Optimal Leverage and Strategic Disclosure

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Abstract

Firms seeking external financing jointly choose what securities to issue, and the extent of their disclosure commitments. The literature shows that enhanced disclosure reduces the cost of financing. This paper analyses how disclosure affects the optimal composition of financing means. It considers a market where firms compete for external financing under costly-state-verification, but, in contrast to the standard model: (i) the degree of asymmetric information between firms and outside investors is variable, and (ii) firms can affect it through a disclosure policy, modeled as a verifiable signal with a cost decreasing in its noise component. Two central predictions emerge.

On the positive side, optimal disclosure and leverage are negatively correlated. Efficient equity financing requires that firms are sufficiently transparent, whereas debt does not; it solely relies on the threat of bankruptcy and liquidation. Therefore, more transparent firms issue cheaper equity and face a higher opportunity cost of leveraged external financing. The prediction is shown to be consistent with the behavior of US corporations since the 1980s.

On the normative side, disclosure externalities and time inconsistencies lead to under-disclosure and excessive leverage relative to the constrained best. If mandatory disclosures are feasible – that is, they cannot be easily dodged – they increase welfare. Otherwise, endogenously higher transparency can be triggered if regulators set capital requirements. Capital regulation proves especially useful when (i) firm performances are highly correlated, and (ii) disclosure requirements can be easily dodged – conditions that seem to apply to large financial firms. The view of capital standards as a means to improve the information environment is novel in the literature; its policy implications and challenges are discussed.

Key words: leverage, costly-state-verification, disclosure, asymmetric information, capital requirements, financial regulation, optimal contracting

JEL classification: D82, G21, G32, G38

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1 Introduction

Firms seeking external financing face a multidimensional choice problem. They need to decide both which securities to issue – whether to borrow or to issue stocks, for example – and they choose the extent of their disclosure commitments – for instance, whether to go public or to keep private. Existing evidence suggests that greater disclosure tends to reduce a firm’s cost of financing, as the theory predicts, dampening the degree of asymmetric information in the market.\(^1\) However, the relation between disclosure and the optimal composition of financing means has not been thoroughly investigated. This paper models the inter-linkages between disclosure and security design under asymmetric information – in particular, costly-state-verification. Two central predictions emerge.

On the positive side, disclosure and leverage are negatively correlated. Efficient equity financing requires that firms are sufficiently transparent, and that they disclose audited earnings on a timely basis. In contrast, debt does not need transparency and it sustains repayments by the hard threat of bankruptcy.\(^2\) Indeed, I uncover new firm-level evidence supporting a negative correlation between transparency and leverage for US corporates over the last 40 years, and argue that the prediction is consistent with the early development of modern stock markets, in the 19th century.\(^3\)

On the normative side, potential externalities in disclosure across firms and time inconsistencies lead to insufficient voluntary disclosures and excessive leverage relative to the constrained best. The inefficiency is reduced if regulators can credibly mandate truthful disclosures, but this is often not possible.\(^4\) Modeling explicitly the inter-linkage between disclosure and leverage suggests an alternative policy: setting capital requirements. Higher capital standards encourage firms to be more transparent, in an effort to reduce the otherwise prohibitive costs of equity financing, and are especially useful when profits are highly correlated across firms and mandatory disclosures can be dodged. Both conditions apply to financial firms, which – consistent with the model’s predictions – are

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\(^2\)Both disclosure and bankruptcy are costly. Costs of disclosing information include hiring independent auditors and losing proprietary information – see Bushee and Leuz (2005), Iliev (2010), Ellis et al. (2012), Alexander et al. (2013) and Dambra et al. (2015). Bankruptcy costs stem from the need to formally verify the value of a borrower’s assets (e.g., illiquidity, loss of market share and reputation, legal expenses, supply chain disruption, uncertainty). Almeida and Philippon (2007) find that for a BBB-rated firm the net present value of distress is around 4.5% of the pre-distress value. See also Molina (2005).

\(^3\)Existing evidence that is consistent with the model’s predictions includes: (i) cross-country comparisons that show how more transparent financial systems encourage the use of equity as opposed to fixed-income securities (e.g., Aggarwal and Kyaw (2009)), and (ii) comparisons of private and public firms that show how private firms being more opaque are more likely to rely on debt instruments as opposed to equity when raising additional external financing (e.g., Brav (2009)). To my knowledge, firm level evidence within the context of public firms is still lacking in the empirical literature.

\(^4\)For example, Sloan (2007) documents that a typical RMBS (Residential Mortgage Backed Security) sold prior to 2008 had a disclosure prospectus of more than 300 pages. Though it complied with regulation, the prospectus hardly made such security transparent. Paul Singer (founder of Elliott Associates) does not invest in banks equity because “There is no major financial institution today whose financial statements provide a meaningful clue [about its risks]” (see Partnoy and Eisinger (2013)).
both highly leveraged and opaque.\footnote{In the US, the median leverage ratio for financial firms after the 1980s ranges between 0.88 and 0.93 (Source: author’s calculation on Compustat data).}

More specifically, I consider a financial market in which firms seek financing from a competitive pool of investors under costly-state-verification (CSV). Firms and investors are symmetrically informed at the contracting stage, but acquire different information about the realized output ex post. Previous CSV models assume an extreme type of hidden information: entrepreneurs learn the output perfectly ex post; investors learn nothing but can verify the output at a cost. I relax this assumption, supposing that investors learn the realized output with some probability $\pi \in [0, 1]$.\footnote{The model generalises Gale and Hellwig (1985), who restrict attention to $\pi = 0$. In Appendix C, I show that more general signal structures maintain similar qualitative properties as those derived here. In particular, exactly the same results hold if the signal’s distribution is uniform and it reveals a lower bound on what the realized output can be. The more general case of FOSD is also considered and solved.} Disclosure of verifiable information that increases $\pi$ is privately costly, and the cost increases in the signal to noise ratio. In addition, the disclosure of a firm might convey information about its competitors.\footnote{Recent work of Badertscher et al. (2013), Shroff et al. (2013) and Durnev and Mangen (2009) identifies substantial information externalities across firms. See also Pyke et al. (1990) on industrial districts. I am currently studying the case of non-verifiable disclosed information in another project.}

**Optimal securities.** The optimal capital structure is a mixture of debt and equity and the amount of assets backed by debt (i.e., the leverage ratio) monotonically decreases in the probability $\pi$ that the investors are informed, which measures the degree of asymmetric information. If $\pi = 0$, we have full leverage as in Gale and Hellwig (1985). The intuition is as follows: (i) the financier must verify low messages to prevent cheating by the entrepreneur when output is higher; (ii) whenever there is verification, the optimal repayment equals the full realised output;\footnote{These two features jointly resemble bankruptcy, where costly verification (or liquidation) takes place, and debt holders are senior claimants on the assets of defaulting borrowers. The interpretation of bankruptcy costs as the costs of verification is discussed in Gale and Hellwig (1985) and Tirole (2010).} (iii) whenever there is no verification, the repayment is incentive compatible if and only if it equals a fixed constant (the face value of debt), regardless of the realized output.\footnote{More precisely, Townsend (1979) and Gale and Hellwig (1985) show that debt is the optimal contract among those that feature commitment to deterministic audits. Different results hold if one allows for random audits (Border and Sobel (1987) and Mookherjee and Png (1989)) or lack of commitment (Gale and Hellwig (1989)). Krasa and Villamil (2000, 2003) argue that debt is optimal if both lack of commitment and random audits are assumed.}

Now suppose that $\pi > 0$, i.e. with some probability investors know the state prior to verification. Property (iii) no longer holds: the highest incentive compatible repayment strictly increases with the output, because firms with higher output ex post have more to lose if caught cheating by the financiers. As this happens with probability $\pi$, the incentive constraint is linear in the type space and corresponds to the payoff of selling a fraction $\pi$ of firm’s shares to investors. Moreover, the incentive constraint must be binding outside of bankruptcy in order to minimize the ex ante need for costly verification. Therefore, optimal contracts have an equity component. Pure debt does not work because upon default the firm gets nothing, whereas if output is high it retains a needlessly large fraction of it. In other words, ex post debt imposes an inefficient subsidy across states of nature.
Eventually, when $\pi$ is high enough, there is no need for verification on-the-equilibrium path and the optimal contract is pure equity.\footnote{Only in the limit, when $\pi = 1$, hidden information vanishes and Modigliani and Miller (1958) holds (i.e., the security design problem becomes irrelevant).}

Importantly, whenever there is verification on-the-equilibrium path the optimal capital structure is unique, for every $\pi$. Otherwise, though there may be multiple optimal securities, they are ex ante identical to issuing no debt, and selling a fraction $s\pi$ of shares, for some $s \in (0, 1)$ that is pinned down by the zero profit condition of investors. As a result, the feasible strategies of a firm can be reduced to selecting the extent of its disclosure commitments, as this immediately maps into an optimal capital structure.\footnote{The same, efficient allocation can be technically implemented by a contract that is a function of signals realizations rather than messages, and for example it would require that the investors seizes the realized signal amount. However, introducing an infinitesimal degree of risk-aversion on the investors side would select the contract I discuss as dominating any contract that is a function of the signal instead.}

**Privately and socially optimal disclosure.** The optimal amount of disclosure reflects the following trade-off: on the one hand, enhanced disclosure brings about higher costs; on the other hand, committing to disclose audited information decreases the degree of asymmetric information between firm’s insiders and outside investors, enabling the firm to issue cheaper equity, lowering its leverage and hence reducing the expected bankruptcy costs. Each firm optimally solves this trade-off, best responding to its competitors who move simultaneously. I characterize of the set of Pure Strategy Nash Equilibria (PSNE) of the disclosure game.\footnote{The game might be discontinuous, since optimal leverage ratios could jump discretely as disclosure changes infinitesimally, and it need not be quasi-concave. Therefore, a PSNE is not guaranteed to exist in general. However, two relatively mild assumptions are sufficient for continuity and quasi-concavity: (i) at any optimal leverage ratio, a marginally higher interest rate increases the expected profits of investors (i.e., it more than compensates for the expected increase in verification costs); and (ii) earnings densities are continuously differentiable, and first derivatives are uniformly bounded below by a constant $z < 0$.}

Comparing disclosure at any PSNE to the socially efficient one, I find that, whenever information is correlated across firms, *private provision of information is inefficiently low*, and *leverage is excessively high*. Firms under-disclose because they free ride on the information revealed by their competitors, and eventually the market gets stuck in a Pareto suboptimal Nash equilibrium. The public good nature of information leads to the possibility of designing Pareto improving government interventions in financial markets.

A government that seeks to restore social optimality should consider two instruments. First, it could mandate a certain degree of disclosure, as in Admati and Pfleiderer (2000). To the extent that firms cannot dodge the disclosure requirements, mandatory disclosures restore optimality. Indeed, we observe a wide range of disclosure requirements in developed economies, especially when shares are traded in stock exchanges (Leuz (2010)). However, as Ben-Shahar and Schneider (2010) document, disclosure regulation is often ineffective. In particular, ‘mandating transparency through disclosure’ proves harder (i) the more complex the underlying firm, and (ii) the greater the opportunity cost of disclosure. Large financial firms are a perfect example, being both highly complex and subject to pervasive information externalities (e.g., due to correlated shocks or proprietary information). The question becomes: are there indirect regulatory tools to promote greater
transparency of complex and interconnected firms, such as large financial institutions?

The last result of this paper suggests that capital requirements are a suitable instrument to this end. Through the lens of my model, firms facing stringent capital standards are encouraged to disclose better (and socially desirable) information to the market in an effort to reduce the otherwise prohibitive costs of equity financing. Although the argument is simple and plausible, it is strikingly absent from the current debate on capital standards, which I believe should not be as separated from that on information requirements as it is at present.\footnote{The mechanism requires that regulators shares with market participants some knowledge about individual firms’ covariates. Otherwise, the Pareto gains or losses in setting capital requirements depend on the average effect on firms, as in Admati and Pfleiderer (2000). Though this presumption is often implausible, observe that at present Basel III distinguishes firms that are too-big (or interconnected)-to-fail, and consistently with my findings it imposes a capital surcharge on them.}

Consider, for instance, the recent discussion around capital standards in the US. The Federal Reserve justifies its regulation as follows:

The primary function of capital is to (i) support the bank’s operations, (ii) act as a cushion to absorb unanticipated losses and declines in asset values that could otherwise cause a bank to fail, and (iii) provide protection to uninsured depositors and debt holders in the event of liquidation. [emphasis and numbers not in the original]

FED Supervisory Policy and Guidance Topics, as of 14.09.2015

The FED’s statement highlights three objectives. The first is to ‘support the bank’s operations’, a relatively vague proposition which is absent from much of the political and academic debate on the matter. The second objective is consistent with the position of many prominent economists, who emphasize the importance of requiring a sufficient ‘loss absorbing’ capital buffer, and is at the center stage of both the public and the academic debate.\footnote{See especially the Squam Lake Report (French et al. (2010)); recent influential books by Kotlikoff (2010), Sinn (2012), Admati and Hellwig (2014) and Stiglitz et al. (2015); academic papers such as Admati et al. (2013), Chamley et al. (2012) and Miles et al. (2013). The general discontent among academics (and a few politicians) with the outcome of Basel III, which sets capital requirements to less than 5%, shifted much of the debate at the national level.}

However, it offers a natural counterargument to finance lobbyists and sceptics of regulation. Despite the ex post virtues of capital buffers, in crises times, they counter argue that stringent requirements curb investment during booms, making it more expensive for firms to obtain external financing. So, from an ex ante perspective they need not be desirable.\footnote{A few examples: Josef Ackermann – former CEO of Deutsche Bank – claims that capital requirements ‘would restrict bank’s ability to provide loans to the rest of the economy’, which ‘reduces growth and has a negative effect for all’; Jamie Dimon – CEO of JP Morgan – argues that capital requirements would ‘greatly diminish growth’. Similar positions have been expressed by Vikram Pandit – former CEO of Citigroup – as well as by the Institute for International Finance (see Admati and Hellwig (2014), pagg. 97, 232 and 274). Van den Heuvel (2008) quantifies the growth loss from capital requirements in a DSGE framework. DeAngelo and Stulz (2015) argue in favor of high leverage for banks, relating leverage to liquidity provision.} Finally, the third argument can be considered as another
subsidy to debt instruments relative to alternatives, in much the same spirit as the tax
deductibility of interest payments, or the recent wave of bond-holders bailouts.16

This paper wishes to shift spotlight toward the first goal, offering an argument that
substantiates how capital requirements might ‘support the bank’s operations’. The mech-
anism I suggest starts with a coordination failure in information provision across banks,
aggravated by (i) systemic risk and correlation of assets portfolios, and (ii) the ease with
which banks can dodge mandatory disclosures. The under-provision of information not
only leads to opacity of financial intermediaries, but it also promotes an excessive re-
liance on debt instruments for funding. Capital requirements induce firms to be more
transparent, in order to reduce the costs of equity financing, and this is unambiguously
beneficial ex ante because of the public good nature of information.

Comparative statics and the evidence.

(1) Leverage is monotonically decreasing in the degree of transparency. The prediction
is novel, to my knowledge, and indeed its empirical validity has not been thoroughly
investigated.17 This leads me to introduce a measure of transparency in an otherwise
standard capital structure regression. In particular, I merge COMPUSTAT with IBES
analysts’ forecast and CRSP prices,18 and add to the standard variables considered in
Frank and Goyal (2009) various market measures of transparency, such as the coefficient
of variation of analysts’ Earnings Per Share (EPS) forecasts. The intuition behind this
measure of transparency is that disagreement among analysts should decrease with the
amount of public information about a firm (i.e., its transparency), and hence the variance
of forecasts is likely to reflect – or at least be correlated with – the degree of asymmetric
information between a firm’s insiders and analysts.19 The regression analysis reveals: (i)
a strong, statistically significant negative correlation between leverage and transparency;
(ii) robustness of the correlation to the inclusion of standard control variables and time-
firm fixed effects. As a result, even if one restricts attention to variation within a firm
across time in leverage and transparency, the two remain reliably negatively correlated.20

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16 An often mentioned force pushing firms toward increasing their leverage is the tax deductibility of
interest payments, but not of dividends. Observe, though, that such factor cannot account for the vast
cross-sectional variation in leverage across firms in the US. It is therefore overlooked here. On the
contradiction between capital requirements and tax advantages of debt, see especially De Mooij (2012)
and Fleischer (2013). Both scholars promote the abolition of any tax advantage of debt.
17 An exception is Aggarwal and Kyaw (2009), who compare leverage and transparency across 14 EU
countries and find a negative correlation. However, we still lack firm level evidence.
18 COMPUSTAT contains both balance sheet and cash flow (annual) information or the universe of US
public firms. IBES (acronym for ‘Institutional Brokers’ Estimate System’) contains analysts’ estimates
of earnings per share for several US corporations. Finally, CRSP (acronym for ‘Centre for Research in
Security Prices’) offers equity prices used to calculate market-based equity measures.
19 The idea of measuring transparency in this way is not new – e.g., Thomas (2002), Tong (2007),
Chang et al. (2007). Many other factors, such as herding or contrarianism – as well as personal opinions
– enter the forecast process. Such factors are discussed in greater depth in Bernhardt et al. (2006).
I implicitly assume that these additional sources of disagreement are orthogonal to leverage. Bhat et
al. (2006) show that analysts’ forecasts error and dispersion are strongly positively correlated with the
country-level transparency measures of Bushman et al. (2004).
20 A possible alternative explanation for the results is that they are driven by belief disagreement among
analysts, which is mechanically magnified by leverage. Although I cannot rule out this alternative
explanation altogether, in Appendix B the panel structure of the data is exploited in order to show
Further, qualitative, evidence in favour of a negative correlation between leverage and transparency comes from the 19th century, when early stock markets developed in northern Europe and the UK. Historical research (e.g., Bordo et al. (1999)) highlights that these developments were largely driven by: (i) improvements in the information environment (e.g., the telegraph), and (ii) the growing financing needs of relatively more transparent industries such as the infrastructure sector (railways and canals, especially). A prominent example is the London Stock Exchange (LSE). Prior to the 1840s, the LSE was essentially a market for government debt. After the telegraph became operational (in the early 1850s), stock trading took off, and by the 1870s the LSE became the largest market for stocks of its time. Soon enough, the system of British provincial stock exchanges disappeared. Railways and infrastructure companies dominated the LSE market, accounting for more than 75% of its capitalization (Grossman (2002)). Arguably, these companies’ revenues streams were easier to monitor and verify, compared to those trade (and military) ventures that dominated financial markets throughout the 18th century.

(2) Consistent with the existing empirical evidence, the model predicts that leverage should monotonically decrease in profitability.\(^{21}\) The intuition is that more profitable firms need to issue less shares (for a given price-per-share) to finance any given investment. Therefore, they are more likely to be able to issue incentive-compatible equity. The result is of interest from a theory perspective, as it reconciles the theory of optimal capital structure based on bankruptcy costs with the evidence.\(^{22}\) My regression analysis confirms the negative relationship between leverage and profitability.

2 Related Literature.

My paper contributes to the existing literature on the links between security design and disclosure. So far, the issue has been mostly studied in noisy rational expectation models subject to adverse selection, where the information sensitivity of securities affects the value of acquiring information about fundamentals for traders (e.g., Fulghieri and Lukin (2001)).\(^{23}\) Equity and call options provide better incentives to acquire information because they cross debt from the right. Differently from these papers, I study information disclosure by firms and focus on ex post asymmetric information.

On the security design side, this paper builds on Townsend (1979) and Gale and Hellwig (1985) CSV framework. The idea that outside information leads to the optimality of issuing some equity in a CSV model dates back to Chang (1999), who considers a firm with two technologies: one subject to CSV and one observable and verifiable (for which Modigliani and Miller (1958) holds). Although my interpretation in terms of signals is that this alternative explanation is less likely. In particular, I find that lagged disagreement correlates significantly with leverage, whereas leverage does not correlate with future disagreement.

\(^{21}\)The negative correlation between leverage and profitability has been documented in several previous studies, such as Frank and Goyal (2009), Welch (2011) and Graham and Leary (2011).

\(^{22}\)Indeed, the static trade-off theory would suggest the exact opposite should hold (e.g., Kraus and Litzenberger (1973)). On the role of taxation in determining optimal capital structures, see also DeAngelo and Masulis (1980).

\(^{23}\)See also Gorton and Pennacchi (1990) and Boot and Thakor (1993) and Pagano and Volpin (2012).
different, and in general it yields different conclusions from those in Chang (see Trigilia (2015)), the intuition is similar: the presence of some reliable information ex post leads to optimal contracts that cross debt from the right.

As such, the rationale for equity in the model I present is distinct from other explanations that involve risk-aversion and transaction costs (Cheung (1968)), costly-state-falsification (Lacker and Weinberg (1989) and Ellingsen and Kristiansen (2011)), double-sided moral hazard (Bhattacharyya and Lafontaine (1995)), control rights and infinite investment horizon (Fluck (1998)) or the combination of ambiguity and ex ante moral hazard (Carroll (2015) and Antic (2014)).

On the disclosure side, my model builds on two different blocks. As Fishman and Hagerty (1989, 1990), Admati and Pfleiderer (2000) and Alvarez and Barlevy (2014), disclosure is privately costly and it bring about externalities due to its public good nature. That is, the disclosure made by one firm affects that of its competitors, and this consideration feeds back to affect the initial optimal disclosure decision. In such settings, the private provision of information is typically socially inefficient, and Leuz (2010) argues that indeed the presence of information externalities is a major justification for the existence of mandatory disclosure requirements in practice. My paper highlights that a similar argument also leads naturally to capital requirements.

Unlike the aforementioned papers, though, I model disclosure as a commitment to revealing information the firm’s management does not privately holds at the financing stage (e.g., the disclosure and audit requirements that come with the decision of going public). In this respect, my model is closer to Kamenica and Gentzkow (2011) and Rayo and Segal (2010), who characterize the optimal disclosure as a commitment (ex ante) to a mapping from future states of nature to signals sent to outsiders, the realizations of which cannot be strategically manipulated ex post.

### 3 Setup

There are two dates $t \in \{0, 1\}$, $N \geq 1$ identical firms and a large number of competitive investors. Both firms and investors are risk-neutral and maximise date one consumption.

Each firm has access to an investment technology at $t = 0$ that requires a fixed input $K > 0$ and generates stochastic output $\hat{x}$ at $t = 1$. I assume that $\hat{x} \in X \equiv [0, \bar{x}]$, and denote by $F(x)$ the cumulative distribution of $\hat{x}$, and by $f(x)$ its density. For simplicity let $f(x) > 0$ for all $x$ and suppose it is continuous. Firms have no initial wealth and hence must seek external financing of $K$. To make the problem interesting, Assumption

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24Explanations for optimal equity based on control rights face increasing difficulties in accounting for the empirical evidence that many corporations are adopting a two-tiered equity structure, whereby investors are offered non-voting stocks (e.g., Google and Facebook). On this point, see also Zingales (2000). In contrast, explanations based on cash-flow rights used to require that the investors play an active role. In my model, equity emerges as optimal contract even when investors are relatively passive.

25The rationale for capital requirement presented in my paper differs from general equilibrium arguments based on pecuniary externalities (see e.g., Korinek and Simsek (2014) and Geanakoplos and Kubler (2015)). It also differs from arguments based on excessive risk taking and ‘collective moral hazard’ (see e.g., Farhi and Tirole (2012) or Admati and Hellwig (2014))

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1 guarantees that the project has positive net present value (NPV) under full information.

**Assumption 1.** \( K < \mathbb{E}_f[\tilde{x}] \). (Positive NPV)

In this paper, I overlook the presence of agency problems within the firm, and refer to the owner/manager of each firm as the *entrepreneur*. I intend to explore the issue in future research.

The representative investor is endowed with a large initial wealth. The investor chooses how much to lend to firms, and how much to invest in a risk-less bond with interest factor normalised to unity.

Investments occur under *symmetric* information. Hidden information comes ex post, when the state of the project is privately observed by the entrepreneur. The investors observe the state with some probability \( \pi \in [0,1] \), which I will motivate later. If the investors do not observe the state, they still have the option of verifying it at a fixed cost \( \mu \geq 0 \). The entrepreneur can affect \( \pi \) at \( t = 0 \) by committing to a disclosure policy – e.g., hiring an independent and trustworthy auditor or going public.

The timing of the game is as follows:

1. At \( t=0 \) each entrepreneur offers a contract (take-it-or-leave-it) to the investors. If the investors accept, \( K \) is invested in the firm;\(^{26}\)
2. Nature determines the realised state \( x \in X \). Then, in sequence:
   1. Each entrepreneur privately observes \( x \) and sends a public message \( m \in M \) about it (e.g., a balance sheet statement);
   2. Investors observe \( x \) with probability \( \pi \), and observe nothing otherwise;
   3. If the investors do not observe the state, they can verify it at a cost \( \mu \);
   4. Transfers occur and the game ends.

I now describe the feasible portfolio of securities and disclosure policies.

### 3.1 Securities

For a given set of public messages \( M \), the aggregate payout from firm \( i \) to its investors can be decomposed in three parts:

1. A *repayment* function \( s_i(m) : M \to \mathbb{R} \) specifies the payout when investors are *uninformed* about the state;
2. A *clawback* function \( z_i(m,x) : M \times X \to \mathbb{R} \) specifies the payout when investors are *informed* about the state;
3. A *verification* function \( \sigma_i(m) : M \to [0,1] \) specifies the probability that the state is verified for every message, when the investors are uninformed otherwise.\(^{27}\)

I impose two restrictions on admissible securities: (i) limited liability; (ii) deterministic verification. Limited liability has two consequences: first, it implies that repayments and clawbacks cannot be negative; second, it imposes that their upper bound depends on

\(^{26}\) Investment is assumed to be an observable and verifiable action.

\(^{27}\) It is easy to establish that no gains can be derived by distinguishing two clawbacks, one for the case where investors are informed due to the signal, and another one for the alternative case where their information comes from costly verification. I omit this discussion for the sake of brevity.
the verifiable output. That is, if the investors are informed, the upper bound is the realised output $x$, otherwise it is the message $m$. This standard assumption guarantees the existence of an optimal contract.\footnote{What is important is that the verifiable output lies in a compact set for every state $x$. One could therefore easily accommodate the monetary equivalent of a bounded non-pecuniary penalty.}

Deterministic verification is commonly assumed in CSV models, but it is a restrictive assumption. Indeed, Border and Sobel (1987) and Mookherjee and Png (1989) show that the optimal random contract is not debt. I make the assumption for two reasons: (i) the optimal random contracts still exhibit the key features derived here;\footnote{The key features are that: (i) lower messages are generally associated with higher verification, and (ii) higher states are not verified and repay a flat rate (in the absence of signals).} and (ii) they cannot be fully characterised, because local incentive compatibility does not suffice for global (see Border and Sobel (1987)). Formally:

**Assumption 2.** A portfolio of securities is feasible only if, $\forall m, x$:

- \( (LL) \) Payments satisfy limited liability: $s_i(m) \in [0, m]$, $z_i(m, x) \in [0, x]$
- \( (DV) \) Verification is deterministic: $\sigma_i(m) \in \{0, 1\}$

### 3.2 Disclosure policies

The disclosure policy of firm $i$ consists of the choice of a binary signal, which reveals the state of nature ex-post with probability $p_i \in [0, 1]$ to the investors public at a cost $c(p_i)$.

In the absence of correlation across firms, the probability that the investors observe $x$ for a given firm – denoted by $\pi_i$ – equals $p_i$. In contrast, when there is more than one firm and output is correlated across firms, we may have $p_i < \pi_i$. In this case, observing the output of other firms might be informative about firm $i$’s realised output as well.

I assume that the correlation between firm $i$ and firm $j$ is captured by a parameter $q_{i,j} \in [0, 1]$. In particular, the probability that the signal sent by firm $j$ is informative about firm $i$ is $q_{i,j} p_j$.\footnote{Evidently, it must be that $q_{i,i} = 1$ for every $i$. Observe that $q$ is not a statistical correlation coefficient, it just captures the presence of spillovers in information provision. Hence, it being positive is without loss of generality.} In aggregate, the probability of having at least one informative signal out of $N$ independent but not identically distributed Bernoulli trials is described by the inverse cdf of a Poisson Binomial distribution evaluated at zero successes:

$$\pi_i(p_i, p_i, q_i) = 1 - \left(1 - p_i\right) \prod_{j \neq i} (1 - q_{i,j} p_j) \quad (1)$$

The formula captures a positive externality coming from each firm’s disclosure policy, because $\partial \pi_i(p_i, p_i, q_i) / \partial p_j \geq 0$ and $\partial \pi_i(p_i, p_i, q_i) / \partial q_{i,j} \geq 0$. However, one could envision the presence of negative externalities as well. For instance, in a model where the feasible aggregate media coverage is limited, the disclosures made by other firms may limit the attention that firm $i$ can attract, reducing the information that the investors can acquire about its output. The analysis of such scenarios, which may give rise to strategic complementarities across firms, is left for future research.
3.3 Equilibrium concept and preliminary lemmas

First, notice that in this model the revelation principle holds, because all investor’s actions ex post are contractible.

**Lemma 1.** *Without loss of generality, we can restrict attention to direct revelation mechanisms.*

As a result, from now onwards I let $M = X$ and focus on truthful implementation. A *type* of firm refers to the *state* $x$ of the project that the entrepreneur observes before sending a public message. The driving force of the optimal portfolio of securities for a firm is the continuum $[0, \pi]$ of incentive compatibility constraints for each ex-post type $x$, which I now describe.

The expected payout from the firm to investors when the realised state is $x$ and the message is $x'$ is denoted by:

$$r_i(x', x) \equiv [\pi_i + (1 - \pi_i)\sigma_i(x')]z_i(x', x) + (1 - \pi_i)(1 - \sigma_i(x'))s_i(x')$$

where we omit the dependence of $\pi_i$ on $p_i, p_{-i}$ and $q_{-i}$, in our notation (i.e., we write $\pi_i$ instead of $\pi_i(p_i, p_{-i}, q_{-i})$). To understand the above expression, observe that:

1. The payout equals $z_i(x', x)$ whenever: (i) there is verification, which happens with probability $(1 - \pi_i)\sigma_i(x')$; or (ii) the investor is informed, which happens with probability $\pi_i$;
2. The payout is equal to $s_i(x')$ otherwise – i.e., when the signal is uninformative and no verification takes place. The probability of this event is $(1 - \pi_i)(1 - \sigma_i(x'))$.

To simplify notation, I let $r_i(x, x) \equiv r_i(x)$ denote the expected payout to investors from truthful revelation in state $x$.

As a consequence of Lemma 1, incentive compatibility requires that, for every $x$, at any optimal contract the expected payoff for the entrepreneur under truthful reporting, $x - r(x)$, exceeds the expected payoff by pretending to be any other type $x' \neq x$, i.e.:

$$x - r_i(x) \geq x - r_i(x, x'), \quad \forall (x, x') \in X^2$$

(2)

It is useful to refer to the incentive compatibility constraint when (i) the true state is $x$ and (ii) the message sent is $x'$, as $IC(x, x')$.

Any contract that implements investment must also satisfy the participation constraint (PC) for the investor, often referred to as the zero profit condition, which by Lemma 1 reads:

$$\int_X [r_i(x) - (1 - \pi_i)\sigma_i(x)\mu] dF(x) \geq K$$

(3)

I shall restrict attention to pure strategy Nash equilibria:

\[\text{The validity of the revelation principle follows from the exact same logic as in Gale and Hellwig (1985); the proof is omitted.}\]
**Definition 1.** A Pure Strategy Nash Equilibrium (PSNE) of the game consists in a set of strategies \( \{s^*_i, z^*_i, \sigma^*_i, p^*_i\} \) for all firms \( i = 1, \ldots, N \) such that, for each firm \( i \) and for a given vector \( p^*_{-i} \), both the portfolio of securities issued and the disclosure policy are optimal:

\[
\{s^*_i, z^*_i, \sigma^*_i, p^*_i\} \in \arg \max \int_X [x - r_i(x)]dF(x) - c(p_i)
\]

s.t. \( LL; \ DV; \ IC(x, x') \forall x, x' \); PC.

It is easy to see that PC must bind at any optimal contract. This is because whenever a contract \( \{s, z, \sigma\} \) is feasible and incentive compatible, so is a contract \( \{s', z', \sigma'\} \) such that (i) \( \sigma' = \sigma \), (ii) \( s' = \alpha s \), and (iii) \( z' = \alpha z \) for some \( \alpha \in [0, 1) \). By substitution, the contracting problem can be rewritten as:

\[
\{s^*_i, z^*_i, \sigma^*_i, p^*_i\} \in \arg \max \int_X [x - (1 - \pi_i)\sigma_i(x)\mu]dF(x) - c(p_i) - K
\]

s.t. \( LL; \ DV; \ IC(x, x') \forall x, x' \)

The latter formulation highlights that the objective function is simply to minimise the expected deadweight costs of verification and disclosure. Two intuitive lemmas hold regardless of \( p_i \), and prove useful in characterising the optimal contracts.

The first lemma deals with off-equilibrium clawback provisions, and shows that we can restrict attention to contracts that impose the harshest feasible clawbacks after cheating by the entrepreneur has been verified. That is, optimal contracts are such that verification takes place when \( m < y \), which proves that the entrepreneur is cheating with certainty, and \( z(m, x) = x \) whenever \( m \neq x \).

**Lemma 2.** Any optimal contract is payoff-equivalent to a contract such that:

(i) All assets are seized upon verified cheating: \( z^*(m, x) = x \) whenever \( m \neq x \);
(ii) Messages revealed to be false are verified.

**Proof.** See the Appendix.

Thus, it is without loss of generality to identify an optimal contract that satisfies those two properties. Observe that we have one degree of freedom in setting \( s^*(m) \) whenever \( \sigma_i(m) = 1 \). As a consequence of Lemma 8, I let \( s^*(m) = z^*(m, x) = x \) in such events.

Next, Lemma 3 shows that we can restrict attention to securities such that both the aggregate payout and the repayment function are weakly increasing on \( X \). The intuition is that having a non-monotonic optimal contract implies that incentive compatibility is not binding in some states, and one can always construct a monotonic contract that replicates the same ex ante allocation satisfying all constraints.

**Lemma 3.** Any optimal contract is payoff-equivalent to a contract such that: (i) \( r(x) \geq r(x') \), and (ii) \( s(x) \geq s(x') \) whenever \( x > x' \).

**Proof.** See the Appendix.
4 Privately Optimal Leverage and Disclosure

The roadmap of my analysis is as follows. In section 4.1 I characterise the optimal portfolio of securities issued for a given $\pi_i$. In particular, I show that the optimal security is a mixture of debt and equity, and that the optimal leverage decreases monotonically with $\pi_i$. Moreover, $\pi_i$ is a sufficient statistic for the optimal leverage ratio.

Next, in section 4.2, I characterise the set of Pure Strategy Nash Equilibria (PSNE) of the disclosure game, where the strategy set of each firm consists of choosing a $p_i \in [0, 1]$. Despite the simple structure of optimal contracts in the model, the game is generally discontinuous and not quasi-concave. I introduce two mild restrictions on the distribution $f(\cdot)$ of output, and show that they are sufficient to obtain a well-behaved – i.e., continuous and quasi-concave – game. Finally, I derive the comparative statics of the model.

4.1 Optimal securities for a given disclosure policy

In this section, I take $p_i$ as given for every $i$, and focus on the associated optimal portfolio of securities. The analysis is of independent interest because it generalises Gale and Hellwig (1985) – who restricted attention to $\pi_i = 0$ for all $i$ – and it highlights the key driving forces behind optimal securities in a CSV model with signals. For ease of notation, in this section I omit the subscript $i$ and any reference to the disclosure cost $c(\cdot)$.

To set a benchmark, consider the case of either $\pi = 1$ or $\mu = 0$. Then, the participation constraint for investors becomes $\int_X r(x)dF(x) \geq K$, and $IC(x, x')$ becomes $r(x) \leq x$. It follows that:

**Remark 1.** When either $\pi = 1$ or $\mu = 0$, Modigliani and Miller (1958) holds, and every feasible security for which PC binds is optimal.

**Proof.** Immediate from the above reasoning.

Henceforth, I restrict attention to $\pi < 1$ and $\mu > 0$. I next define the two securities that will be part of any optimal contract:

**Definition 2.** A security is debt if and only if $s(m) = \min\{m, d\}$ for some $d \in X$.

**Definition 3.** A security is equity if and only if $s(m) = \alpha m$ for $\alpha \in [0, 1]$.

The two securities are depicted in Figure 1. It is important to stress that because investment is risky, any feasible debt contract that implements investment must be such that $d > K$, as depicted in the left panel of the Figure. The following proposition characterises the optimal contract.

**Proposition 1.** If $E_f[\pi \hat{x}] \geq K$, then equity is optimal and debt is suboptimal. If $E_f[\pi \hat{x}] < K$ the uniquely optimal contract is a mixture of debt and equity.
Figure 1: The Relevant Securities

Proof. See the Appendix. □

The result follows from establishing three properties of optimal contracts:

Property 1: when the signal is not informative it is optimal to verify only a convex set of low messages that includes the message zero. This is because verifying higher messages imposes a cost and no gains in terms of increasing the feasible and incentive compatible payout from the firm to the investors. Define the following sets: the set of messages that trigger verification, \( V \equiv \{ m | \sigma (m) = 1 \} \), and its complement \( NV \equiv \{ m | \sigma (m) = 0 \} \).

Because \( X \) is bounded, there must exist \( x_{NV} \equiv \inf_{x \in NV} \{ x \} \) and \( x_{V} \equiv \sup_{x \in V} \{ x \} \). The first property implies that at the optimal contract \( x_{NV} > x_{V} \).

Property 2: whenever \( x_{NV} > 0 \), the optimal repayment function for every \( x \in NV \) is given by:

\[
s^*(x) = (1 - \pi_i) x_{NV} + \pi_i x
\]

The expression follows from two considerations. First, \( r^*(x_{NV}) = x_{NV} \) by monotonicity (i.e., Lemma 3) and the fact that all states \( x < x_{NV} \) are verified and hence cannot be profitable deviations by Lemma 8. Second, it is optimal to extract the highest incentive compatible repayment in the no-verification region to push \( x_{NV} \) to the minimum possible level that satisfies PC with equality. When \( s^*(x) = (1 - \pi_i) x_{NV} + \pi_i x \), incentive compatibility binds for every \( x \in NV \) and hence it is optimal.

If instead \( x_{NV} = 0 \), there exist multiple optimal repayment functions. They only need to be such that the slope is less than or equal to \( \pi_i \) for every state in the no-verification region. Therefore, a pure equity contract with \( \alpha \leq \pi_i \) is optimal.

Property 3: for every \( x \in V \), \( z^*(x, x) = s^*(x) = x \). That is, investors are senior claimants in verification states (that are the model equivalent of bankruptcy). This holds because bankrupt firms have no feasible deviation such that they can repay less (in expectation) than their realised output. As a result, minimisation of bankruptcy costs requires them to payout all their output.

Figure 2, Panel (a), depicts the firm’s payout at the optimal mixture of debt and
equity. Panel (b) sketches the characterisation of the optimal contract as a function of both transparency (measured by $\pi$) and profitability (measured as the ratio $K/E_f[\tilde{x}]$). Moving from the bottom-right corner – high profitability, high transparency – toward the top-left corner – low profitability, low transparency – the amount of debt in the optimal contract rises. The gray area denotes the parameter region where the first-best (no verification on-the-equilibrium path) can be implemented and firms have zero leverage at the optimal contract. In the upper-left triangle, instead, the solution is second-best and the amount of debt in the contract is increasing in $K/E_f[\tilde{x}]$ and decreasing in $\pi$.

The comparative static results behind the graph will be formally stated and proved in Corollary 3. First, observe that Proposition 5 implies that pure debt is optimal if and only if $\pi_i = 0$. 

Corollary 1. Pure debt is optimal if and only if $\pi = 0$.

Proof. Immediate from Proposition 5, since whenever $x_{NV} > 0$ we must have $\alpha = \pi$. □

Notice that both Proposition 5 and Corollary 1 identify the shape of the optimal contract that implement investment, however they offer no guarantee that investment would be made. I next turn to the question of whether or not investment would be made.

The expected profits of the investors at a given mixture of debt and equity are denoted by $R(x_{NV}) \equiv \mathbb{E}_f[r(x) - (1 - \pi)\sigma(x)\mu] - K$, where:

$$R(x_{NV}) = \int_0^{x_{NV}} [x - (1 - \pi)\mu]dF(x) + \int_{x_{NV}}^{\pi} \pi xdF(x) + (1 - F(x_{NV}))(1 - \pi)x_{NV} - K$$  \hspace{1cm} (6)

$R(x_{NV})$ takes values on a compact subset of the real line. The continuity of $f(\cdot)$ also implies that $R(\cdot)$ is continuous in $x$. As a result, there must exist (at least one) threshold
That maximises $R(x_{NV})$. If there is more than one, pick the smallest. Formally, let:

$$x^* \equiv \min \{ x_{NV} \mid x_{NV} \in \arg \max R(x_{NV}) \}$$

(7)

We obtain the following characterisation of the financing constraint coming from hidden information:

**Corollary 2.** Investment takes place only if $R(x^*) \geq 0$.

**Proof.** It follows from the above reasoning. \qed

In turn, the equilibrium face value of debt $d^*$ is given by:

$$d^* = \min \{ x_{NV} \mid R(x_{NV}) = 0 \}$$

(8)

Although the expected profits of investors do not necessarily increase with the interest rate in a CSV model (due to the presence of verification costs), it must be that $R(d^*)$ is weakly increasing in its argument. That is, the expected equilibrium profits of investors increase at the margin with the interest rate.

**Lemma 4.** $R(d^*)$ is weakly increasing in $d^*$.

**Proof.** See the Appendix. \qed

Because of Lemma 4, the effect of transparency ($\pi$), profitability (lower $K$ for a given $E_f(\hat{x})$) and verification costs ($\mu$) on leverage ($d^*$) are monotonic and can be easily derived.

**Corollary 3.** Cæteris paribus, leverage ($d^*$) is monotonically increasing in profitability and decreasing in transparency. It also increases with the verification cost.

**Proof.** See the Appendix. \qed

The effect of transparency and profitability on optimal leverage ratios is depicted in Figure 2, panel (b). More transparent firms can finance with equity projects of relatively lower profitability. Conversely, firms that are more opaque need to have highly profitable investment opportunities to issue equity, otherwise it is optimal for them to borrow (to some degree).

To provide intuition, I present an example.

**Example.** Suppose that $\hat{x}$ is distributed uniformly and $X = [0, 10]$. If the verification cost is given by $\mu = 1$ and $K = 4$, the optimal leverage ratio (i.e. debt over total assets) is depicted in Figure 3, panel (a). If firms are sufficiently transparent, i.e. $\pi \geq 4/5$, then zero leverage is optimal. If, instead, $\pi < 4/5$, then some debt will be issued, and the optimal amount of debt rises as transparency falls.

Indeed, for $\pi \leq 4/5$ the PC reads:

$$\int_0^d [x - (1 - \pi)]dx + \int_d^{10} [\pi x + (1 - \pi)d]dx = 40$$
which can be rewritten as: \(0.5(1 - \pi)d^2 - 9(1 - \pi)d + 40 - 50\pi = 0\). Of the two roots, it is easy to check that the relevant root is negative:

\[
d^* = 9 - \frac{\sqrt{-19\pi^2 + 18\pi + 1}}{1 - \pi}
\]

Moreover, the derivative of the expression with respect to \(\pi\) reads:

\[
\frac{\partial d^*}{\partial \pi} = \frac{-10}{(1 - \pi)\sqrt{-19\pi^2 + 18\pi + 1}} < 0
\]

Panel (b) plots the firm’s profits as a function of both \(\pi\) and \(K\). In the purple region at the top-left corner, investment does not take place (in fact, firm’s profits would be negative in this region). Otherwise, investment takes place and profits decrease in \(K\) and increase in \(\pi\). In particular, profits are strictly increasing in transparency when some debt is issued (i.e., \(\pi < 0.8\)), and are constant otherwise.

Figure 3: Optimal Contract in the Example

(a) Leverage and Transparency  (b) Firm profits (gross of disclosure costs)

4.2 Optimal disclosure policies

The previous section offered a characterisation of the optimal contract as a function of \(\pi_i\). The optimal contract is unique whenever verification takes place on-the-equilibrium path, and it can be implemented by pure equity otherwise. In this section, we exploit this result to characterise the equilibria of the disclosure game.

To set a benchmark, consider what happens when disclosure is costless. From PC, it is obvious that the entrepreneur only gains from increasing \(p_i\), as it prevents any need
for ex post verification. Therefore:

**Remark 2.** If disclosure is costless (i.e., if \( c(p_i) = 0 \), \( \forall p_i \) and \( \forall i \)), optimal contracts are such that \( p_i^* = p_j^* = 1 \) for all \( i, j \) and Modigliani and Miller (1958) holds.

**Proof.** Immediate from the above reasoning and Remark 1.

A more interesting and realistic scenario occurs when disclosure is costly – e.g., the fee charged by an independent audit firm. Increasing the degree of disclosure raises the disclosure cost \( c(p_i) \). However, it also lowers the costs of financing by enabling the entrepreneur to issue more (cheaper) equity, thereby decreasing the face value of debt and the expected deadweight verification costs.

Observe that (8) allows us to express \( d_i^* \) as a function of \( p_i \) through its dependence on \( \pi_i(p_i, p_{-i}, q_{-i}) \), for any given strategy of the other \( N - 1 \) firms. Moreover, we can disregard every \( p_i \) such that \( p_i > K/E_f[\tilde{x}] \) (regardless of strategy of the opponents), because it is dominated by \( p_i = K/E_f[\tilde{x}] \). To rule out uninteresting corner solution, suppose that the cost function satisfies the following Inada conditions:

**Assumption 3.** The cost function \( c(\cdot) \) is strictly increasing (\( c'(0) > 0 \), strictly convex (\( c''(0) > 0 \)) with:

\[
c(0) = c'(0) = 0 \quad \text{and} \quad c'(1) \to +\infty.
\]

Because the optimal capital structure can be fully described by \( \pi_i \), Program (5) can be rewritten as follows:

\[
p_i^* \in \arg \max_{p_i \in [0, K/E_f[\tilde{x}]]} V(p_i, p_{-i}) \equiv E_f[\tilde{x}] - (1 - \pi_i(p_i, p_{-i}, q_{-i}))F(d^*(\pi_i(p_i, p_{-i}, q_{-i})))\mu - c(p_i) - K
\]

The objective function \( V(p_i, p_{-i}) \) need not be differentiable with respect to \( p_i \), because \( d^*(\pi_i(p_i, p_{-i}, q_{-i})) \) may jump as \( p_i \) changes infinitesimally. This phenomenon happens when the payout to investors does not increase with the face value of debt – that is, when \( (1 - F(d^*)) = f(d^*)\mu \) – and such discontinuities are problematic for the existence of a solution to the program. However, if the set of points such that the equality holds is empty, then \( d^*(\pi_i(p_i, p_{-i}, q_{-i})) \) is differentiable and so is \( V(p_i, p_{-i}) \).

Define the following threshold, which corresponds to the equilibrium face value of debt of a standard CSV model with \( \pi_i = 0 \):

\[
\tilde{d} \equiv \min \left\{ x \in X \left| \int_0^x [x - \mu]dF(x) + (1 - F(d))d = K \right. \right\}
\]

A sufficient condition for differentiability of \( d^*(\pi_i(p_i, p_{-i}, q_{-i})) \) is the following:

**Lemma 5.** The objective function \( V(p_i, p_{-i}) \) is differentiable if the hazard rate \( h(x) \) is uniformly bounded so that:

\[
h(x) \equiv \frac{f(x)}{(1 - F(x))} < \frac{1}{\mu}, \quad \forall x \leq \tilde{d}
\]

\( ^{32} \)Recall that by Lemma 4 it can never be the case that \( (1 - F(d^*)) < f(d^*)\mu \).
Proof. See the Appendix.

The condition has a natural economic interpretation. It guarantees that the gains to investors from an increase in the face value of debt (e.g., a marginally higher interest rate) more than compensate the losses due to verification. The bound becomes tighter when the verification cost \( \mu \) increases, and/or profitability falls.

If (10) holds, Program (9) is guaranteed to have at least one solution by the theorem of the maximum. Moreover, totally differentiating (8) with respect to \( x_{NV} \) and \( p_i \), and evaluating at \( x_{NV} = d_i^* \) yields:

\[
\frac{d}{dp_i} d_i^* = \frac{d}{d \pi_i} \frac{d \pi_i}{dp_i} = -\frac{d}{dp_i} \frac{\mu F(d_i^*) + \int_{d_i^*}^{x} [x - d_i^*]dF(x)}{(1 - \pi_i)[1 - F(d_i^*) - \mu f(d_i^*)]} < 0,
\]

where the inequality follows from three observations: (i) \( \pi_i \) is strictly increasing in \( p_i \); (ii) \( \mu F(d_i^*) + \int_{d_i^*}^{x} [x - d_i^*]dF(x) > 0 \) for every \( d_i^* \in X \); and finally (iii) \( (1 - \pi_i)[1 - F(d_i^*) - \mu f(d_i^*)] > 0 \) by inequality (10) and Assumption 1.\(^{33}\)

As a result, the first derivative of the objective function \( V(p_i, p_{-i}) \) reads:

\[
\frac{\partial V(p_i, p_{-i})}{\partial p_i} = \mu \frac{\partial \pi_i}{\partial p_i} \left[ F(d_i^*) + f(d_i^*) \cdot \frac{\mu F(d_i^*) + \int_{d_i^*}^{x} [x - d_i^*]dF(x)}{1 - F(d_i^*) - \mu f(d_i^*)} \right] - c'(p_i) \equiv \gamma > 0
\]

Equation (12) formalises the trade-off that underpins the choice of an optimal disclosure policy: on the one hand, greater disclosure comes at a higher marginal cost \( c'(\text{due to the strict convexity of the cost functional}), \) on the other hand, it pushes leverage down – enabling the firm to issue a larger fraction of incentive compatible equity – at a gain proportional to \( \gamma > 0 \).

The second derivatives with respect to \( p_j \) for \( j = 1, \ldots, N \) is a relatively long collection of terms. I leave its derivation and explanation to the Appendix (at the beginning of the proof of Lemma 6). Most of the terms can be signed to be negative, suggesting that the problem has a certain degree of concavity built in, and coming from the participation constraint for the investors (the zero profit condition). Indeed, a (strictly negative) lower bound on the derivative of the density function is sufficient for \( V(p_i, p_{-i}) \) to be strictly concave, as the next lemma shows:

**Lemma 6.** A sufficient condition for \( V(p_i, p_{-i}) \) to be strictly concave is the following:

\[
f'(x) > -\frac{1}{h(x)^{-1} - \mu}, \quad \forall x \in [0, d]
\]

Proof. See the Appendix.

The condition in Lemma 6 is not very restrictive if (10) holds, as \( h(x)^{-1} > \mu \) and the lower bound is negative. Moreover, like (10), it is a straightforward property to check.\(^{33}\)

\(^{33}\)Assumption 1 implies that \( K/E f[\tilde{x}] < 1 \), so it is never the case that \( (1 - \pi_i) = 0 \), regardless of \( q \).
Henceforth, I assume that both restrictions on the distribution of output hold, so that the disclosure game is well behaved:

**Assumption 4.** Both (10) and (13) hold. Hence, $V(p_i, p_{-i})$ is $C^2$ and strictly concave.

Define strict submodularity and aggregativity of a game as follows:

**Definition 4.** A game is strictly submodular if $\partial^2 V(p_i, p_{-i})/\partial p_i \partial p_j < 0$ for every $i$ and for every $j \neq i$.

**Definition 5.** A game is aggregative if there exists a continuous and additively separable function $g : [0, 1]^{N-1} \rightarrow [0, 1]$ (the aggregator) and functions $\tilde{V} : [0, 1]^2 \rightarrow \mathbb{R}$ (the reduced payoff functions) such that for each player $i$:

$$V(p_i, p_{-i}) = \tilde{V}(p_i, g(p_{-i})), \quad \forall p \in [0, 1]^N$$

From these definitions, and from Assumption 4, it follows that:

**Lemma 7.** The disclosure game is aggregative and strictly submodular.

**Proof.** See the Appendix.

The aforementioned properties guarantee both the existence of a PSNE, and the presence of monotone comparative statics with respect to the correlation parameter $q$.

**Proposition 2.** The set of Pure Strategy Nash Equilibrium (PSNE) is non-empty, and each firm $i = 1, \ldots, N$ chooses a disclosure policy $p_i^*$ such that:

1. If $V_1(K/E_f[\tilde{x}], p_{-i}^*) > 0$, $p_i^* = K/E_f[\tilde{x}]$;
2. Otherwise $V_1(p_i^*, p_{-i}^*) = 0$ and $p_i^* \in [0, K/E_f[\tilde{x}]]$.

Moreover, the smallest and the largest equilibria, denoted by $Q_*(q)$ and $Q^*(q)$ respectively, are such that: $Q_* : [0, 1]^{N(N-1)/2} \rightarrow \mathbb{R}$ is lower semi-continuous and $Q^* : [0, 1]^{N(N-1)/2} \rightarrow \mathbb{R}$ is upper semi-continuous.

**Proof.** See the Appendix.

Existence of a PSNE follows from three properties of the game: (i) convexity and compactness of the strategy set $[0, 1]$, for all $i$; (ii) continuity of $V(p_i, p_{-i})$ in all arguments; and (iii) quasi-concavity of $V(p_i, p_{-i})$ in $p_i$. Aggregativity and submodularity also imply that monotone comparative statics with respect to the correlation vector $q_{-i}$ can be derived:

**Corollary 4.** *Cæteris paribus, the equilibrium disclosure $p_i^*$ decreases with $q_{i,j}$, for every $i, j$. The equilibrium leverage might decrease or increase with $q_{i,j}$.*

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34 Both definitions are the analogue of those in Acemoglu and Jensen (2013), for the case of one-dimensional strategy sets.

35 As standard, $V_1$ denotes the derivative of $V$ with respect to the first argument.

36 See Acemoglu and Jensen (2013) for general results, of which mine are a special case.
Summing up, the equilibrium disclosure policies are a function of the correlation vector \( q \), and the higher the correlation the lower the disclosure of each firm, because the larger the gains from free riding on the information produced by competitors. It remains to consider the efficiency properties of the private disclosure and leverage policies, which is the subject of the next section.

5 Socially Optimal Leverage and Disclosure

The set of Socially Efficient (SE) disclosure policies is the set of disclosure vectors of length \( N \) that maximise the aggregate surplus:

\[
SE \equiv \left\{ p^e \in [0, 1]^N \mid p^e \in \arg \max_{p \in [0,1]^N} \sum_{i=1}^{N} V(p_i, p_{-i}) \right\}
\]

The set \( SE \) is non-empty, and can be characterised as follows:

**Proposition 3.** There exists a non-empty set of Socially Efficient (SE) disclosure policy vectors. Any optimal disclosure vector \( p^e \) is such that \( p^e > p^* \), where \( p^* \) belong to the largest Nash equilibrium \( Q^*(q) \). In addition, \( p^e >> p^* \) whenever \( q_{-i} > 0 \). That is, in equilibrium there is under-disclosure.

**Proof.** See the Appendix.

Proposition 3 shows that the presence of disclosure spillovers across firms leads to an inefficiently low private provision of information, and consequently inefficiently high leverage ratios. A social planner could increase the aggregate welfare by promoting higher disclosure and lower borrowing. How could the result be achieved?

A first policy would focus on mandatory disclosures, and mandate that firms disclose according to the vector \( p^e \). However, there may be limits in the efficacy of mandatory requirements, especially when dealing with firms that are naturally opaque (such as banks or insurance companies).

In fact, opaque sectors such as the financial industry are regulated according to different principles. In particular, they tend to be subject to mandatory capital requirements – that is, a sufficient fraction of their assets must be backed by equity claims. At present, Basel III confirms capital requirements in the range of 4% of the risk weighted assets for banks.\(^{37}\) This paper shows that mandatory capital requirements may well be welfare increasing, and can be an alternative to disclosure regulation in those instances where reaching effective disclosures may prove daunting. The result is summarised in the following proposition.

\(^{37}\)However, many policy makers and academics called for substantially higher requirements. For instance, Calomiris called for 10%, Admati and Hellwig 20-30%, and Kotlikoff 100%.
**Proposition 4.** When \( q_{-i} > 0 \) for some \( i \), any SE can be implemented as a PSNE, either by mandating a certain amount of disclosure \( p_i^e \), or by mandating capital requirements of size \( l_i^e \) (and setting transfers accordingly).

**Proof.** The case for mandatory disclosure is straightforward: simply solve for the SE, and set \( p_i^e \) equal to the disclosure at an SE.

If disclosure cannot be mandated effectively, consider the leverage at the SE: it would be \( \alpha_i^e = \pi_i^e \) by Proposition 5. Then, compute the corresponding \( d(\pi_i^e) \), and set:

\[
l_i^e = \frac{d(\pi_i^e)}{d(\pi_i^e)(1 - \pi_i^e) + \pi_i^e \mathbb{E}[\tilde{x}]}
\]

Note that: \( l_i^e = 0 \) if \( d(\pi_i^e) = 0 \), and \( l_i^e = 1 \) if \( d(\pi_i^e) = \bar{d} \) (which implies that \( \alpha_i^e = 0 \)).

The increase is social surplus moving from a PSNE to a SE guarantees that there exists at least one set of transfers that support the SE as a Pareto improvement. \( \square \)

An important remark on the implementation of socially efficient outcomes concerns the assumption that the regulator knows the degree of connectedness of individual firms. Although we implicitly assumed that the market knows such information, and can price it correctly, it could be that a regulator does not know it. In such a scenario, it cannot rely on firms disclosing truthfully their *systemic risk*: all firms have strong incentives to underreport so they can avoid the regulatory requirements. Similar problems arise in most models of disclosure under externalities, such as Admati and Pfleiderer (2000).\(^{38}\)

Though this limitation is likely to be relevant in practice, observe that current US regulation is implicitly following the approach sketched here, when it imposes additional capital requirements on *too-big-too-fail* institutions. Effectively, the regulator uses a measure of the *size* of firms to capture their interconnectedness, and requires better capitalisation precisely when the model I presented suggests it would be necessary. Better measures are currently studied by academics and policy makers.

I conclude the section by returning to our example, and solving for the privately and socially optimal disclosure policies.

**Example (cont’d).** Recall from the previous analysis that:

\[
d^* = \begin{cases} 
9 - \sqrt{-19 \pi_i^2 - 18 \pi_i + 1} / \left(1 - \pi_i\right) & \text{if } \pi \leq 4/5 \\
0 & \text{otherwise}
\end{cases}
\]

The function is continuous, and inequality (10) holds because: (i) \( \bar{d} = 8 \); and (ii) the hazard rate is: \( 1/(10 - x) \), which is strictly less than \( 1/\mu = 1 \) for every \( x \in [0,9] \). Moreover, \( f' = 0 \) implies that (13) holds.

\(^{38}\)I overlook them here not because they are unimportant, but because their consequences are obvious: the regulation trades off a distortion due to ‘pooling’ with the benefits of enhanced disclosure and lower leverage. The final result depends on parameter values.
Suppose that $N = 2$, $\pi_i = p_i + q(1 - p_i)p_{j \neq i}$ for both firms, and $c = |1 - 0.8(0.8 - p_i)^{-1}|/100$. Program (5) can be written as:

$$\max_{p_i \in [0, 0.8]} V(p_i, p_{-i}) = -\frac{1 - \pi_i}{10} \left[ 9 - \frac{-19\pi_i^2 + 18\pi_i + 1}{1 - \pi_i} \right] - \frac{|1 - 0.8(0.8 - p_i)^{-1}|}{100} + 1 \quad (14)$$

It is easy to verify that $\partial^2 V(p_i, p_{-i})/\partial p_i^2 < 0$ and $\partial^2 V(p_i, p_{-i})/\partial p_i \partial p_{-i} < 0$. As a result, there exists a unique interior maximum, fully characterised by the first order condition: $\partial V(p_i^*, p_{-i})/\partial p_i = 0$.

In contrast, exploiting symmetry, the socially optimal disclosure can be derived as the solution of a planner’s problem, who maximises aggregate welfare with $p_i = p_{-i} = p$:

$$\max_{p \in [0, 0.8]} W(p) \equiv \frac{p + q(1 - p)p - 1}{5} \left[ 9 - \frac{-19(p + q(1 - p)p)^2 + 18(p + q(1 - p)p) + 1}{1 - p - q(1 - p)p} \right] - \frac{|1 - 0.8(0.8 - p)^{-1}|}{50} + 2 \quad (15)$$

Again, it is easy to verify that the planner’s objective function is strictly concave in $p$. Hence, the socially optimal disclosure level satisfies: $\partial W(p^*)/\partial p = 0$.

The SNE and the planner’s solution are plotted in Figure 4. In the absence of externalities (i.e., when $q = 0$) the private and social optimum coincide. However, for every $q > 0$ the SNE displays an inefficiently low level of disclosure, relative to the social optimum. Moreover, the divergence between private and social optimum increases with the externality parameter $q$.\textsuperscript{39} From Proposition 5, it follows that leverage is inefficiently high whenever $q > 0$, and the inefficiency is increasing in $q$.

Figure 4: Privately and Socially optimal disclosure policies

\textsuperscript{39}This is an instance of the monotone comparative static derived in Acemoglu and Jensen (2013) for more general (though still aggregative) submodular games.
6 Empirical analysis

To investigate whether the predictions of the model are consistent with the empirical evidence, I first build a firm-level panel of the universe of US public firms. I then construct various measures of transparency and leverage (as well as other standard controls) to use as inputs in my regression analysis.

To construct my data, I combine two sources: (i) the CRSP/COMPUSTAT merged dataset; and (ii) the IBES analysts’ forecast dataset. To do this, I follow the path described below.

I first collect the raw CRSP/Compustat merged dataset, which contains balance sheet information about the universe of US public corporations, as well as the prices of their securities for the period 1979-2014. From the original file, I drop observations that satisfy at least one of the following requirements: (i) total assets (AT) are missing or negative; (ii) the firm is not US based (i.e. FIC ≠ USA); (iii) total liabilities (LT) are missing or negative; (iv) total liabilities exceed total assets (LT > AT); (v) either the equity price (PRCC) or the market capitalisation (CSHO) are missing.

Then I collect the detailed IBES dataset (adjusted for stock splits), which contains individual forecasts by analysts of EPS (Earnings per share). For any given firm-year pair, I generate the following summary statistics: (i) NUMEST – the number of analysts’ estimates of expected EPS; and (ii) CV – the coefficient of variation of analysts’ forecasts (i.e. their standard deviation normalised by the mean). I drop firm-date pairs for which there are less than five forecasts, and I collapse the data at the firm-year level.40

The procedure ends with 32,361 matched firm-year pairs such that (i) both Compustat and IBES data is successfully merged, and (ii) more than five forecasts are available.

Table 1: Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>LT/AT</td>
<td>0.56</td>
<td>0.24</td>
<td>0</td>
<td>1</td>
<td>32361</td>
</tr>
<tr>
<td>CV forecasts</td>
<td>0.07</td>
<td>0.15</td>
<td>0</td>
<td>7.92</td>
<td>32361</td>
</tr>
<tr>
<td>Estimates</td>
<td>13.12</td>
<td>8.19</td>
<td>5</td>
<td>59</td>
<td>32361</td>
</tr>
<tr>
<td>Total Assets</td>
<td>7.29</td>
<td>1.87</td>
<td>-0.03</td>
<td>14.7</td>
<td>32361</td>
</tr>
<tr>
<td>Profitability</td>
<td>0.01</td>
<td>0.18</td>
<td>-5.88</td>
<td>4.1</td>
<td>32361</td>
</tr>
<tr>
<td>Book-to-Market</td>
<td>0.59</td>
<td>0.56</td>
<td>0</td>
<td>21.26</td>
<td>32361</td>
</tr>
<tr>
<td>Intangibles</td>
<td>0.13</td>
<td>0.18</td>
<td>0</td>
<td>0.92</td>
<td>28949</td>
</tr>
<tr>
<td>Industry Leverage</td>
<td>0.57</td>
<td>0.18</td>
<td>0.17</td>
<td>0.94</td>
<td>32361</td>
</tr>
</tbody>
</table>

Table 1 reports the descriptive statistics for the variables of interest. The definition I adopt of leverage includes both financial and non-financial liabilities (as suggested by Welch (2011)), which is easily computed as the ratio of total liabilities (LT) over total as-

40In the empirical Appendix, I conduct robustness exercises where I let the cutoff run from 1 to 4, and show the results are unchanged. Moreover, I consider alternatives to the coefficient of variation, such as the median absolute deviation from the mean (both normalised and not). Again, the results do not change.
The Book-to-Market ratio is computed as the book value of a share (PRCCF) multiplied by the total number of outstanding shares (CSHO), and then divided by the market value of equity (MEQ). Intangibles are measured as a fraction of total assets, i.e., INTAN/AT. Finally, Total Assets are reported as the natural logarithm of AT, hence the negative minimum numbers which obtain for AT $\in (0, 1)$.

I now proceed to the regression analysis. I follow the procedure of gradually introducing independent variables, to check how the sensitivity and significance of the coefficients of interest evolve. The general linear regression that I estimate takes the following form (where $i$ indexes firms and $t$ years):

$$
\text{Leverage}_{i,t} = \alpha + \beta \text{X}_{i,t-1} + \gamma_i + \gamma_t + \epsilon_{i,t}
$$

where the matrix $\text{X}_{i,t}$ includes various covariates of a firm-date pair, among which the main regressor of interest (i.e. CV – the coefficient of variation of analysts’ forecasts).

Table 2 reports the regression results. I first regress leverage on CV, a constant and time dummies (column (1)). Then, in column (2) I add controls used in previous papers (e.g., Frank and Goyal (2009)) that are identified as reliable predictors of the leverage of a firm. In column (3), I regress leverage on CV, a constant, time dummies and firm fixed effects. Column (4) adds the controls to the fixed-effect regression of column (3). Next, I present two robustness checks: in column (5) I restrict attention to non-financial firms; in column (6) I increase the cutoff on the number of forecasts to ten. In both cases, the coefficient of interest remains significant, and it even marginally increases in magnitude relative to that of column (4).

The signs of most other controls are consistent with previous studies. Profitability is strongly negatively correlated with leverage. Average industry leverage is strongly positively correlated with leverage. Total assets (i.e., size) are positively correlated with leverage, though the correlation vanishes when firm fixed effects are included. Both the Book-to-Market ratio and the fraction of intangible assets are not robustly signed. The inclusion of firm fixed effects explains about 50% of the observed variation in leverage, consistent with studies such as Lemmon et al. (2008).

Comparing columns (4) and (5) of Table 2, observe that significance of my regressor of interest (i.e., lagged CV) increases. This is presumably due to the lack of variation in leverage and transparency of financial firms, and it suggests that the inclusion of firm fixed-effects reduces substantially the variation that can be used to capture the effect of transparency on leverage. Therefore, presumably the best specification might be that of column (2), with time dummies and controls but without firm fixed-effects. Nevertheless, the fact that coefficients remain significant in the fixed effects regressions suggests that

---

41 Other definitions I consider in the Appendix are: (i) LT/AM, where AM stands for market value of assets; (ii) DT/AT, where DT = DLC + DLTT refers to the aggregate financial liabilities (debt); and finally (iii) DT/AM. Overall, the qualitative results are not very sensitive to the leverage measure chosen, although they are more statistically significant when book values are considered rather than market values.

42 In the Appendix, I run additional robustness exercises and show that the results are qualitatively similar throughout various specifications.
Table 2: Regression table

<table>
<thead>
<tr>
<th></th>
<th>(1) LT/AT</th>
<th>(2) LT/AT</th>
<th>(3) LT/AT</th>
<th>(4) LT/AT</th>
<th>(5) LT/AT</th>
<th>(6) LT/AT</th>
</tr>
</thead>
<tbody>
<tr>
<td>L.CV forecasts</td>
<td>0.0983***</td>
<td>0.0427**</td>
<td>0.0432***</td>
<td>0.0217*</td>
<td>0.0342**</td>
<td>0.0354**</td>
</tr>
<tr>
<td></td>
<td>(5.11)</td>
<td>(2.90)</td>
<td>(4.18)</td>
<td>(2.53)</td>
<td>(2.99)</td>
<td>(2.72)</td>
</tr>
<tr>
<td>L.Total Assets</td>
<td>0.0589***</td>
<td>0.00934</td>
<td>0.00596</td>
<td>0.00539</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(31.28)</td>
<td>(1.73)</td>
<td>(1.06)</td>
<td>(0.82)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L.Profitability</td>
<td>-0.193***</td>
<td>-0.170***</td>
<td>-0.160***</td>
<td>-0.216***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-6.28)</td>
<td>(-7.96)</td>
<td>(-7.33)</td>
<td>(-7.79)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L.Book-to-Market</td>
<td>-0.000492</td>
<td>0.00132</td>
<td>-0.00218</td>
<td>-0.0143**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.10)</td>
<td>(0.33)</td>
<td>(-0.50)</td>
<td>(-2.75)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L.Intangibles</td>
<td>-0.000937</td>
<td>0.0260</td>
<td>0.0303</td>
<td>0.00306</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.05)</td>
<td>(1.06)</td>
<td>(1.19)</td>
<td>(0.10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L.Industry Leverage</td>
<td>0.466***</td>
<td>0.146**</td>
<td>0.135*</td>
<td>0.123</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(22.09)</td>
<td>(2.67)</td>
<td>(2.27)</td>
<td>(1.77)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.544***</td>
<td>-0.111***</td>
<td>0.592***</td>
<td>0.424***</td>
<td>0.420***</td>
<td>0.504***</td>
</tr>
<tr>
<td></td>
<td>(49.22)</td>
<td>(-8.19)</td>
<td>(133.26)</td>
<td>(7.55)</td>
<td>(7.42)</td>
<td>(6.49)</td>
</tr>
<tr>
<td>Time FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm FE</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Exclude Finance</td>
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<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
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<td>10 forecasts or more</td>
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<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
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<td>23499</td>
<td>26337</td>
<td>23499</td>
<td>19121</td>
<td>13395</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.010</td>
<td>0.479</td>
<td>0.847</td>
<td>0.846</td>
<td>0.778</td>
<td>0.856</td>
</tr>
</tbody>
</table>

$ t $ statistics in parentheses. $ * p<0.05, ** p<0.01, *** p<0.001 $

Sources: Compustat merged with CRSP (annual), IBES (detail, adjusted for stock splits).

Notes: all independent variables are lagged by one year. Standard errors are clustered at the firm level.
the results are not entirely driven by time-invariant, firm-specific covariates.

Overall, the results support the predictions of the model I propose, although a validation of the my hypotheses with statistical causality is left for future research.

7 Conclusions

This paper analyses the effect of disclosure on the composition of the means of financing for firms. I develop a novel costly-state-verification setting with variable and endogenous degrees of asymmetric information between firms and investors. I derive the optimal securities, showing that it consists of a mixture of debt and equity, and that disclosure and leverage should be negatively correlated. Higher disclosure leads to the possibility of issuing cheaper incentive-compatible stocks, hence increasing the opportunity of leveraged financing and its bankruptcy costs.

My empirical analysis for US public firms after the 1980s provides confirmatory evidence for my model, as long as effective transparency is negatively correlated to the dispersion in analysts’ EPS forecasts. Of course, the dispersion in forecasts is a noisy proxy of transparency, and one should confirm that the results are robust across alternative measures. Nevertheless, the validity of the correlation derived in the paper hinges on the observation that most factors that influence the dispersion of forecasts, such as herding or contrarianism, do not seem to be linked to leverage ratios by existing theories.

The presence of disclosure externalities across firms yields insufficient voluntary disclosure and excessive leverage, relative to the constrained best. Therefore, it brings about the question of regulation. If regulators can effectively mandate truthful disclosures, then social efficiency can be restored. However, the explicit treatment of the interlinkage between disclosure and financing policies suggests an additional tool that regulators should explore when truthful disclosures prove hard to implement: capital requirements. By setting higher capital requirements, regulators promote endogenously-enhanced transparency and can restore social efficiency.

The argument for mandatory capital standards that I put forward relies on two pillars: (i) firms’ output should be sufficiently correlated (e.g., in the presence of high systemic risk); and (ii) mandatory disclosures are hard to translate into greater transparency, because they can be dodged to a large extent. Both conditions plausibly apply to financial firms, and indeed they are the subject of regulatory capital requirements.

Moreover, my argument is immune from the most common critique of the existing, alternative, stories based on the absorbing of losses in crises (e.g., Admati and Hellwig (2014)). Banking lobbyists commonly counter argue that, although ex-post desirable in crises times, capital requirement are ex-ante detrimental to credit extension and would dampen growth during boom times, because they increase the cost of funding for banks. Indeed, were this not so we would observe already much higher equity financing in the financial industry. The model I present is not subject to this critique, because capital requirements are efficient ex ante, and solve a coordination failure in information provisions across firms.
As always with regulation, the devil lies in the details. Moreover, I ignore important aspects such as agency problems within firms and the government, for the sake of simplicity. Any regulatory effort must confront such issues convincingly in order to be credible. What my paper highlights is that debates around capital requirements and mandatory disclosures for financial firms should be more closely connected, as their consequences are deeply intertwined.
References


A Proofs

Lemma 8

Proof. Claim (i). Suppose there exists an optimal contract \( \{s, z, \sigma, p\} \) such that:

\[
\{x \mid z(m, x) < x, \text{ for some } m \neq x\} \neq \emptyset
\]

Consider replacing it with another contract \( \{s', z', \sigma', p'\} \) such that \( \sigma = \sigma', s = s', p = p' \) and:

\[
z'(m, x) = \begin{cases} x & \text{if } m \neq x \\ z(m, x) & \text{otherwise} \end{cases}
\]

Clearly, the new contract is feasible because when \( \sigma = 1 \) the maximum feasible clawback equals \( x \). To see that it is incentive compatible, observe that because \( \{s, z, \sigma, p\} \) is optimal, we know that \( r(x) \leq r(x', x') \) for every pair \( x, x' \). We also know that (i) \( r(x) = r'(x) \) for every \( x \), and (ii) \( r(x, x') \leq r'(x, x') \) for every \( x, x' \) by construction. Hence, \( \{s', z', \sigma', p'\} \) is incentive compatible. The participation constraint remains binding because \( E_f[r'(x)] = E_f[r(x)] \), and the deadweight loss due to verification and disclosure do not change. Therefore, the entrepreneur is indifferent between \( \{s, z, \sigma, p\} \) and \( \{s', z', \sigma', p'\} \), proving our claim.

Claim (ii). It mirrors the proof of claim (i): start with an optimal \( \{s, z, \sigma, p\} \) that does not satisfy the property (i.e., \( \sigma(m) = 0 \) for some \( m < y \)). For all such cases, replace \( \sigma \) with \( \sigma' = 1 \). Otherwise, set \( \sigma = \sigma' \), \( z = z' \) and \( s = s' \) and \( p = p' \). Because the change occurs only off-equilibrium path, the participation constraint remains unchanged. Furthermore, incentive compatibility and feasibility are trivially satisfied, proving the claim. \( \square \)

Lemma 3

Proof. Claim (i). First, we know that when \( \pi = 0 \) the optimal contract is debt, and it is monotonic (Gale and Hellwig (1985)). Therefore, we can restrict attention to \( \pi > 0 \) and consider an optimal contract \( \{s, z, \sigma, p\} \). Suppose that under \( \{s, z, \sigma, p\} \) there exists a set \( A \subset X \) and an \( \hat{x} \) such that \( A \equiv \{x > \hat{x} \mid r(\hat{x}) > r(x)\} \). Evidently, the contract is not monotonic. Without loss of generality, suppose there only exists one such \( \hat{x} \) (if there was more than one, the same reasoning could be iterated).

Consider another contract \( \{s', z', \sigma', p'\} \) such that \( \sigma = \sigma' \), \( p = p' \), \( s'(x) \in [s(x), x] \), and:

\[
r'(x) = \begin{cases} r(x) & \text{if } x \notin A \\ r(\hat{x}) & \text{otherwise} \end{cases}
\]

The new contract is feasible because \( r(\hat{x}) \leq \hat{x} < x \) for every \( x \in A \). To show that it is also incentive compatible, I partition the state according to whether they belong to \( A \) or not.

First, consider \( x \notin A \). By construction (i) \( r'(x) = r(x) \), and (ii) \( r'(x, x') \geq r(x', x) \) for every \( x' \). Hence, because \( \{s, z, \sigma, p\} \) was incentive compatible, incentive compensation holds also under \( \{s', z', \sigma', p'\} \).

Second, consider \( x \in A \). From the way I constructed \( r' \), I know that \( r'(x') = r(x') \). First, IC(\( \hat{x}, x' \)) under the old contract reads:

\[
\hat{x} - r(\hat{x}) \geq (1 - \pi)(1 - \sigma(x'))[\hat{x} - s(x')] \quad \Rightarrow \quad \hat{x} \geq \frac{r(\hat{x}) - (1 - \pi)(1 - \sigma(x'))s(x')}{\pi + (1 - \pi)\sigma(x')}
\]

The ratio is well defined because \( \pi > 0 \). Under the new contract, by construction we have: \( \sigma'(x') = \sigma(x') \);
\(s'(x') = s(x')\) and \(r'(\hat{x}) = r(\hat{x})\), so we can write:
\[
\hat{x} \geq \frac{r(\hat{x}) - (1 - \pi)(1 - \sigma(x'))s(x')}{\pi + (1 - \pi)\sigma(x')} = \frac{r'(\hat{x}) - (1 - \pi)(1 - \sigma'(x'))s'(x')}{\pi + (1 - \pi)\sigma'(x')}
\]
Observe that \(IC(x, x')\) under the new (prime) contract reads:
\[
x \geq \frac{r'(x) - (1 - \pi)(1 - \sigma'(x'))s'(x')}{\pi + (1 - \pi)\sigma'(x')} = \frac{r'(\hat{x}) - (1 - \pi)(1 - \sigma'(x'))s'(x')}{\pi + (1 - \pi)\sigma'(x')}
\]
where the last equality holds by construction of the new contract \(\{s', z', \sigma', p'\}\) i.e., the fact that, for every \(x \in A\), \(r'(x) = r(\hat{x})\). Since \(x > \hat{x}\) the prime contract is incentive compatible as well.

Now consider the participation constraint. Regardless of the measure of the set \(A\), at the prime contract the investors make strictly positive profits. Define a third contract \(\{s''', z'', \sigma'', p''\}\) such that \(p'' = p', \sigma'' = \sigma' = \sigma, z'' = \alpha z'\) and \(s'' = \alpha s'\) for some \(\alpha \in [0, 1]\) such that: \(\mathbb{E}_f[r''(x) - (1 - \pi)\sigma''(x)] = K\). We know that such an \(\alpha\) exists because: (i) when \(\alpha = 1\) we have \(\mathbb{E}_f[r''(x) - (1 - \pi)\sigma''(x)] \geq K\); (ii) when \(\alpha = 0\) we have \(\mathbb{E}_f[-(1 - \pi)\sigma''(x)] < 0\); and (iii) the left hand side of the equation is continuous in \(\alpha\). The new (double-prime) contract is feasible because \(\alpha \in (0, 1)\), and it is trivially incentive compatible. Because the deadweight loss does not change and the investors make zero profits, the firm must be indifferent between \(s, z, \sigma, p\) and \(\{s'', z'', \sigma'', p''\}\), proving the claim.

**Claim (ii)** Consider an optimal contract \(\{s, z, \sigma, p\}\) that satisfies Claim (i). Suppose there exists an interval \(A \subset X\), such that \(s(x) < s(\hat{x})\) for every \(x \in A\) and some \(\hat{x} \in \text{inf}\{x | x \in A\}\). The repayment function is not monotonic. Introduce another contract \(\{s', z', \sigma', p'\}\) such that: \(p = p', \sigma = \sigma', r = r'\) but:
\[
s'(m) = \begin{cases} s(m) & \text{if } m \notin A \\ s(\hat{x}) & \text{otherwise} \end{cases}
\]
Of course, for all \(x \in A\) the fact that \(r = r'\) and the shape of \(s'\) imply that:
\[
z'(x, x) = z(x, x) - \frac{(1 - \pi)}{\pi}[s(\hat{x}) - s(x)] < z(x, x)
\]
The new repayment function is monotonic. To see that the prime contract is feasible, notice that (i) the original contract was feasible; (ii) \(s(\hat{x}) \leq \hat{x} < x\) and (iii) by the monotonicity of \(r\) we have:
\[
r(x) \geq r(\hat{x}) \geq (1 - \pi)s(\hat{x}) \Rightarrow z'(x, x) = z(x, x) - \frac{(1 - \pi)}{\pi}[s(\hat{x}) - s(x)] \geq 0, \quad \forall x \in A
\]
To show it is also incentive compatible, partition the incentive constraints in the following categories:

**First, consider** \(x \notin A\). All incentive constraints hold because \(\{s, z, \sigma, p\}\) was incentive compatible and: (i) \(s(x) = s'(x)\) for all \(x' \notin A\); (ii) \(s(x) < s(\hat{x}) = s'(x')\) for all \(x' \in A\).

**Second, consider** \(x \in A\). If message \(x' \neq x\) is such that \(\sigma'(x') = 1\) incentive compatibility trivially holds. Moreover, if \(x\) is such that \(\sigma'(x) = 1\) incentive compatibility holds because \(s'(x)\) is irrelevant (i.e., \(r(x) = z(x, x)\)). Finally, if \(x, x'\) are such that \(\sigma'(x) = 0 = \sigma'(x)\), we have:
\[
\pi z'(x, x) + (1 - \pi)s'(x) \leq \pi x + (1 - \pi)s'(x')
\]
If \(x' \in A\), then \(s'(x) = s'(x') = s(\hat{x})\) by construction and since \(z'(x, x) \leq x\) by limited liability incentive compatibility holds. If \(x' \notin A\) and \(x' > x\), incentive compatibility follows from \(s'(x') = s'(x') \geq s(\hat{x}) = s'(x)\), by definition of the set \(A\). Finally, if \(x' \notin A\) and \(x' < x\), incentive compatibility follows from \(r = r'\) and \(s'(x') = s'(x')\). So, the prime contract is incentive compatible.

In conclusion, observe that: (i) because \(\sigma = \sigma'\) the deadweight verification cost does not change; and (ii) because \(r = r'\) the investors revenues do not change. As a result, the two contracts are equivalent from the firm’s perspective and because \(\{s, z, \sigma, p\}\) is optimal, so is \(\{s', z', \sigma', p'\}\).
Proposition 5

Proof. Case 1: \( E_f[\pi x] \geq K \). The contract with minimum possible verification on-the-equilibrium path is such that \( \sigma(m) = 0 \) for every \( m \). Because of Lemmas 8-3, when \( \sigma(m) = 0 \) for every \( m \) there is at most one binding incentive constraint for each type \( x \in X \), \( IC(x,0): x - r(x) \geq (1 - \pi)x \), or equivalently: \( r(x) \leq \pi x \) where I substituted \( s(0) = 0 \) by limited liability. In addition, evidently one can set \( s(x) = r(x) \) for every \( x \). If \( \sigma(m) = 0 \) for every \( m \) and incentive compatibility holds, the fraction of equity that needs to be sold is \( \alpha = K/E_f[\pi x] \), and because \( \alpha \leq \pi \) equity is optimal.\(^{43}\)

Debt is suboptimal because the incentive constraint for a type \( x \leq d \) reads \( x \leq \pi x \), which is never satisfied because \( \pi < 1 \). Moreover, \( d > K \) because investment is risky, and hence the set of \( x < d \) is nonempty.

Case 2: \( E_f[\pi x] < K \). The proof proceeds in three steps:

Step 1: Any optimal contract is such that \( x \leq x_{NV} \).

Proof. Divide \( X \) into intervals \( X_1, X_2, \ldots, X_n \) such that (i) \( \min X_1 = 0, \max X_n = \bar{x}, \cup_{i=1}^n X_i = X \), and (ii) for every \( i \), and for every pair \( x, x' \in X_i^2 \), \( \sigma(x) = \sigma(x') \). By contradiction, suppose that at the optimal contract \( \{s, z, \sigma\} \) we have \( x_{NV} > x \). Without loss of generality, suppose that \( X_1 \subseteq NV \), so that (i) \( X_2 \neq \emptyset \) and \( X_2 \subseteq V \), (ii) \( X_3 \neq \emptyset \) and \( X_3 \subset NV \), and so on. For \( x \in X_3 \), incentive compatibility of \( \{s, z, \sigma, p\} \) requires that (i) for every \( x' \in X_1 \) we have \( r(x) \leq \pi x + (1 - \pi)s(x') \); and (ii) for every \( x'' \in X_2 \) we have \( r(x) \leq x \).

Consider another contract \( \{s', z', \sigma'\} \) such that \( s' = s, z' = z, p = p' \) and:

\[
\sigma'(m,0) = \begin{cases} 
\sigma(m) & \text{if } m \notin X_2 \\
0 & \text{otherwise}
\end{cases}
\]

By Lemma 1 the new contract is feasible, because \( \max\{m^*(x), y\} = x \) for every \( x \). Now I prove it is incentive compatible.

If \( x \in X_2 \), incentive compatibility of \( \{s, z, \sigma, p\} \), \( s = s' \) and \( z = z' \) jointly imply that \( IC(x, x') \) is satisfied at the prime contract for every \( x' \in X \). If \( x \in X_1 \), incentive compatibility follows from the monotonicity of \( s(m) \) – by Lemma 3. If \( x \in X_3 \) we have two cases: (i) if \( x' \in X_1 \) or \( x' \in X_4 \) and \( i \geq 3 \) then we have \( r'(x) \leq r'(x, x') \) because \( r \neq r' \) and \( \sigma'(x') = \sigma(x') \); (ii) if instead \( x' \in X_2 \) incentive compatibility reads: \( \pi z'(x, x) + (1 - \pi)s(x') \leq \pi x + (1 - \pi)s(x') \). Because \( x' > x'' \) for every \( x'' \in X_1 \), and since \( X_1 \subseteq V \), we also have: \( \pi x + (1 - \pi)s(x') = \pi x + (1 - \pi)s(x') \geq \pi x + (1 - \pi)s(x'') = \pi x + (1 - \pi)s(x'') \), where the inequality follows from incentive compatibility of \( \{s, z, \sigma, p\} \). Similar arguments can be used for \( x \in X_1 \) and \( i > 3 \), proving the claim. \( \Box \)

Step 2: For every \( x \geq x_{NV} \), \( z'(x,x) = s^*(x) = (1 - \pi)x_{NV} + \pi x \).

Proof. First I show that \( s(x_{NV}) = x_{NV} \). Suppose not, i.e. there exists an optimal contract \( \{s, z, \sigma, p\} \) such that \( x_{NV} > r(x_{NV}) \) (the case of the opposite inequality is prevented by limited liability). Define the set \( B \equiv \{ x \in NV | r(x) < x_{NV} \} \). Design a new contract \( \{s', z', \sigma', p'\} \) such that \( z = z', \sigma = \sigma', p = p' \) and:

\[
s'(m) = \begin{cases} 
s(m) & \text{if } x \notin A \\
x_{NV} & \text{otherwise}
\end{cases}
\]

Clearly, the prime contract is feasible. It is also incentive compatible because \( \{s, z, \sigma, p\} \) is incentive compatible. It remains to show that from the optimality of \( \{s, z, \sigma, p\} \) it follows that \( B \) is of zero measure, hence PC remains binding. By contradiction, suppose not. Define the following threshold:

\[ \hat{x} \equiv \left\{ x \in X \left| \int_0^i [x - (1 - \pi)]dF(x) + \int_{\hat{x}}^x \min\{s'(x), x\}dF(x) = K \right\} \]

\(^{43}\)In the limit, when \( E_f[\pi x] = K \), pure equity is the uniquely optimal contract.
We know that \( \hat{x} \) exists and \( 0 < \hat{x} < x_{NV} \) because if \( \hat{x} = 1 \) we have:

\[
\int_0^{\hat{x}} [x - (1 - \pi)\mu]dF(x) + \int_{\hat{x}}^{x} \min\{s'(x), x\}dF(x) = \int_0^{x_{NV}} [x - (1 - \pi)\mu]dF(x) + \int_{x_{NV}}^{\hat{x}} s'(x)dF(x) > K
\]

if, instead, \( \hat{x} = 0 \) we have \( \int_0^{\hat{x}} \min\{s'(x), x\}dF(x) < K \), where the inequality follows from the fact that \( \mathbb{E}_f[\pi \hat{x}] < K \). Observe that a contract \( \{s'', z', \sigma'', p''\} \) such that \( z'' = z' = z, p'' = p, s'' = \min\{s', x\} \) and:

\[
\sigma''(m) = \begin{cases} 
\sigma(m) & \text{if } m \notin [\hat{x}, x_{NV}] \\
0 & \text{otherwise}
\end{cases}
\]

would be both feasible and incentive compatible. Moreover, it would make the participation constraint for the investors binding, strictly reducing the expected verification costs relative to \( \{s, z, \sigma, p\} \).\(^{44}\) As a result, \( \{s, z, \sigma, p\} \) cannot be optimal, proving our claim.

That \( s(x) = (1 - \pi)x_{NV} + \pi x \) follows from three observations. First, incentive compatibility for \( x, x' \in NV^2 \) reads:

\[
s(x) \leq \pi x + (1 - \pi)s(x')
\]

Second, because \( r(x_{NV}) = x_{NV} \) and by Lemma 3 (i.e., monotonicity of \( s(\cdot) \)) we have: \( \min\{s(m)|m \in NV\} = x_{NV} \). Third, incentive compatibility must be binding almost surely for every \( x \in NV \) (that is, up to sets of zero measure). To see the latter observation must hold, simply observe that if there is a set of strictly positive measure where incentive compatibility does not hold at any candidate optimal contract, one can repeat the argument given for the previous claim (i.e., \( r(x_{NV}) = x_{NV} \)) and show that the candidate contract cannot be optimal.

\( \square \)

**Step 3:** For every \( x \) such that \( \sigma(x) = 1 \), we have \( z^*(x, x) = s^*(x) = x \).

**Proof.** The proof is identical to that of Step 2. It consists in showing that if a contract is such that \( z^*(x, x) < x \) for a set of states of strictly positive measure, such contract cannot be optimal because the deadweight verification costs can be reduced moving to \( z^*(x, x) = x \) for every \( x \in V \) with another feasible, incentive compatible contract that makes PC binding.

\( \square \)

Summing up, steps 1-3 imply that the optimal contract is a mixture of debt and equity with \( \alpha^* = \pi \) and \( d^* = \min\{x_{NV}| PC\ binds\} \).

\( \square \)

**Lemma 4**

First notice that the repayment to investors when \( x^* = 0 \) is equal to \( \mathbb{E}_f[\pi \hat{x}] \), and it must be strictly less than \( K \) when \( x^* > 0 \) by Proposition 5. Suppose that \( - \) by contradiction \( - \) the derivative at \( x^* \) of the objective function in (7) is strictly negative, i.e.: \( (1 - F(x^*)) < f(x^*)\mu \). Because the function is continuous, and it starts at a positive value below strictly below \( K \), then whenever the derivative is negative it must be that there exists an \( x' < x^* \) such that the repayment to investors equals \( K \). But this contradicts the definition of \( x^* \), proving our claim.

\( \square \)

**Corollary 3**

**Proof.** Consider profitability first. We have two cases: \( d = 0 \) and \( d > 0 \). If \( d = 0 \), it means that \( K/\mathbb{E}_f[\hat{x}] = \alpha \leq \pi \). If \( K' < K \) I have \( K'/\mathbb{E}_f[\hat{x}] = \alpha' < K/\mathbb{E}_f[\hat{x}] = \alpha \leq \pi \) and \( d' = d = 0 \). Now consider the case of \( d > 0 \). At any optimal contract that sustains investment where \( d > 0 \), (2) holds with equality at \( x^* = d \). We can rewrite (2) at the optimum as:

\[
\mathbb{E}_f[\hat{x}] - K - (1 - \pi)\mu F(x^*) - \int_{x^*}^{\infty} (1 - \pi)x dF(x) + (1 - F(x^*)) (1 - \pi)x^* = 0
\]

\(^{44}\)Strictly because we supposed that \( B \) had a strictly positive measure.
Suppose that $K$ increases for a given $\mathbb{E}_f[\bar{x}]$. By Lemma 4 I know that $(1 - F(x^*)) \geq f(x^*)\mu$. If the inequality is strict, totally differentiating the expression with respect to $K$ and $x^*$ I get:

$$-dK + dx^*(1 - \pi)[1 - F(x^*) - f(x^*)\mu] = 0$$

and $dx^*/dK > 0$ implies that either $d$ increases as profitability falls, or at the new $K$ there is no investment. If, instead, $(1 - F(x^*)) = f(x^*)\mu$, then $d$ must jump to the right and again either there exists a higher $d$ that satisfies PC, or there is no investment.

As for transparency, suppose it decreases to $\pi' < \pi$. If $\pi' \geq K/\mathbb{E}_f[\bar{x}]$, then $d' = d = 0$. If $\pi' < K/\mathbb{E}_f[\bar{x}] \leq \pi$, then either at $\pi'$ there is no investment or it must be that $d' > d = 0$. Finally, if $\pi' < \pi < K/\mathbb{E}_f[\bar{x}]$, I must have that again either at $\pi'$ there is no investment or $d' > d$ because the derivative of $(2)$ with respect to $\pi$ is equal to $\mu F(x^*) + \int_{x^*}^\pi [x - x^*] f(x) dx > 0$.

Finally, that $x^*$ increases with $\mu$ is immediate from inspection. \hfill $\Box$

**Lemma 5**

**Proof.** First, recall that by Lemma 3 the equilibrium face value of debt is monotonically decreasing with $p_i$. Therefore, we must have $d^* \leq d$.

Second, observe that the derivative of $(8)$ (conditional on $\mathbb{E}_f[\pi, \bar{x}] \leq K$) with respect to $x_{NV}$ is given by $(1 - \pi_i)[(1 - F(x_{NV})) - \mu f(x_{NV})]$, and it is strictly positive when (i) $h(x) < 1/\mu$ for every $x \leq d$; and (ii) $\pi_i \leq K/\mathbb{E}_f[\bar{x}]$.

As a result, the change in $d^*$ as $p_i$ increases infinitesimally can be computed simply total differentiating $(8)$ with respect to $x_{NV}$ and $p_i$, and evaluating at $x_{NV} = d$. \hfill $\Box$

**Lemma 6**

**Proof.** The second derivative of $V(p_i, p_{-i})$ with respect to $p_i$ reads:

$$\frac{\partial^2 V(p_i, p_{-i})}{\partial p_i^2} = \mu^2 \frac{\partial^2 \pi_i}{\partial p_i^2} \left[ F(d_i^*) + f(d_i^*) \cdot \frac{\mu F(d_i^*) + \int_{d_i^*}^{x_i^*} [x - d_i^*] dF(x)}{1 - F(d_i^*) - \mu f(d_i^*)} \right] +$$

$$+ \mu \frac{\partial \pi_i}{\partial p_i} \frac{\partial d_i^*}{\partial \pi_i} \left\{ \int_{d_i^*}^{x_i^*} f(d_i^*) \cdot \frac{\mu F(d_i^*) + \int_{d_i^*}^{x_i^*} [x - d_i^*] dF(x)}{1 - F(d_i^*) - \mu f(d_i^*)} \right\} +$$

$$+ \frac{\mu F(d_i^*) + \int_{d_i^*}^{x_i^*} [x - d_i^*] dF(x)}{1 - F(d_i^*) - \mu f(d_i^*)} \left\{ \frac{(f(d_i^*))^2}{\mu f(d_i^*)} \right\} - c'(p_i)$$

Though the expression looks frightening, observe that we can sign all terms but those that involve the derivative of the density function $f(\cdot)$. Moreover, all terms are negative, suggesting that the problem has a certain degree of concavity built in from the zero profit condition for investors.

Strict concavity requires $\frac{\partial^2 V(p_i, p_{-i})}{\partial p_i^2} < 0$. From (16):

$$f'(x) > -\frac{f(x)}{1 - F(x) - \mu f(x)}, \ \forall x \in [0, d] \ \Rightarrow \ \frac{\partial^2 V(p_i, p_{-i})}{\partial p_i^2} < 0$$

dividing through the fraction in the right hand side by $1 - F(x) > 0$ and applying the definition of $h(x)$ yields the result. \hfill $\Box$
Lemma 7

Proof. **Strict Concavity:** The second cross derivative of \( V(p_i, p_{-i}) \) with respect to \( p_j \neq i \), for every such \( j \), reads:

\[
\frac{\partial^2 V(p_i, p_{-i})}{\partial p_i \partial p_j} = \mu \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} \left[ F(d_i^*) + f(d_i^*) \cdot \frac{\mu F(d_i^*) + \int_{d_i^*}^{x} [x - d_i^*]dF(x)}{1 - F(d_i^*) - \mu f(d_i^*)} \right] + \\
+ \mu \left( \frac{\partial \pi_i}{\partial p_j} \right)^2 \frac{\partial^2 d_i^*}{\partial p_j} \left\{ f(d_i^*) \cdot \frac{\mu F(d_i^*) + \int_{d_i^*}^{x} [x - d_i^*]dF(x)}{1 - F(d_i^*) - \mu f(d_i^*)} \right\} + \\
+ \frac{\mu F(d_i^*) + \int_{d_i^*}^{x} [x - d_i^*]dF(x)}{1 - F(d_i^*) - \mu f(d_i^*)} \frac{\partial f(d_i^*)}{\partial d_i^*} \left\{ \left( f(d_i^*) \right)^2 + \frac{\partial f(d_i^*)}{\partial d_i^*} \mu f(d_i^*) \right\} - c'(p_i)
\]

The expression in curly brackets is the same that we found in (16), hence it is strictly positive under Assumption 4. As a result, the game is strictly concave.

**Aggregativity:** It follows immediately from the definition of \( \pi_i(p_i, p_{-i}, q_{-i}) \) (i.e., equation (1)). □

Proposition 2

Proof. Define the best response correspondence for firm \( i \) as follows:

\[
b_i(p_{-i}) \equiv \arg \max_{p_i \in [0, K/E_f(x)]} V(p_i, p_{-i})
\]

We know \( b_i(p_{-i}) \) is nonempty by the theorem of the maximum because \( V(p_i, p_{-i}) \) is continuous and the set \([0, K/E_f(x)]\) is compact. Moreover, \( b_i(p_{-i}) \) is a singleton because \( V(p_i, p_{-i}) \) is strictly concave. Hence, \( b_i(p_{-i}) \) is convex and upper hemicontinuous. It follows by Kakutani fixed point theorem that a PSNE exists.

As for the properties of \( Q_* \) and \( Q^* \), they follow from Lemma 7, which guarantees that my game is a special case of those to which Theorem 1 in Acemoglu and Jensen (2013) applies. □

Corollary 4

Proof. Observe first that the FOC can be written as:

\[
\mu \frac{\partial \pi_i}{\partial p_i} \bigg|_{p_i = p} \left[ F(d(p^*)) + f(d(p^*)) \cdot \frac{\mu F(d(p^*)) + \int_{d(p^*)}^{x} [x - d(p^*)]dF(x)}{1 - F(d(p^*)) - \mu f(d(p^*))} \right] = c'(p^*)
\]

The right hand side is not a function of \( q_{-i} \). In contrast, the left hand side is a function of \( q_{-i} \), through its effect on \( \pi_i \). Moreover, the sign of the derivative of the left hand side with respect to \( q_{i,j} \) is the same as that in (17), hence it is strictly positive. Evidently, \( p^*_i \) must decrease for the equation to keep holding, proving that equilibrium disclosure decreases with \( q_{i,j} \).

As a shock to \( q \) hits the aggregator, in the sense of Acemoglu and Jensen (2013), both \( Q_* \) and more importantly \( Q^* \) decrease with it.

Coming to leverage, from Proposition 5 we know that leverage increases with \( q_{i,j} \) if and only if \( \partial^2 \pi_i/\partial q_{i,j} < 0 \). However, this derivative embeds two effects: on the one hand, a higher correlation directly increases \( \pi_i^* \). On the other hand, it lowers the equilibrium disclosure which in turn lowers \( \pi_i^* \). The elasticities cannot be signed a priori. □

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Proposition 3

Proof. Existence is immediate from continuity. Moreover, \( \frac{\partial V(p_i, p_{-i})}{\partial p_j} > 0 \) whenever \( q_{i,j} > 0 \) implies that the private disclosure is inefficiently lower than that at the SE. \( \square \)

B Empirics: Robustness Checks

In this appendix, I present and discuss additional empirical exercises to confirm that the correlations presented in the paper are robust.

The first exercise pertain the cutoff in the number of analysts’ forecast required for an observation to be included in the data. In the man text, I consider a cutoff of 5, but I claim this choice does not affect the results. To show that this is the case, Table 3 presents the fixed effect regression results for cutoffs ranging from 2 to 7.\(^{45}\)

From now onwards, by ‘Usual Controls’ I shall refer to those included in the regressions of Table 3.

The second set of robustness checks, presented in Table 4, studies how the results change with different measures of analysts’ forecast dispersion. Column (1) reports the benchmark estimate using the coefficient of variation (it is equivalent to column (4) of Table 2). Column (2) clarifies the importance of normalising the standard deviation by the mean: without the normalisation the significance is lost. Column (3) and (4) do the same replacing CV with MAD (the median absolute deviation from the mean forecast). Similar results attain. Finally, column (5) shows that one could also use directly the number of analysts following the firm in a given year. As expected, the number is negatively correlated with leverage, suggesting that the higher the number of analysts following a firm, the lower its subsequent leverage ratio.

The third series of robustness checks is presented in Table 5. It considers the effects on the estimates of changing the definition of leverage. In particular, column (1) presents again the estimates shown in the main text, where leverage is defined as in Welch (2011), to equal the ratio of Total Liabilities (LT) over Total Assets (AT). Column (2) replaces AT with the market value of assets (AM = MEQ + LT). The coefficient of interest is positive but looses a one degree of significance. Column (3) shows what happens when leverage is defined as the ratio of Total debt (DT) – defined as the sum of Debt in Current Liabilities (DLC) and Long Term Debt (DLTT) – over the book value of assets. The result is similar to that of column (2). Finally, column (4) shows what happens when leverage is defined as DT/AM. The coefficient looses significance altogether. Columns (5)-(7) repeat the exercise of substituting LT/AT with alternative measures of leverage for the independent variable MAD. Similar results attain.

Finally, Table 6 explores the leads and lags structure of the data. Although CV is serially correlated, the Table shows that the results are stronger when CV is assumed to precede leverage than the other way around. Of course, the results do not rule out

\(^{45}\)Evidently, two is the minimum number of forecasts needed to be able to actually compute a coefficient of variation. Robustness to even higher cutoffs (in particular, ten) is presented in Table 2 in the main text.
Table 3: Robustness Check (1): different cutoffs

<table>
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<tr>
<th></th>
<th>Cutoff 2</th>
<th>Cutoff 3</th>
<th>Cutoff 4</th>
<th>Cutoff 5</th>
<th>Cutoff 6</th>
<th>Cutoff 7</th>
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<td>0.0215**</td>
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<td>0.0133**</td>
<td>0.0127*</td>
<td>0.0136*</td>
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<td>(3.27)</td>
<td>(2.91)</td>
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<td>-0.143***</td>
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<td>(2.31)</td>
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<td>(6.53)</td>
<td>(5.88)</td>
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</tr>
<tr>
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<td>0.846</td>
<td>0.845</td>
<td>0.848</td>
<td>0.848</td>
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</tbody>
</table>

$t$ statistics in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Sources: Compustat merged with CRSP (annual), IBES (detail, adjusted for stock splits).
Notes: all independent variables are lagged by one year. Standard errors are clustered at the firm level.
Table 4: Robustness Check (2): different independent variables

<table>
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<th>(4)</th>
<th>(5)</th>
</tr>
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<td>L.CV forecasts</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.86)</td>
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<tr>
<td></td>
<td>(0.03)</td>
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<td></td>
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<tr>
<td>L.MAD forecasts</td>
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<td></td>
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<tr>
<td></td>
<td>(3.31)</td>
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<tr>
<td>L.MAD*MEAN</td>
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<td></td>
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</tr>
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</tr>
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<tr>
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<tr>
<td>Adjusted $R^2$</td>
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<td>0.848</td>
<td>0.848</td>
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</table>

* t statistics in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Sources: Compustat merged with CRSP (annual), IBES (detail, adjusted for stock splits).
Notes: all independent variables are lagged by one year.
Standard errors are clustered at the firm level.

Table 5: Robustness Check (3): different dependent variables

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<th>(6)</th>
<th>(7)</th>
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<td>0.0218*</td>
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<td>(1.76)</td>
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</tr>
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<td>0.813</td>
<td>0.884</td>
<td>0.794</td>
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* t statistics in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Sources: Compustat merged with CRSP (annual), IBES (detail, adjusted for stock splits).
Notes: all independent variables are lagged by one year. Standard errors are clustered at the firm level.
reverse causality, and a statistically causal analysis is still required in future work.

Table 6: Robustness Check (4): lags and leads

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<td>0.848</td>
<td>0.849</td>
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$t$ statistics in parentheses. $^* p < 0.05$, $^{**} p < 0.01$, $^{***} p < 0.001$

Sources: Compustat merged with CRSP (annual), IBES (detail, adjusted for stock splits).
Notes: all independent variables are lagged by one year. Standard errors are clustered at the firm level.

C More general, exogenous signals

In this appendix I show that the qualitative results of interest do not depend on the simple binary signals structure assumed in my baseline model. To this end, I extend the analysis to more general signals that reveal a lower bound on the realized output, and satisfy First-Order-Stochastic-Dominance (FOSD) – that is, I study the case where higher signals are consistently more likely the higher the realized output.

The main results can be summarized as follows: (i) debt is optimal only if all no-bankruptcy states under full leverage are indistinguishable in expectation; (ii) the optimal contract crosses debt from the right – that is, it requires that the parties share more evenly profits and losses; (iii) in the special case of uniform signals, the optimal contract is exactly a mixture of debt and equity. Otherwise, it might involve call options or be non-linear, depending on the signal’s distribution.
In order to set up the problem, I need to introduce a bit of notation. In doing so, I repeat the already defined notation in order for this section to be self contained.

C.1 Setup

There are two dates \( t = 0, 1 \), and two agents: a manager and an outside investor. Both are risk-neutral and consume only at \( t = 1 \). The manager has a project that generates stochastic date one output \( \tilde{x} \) and requires investing \( i \) at \( t = 0 \). The investor has capital in excess of \( i \), and can either lend it to the manager or invest in a risk-less bond with interest factor normalised to one. Investment by managers is observable and verifiable, and at this stage the parties are symmetrically informed. Hidden information comes ex-post, when realized output is privately observed by the manager. The investor receives a noisy signal about it, after which he decides whether or not to verify the output at a fixed cost \( c \). To make the problem interesting, assume that implementing the project is of positive net present value under full information: \( i < \mathbb{E}[\tilde{x}] \).

Properties of optimal contracts do not depend on the ex-ante distribution of bargaining power across agents. Therefore, assume that managers have all the bargaining power. Timing is as follows:

- **t=0** The manager offers a contract to the investor. Upon rejection, the game ends. Otherwise, \( i \) is invested in the project;
- **t=1** The state \( x \) realizes. The manager observes \( x \), and sends a public message \( m \) about it. The investor observes \( m \), and also receives an independent stochastic signal \( \tilde{y} \). Costly verification takes place (or not), transfers occur and the game ends.

\( F(x) \) denotes the cumulative distribution of output \( \tilde{x} \), and \( f(x) \) its density. The support of \( \tilde{x} \) is \( X \), a bounded subset of \( \mathbb{R}_+ \). The signal \( \tilde{y} \) reveals a lower bound on realized output – e.g., the value of those assets that are tangible. Denote by \( G(y|x) \) the cumulative distribution of \( \tilde{y} \), conditional on the realised state being \( x \), by \( g(y|x) \) the associated density and let \( y \) describe a realization of the signal \( \tilde{y} \). A natural assumption to make is that lower signals are consistently more likely when realised output is lower, in the first-order stochastic dominance sense:

**Assumption 5.** \( G(y|x) \leq G(y|x'), \forall x, x' \text{ such that } x > x', \text{ and } \forall y \) \hspace{1cm} (FOSD)

For example, uniform signals are such that \( G(y|x) = y/x \) for every \( y \leq x \) and for every \( x > 0 \). Bernoulli signals require that, for every \( x \), \( g(0|x) = 1 - \pi \), \( g(x|x) = \pi \), and \( g(y|x) = 0 \) otherwise, for \( \pi \in [0, 1] \). It is easy to check that both satisfy FOSD.

Contracts regulate (i) public communication, (ii) verification, (iii) allocation of output. Communication consists on a public message \( m \in M \) that the manager sends at \( t = 0 \).
1, after he knows the state but before verification – e.g., a balance-sheet statement. Verification is a function of signals and messages, $\sigma(m, y) : M \times X \rightarrow [0, 1]$, and it describes the probability that each message is verified depending on the realized signal.\footnote{Among the extensions, I consider the case of non-verifiable signals. Contractibility of signals is not necessarily realistic. I assume it to clarify that the suboptimality of debt does not depend on those commitment issues that arise when signals are private information of the investor.}

Finally, allocation of output can be described by two functions:

1. The repayment function $s(m, y) : M \times X \rightarrow \mathbb{R}$ denotes the payment from manager to investor when verification does not take place;

2. The clawback function $z(m, x) : M \times X \rightarrow \mathbb{R}$ denotes the payment from manager to investor when verification does take place.\footnote{Evidently, nothing to be gained by conditioning repayments also on realised signals in this case.}

I impose two restrictions on feasible contracts: (i) limited liability; (ii) deterministic verification:

**Assumption 6.** A contract is feasible if and only if, $\forall m, y, x$:

Payments satisfy limited liability: $s(m, y) \in [0, \max\{m, y\}]$, $z(m, x) \in [0, x]$  

Verification is deterministic: $\sigma(m, y) \in \{0, 1\}$  

Assumption 6 generates two sets: $V \equiv \{m, y | \sigma(m, y) = 1\}$ and $NV \equiv X^2 \setminus V$. The set $V$ consists in all those information sets $(m, y)$ that trigger verification at $t = 1$.

A type of manager in this model corresponds to the realized state $x$ that he privately observes ex-post. From now onwards, type and state both make reference to the realized $x$. Before stating the contracting problem, observe that because of commitment the revelation principle holds here: one can restrict attention contracts such that $M = X$ and each type reports truthfully $x$ on-the-equilibrium path.\footnote{The proof is standard and I omit it.}

Define the expected payment when type $x$ sends, respectively, (i) a truthful message $x$, and (ii) a message $x' \neq x$, as:

(i) $S(x) \equiv \int_X [\sigma(x, y)s(x, y) + (1 - \sigma(x, y))z(x, x)] \, dG(y|x)$

(ii) $S(x, x') \equiv \int_X [\sigma(x', y)s(x', y) + (1 - \sigma(x', y))z(x', x)] \, dG(y|x)$

Incentive compatibility requires that:

$$S(x) \leq S(x, x'), \ \forall x, x'$$ (18)

Define the expected verification for a truthful message $x$ as $\Sigma(x) \equiv E_G[\sigma(x, y)|x]$. The participation constraint (or zero profit condition) of the financier reads:

$$E_f[S(x) - \Sigma(x)]c \geq i$$ (19)
The contracting problem can be stated as follows:

**Definition 6.** A contract \( \{s^*, z^*, \sigma^*\} \) is optimal if:

\[
\{s^*, z^*, \sigma^*\} \in \arg \max E_f[x - S(x)] \quad \text{s.t. LL, DV, (1) and (2)}. \tag{20}
\]

It is easy to see that (19) must be binding at any optimal contract. This is because whenever a contract \( \{s, z, \sigma\} \) is feasible and incentive compatible, so is a contract \( \{s', z', \sigma'\} \) such that (i) \( \sigma' = \sigma \), (ii) \( s' = \alpha s \), and (iii) \( z' = \alpha z \) for some \( \alpha \in [0,1] \).

By substitution, I can rewrite the contracting problem as:

\[
\{s^*, z^*, \sigma^*\} \in \arg \max E_f[x - \Sigma(x)\mu] - i \quad \text{s.t. LL, DV, (1)}. \tag{21}
\]

Program (21) makes it clear that a feasible and incentive compatible contract is optimal if it minimises the expected cost of verification. I refer to an optimal allocation as **first-best** if \( E_f[\Sigma^*(x)c] = 0 \), as **second-best** otherwise. The condition is equivalent almost surely to having \( \Sigma^*(x) = 0 \) for every \( x \) (that is, up to sets of measure zero).

It is useful to start by analysing the optimal clawback provisions and verification off-equilibrium path. As the next lemma shows, it is always beneficial to set the harshest feasible clawback provisions whenever \( m \neq x \) is revealed, in order to implement truthtelling in equilibrium.

**Lemma 8.** We can restrict attention to contracts such that:

(i) All assets are seized upon verified cheating: \( z^*(m,x) = x \) whenever \( m \neq x \);

(ii) Messages revealed to be false are verified: \( \sigma^*(m,y) = 1 \) whenever \( m < y \).

**Proof.**

As in the baseline model.

As a consequence of Lemma 8, we have one degree of freedom in setting \( s^*(m,y) \) whenever \( m < y \). I let \( s^*(m,y) = z^*(m,y) = x \) in such events.

### C.2 First-Best

As a benchmark, consider the case of either \( g(x|x) = 1 \) or \( c = 0 \). The participation constraint reads \( \int_X s(x,x) \, dF(x) \geq K \), and incentive compatibility becomes \( s(x,x) \leq x \). Any feasible repayment function \( s(\cdot) \) is optimal as long as it makes the participation constraint binding. In words, when earnings are observable and verifiable with certainty at no cost Modigliani and Miller (1958) holds: the security design question is irrelevant. From now onwards, I restrict attention to \( g(x|x) < 1 \) and \( c > 0 \).

The next Proposition shows that: (i) if signals are sufficiently informative the First-Best can be implemented despite the presence of hidden information; and (ii) to implement the First-Best one should simply look at the properties of a contract \( s^*(m,y) = y \) and \( \sigma^*(m,y) = 0 \) for every \( m \geq y \).

**Proposition 5.** The First-Best can be implemented if and only if \( E_f[\mathbb{E}_G[y|x]] \geq i \).
Proof.

Sufficiency. Consider the contract \( \{ s, z, \sigma \} \) such that \( s(m, y) = \alpha y \) and \( \sigma(m, y) = 0 \) for every \( m \geq y \) and for some \( \alpha \in (0, 1) \). Recall that (i) \( \sigma(m, y) = 1 \) and (ii) \( z(m, x) = x \) whenever \( m < y \) by Lemma 8.

The contract is feasible, and it is trivially incentive compatible. Moreover, we know there exists a number \( \alpha \in (0, 1) \) such that it makes the participation constraint binding. Because it does so without verification on-the-equilibrium path, the contract maximises the entrepreneur’s payoff and therefore it is optimal.

Necessity. Incentive compatibility requires:

\[
\int_0^x \left[ s(x, y) - s(x', y) \right] \, dG(y|x) \leq 0
\]

Under \( s(m, y) = \alpha y \) any sequence of incentive constraints tends to be binding when \( x' \to x \), for every \( x \). Further, by feasibility we must have \( s(0, y) = 0 \) and \( s(x, x) \leq x \). Therefore, increasing \( s(x, y) \) would violate either feasibility or incentive compatibility, proving our claim.

Corollary 5. Suppose that, for every \( x \), \( G(y|x) \) is either uniformly distributed, or it is Bernoulli distributed with positive mass only on the pair \( \{ 0, x \} \). Then, whenever \( \mathbb{E}_f[\mathbb{E}_G[y|x]] \geq K \), pure equity is optimal and it implements the First-Best.

Proof. Consider the uniform case first. We have that, for every \( x \):

\[
\mathbb{E}_G[y|x] = \frac{1}{x} \left[ \int_0^x y \, dy \right] = \frac{x}{2}
\]

We have to show that a contract \( s'(x) = \beta x/2 \) for some \( \beta \in (0, 1) \) (and no verification on-the-equilibrium path) does just as well as a contract \( s(y) = \beta y \).

Evidently, by he law of total probability the two contracts would yield exactly the same expected revenues to the financier. Moreover, \( s' \) is feasible because \( \beta \in (0, 1) \). To see that it is also incentive compatible observe that, for every type \( x \) and message \( x' \neq x \), incentive compatibility reads:

\[
\frac{\beta x}{2} \leq \frac{\beta (x')^2}{2x} + x - x' \iff 0 \leq \frac{(2 - \beta)x}{2} + \frac{\beta (x')^2}{2x} - x'
\]

The right hand side is a weakly decreasing function of \( x' \) so we simply need to evaluate it at \( x' \to x \), in which case the inequality reads \( 0 \leq 0^+ \) and it is satisfied.

As for the Bernoulli case, we have \( \mathbb{E}_G[y|x] = \pi x \) for some \( \pi \in (0, 1) \). We have to show that a contract \( s'(x) = \beta \pi x \) for some \( \beta \in (0, 1) \) (and no verification on-the-equilibrium path) does just as well as a contract \( s(y) = \beta y \).

Revenues are the same by the law of total probability, and \( s' \) is clearly feasible. Incentive compatibility reads:

\[
\beta \pi x \leq \pi x + (1 - \pi) \beta \pi x'
\]
The right hand side is an increasing function of $x'$ so we simply need to evaluate it at $x' \to 0$. In such case it reads $\beta \leq 1$, which is satisfied by construction. 

Finally, as this is an extended CSV model, it is important to observe that a debt contract never implements the First-Best. This is because $d > K > 0$ and for every type $x < d$ debt requires $\sigma(x) = 1$.

**Corollary 6.** Debt never implements the First-Best.

I now turn to the derivation of the optimal contract that implements the Second-Best.

### C.3 Second-Best

As a starting point, I consider the efficiency properties of debt contracts. Pure debt is suboptimal whenever signals convey some information in the no-default states: the optimal contract crosses debt from the right. Recall that we denote the face value of a pure debt contract by $d$. For this section, define also:

$$\hat{d} = \inf \{ d \in X | \mathbb{E}[\min\{x, d\}] - F(d)\mu = K \}$$

and suppose the infimum exists.\(^{52}\)

**Proposition 6.** If the set $S \equiv \{ x | \mathbb{E}[y|x] > \hat{d} \}$ is non-empty and of strictly positive measure, then pure debt is suboptimal.

*Proof.* By contradiction, suppose the contract $\{ s, z, \sigma \}$ is pure debt and it is optimal. This implies that (i) $s = \min\{m, \hat{d}\}$, and (ii) $S \subseteq V$. Consider a contract $\{ s', z', \sigma' \}$ such that $z' = z$, $\sigma = \sigma'$ and:

$$s'(m, y) = \begin{cases} s(m, y) & \text{if } \sigma = 1 \\ \min\{\hat{d}, y\} & \text{otherwise} \end{cases}$$

The contract is clearly feasible and incentive compatible. However, because (i) $S$ is of strictly positive measure; and (ii) for every element of $S$ we have $\mathbb{E}[y|x] > \hat{d}$, the financier is now making strictly positive profits.

To conclude the proof, I show that there exists another contract $\{ s'', z'', \sigma'' \}$ such that the financier makes the same profits as the pure debt contract we started with, at a strictly lower verification cost. This contradicts the optimality of pure debt.

Indeed, define:

$$\hat{d}'' = \inf \{ d \in X | \int_0^x \int_0^y \min\{d, y\} dG(y|x) dF(x) = K \}$$

\(^{52}\)A sufficient condition for this to be the case is $\mathbb{E}[x] - \mu \geq K$. The case where the infimum does not exist would trivially imply that debt cannot be optimal, as no debt contract satisfies the participation constraint for the financier.
We know that \( \hat{d}'' \) exists whenever \( \hat{d} \) exists. Further, suppose that \( z'' = z' = z, s'' = s' \) and:

\[
\sigma'' = \begin{cases} 
\sigma & \text{if } m \leq \hat{d}'' \\
0 & \text{otherwise}
\end{cases}
\]

The double-prime contract is clearly feasible and incentive compatible. Moreover, because \( S \) is of strictly positive measure we know that \( \hat{d}'' < \hat{d} \). Therefore, expected verification costs are lower. But, by construction, the financier makes zero profits on the double-prime contract, which implies the entrepreneur strictly prefers it to the pure debt contract we started with, proving our claim. \( \square \)