A Behavioural Power Index

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Abstract

We propose an empirically informed measure of the voting power that relaxes the assumptions of equally probable and independent votes. The behavioral power index measures the voter's ability to swing a decision based on the probability distributions of the others' behavior. We apply it to the Supreme Court of the United States using roll-call data to estimate voting probability distributions, which lead us to refute the assumption of equally probable and independent votes, and estimate the equivalent number of independent Justices for the Warren, Burger and Rehnquist benches, which turns out to be very low.

JEL-Codes: D72
Key Words: behavioral voting power, constitutional voting power, U.S. Supreme Court

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1. Introduction

A common criticism of the widely used Penrose (1946) or Banzhaf (1965) measure of voting power is that it fails to take account of preferences.[1] It is a measure of the *a priori* voting power of an individual member of a voting body defined as the probability of her being decisive, considering all possible voting behavior by the other members. Power is assumed to arise purely from the voting rules and its measurement is based on an examination of the possibilities that exist for voters to affect decisions by switching their votes. All outcomes permitted under the rules are treated as being equiprobable, rather than their probability distribution being derived empirically from evidence on actual behavior. A standard power index is therefore best described as a measure of *constitutional* voting power: power that is inherent in the formal rules of the voting body only.

The assumption that all voting profiles are equally probable is equivalent to the binomial model of probabilistic voting in which each vote has an equal probability of being cast for or against a motion, and all votes are independent. Felsenthal and Machover (2004) defend the binomial model on three counts. First, it is a reasonable assumption in the absence of prior knowledge about the future issues on the ballot and how divided over these issues voters will be. Second, it therefore suits the purpose of measuring the distribution of a priori or constitutional voting power that follows from the rules of the voting body only in a purely formal sense. Third, it is attractive on normative grounds because it presumes the maximum freedom of choice for the voter.[2] It therefore is the benchmark model to use in constitutional design but lacks descriptive

[1] See, for example, Garrett and Tsebelis (1999), Gelman, Katz and Bafumi (2004). This topic has been extensively debated in Journal of Theoretical Politics; see Napel and Widgrén (2004) and the critique in Braham and Holler (2005a), as well as the reply in Napel and Widgrén (2005) and the rejoinder in Braham and Holler (2005b).

[2] In a binary voting game, the assumption of equally probable and independent votes maximizes the information entropy of the distribution on the choice set of each voter $\{0, 1\}$, and also maximizes the entropy of the distribution on the set of all 2^n voting profiles. The model can therefore be interpreted as presuming the maximum freedom of choice for the voter and the maximum freedom of choice for the voting assembly.

realism.

In this paper we propose an empirically informed power measure by relaxing this very strong assumption of binomial voting and replacing it by probability distributions that reflect real preferences. Since, for each individual, it is the behavior of the other voters that determines the likelihood that she will be decisive, this approach makes it necessary to base the definition of the power index for each voter on a *different* probability distribution. These probability distributions can be estimated empirically from a relative frequency distribution over the set of all theoretically possible voting profiles; that is, from observation of the frequency of occurrence of every one of the large number of possible roll-calls over a suitable period of time.

We apply this approach to data for the Supreme Court of the United States. Using a large sample of voting data we are led to reject the binomial voting model. A major reason for this is evidence of positive correlations between votes. There is a large discrepancy between the Penrose measure of constitutional power and the behavioral power index.

The plan of the paper is as follows. In the next section we introduce the behavioral index of voting power. Section 3. describes the data and presents computations of behavioral power for the justices in the U.S. Supreme Court. Section 4. presents estimates of Coleman's measure of the overall dependence and the correlation coefficients between justices' votes. Section 5. summarizes the paper.

2. A behavioral power index

We assume that the voting body consists of n members, labeled $i = 1, 2, \ldots, n$; the set of all members is $N = \{1, 2, \ldots, n\}$. Each member has a number of votes (or voting weight) w_i and must cast all her votes as a bloc either for or against a motion. The decision rule is defined by a majority quota or threshold number of votes, q, such that, if the total number of votes cast by *Yes* voters is greater than or equal to q, the decision is *Yes*, and otherwise it is *No*. A decisive voting rule requires $0.5\sum_{i=1}^{n} w_i < q \leq \sum_{i=1}^{n} w_i$, which is assumed.[3]

We seek to define a measure of the voting power of a particular voter i in relation to decisions taken under the rules, recognizing the behavior of all the other voters.

In defining a behavioral power index for an individual voter we follow Braham and Holler (2005a) and Morriss (2002), who argue that, because power is fundamentally *dispositional* in nature, its possession or its magnitude do not depend on its *exercise*. That is, power exists as a potential whether or not its possessor uses it. This means that the *behavior* of voter i has no relevance to a measure of the power of i, which depends on the behavior of the other voters. That is, the actual behavior of voter i, in the sense of how frequently she votes *Yes* or *No*, is not part of the measure.

Thus, for example, if it is known that voter i almost always votes Yes whenever the other voters are tied, and almost never votes No in the same circumstances, this information is irrelevant to the measurement of the power of i. Voter i's power depends only on the potential she faces to affect decisions, which arises due to the behavior of the other voters (as well as her voting weight and the decision rule).

2.1. Definition of the power index

We refer to any particular voting profile, or the result of a ballot, as a division. We represent a division of all n voters by the set D (where $D \subseteq N$). The members of D are those voting Yes. The total number of *Yes* votes cast is $w(D) = \sum_{j \in D} w_j$.[4]

[3]Later it will be convenient to let w_i represent the number of *Yes* votes cast by member i, and set $w_i = 0$ if she votes *No*.

[4]It is common in the literature to refer to a division D as 'winning' if $w(D) \geq q$ and 'losing' otherwise. We will here avoid this terminology and instead refer to a *Yes* decision and a *No* decision respectively.

Now consider the measurement of the voting power of member i. In general a member's power index measures the likelihood of her being able to bring about a change in a decision by changing her vote, that is, the probability of being a swing voter. This probability depends on the distribution of divisions of *all voters other than* i. Thus in our probability model the number of elements in the set of elementary outcomes equals 2^{n-1}. We represent such a division by D_i, where $D_i \subseteq N\backslash\{i\}$. D_i is a swing for i if $q - w_i \leq w(D_i) < q$. Let the set of swings for i be H_i and the total number of swings be η_i.

We assume a probability distribution over D_i, with the probability π_{D_i} such that $\pi_{D_i} \geq 0, \forall D_i \subseteq N\backslash\{i\}$ and $\sum_{D_i} \pi_{D_i} = 1$.

Then we can define a behavioral power index as the probability of a swing for voter i:

$$\gamma_i = \sum_{D_i \in H_i} \pi_{D_i} \tag{1}$$

A special case, of course, is the Penrose-Banzhaf measure, where we take all divisions as equally probable, that is $\pi_{D_i} = 2^{1-n}$ for all D_i. Then

$$\gamma_i = \sum_{D_i \in H_i} 2^{1-n} = \frac{\eta_i}{2^{n-1}} = \beta_i' \tag{2}$$

using the usual notation β_i' for the index, also known as the absolute Banzhaf index.

We follow Morriss (2002) in making an important distinction between 'power as ability' and 'power as ableness'. A voter's ability is defined as her power to affect decisions whatever the actions of others. On the other hand, the term ableness refers to the situation in which the voter finds herself given specific actions by others. The suggestion is that the former leads to a power measure which makes no assumptions about the actual or likely voting behavior of the voters other than i, treating all divisions that could possibly occur as equiprobable: the Penrose-Banzhaf measure. The latter means taking account of the actual behavior of the voters other than i, as in definition (1).

2.2. Allowing for asymmetries in voting behavior

Definition (1) assumes that the probability distribution of divisions of voters other than i does not depend on how voter i votes. Empirically this is unlikely to be the case because of both asymmetries in the way in which issues are presented and correlation of voter behavior. That is, certain D_i will be more/less likely to occur when voter i votes Yes than when she votes No and this fact is relevant to how the measure is constructed. This kind of dependence can arise in various ways. It can be causal in either direction or reflect common influences; but that is immaterial to a measure of voting power which is merely a description of the potential to swing a vote.

It may be, to take an example, that voter i happens to be some kind of leader on issues on which she votes Yes and that the other $n-1$ voters are likely to follow her in also voting Yes, while on an issue where she votes No, there is high probability that they are evenly split, with similar numbers of votes on each side. This is observationally equivalent to a situation where, for whatever reason, voter i votes Yes when there is often a large Yes vote anyway, but No when the others are evenly divided. But in both cases i's voting power is the same.

It is therefore necessary to allow for this kind of asymmetry by the use of two different probability distributions for divisions D_i.

Definition 1. *Let:* $\phi_{D_i} = Pr[D_i]$ *when* i *votes* Yes*; and* $\psi_{D_i} = Pr[D_i]$ *when* i *votes* No*.*

These two probability distributions can be used to define two different behavioral power indices, one for each way that i votes, by substituting ϕ_{D_i} and ψ_{D_i} for π_{D_i} in (1). However, we seek a single measure that is independent of voter i's behavior and so we need some way of combining them.

The measure we propose combines the two probabilities for each D_i equally, on the grounds that, since power is inherent in its disposition rather than its exercise, there is no reason to regard voter i's power as greater *because* she has chosen to vote in a certain way. We must allow for the fact that voter i has a sovereign right to choose whether to vote Yes or No equally. Therefore the two probability distributions must have equal weight in the behavioral power index.

We therefore replace π_{D_i} in (1) by a simple mean of these probabilities, and define the behavioral power index α_i in terms of the two distributions as:[5]

$$\alpha_i = \frac{1}{2} \sum_{D_i \in H_i} (\phi_{D_i} + \psi_{D_i}). \tag{3}$$

It might be objected that this leads to inconsistency in that, when measuring the power of another voter, j, the behavior of voter i is then treated differently, as random. However such inconsistency is inherent in the fundamental approach to the measurement of power we are employing and the idea of power as ableness.[6]

An alternative measure has been proposed by Morriss (2002). He suggests a measure of power equal to the probability of a swing for voter i where the basic probability model is a

[5]In basing α_i on the key distinction that the behavior of all the members of $N\backslash\{i\}$ is random, but that of i is not, means that our approach differs fundamentally from that of Laruelle and Valenciano (2005), whose starting point is to assume a probability model for all divisions of N. The set of elementary outcomes in their model contains 2^n instead of 2^{n-1} elements. Consistent with their model they speak of conditional probabilities of being decisive, conditioned on whether i votes *Yes* or *No*.

[6]The Penrose, or absolute Banzhaf, measure has the same logical inconsistency in the treatment of i and D_i but it is less apparent because the probabilities of the D_is are equal. This only becomes an important issue when the index is normalized, as shown by Dubey and Shapley (1979).

distribution over all divisions, $D \subseteq N$, rather than, as here, $D_i \subseteq N\backslash\{i\}$. It therefore treats the voting behavior of i as probabilistic. In our view this measure fails as a power index because it depends on the behavior of i herself, and therefore is subject to the exercise fallacy.

Suppose, for example, that voter i's probability of a swing is 0.9 when she votes *Yes*, and 0 when she votes *No*. If she votes *Yes* on 90% of occasions we might say that the swing probability index is equal to $0.9 \cdot 0.9 + 0 \cdot 0.1 = 0.81$. On the other hand, if she decides to vote *No* with probability 0.9, the index will be $0.9 \cdot 0.1 + 0 \cdot 0.9 = 0.09$. The value of this particular power measure therefore depends on the exercise of power. The index α_i, however, disregards the behavior of voter i and gives a value of 0.45 reflecting only the probabilities of swings.

2.3. Estimation and computation

The calculation of the power index α_i using the definition in (3) requires knowledge of the probabilities of divisions, the ϕ_{D_i}s and ψ_{D_i}s. In practice these must be estimated from the observed frequencies with which each division occurs, separately for each i.

Let assume that we know the relative frequencies of occurrence of divisions of all voters, obtained from historical data. That is, for each division $D \subseteq N$ we know its relative frequency f_D (where $0 \leq f_D \leq 1, \sum f_D = 1$).

We estimate the ϕ_{D_i}s and ψ_{D_i}s from the conditional frequencies[7] for the cases where i votes *Yes* and *No* respectively. Let the relative frequency with which i votes *Yes* be denoted by f_i; that is $f_i = \sum_{D;i \in D} f_D$. The frequency with which i has voted *No* is equal to $1 - f_i$.[8]

[7]Note the fact that these are conditional frequencies does not imply that the probabilities ϕ_{D_i} and ψ_{D_i} should be regarded as conditional probabilities. Such terminology would be inappropriate since the vote of i is not probabilistic with respect to the measure of voting power of i.

[8]The use of this approach to estimate probabilities requires $f_i \neq 0$ and $f_i \neq 1$. The probabilities cannot be retrieved from observed data if i never votes either Yes or No.

Now, our estimates of the probabilities are:

$$\begin{aligned}
\phi_{D_i} &= \frac{f_D}{f_i}, \quad \text{when} \quad i \in D, \text{ that is, when } D = D_i \cup \{i\}, D_i \subseteq N \backslash \{i\} \quad (i \text{ votes } \textit{Yes}) \\
\psi_{D_i} &= \frac{f_D}{(1 - f_i)} \quad \text{when} \quad i \notin D, \text{ that is, when } D = D_i, D_i \subseteq N \backslash \{i\} \quad (i \text{ votes } \textit{No}).
\end{aligned}$$

Table 1 gives an example illustrating the above calculation for a weighted voting game with $n = 3$, $q = 4$, $w_1 = 3$, $w_2 = 2$ and $w_3 = 1$. The discrepancy between the behavioral and constitutional power measures is evident in this example.

2.4. Preferences

In seeking to allow for preferences in a power measure, we agree with Braham and Holler (2005a: 138) that "the basic concept of power as a potential or capacity cannot accommodate the preferences of the players whose power we are measuring". But we believe that a power measure should accommodate the preferences of the players other that the one whose power is being measured. In this we hope to reconcile the fundamental critique by Braham and Holler with the counter-argument by Napel and Widgrén (2005: 379) that "preferences are needed to screen outcomes that are possible and can potentially be affected or forced by a given player from those that are not possible given all players' strategic efforts to realize their own will".

We do not model preferences in the way true preference-based measures do. Typically these measures model preferences as points in a suitably labeled Euclidean space.[9] Our measure does not specify preferences, strategies, informational asymmetries that may influence the distribution of votes, and yet it fully accounts for the statistical consequences of the preferences conveyed by the roll-call data on voting. Whether it was differences in preferences, a plot on the part of

[9] For example, Steunenberg, Schmidtchen and Koboldt (1999), or Napel and Widgrén (2004).

other voters, or a basic lack of information that caused this distribution may be important to the voter, but has no bearing on the measurement of voting power. We therefore do not call our measure preference-based, but rather empirically informed. Since our measure is based on the probability space of the classic Penrose-Banzhaf-Coleman measures, it cannot be compared directly to measures based on a spatial representation of preferences as they have different spaces of elementary outcomes, and hence also different probability spaces.

3. Voting power in the U.S. Supreme Court

The U.S. Supreme Court is the highest judicial authority of the United States. The Court comprises the Chief Justice and eight Associate Justices. The justices are nominated by the President and appointed with the advice and consent of the Senate to serve for life.

The Court votes under simple majority rule provided at least six justices are present for the Court to be in session. Justices vote in the order of their seniority, starting with the Chief Justice. Although the Chief Justice has a number of exclusive prerogatives and responsibilities, all justices have equal a priori voting power.

It is customary to refer to the Court by the name of the presiding Chief Justice, who usually is the longest-serving member of the Court. We define a bench as a Court in which the same group of persons sit as justices, and study three benches from the Chief Justice Warren (1953-1969), Burger (1969-1986) and Rehnquist's (1989-2005) eras.

3.1. The data

The Supreme Court Justice-Centered Judicial Databases sponsored by the University of Kentucky and maintained by Benesh and Spaeth is a comprehensive source of data on the decisions and opinions of individual justices during the three eras.[10]

Several decisions had to be made when preparing the data. Our focus is on the relative frequencies of divisions. For a meaningful analysis we therefore confine the analysis to cases in which the same bench of justices voted. Out of 228 observed benches we chose the three which had the largest number of cases, one for each era. The high number of different benches arises from the combinations of six to nine justices possibly attending a session, and from the fact that associate justices retired and were replaced in the same era. Our selection includes 416, 1110, and 683 cases from respectively the Warren, Burger and Rehnquist eras, with all nine justices voting on each case. Since we would like the cases to be as independent from each other as possible, we exclude those with multiple legal issues or multiple legal provisions. Finally, we define a *Yes* vote as one in favor of the petitioning party.[11]

3.2. Relative frequencies of divisions

Figure 1 shows the relative frequencies of divisions. With nine justices there will be 512 possible divisions. The abscissa scale shows the divisions ordered from a unanimous vote against the petitioning party to a unanimous vote in favor of the petitioning party. Note that:

(i) only a small minority of possible divisions actually has occurred, the percentage in each bench being Warren (9.4), Burger (31.6) and Rehnquist (21.5);

(ii) the relative frequencies of those which are observed are very unequal;

(iii) unanimity is by far the most common of all voting profiles. The Burger bench is an exception with the 2:7 divisions against the petitioning party occurring slightly more frequently than the 0:9 divisions;

[10]These data are available at `http://www.cas.sc.edu/poli/juri/sctdata.htm`.

[11]In terms of the variables in the Benesh and Speath databases, we restrict $ANALU = 0$, delete the duplicate LED numbers and compute a vote variable which takes the value 1 if $MAJ\_MIN = WIN\_DUM$, and 0 otherwise.

(iv) a unanimous vote in favor of the petitioning party is more frequent than a unanimous vote against the petitioning party. This suggests a selection bias in the data in favor of petitions with a reasonable chance of success.

These observations refute the binomial model of equally probable and independent votes as an empirical description. Although comprehensive voting records such as that for the U.S. Supreme Court are scarce, data on several other voting bodies reveal that divisions typically occur with very different frequencies. This is true of the Supreme Court of Canada (Heard and Swartz 1998), the European Union Council of Ministers (Hayes-Renshaw, van Aken and Wallace 2006) and the institutions of the United Nations (Newcombe, Ross and Newcombe 1970). In the next section we show that positive correlations between votes are the primary cause of the observed pattern.

3.3. The behavioral power index

In a voting game with equal weights all voters have equal a priori powers. Under simple-majority rule with an odd number of votes the Banzhaf absolute measure equals the binomial probability of $(n-1)/2$ successes in $n-1$ trials with 0.5 as the probability of success. For $n = 9$ this probability equals 0.273.

Table 2 reports the behavioral power index α_i for each justice. It is evident that the justices' behavioral powers are not equal. We quantify this inequality by comparing the Gini coefficient of α_i. The largest inequality is observed in the Warren bench, followed by Rehnquist and Burger. The biggest losers in terms of behavioral versus a priori voting power were justices Black, Douglas and Clark of the Warren bench. None of the justices had more behavioral power than a priori power.

The actual distribution of voting power in the U.S. Supreme Court has been studied before. Edelman and Chen (1996) compute what essentially is a generalization of the normalized

Banzhaf measure of voting power, in which a devision is defined by the group of justices who join the Court's opinion. They identify Justice Kennedy as the one who wielded the most voting power during this period of time. This conclusion is not supported by our calculations for the Rehnquist bench. We find that it was Justice Souter who had the most power. The difference can be explained by three facts. First, Edelman and Chen (1996) based their calculations on a subsample of the data we used, covering the Court's 1994 and 1995 October Terms only. Second, their calculations account for the Court opinions joined by the Justices. Opinions and votes differ occasionally. A justice may vote with the majority and yet write a dissenting opinion. Defining divisions based on opinions in the context of a binary choice is also problematic because the Court may deliver more than two opinions ('seriatim opinion'). And third, Edelman and Chen's measure differs conceptually in that it is not free from the effect of the own vote on the collective outcome.

4. Coleman's measure of dependence

The fact that unanimity is by far the most frequent of all divisions points to a positive correlation between the justices' votes. The correlation coefficient is a pairwise measure of linear dependence. But is there a measure of the overall dependence between votes? One such measure has been proposed in Coleman (1973).

Under the binomial model the variance of the fraction of *Yes* votes is $\sigma^2 = p(1-p)/n$, where p is the probability of a *Yes* vote, here common to all voters. The sample variance s^2 will differ systematically from σ^2 if the assumptions of the binomial model are not met. This will be the case if votes are not independent, or if the true probability is not p, or both. For example, if each pair of votes correlates with the correlation coefficients c, then $s^2 \approx \sigma^2[1 + (n-1)c]$, provided $(1-n)^{-1} \leq c \leq 1$. The equality is only approximate since s^2 depends on the sample of voting data, i.e., is a statistic. Assuming that the true probability is indeed p, the ratio σ^2/s^2 is a rough measure of the overall dependence in a voting body of size n.

Coleman used this fact to compute the equivalent number of independent voters m. Setting $s^2 = p(1-p)/m$ yields $m = p(1-p)/s^2$. For independent voters $s^2 \approx p(1-p)/n$ so that $m \approx n$. Compared to $n = 9$, low values of m for the Warren (1.48), Burger (2.21) and Rehnquist (1.79) benches suggest a high positive correlations between votes.

A measure of the overall dependence similar to Coleman's has been proposed in Sirovich (2003). Sirovich claims that the second Rehnquist U.S. Supreme Court "acts as if composed of 4.68 ideal Justices". His measure is based on the concept of informational entropy and is computed for a sub-sample of the data for the Rehnquist bench analyzed in this paper. We find an even lower equivalent number of independent Justices on the Rehnquist bench.[12]

Coleman's is a measure of the overall dependence. Table 3 reports estimates of the individual marginal probabilities and correlation coefficients. They show that in most cases the probability of a *Yes* vote is not far from 0.5 but that votes are highly positively correlated. For the three Courts the average probabilities of a *Yes* vote are 0.63, 0.47 and 0.54. The spreads of the estimated probabilities range from (0.44, 0.73) for the Warren bench to (0.53, 0.55) for the Rehnquist bench. The average correlation coefficients between two *Yes* votes for the three benches are 0.67, 0.39 and 0.5.

Fisher's Exact Test[13] cannot reject the null hypothesis of conditional independence at the 5% level of significance for only 7 of 108 pairs of justices indicated by an asterisk. There is significant positive correlation in justices' decisions.

[12]For a critique of the equivalent number of independent (or, as Sirovich calls them, "platonic" justices), see a commentary by Edelman (2004).

[13]See Everitt (1992).

One consequence of high positive correlations is that voters often see their preferred outcome prevail. Several authors have proposed to measure voting power using this probability (Satisfaction index by Straffin (1978), EPW index by Morriss (2002)). We believe that the probability of being on the winning side is not a valid measure of power because it does not distinguish between power and luck.[14] For example, it assigns power to a voter who cannot swing in any theoretically conceivable division of votes, a dummy. Our approach is to define power in terms of the capacity to affect a decision, based on the concept of a swing.

5. Summary

The standard approach to measuring the voting power of a particular member of the voting body is to compute the probability of her casting a decisive vote. This analysis must encompass both the formal rules of the voting body and also the behavior of all the voters. The member is more powerful the more frequently her vote is decisive. But this will depend in practice on circumstances created by others casting their votes so that she has opportunities to be decisive. Thus an empirical voting power measure must be based on the frequencies with which the various voting profiles occur. As is well known, the Penrose-Banzhaf measure ignores behavior and takes into account only power deriving from the rules construed in a purely formal, a priori sense.

We propose an empirically informed power measure by relaxing this very strong assumption and replacing it by the use of information about real or assumed voting patterns. The behavioral power index is based on the probability distribution of divisions of all other voters except the one whose power is being measured, which can be assumed or estimated using ballot data.

The voting behavior of justices in the U.S. Supreme Court clearly contradicts the assumptions of equally probable and independent votes. Over a range of cases, voting profiles occur with

[14] This point has been extensively discussed by Dowding (1996).

very unequal relative frequencies. The observed patterns of relative frequencies point to positive correlation as their main cause, and indeed we find significant positive correlation in justices' decisions.

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Table 1: The behavioral power index for the voting body $\{4; 3, 2, 1\}$

D	f_D			VOTER 1			
$\{1,2,3\}$	0.309	D_1	$\{2,3\}$*	$\{2\}$*	$\{3\}$*	$\{\emptyset\}$	
$\{1,2\}$	0.016	ϕ_{D_1}	0.618*	0.032*	0.262*	0.088	
$\{1,3\}$	0.131	ψ_{D_1}	0.498*	0.052*	0.222*	0.228	
$\{1\}$	0.044	π_{D_1}	0.558*	0.042*	0.242*	0.158	$\alpha_1 = 0.842$
$\{2,3\}$	0.249			VOTER 2			
$\{2\}$	0.026	D_2	$\{1,3\}$	$\{1\}$*	$\{3\}$	$\{\emptyset\}$	
$\{3\}$	0.111	ϕ_{D_2}	0.515	0.027*	0.415	0.043	
$\{\emptyset\}$	0.114	ψ_{D_2}	0.328	0.110*	0.278	0.285	
f_1	0.5	π_{D_2}	0.421	0.068*	0.346	0.164	$\alpha_2 = 0.068$
f_2	0.6			VOTER 3			
f_3	0.8	D_3	$\{1,2\}$	$\{1\}$*	$\{2\}$	$\{\emptyset\}$	
w_1	3	ϕ_{D_3}	0.386	0.164*	0.311	0.139	
w_2	2	ψ_{D_3}	0.080	0.220*	0.130	0.570	
w_3	1	π_{D_3}	0.233	0.192*	0.221	0.354	$\alpha_3 = 0.192$

Behavioral powers equal 0.842, 0.068 and 0.192. A priori absolute Banzhaf powers equal 0.75, 0.25 and 0.25. The swings are indicated with an asterisk.

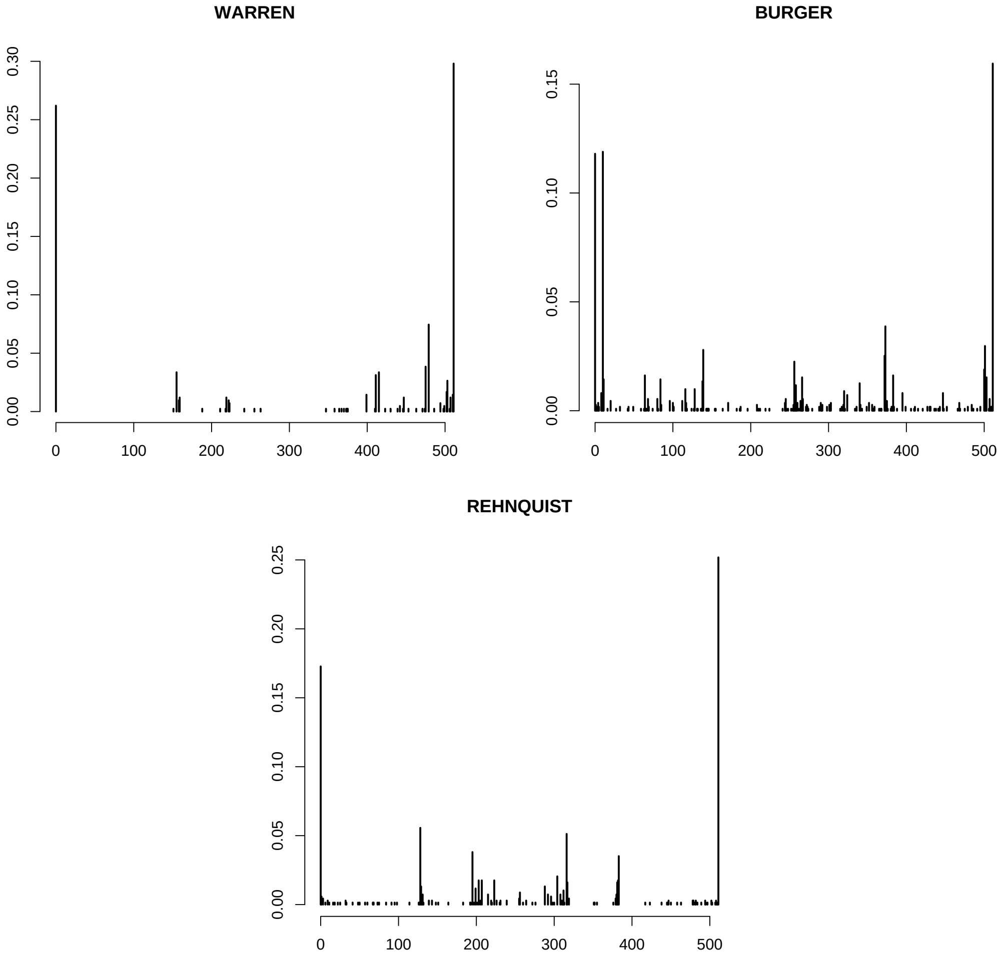
Table 2: The behavioral power index

WARREN		BURGER		REHNQUIST	
Black	0.009	Blackmun	0.106	Breyer	0.118
Brennan	0.046	Brennan	0.073	Ginsburg	0.072
Clark	0.017	Burger	0.089	Kennedy	0.074
Douglas	0.013	Marshall	0.106	Oconnor	0.112
Goldberg	0.047	Oconnor	0.096	Rehnquist	0.116
Harlan	0.063	Powell	0.059	Scalia	0.135
Stewart	0.019	Rehnquist	0.098	Souter	0.142
Warren	0.042	Stevens	0.060	Stevens	0.071
White	0.032	White	0.061	Thomas	0.070
Gini	0.306	Gini	0.126	Gini	0.150

Table 3: Marginal probabilities and correlation coefficients

WARREN				BURGER				REHNQUIST			
(i,j)	p_i	p_j	$c_{i,j}$	(i,j)	p_i	p_j	$c_{i,j}$	(i,j)	p_i	p_j	$c_{i,j}$
Black-Brennan	0.64	0.73	0.80	Blackmun-Brennan	0.44	0.51	0.36	Breyer-Ginsburg	0.55	0.53	0.72
Black-Clark	0.64	0.58	0.56	Blackmun-Burger	0.44	0.49	0.47	Breyer-Kennedy	0.55	0.55	0.48
Black-Douglas	0.64	0.65	0.74	Blackmun-Marshall	0.44	0.49	0.35	Breyer-O'Connor	0.55	0.55	0.54
Black-Goldberg	0.64	0.70	0.77	Blackmun-O'Connor	0.44	0.47	0.49	Breyer-Rehnquist	0.55	0.54	0.38
Black-Harlan	0.64	0.44	0.37	Blackmun-Powell	0.44	0.46	0.54	Breyer-Scalia	0.55	0.55	0.27
Black-Stewart	0.64	0.57	0.55	Blackmun-Rehnquist	0.44	0.50	0.37	Breyer-Souter	0.55	0.53	0.71
Black-Warren	0.64	0.72	0.79	Blackmun-Stevens	0.44	0.38	0.52	Breyer-Stevens	0.55	0.53	0.58
Black-White	0.64	0.64	0.69	Blackmun-White	0.44	0.54	0.46	Breyer-Thomas	0.55	0.55	0.24
Brennan-Clark	0.73	0.58	0.67	Brennan-Burger	0.51	0.49	0.00*	Ginsburg-Kennedy	0.53	0.55	0.52
Brennan-Douglas	0.73	0.65	0.82	Brennan-Marshall	0.51	0.49	0.81	Ginsburg-O'Connor	0.53	0.55	0.50
Brennan-Goldberg	0.73	0.70	0.90	Brennan-O'Connor	0.51	0.47	0.05*	Ginsburg-Rehnquist	0.53	0.54	0.41
Brennan-Harlan	0.73	0.44	0.51	Brennan-Powell	0.51	0.46	0.13	Ginsburg-Scalia	0.53	0.55	0.30
Brennan-Stewart	0.73	0.57	0.67	Brennan-Rehnquist	0.51	0.50	-0.10	Ginsburg-Souter	0.53	0.53	0.79
Brennan-Warren	0.73	0.72	0.95	Brennan-Stevens	0.51	0.38	0.36	Ginsburg-Stevens	0.53	0.53	0.62
Brennan-White	0.73	0.64	0.78	Brennan-White	0.51	0.54	0.12	Ginsburg-Thomas	0.53	0.55	0.27
Clark-Douglas	0.58	0.65	0.56	Burger-Marshall	0.49	0.49	-0.03*	Kennedy-O'Connor	0.55	0.55	0.72
Clark-Goldberg	0.58	0.70	0.59	Burger-O'Connor	0.49	0.47	0.76	Kennedy-Rehnquist	0.55	0.54	0.75
Clark-Harlan	0.58	0.44	0.62	Burger-Powell	0.49	0.46	0.73	Kennedy-Scalia	0.55	0.55	0.65
Clark-Stewart	0.58	0.57	0.62	Burger-Rehnquist	0.49	0.50	0.75	Kennedy-Souter	0.55	0.53	0.57
Clark-Warren	0.58	0.72	0.66	Burger-Stevens	0.49	0.38	0.40	Kennedy-Stevens	0.55	0.53	0.3
Clark-White	0.58	0.64	0.69	Burger-White	0.49	0.54	0.59	Kennedy-Thomas	0.55	0.55	0.63
Douglas-Goldberg	0.65	0.70	0.77	Marshall-O'Connor	0.49	0.47	0.02*	O'Connor-Rehnquist	0.55	0.54	0.71
Douglas-Harlan	0.65	0.44	0.34	Marshall-Powell	0.49	0.46	0.10	O'Connor-Scalia	0.55	0.55	0.63
Douglas-Stewart	0.65	0.57	0.52	Marshall-Rehnquist	0.49	0.50	-0.13	O'Connor-Souter	0.55	0.53	0.58
Douglas-Warren	0.65	0.72	0.83	Marshall-Stevens	0.49	0.38	0.33	O'Connor-Stevens	0.55	0.53	0.25
Douglas-White	0.65	0.64	0.63	Marshall-White	0.49	0.54	0.05*	O'Connor-Thomas	0.55	0.55	0.64
Goldberg-Harlan	0.70	0.44	0.49	O'Connor-Powell	0.47	0.46	0.72	Rehnquist-Scalia	0.54	0.55	0.76
Goldberg-Stewart	0.70	0.57	0.66	O'Connor-Rehnquist	0.47	0.50	0.75	Rehnquist-Souter	0.54	0.53	0.45
Goldberg-Warren	0.70	0.72	0.89	O'Connor-Stevens	0.47	0.38	0.46	Rehnquist-Stevens	0.54	0.53	0.12
Goldberg-White	0.70	0.64	0.69	O'Connor-White	0.47	0.54	0.53	Rehnquist-Thomas	0.54	0.55	0.75
Harlan-Stewart	0.44	0.57	0.65	Powell-Rehnquist	0.46	0.50	0.67	Scalia-Souter	0.55	0.53	0.37
Harlan-Warren	0.44	0.72	0.48	Powell-Stevens	0.46	0.38	0.47	Scalia-Stevens	0.55	0.53	0.05
Harlan-White	0.44	0.64	0.63	Powell-White	0.46	0.54	0.58	Scalia-Thomas	0.55	0.55	0.87
Stewart-Warren	0.57	0.72	0.63	Rehnquist-Stevens	0.50	0.38	0.35	Souter-Stevens	0.53	0.53	0.58
Stewart-White	0.57	0.64	0.70	Rehnquist-White	0.50	0.54	0.50	Souter-Thomas	0.53	0.55	0.35
Warren-White	0.72	0.64	0.74	Stevens-White	0.38	0.54	0.34	Stevens-Thomas	0.53	0.55	0.03

Figure 1: Relative frequencies of divisions



A division is represented by a binary vector $(v_1, v_2, \ldots, v_9)$, where $v_i = 1$ if i votes in favor of the petitioning party, and $v_i = 0$ otherwise. The abscissa scale shows the divisions ordered in the ascending order of the decimals their binary vectors represent, starting from the zero vector.