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Summary

We make four main contributions to the theory of network formation. (1) The problem of network formation with farsighted agents can be formulated as an abstract network formation game. (2) In any farsighted network formation game the feasible set of networks contains a unique, finite, disjoint collection of nonempty subsets having the property that each subset forms a strategic basin of attraction. These basins of attraction contain all the networks that are likely to emerge and persist if individuals behave farsightedly in playing the network formation game. (3) A von Neumann Morgenstern stable set of the farsighted network formation game is constructed by selecting one network from each basin of attraction. We refer to any such von Neumann-Morgenstern stable set as a farsighted basis. (4) The core of the farsighted network formation game is constructed by selecting one network from each basin of attraction containing a single network. We call this notion of the core, the farsighted core. We conclude that the farsighted core is nonempty if and only if there exists at least one farsighted basin of attraction containing a single network. To relate our three equilibrium and stability notions (basins of attraction, farsighted basis, and farsighted core) to recent work by Jackson and Wolinsky (1996), we define a notion of pairwise stability similar to the Jackson-Wolinsky notion and we show that the farsighted core is contained in the set of pairwise stable networks. Finally, we introduce, via an example, competitive contracting networks and highlight how the analysis of these networks requires the new features of our network formation model.

Keywords: Basins of attraction, Network formation, Supernetworks, Farsighted core, Nash networks

JEL Classification: A14, D20, J00

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1 Introduction

Overview

In many economic and social situations, the totality of interactions between individuals and coalitions can be modeled as a network. The question we address in this paper is the following: if individuals are concerned with the long run consequences of their immediate actions in forming networks with other individuals, that is, if individuals are farsighted in choosing their network formation strategies, what networks are likely to emerge and persist? One possible approach to this question is to think of each possible network representation of individual connections and interactions as a node in a larger network, called a supernetwork, in which the arcs represent coalitional preferences over networks and possible coalitional moves from one network to another.[1] Given the supernetwork representation of agent preferences and the rules governing network formation, it is then possible to define a farsighted dominance relation over the networks composing the nodes of the supernetwork and to address the issue of farsighted equilibrium and stability in network formation. Our first contribution is a model of network formation with farsighted agents as an abstract game with respect to farsighted dominance.[2] In this game, starting from any status quo network the supernetwork plays the role of a global constraint set (or an effectivity network), specifying which networks can be formed by coalitions, as well as which networks are farsightedly preferred by coalitions.

Using the farsighted network formation game induced by the supernetwork as our basic analytic tool, we make three additional contributions to the theory of network formation. First, we demonstrate that in any farsighted network formation game the feasible set of networks contains a unique, finite, disjoint collection of nonempty subsets having the property that each set of networks in the collection forms a *basin of attraction* in the farsighted network formation game. These farsighted basins of attraction contain all the networks that are likely to emerge and persist if individuals behave farsightedly in choosing their network formation strategies. Second, we show that by selecting one network from each basin of attraction, we construct a von Neumann Morgenstern stable set of the farsighted network formation game. Thus, we show that given any set of rules governing network formation and given any profile of individual preferences over the feasible set of networks (i.e., given any supernetwork), the corresponding farsighted network formation game possesses a von Neumann-Morgenstern stable set of networks. We refer to any such von Neumann-Morgenstern stable set as a *farsighted basis* of the network formation game and we

[1] The supernetwork approach to network formation is introduced in Page, Wooders, and Kamat (2001).

[2] An abstract game in the sense of von Neumann and Morgenstern (1953) (also, see Roth (1976)) consists of a pair $(D, >)$ where D is a set of outcomes and $>$ is an ordering defined on D.

refer to any network contained in a farsighted basis as a *farsightedly basic network*. Finally, we show that by selecting one network from each basin of attraction containing a *single* network, we construct the core of the farsighted network formation game - that is, we construct a set of networks having the property that no network in the set is farsightedly dominated by any other network in the supernetwork. Thus, at each network in the farsighted core no agent or coalition of agents has an incentive to alter the given network. We call this notion of the core, the *farsighted core*. Given the way in which the farsighted core is constructed, we conclude that the farsighted core is contained in each farsighted basis of the network formation game and that the farsighted core is nonempty if and only if there is a farsighted basin of attraction containing a single network.[3]

We illustrate our three notions of equilibrium and stability in network formation games (i.e., basins of attraction, farsighted bases, and the farsighted core) via an example of strategic competitive contracting. In particular, we introduce a competitive contracting network in which farsighted firms compete to contract with a single, privately informed agent. Each contracting network in the supernetwork corresponds to a unique profile of contracting strategies. In our first example, the farsighted network formation game over contracting networks has one basin of attraction (with respect to farsighted dominance) consisting of a single contracting network. This single contracting network constitutes the farsighted core of the network formation game and thus identifies a unique profile of contracting strategies which is likely emerge and persist if firms behave farsightedly. In our second example, there is again one basin of attraction, but this time consisting of multiple contracting networks. Thus, the farsighted core is empty. However, each contracting network contained in this single basin of attraction is a farsightedly basic network, and each of these basic networks taken as a singleton set constitutes a farsighted basis (i.e., a von Neumann-Morgenstern stable set) of the farsighted network formation game. Taken together the contracting networks contained in this single basin of attraction identify a set of contracting strategy profiles each of which is likely to emerge and persist if firms behave farsightedly.

Directed Networks vs Linking Networks

We focus on directed networks, and in particular, on the extended notion of directed networks introduced in Page, Wooders, and Kamat (2001). In a directed network, each arc possesses an orientation or direction: arc j connecting nodes i and i' must either go from node i to node i' or must go from node i' to node i.[4] For example, an individual may have a link on his web page to the web pages of all Nobel Laureates in economics but no Nobel Laureate may have a link to the individual's web page. In an undirected (or linking) network, an arc (or a link) is identified by a nonempty subset of nodes consisting of exactly two distinct nodes, for example $\{i, i'\}$, $i \neq i'$. Thus, in an undirected network, a link has no orientation and would

[3] Put differently, the farsighted core is empty if and only if all basins of attraction contain multiple networks. This equivalency holds because each pair of networks contained in a basin consisting multiple networks must lie on the same circuit (or cycle) with respect to farsighted dominance.

[4] We denote arc j going from node i to node i' via the ordered pair $(j, (i, i'))$, where (i, i') is also an ordered pair. Alternatively, if arc j goes from node i' to node i, we write $(j, (i', i))$.

simply indicate a connection between nodes i and i'. Moreover, in an undirected network links are not distinguished by type - that is, links are homogeneous. Under our extended definition of a directed network connections between nodes (i.e., arcs), besides having an orientation, are allowed to be heterogeneous. For example, if the nodes in a given network represent agents, an arc j going from agent i to agent i' might represent a particular type and intensity of interaction (identified by the arc label j) initiated by agent i towards agent i'. For example, agent i might direct great affection toward agent $i^{'}$ as represented by arc j, but agent $i^{'}$ may direct only lukewarm affection toward agent i as represented by arc $j^{'}$. Also, under our extended definition nodes are allowed to be connected by multiple, distinct arcs. Thus, we allow nodes to interact in multiple, distinct ways. For example, nodes i and i' might be connected by arcs j and j', with arc j running from node i to i' and arc j' running in the opposite direction (i.e., from node i' to node i).[5] If node i represents a seller and node i' a buyer, then arc j might represent a contract offer by the seller to the buyer, while arc $j^{'}$ might represent the acceptance or rejection of that contract offer. Finally, under our extended definition loops are allowed and arcs are allowed to be used multiple times in a given network.[6] For example, arc j might be used to connect nodes i and i' as well as nodes i' and i''. Thus, under our definition nodes i and i' as well as nodes i' and i'' are allowed to engage in the same type of interaction as represented by arc j. Allowing each type of arc to be used multiple times makes it possible to distinguish coalitions by the type of interaction taking place between coalition members and to give a network representation of such coalitions. For example, if the nodes in a given network represent agents, a j coalition would consist of all agents i having a j connection with at least one other agent $i^{'}$. Such a j coalition would then have a network representation as the directed subnetwork consisting of pairs of nodes, i and $i^{'}$, connected by a j arc. Until now, most of the economic literature on networks has focused on linking networks (see Jackson (2001) for an excellent survey). By allowing arcs to possess direction and be used multiple times and by allowing loops and nodes to be connected by multiple arcs, our extended definition makes possible the application of networks to a richer set of economic environments.

Given a particular directed network, an agent or a coalition of agents can change the network to another network by simply adding, subtracting, or replacing arcs from the existing network in accordance with certain rules represented via the supernetwork.[7] For example, suppose the nodes in a network represent agents and the rule for adding an arc j from node i to node i' requires that both agents i and i' agree to add arc j. Suppose also the rule for subtracting arc j from node i to node i' requires that only agent i or agent i' agree to dissolve arc j. We refer to this particular set of rules as Jackson-Wolinsky rules (see Jackson and Wolinsky (1996)). Other rules are

[5] Under our extended definition, arc j' might also run in the same direction as arc j. However, our definition does not allow arc j to go from node i to node i' multiple times.

[6] A loop is an arc going *from* a given node *to* that same node. For example, given arc j and node i, the ordered pair $(j,(i,i))$ is a loop.

[7] Put differently, agents can change one network to another network by adding, subtracting, or replacing ordered pairs, $(j,(i,i'))$, in accordance with certain rules.

possible. For example, the addition of an arc might require that a simple majority of the agents agree to the addition, while the removal an arc might require that a two-thirds majority agree to the removal. Given the flexibility of the supernetwork framework, any set rules governing network formation can be represented.

In order to relate our approach to network formation to the seminal work by Jackson and Wolinsky (1996) we define a notion of pairwise stability for farsighted network formation games over directed networks similar to the notion of pairwise stability introduced by Jackson and Wolinsky (1996) for myopic network formation games over linking networks. We show that in any farsighted network formation game induced by any rules of network formation, including the Jackson-Wolinsky (1996) rules, the farsighted core is a subset of the set of pairwise stable networks.[8] Thus, nonemptiness of the farsighted core implies nonemptiness of the set of pairwise stable networks and, given our result on the equivalence of nonemptiness of the farsighted core and the existence of at least one strategic basin of attraction containing a single network, the existence of a pairwise stable network is guaranteed by the existence of at least one basin of attraction containing a single network. This result can be viewed as an extension of a result due Jackson and Watts (2002) on the existence of pairwise stable linking networks for myopic network formation games induced by Jackson-Wolinsky rules (i.e., by Jackson-Wolinsky supernetworks). In particular, Jackson and Watts (2002) consider myopic improving paths through the supernetwork (our terminology) further assuming that moves from one network to another take place one link at a time. They show that for any myopic network formation game induced by such a supernetwork there exists a pairwise stable network if and only if there does not exist a closed cycle of networks. Specializing to myopic network formation games over linking networks induced by Jackson-Wolinsky supernetworks, our notion of a strategic basin of attraction containing *multiple* networks corresponds to their notion of a closed cycle of networks. Thus, stated in our terminology Jackson and Watts show that for myopic Jackson-Wolinsky network formation games, there exists a pairwise stable network if and only if there does not exist a strategic basin of attraction containing multiple networks. In fact, following our approach, if we specialize to myopic Jackson-Wolinsky network formation games (and strategic basins of attraction generated by myopic improving paths), then we can conclude that the existence of at least one strategic basin containing a single network is both necessary and sufficient for the existence of a pairwise stable network.

We also define a notion of strong stability for farsighted network formation games over directed networks similar to the strong stability notion of Jackson and van den Nouweland (2001) for network formation games over linking networks. We show that for any farsighted network formation game induced by any supernetwork, the farsighted core is a subset of the set of strongly stable networks. Thus, nonemptiness of the farsighted core implies nonemptiness of the set of strongly stable networks.

Finally, we show that in any farsighted network formation game, including those

[8] According to Jackson and Wolinsky, a network is pairwise stable if each pair of agents directly connected by an arc in the network weakly prefer to remain directly connected, and if for each pair of agents not directly connected, a direct connection preferred by one of the agents makes the other agent strictly worse off (i.e., if one agent prefers to be directly connected, the other does not).

induced by Jackson-Wolinsky supernetworks, each strategic basin of attraction has a nonempty intersection with the largest consistent set of networks (i.e., the Chwe set of networks, see Chwe (1994)).[9] Given the way in which the farsighted core is constructed from the basins of attraction, we conclude as an immediate corollary of this result that the farsighted core is a subset of the largest consistent set of networks. This corollary together with our result on the farsighted core and strong stability imply that any network contained in the farsighted core is not only farsightedly consistent but also strongly stable.

[9] Consistency with respect to farsighted dominance and the notion of a largest consistent set were introduced by Chwe (1994) in an abstract game setting. We provide a detailed discussion of Chwe's notion in Section 5.3.

2 Directed Networks

2.1 The Extended Definition

We begin by giving the formal definition of a directed network introduced in Page, Wooders, and Kamat (2001). Let N be a finite set of nodes, with typical element denoted by i, and let A be a finite set of arcs, with typical element denoted by j. Arcs represent potential connections between nodes, and depending on the application, nodes can represent economic agents or economic objects such as markets or firms.[10]

Definition 1 *(Directed Networks)*
Given node set N and arc set A, a directed network, G, is a nonempty subset of $A \times (N \times N)$.

The collection of all directed networks given N and A is given by $P(A \times (N \times N))$, where $P(A \times (N \times N))$ denotes the collection of all *nonempty* subsets of $A \times (N \times N)$.

A directed network $G \in P(A \times (N \times N))$ specifies how the nodes in N are connected via the arcs in A. Note that in a directed network order matters. In particular, if $(j,(i,i')) \in G$, this means that arc j *goes from* node i *to* node i'. Also, note that under our definition of a directed network, loops are allowed - that is, we allow an arc to go from a given node back to that given node. Finally, note that under our definition an arc can be used multiple times in a given network and multiple arcs can go from one node to another. However, our definition does not allow an arc j to go from a node i to a node i' multiple times.

The following notation is useful in describing networks. Given directed network $G \in P(A \times (N \times N))$, let

$$\left.\begin{array}{c} G(j) := \left\{(i,i') \in N \times N : (j,(i,i')) \in G\right\}, \\ \text{and} \\ G(i) := \left\{ j \in A : (j,(i,i')) \in G \text{ or } (j,(i',i)) \in G \right\}. \end{array}\right\} \tag{1}$$

Thus,

$G(j)$ is the *set of node pairs* connected by arc j in network G, and
$G(i)$ is the *set of arcs* going from node i or coming to node i in network G.

Note that if for some arc $j \in A$, $G(j)$ is empty, then arc j is not used in network G. Moreover, if for some node $i \in N$, $G(i)$ is empty then node i is not used in network G, and node i is said to be isolated relative to network G.

[10] Of course in a supernetwork, nodes represent networks.

Suppose that the node set N is given by $N = \{i_1, i_2, \ldots, i_5\}$, while the arc set A is given by $A = \{j_1, j_2, \ldots, j_5, j_6, j_7\}$. Consider the network, G, depicted in Figure 1.

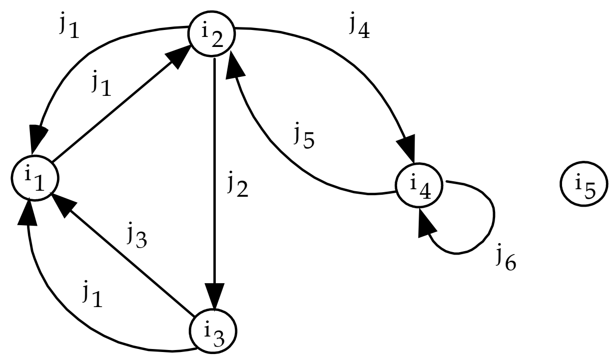


Figure 1: Network G

Note that in network G nodes i_1 and i_2 are connected by two j_1 arcs running in opposite directions and that nodes i_1 and i_3 are connected by two arcs, j_1 and j_3, running in the same directions from node i_3 to node i_1. Thus, $G(j_1) = \{(i_1, i_2), (i_2, i_1), (i_3, i_1)\}$ and $G(j_3) = \{(i_3, i_1)\}$. Observe that $(j_6, (i_4, i_4)) \in G$ is a loop. Thus, $G(j_6) = \{(i_4, i_4)\}$. Also, observe that arc j_7 is not used in network G. Thus, $G(j_7) = \emptyset$.[11] Finally, observe that $G(i_4) = \{j_4, j_5, j_6\}$, while $G(i_5) = \emptyset$. Thus, node i_5 is *isolated* relative to G.[12]

2.2 Linking Networks and Directed Graphs

Our extended notion of a directed network can be formally related to the notion of a linking network as follows. As before, let N denote a finite set of nodes. A linking network, say g, consists of a finite collection of subsets of the form $\{i, i'\}$, $i \neq i'$. For example, g might be given by $g = \{\{i, i'\},\{i', i''\}\}$ for i, i', and i'' in N. Note that in a linking network all connections or links are the same (i.e., connection types are homogeneous), direction does not matter, and loops are ruled out by definition. Letting g^N denote the collection of all subsets of N of size 2, the collection of all linking networks given N is given by $P(g^N)$ where, recall, $P(g^N)$ denotes the collection of all nonempty subsets of g^N (e.g., see the definition in Jackson and Wolinsky (1996)).

A directed graph, say E, consists of a finite collection of ordered pairs $(i, i') \in N \times N$. For example, E might be given by $E = \{(i, i'), (i', i')\}$ for (i, i') and (i', i')

[11] The fact that arc j_7 is not used in network G can also be denoted by writing

$$j_7 \notin proj_A G,$$

where $proj_A G$ denotes the projection onto A of the subset

$$G \subseteq A \times (N \times N)$$

representing the network.

[12] If the loop $(j_7, (i_5, i_5))$ were part of network G in Figure 1, then node i_5 would no longer be considered isolated under our definition. Moreover, we would have $G(i_5) = \{j_7\}$.

in $N \times N$. Stated more compactly, a directed graph E is simply a subset of $N \times N$. Thus, in any directed graph connection types are again homogeneous but direction does matter and loops are allowed.

Under our definition, a directed network G is a subset of $A \times (N \times N)$, where as before A is a finite set of arcs. Thus, in a directed network, say $G \in P(A \times (N \times N))$, connection types are allowed to be heterogeneous (distinguished by arc labels), direction matters, and loops are allowed.

3 Supernetworks

3.1 The Definition of a Supernetwork

Let D denote a nonempty set of agents (or economic decision making units) with typical element denoted by d, and let $P(D)$ denote the collection of all coalitions (i.e., nonempty subsets of D) with typical element denoted by S.

Given a feasible set of directed networks $\mathbb{G} \subseteq P(A \times (N \times N))$, we shall assume that each agent's preferences over networks in $\mathbb{G}$ are specified via a real-valued network payoff function,

$$v_d(\cdot) : \mathbb{G} \rightarrow R.$$

For each agent $d \in D$ and each directed network $G \in \mathbb{G}$, $v_d(G)$ is the payoff to agent d in network G. Agent d then prefers network G' to network G if and only if

$$v_d(G') > v_d(G).$$

Moreover, coalition $S' \in P(D)$ prefers network G' to network G if and only if

$$v_d(G') > v_d(G) \text{ for all } d \in S'.$$

Note that the payoff function of an agent depends on the entire network. Thus, the agent may be affected by directed links between other agents even when he himself has no direct or indirect connection with those agents. Intuitively, 'widespread' network externalities are allowed.

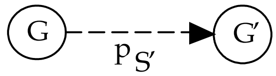
By viewing each network G in a given collection of directed networks $\mathbb{G} \subseteq P(A \times (N \times N))$ as a node in a larger network, we can give a precise network representation of the rules governing network formation as well as agents' preferences. To begin, let

$$\mathcal{M} := \{m_S : S \in P(D)\} \text{ denote the set of move arcs (or } m\text{-arcs for short),}$$
$$\mathcal{P} := \{p_S : S \in P(D)\} \text{ denote the set of preference arcs (or } p\text{-arcs for short),}$$
$$\text{and}$$
$$\mathcal{A} := \mathcal{M} \cup \mathcal{P}.$$

Given networks G and G' in $\mathbb{G}$, we shall denote by $(m_{S'}(G, G'))$ (i.e., by an m-arc, belonging to coalition S', going from node G to node G') the fact that coalition $S' \in P(D)$ can change network G to network G' by adding, subtracting, or replacing arcs in network G. Graphically, $(m_{S'}, (G, G'))$ is represented by

$$\text{G} \xrightarrow[\text{m}_{S'}]{} \text{G}'$$

Moreover, we shall denote by $(p_{S'}, (G, G'))$ (i.e., by a p-arc, belonging to coalition S', going from node G to node G') the fact that each agent in coalition $S' \in P(D)$ prefers network G' to network G. Graphically, $(p_{S'}, (G, G'))$ is represented by



Definition 2 *(Supernetworks)*

Given directed networks $\mathbb{G} \subseteq P(A \times (N \times N))$, *agent payoff functions* $\{v_d(\cdot) : d \in D\}$, *and arc set* $\mathcal{A} := \mathcal{M} \cup \mathcal{P}$, *a supernetwork,* $\mathbf{G}$, *is a nonempty subset of* $\mathcal{A} \times (\mathbb{G} \times \mathbb{G})$ *such that for all networks* G *and* G' *in* $\mathbb{G}$ *and for all coalition* $S' \in P(D)$,

$(m_{S'}, (G, G')) \in \mathbf{G}$ *if and only if coalition* S' *can change network* G *to network* G', $G' \neq G$, *by adding, subtracting, or replacing arcs in network* G,

and

$(p_{S'}, (G, G')) \in \mathbf{G}$ *if and only if* $v_d(G') > v_d(G)$ *for all* $d \in S'$.

Thus, a supernetwork $\mathbf{G}$ specifies how the networks in $\mathbb{G}$ are connected via coalitional moves and coalitional preferences - and thus provides a *network representation* of agent preferences and the rules governing network formation. Note that for all coalitions $S' \in P(D)$ and networks G and G' contained in $\mathbb{G}$, if $(p_{S'}, (G, G')) \in \mathbf{G}$, then $(p_S, (G, G')) \in \mathbf{G}$ for all subcoalitions S of S'.

Under our definition of a supernetwork, multiple m-arcs, as well as multiple p-arcs, connecting networks G and G' in supernetwork $\mathbf{G}$ are allowed. However, multiple m-arcs, or multiple p-arcs, from network $G \in \mathbf{G}$ to network $G' \in \mathbf{G}$ belonging to the *same* coalition are not allowed - and moreover, are unnecessary. Multiple m-arcs (not belonging to the same coalition) connecting networks G and G' in a given supernetwork $\mathbf{G}$ indicate that in supernetwork $\mathbf{G}$ there is more than one coalition capable of changing network G to network G'. At the other extreme, if network $G \in \mathbb{G}$ is such that no m-arcs or p-arcs go to or come from G, then network G cannot be changed and is said to be isolated relative to supernetwork $\mathbf{G}$.

Finally, it is important to note that in many economic applications, the set of nodes, N, used in defining the networks in the collection $\mathbb{G}$, and the set of economic agents D are one and the same (i.e., in many applications $N = D$). However, under our approach to network formation via supernetworks, it is not required that $N = D$.

3.2 The Farsighted Dominance Relation Induced by a Supernetwork

Given supernetwork $\mathbf{G} \subset \mathcal{A} \times (\mathbb{G} \times \mathbb{G})$, we say that network $G' \in \mathbb{G}$ farsightedly dominates network $G \in \mathbb{G}$ if there is a finite sequence of networks,

$$G_0, G_1, \ldots, G_h,$$

with $G = G_0$, $G' = G_h$, and $G_k \in \mathbb{G}$ for $k = 0, 1, \ldots, h$, and a corresponding sequence of coalitions,

$$S_1, S_2, \ldots, S_h,$$

such that for $k = 1, 2, \ldots, h$

$$(m_{S_k}, (G_{k-1}, G_k)) \in \mathbf{G},$$
$$\text{and}$$
$$(p_{S_k}, (G_{k-1}, G_h)) \in \mathbf{G}.$$

We shall denote by $G \lhd\lhd G'$ the fact that network $G' \in \mathbb{G}$ farsightedly dominates network $G \in \mathbb{G}$.

Figure 2 below provides a network representation of the farsighted dominance relation in terms of m-arcs and p-arcs. In Figure 2, network G_3 farsightedly dominates network G_0.

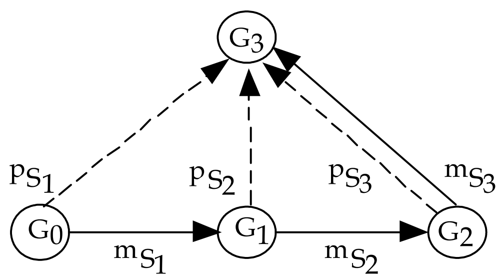


Figure 2: G_3farsightedly dominates G_0

Note that what matters to the initially deviating coalition S_1, as well as coalitions S_2 and S_3, is the ultimate network outcome G_3. Thus, the initially deviating coalition S_1 will not be deterred even if

$$(p_{S_1}, (G_0, G_1)) \notin \mathbf{G}$$

as long as the ultimate network outcome G_3 is preferred to G_0, that is, as long as G_3 is such that

$$(p_{S_1}, (G_0, G_3)) \in \mathbf{G}.$$

Finally, we say that a network $G \in \mathbb{G}$ is farsightedly isolated relative to supernetwork $\mathbf{G}$, if there does not exist a network $G' \in \mathbb{G}$ with $G' \rhd\rhd G$ or $G \rhd\rhd G'$. Note that if network G is isolated relative to $\mathbf{G}$, then it is farsightedly isolated relative to $\mathbf{G}$.

3.3 The Farsighted Domination Path Relation Induced by a Supernetwork

Given supernetwork $\mathbf{G} \subset \mathcal{A} \times (\mathbb{G} \times \mathbb{G})$, we say that a sequence of networks $\{G_k\}_k$ in $\mathbb{G}$ is a *farsighted domination path through supernetwork* $\mathbf{G}$ (i.e., a $\lhd\lhd$-path through $\mathbf{G}$) if for any two consecutive networks G_{k-1} and G_k, G_k farsightedly dominates G_{k-1}, that is, if for any two consecutive networks G_{k-1} and G_k,

$$G_{k-1} \lhd\lhd G_k.$$

We can think of the farsighted dominance relation $G_{k-1} \lhd\lhd G_k$ between networks G_k and G_{k-1} as defining a $\lhd\lhd$-arc *from* network G_{k-1} *to* network G_k. Given $\lhd\lhd$-path

$\{G_h\}_h$ through $\mathbf{G}$, the *length* of this path is defined to be the number of $\lhd\lhd$-arcs in the path. We say that network $G_1 \in \mathbb{G}$ is $\lhd\lhd$-*reachable* from network $G_0 \in \mathbb{G}$ in $\mathbf{G}$ if there exists a finite $\lhd\lhd$-path in $\mathbf{G}$ from G_0 to G_1 (i.e., a $\lhd\lhd$-path in $\mathbf{G}$ from G_0 to G_1 of finite length). If network $G_0 \in \mathbb{G}$ is $\lhd\lhd$-reachable from network G_0 in $\mathbf{G}$, then we say that supernetwork $\mathbf{G}$ contains a $\lhd\lhd$-*circuit*. Thus, a $\lhd\lhd$-circuit in $\mathbf{G}$ starting at network $G_0 \in \mathbb{G}$ is a finite $\lhd\lhd$-path from G_0 to G_0. A $\lhd\lhd$-circuit of length 1 is called a $\lhd\lhd$-loop. Note that because preferences are irreflexive, $\lhd\lhd$-loops are in fact ruled out. However, because the farsighted dominance relation, $\lhd\lhd$, is *not transitive*, it is possible to have $\lhd\lhd$-circuits of length greater than 1.

Given supernetwork $\mathbf{G} \subset \mathcal{A}\times(\mathbb{G}\times\mathbb{G})$, we can use the notion of $\lhd\lhd$-reachability to define a new relation on the set of networks $\mathbb{G}$. In particular, for any two networks G_0 and G_1 in $\mathbb{G}$ define

$$G_1 \trianglerighteq_{\mathbf{G}} G_0 \text{ if and only if } \begin{cases} G_1 \text{ is } \lhd\lhd \text{-reachable from } G_0 \text{ in supernetwork } \mathbf{G} \text{ , or} \\ G_1 = G_0. \end{cases} \tag{2}$$

The relation $\trianglerighteq_{\mathbf{G}}$ is a weak ordering on the set of networks $\mathbb{G}$. In particular, $\trianglerighteq_{\mathbf{G}}$ is reflexive ($G \trianglerighteq_{\mathbf{G}} G$) and $\trianglerighteq_{\mathbf{G}}$ is transitive ($G_2 \trianglerighteq_{\mathbf{G}} G_1$ and $G_1 \trianglerighteq_{\mathbf{G}} G_0$ implies that $G_2 \trianglerighteq_{\mathbf{G}} G_0$). We shall refer to the relation $\trianglerighteq_{\mathbf{G}}$ as the *farsighted domination path (FDP) relation induced by supernetwork* $\mathbf{G}$.[13]

4 Farsighted Network Formation Games

Given any collection of directed networks $\mathbb{G} \subseteq P(A\times(N\times N))$ and any supernetwork $\mathbf{G} \subseteq \mathcal{A}\times(\mathbb{G}\times\mathbb{G})$, where arc set $\mathcal{A}$ is the union of coalitional move arcs $\mathcal{M}$ and coalitional preference arcs $\mathcal{P}$, the corresponding farsighted network formation game is given by the pair

$$(\mathbb{G}, \trianglerighteq_{\mathbf{G}}),$$

where $\trianglerighteq_{\mathbf{G}}$ is the farsighted domination path (FDP) relation on $\mathbb{G}$ induced by supernetwork $\mathbf{G}$ (see expression (2)).

4.1 Descendance Relations, Maximal Networks, and Networks Without Descendants

If $G_1 \trianglerighteq_{\mathbf{G}} G_0$ and $G_0 \trianglerighteq_{\mathbf{G}} G_1$, we say that networks G_1 and G_0 are equivalent and we write $G_1 \equiv_{\mathbf{G}} G_0$. If networks G_1 and G_0 are equivalent this means that either networks G_1 and G_0 coincide or that G_1 and G_0 are on the same $\lhd\lhd$-circuit in supernetwork $\mathbf{G}$. If networks G_1 and G_0 are such that $G_1 \trianglerighteq_{\mathbf{G}} G_0$ but G_1 and G_0 are not equivalent (i.e., but not $G_1 \equiv_{\mathbf{G}} G_0$), we say that network G_1 is a *descendant* of network G_0 and we write

$$G_1 \vartriangleright_{\mathbf{G}} G_0. \tag{3}$$

[13] The relation $\trianglerighteq_{\mathbf{G}}$ is sometimes referred to as the transitive closure of the farsighted dominance relation, $\lhd\lhd$.

We say that a directed network $G' \in \mathbb{G}$ is *maximal in* $\mathbb{G}$ if for any $G \in \mathbb{G}$

$$G \trianglerighteq_{\mathbf{G}} G' \text{ implies that } G \equiv_{\mathbf{G}} G',$$

that is, if G' is maximal then $G \trianglerighteq_{\mathbf{G}} G'$ implies that G and G' coincide or lie on the same $\triangleleft\triangleleft$-circuit. Thus, given the definition of descendance, maximal networks are precisely those networks *without descendants*. Letting

$$\Gamma_{\triangleright_{\mathbf{G}}}(G') := \left\{G \in \mathbb{G} : G \triangleright_{\mathbf{G}} G'\right\},$$

a network $G' \in \mathbb{G}$ is without descendants or is maximal in the farsighted network formation game $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ if and only if

$$\Gamma_{\triangleright_{\mathbf{G}}}(G') = \emptyset.$$

Note that any farsightedly isolated network is by definition a network without descendants. Recall that a network $G' \in \mathbb{G}$ is farsightedly isolated relative to $\mathbf{G}$, if there does not exist a network $G \in \mathbb{G}$ with $G' \triangleright\triangleright G$ or $G \triangleright\triangleright G'$.

In attempting to identify those networks which are likely to emerge and persist if agents are farsighted, networks *without descendants* are of particular interest. Here is our main result concerning networks without descendants.

Theorem 1 *(All farsighted network formation games have networks without descendants)*

Let $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ be a farsighted network formation game. For every network $G \in \mathbb{G}$ there exists a network $G' \in \mathbb{G}$ such that

1. *$G' \trianglerighteq_{\mathbf{G}} G$, and*
2. *$\Gamma_{\triangleright_{\mathbf{G}}}(G') = \emptyset$.*

Proof. Let G_0 be any network in $\mathbb{G}$. If $\Gamma_{\triangleright_{\mathbf{G}}}(G_0) = \emptyset$, we are done. If not choose $G_1 \in \Gamma_{\triangleright_{\mathbf{G}}}(G_0)$. If $\Gamma_{\triangleright_{\mathbf{G}}}(G_1) = \emptyset$, we are done. If not, continue by choosing $G_2 \in \Gamma_{\triangleright_{\mathbf{G}}}(G_1)$. Proceeding iteratively in this way, we can generate a sequence, $G_0, G_1, G_2, \ldots$. Now observe that in a finite number of iterations we must come to a network $G_{k'}$ such that $\Gamma_{\triangleright_{\mathbf{G}}}(G_{k'}) = \emptyset$. Otherwise, we could generate an infinite sequence, $\{G_k\}_k$ such that for all k,

$$G_k \triangleright_{\mathbf{G}} G_{k-1}.$$

However, because $\mathbb{G}$ is finite this sequence would contain at least one network, say $G_{k'}$, which is repeated an infinite number of times. Thus, all the networks in the sequence lying between any two consecutive repetitions of $G_{k'}$ would be on the same $\triangleleft\triangleleft$-circuit in supernetwork $\mathbf{G}$, contradicting the fact that for all k, G_k is a descendant of G_{k-1} (i.e., $G_k \triangleright_{\mathbf{G}} G_{k-1}$). ■

By Theorem 1, in any farsighted network formation game $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$, corresponding to any network $G \in \mathbb{G}$ there is a network $G' \in \mathbb{G}$ without descendants which is $\triangleleft\triangleleft$-reachable from G. Thus, in any farsighted network formation game the set of networks without descendants given by

$$\mathbb{Z} := \{G \in \mathbb{G} : \Gamma_{\triangleright_{\mathbf{G}}}(G) = \emptyset\}$$

is nonempty.

4.2 Basins of Attraction

Stated loosely, a basin of attraction is a set of *equivalent* networks to which the strategic network formation process represented by the game might tend and from which there is no escape. Formally, we have the following definition.

Definition 3 *(Basin of Attraction)*
Let $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ be a farsighted network formation game. A set of networks $\mathbb{A} \subseteq \mathbb{G}$ is said to be a basin of attraction for $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ if

1. *the networks contained in $\mathbb{A}$ are equivalent (i.e., for all G' and G in $\mathbb{A}$, $G' \equiv_{\mathbf{G}} G$), and*
2. *no network in $\mathbb{A}$ has descendants (i.e., there does not exist a network $G' \in \mathbb{G}$ such that $G' \triangleright_{\mathbf{G}} \mathbb{A}$ where $G' \triangleright_{\mathbf{G}} \mathbb{A}$ if and only if $G' \triangleright_{\mathbf{G}} G$ for some $G \in \mathbb{A}$).*

As the following characterization result shows, there is a very close connection between networks without descendants and basins of attraction.

Theorem 2 *(A characterization of basins of attraction)*
Let $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ be a farsighted network formation game and let $\mathbb{A}$ be a subset of networks in $\mathbb{G}$. The following statements are equivalent:

1. *$\mathbb{A}$ is a basin of attraction for $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$.*
2. *There exists a network without descendants, $G \in \mathbb{Z}$, such that*

$$\mathbb{A} = \left\{G' \in \mathbb{Z} : G' \equiv_{\mathbf{G}} G\right\}.$$

Proof. (1) implies (2): Because the sets $\mathbb{A}$ and $\{G' \in \mathbb{Z} : G' \equiv_{\mathbf{G}} G\}$, $G \in \mathbb{Z}$, are equivalence classes, $\mathbb{A} \neq \{G' \in \mathbb{Z} : G' \equiv_{\mathbf{G}} G\}$ implies that

$$\mathbb{A} \cap \left\{G' \in \mathbb{Z} : G' \equiv_{\mathbf{G}} G\right\} = \emptyset \text{ for all } G \in \mathbb{Z}.$$

Thus, if (2) fails, this implies that $\mathbb{A}$ contains a network with descendants. Thus, $\mathbb{A}$ cannot be a basin of attraction for $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$, and thus, (1) implies (2).[14]

(2) implies (1): Suppose now that

$$\mathbb{A} = \left\{G' \in \mathbb{Z} : G' \equiv_{\mathbf{G}} G\right\}$$

for some network $G \in \mathbb{Z}$. If $\mathbb{A}$ is not a basin of attraction, then for some network $G'' \in \mathbb{G}$, $G'' \triangleright_{\mathbf{G}} G'$ for some $G' \in \mathbb{A}$. But now $G'' \triangleright_{\mathbf{G}} G'$ and $G' \equiv_{\mathbf{G}} G$ imply that $G'' \triangleright_{\mathbf{G}} G$, contradicting the fact that $G \in \mathbb{Z}$. Thus, (2) implies (1). ■

In light of Theorem 2, we conclude that in any farsighted network formation game $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$, $\mathbb{G}$ contains a *unique*, finite, disjoint collection of basins of attraction, say $\{\mathbb{A}_1, \mathbb{A}_2, \ldots, \mathbb{A}_m\}$, where for each $k = 1, 2, \ldots, m$ $(m \geq 1)$

$$\mathbb{A}_k = \mathbb{A}_G := \left\{G' \in \mathbb{Z} : G' \equiv_{\mathbf{G}} G\right\}$$

[14] Note that if $G \in \mathbb{Z}$ and $G' \equiv_{\mathbf{G}} G$, then $G' \in \mathbb{Z}$.

for some network $G \in \mathbb{Z}$. Note that for networks G' and G in $\mathbb{Z}$ such that $G' \equiv_{\mathbf{G}} G$, $\mathbb{A}_{G'} = \mathbb{A}_G$ (i.e. the basins of attraction $\mathbb{A}_{G'}$ and $\mathbb{A}_G$ coincide). Also, note that if network $G \in \mathbb{G}$ is farsightedly isolated relative to $\mathbf{G}$, then $G \in \mathbb{Z}$ and

$$\mathbb{A}_G := \left\{ G' \in \mathbb{Z} : G' \equiv_{\mathbf{G}} G \right\} = \{G\}$$

is, by definition, a basin of attraction - but a very uninteresting one.

Example 1 *(The farsighted dominance relation and basins of attraction)*

Figure 3 depicts the graph of the farsighted dominance relation induced by a supernetwork $\mathbf{G}$.

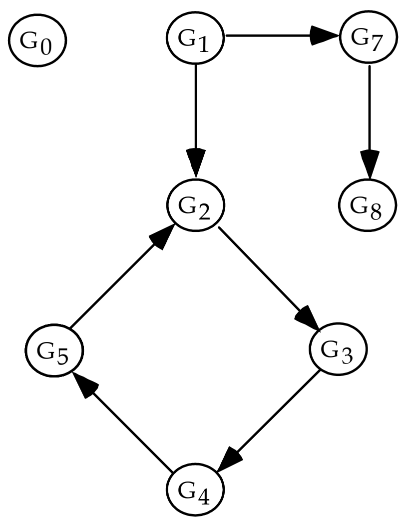


Figure 3: Graph of the farsighted dominance relation induced by supernetwork G

In Figure 3, a network at the end of an arrow (a network by the arrowhead) farsightedly dominants the network at the beginning of the arrow. Thus, in Figure 3, network G_7 farsightedly dominants network G_1 (i.e., $G_7 \rhd\rhd G_1$). First, note that network G_0 is farsightedly isolated relative to the supernetwork. Second, note that the set of networks without descendants is given by

$$\mathbb{Z} = \{G_0, G_2, G_3, G_4, G_5, G_8\}.$$

Third, note that even though there are nine networks without descendants, because networks $G_2, G_3, G_4,$ and G_5 are equivalent, there are only three basins of attraction:

$$\mathbb{A}_1 = \{G_0\}, \ \mathbb{A}_2 = \{G_2, G_3, G_4, G_5\}, \text{ and } \mathbb{A}_3 = \{G_8\}.$$

Because $G_2, G_3, G_4,$ and G_5 are equivalent,

$$\mathbb{A}_{G_2} = \mathbb{A}_{G_3} = \mathbb{A}_{G_4} = \mathbb{A}_{G_5} = \{G_2, G_3, G_4, G_5\}.$$

4.3 Farsighted Bases

In this subsection we show that the fact that all farsighted network formation games possess a unique, finite, disjoint collection of basins of attraction implies that all farsighted network formation games possess von Neumann-Morgenstern stable sets with respect to the farsighted domination path relation, $\trianglerighteq_{\mathbf{G}}$. We refer to these $\trianglerighteq_{\mathbf{G}}$-stable sets as *farsighted bases* and we refer to any network contained in a farsighted basis as a *farsightedly basic network*. The formal definition of a farsighted basis is as follows.

Definition 4 *(Farsighted Basis)*

Let $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ be a farsighted network formation game. A subset $\mathbb{B}$ of directed networks in $\mathbb{G}$ is said to be a farsighted basis for $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ if

(a) (internal $\trianglerighteq_{\mathbf{G}}$ -stability) whenever G_0 and G_1 are in $\mathbb{B}$, with $G_0 \neq G_1$, then neither $G_1 \trianglerighteq_{\mathbf{G}} G_0$ nor $G_0 \trianglerighteq_{\mathbf{G}} G_1$ hold, and

(b) (external $\trianglerighteq_{\mathbf{G}}$ -stability) for any $G_0 \notin \mathbb{B}$ there exists $G_1 \in \mathbb{B}$ such that $G_1 \trianglerighteq_{\mathbf{G}} G_0$.

In other words, a nonempty subset of networks $\mathbb{B}$ is a farsighted basis for $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ if G_0 and G_1 are in $\mathbb{B}$, with $G_0 \neq G_1$, then G_1 is not reachable from G_0, nor is G_0 reachable from G_1, and if $G_0 \notin \mathbb{B}$, then there exists $G_1 \in \mathbb{B}$ reachable from G_0.

We now have our main results on the existence, construction, and cardinality of farsighted bases.[15]

Theorem 3 *(Farsighted bases: existence, construction, and cardinality)*

Let $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ be a farsighted network formation game, and without loss of generality assume that $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ has basins of attraction given by

$$\{\mathbb{A}_1, \mathbb{A}_2, \ldots, \mathbb{A}_m\},$$

where basin of attraction $\mathbb{A}_k$ contains $|\mathbb{A}_k|$ many networks (i.e., $|\mathbb{A}_k|$ is the cardinality of $\mathbb{A}_k$). Then the following statements are true:

1. *$\mathbb{B} \subseteq \mathbb{G}$ is a farsighted basis for $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ if and only if $\mathbb{B}$ is constructed by choosing one network from each basin of attraction, that is, if and only if $\mathbb{B}$ is of the form*
$$\mathbb{B} = \{G_1, G_2, \ldots, G_m\},$$
where $G_k \in \mathbb{A}_k$ for $k = 1, 2, \ldots, m$.

2. *$(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ possesses*
$$|\mathbb{A}_1| \cdot |\mathbb{A}_2| \cdot \cdots \cdot |\mathbb{A}_m| := M$$
many farsighted bases and each basis, $\mathbb{B}_q$, $q = 1, 2, \ldots, M$, has cardinality
$$|\mathbb{B}_q| = |\{\mathbb{A}_1, \mathbb{A}_2, \ldots, \mathbb{A}_m\}| = m.$$

[15] These results can be viewed as extensions of some classical results from graph theory (e.g., see Berge (2001), Chapter 2) to the theory of farsighted network formation games.

Proof. It suffices to prove (1). Given (1), the proof of (2) is straightforward. To begin, let

$$\mathbb{B} = \{G_1, G_2, \ldots, G_m\},$$

where $G_k \in \mathbb{A}_k$ for $k = 1, 2, \ldots, m$, and suppose that for G_k and $G_{k'}$ in $\mathbb{B}$, $G_{k'} \trianglerighteq_{\mathbf{G}} G_k$. Since $G_k \in \mathbb{A}_k$ has no descendants, this would imply that $G_{k'} \equiv_{\mathbf{G}} G_k$. But this is a contradiction because $G_k \in \mathbb{A}_k$ and $G_{k'} \in \mathbb{A}_{k'}$ and the basins of attraction $\mathbb{A}_k$ and $\mathbb{A}_{k'}$ are disjoint. Thus, $\mathbb{B}$ is internally $\trianglerighteq_{\mathbf{G}}$-stable. Now suppose that network G is not contained in $\mathbb{B}$. By Theorem 1, there exists a network $G' \in \mathbb{G}$ such that $G' \trianglerighteq_{\mathbf{G}} G$, and $\Gamma_{\triangleright_{\mathbf{G}}}(G') = \emptyset$ (i.e., G' is a network without descendants). By Theorem 2, G' is contained in some basin of attraction $\mathbb{A}_k$ and therefore $G' \equiv_{\mathbf{G}} G_k$ where G_k is the k^{th} component of $\{G_1, G_2, \ldots, G_m\}$. Thus, we have $G_k \trianglerighteq_{\mathbf{G}} G' \trianglerighteq_{\mathbf{G}} G$ implying that $G_k \trianglerighteq_{\mathbf{G}} G$, and thus $\mathbb{B}$ is externally $\trianglerighteq_{\mathbf{G}}$-stable.

Suppose now that $\mathbb{B} \subseteq \mathbb{G}$ is a farsighted basis for $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$. First note that each network G in $\mathbb{B}$ is a network without descendants. Otherwise there exists $G' \in \mathbb{G}\backslash\mathbb{B}$ such that $G' \triangleright_{\mathbf{G}} G$. But then because $\mathbb{B}$ is externally $\trianglerighteq_{\mathbf{G}}$-stable, there exists $G'' \in \mathbb{B}$, $G'' \neq G$, such that $G'' \trianglerighteq_{\mathbf{G}} G'$ implying that $G'' \trianglerighteq_{\mathbf{G}} G$ and contradicting the internal $\trianglerighteq_{\mathbf{G}}$-stability of $\mathbb{B}$. Because each $G \in \mathbb{B}$ is without descendants, it follows from Theorem 2 that each $G \in \mathbb{B}$ is contained in some basin of attraction $\mathbb{A}_k$. Moreover, because $\mathbb{B}$ is internally $\trianglerighteq_{\mathbf{G}}$-stable and because all networks contained in any one basin of attraction are equivalent, no two distinct networks contained in $\mathbb{B}$ can be contained in the same basin of attraction. It only remains to show that for each basin of attraction, $\mathbb{A}_k$, $k = 1, 2, \ldots, m$,

$$\mathbb{B} \cap \mathbb{A}_k \neq \emptyset.$$

Suppose not. Then for some k', $\mathbb{B} \cap \mathbb{A}_{k'} = \emptyset$. Because all networks in $\mathbb{A}_{k'}$ are without descendants, for no network $G \in \mathbb{A}_{k'}$ is it true that there exists a network $G' \in \mathbb{B}$ such that $G' \trianglerighteq_{\mathbf{G}} G$. Thus, we have a contradiction of the external $\trianglerighteq_{\mathbf{G}}$-stability of $\mathbb{B}$. ■

Example 2 *(Basins of attraction and farsighted bases)*

Referring back to the graph of the farsighted dominance relation induced by supernetwork $\mathbf{G}$ *given in Figure 3, it follows from Theorem 3 that because*

$$|\mathbb{A}_1| \cdot |\mathbb{A}_2| \cdot |\mathbb{A}_3| = 1 \cdot 4 \cdot 1 = 4,$$

the farsighted network formation game $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ *has 4 farsighted bases, each with cardinality 3. By examining Figure 3 in light of Theorem 3, we see that the farsighted bases for* $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ *are given by*

$$\begin{aligned} \mathbb{B}_1 &= \{G_0, G_2, G_8\}, \\ \mathbb{B}_2 &= \{G_0, G_3, G_8\}, \\ \mathbb{B}_3 &= \{G_0, G_4, G_8\}, \\ \mathbb{B}_4 &= \{G_0, G_5, G_8\}. \end{aligned}$$

4.4 The Farsighted Core

One of the most fundamental stability notions in game theory is the core. Here we define the notion of core for farsighted network formation games. We call this notion of the core the *farsighted core.*

Definition 5 *(The Farsighted Core)*

Let $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ be a farsighted network formation game. A subset $\mathbb{C}$ of directed networks in $\mathbb{G}$ is said to be the farsighted core of $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ if for each network $G \in \mathbb{C}$ there does not exist a network $G' \in \mathbb{G}$, $G' \neq G$, such that $G' \trianglerighteq_{\mathbf{G}} G$.

Our next results give necessary and sufficient conditions for the core of a farsighted network formation game to be nonempty, as well as a recipe for constructing the farsighted core.

Theorem 4 *(Farsighted core: nonemptiness, construction, and cardinality)*

Let $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ be a farsighted network formation game, and without loss of generality assume that $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ has basins of attraction given by

$$\{\mathbb{A}_1, \mathbb{A}_2, \ldots, \mathbb{A}_m\},$$

where basin of attraction $\mathbb{A}_k$ contains $|\mathbb{A}_k|$ many networks (i.e., $|\mathbb{A}_k|$ is the cardinality of $\mathbb{A}_k$). Then the following statements are true:

1. *$(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ has a nonempty farsighted core if and only if there exists a basin of attraction containing a single network, that is, if and only if for some basin of attraction $\mathbb{A}_k$, $|\mathbb{A}_k| = 1$.*

2. *Suppose there exist basins of attraction with cardinality 1, and let*

$$\{\mathbb{A}_{k_1}, \mathbb{A}_{k_2}, \ldots, \mathbb{A}_{k_n}\} \subseteq \{\mathbb{A}_1, \mathbb{A}_2, \ldots, \mathbb{A}_m\},$$

where $\mathbb{A}_k \in \{\mathbb{A}_{k_1}, \mathbb{A}_{k_2}, \ldots, \mathbb{A}_{k_n}\}$ if and only if $|\mathbb{A}_k| = 1$, $k = 1, 2, \ldots, m$.

$\mathbb{C} \subseteq \mathbb{G}$, $\mathbb{C} \neq \emptyset$, is the farsighted core of $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ if and only if $\mathbb{C}$ is given by

$$\mathbb{C} = \{G_{k_1}, G_{k_2}, \ldots, G_{k_n}\},$$

where $G_{k_i} \in \mathbb{A}_{k_i}$, for $i = 1, 2, \ldots, n$. Moreover, if $\mathbb{C} \neq \emptyset$ is the farsighted core of $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$, then $\mathbb{C}$ has cardinality

$$|\mathbb{C}| = |\{\mathbb{A}_{k_1}, \mathbb{A}_{k_2}, \ldots, \mathbb{A}_{k_n}\}| = n.$$

Proof. It suffices to show that a network G is contained in the farsighted core $\mathbb{C}$ if and only if $G \in \mathbb{A}_k$ for some basin of attraction $\mathbb{A}_k$, $k = 1, 2, \ldots, m$, with $|\mathbb{A}_k| = 1$. First note that if G is in the farsighted core, then G is a network without descendants. Thus, $G \in \mathbb{A}_k$ for some basin of attraction $\mathbb{A}_k$. If $|\mathbb{A}_k| > 1$, then there exists another network $G' \in \mathbb{A}_k$ such that $G' \equiv_{\mathbf{G}} G$. Thus, $G' \trianglerighteq_{\mathbf{G}} G$ contradicting the fact that G is in the farsighted core. Conversely, if $G \in \mathbb{A}_k$ for some basin of attraction $\mathbb{A}_k$ with $|\mathbb{A}_k| = 1$, then there does not exist a network $G' \neq G$ such that $G' \trianglerighteq_{\mathbf{G}} G$. ■

Example 3 *(Basins of attraction and the farsighted core)*

Referring back to the graph of the farsighted dominance relation induced by supernetwork $\mathbf{G}$ *given in Figure 3, it follows from Theorem 4 that*

$$\mathbb{C} = \{G_0, G_8\},$$

is the farsighted core of the network formation game $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$*. Consider the graph of the farsighted dominance relation induced by a different supernetwork* $\mathbf{G}'$ *given in Figure 4.*

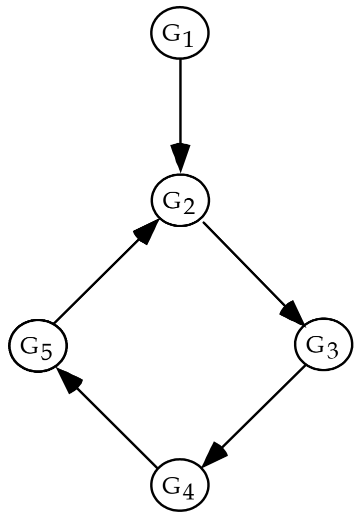


Figure 4: Graph of the farsighted dominance relation induced by supernetwork G'

Here there is only one basin of attraction,

$$\mathbb{A} = \{G_2, G_3, G_4, G_5\},$$

and because $|\mathbb{A}| > 1$*, the farsighted core of the network formation game* $(\mathbb{G}, \trianglerighteq_{\mathbf{G}'})$ *is empty.*

5 Other Stability Notions for Network Formation Games

5.1 Strongly Stable Networks

In this subsection we extend the Jackson-van den Nouweland (2000) notion of strong stability to farsighted network formation games over directed networks induced by arbitrary supernetworks. We show that if the farsighted core is nonempty then the set of strongly stable networks is nonempty and contains the farsighted core. It then follows from Theorem 4 that the existence of a basin of attraction containing a single network implies nonemptiness of the set of strongly stable networks.

We begin with a formal definition of strong stability in farsighted network formation games.

Definition 6 *(Strong stability)*

Let $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ be a farsighted network formation game. A network $G \in \mathbb{G}$ is said to be strongly stable in $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$, if $(m_S, (G, G')) \in \mathbf{G}$ for some coalition S and network $G' \in \mathbb{G}$, implies that $(p_S, (G, G')) \notin \mathbf{G}$. We shall denote by $\mathbb{SS}$ the set of strongly stable networks in $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$.

We now have our main result on the farsighted core and strong stability.

Theorem 5 *(The Farsighted Core and Strong Stability)*

Let $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ be a farsighted network formation game induced by any supernetwork $\mathbf{G}$. If the farsighted core, $\mathbb{C}$, of $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ is nonempty, then $\mathbb{SS}$ is nonempty and

$$\mathbb{C} \subseteq \mathbb{SS}.$$

Proof. Let $\mathbb{C} \subseteq \mathbb{G}$, $\mathbb{C} \neq \emptyset$, be the farsighted core of $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ and let network G be contained in $\mathbb{C}$. Then $\{G\}$ is a basin of attraction. Thus, there does not exist a network $G' \in \mathbb{G}$, $G' \neq G$, such that $G' \trianglerighteq_{\mathbf{G}} G$. If for some coalition S and some network $G' \in \mathbb{G}$, $(m_S, (G, G')) \in \mathbf{G}$, then it must be true that $(p_S, (G, G')) \notin \mathbf{G}$, otherwise, we would have $G' \trianglerighteq_{\mathbf{G}} G$, a contradiction. Thus, $G \in \mathbb{SS}$. $\blacksquare$

5.2 Pairwise Stable Networks

In this subsection, we assume that the set of nodes N and the set of agents D are one and the same (i.e., $N = D$) and we extend the Jackson-Wolinsky (1996) notion of pairwise stability to farsighted network formation games over directed networks induced by arbitrary supernetworks (including Jackson-Wolinsky supernetworks in which arc addition is bilateral and arc subtraction is unilateral). We then show that if the farsighted core is nonempty, then the set of pairwise stable networks is nonempty and contains the farsighted core. It then follows from Theorem 4 that the existence of a basin of attraction containing a single network implies nonemptiness of the set of pairwise stable networks.

Definition 7 *(Pairwise stability)*

Let $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ be a farsighted network formation game. A network $G \in \mathbb{G}$ is said to be pairwise stable in $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$, if

1. *$(m_{\{i,i'\}}, (G, G \cup (j, (i, i'))) \in \mathbf{G}$ for some agents i and i' in $N = D$ and some arc $j \in A$, implies that $(p_{\{i,i'\}}, (G, G \cup (j, (i, i'))) \notin \mathbf{G}$ (i.e., implies that either $v_i(G \cup (j, (i, i')) \leq v_i(G)$ or that $v_{i'}(G \cup (j, (i, i')) \leq v_{i'}(G)$);*

2. *(a) $(m_{\{i\}}, (G, G \setminus (j, (i, i'))) \in \mathbf{G}$ for some agent i in $N = D$ and some arc $j \in A$, implies that $(p_{\{i\}}, (G, G \setminus (j, (i, i'))) \notin \mathbf{G}$ (i.e., implies that $v_i(G \setminus (j, (i, i')) \leq v_i(G)$), and*

   (b) $(m_{\{i'\}}, (G, G \setminus (j, (i, i'))) \in \mathbf{G}$ for some agent i' in $N = D$ and some arc $j \in A$, implies that $(p_{\{i'\}}, (G, G \setminus (j, (i, i'))) \notin \mathbf{G}$ (i.e., implies that $v_{i'}(G \setminus (j, (i, i')) \leq v_{i'}(G)$).

Let $\mathbb{PS}$ denote the set of pairwise stable networks in $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$, where $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ is a farsighted network formation game with $N = D$ (i.e., nodes = agents) induced by an arbitrary supernetwork $\mathbf{G}$. It follows from the definitions of strong stability and pairwise stability that

$$\mathbb{SS} \subseteq \mathbb{PS}.$$

Moreover, if $\mathbf{G}$ is a Jackson-Wolinsky supernetwork, then $\mathbb{SS} = \mathbb{PS}$. Also, under our definition of pairwise stability a network $G \in \mathbb{G}$ that cannot be changed to another network by any coalition (including a coalition consisting of one or two agents) in supernetwork $\mathbf{G}$ is pairwise stable. Stately formally, a network $G \in \mathbb{G}$ such that

$$(m_S, (G, G^{'})) \notin \mathbf{G} \text{ for all coalitions } S \text{ and all networks } G' \in \mathbb{G},$$

is pairwise stable in $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$.

We now have our main result on the farsighted core and pairwise stability. The proof of this result is similar to the proof of Theorem 5 above.

Theorem 6 *(The Farsighted Core and Pairwise Stability)*

Let $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ be a farsighted network formation game with $N = D$ induced by an arbitrary supernetwork $\mathbf{G}$. If the farsighted core, $\mathbb{C}$, of $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ is nonempty, then $\mathbb{PS}$ is nonempty and

$$\mathbb{C} \subseteq \mathbb{PS}.$$

5.3 Farsightedly Consistent Networks

In this subsection, we show that in any farsighted network formation game induced by an arbitrary supernetwork, each basin of attraction has a nonempty intersection with the largest consistent set (i.e., the Chwe set - see Chwe (1994)). This fact implies that if the farsighted network formation game has a nonempty farsighted core, then it is contained in the largest consistent set. In Page, Wooders, and Kamat (2001), Chwe's notions of farsighted consistency and largest consistent set are extended supernetworks and it is shown that any farsighted network formation game has a nonempty, largest consistent set. In light of Theorem 6 above, we can conclude therefore that any network contained in the farsighted core (i.e., in a basin of attraction containing a single network) is not only farsightedly consistent but also strongly stable.

We begin with a formal definition of farsighted consistency.

Definition 8 *(Farsightedly Consistent Sets)*

Let $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ be a farsighted network formation game. A subset $\mathbb{F}$ of directed networks in $\mathbb{G}$ is said to be farsightedly consistent in $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ if

$$\begin{gathered}\text{for all } G_0 \in \mathbb{F}, \\ (m_{S_1}, (G_0, G_1)) \in \mathbf{G} \text{ for some } G_1 \in \mathbb{G} \text{ and some coalition } S_1, \text{ implies that} \\ \text{there exists } G_2 \in \mathbb{F} \\ \text{with } G_2 = G_1 \text{ or } G_2 \rhd\rhd G_1 \text{ such that,} \\ (p_{S_1}, (G_0, G_2)) \notin \mathbf{G}.\end{gathered}$$

In words, a subset of directed networks $\mathbb{F}$ is said to be farsightedly consistent in $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ if given any network $G_0 \in \mathbb{F}$ and any m_{S_1}-deviation to network $G_1 \in \mathbb{G}$ by coalition S_1 (via adding, subtracting, or replacing arcs in accordance with $\mathbf{G}$), there exists further deviations leading to some network $G_2 \in \mathbb{F}$ where the initially deviating coalition S_1 is not better off - and possibly worse off. A network $G \in \mathbb{G}$ is said to be farsightedly consistent if $G \in \mathbb{F}$ where $\mathbb{F}$ is a farsightedly consistent set in $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$. There can be many farsightedly consistent sets in $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$. We shall denote by $\mathbb{F}^*$ is largest farsightedly consistent set (or simply, the *largest consistent set*). Thus, if $\mathbb{F}$ is a farsightedly consistent set, then $\mathbb{F} \subseteq \mathbb{F}^*$.

Two questions arise in connection with the largest consistent set: (i) does there exist a largest consistent set of networks in $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$, and (ii) is it nonempty? As shown in Page, Wooders, and Kamat (2001) existence follows from Proposition 1 in Chwe (1994), while nonemptiness (and also external stability) follow from the Corollary to Proposition 2 in Chwe (1994). We now have our main result on the relationship between basins of attraction, the farsighted core, and the largest consistent set for farsighted network formation games induced by arbitrary supernetworks.

Theorem 7 *(Basins of Attraction, the Farsighted Core, and the Largest Consistent Set)*

Let $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ be a farsighted network formation game, and without loss of generality assume that $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ has nonempty largest consistent set given by $\mathbb{F}^*$ and basins of attraction given by

$$\{\mathbb{A}_1, \mathbb{A}_2, \ldots, \mathbb{A}_m\}.$$

Then the following statements are true:

1. *Each basin of attraction $\mathbb{A}_k$, $k = 1, 2, \ldots, m$, has a nonempty intersection with the largest consistent set $\mathbb{F}^*$, that is*

$$\mathbb{F}^* \cap \mathbb{A}_k \neq \emptyset, \text{ for } k = 1, 2, \ldots, m.$$

2. *If $(\mathbb{G}, \trianglerighteq_{\mathbf{G}})$ has a nonempty farsighted core $\mathbb{C}$, then*

$$\mathbb{C} \subseteq \mathbb{F}^*.$$

Proof. In light of Theorem 4, (2) easily follows from (1). Thus, it suffices to prove (1). Suppose that for some basin of attraction $\mathbb{A}_{k'}$

$$\mathbb{F}^* \cap \mathbb{A}_{k'} = \emptyset.$$

Let G' be a network in $\mathbb{A}_{k'}$. Because $\mathbb{F}^*$ is externally stable with respect to the farsighted dominance relation $\triangleright\triangleright$, $G' \notin \mathbb{F}^*$ implies that there exists some network $G^* \in \mathbb{F}^*$ such that $G^* \triangleright\triangleright G'$. Thus, $G^* \trianglerighteq_{\mathbf{G}} G'$. Because the networks in $\mathbb{A}_{k'}$ are without descendants, it must be true that $G' \trianglerighteq_{\mathbf{G}} G^*$. But this implies that $G^* \equiv_{\mathbf{G}} G'$, and therefore that $G^* \in \mathbb{A}_{k'}$, a contradiction. ■

6 Competitive Contracting Networks

In this section we introduce the notion of a competitive contracting network via a relatively simple example. The sort of nodes in the network we have in mind are, for example, L.L. Bean and Land's End, or competitors offering mutual funds or insurance contracts and also potential customers of the competing firms. In principle, there may be many firms (nodes) offering catalogs of products to potential consumers or 'the market' (also nodes). For this application, since arcs represent contracts, it is essential that arcs be allowed to be labelled and be heterogeneous.[16] We consider an example in which two firms compete for the services of a single, privately informed agent via catalogs of contracts. Our objective is to identify those competitive contracting strategies (i.e., catalog strategies) that are likely to emerge and persist if firms behave farsightedly in choosing their catalogs.

6.1 Contracts and Catalogs

To begin, suppose that there are only two contracts, f_A and f_B, and that each firm, F_1 and F_2, can offer the agent, M, a catalog of contracts from the following set of catalogs:

$$\{\{0\}, \{f_A\}, \{f_B\}, \{f_A, f_B\}\}.$$

Here, contract 0 denotes *no contracting*. Thus, if firm 1 offers the agent catalog $\{0\}$, while firm 2 offers the agent catalog $\{f_A, f_B\}$, then firm 1 has chosen not to enter the competition - or put differently, has chosen not to enter the industry. In this case, the catalog profile offered by firms is given by

$$(\{0\}, \{f_A, f_B\}).$$

Given the catalog profile offered by firms, the privately informed agent then chooses a firm with which to contract and a particular contract from the catalog offered by that firm (i.e., the agent can contract with one and only one firm). In order to take into account the possibility that the agent may wish to abstain from contracting altogether, we assume that there is a fictitious firm $i = 0$ that offers the catalog $\{0\}$. Thus, if firms 1 and 2 offer catalog profile $(\{f_A\}, \{f_A, f_B\})$, then the full catalog profile is given by

$$\Big(\underset{\text{firm } 0}{\{0\}}, \underset{\text{firm } 1}{\{f_A\}}, \underset{\text{firm } 2}{\{f_A, f_B\}}\Big),$$

and the agent's mutually exclusive choices can be summarized as follows:

contract with firm 0 and choose from catalog $\{0\}$,
contract with firm 1and choose from catalog $\{f_A\}$,
contract with firm 2 and choose from catalog $\{f_A, f_B\}$.

The timing in the contracting game is as follows: First, each firm simultaneously and confidentially chooses and commits to a particular catalog offer. Next, the agent

[16] To the best of our knowledge, prior models of networks in the economics and game theoretic literature do not permit such an application.

chooses a firm and a contract from that firm's catalog. Ex ante each firm knows the agent up to a distribution of agent types, and therefore, given any catalog profile offered by the firms, each firm is able to deduce the agent's type-dependent best responses and compute the firm's expected payoff.

6.2 Catalog Strategies and Contracting Networks

Each profile of catalog strategies by firms can be uniquely represented by a contracting network. Let the set of nodes be given by $N = \{F_1, F_2, M\}$, the set of arcs by $A = \{\{0\}, \{f_A\}, \{f_B\}, \{f_A, f_B\}\}$, and the set of agents *participating in the network formation game* by $D = \{F_1, F_2\}$.

Figure 5 depicts the contracting network G corresponding to a particular profile of catalog offers by the firms.

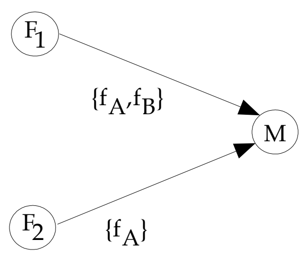


Figure 5: A Competitive Contracting Network

In Figure 5, the arc labeled $\{f_A, f_B\}$ going from node F_1 to node M indicates that in network G firm 1 is offers catalog $\{f_A, f_B\}$ to the agent, while the arc labeled $\{f_A\}$ going from node F_2 to node M indicates that firm 2 offers catalog $\{f_A\}$.

6.3 The Competitive Contracting Supernetwork

6.3.1 Network Payoffs

For the competitive contracting game, the feasible set of contracting networks $\mathbb{G}$ consists of 16 networks. Table 1 summarizes the expected payoffs to the firms in each possible contracting network.

Firm 1 ↓ / Firm 2 →	$\{0\}$	$\{f_A\}$	$\{f_B\}$	$\{f_A, f_B\}$
$\{0\}$	$\boxed{(0,0)}_1$	$\boxed{(0,-2)}_2$	$\boxed{(0,5)}_3$	$\boxed{(0,3)}_4$
$\{f_A\}$	$\boxed{(3,0)}_5$	$\boxed{(1,-3)}_6$	$\boxed{(1,3)}_7$	$\boxed{(1,1)}_8$
$\{f_B\}$	$\boxed{(0,0)}_9$	$\boxed{(-1,-2)}_{10}$	$\boxed{(0,3)}_{11}$	$\boxed{(-2,3)}_{12}$
$\{f_A, f_B\}$	$\boxed{(2,0)}_{13}$	$\boxed{(1,-3)}_{14}$	$\boxed{(0,1)}_{15}$	$\boxed{(-1,1)}_{16}$

Table 1: Contracting Network Payoffs

For example, the contracting network depicted in Figure 5, call it network G_{14}, generates the payoffs in cell 14 of the payoff matrix. Thus, the expected payoffs to firm 1 (F_1) and firm 2 (F_2) in network G_{14} are given by

$$v_{F_1}(G) := \Pi_1(\{f_A, f_B\}, \{f_A\}) = 1,$$
and
$$v_{F_2}(G) := \Pi_2(\{f_A, f_B\}, \{f_A\}) = -3.$$

This means that if firm 1 offers the agent catalog $\{f_A, f_B\}$ while firm 2 offers catalog $\{f_A\}$, then firm 1's expected payoff is 1, while firm 2's expected payoff is -3.[17]

6.3.2 Network Formation Rules

We shall assume that rules of network formation corresponding to the contracting game are purely unilateral. Thus, for example, firm 1 can alter the status quo contracting network by unilaterally replacing the arc from F_1 to M representing the firm's current catalog offer by another arc representing a different catalog offer. Figure 6 below depicts the m-arc, as well as the p-arc, connections between networks G_{14} and G_7 (corresponding to cells 14 and 7 in Table 1) in the competitive contracting supernetwork.

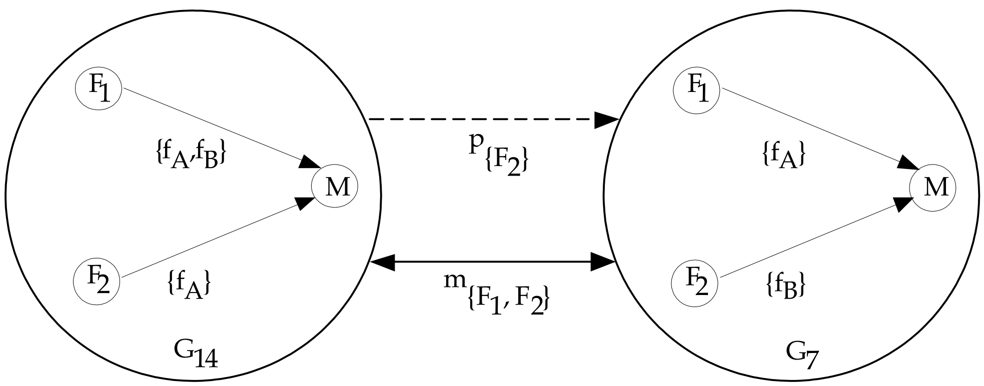


Figure 6: The Connections Between $G_{14}$and G_7in the Competitive Contracting Supernetwork

Note that firms 1 and 2 can change network G_{14} to network G_7 - as well as change network G_7 back to network G_{14} (hence the two-headed m-arc, $m_{\{F_1,F_2\}}$, connecting networks G_{14} and G_7).[18] In moving from G_{14} to G_7, firm 1 unilaterally replaces arc $\{f_A, f_B\}$ from F_1 to M with arc $\{f_A\}$ from F_1 to M, while firm 2 unilaterally replaces arc $\{f_A\}$ from F_2 to M with arc $\{f_B\}$ from F_2 to M. Thus, moving from G_{14} to G_7 requires that firms 1 and 2 act unilaterally and simultaneously.[19] Referring to Table 1, note that firm 2 prefers G_7 to G_{14} because

$$v_{F_2}(G_7) := \Pi_2(\{f_A\}, \{f_B\}) = 3,$$
and
$$v_{F_2}(G_{14}) := \Pi_2(\{f_A, f_B\}, \{f_A\}) = -3,$$

[17] Here, we have spared the reader the tedious details of computing the expected payoffs appearing in Table 1.

[18] Thus, the competitive contracting supernetwork is said to be symmetric.

[19] It is important to note that the rules of network formation, even though they are unilateral, do not rule out the possibility that firms act cooperatively.

while firm 1 is indifferent to networks G_7 and G_{14} because

$$v_{F_1}(G_7) := \Pi_2(\{f_A\}, \{f_B\}) = 1,$$
and
$$v_{F_1}(G_{14}) := \Pi_2(\{f_A, f_B\}, \{f_A\}) = 1.$$

Hence, given the definition of p-arcs, in Figure 6 there is a p-arc, $p_{\{F_2\}}$, from G_{14} to G_7, but there is no p-arc, $p_{\{F_1\}}$, from G_{14} to G_7 or from G_7 to G_{14}.

6.4 The Farsighted Core of the Competitive Contracting Game

Given the simplicity of the network formation rules, we conclude by inspection of Table 1 that the farsighted network formation game over contracting networks has one strategic basin of attraction consisting of a single network, G_7. Thus, the farsighted core of the network formation game over contracting networks is given by

$$\mathbb{C} = \{G_7\},$$

and thus the catalog profile

$$(\{f_A\}, \{f_B\})$$

corresponding to contracting network G_7 is likely to emerge and persist if firms behave farsightedly in choosing their catalogs. Note that network G_7 is also strongly stable and farsightedly consistent. In fact, network G_7 is the only strongly stable network as well as the only farsightedly consistent network. Thus in this example.

$$\mathbb{C} = \mathbb{SS} = \mathbb{F}^* = \{G_7\}.$$

6.5 A Variation on the Example

Suppose now that we change the example by changing the payoffs. Table 2 summaries the new payoffs.

Firm 1 ↕ \ Firm 2 ⟷	$\{0\}$	$\{f_A\}$	$\{f_B\}$	$\{f_A, f_B\}$
$\{0\}$	$\boxed{(0,0)}_1$	$\boxed{(0,-2)}_2$	$\boxed{(0,5)}_3$	$\boxed{(0,1)}_4$
$\{f_A\}$	$\boxed{(3,0)}_5$	$\boxed{(1,-3)}_6$	$\boxed{(1,1)}_7$	$\boxed{(1,-1)}_8$
$\{f_B\}$	$\boxed{(0,0)}_9$	$\boxed{(-1,-2)}_{10}$	$\boxed{(0,1)}_{11}$	$\boxed{(-2,1)}_{12}$
$\{f_A, f_B\}$	$\boxed{(2,0)}_{13}$	$\boxed{(1,-3)}_{14}$	$\boxed{(0,-1)}_{15}$	$\boxed{(-1,-1)}_{16}$

Table 2: The New Contracting Network Payoffs

By a careful inspection of Table 2 we conclude that in our new farsighted network formation game there is again only one strategic basin of attraction - but this time consisting of multiple networks. In particular, this single basin of attraction is given by

$$\mathbb{A} = \{G_5, G_7, G_{13}\}.$$

Thus, in our new farsighted network formation game the farsighted core is empty. Despite this, we can conclude that the catalog profiles corresponding to the networks contained in $\mathbb{A}$ are those that are likely to emerge and persist if firms behave farsightedly in choosing their catalogs. In particular, each network in $\mathbb{A}$ constitutes a farsighted basis of the network formation game (with respect to the farsighted domination path relation induced by the contracting supernetwork $\mathbf{G}$). Thus, in the competitive contracting game the set catalog profiles corresponding to the farsightedly basic networks, G_5, G_7,and G_{13}, listed in Table 3, is likely to emerge and persist if firms behave farsightedly in choosing their catalog strategies.

G_5	G_7	G_{13}
$(\{f_A\}, \{0\})$	$(\{f_A\}, \{f_B\})$	$(\{f_A, f_B\}, \{0\})$

Table 3: The Farsightedly Basic Networks and Their Corresponding Catalog Profiles

We close by noting that network G_7 is the only strongly stable network and that networks G_7 and G_{13} are the only farsightedly consistent networks. Thus, in our new example $\mathbb{SS} = \{G_7\}$ and $\mathbb{F}^* = \{G_7, G_{13}\}$.[20]

7 Further research

While we have related and characterized a number of solution concepts for our new model of networks and network formation, a number of questions remain. For example, is there an analogue of 'balancedness,' ensuring nonemptiness of the farsighted core of a game?[21] Is there an analogue of the 'partnered core'[22] for networks? Are there any conditions on 'admissible' networks which ensure that the farsighted core is nonempty independently of the structure of payoffs?[23] Current research is directed towards investigating these issues.

We close by noting that a number of other economic situations might provide interesting possibilities for analysis as abstract networks as developed in this paper. We have in mind, for example, problems from industrial organization, such as cartel formation, the formation of networks of collaboration, and trade networks. See, for example, Casella and Rauch (2001) and Bloch (2001) or other articles in the same volume for further potential applications.

References

[1] Berge, C. (2001) *The Theory of Graphs*, Dover, Mineola, New York. (reprint of the translated French edition published by Dunod, Paris, 1958).

[20] The farsightedly consistent sets in both versions of this example were computed using a *Mathematica* package developed by Kamat and Page (2001).

[21] A result due to Bondareva (1962), Shapley (1967) and Scarf (1967).

[22] See Reny and Wooders (1996) for an introduction to the partnered core of a nontransferable utility game.

[23] A famous class of games satisfying this property is assignment, or matching games; see for example Shapley and Shubik (1972). See also Demange (2001) and references therein for discussions of other classes of games with this property.

[2] Bloch, F. (2001) "Group and Network Formation in Industrial Organization; A Survey," *Group Formation in Economics: Networks, Clubs, and Coalitions*, G. Demange and M. Wooders (eds.). Cambridge University Press, forthcoming.

[3] Bondareva, O. (1962) "Theory of the Core in an n-Person Game," Vestnik, LGU13, 141-142 (in Russian), (Leningrad State University, Leningrad).

[4] Casella, A. and J. Rauch (2001) *Networks and Markets*, The Russel Sage Foundation, New York.

[5] Chwe, M. (1994) "Farsighted Coalitional Stability," *Journal of Economic Theory* 63, pp. 299-325.

[6] Demange, G. (2001) "Group Formation; The Interaction of Increasing Returns and Preference Diversity," In: Demange, G. and M. H. Wooders. (eds.) *Group Formation in Economics: Networks, Clubs, and Coalitions*. Cambridge University Press, forthcoming.

[7] Jackson, M. O. (2001) "A Survey of Models of Network Formation: Stability and Efficiency." In: Demange, G. and M. H. Wooders. (eds.) *Group Formation in Economics: Networks, Clubs, and Coalitions*. Cambridge University Press, forthcoming.

[8] Jackson, M. O. and A. van den Nouweland (2001) "Strongly Stable Networks," typescript, Caltech, forthcoming in *Games and Economic Behavior*.

[9] Jackson, M. O. and A. Watts (2001) "The Evolution of Social and Economic Networks," *Journal of Economic Theory* 106, pp. 265-295.

[10] Jackson, M. O. and A. Wolinsky (1996) "A Strategic Model of Social and Economic Networks," *Journal of Economic Theory* 71, pp. 44-74.

[11] Kamat, S. and F. H. Page, Jr. (2001) "Computing Farsighted Stable Sets," typescript, University of Alabama.

[12] Page, Jr., F. H., M. H. Wooders and S. Kamat (2001) "Networks and Farsighted Stability," Warwick Economic Research Papers, No 621, University of Warwick, forthcoming in the *Journal of Economic Theory*.

[13] Reny, P.J. and M.H. Wooders (1996) "The partnered core of a game without side payments," *Journal of Economic Theory* 70, 298-311.

[14] Scarf, H. (1967) "The Core of an n-person Game," *Econometrica* 35, 50-69.

[15] Shapley, L. S.. and M. Shubik (1972) "The Assignment Game 1; The Core," *International Journal of Game Theory* 1, 11-30.

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