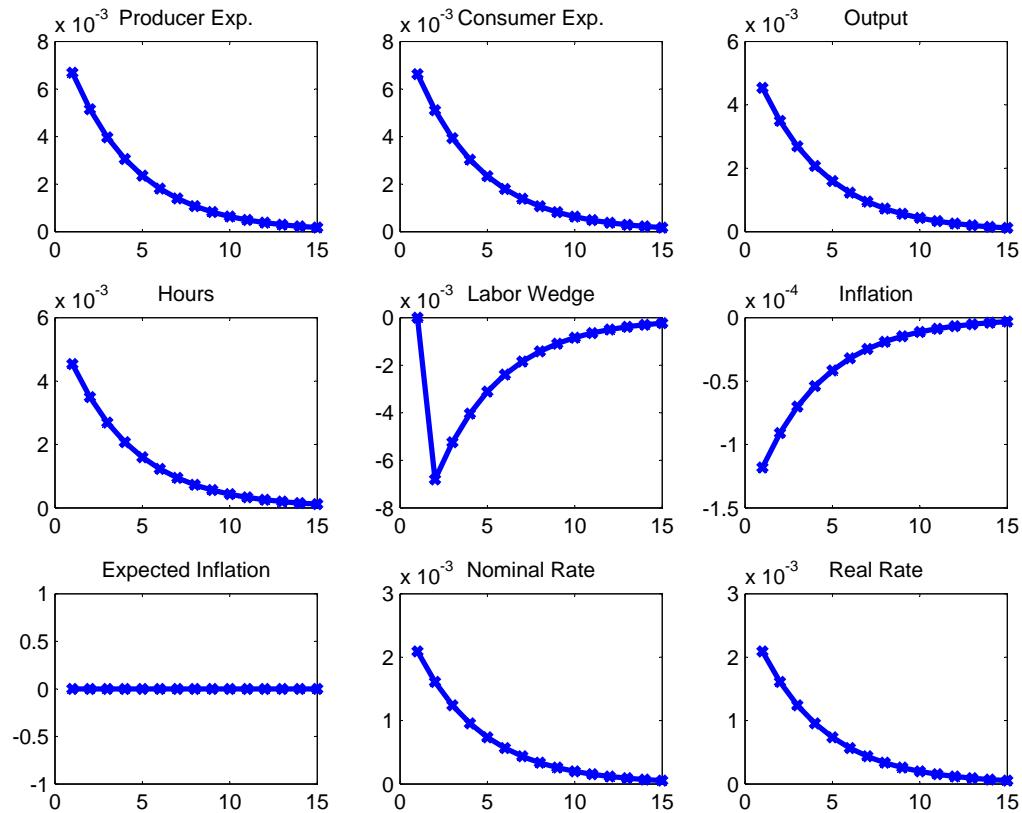


Figure 6: Impulse responses to a positive permanent productivity shock for rule 1 with $\phi_y = 0.5$

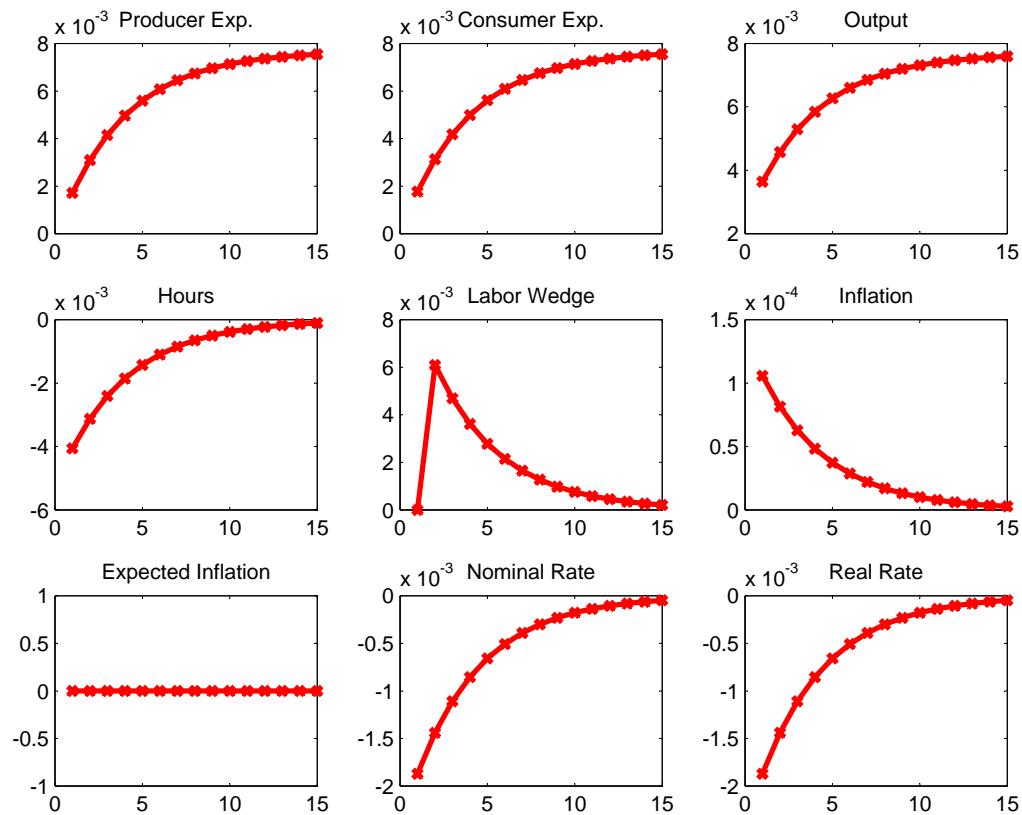


Figure 7: Impulse responses to a positive temporary productivity shock for rule 1 with $\phi_y = 0.5$

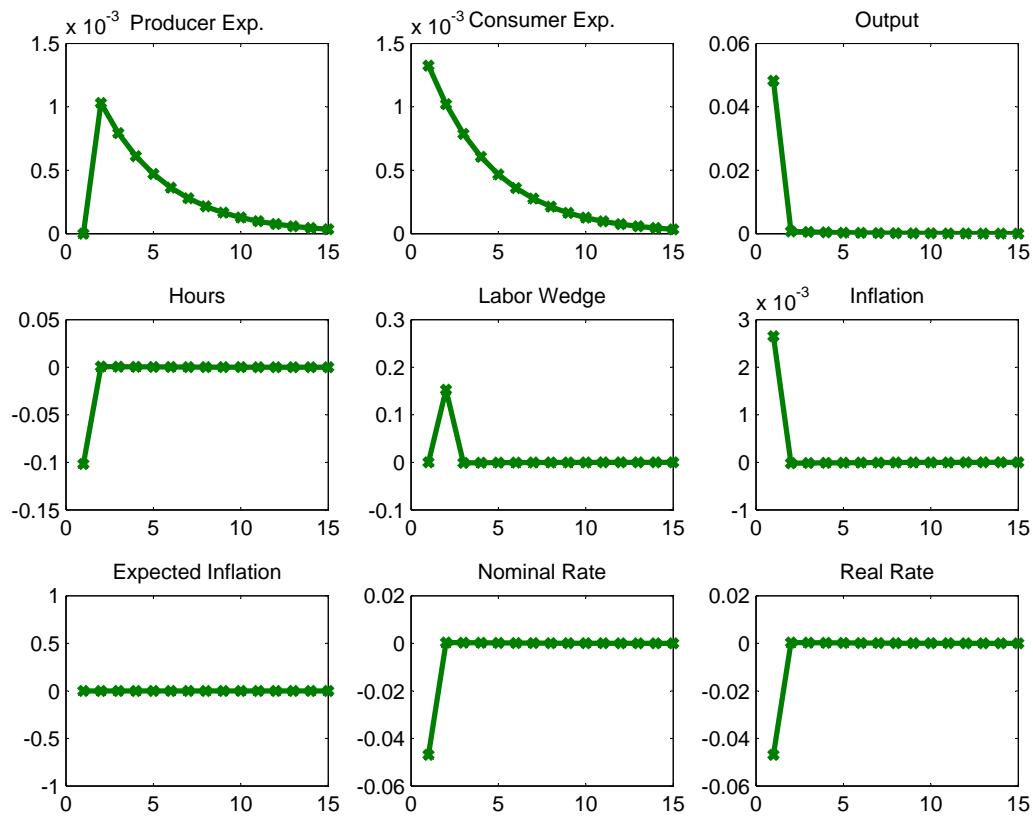


Figure 8: Impulse responses to a positive expectational shock for rule 1 with $\phi_y = 0$

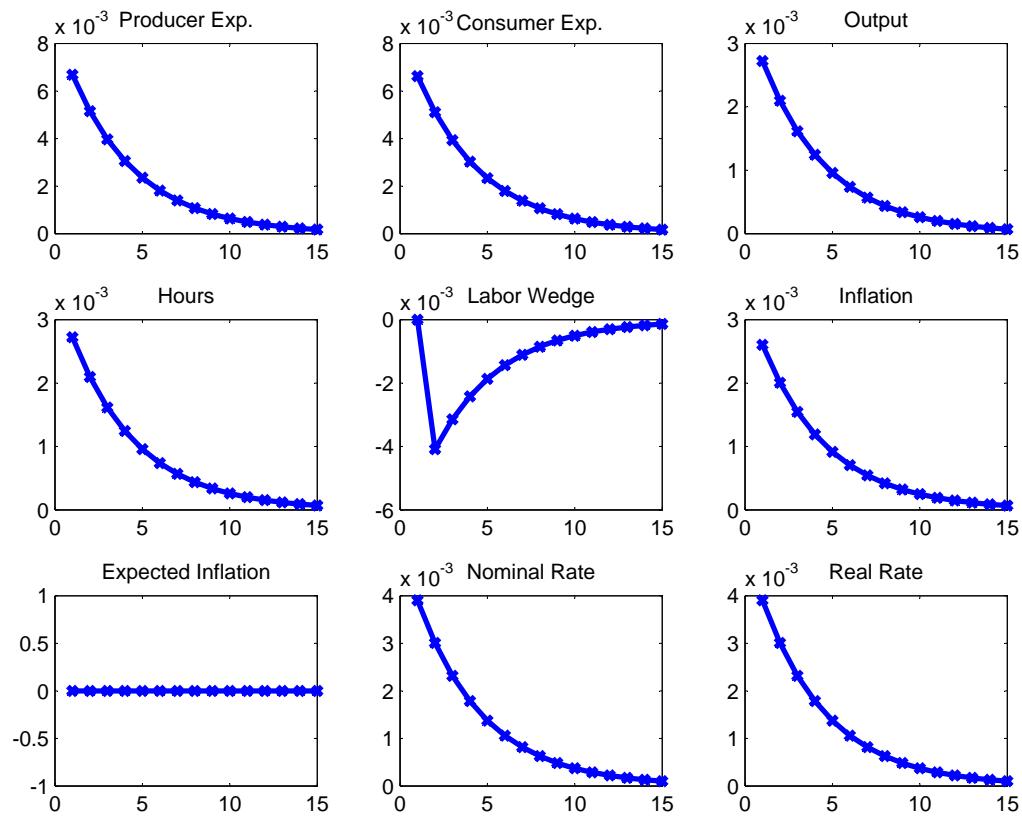


Figure 9: Impulse responses to a positive permanent productivity shock for rule 1 with $\phi_y = 0$

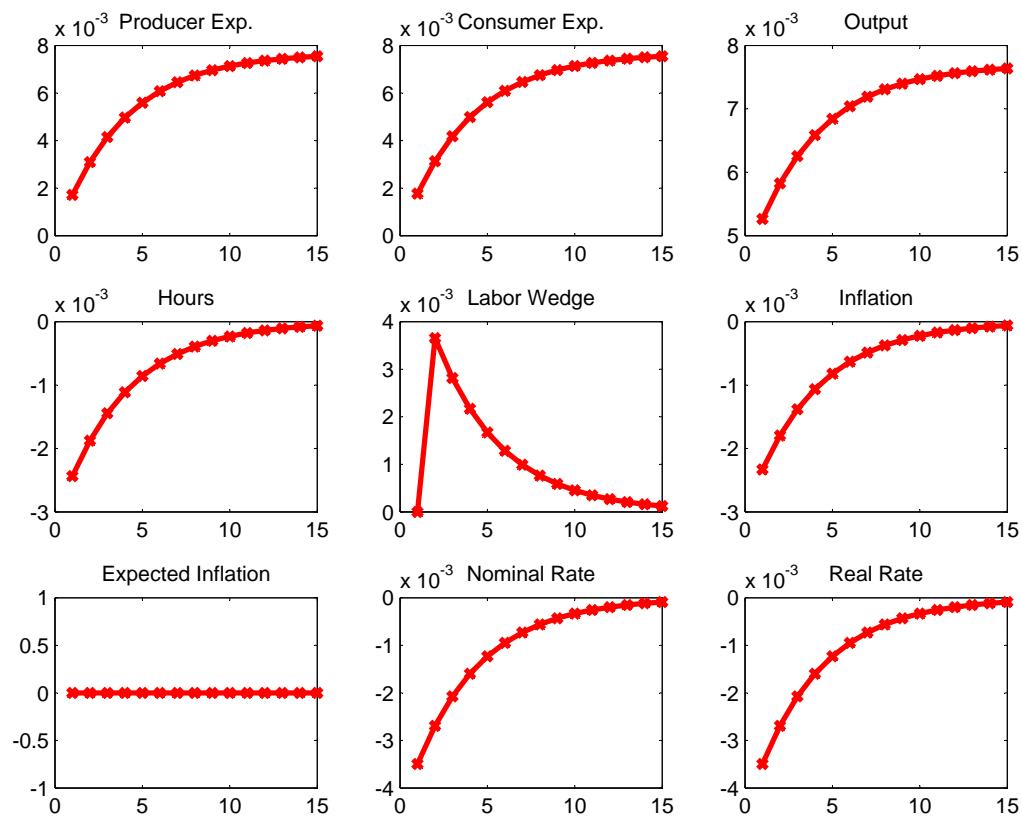


Figure 10: Impulse responses to a positive temporary productivity shock for rule 1 with $\phi_y = 0$

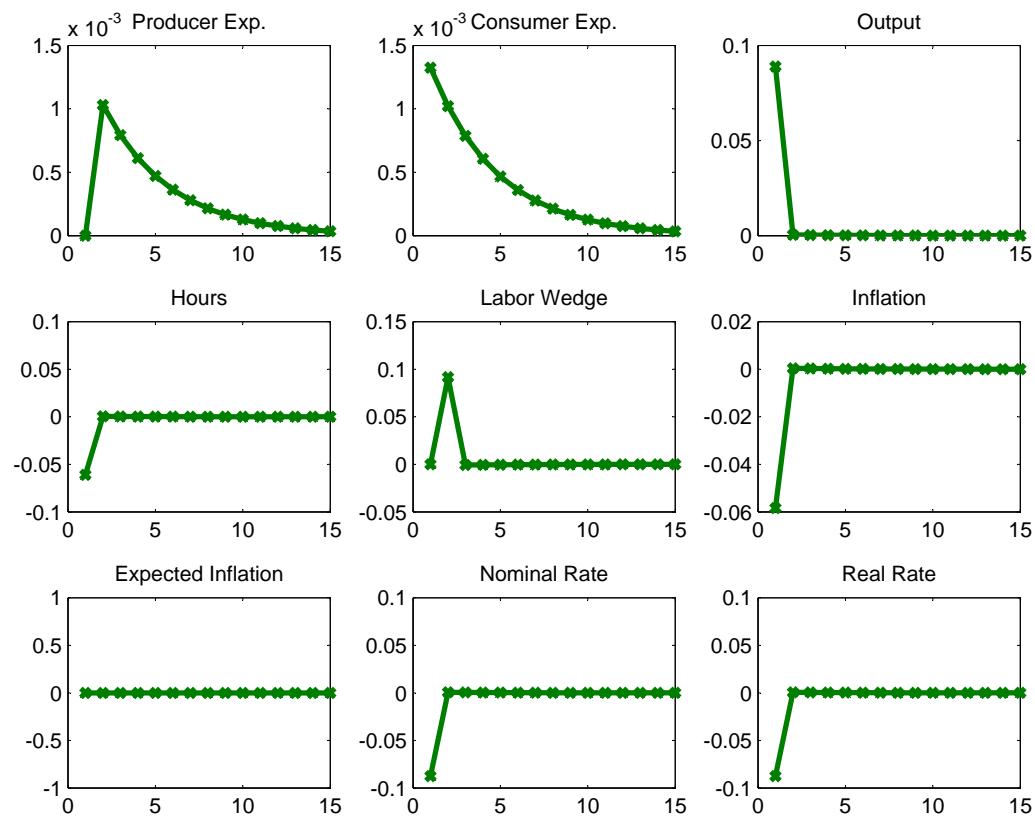


Figure 11: Impulse responses to a positive expectational shock for baseline rule 2 (1)

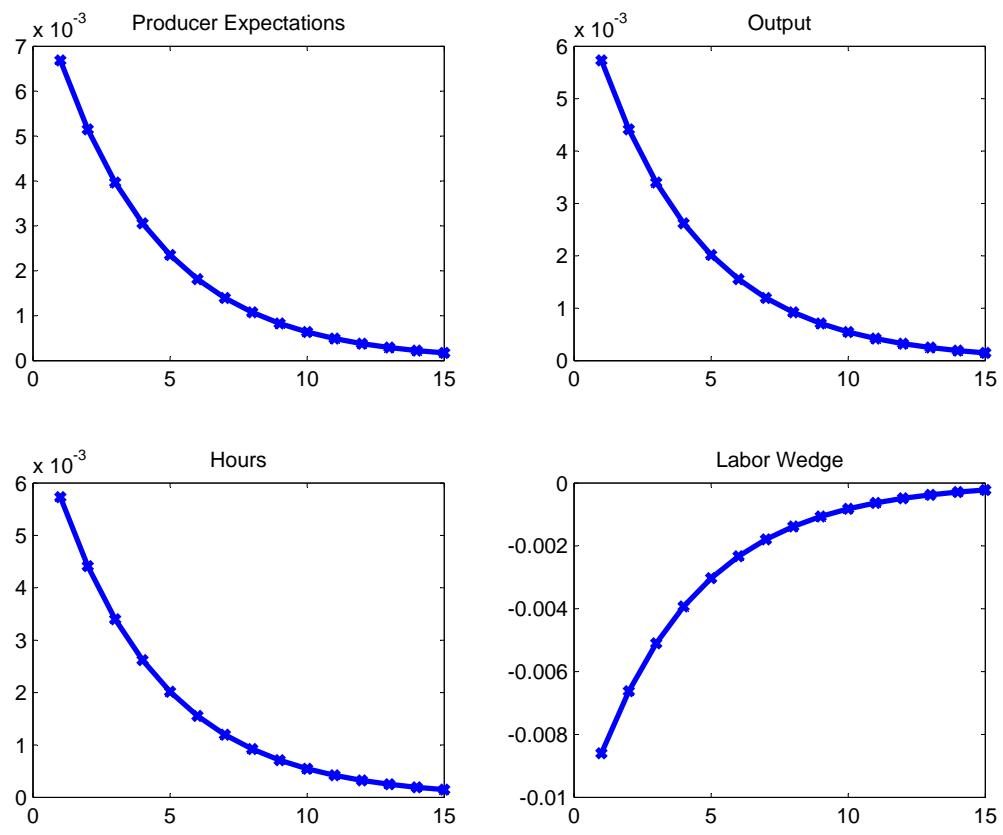


Figure 12: Impulse responses to a positive expectational shock for baseline rule 2 (2)

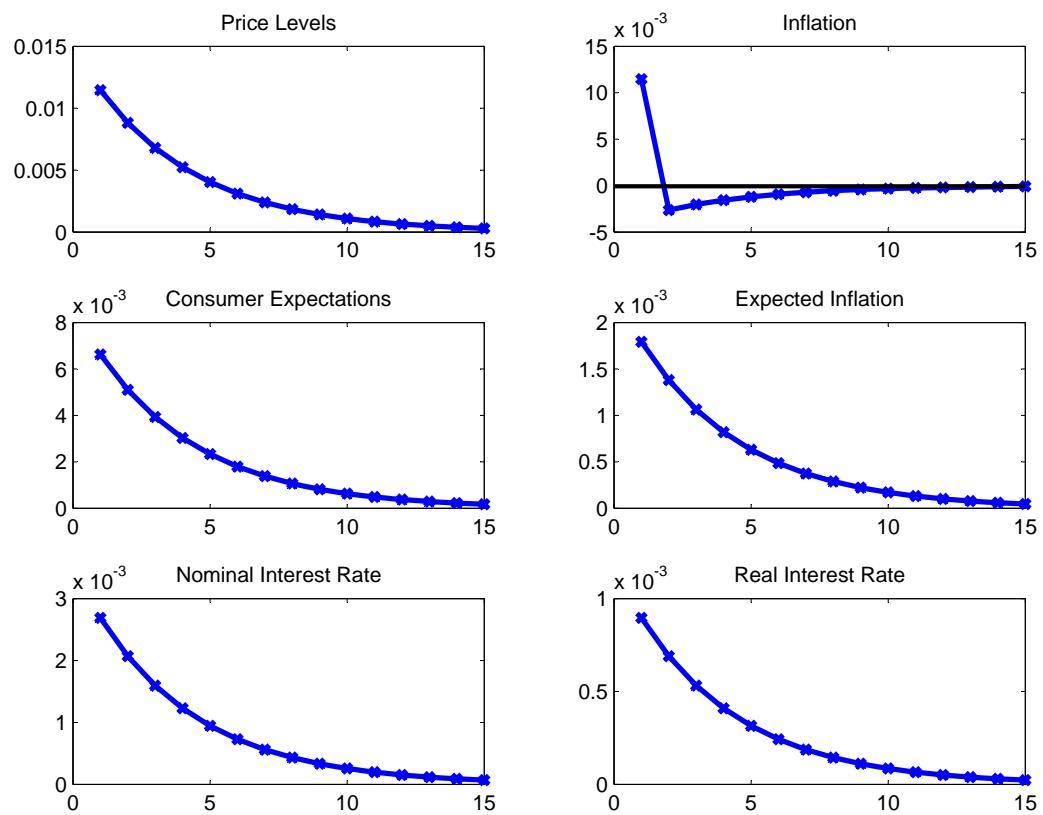


Figure 13: Impulse responses to a positive permanent productivity shock for baseline rule 2 (1)

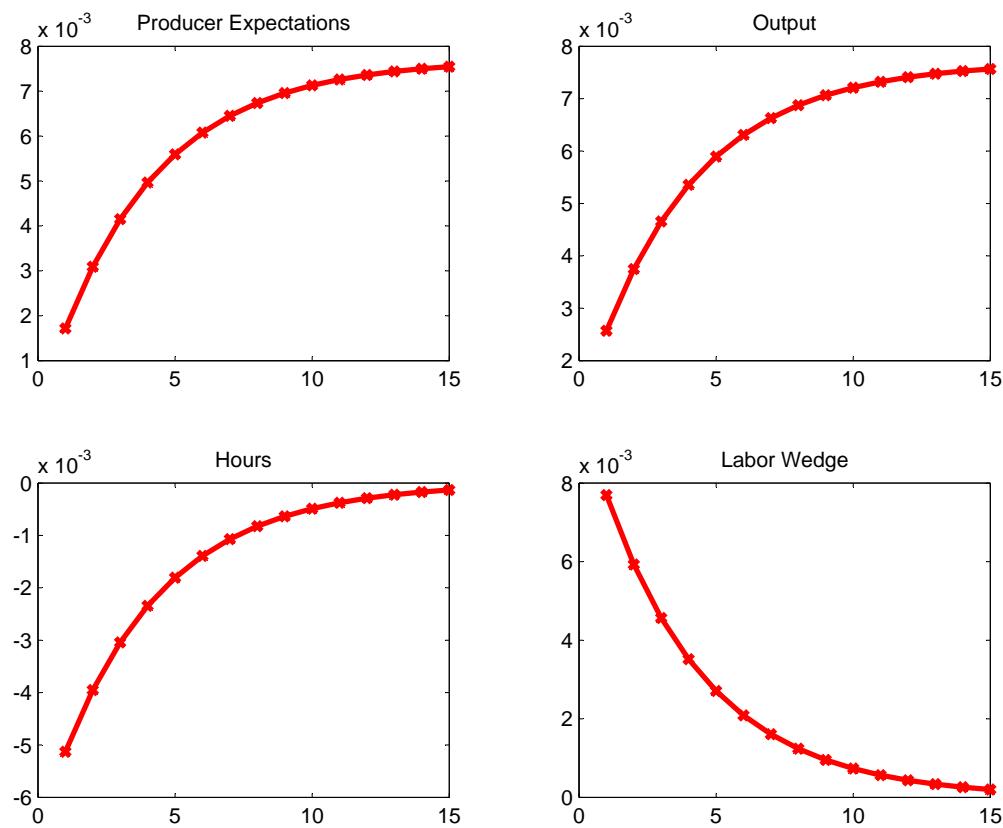


Figure 14: Impulse responses to a positive permanent productivity shock for baseline rule 2 (2)

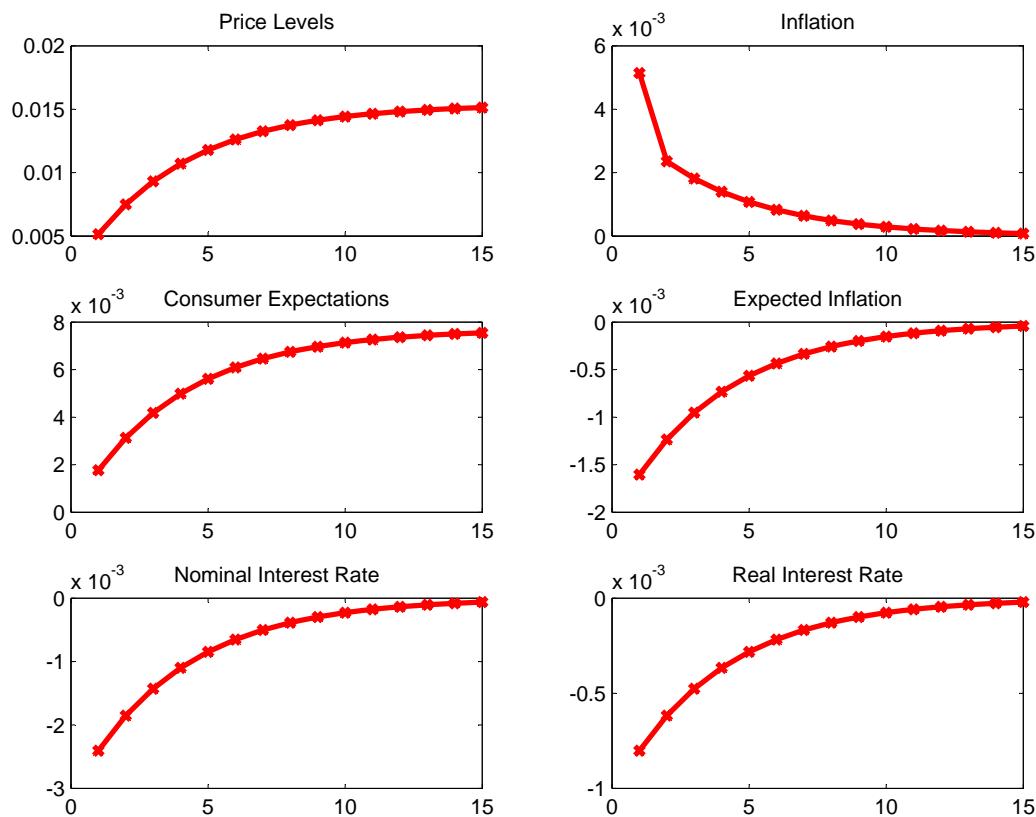


Figure 15: Impulse responses to a positive temporary productivity shock for baseline rule 2 (1)

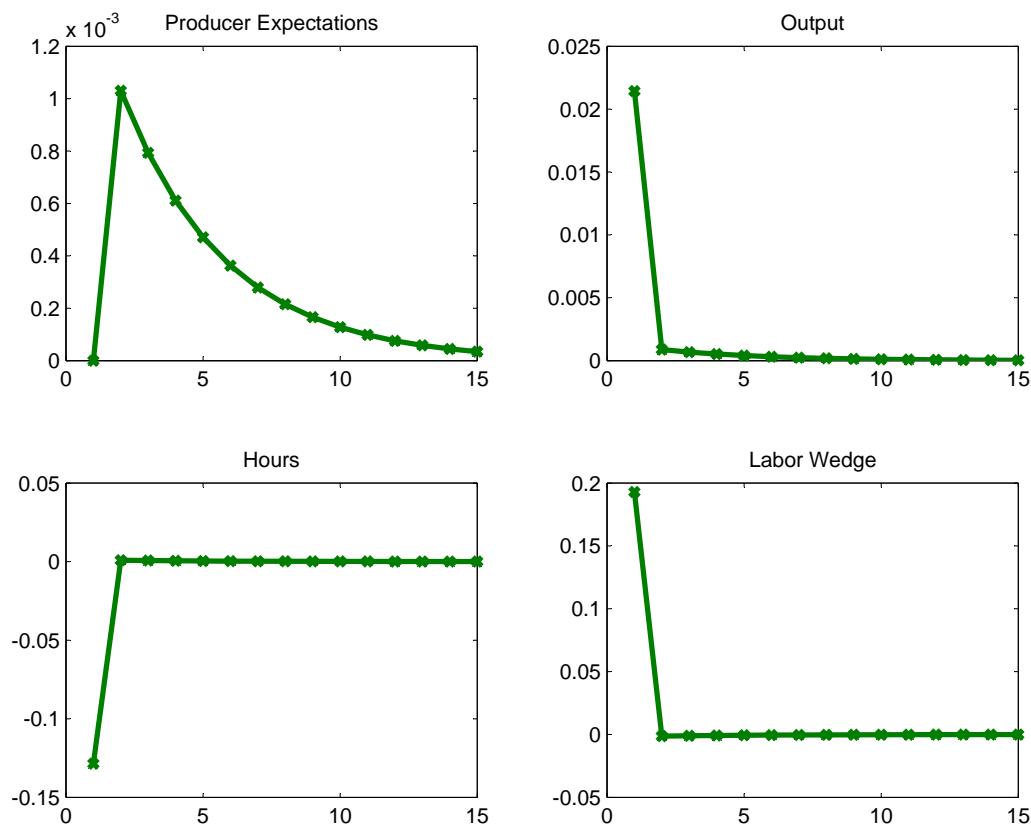


Figure 16: Impulse responses to a positive temporary productivity shock for baseline rule 2 (2)

