The KPSS Test with Outliers

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ABSTRACT

We investigate the effects of outliers on the KPSS tests. We find that for nonstationary series outliers induce spurious stationarity by lowering the power of these tests. The empirical size of these tests is also found to be sensitive to the location of the outlier.

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1. Introduction

Kwiatkowski *et. al.* (1992) (KPSS) developed a test of the null hypothesis that a series is stationary around a deterministic trend. In Monte Carlo simulations they analysed the power of this test against the alternative of a unit root in finite samples. The KPSS test is complementary to the, for example, Augment Dickey Fuller (ADF) and Phillips and Perron unit root tests, that are used to analyse the time series properties of individual series.

Investigating the performance of the ADF test, Perron (1989) showed that the power of this test to reject the null hypothesis of a unit root, when the underlying series is stationary with a break in either its mean or trend, fell as the size of the break increased. By contrast, Franses and Haldrup (1994) showed that the size of the ADF test was sensitive to the presence of outliers. In particular, they showed that at a nominal 5% significance level, the empirical size of the ADF test increases with an increase in either (i) the frequency of outliers, or (ii) the size of the outlier, concluding that "the implications of this type of data irregularities will tend to produce spurious stationarity" (p. 476).

Developing the idea of Perron (1989), Chen (2002) investigated the behaviour of the KPSS test in the presence of permanent changes in the intercept and/or trend of a stationary series. The author found that the KPSS test had power to reject the null hypothesis of level and trend stationarity of a series in the presence of these breaks.

In this paper we investigate the performance of the KPSS tests in the presence of outliers. We find that the power of these tests to reject the null hypothesis of stationarity falls when the underlying series has a unit root with outliers and this effect is greater the more persistent are the outliers. We find that the location of the outlier within the sample can have markedly differing effects on the empirical size of the KPSS tests.

2. Outlier models and the KPSS test

Following KPSS we write out the model as:

$$y_t^* = \beta t + r_t + \phi y_{t-1}^* + \varepsilon_t \quad t = 1, \ldots, T \tag{1}$$
$$r_t = r_{t-1} + u_t$$

where ε_t are iid $(0, \sigma_\varepsilon^2)$ and $\sigma_\varepsilon^2 = 1$, u_t are iid $(0, \sigma_u^2)$ and $|\phi| < 1$. If $\sigma_u^2 = 0$ (assuming $|\phi| < 1$) and the initial value of r_0 is fixed, then y_t^* is a trend stationary process. If $\beta = 0$, the process is stationary around its mean (r_0) rather than around a trend. For $\sigma_u^2 > 0$ the process has a unit root.

To allow for breaks in the underlying process, y_t, and following the approach of Chen and Liu (1993), we introduce the following equation:

$$y_t = y_t^* + \theta \left[\frac{\delta_t}{(1 - \rho L)} \right] \tag{2}$$

where δ_t is an indicator function for the occurrence of an outlier (of size θ) and L is a lag operator. Of the four outlier models considered by Chen and Liu (1993), we look at two cases: (i) the additive outlier (AO) model, $\rho = 0$, and (ii) the temporary change (TC) model, $0 < |\rho| < 1$.

The KPSS test for stationarity is constructed as:

$$\eta_j = \frac{T^{-2} \sum_{t=1}^{T} S_t^2}{s_T^2(l)} \quad j = 1, 2 \tag{3}$$

where S_t is the partial sum of residuals (e_t) from a regression of y_t on an intercept, for $j=1$, or a regression of y_t on an intercept and trend for $j=2$, according to whether one is testing for level or trend stationarity, and $s_T^2(l)$ is an estimate of the error variance from the appropriate regression. KPSS use an estimator of the form:

$$s_T^2(l) = T^{-1} \sum_{t=1}^{T} e_t^2 + 2T^{-1} \sum_{s=1}^{l} w(s,l) \sum_{t=s+1}^{T} e_t e_{t-s} \tag{4}$$

where, $w(s,l)$ is the weighting function corresponding to the choice of the spectral window, and KPSS use the Bartlett window $w(s,l) = 1 - s/(l+1)$, as in Newey and West (1987), where l is the lag truncation parameter. In the experiments that follow we take $l = \text{integer}\left[12(T/100)^{1/4} \right]$.

3. Monte Carlo simulations

In the Monte Carlo simulations we consider a range of experiments from equations (1) and (2). To investigate the empirical size of the two KPSS tests (η_1 and

η_2), we assume we have a level stationary process in equation (1), with $\beta = 0$, $\sigma_u^2 = 0$ and $\phi = (0.0, \pm 0.2, \pm 0.5, \pm 0.8)$, whereas to investigate power we assume we have a nonstationary process in equation (1), with $\beta = 0$, $\phi = 0.0$, $\sigma_u^2 > 0$, and $\lambda = \dfrac{\sigma_u^2}{\sigma_\varepsilon^2} = (0.0001, 0.001, 0.1, 1, 10, 100, 1000)$, where $\lambda = \infty$ corresponds to a pure random walk in y_t^*. We consider a range of sample sizes, $T = (30, 50, 100, 200, 500)$. These parameter values are similar to those used by KPSS.

In both investigating the size and power of the KPSS tests, we consider two cases for the generation of δ_t in equation (2). Following Franses and Haldrup (1994), we take δ_t as a Bernoulli variable taking the value 1 or -1 with the specified probability, $\pi = (0.05, 0.10)$, and zero otherwise. In this case the outliers are assumed to be uniformly distributed over the entire sample, therefore in order to investigate the effect of the location of the outlier on the performance of these tests, we define $\delta_t = \begin{cases} 1 & t = \tau T \\ 0 & \text{otherwise} \end{cases}$, $0 < \tau < 1$ and we take $\tau = (0.1, 0.2, \ldots, 0.9)$. For both types of outliers the persistence of the outliers is controlled by the value of the parameter ρ, in equation (2), and we take $\rho = (0.0, 0.3, 0.5, 0.7, 0.9)$. The magnitude of the outliers is determined by θ, which we take as $\theta = (0, 3, 4, 5, 10)$. All results are based on 10,000 replications.

4. Results of the Monte Carlo experiments

Table 1 reports the results from those experiments when the outliers, δ_t, are generated as a Bernoulli variable with $\pi = 0.1$. We will refer to the results for $\pi = 0.05$ only in passing.

For the test of level stationarity, η_1, our results for $\theta = 0$, are very similar to those reported by KPSS (Table 3, p.171), with the empirical size of the test too large for $\phi > 0$ and too small for $\phi < 0$. Introducing outliers which are purely additive (having no persistence, ρ=0) we find that the empirical size of the test falls (rises) as the magnitude of the outliers, θ, increases for $\phi \geq 0$ ($\phi < 0$), compared to those results reported for $\theta = 0$; for example, for T=100 and $\phi = 0.8$ size falls from 9.2% for $\theta = 0$ to 6.14% for $\theta = 10$, whereas it increases from 1.06% to 2.72% for $\phi = -0.8$.

An increase in the persistence of the outliers, ρ, has only a limited effect on empirical size of the test, for $\rho \leq 0.7$. However, for a relatively persistent outlier, $\rho = 0.9$, but which is not a permanent shift in the intercept (results for $\rho = 1$ are reported in Chen (2002)), the empirical size of the test increases markedly above the nominal 5% level, such that for T=200 and $\theta = 4$ the empirical size is around 18% irrespective of ϕ. Suggesting that the KPSS can be quite sensitive to persistent outliers.

The results for the test of trend stationarity, η_2, are very similar in nature to those discussed above for the η_1 test. The results for outliers which occur with probability, $\pi = 0.05$, are similar to those reported in Table 1, although the influence of the outliers on the size of the tests are slightly reduced.

Table 2 reports the results of these Bernoulli outliers on the power of the η_1 and η_2 tests when there is a unit root in underlying process, y_t. The results for $\theta = 0$ are very similar to those reported in KPSS (Table 4, p.172). The results in Table 2 show that the effect of the outliers on the power of the test is minimal for $\lambda \geq 100$, irrespective of the magnitude of the outliers, θ, or their persistence, ρ, due to the dominant nature of the nonstationary component. For $\lambda = 0.0001$ the effect of outliers on the power of the test is again minimal, although this is because for T<200, the test has almost no power even when $\theta = 0$. For $0.001 \leq \lambda \leq 1$, power falls markedly in the presence of an outlier and the fall in power increases as the magnitude of the outliers, θ, increases. We also note that the reduction in power is greater as the persistence of the outliers, ρ, increases up to 0.7. The power results for $\rho = 0.9$ over-estimate the true power of the test, due to the large empirical sizes observed in Table 1.

These findings are consistent with those observed in Franses and Haldrup (1994, p.476), as the presence of outliers leaves size only slightly affected (for $\rho < 0.9$), but lowers the power of the tests when the underlying process is nonstationary and so tends "to produce spurious stationarity".

In Tables 1 and 2 the outliers are uniformly distributed throughout the sample. In Table 3 we report the results of the experiments when the outlier occurs at the point τT, in order to investigate the sensitivity of the tests to the location of the outlier. In Table 3 we only report results for T=100 and T=200 and $\phi = 0.0$, additionally as the results of the η_1 and η_2 tests were approximately symmetric around $\tau = 0.5$, we only

report the results for $\tau = (0.1, 0.2, 0.3, 0.4, 0.5)$. Strikingly the empirical size of the η_1 test falls towards zero as τ increases. In the presence of persistence in the outlier, $\rho > 0$, so the empirical size of the test falls even faster towards zero as τ increases, such that we are almost never rejecting the true null hypothesis of level stationarity. For $\rho = 0.9$ there is evidence to suggest that the empirical size of the test, η_1, becomes too big implying that we reject stationarity too frequently.

The test for trend stationarity, η_2, exhibits similar problems to that of the η_1 test, with the empirical size of the test approaching zero as τ increases. However, for this test there is greater evidence of the increase in the empirical size of the test for $\rho = 0.9$.

The power results (not reported) show that for $0.001 \le \lambda \le 1$, power falls with an increase in the magnitude of the outlier and that this effect is greater as τ increases. However, these power results under estimate the true power of these tests, given the substantial under-size problems reported in Table 3.

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Table 1: Sensitivity of the size of the KPSS tests to outliers, $\pi = 0.1$, 5% level

		T=50						T=100					
		η_1			η_2			η_1			η_2		
ϕ	ρ	$\theta=0$	$\theta=4$	$\theta=10$	$\theta=0$	$\theta=4$	$\theta=10$	$\theta=0$	$\theta=4$	$\theta=10$	$\theta=0$	$\theta=4$	$\theta=10$
0.8	0.0	5.12	4.54	3.32	7.37	6.90	5.69	9.22	8.39	6.14	9.66	9.14	6.97
0.5		2.25	2.05	1.56	5.75	5.06	4.44	4.89	3.92	3.21	4.99	4.51	4.13
0.2		1.55	1.58	1.27	5.17	4.63	4.07	4.01	3.13	2.94	3.78	3.93	3.77
0.0		1.28	1.39	1.21	4.86	4.36	3.99	3.40	2.84	2.86	3.47	3.61	3.69
-0.2		1.05	1.22	1.20	4.37	4.17	4.06	3.09	2.84	2.86	3.02	3.52	3.60
-0.5		0.72	1.07	1.22	3.71	4.03	4.05	2.50	2.70	2.79	2.38	3.41	3.54
-0.8		0.35	0.90	1.16	3.50	3.39	3.86	1.06	2.29	2.72	1.17	2.91	3.44
0.8	0.7	5.12	4.48	3.96	7.37	6.21	5.54	9.22	7.45	5.93	9.66	8.26	7.72
0.5		2.25	3.26	3.74	5.75	5.24	5.59	4.89	5.12	5.64	4.99	6.60	6.94
0.2		1.55	3.38	3.75	5.17	5.29	5.66	4.01	5.30	5.67	3.78	6.76	6.94
0.0		1.28	3.45	3.77	4.86	5.46	5.61	3.40	5.51	5.62	3.47	6.82	7.01
-0.2		1.05	3.49	3.84	4.37	5.48	5.62	3.09	5.56	5.62	3.02	6.76	6.94
-0.5		0.72	3.54	3.78	3.71	5.51	5.74	2.50	5.56	5.64	2.38	6.67	7.01
-0.8		0.35	3.47	3.73	3.50	5.13	5.71	1.06	5.51	5.61	1.17	6.68	6.95
0.8	0.9	5.12	9.55	10.85	7.37	9.29	10.32	9.22	15.31	16.93	9.66	16.50	18.29
0.5		2.25	10.20	11.25	5.75	9.75	10.60	4.89	16.44	17.31	4.99	17.75	18.92
0.2		1.55	10.72	11.42	5.17	10.16	10.88	4.01	16.98	17.57	3.78	18.54	18.95
0.0		1.28	10.96	11.36	4.86	10.35	10.94	3.40	17.15	17.60	3.47	18.64	18.86
-0.2		1.05	11.08	11.41	4.37	10.58	10.99	3.09	17.21	17.53	3.02	18.63	18.84
-0.5		0.72	11.19	11.42	3.71	10.63	10.96	2.50	17.34	17.64	2.38	18.69	18.94
-0.8		0.35	11.12	11.39	3.50	10.43	10.94	1.06	17.34	17.59	1.17	18.63	19.00

		T=200						T=500					
		η_1			η_2			η_1			η_2		
ϕ	ρ	$\theta=0$	$\theta=4$	$\theta=10$	$\theta=0$	$\theta=4$	$\theta=10$	$\theta=0$	$\theta=4$	$\theta=10$	$\theta=0$	$\theta=4$	$\theta=10$
0.8	0.0	9.55	8.40	6.73	10.73	10.10	7.99	9.09	8.83	7.62	11.28	10.93	8.98
0.5		5.01	4.42	4.13	5.53	4.92	4.36	5.70	5.43	4.75	6.21	6.10	5.38
0.2		4.22	3.99	3.90	4.43	4.04	4.08	4.93	4.76	4.49	5.36	5.29	4.86
0.0		3.92	3.81	3.87	4.01	3.84	4.05	4.61	4.59	4.59	4.92	5.09	4.81
-0.2		3.65	3.78	3.79	3.65	3.71	4.03	4.26	4.47	4.57	4.49	4.83	4.82
-0.5		3.05	3.57	3.78	2.99	3.64	3.95	3.66	4.27	4.51	3.78	4.65	4.74
-0.8		1.48	3.19	3.68	1.54	3.24	3.86	2.26	3.90	4.39	2.18	4.19	4.65
0.8	0.7	9.55	7.90	7.08	10.73	9.66	8.51	9.09	8.36	7.46	11.28	10.54	8.99
0.5		5.01	6.34	6.68	5.53	7.57	7.69	5.70	6.90	6.79	6.21	7.97	8.38
0.2		4.22	6.55	6.77	4.43	7.43	7.66	4.93	6.69	6.95	5.36	8.09	8.39
0.0		3.92	6.49	6.86	4.01	7.51	7.44	4.61	6.77	6.90	4.92	8.25	8.37
-0.2		3.65	6.53	6.88	3.65	7.65	7.48	4.26	6.80	6.80	4.49	8.26	8.40
-0.5		3.05	6.59	6.79	2.99	7.57	7.57	3.66	6.91	6.76	3.78	8.29	8.39
-0.8		1.48	6.51	6.71	1.54	7.43	7.53	2.26	6.85	6.77	2.18	8.28	8.33
0.8	0.9	9.55	16.19	17.69	10.73	20.42	22.50	9.09	15.00	16.24	11.28	20.93	22.78
0.5		5.01	17.13	17.85	5.53	21.88	22.72	5.70	16.05	16.46	6.21	22.33	22.87
0.2		4.22	17.63	17.90	4.43	22.31	22.79	4.93	16.31	16.57	5.36	22.67	22.80
0.0		3.92	17.70	17.85	4.01	22.60	22.77	4.61	16.35	16.57	4.92	22.79	22.77
-0.2		3.65	17.72	17.80	3.65	22.62	22.82	4.26	16.41	16.62	4.49	22.74	22.78
-0.5		3.05	17.82	17.78	2.99	22.73	22.82	3.66	16.54	16.60	3.78	22.71	22.81
-0.8		1.48	17.80	17.79	1.54	22.70	22.84	2.26	16.54	16.62	2.18	22.67	22.83

Table 2: Power of the KPSS tests to a unit root with outliers, π = 0.1, 5% level

			η_1			η_2			η_1			η_2	
			T=50						T=100				
λ	ρ	θ=0	θ=4	θ=10	θ=0	θ=4	θ=10	θ=0	θ=4	θ=10	θ=0	θ=4	θ=10
0.0001	0.0	1.41	1.34	1.23	5.00	4.39	4.03	4.09	3.21	2.94	3.69	3.68	3.77
0.001		1.97	1.45	1.32	5.13	4.39	4.07	10.61	5.55	3.55	5.11	4.21	3.83
0.01		8.72	4.43	1.94	6.53	4.93	4.33	38.09	25.23	9.70	17.09	9.87	5.20
0.1		26.58	19.95	8.53	13.15	9.65	6.00	55.51	50.94	37.49	36.03	29.31	15.99
1		33.63	31.82	26.54	17.22	16.25	12.74	58.39	57.74	54.87	40.67	39.25	35.08
100		34.52	34.51	34.51	17.73	17.64	17.65	58.84	58.82	58.82	41.19	41.22	41.15
10000		34.51	34.52	34.56	17.66	17.66	17.64	58.88	58.87	58.88	41.16	41.17	41.18
0.0001	0.7	1.41	3.49	3.80	5.00	5.39	5.60	4.09	5.56	5.68	3.69	6.80	7.02
0.001		1.97	3.59	3.81	5.13	5.42	5.57	10.61	6.03	5.78	5.11	6.84	6.90
0.01		8.72	4.49	4.00	6.53	5.67	5.62	38.09	11.49	6.80	17.09	8.29	7.23
0.1		26.58	10.97	5.41	13.15	7.36	6.06	55.51	36.08	15.69	36.03	17.50	9.45
1		33.63	26.87	14.95	17.22	13.01	8.58	58.39	54.44	42.35	40.67	34.63	21.94
100		34.52	34.44	33.79	17.73	17.77	17.70	58.84	58.83	58.40	41.19	41.12	40.44
10000		34.51	34.58	34.59	17.66	17.62	17.55	58.88	58.87	58.87	41.16	41.20	41.20
0.0001	0.9	1.41	7.06	7.29	5.00	8.58	9.04	4.09	14.72	14.88	3.69	16.81	17.32
0.001		1.97	7.11	7.29	5.13	8.70	9.07	10.61	15.05	15.07	5.11	16.86	17.28
0.01		8.72	7.35	7.37	6.53	8.81	9.11	38.09	16.46	15.38	17.09	17.19	17.38
0.1		26.58	10.46	8.16	13.15	9.35	9.35	55.51	26.89	17.63	36.03	20.32	17.98
1		33.63	22.87	12.81	17.22	12.78	9.87	58.39	48.73	31.51	40.67	31.36	22.13
100		34.52	34.13	32.99	17.73	17.82	17.23	58.84	58.39	57.70	41.19	40.84	40.22
10000		34.51	34.61	34.65	17.66	17.65	17.64	58.88	58.87	58.81	41.16	41.18	41.11

			η_1			η_2			η_1			η_2	
			T=200						T=500				
λ	ρ	θ=0	θ=4	θ=10	θ=0	θ=4	θ=10	θ=0	θ=4	θ=10	θ=0	θ=4	θ=10
0.0001	0.0	7.57	5.15	4.00	5.05	4.53	4.15	27.38	14.94	6.78	11.78	7.71	5.53
0.001		31.27	17.71	7.04	13.46	8.00	5.14	67.53	52.98	27.09	50.99	30.00	11.84
0.01		62.71	52.67	29.95	44.79	30.74	12.61	86.57	81.60	66.93	84.37	75.24	48.96
0.1		70.98	69.41	61.42	63.44	58.72	43.83	89.43	88.92	85.87	90.39	89.17	83.39
1		72.38	72.01	71.09	66.40	65.78	62.90	89.68	89.51	89.32	90.99	90.79	90.05
100		72.36	72.38	72.46	66.83	66.89	66.90	89.75	89.74	89.76	90.95	90.93	90.90
10000		72.40	72.40	72.38	66.86	66.88	66.91	89.75	89.75	89.76	90.96	90.96	90.95
0.0001	0.7	7.57	6.61	6.61	5.05	7.70	7.67	27.38	8.42	7.25	11.78	8.79	8.38
0.001		31.27	8.89	6.81	13.46	8.41	7.95	67.53	23.25	9.94	50.99	13.06	9.28
0.01		62.71	27.50	10.78	44.79	14.25	9.18	86.57	62.33	31.24	84.37	43.64	16.76
0.1		70.98	59.65	35.65	63.44	41.82	18.49	89.43	84.60	69.76	90.39	81.02	54.62
1		72.38	70.57	63.55	66.40	62.21	49.11	89.68	89.13	86.47	90.99	89.84	84.56
100		72.36	72.41	72.27	66.83	66.88	66.48	89.75	89.75	89.73	90.95	90.89	90.83
10000		72.40	72.38	72.37	66.86	66.88	66.89	89.75	89.76	89.78	90.96	90.94	90.90
0.0001	0.9	7.57	16.39	16.64	5.05	21.58	21.93	27.38	16.31	16.35	11.78	22.42	22.39
0.001		31.27	16.91	16.61	13.46	21.71	21.85	67.53	19.27	16.66	50.99	23.52	22.74
0.01		62.71	21.75	17.22	44.79	23.50	22.20	86.57	38.82	21.35	84.37	31.18	24.34
0.1		70.98	44.32	24.93	63.44	33.35	25.16	89.43	72.76	46.33	90.39	63.30	36.35
1		72.38	66.05	50.50	66.40	56.05	37.89	89.68	87.07	77.49	90.99	86.41	70.84
100		72.36	72.26	72.01	66.83	66.64	65.60	89.75	89.68	89.50	90.95	90.86	90.62
10000		72.40	72.37	72.43	66.86	66.89	66.92	89.75	89.75	89.76	90.96	90.93	90.93

Table 3: Sensitivity of the size of the KPSS tests to outliers, ϕ=0.0, 5% level

ρ	τ	T=100 η_1			T=100 η_2			T=200 η_1			T=200 η_2		
		θ=0	θ=4	θ=10	θ=0	θ=4	θ=10	θ=0	θ=4	θ=10	θ=0	θ=4	θ=10
0.0	0.1	3.40	3.89	4.49	3.47	3.34	3.16	3.92	4.74	5.92	4.01	4.39	4.26
0.3		3.40	4.49	4.15	3.47	3.49	2.57	3.92	5.27	6.29	4.01	4.39	3.98
0.5		3.40	5.24	3.25	3.47	3.72	1.38	3.92	6.05	6.36	4.01	4.58	3.25
0.7		3.40	6.69	2.04	3.47	3.30	0.32	3.92	7.63	5.70	4.01	4.94	1.38
0.9		3.40	26.39	27.69	3.47	0.50	0.00	3.92	26.76	33.52	4.01	4.54	0.08
0.0	0.2	3.40	3.54	2.57	3.47	2.23	0.47	3.92	4.20	4.15	4.01	3.86	1.91
0.3		3.40	3.50	1.66	3.47	1.55	0.06	3.92	4.43	3.75	4.01	3.35	0.90
0.5		3.40	3.75	0.88	3.47	0.88	0.00	3.92	4.52	2.91	4.01	2.69	0.23
0.7		3.40	3.21	0.09	3.47	0.23	0.00	3.92	4.68	1.29	4.01	1.38	0.01
0.9		3.40	1.98	0.00	3.47	0.01	0.00	3.92	6.87	0.36	4.01	0.05	0.00
0.0	0.3	3.40	3.23	1.45	3.47	2.24	0.45	3.92	3.86	2.70	4.01	3.82	1.90
0.3		3.40	2.75	0.52	3.47	1.85	0.11	3.92	3.60	1.85	4.01	3.48	1.00
0.5		3.40	2.26	0.09	3.47	1.46	0.01	3.92	3.27	1.00	4.01	2.86	0.50
0.7		3.40	1.31	0.00	3.47	0.82	0.00	3.92	2.57	0.16	4.01	2.03	0.11
0.9		3.40	0.02	0.00	3.47	4.93	0.06	3.92	0.82	0.00	4.01	5.57	0.22
0.0	0.4	3.40	2.51	0.55	3.47	3.03	2.13	3.92	3.55	1.79	4.01	4.70	4.53
0.3		3.40	2.07	0.14	3.47	3.18	1.49	3.92	3.16	0.79	4.01	4.86	4.17
0.5		3.40	1.38	0.01	3.47	3.40	0.77	3.92	2.57	0.22	4.01	5.12	3.51
0.7		3.40	0.39	0.00	3.47	3.96	0.33	3.92	1.39	0.00	4.01	6.15	2.32
0.9		3.40	0.00	0.00	3.47	25.10	19.25	3.92	0.03	0.00	4.01	26.83	31.61
0.0	0.5	3.40	2.47	0.38	3.47	3.79	3.48	3.92	3.40	1.48	4.01	4.89	5.65
0.3		3.40	1.87	0.04	3.47	3.99	2.76	3.92	2.94	0.52	4.01	5.26	5.82
0.5		3.40	0.99	0.00	3.47	4.50	2.02	3.92	2.21	0.07	4.01	6.00	5.48
0.7		3.40	0.22	0.00	3.47	5.83	0.82	3.92	1.04	0.00	4.01	7.95	4.64
0.9		3.40	0.01	0.00	3.47	14.90	4.16	3.92	0.02	0.00	4.01	29.47	37.29

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