Incentives to Innovate, Compatibility and Efficiency in Durable Goods Markets with Network Effects

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No 1054
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October 10, 2014

Abstract

This paper investigates the relation between firms’ R&D incentives and their compatibility decisions regarding durable, imperfectly substitutable network goods in the presence of forward looking consumers. Non drastic product innovation is sequential and both an initially dominant firm and a smaller rival are potential inventors. For sufficiently innovative future products, our first key result is that the dominant firm invests more when there is compatibility and voluntarily decides to supply interoperability information. This happens as the probability that he is the only inventor increases, allowing him to enjoy a higher expected future profit that outweighs the current lost revenue. For economies whose initial market size is considerably large, the rival also demands compatibility but this is no longer true in industries with a relatively smaller number of existing consumers. For less innovative new versions, the dominant firm rejects compatibility and there is a cutoff in network externalities below which he invests more when there is incompatibility. Regarding welfare, we find that a laissez faire Competition Law with respect to the IPR holders is socially preferable.

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2 I would like to express my gratitude to Claudio Mezzetti, Daniel Sgroi, Dan Bernhardt and Udara Peiris. All the errors in this work are solely mine.
1 Introduction

Do dominant firms always refuse to supply interoperability information of their durable, network products and do smaller rivals always demand to be compatible with the current market leader? Even if dominant firms decide not to allow compatibility, does this necessarily imply that their R&D incentives are curbed? Which is the economy that offers the socially preferable balance of aggregate R&D incentives: one that operates under mandatory compatibility or under a laissez faire Competition Law? These questions are certainly not new but this is the first paper they are examined in an environment where technological progress is modelled in a scenario with sequential innovations of durable, network products.

Although standard economic theory predicts that dominant firms may refuse to reveal interoperability information to smaller rivals\(^3\), there are many cases in technology markets where firms with leading market shares welcome compatibility even from direct competitors.\(^4\) Absent network effects, a potential explanation is sequential, important innovation: the initial inventor allows imitation instead of getting a patent as the (exogenous) probability of future inventions increases allowing him to enjoy a higher expected payoff which outweighs the loss from a lower current profit.\(^5\)

In this paper, we provide an alternative explanation: we endogenize firms’ probability of successful innovation by studying the competitors’ R&D incentives as well as their compatibility choices in the presence of durable, network goods and we show that sequential important innovation may lead the dominant firm to voluntarily support compatibility even if it may compete directly with its rival in the future. In this case, dominant firms invest less if the Intellectual Property Rights system is very strong. In particular, we consider a model where substitutable, sequential innovations result from a discrete time R&D stochastic process and technological progress is modelled with exogenous quality improvements.\(^6\)

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\(^3\)See Chen, Doraszelski and Harrington (2009).
\(^4\)See Bessen and Maskin (2009) for examples concerning dominant firms’ welcoming competition.
\(^5\)See Bessen and Maskin (2009).
\(^6\)Further work will endogenize the quality improvements.
give both an initially dominant and a rival firm the ability to come up with valuable ideas following a commonly observed scenario of creative destruction in the Hi-tech and software industry where smaller innovative rivals often displace initial market leaders.\footnote{For example, Microsoft Excel replaced Lotus 1-2-3 and Microsoft Word replaced WordPerfect.} We find that for important innovative products, the leader invests more when compatibility is present and in fact, he voluntarily chooses to offer interoperability information to the rival who in turn accepts it only when the current market size is relatively large. For less innovative products, we find that when network effects are larger than a cutoff, a laissez faire Competition Law with respect to Intellectual Property holders leads the dominant firm to reject compatibility and also invest less than in the economy that compatibility is mandatory. Our welfare analysis, based on a more "economically sound" comparison of the market outcome with the socially optimal level of investment, shows that a laissez faire Competition Law is socially preferable compared to an economy operating under mandatory compatibility, especially when network effects are relatively weak. These results cast some doubts on whether mandated compatibility by Competition Authorities may lead to socially undesirable results.

This paper is organized as follows: the next subsection discusses the Related Literature. Section 2 presents the Model. In Section 3, we solve for equilibrium outcomes when compatibility is either mandatory and under a laissez faire Competition Law. Section 4 provides the socially efficient investment level that a social planner would induce and a comparison with the market equilibrium investment under the economies that operate under mandatory compatibility or under a laissez faire Competition Law and Section 5 concludes.

1.1 Related Literature

This paper relates to the literature regarding firms’ attitude towards compatibility. In an economy where network effects exist and product quality is constant, Chen, Doraszelski and Harrington (2009), Malueg and Schwartz (2006), Economides and Flyer (1998), Cremer, Rey and Tirole (2000) investigate whether compatibility is supported by dominant firms where
modelling usually consists of a two-stage structure: first firms make compatibility decisions and then they engage into price or quantity competition. Although compatibility increases the number of potential buyers because of a larger network, the market leader prefers not to support it because otherwise, he would lose the advantage of the larger installed base. When sequential innovation occurs with certainty and products are substitutable, Athanasopoulos (2014) showed that a dominant market player offers interoperability information of his durable products to a smaller innovative rival when he expects a moderately large, future quality improvement from his competitor. Thus, strategic pricing allows the market leader to extract more of the higher expected total surplus when he supports compatibility. An important assumption in this model is that the rival is the only firm that can innovate in the future. Our work differs because unlike that paper, both competitors are potential future innovators. Moreover, innovation is no longer certain and we assume that the investment cost is a function of the probability of success. For sufficiently innovative future products, we also find that the dominant firm voluntarily supports compatibility.

When network effects are not present and innovations are sequential and complementary, Bessen and Maskin (2009) showed that the initial innovator may welcome imitation because it allows both competitors to invest, increasing the exogenous probability of successful innovation and his second period profit, outweighing the loss from the foregone first period revenues. We depart from their work in a number of ways: First, we assume that direct network effects exist and products are durable. Second, there is an alternative process that allows for product innovation even if there is incompatibility in the market. Third, unlike their paper where the probability of successful innovation is a parameter, we adopt a game theoretical approach where firms’ R&D cost is a function of the probability of success. We also consider forward looking customers and their role in determining equilibrium outcomes and the social optimum. We agree with the message of their paper: dominant firms welcome compatibility when future products are sufficiently innovative while interestingly, we find that the smaller rival may reject compatibility if the initial market size is relatively
small. We also show that the initial market leader rejects interoperability for less important expected new products.

This work also relates to a threatened incumbent’s and a smaller rival’s R&D incentives when the economy operates under mandatory compatibility or a laissez faire Competition Law with respect to Intellectual Property Rights holders when network effects exist and substitutable products are durable. The Literature has focused mostly on the initial market structure and assesses whether a monopolist with perfectly exclusive Property Rights has higher or lower R&D incentives than his counterpart under perfect or imperfect competition.\footnote{See Gilbert (2006) for an excellent survey on issues related to the initial market structure and the firms’ incentives.} In this work, we assume an initial monopolist who is threatened to be displaced by a smaller innovative rival. We find that when network effects are relatively weak and for less innovative products, the dominant firm invests more when he does not supply interoperability, not allowing the rival to use his network. When the new versions are relatively important, the market leader initially invests more when compatibility is supported.\footnote{Current work looks at the initial market structure and the competitors’ incentives when network effects are present and products are durable.}

Regarding welfare, Bessen and Maskin (2009) showed that for important complementary innovations, imitation raises welfare and patents may impede innovation. When network effects are present, Economides (2006) also found that compatibility raises social and consumers’ welfare. We find that a laissez faire Competition Law either leads to compatibility or offers a socially preferable balance of both competitors’ R&D incentives compared to the economy that operates under mandatory compatibility.

## 2 The Model

Consider the market for computer software applications where the current market leader must choose how much to invest into improving its durable, network product. The firm also needs to decide whether to support compatibility of its current and future version with a
smaller rival that can also potentially innovate and has the same set of possible strategies regarding compatibility of her future product and investment decisions.

On the supply side, the sequence of events is as follows: at date $t=0$, competitors simultaneously decide their investment levels as well as their attitude towards compatibility. Compatibility is a binary decision, is achieved bilaterally and comes free of charge.\textsuperscript{10} The two research lines are independent and no firm has a cost advantage in its R&D process over its opponent. More precisely, we assume that R&D spending is quadratic in the probability of successfully improving product quality.

At date $t=1$, the dominant firm chooses the price for its initial version of quality $q_1$\textsuperscript{11} while in the second ($t=2$), the two firms compete a la Bertrand. If both research lines are successful, firms sell an improved product of expected quality $q_2^e$ ($q_2^e > q_1$), which is considered as exogenous in the model.\textsuperscript{12} Forward incompatibility of the product of quality $q_1$ prevents its users from working with a file that is created with a product of higher quality $q_2$. If compatibility is supported and because of backward compatibility, buyers of a product of quality $q_2$ join a network of maximum size.\textsuperscript{13} In contrast, when there is incompatibility, purchasers of a product of quality $q_2$ join only their seller’s network. Note that both firms’ goal is to maximise their expected profits where the marginal cost of production for all product versions is normalized to zero.\textsuperscript{14}

On the demand side, consumers are identical and arrive in constant flows $\lambda_t$ ($t=1, 2$). At date $t=1$, $\lambda_1$ observe the price for the product of quality $q_1$ and must decide whether to buy it. Their utility is partially dependent on network effects, captured by the parameter $\alpha$. Thus, if they buy the product of quality $q_1$, their utility (gross of price) is $q_1 + \alpha \lambda_1 x_1 - c$, where $x_1$ is the $\lambda_1$ customers’ fraction that also buys $q_1$ and $c$ is these customers’ adoption

\textsuperscript{10}See Malueg and Schwartz (2006).
\textsuperscript{11}We follow Ellison and Fudenberg (2000) who also considered quality as a positive, real number $q$.
\textsuperscript{12}Current work endogenizes the quality improvement.
\textsuperscript{13}See Ellison and Fudenberg (2000) for a paper where backward compatibility and forward incompatibility are present.
\textsuperscript{14}Zero marginal cost is consistent with the applications in the computer software market industry.
cost.\textsuperscript{15} Of course, these customers’ overall benefit depends on their forecasts regarding the second period play.

At date \(t=2\) and if there is a new product in the market, the new \((\lambda_2)\) and the old \((\lambda_1)\) customers make their purchasing decisions after they observe the rival firms’ prices. Old customers’ purchasing decision given announced prices resembles a coordination game and can have multiple equilibria. Following the literature, old consumers may be able to coordinate either to the Pareto optimal outcome or to what all the members of their class prefer.\textsuperscript{16} In the similar coordination problem related to the new customers’ purchasing decisions, the standard assumption is that buyers with the same preferences act as if they were a single player. Thus, after observing the prices, they coordinate to what is best for all of them. Since price discrimination is possible, both competitors can offer lower prices to old customers. We restrict attention to pure firms’ strategies and all consumers make their purchasing decisions simultaneously while we assume that they decide to purchase a superior product rather than an old version and join a network of superior than a smaller size even when their net utility may be equivalent. We also assume the same discount factor \(\delta\) for all the agents in the economy.

Figure 1 summarizes the timing of the agents’ moves:

3 Market outcome

In this section, we will solve for equilibrium outcomes; that is, firms’ investment decisions, their prices in both the first and the second period as well as customers’ choices. We will start our analysis by considering the case where compatibility is mandatory.

\textsuperscript{15}Note that the utility function may not be necessarily linear in income (any monotonic transformation would suffice) but linear utility simplifies the analysis.

\textsuperscript{16}See Ellison and Fudenberg (2000).
3.1 Mandatory compatibility

We will solve the model using backwards induction, starting from the second period firms’ pricing decisions \((t = 2)\), going back to calculating the dominant firm’s price for his initial version of quality \(q_1\) \((t = 1)\) as well as the competitors’ optimal investment and compatibility decisions \((t = 0)\).

3.1.1 Second period \((t = 2)\)

Imagine that both firms innovate and think first of the new customers \((\lambda_2)\) who join a network of maximum size independently of where they purchase their new product of quality \(q_2\). Thus, given the competitors’ prices and if we restrict attention to linear utility in income\(^{17}\), their utility by purchasing any of the two new products is \(q_2 + \alpha - c - p_{22i}\), after normalizing the second period market size to unity where the three subscripts in the price charged denote

\(^{17}\)The same results hold for any utility function, \(V(.)\).
the quality of the product \((q_2)\), the type of consumers \((\lambda_2)\) and the product maker \((i = 1\) for the leader and \(i = 2\) for the smaller rival), respectively. If all these customers purchase the dominant firm’s initial version, their utility given his price \(p_{12}\) is \(q_1 + \alpha \lambda_1 x_1 + \alpha \lambda_2 - c - p_{12}\), where \(x_1\) is the \(\lambda_1\) customers’ fraction that sticks to \(q_1\).

Old customers \((\lambda_1)\) observe the prices set by the competitors and their utility is \(q_2 + \alpha - c_u - p_{21i}\), if they buy \(q_2\) from competitor \(i\) and \(q_1 + \alpha \lambda_1 x_1 + \alpha \lambda_2 x_2\) if they stick to the product of quality \(q_1\), where \(x_1, x_2\) are the customers’ fractions that either stick or buy \(q_1\) in the second period. If old customers make their purchasing decisions independently of what other old customers do, they will buy either the dominant or the smaller firm’s product when:

\[ p_{21i} \leq \Delta q + \alpha \lambda_2 (1 - x_2) - c_u, \ \forall i = 1, 2. \]

We make the following assumption:

**Assumption 1 (A1):** \(\Delta q + \alpha \lambda_2 x_2 - c_u \geq 0, 0 \leq x_2 \leq 1\).

This assumption says that the old second period customers’ expected benefit from buying any new product is at least greater than the cost of learning how to use it and allows us to isolate the role of network externalities and the expected quality improvements in firms’ strategies and welfare.

Thus, in such a case, all customers buy or purchase any new version for free due to Bertrand competition.

If only the dominant firm innovates, he remains the sole supplier in both periods. Thus, given his prices, new customers’ utility if they purchase the new product \((q_2)\) is \(q_2 + \alpha - c - p_{221}\), while if they all buy his initial version \((q_1)\), their utility is \(q_1 + \alpha \lambda_1 x_1 + \alpha \lambda_2 - c - p_{121}\), where \(x_1\) is the old customers’ fraction that sticks to the initial version. Old customers’ utility if they upgrade to the dominant firm’s \(q_2\) is \(q_2 + a - c_u - p_{211}\) while their utility if they stick to

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18 These customers are induced to buy the initial product of quality \(q_1\) at \(t = 1\) (see the Appendix for the first period analysis).

19 See the Appendix for the prices these customers are willing to pay if they coordinate to what all the other members of their class prefer.
the old version is \( q_1 + \alpha \lambda_1 x_1 + \alpha \lambda_2 x_2 \). If these customers coordinate on the Pareto optimal outcome, they will buy the new product even if everyone else sticks to \( q_1 \) \((x_1 = 1)\) if:

\[
p_{211} \leq \Delta q + \alpha \lambda_2 (1 - x_2) - c_u.
\]

Thus, since the dominant firm’s second period profit is a decreasing function of the number of new customers who stick to the initial version, the market leader’s optimal choice is to stop selling his initial version and the prices he charges to customers are \( p_{221} = q_2 + \alpha - c \), \( p_{211} = \Delta q + \alpha \lambda_2 - c_u \).

When the rival firm is the sole inventor, the dominant firm can no longer stop selling his initial version in the second period as such a choice would imply a potentially collusive behaviour. Thus, in this scenario, the competitors’ optimal prices are \( p_{222} = \Delta q + \alpha \lambda_1 \), \( p_{121} = 0 \), \( p_{212} = \Delta q + \alpha \lambda_2 - c_u \) and all customers buy the rival firm’s innovative product.

The last case occurs when none of the competitors innovates where the new customers face a price \( p_{121} = q_1 + a - c \) from the dominant firm which extracts all their surplus.\(^{20}\)

### 3.1.2 First and initial period (\( t = 1 \) and \( t = 0 \))

In the first period \((t = 1)\), the dominant firm decides on the optimal price of his initial version of quality \( q_1 \) wishing to extract consumers’ total expected surplus and potential buyers \((\lambda_1)\) make their purchasing decisions, depending on their expectations regarding the market participants’ second period behaviour.

Moving to the initial period \((t = 0)\), both firms decide their optimal investment taking into consideration that the rival is also maximising his/her expected total profits.\(^{21}\) We will consider the following possible scenarios:

**Scenario 2 (A2):** \( \Delta q^e < \alpha \lambda_1 \), \( \Delta q^e \geq \lambda_1 c_u \). This scenario occurs when the expected quality improvement is smaller relative to the network effects.

\(^{20}\)See the Appendix for the table containing the second period prices in the different scenarios.

\(^{21}\)See the Appendix for the rival’s maximization problems and their optimal investment levels.
Scenario 3 (A3): \( \Delta q^e > \alpha \lambda_1, \Delta q^e \geq \lambda_1 c_u \). In this case, the expected quality improvement is larger than the extent of network externalities.

The next lemma summarizes the market equilibrium outcome when compatibility is mandatory:

**Lemma 1** Both competitors’ optimal choice is to invest into developing the product of quality \( q_2 \). If A2 holds, the dominant firm’s investment decision is an increasing function of the rival’s optimal choice. When A3 holds, the dominant firm’s investment is a decreasing function of the rival’s optimal choice. Customers in the first period purchase the product of quality \( q_1 \) and in the second, the whole market purchases the superior product of quality \( q_2 \).

When network effects are larger than the expected quality improvement (A2), the dominant firm’s reaction function is an increasing function of the rival’s investment decision.\(^{22}\) On the other hand, when network effects are relatively weak (A3), the dominant firm seems to free ride on the rival’s investment choice as his expected second period benefit by increasing his probability of success would be outweighed by the additional first period cost.

### 3.2 Laissez faire Competition Law

Under a laissez faire Competition Law, both firms initially choose their investment levels as well as whether they will support compatibility in the future period.

**3.2.1 Second period \((t = 2)\)**

Think first of the scenario where only the rival innovates and consider the new second period customers \((\lambda_2)\). After they observe the prices, if they all purchase the rival’s product of quality \( q_2 \), their utility is \( q_2 + \alpha \lambda_2 + \alpha \lambda_1 (1 - x_1) - c - p_{222} \), where \( 1 - x_1 \) is the old customers’ fraction that purchases \( q_2 \). If they all buy \( q_1 \), their utility is \( q_1 + \alpha \lambda_2 + \alpha \lambda_1 x_1 - c - p_{121} \).

\(^{22}\)See the Appendix for the graphical representation of the different cases.
Thus, these customers prefer the rival’s superior product of quality $q_2$ if:

$$p_{222} - p_{121} \leq \Delta q + \alpha \lambda_1 (1 - 2x_1).$$

Old customers also observe the prices and decide whether to buy the superior product or stick to the initial version.\textsuperscript{23} If they purchase $q_2$, their utility is $q_2 + \alpha \lambda_1 (1 - x_1) + \alpha \lambda_2 (1 - x_2) - c_u - p_{212}$ while if they stick to $q_1$, their utility is $q_1 + \alpha \lambda_1 x_1 + \alpha \lambda_2 x_2$, where $x_1, x_2$ are the old and new customers’ fractions that stick or buy $q_1$, respectively. If these customers coordinate on the Pareto optimal outcome, they will buy $q_2$ even when all the other old customers stick to $q_1$ if:

$$p_{212} \leq \Delta q + \alpha \lambda_2 (1 - 2x_2) - \alpha \lambda_1 - c_u.\textsuperscript{24}

We will consider the following two scenarios:

**Scenario 4 (A4):** $\Delta q + \alpha \lambda_2 - \alpha \lambda_1 - c_u < 0$. In this scenario and in equilibrium, old customers do not buy the rival’s product of quality $q_2$ as they are better-off by retaining the dominant firm’s initial version.

**Scenario 5 (A5):** $\Delta q + \alpha \lambda_2 - \alpha \lambda_1 - c_u > 0$. In this case and in equilibrium, the first period customers are better off by purchasing the rival firm’s new product.

If the quality improvement from the rival’s new product is relatively small (A4), new customers prefer the product of quality $q_2$ if:

$$p_{222} - p_{121} \leq \Delta q - \alpha \lambda_1,$$

and thus, the optimal firms’ prices are: $p_{222} = \Delta q - \alpha \lambda_1$, $p_{121} = 0$.

If old customers buy the rival’s version (A5 holds), new customers prefer the new product

\textsuperscript{23}We consider here that these customers were already induced to buy $q_1$ in the previous period (see the Appendix).

\textsuperscript{24}See the Appendix for the price these customers are willing to pay to purchase the product of quality $q_1$ if they coordinate to what all the other members of their class prefer.
rather than the old if:

\[ p_{222} - p_{121} \leq \Delta q + \alpha \lambda_1, \]

and the competitors’ optimal choices are: \( p_{222} = \Delta q + \alpha \lambda_1, \) \( p_{121} = 0, \) \( p_{212} = \Delta q + \alpha \lambda_2 - \alpha \lambda_1 - c_u. \)

In the scenario that both competitors’ R&D processes are successful, consider first the new customers. After they observe the competitors’ prices, their utility if they all purchase the dominant firm’s or the rival’s \( q_2 \) is \( q_2 + \alpha \lambda_2 + \alpha \lambda_1 x_1 - c - p_{221}, \) \( q_2 + \alpha \lambda_2 + \alpha \lambda_1 x_1’ - c - p_{222}, \) respectively, where \( x_1, x_1’ \) are the old customers’ fractions that belong to either of the rivals’ network. If they all buy the dominant firm’s initial version, their utility is \( q_1 + \alpha \lambda_2 + \alpha \lambda_1(1 - x_1 - x_1’), \) \( q_1 + \alpha \lambda_2 - c - p_{121}. \) Thus, these customers will choose to purchase the dominant firm’s superior product if:

\[
q_2 + \alpha \lambda_2 + \alpha \lambda_1 x_1 - c - p_{221} \geq \\
\geq \max\{q_2 + \alpha \lambda_2 + \alpha \lambda_1 x_1’ - c - p_{222}, \ q_1 + \alpha \lambda_2 + \alpha \lambda_1(1 - x_1 - x_1’), \ q_1 + \alpha \lambda_2(1 - x_2 - x_2’), \ q_1 + \alpha \lambda_2(1 - x_2 - x_2’) + \alpha \lambda_1(1 - x_1 - x_1’). \}
\]

Moving our attention to the old customers, their utility if they purchase \( q_2 \) from either the dominant or the rival firm is \( q_2 + \alpha \lambda_2 x_2 + \alpha \lambda_1 x_1 - c_u - p_{211}, \) \( q_2 + \alpha \lambda_2 x_2’ + \alpha \lambda_1 x_1’ - c_u - p_{212}, \) respectively while if they stick to the initial version \( q_1, \) their utility is \( q_1 + \alpha \lambda_2(1 - x_2 - x_2’), \) \( q_1 + \alpha \lambda_1 + \alpha \lambda_2 x_2’ - c_u - p_{212}. \) Thus, they will choose to buy the dominant firm’s product of quality \( q_2 \) even if all the other old customers either stick to the initial version or buy the rival’s new version if:

\[
q_2 + \alpha \lambda_2 x_2 - c_u - p_{211} \geq q_2 + \alpha \lambda_1 + \alpha \lambda_2 x_2’ - c_u - p_{212}
\]

and

\[
q_2 + \alpha \lambda_1 + \alpha \lambda_2 x_2 - c_u - p_{211} \geq q_1 + \alpha \lambda_1 + \alpha \lambda_2(1 - x_2 - x_2’).
\]

Note that when A4 holds and old customers coordinate on the Pareto optimum, their utility if they purchase the rival’s product of quality \( q_2 \) is not an option for them as it is strictly dominated by their alternative of sticking to \( q_1. \) The dominant firm’s optimal choice is to
stop selling the initial version to the new second period customers, the equilibrium prices are \( p_{221} = \alpha \lambda_1, \ p_{222} = 0, \ p_{211} = \Delta q + \alpha \lambda_2 - c_u \) and all customers buy the dominant firm’s superior product. When A5 holds, the whole market buys either the rival’s or the dominant firm’s new version and Bertrand competition drives all prices to zero.

The scenarios where the dominant firm is the only innovator as well as the case where no firm’s R&D process is successful lead to the same market outcome as in the economy that operates under mandatory compatibility.\(^{25}\)

### 3.2.2 First and initial period \((t = 1 \text{ and } t = 0)\)

In the first period \((t = 1)\), the dominant firm decides the optimal price of his initial version of quality \(q_1\) wishing to extract customers’ expected total surplus and potential buyers \((\lambda_1)\) make their purchasing decisions, depending on their expectations regarding the market participants’ second period behaviour.

Moving to the initial period \((t = 0)\), both competitors choose their investment levels aiming to maximise their expected profits.\(^{26}\)

The next proposition summarizes the market equilibrium outcome in an economy that operates under a laissez faire Competition Law:\(^{27}\)

**Proposition 2** (a) For relatively less innovative future products (A4), the dominant firm’s optimal choice is not to support compatibility. (b) For sufficiently innovative products (A5): (1) both firms welcome compatibility for a relatively large initial market size \((\lambda_1)\), (2) if the first period market size is relatively small, the rival rejects to offer interoperability information to the initial market leader.

**Proof.** See the Appendix. ■

\(^{25}\)See the Appendix for the table containing the equilibrium second period prices under the different scenarios.

\(^{26}\)See the Appendix for the competitors’ maximization problems and their optimal investment choices as functions of the rival’s optimal choice.

\(^{27}\)Note that the dominant firm would be indifferent between supporting and impeding compatibility if old consumers coordinate to what all the other old consumers prefer.
For less innovative products (A4), incompatibility prevails in the market as the dominant firm prefers not to share his network with the smaller innovative rival. More precisely, for relatively weak network effects (A3) and unlike the rival, the dominant firm invests more under incompatibility while for stronger network externalities (A2), the dominant firm would have invested more if compatibility was compulsory. On the other hand, for sufficiently innovative products relative to network externalities (A5), the dominant firm both welcomes compatibility and invests more even if the rival is a direct future competitor. This happens as the gains from sharing its network outweigh the potential costs: more precisely, by supporting compatibility, the probability that he is the only inventor increases allowing him to enjoy a larger second period expected profit exceeding the loss from a lower first period profit. The rival faces a trade-off: if she supports compatibility, the probability of being the sole second period supplier decreases while it allows her to set a higher price to existing customers ($\lambda_1$). Thus, for a relatively large first period market size, her optimal choice is to offer compatibility to the market leader (b1). In a such a case, she also invests more than in a economy that incompatibility is mandatory. When the number of old customers is smaller (b2), unlike the dominant firm, the rival is better-off by not supplying interoperability information to the initial market leader.

4 Social Welfare Maximization

We consider the problem faced by a social planner who wishes to maximise the sum of consumers’ and producers’ total discounted expected surplus. He has access to the firms’ cost functions and can invest into the two research lines as well as choose his attitude towards compatibility.\footnote{We call the initial line whose past R&D success produces $q_1$ as Research line 1.}

If the planner supports compatibility, all customers are expected to buy the improved version of quality $q_2$ in the second period (A1), joining a network of maximum size. For

\footnote{See figures 2 and 3 in the Appendix.}
less innovative products (A4) and unlike the case where innovations are relatively important (A5), if compatibility is not supported, old customers only buy the Research line 1 new version.

The next proposition summarizes the socially optimal investment and compatibility choice and provides a comparison with the market equilibrium obtained in an economy operating under mandatory compatibility or a laissez faire Competition Law:

**Proposition 3** a) If A4 and A2 hold, the social planner decides to support compatibility. Although the economy that operates under mandatory compatibility leads to overinvestment while a laissez faire Competition Law may lead to underinvestment, the laissez faire Competition Law is socially preferable. b) If A4 and A3 hold, the planner may choose to support compatibility. The market equilibrium outcome in a laissez faire economy leads to incompatibility and is always socially preferable compared to the market equilibrium under mandatory compatibility. c) If A5 holds, the planner is indifferent between supporting and impeding compatibility.

**Proof.** See the Appendix.

For less innovative products (A4), although a laissez faire economy leads to the dominant firm rejecting compatibility, the magnitude of the potential inefficiency is smaller compared to the economy that operates under mandatory compatibility. In particular, when network effects are relatively weak (A3), a laissez faire Competition Law leads to more balanced R&D incentives for both rivals and is certainly socially preferable compared to the economy that mandates compatibility where the dominant firm underinvests and the rival overinvests heavily.30 Similarly, when network externalities are relatively stronger (A2), a market where interoperability is compulsory leads to overinvestment and a laissez faire Competition Law is socially preferable although the rival is deterred to invest.31

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30 See figure 3 in the Appendix.
31 See figure 2 in the Appendix.
In conclusion, we could say that a Laissez faire Competition Law is socially preferable compared to an economy that either mandates compatibility or imposes very strong Intellectual Property Rights.

5 Conclusion

The first contribution of this work is that we give an alternative explanation of why dominant firms may welcome compatibility. More precisely, we show that sequential innovation and sufficiently innovative products in an economy with durable, network goods allow the market leader to voluntarily supply interoperability information even to direct future competitors. In fact, when compatibility is present, the dominant firm invests more increasing his probability of success as well as the probability that he is the only inventor in the market. On the other hand, the rival’s optimal choice depends on the market size: if the number of initial customers is sufficiently large, she will also support compatibility while this is no longer true for a smaller initial market size.

Our second contribution relates to the dominant firm’s R&D incentives as we show that they are not curbed under incompatibility for less innovative products. In particular, we find a critical cutoff in network externalities below which the market leader invests more when he refuses to support compatibility to his future potential rival.

Third, we hope to contribute in the discussion with respect to the social desirability of a more interventionist Competition Law: we find that when network effects are weak, a laissez faire Competition Law is socially preferable compared to an economy that operates under mandatory compatibility as a laissez faire market either converges to compatibility or when this doesn’t occur, incompatibility is socially beneficial. When network externalities are strong, unlike a laissez faire market, an economy that operates under mandatory compatibility leads to overinvestment.
We acknowledge limitations of this piece of research. First, current work considers the interaction of different business models when the quality improvement is endogenous and is not modelled as a parameter when network effects and durability are present. Further research could also analyze the competitors’ R&D incentives and compatibility decisions in the face of stochastic demand.

References


6 Appendix

6.1 Second period prices when old customers coordinate to what all the other old customers do

6.1.1 Compatibility

The cases where both or none of the competitors innovate yield the same equilibrium second period prices as in the case that customers coordinate to the Pareto optimum.

Thus, let’s think of the case that the rival is the only firm that innovates in the second period. Old and new customers purchase the product of quality \( q_2 \) if:

\[
q_2 + \alpha - c_u - p_{212} \geq q_1 + \alpha \lambda_2 x_2,
\]

or equivalently:

\[
p_{212} \leq \Delta q + \alpha \lambda_1 + \alpha \lambda_2 (1 - x_2) - c_u
\]

and

\[
q_2 + \alpha - c - p_{222} \geq q_1 + \alpha \lambda_2 + \alpha \lambda_1 x_1 - p_{121}
\]
or

\[ p_{222} - p_{121} \leq \Delta q + \alpha \lambda_1 (1 - x_1) \]

are satisfied, respectively. From A1, old customers buy the new version and the equilibrium second period prices are: \( p_{222} = \Delta q + \alpha \lambda_1, p_{212} = \Delta q + \alpha - c_u, p_{121} = 0 \).

## 6.1.2 Incompatibility

We will show that the second period equilibrium prices are equal to the case of mandatory compatibility.

If only the rival firm innovates, old customers buy the new product and do not stick to \( q_1 \) if:

\[ q_2 + \alpha \lambda_1 + \alpha \lambda_2 x_2 - c_u - p_{212} \geq q_1 + \alpha \lambda_2 (1 - x_2) \]

or equivalently

\[ p_{212} \leq \Delta q + \alpha \lambda_1 + \alpha \lambda_2 (2x_2 - 1) - c_u \]

while the new customers buy the product of quality \( q_2 \) if:

\[ q_2 + \alpha \lambda_2 + \alpha \lambda_1 \psi_1 - c - p_{222} \geq q_1 + \alpha \lambda_2 + \alpha \lambda_1 (1 - \psi_1) - p_{121} \]

or equivalently if:

\[ p_{222} - p_{121} \leq \Delta q + \alpha \lambda_1 (2\psi_1 - 1). \]

Thus, similarly to the case where compatibility is mandatory, the second period equilibrium prices are \( p_{222} = \Delta q + \alpha \lambda_1, p_{212} = \Delta q + \alpha - c_u, p_{121} = 0 \).

Note that if both firms’ R&D processes are successful, we obtain the same second period equilibrium prices as in the case compatibility is mandatory.
### 6.2 Tables regarding the second period prices

The next table summarizes the different potential cases as well as the rivals’ optimal second period prices charged to the new and the old customers under compatibility:

<table>
<thead>
<tr>
<th></th>
<th>Prices to $\lambda_2$</th>
<th>Prices to $\lambda_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both firms innovate</td>
<td>$p_{22i} = 0$, $\forall i = 1, 2$</td>
<td>$p_{21i} = 0$, $\forall i = 1, 2$</td>
</tr>
<tr>
<td>Only the Dominant innovates</td>
<td>$p_{221} = q_2 + \alpha - c$</td>
<td>$p_{211} = \Delta q + \alpha \lambda_2 - c_u$</td>
</tr>
<tr>
<td>Only the Rival innovates</td>
<td>$p_{222} = \Delta q + \alpha \lambda_1$</td>
<td>$p_{212} = \Delta q + \alpha \lambda_2 - c_u$</td>
</tr>
<tr>
<td>Noone innovates</td>
<td>$p_{121} = q_1 + \alpha - c$</td>
<td>already bought at $t = 1$</td>
</tr>
</tbody>
</table>

Under mandatory incompatibility, the following table summarizes all the potential second period cases as well as the rivals’ prices to the different customers’ classes under $A4$ when both firms invest into producing an improved version of quality $q_2$:

<table>
<thead>
<tr>
<th></th>
<th>Prices to $\lambda_2$</th>
<th>Prices to $\lambda_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both firms innovate</td>
<td>$p_{221} = \alpha \lambda_1$</td>
<td>$p_{211} = \Delta q + \alpha \lambda_2 - c_u$</td>
</tr>
<tr>
<td>Only the Dominant innovates</td>
<td>$p_{221} = q_2 + \alpha - c$</td>
<td>$p_{211} = \Delta q + \alpha \lambda_2 - c_u$</td>
</tr>
<tr>
<td>Only the Rival innovates</td>
<td>$p_{222} = \Delta q - \alpha \lambda_1$</td>
<td>$p_{212} = 0$</td>
</tr>
<tr>
<td>Noone innovates</td>
<td>$p_{121} = q_1 + \alpha - c$</td>
<td>already bought at $t = 1$</td>
</tr>
</tbody>
</table>

While under $A5$, the table becomes:

<table>
<thead>
<tr>
<th></th>
<th>Prices to $\lambda_2$</th>
<th>Prices to $\lambda_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both firms innovate</td>
<td>$p_{222} = 0, p_{221} = 0$</td>
<td>$p_{21i} = 0$, $\forall i = 1, 2$</td>
</tr>
<tr>
<td>Only the Dominant innovates</td>
<td>$p_{221} = q_2 + \alpha - c$</td>
<td>$p_{211} = \Delta q + \alpha \lambda_2 - c_u$</td>
</tr>
<tr>
<td>Only the Rival innovates</td>
<td>$p_{222} = \Delta q + \alpha \lambda_1$</td>
<td>$p_{212} = \Delta q + \alpha \lambda_2 - \alpha \lambda_1 - c_u$</td>
</tr>
<tr>
<td>Noone innovates</td>
<td>$p_{121} = q_1 + \alpha - c$</td>
<td>already bought at $t = 1$</td>
</tr>
</tbody>
</table>
6.3 Calculating firms’ investment decisions as a function of the rival’s optimal choices

6.3.1 Mandatory compatibility

Given the market leader’s price \( p_{11} \), first period customers’ total discounted expected utility if they purchase the product \( q_1 \) is:

\[
q_1 + \alpha \lambda_1 - c + \delta s_1 (1 - s_2) \left( q_2^e + \alpha - c_u - p_{211}^e \right) + \\
+ \delta (1 - s_1) s_2 (q_2^e + \alpha - c_u - p_{212}^e) + \delta s_1 s_2 (q_2^e + a - c_u) + \\
+ \delta (1 - s_1)(1 - s_2)(q_1 + a) - p_{11},
\]

where \( s_1, s_2 \) are the dominant firm’s and the rival’s probabilities of successfully innovating, respectively and the subscript \( e \) denotes the expectation for the quality improvement and the second period prices. Note that the market leader wishes to extract \( \lambda_1 \) customers’ expected total surplus by setting the highest price \( p_{11} \) that would induce them to buy \( q_1 \) and thus, his optimal first period choice is:

\[
p_{11} = q_1 + \alpha \lambda_1 - c + \delta s_1 (1 - s_2) \left( q_2^e + \alpha - c_u - p_{211}^e \right) + \delta (1 - s_1) s_2 (q_2^e + \alpha - c_u - p_{212}^e) + \\
+ \delta s_1 s_2 (q_2^e + a - c_u) + \delta (1 - s_1)(1 - s_2)(q_1 + a). \tag{1}
\]

Moving back to the initial period \( (t = 0) \), the two firms simultaneously choose their investment levels. Thus, the smaller firm’s maximization problem is:

\[
\max_{s_2 \geq 0} \begin{cases} \\
\delta s_2 (1 - s_2^*) (\lambda_2 p_{222}^* + \lambda_1 p_{212}) - s_2^2 / 2 \text{ if } s_2 > 0 \\
0, \text{ otherwise} 
\end{cases}.
\tag{2}
\]
The similar maximization problem for the dominant firm is:

$$\max_{s_1 \geq 0} \begin{cases} 
\lambda_1 p_{11} + \delta \lambda_1 s_1 (1 - s_2^*) p_{211} + \delta \lambda_2 s_1 (1 - s_2^*) p_{221} + \\
+ \delta \lambda_2 (1 - s_1) (1 - s_2^*) p_{121} - s_1^2 / 2, \text{ if } s_1 > 0 \\
\lambda_1 p_{11} + \delta \lambda_2 (1 - s_2^*) p_{121}, \text{ otherwise}
\end{cases}$$

(3)

where $p_{11}$ is given in (1) and $s_2^*$ is the rival’s optimal investment choice.

The rival and the dominant firm’s investment decisions as a function of the competitor’s optimal choice are:

$$s_2 = \delta (1 - s_1^*)(\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u),$$

(4)

$$s_1 = -s_2^* (\delta \lambda_2 \Delta q^e - \delta \alpha \lambda_1 \lambda_2) + \delta (\Delta q^e - \lambda_1 c_u),$$

(5)

respectively.

### 6.3.2 Mandatory incompatibility

If A4 holds, the rival’s optimization problem is:

$$\max_{s_4 \geq 0} \begin{cases} 
\delta (1 - s_3^*) s_4 \lambda_2 p_{221}^e - s_4^2 / 2, \text{ if } s_4 > 0 \\
0, \text{ otherwise}
\end{cases}$$

(6)

where $s_4$ is her investment choice and $s_3^*$ is the dominant firm’s optimal investment decision.

The similar maximization problem faced by the market leader is:

$$\max_{s_3 \geq 0} \begin{cases} 
\lambda_1 p_{11} + \delta \lambda_1 s_3 (1 - s_4^*) p_{211}^e + \delta \lambda_2 s_3 (1 - s_4^*) p_{221}^e + \\
+ \delta \lambda_2 (1 - s_3) (1 - s_4^*) p_{121}^e + \delta \lambda_2 s_3 s_4 \alpha \lambda_1 - s_3^2 / 2, \text{ if } s_3 > 0 \\
\lambda_1 p_{11} + \delta \lambda_2 (1 - s_4^*) p_{121}^e, \text{ otherwise}
\end{cases}$$

(7)
where the price $p_{11}$ extracts the first period customers expected surplus and is given by the expression:

$$p_{11} = q_1 + \alpha \lambda_1 - c + \delta s_3 (1 - s_4) (q_2^e + \alpha - c_u - p_{211}^e) + \delta s_3 s_4 (q_2^e + \alpha - c_u - p_{211}^e) + \delta (1 - s_3) (1 - s_4) (q_1 + \alpha) + s_3 s_4 (q_1 + \alpha \lambda_1).$$

When old customers expect to purchase the product of quality $q_2$ in the second period independently of which firm innovates (A5 holds), the competitors' problems become:

$$\max_{s_4 \geq 0} s_4 (1 - s_3^*) (\lambda_2 p_{222}^e + \lambda_1 p_{212}^e) - s_4^2/2, \quad (6'$$

$$\max_{s_3 \geq 0} \lambda_1 p_{11} + \delta \lambda_1 s_3 (1 - s_4^*) p_{211}^e + \delta \lambda_2 s_3 (1 - s_4^*) p_{221}^e + \delta \lambda_2 (1 - s_3) (1 - s_4^*) p_{121}^e - s_3^2/2, \quad (7'$$

for the rival and the dominant firm, respectively and the first period price is:

$$p_{11} = q_1 + \alpha \lambda_1 - c + \delta s_3 (1 - s_4) (q_1 + \alpha \lambda_1) + \delta s_3 s_4 (q_2^e - c_u) + \delta (1 - s_3) (1 - s_4) (q_1 + \alpha) + \delta (1 - s_3) s_4 (q_1 + 2 \alpha \lambda_1).$$

Note that when both firms’ R&D is successful, all customers are expected to buy the product of quality $q_2^e$ from either of the incompatible competitors. Thus, they will be part of a network of size $x$, with $x$ being any non-negative number. Thus, in the first period, the dominant firm may risk losing these customers if he charges a price greater than $p_{11}$ defined above.

### 6.4 Proof of Proposition 2

a) If A3 and A4 hold, the dominant firm always refuses to support compatibility. To see this, let $E(\Pi_{no\_compatibility}) = f(s)$ and $E(\Pi_{compatibility}) = g(s)$ denote the dominant firm’s expected
profit under incompatibility and compatibility, respectively, where \( f(0) > g(0) \).

We take the derivative of the two functions with respect to the dominant firm’s choice (s):

\[
\frac{\partial f}{\partial s} = -\delta[s_4(\Delta q^e - \alpha \lambda_1 \lambda_2 - \lambda_1 c_u) - (\Delta q^e - \lambda_1 c_u)] - s
\]

while

\[
\frac{\partial g}{\partial s} = \delta[\Delta q^e - \lambda_1 c_u - s_2 \lambda_2(\Delta q^e - \alpha \lambda_1)] - s,
\]

where \( s_4, s_2 \) are the rival’s optimal choices under incompatibility and compatibility, respectively (see figure 3).

The dominant firm is better-off by not supporting compatibility when:

\[
s_2 \lambda_2(\Delta q^e - \alpha \lambda_1) - s_4(\Delta q^e - \alpha \lambda_1 \lambda_2 - \lambda_1 c_u) > 0,
\]

where the rival’s choices lie on the lines:

\[
s_2 = \delta(1 - s)(\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u), \quad \text{and} \quad s_4 = \delta(1 - s)\lambda_2(\Delta q^e - \alpha \lambda_1).
\]

Without loss of generality, we assume that the discount factor is large (\( \delta = 1 \)). After substituting \( s_2 \) and \( s_4 \) in * we get:

\[
(1 - s)(\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u)\lambda_2(\Delta q^e - \alpha \lambda_1) - (1 - s)\lambda_2(\Delta q^e - \alpha \lambda_1)(\Delta q^e - \alpha \lambda_1 \lambda_2 - \lambda_1 c_u) > 0
\]

which always holds and thus \( f_s > g_s \quad \forall s \).

After solving for \( s_1^*, s_3^* \), one gets:

\[
s_1^* = \frac{\Delta q^e - \lambda_1 c_u - (\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u)(\lambda_2 \Delta q^e - \lambda_2 \alpha \lambda_1)}{1 - (\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u)(\lambda_2 \Delta q^e - \lambda_2 \alpha \lambda_1)}
\]

and

\[
s_3^* = \frac{\Delta q^e - \lambda_1 c_u - (\Delta q^e + \alpha \lambda_1 \lambda_2 - \lambda_1 c_u)(\lambda_2 \Delta q^e - \lambda_2 \alpha \lambda_1)}{1 - (\Delta q^e + \alpha \lambda_1 \lambda_2 - \lambda_1 c_u)(\lambda_2 \Delta q^e - \lambda_2 \alpha \lambda_1)}
\]

and after substituting to the expressions for \( s_2^*, s_4^* \), we get:

\[
s_2^* = [1 - \frac{\Delta q^e - \lambda_1 c_u - (\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u)(\lambda_2 \Delta q^e - \lambda_2 \alpha \lambda_1)}{1 - (\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u)(\lambda_2 \Delta q^e - \lambda_2 \alpha \lambda_1)}](\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u)
\]
\[ s_4^* = \left[ 1 - \frac{\Delta q^e - \lambda_1 c_u - (\Delta q^e - \alpha \lambda_1 \lambda_2) (\lambda_2 \Delta q^e - \lambda_2 \alpha \lambda_1)}{1 - (\Delta q^e - \alpha \lambda_1 \lambda_2 - \lambda_1 c_u) (\lambda_2 \Delta q^e - \lambda_2 \alpha \lambda_1)} \right] \lambda_2 (\Delta q^e - \alpha \lambda_1). \]

Thus, * becomes after some algebraic manipulation:

\[
(\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u) \left[ 1 - (\Delta q^e - \alpha \lambda_1 \lambda_2 - \lambda_1 c_u) \lambda_2 (\Delta q^e - \alpha \lambda_1) \right] > \\
(\Delta q^e - \alpha \lambda_1 \lambda_2 - \lambda_1 c_u) \left[ 1 - (\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u) \lambda_2 (\Delta q^e - \alpha \lambda_1) \right]
\]

which simply verifies that \( s_3^* > s_1^* \).

Thus, the dominant firm impedes compatibility.

Note that if A4 and A2 hold, the rival’s optimal choice is not to invest \( (s_4 = 0) \) and the dominant firm chooses not to support compatibility. Think for example the following parameter values that satisfy A4 and A2: \( \Delta q^e = 0.3, \alpha = 1, \lambda_1 = 0.7, \lambda_2 = 0.3, c_u = 0.1, c = 0.2, q_1 = 0.1, q_2 = 0.4, \delta = 1. \) Direct comparison of the dominant firm’s values of maximised expected profit show that he impedes compatibility.

b1) Think for the example the case where: \( \Delta q^e = 0.9, \alpha = 1, \lambda_1 = 0.8, \lambda_2 = 0.2, c_u = 0.2, c = 0.3, q_1 = 0.1, q_2 = 1, \delta = 1. \)

Direct comparison of the two firms’ expected profits lead to the conclusion that they both support compatibility.

b2) Think of the parameter values: \( \Delta q^e = 0.4, \alpha = 1, \lambda_1 = 0.3, \lambda_2 = 0.7, c_u = 0.2, c = 0.3, q_1 = 0.1, q_2 = 0.5, \delta = 1. \)

Direct comparison of the firms’ expected profits yields that the dominant firm supports compatibility while the rival firm rejects it.

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6.5 Proof of Proposition 3

Depending on whether the planner invests or not and whether he supports compatibility or not, the social welfare function if A4 holds is:

\[
\max_{\rho, \rho' \geq 0} \begin{cases} 
\lambda_1(q_1 + \alpha \lambda_1 - c) + \delta \lambda_1(\rho + \rho' - \rho\rho')(q_2^* + \alpha - c_u) + \\
\delta \lambda_1(1 - \rho)(1 - \rho')(q_1 + \alpha) + \delta \lambda_2(\rho + \rho' - \rho^2\rho')(q_2^* + \alpha - c) + \\
+\delta \lambda_2(1 - \rho)(1 - \rho')(q_1 + \alpha - c) - \rho^2/2 - \rho^2/2, \ \rho, \rho' > 0 \text{ if he supports} \\
\text{compatibility,} \\
\lambda_1(q_1 + \alpha \lambda_1 - c) + \delta \lambda_1\rho(q_2^e + \alpha - c_u) + \delta \lambda_1\rho'(1 - \rho)(q_1 + \alpha \lambda_1) + \\
+\delta \lambda_1(1 - \rho)(1 - \rho')(q_1 + \alpha) + \delta \lambda_2(\rho + \rho' - \rho\rho')(q_2^e + \alpha - c) + \\
+\delta \lambda_2(1 - \rho)(1 - \rho')(q_1 + \alpha - c) - \rho^2/2 - \rho^2/2, \ \rho, \rho' > 0 \text{ if he} \\
\text{does not support compatibility,} \\
\lambda_1(q_1 + \alpha \lambda_1 - c + \delta q_1 + \delta \alpha) + \delta \lambda_2(q_1 + \alpha - c), \ \rho = \rho' = 0,
\end{cases}
\]

where \(\rho, \rho'\) are the planner’s investment choices in Research lines 1 and 2, respectively.

a) If A4 and A2 hold, the social planner will make the two products in the second period compatible if the maximum value of the social welfare function is higher compared to the scenario he makes incompatible products. If \(\max \text{SW}_{com}\), \(\max \text{SW}_{incom}\) are the highest values in the social welfare if he supports compatibility or not, it is immediate to see that in the latter case, he only invests in improving the dominant firm’s product.

The planner supports compatibility when:

\[
\max \text{SW}_{com} > \max \text{SW}_{incom}
\]

or equivalently when the expression:

\[
\lambda_1(\rho + \rho' - \rho\rho' - \rho'')(q_2^* + \alpha - c_u) + \lambda_1[(1 - \rho)(1 - \rho') - (1 - \rho'')(q_1 + \alpha)] + \\
\lambda_2(\rho + \rho' - \rho\rho' - \rho'')(q_2^* + \alpha - c) + \lambda_2[(1 - \rho)(1 - \rho') - (1 - \rho'')(q_1 + \alpha - c)]
\]
is positive, where $\rho, \rho'$ are his optimal investment choices when he chooses compatibility and $\rho''$ is his optimal investment if he chooses to have incompatible products satisfying the equations:

$$
\rho = -\delta \rho' (\Delta q^e - \lambda_1 c_u) + \delta (\Delta q^e - \lambda_1 c_u), \\
\rho' = -\delta \rho (\Delta q^e - \lambda_1 c_u) + \delta (\Delta q^e - \lambda_1 c_u), \\
\rho'' = \delta (\Delta q^e - \lambda_1 c_u).
$$

Note that $\rho = \rho' = \frac{\kappa}{\kappa + 1}$, where $\kappa = \delta (\Delta q^e - \lambda_1 c_u), 0 < \kappa < 1$.

Thus, we need to show that:

$$
\lambda_1 \left\{ \frac{2\kappa}{\kappa + 1} - \frac{\kappa^2}{(\kappa + 1)^2} - \kappa (q_2^e + \alpha - c_u) + [(1 - \frac{\kappa}{\kappa + 1})^2 - (1 - \kappa)](q_1 + \alpha) \right\} + \\
\lambda_2 \left\{ (\frac{2\kappa}{\kappa + 1} - \frac{\kappa^2}{(\kappa + 1)^2} - \kappa) (q_2^e + \alpha - c) + [(1 - \frac{\kappa}{\kappa + 1})^2 - (1 - \kappa)](q_1 + \alpha - c) \right\} - 2 \frac{\kappa^2}{(\kappa + 1)^2} \kappa^2 > 0
$$

or equivalently:

$$
\lambda_1 \frac{1 - \kappa^2 - \kappa}{(\kappa + 1)^2} (\Delta q^e - c_u) + \lambda_2 \frac{1 - \kappa^2 - \kappa}{(\kappa + 1)^2} \Delta q^e + \kappa^2 - \frac{2\kappa^2}{(\kappa + 1)^2} > 0.
$$

For parameter values satisfying A1, A2, A4, the above expression takes a positive sign. Think for example the parameter values satisfying A1, A2 and A4 ($\alpha = 1, \lambda_1 = 0.7, \Delta q^e = 0.3, c_u = 0.01$). Direct calculation leads to the conclusion that the above expression is positive and thus the planner chooses compatibility.

b) If A4 and A3 hold, the social planner decides to support compatibility if:

$$
max. SW_{com} > max. SW_{incom}
$$

or equivalently the expression:
\[
\delta \lambda_1 (\rho + \rho' - \rho \rho' - \rho'') (q_2^2 + \alpha - c_u) + \delta \lambda_1 [(1 - \rho)(1 - \rho') - (1 - \rho'')(1 - \rho''')] (q_1 + \alpha) - \\
-\delta \lambda_1 \rho''' (1 - \rho'') (q_1 + \alpha \lambda_1) + \delta \lambda_2 (\rho + \rho' - \rho \rho' - \rho'' - \rho''' + \rho'''' + \rho'''') (q_2^2 + \alpha - c) + \\
+\delta \lambda_2 [(1 - \rho)(1 - \rho') - (1 - \rho'')(1 - \rho''')] (q_1 + \alpha - c) - \rho^2/2 - \rho'^2/2 - \rho''^2/2 - \rho'''^2/2 
\]

takes a positive sign. It is straightforward to see that for parameter values satisfying A4 and A3 (for example, take \(\Delta q_1 = 0.4\), \(\lambda_1 = 0.7, \alpha = 0.5, \delta = 1, c = 0.4, c_u = 0.3, q_1 = 0.1\)), the planner supports compatibility as the social welfare function is maximised.

### 6.6 Figures regarding the competitors’ and the planner’s optimal investment decisions

The next figures summarize the market equilibrium outcome under a laissez faire Competition Law and under mandatory compatibility as well the social optimum level of investment:
Figure 2: A4 and A2

Figure 3: A4 and A3