

Liquidity, moral hazard and bank runs

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Abstract

In a model of banking with moral hazard, efficient risk-sharing between depositors may no longer be implementable. When depositors have all the bargaining power, we show that (i) with costless and perfect monitoring, contracts with the threat of bank runs off the equilibrium path of play improve on contracts with transfers, (ii) when the bank's actions are non-contractible, equilibrium bank runs driven by incentives are linked to liquidity provision by banks. When the bank has all the bargaining power, there is production efficiency but no liquidity provision to depositors. The model is extended to allow for general monitoring scenarios and partial contractibility of the bank's payoffs. Our results provide a theoretical foundation for the doctrine of "*creative ambiguity*".

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1 Introduction

A key issue in the theoretical literature on banking is the link between illiquid assets, liquid liabilities and bank runs. In the seminal paper by Diamond and Dybvig (1983) (see also Bryant (1980)) efficient risk-sharing between depositors with idiosyncratic, privately observed taste shocks creates a demand for liquidity. Banks invest in illiquid assets but take on liquid liabilities by issuing demand deposit contracts with a sequential service constraint. Although demand deposit contracts support the efficient risk-sharing between depositors, the use of such contracts makes banks vulnerable to runs driven by depositor coordination failure.

However, as Diamond and Dybvig point out, when aggregate taste shocks are common knowledge, a demand deposit contract with an appropriately chosen threshold for suspension of convertibility eliminates bank runs while supporting efficient risk-sharing. This result raises the following question: without any a priori restrictions on banking contracts, are there scenarios where equilibrium bank runs occur with positive probability in a banking contract?

This paper studies the connection between the possible misalignment of the incentives of the bank and depositors, and the role of bank runs as a disciplining device. We study a model of banking *with* moral hazard but *without* aggregate payoff-relevant uncertainty. In our model, the bank has no investment funds of its own but has a comparative advantage in operating illiquid assets: no other agent in the economy has the human capital to operate illiquid assets. Consequently, the bank controls any investment made in illiquid assets. The bank has a choice of two illiquid assets to invest in. After depositors' endowments have been mobilized, but before the realization of idiosyncratic taste shocks, the bank makes an investment decision. Each illiquid asset generates a stream of "public" and "private" returns. We think of "public" returns as cash flows generated by the asset that the bank cannot access without depositors' consent (for instance, such cash flows are generated by physical capital which can be monitored and seized by depositors). "Private" returns, then, are cash flows generated by the asset which can be accessed by the bank without depositors' consent.

Consider first the case when depositors have all the bargaining power. Even with costless, perfect monitoring of the bank's actions, we show that using transfers to provide the bank with appropriate incentives can result in narrow banking and no liquidity provision. More generally, incentive compatible transfers to the bank will lower consumption for all types of depositors. Nevertheless, our first result shows that it is still possible to implement efficient risk sharing between depositors, without sacrificing consumption, by using a contract which embodies the threat of a bank run off the equilibrium path of play.

When the investment decision of the bank is non-contractible¹, we show that efficient risk-sharing between depositors is no longer implementable. Even with forward looking depositors, the positive probability of an equilibrium bank run is

¹In particular, any transfer to the bank cannot be made contingent on the actions chosen by the bank.

necessary and sufficient to resolve incentive problems in banking. Although the second-best incentive compatible contract improves on autarky, it also generates, endogenously, the risk of a banking crisis.

Next, we note that when the bank has all the bargaining power and is able to appropriate both public and private returns generated by assets, there is productive efficiency (the bank invests all mobilized deposits in the asset with highest combined public and private return) but narrow banking and no liquidity provision (equivalently, efficient risk-sharing between depositors).

Reverting to the assumption that the depositors have all the bargaining power, we extend the model in two ways, first, by studying more general monitoring scenarios and second, by allowing the use of collaterals in banking. With costly but perfect monitoring, the trade-off between risk-sharing between depositors and providing appropriate incentives to the bank, continues to apply. With costly and imperfect monitoring, we show that the threat of bank runs off-the-equilibrium path of play and equilibrium bank runs, are both features of a banking contract.

Next, we study a different monitoring scenario where all monitoring takes place conditional on a bank run. In such situations, we show that positive probability of bank runs *and* monitoring along the equilibrium path of play are essential features of an incentive compatible contract. Our results rationalize the sequence of events involved in such interventions (temporary bailout measures, followed by a discovery phase and finally, a restructuring phase).

We then extend the model to scenarios where a small portion of the bank's non-contractible payoffs can be seized by an outside agent (a court). In such scenarios, we show that random demandable debt contracts studied here Pareto improve on autarky.

As there is no aggregate uncertainty in preferences and technology, the randomness introduced by banking contracts studied here is uncorrelated with fundamentals and is driven purely by incentives. We believe this is a more primitive explanation for bank runs. In the formal model studied here, bailouts are equivalent to building in a suspension of convertibility clause in the banking contract. In this sense, the random second-best contract studied here provides a rationale for the doctrine of "*creative ambiguity*" when the banking regulator makes no ex-ante commitment to a particular bailout policy but instead leaves the banking sector in doubt about its intentions (Goodhart (1999)). When all monitoring takes place conditional on a bank run, as both bank runs and monitoring are triggered by payoff irrelevant uncertainty, our results provide a further justification for the doctrine of "*creative ambiguity*".

The rest of the paper is structured as follows. The remainder of the introduction relates the results obtained here with other papers on bank runs. Section 2 studies a simple model of banking with moral hazard and leads up to the main result of the paper. Section 3 is devoted to robustness issues. The final section concludes.

1.1 Related literature

Although to the best of our knowledge, both the model and the results of our paper are new, in what follows, we situate our analysis in the context of related work².

Perhaps the paper closest to the approach we adopt here is Diamond and Rajan (2001) who show that the threat of bank runs off the equilibrium path of play impacts on the bank's ability and incentives to renegotiate loan contracts with borrowers. We obtain a similar result: the threat of bank runs off the equilibrium path of play (when monitoring is both costless and perfect) impacts on the investment decision of the bank. However, they do not obtain equilibrium bank runs as, in their model, whether or not banks renegotiate is observable (though not verifiable and therefore, non-contractible ex-ante).

Calomiris and Kahn (1991) study a model of embezzlement in banking where the bank's temptation to embezzle depends on the realization of an exogenous move of nature and depending on the prevailing state, either the bank will never be tempted to embezzle or will always be tempted to embezzle. Therefore, in Calomiris and Kahn (1991), the positive probability of a bank run relies on the existence of aggregate payoff-relevant uncertainty. Diamond and Rajan (2000), in a framework similar to Diamond and Rajan (2001), also require the additional feature of exogenous uncertainty to obtain equilibrium bank runs. In contrast, in our paper the existence of equilibrium bank runs doesn't rely on aggregate payoff relevant uncertainty. Here bank runs are driven purely by incentives.

Holmström and Tirole ((1997), (1998)), study a model where conditional on the realization of an exogenous liquidity shock, banks incentives have to be aligned with those of the depositors. In their model, ex-ante (before the realization of the exogenous liquidity shock), the threshold (in the space of liquidity shocks) below which the bank is liquidated is set. They show that this threshold will be higher than the first-best threshold when agency costs are taken into account. In this sense, their inefficient termination (relative to the first-best) is driven by exogenous payoff-relevant uncertainty while in our paper inefficient termination doesn't require exogenous payoff-relevant uncertainty.

It is worth remarking that a common feature of Calomiris and Kahn (1991), Holmström and Tirole ((1997), (1998)), and Diamond and Rajan (2001), is their focus on issues of moral hazard that arise conditional on the realization of the liquidity shock. In contrast, here, we study moral hazard issues that arise ex-ante before the realization of the liquidity shock.

A related branch has focused on the relation between incomplete information about the distribution of taste shocks across depositors and bank runs in banking scenarios with a finite number of depositors. Under the assumption that the social planner can condition allocations on the position a depositor has in the queue of depositors attempting to withdraw their deposits, Green and Lin (2003), building on Wallace ((1998), (1990)), show that it is possible to implement the first-best socially optimal risk-sharing allocation without bank

²Through out this paper we focus on the case with a single monopoly bank and therefore ignore issues of financial contagion (see, for instance, Allen and Gale (2000)).

runs. On the other hand, by imposing further restrictions on banking contracts, Peck and Shell (2003) obtain equilibrium bank runs as a feature of the optimal banking contract.

Another branch of the literature has focused on the relation between incomplete information about the future returns of the illiquid asset and bank runs (see, for instance, Gorton (1985), Gorton and Pennacchi (1990), Postlewaite and Vives (1987), Chari and Jaganathan (1988), Jacklin and Bhattacharya (1988), Allen and Gale (1998)). However, in these papers, the variation in the future returns of the illiquid asset is exogenous while here the variation in future returns is a function of the investment decision of the bank and is hence endogenous.

Finally, in our paper, as in Aghion and Bolton (1992), bank runs can be interpreted as a way of allocating control of over banking assets to depositors. However, unlike Aghion and Bolton (1992), the reallocation of control rights isn't triggered by some exogenous event but endogenously via depositor's actions in the second-best banking contract.

2 Banking with moral hazard

2.1 The model

The model extends Diamond-Dybvig (1983) to allow for moral hazard in banking. There are three time periods, $t = 0, 1, 2$. In each period there is a single perishable good x_t . There is a continuum of identical depositors in $[0, 1]$, indexed by i , of mass one, each endowed with one unit of the perishable good at time period $t = 0$ and nothing at $t = 1$ and $t = 2$. Each depositor has access to a storage technology that allows him to convert one unit of the consumption good invested at $t = 0$ to 1 unit of the consumption good at $t = 1$ or to 1 unit of the consumption good at $t = 2$.

Depositors preferences over consumption are identical ex-ante, i.e. as of period 0. Each faces a privately observed uninsurable risk of being type 1 or type 2. In period 1, each consumer learns of his type. Type 1 agents care only about consumption in period 1 while for type 2 agents, consumption in period 1 and consumption in period 2 are perfect substitutes. For each agent, only total consumption (and not its period-wise decomposition) is publicly observable. Formally, at $t = 1$, each agent has a state dependent utility function which has the following form:

$$U(x_1, x_2, \theta) = \begin{cases} u(x_1) & \text{if } i \text{ is of type 1 in state } \theta \\ u(x_1 + x_2) & \text{if } i \text{ is of type 2 in state } \theta \end{cases}$$

In each state of nature, there is a proportion λ of the continuum of agents who are of type 1 and conditional on the state of nature, each agent has an equal and independent chance of being type 1. It is assumed that λ is commonly known.

In addition, there is a bank, denoted by b . The bank's preferences over consumption is represented by the linear utility function $U^b(x_0, x_1, x_2) = x_1 +$

x_2^3 . Unlike depositors, the bank has no endowments of the consumption good at $t = 0$. However, the bank is endowed with two different asset technologies, $j = A, B$, that convert inputs of the perishable good at $t = 0$ to outputs of the perishable consumption good at $t = 1$ or $t = 2$. We will assume that the size of the bank is large relative to the size of an individual depositor⁴. As each individual depositor has a (Lebesgue) measure zero, if the bank has the same size as an individual depositor, transfers to the bank can be made without affecting the overall resource constraint. In order to capture the trade-off between making transfers to the bank and efficient risk sharing between depositors, the bank has to be large relative to the depositors.

The output of the perishable consumption good produced by either asset technology has two components: a “private” non-contractible component that only the bank can access and consume and a “public” component which depositors can access and consume as well. Both the “public” and the “private” component of both asset technologies are characterized by constant returns to scale. For each unit of the consumption good invested in $t = 0$, asset technology j , $j = A, B$, yields either 1 unit of the “public” component of the consumption good if the project is terminated at $t = 1$ or $R_j > 0$ units of the “public” component of the consumption good at $t = 2$ if the project continues to $t = 2$. In addition, for each unit of the consumption good invested in $t = 0$, asset technology j , $j = A, B$, yields 1 unit of the “private” non-contractible component of the consumption good if the project is terminated at $t = 1$, or $R_j^b > 0$ units of the “private” component of the consumption good at $t = 2$ if the project continues to $t = 2$ ⁵. In addition, at $t = 0$, the bank incurs a direct private utility cost c_j per unit of the consumption good invested in asset j at $t = 0$.

In order to operate either of these two asset technologies, the bank has to mobilize the endowments of the depositors. At $t = 0$, we assume that mobilizing depositors’ endowments requires a banking contract which specifies an allocation for each type of depositor and an investment portfolio for the bank.

Any contract used must satisfy the following constraints:

- (a) the bank controls any investment that is made into either of these two asset technologies and the operation of both these two asset technologies,
- (b) no other agent in the economy has the human capital to operate either of these two technologies,
- (c) no other agent can replace the bank to take over the operation of either illiquid asset from the bank at $t = 1$,
- (d) at $t = 1$ verifying or observing the investment decision of the bank, made

³The assumption that $u^b(\cdot)$ is linear simplifies the computations and the notation considerably. All the results stated here extend, with appropriately modified computations, to the case where $u^b(\cdot)$ is a strictly increasing in consumption.

⁴Technically, the set of agents is modelled as a mixed measure space where each individual depositor has a Lebesgue measure zero (and therefore is part of an atomless continuum of depositors) while the bank is an atom with measure one. For details on how construct such a measure space see Codognato and Ghosal (2001).

⁵The assumption that within a technology there is no choice as to how much of the investment goes into the public component and how much into the private component is a simplification and nothing essential in our results depends on this analysis.

at $t = 0$, is possible only if an appropriate monitoring technology is available,

(e) the public return at $t = 1$ is observed by the depositors and/or an outside agent (a court) only if the asset technology is terminated at $t = 1$ and the public return at $t = 2$ is observed by the depositors and/or the outside agent only at $t = 2$.

The consequence of making these assumptions is that, in the absence of a perfect monitoring technology, the investment decision of the bank at $t = 0$ is non-contractible. The combination of non-contractible actions together with the private non-contractible component to asset payoffs is the source of the moral hazard problem in banking.

In addition, we make some further assumptions on depositor's preferences and the two asset technologies:

(A1) $u(\cdot)$ is strictly increasing, strictly concave, smooth utility function,

(A2) $-\frac{u''(x)x}{u'(x)} > 1$ for all $x > 0$,

(A3) $R_A > 1 > R_B > 0$,

(A4) $R_A + R_A^b > R_B + R_B^b$,

(A5) $1 < R_j^b, j = A, B$,

(A6) $c_A < c_B$.

Assumption (A1) implies that each individual type 1 and type 2 depositor is risk-averse. Assumption (A2) implies that whenever there is efficient risk-sharing, the bank has to provide liquidity services: narrow banking is ruled out. Under assumption (A3), it can never be in the depositor's interest for the bank to invest in asset B : depositors will prefer to invest their endowments of the consumption good in the storage technology. Assumption (A4) implies that production efficiency requires investment in asset A . Assumption (A5) implies that for either asset, the bank prefers the project to continue to $t = 2$. Finally, assumption (A6) implies that the effort cost to the bank of investing in asset A is less than the effort cost of investing in asset B .

An allocation is a vector $(\gamma_s, \gamma, x, x^b)$ where (γ_s, γ) is the asset (equivalently, investment) portfolio (chosen at $t = 0$) and describes the proportion of endowments invested in the storage technology and asset technology A (with proportion $1 - \gamma_s - \gamma$ invested in asset technology B), $x = (x_1^1, x_1^2, x_2^1, x_2^2)$ is the consumption allocation of the depositors (x_t^k is the consumption of type k depositor in time period t , $k = 1, 2$ and $t = 1, 2$) and describes what each type of depositor consumes in each period and $x^b = (x_1^b, x_2^b)$ describes the consumption allocation to the bank. A consequence of assumptions (A4) and (A5) is that productive efficiency, and hence social efficiency, requires that $\gamma = 1$.

We study two different scenarios in this paper: one in which depositors have all the bargaining power and other in which the bank has all the bargaining power. When depositors have all the bargaining power, as all depositors are identical ex-ante, a representative depositor, acting on behalf of all other depositors, makes a "take-it-or-leave-it" offer of a banking contract to the bank, which the bank can either accept or reject. The roles of the bank and the representative depositor are reversed when the bank has all the bargaining power.

In all cases, we first characterize the (constrained) efficient allocation and

then, examine the implementation of this allocation using contracts (games).

Given the sequential structure of the banking scenario studied here, our notion of implementation requires that agents use dominant actions in every subgame of the banking contract.

In the first three subsections of this section, we study the case when depositors have all the bargaining power. The case where the bank has all the bargaining power is studied in the final subsection of this section.

2.2 Depositor control and contracts without bank runs

Clearly, the objective function of the representative depositor is

$$g(x) = \lambda u(x_1^1) + (1 - \lambda)u(x_1^2 + x_2^2)$$

where $g(x)$ is a weighted sum of type 1 and type 2 depositors preferences where the weights used reflect the aggregate proportions of type 1 and type 2 depositors. When there is no monitoring technology available, the representative depositor cannot condition transfers to the bank at $t = 1$ or $t = 2$, on the investment portfolio chosen by the bank at $t = 0$. In this case, making transfers to the bank will have no impact on the bank's incentives. Without a monitoring technology, in any banking contract written by the representative depositor, no transfers, over and above the private non-contractible payoff the bank receives by operating either asset technology, will be made to the bank.

Consider the case when $R_A^b \geq R_B^b$. By assumption, $c_A < c_B$, and therefore, $R_A^b - c_A \geq R_B^b - c_B$. In this case, we claim that the representative depositor can design a banking contract that implements the efficient risk-sharing without bank runs. The representative depositor solves the following maximization problem (labelled (P) for later reference):

$$\max_{\{\gamma, x, x^b\}} g(x)$$

subject to

$$(P1) \quad R_A \geq R_A (\lambda x_1^1 + (1 - \lambda) x_1^2) + (\lambda x_2^1 + (1 - \lambda) x_2^2),$$

$$(P2) \quad (\hat{A}2) \quad x_t^k \geq 0, k = 1, 2, t = 1, 2,$$

$$(P3) \quad u(x_1^1) \geq u(x_1^2),$$

$$(P4) \quad u(x_1^2 + x_2^2) \geq u(x_1^1 + x_2^1).$$

The solutions to (P) satisfy the equations

$$(1) \quad x_1^{2*} = x_2^{1*} = 0,$$

$$(2) \quad u'(x_1^{1*}) = R_A u'(x_2^{2*}),$$

$$(3) \quad R_A = \lambda R_A x_1^{1*} + (1 - \lambda) x_2^{2*},$$

while for the bank

$$(4a) \quad \gamma^* = 1,$$

$$(4b) \quad x_1^{b*} = 0,$$

$$(4c) \quad x_2^{b*} = R_A^b.$$

Allocations characterized by (1)–(4) correspond to the first-best allocations in Diamond and Dybvig (1983). As in their paper, under the assumption (A1),

$u''(x) < 0$ while under assumption (A3), $R_A > 1$. Therefore, using (2), it follows that $x_2^{2*} > x_1^{1*}$. This ensures that the truth telling constraints (P3) is satisfied. Under the additional assumption that $-\frac{u''(x)x}{u'(x)} > 1$ it also follows that $x_1^{1*} > 1$ while $x_2^{2*} < R_A$. This implies that whenever there is efficient risk-sharing, the bank has to provide liquidity services: narrow banking is ruled out.

Again, as in Diamond and Dybvig (1983), there is a banking contract $(\hat{\gamma}, \hat{r}, \hat{k})$, satisfying a sequential service constraint and with suspension of convertibility⁶, that implements (γ^*, x^*) . Each depositor who withdraws in period 1 obtains a fixed claim $\hat{r}_1 = x_1^{1*}$ per unit deposited at $t = 0$ and convertibility is suspended at $\hat{k} = \lambda$. If banking continues to $t = 2$, each agent who withdraws at $t = 2$, obtains a fixed claim $\hat{r}_2 = x_2^{2*}$ per unit deposited at $t = 0$ and not withdrawn at $t = 1$. Moreover, $\hat{\gamma} = 1$. The argument establishing how such a contract implements first-best risk sharing follows Diamond and Dybvig (1983) and is reported in the appendix.

What happens if $R_A^b < R_B^b$? As long as $R_A^b - c_A \geq R_B^b - c_B$, nothing essential in the preceding argument changes and efficient risk-sharing without bank runs can still be implemented. On the other hand, when $R_A^b - c_A < R_B^b - c_B$ ⁷, if costless and perfect monitoring of the bank's portfolio choice, made at $t = 0$, is possible at $t = 1$, the depositor can write a banking contract that conditions transfers at $t = 1$ on portfolio choices made by the bank at $t = 0$. Whether the representative depositor will actually choose to do so is an issue examined in the next subsection.

2.3 Depositor control and bank runs with costless and perfect monitoring

In this subsection, we examine the case where at time $t = 0$, it becomes common knowledge that the representative depositor invested in the monitoring technology and study the case of *costless and perfect* monitoring. With monitoring, we assume that (a) before the bank makes its investment decision, it becomes common knowledge that depositors have invested in the monitoring technology, and (b) the results of monitoring are revealed, publicly, before depositors choose whether or not to withdraw their deposits.

Specifically, we assume that at $t = 0$, it is common knowledge that at the be-

⁶The sequential service constraint implies that (a) withdrawal tenders are served sequentially in random order until the bank runs out of assets and (b) the bank's payoff to any agent can depend only on the agent's place in the line and not on any future information about agents behind him in the line while suspension of convertibility implies that any agent attempting to withdraw at $t = 1$ will receive nothing at $t = 1$ if he attempts to withdraw at $t = 1$ after a fraction \hat{k} of depositors. Note that along the equilibrium path of play, neither the sequential service constraint nor the suspension of convertibility constraint ever binds in any of the banking contracts, whether random or deterministic, studied in this paper.

⁷Taken together, the inequalities $c_A < c_B$ and $R_A^b - c_A < R_B^b - c_B$, imply that from the bank's perspective the project with higher net private utility return at $t = 2$ is also the one with the higher effort cost at $t = 0$. When $R_A^b - c_A < R_B^b - c_B$, as $R_A > R_B$, the long-run interests of the depositors and the bank are no longer aligned.

ginning of $t = 1$, the representative depositor observes the investment allocation across assets made by the bank at $t = 0$. We assume that $R_A^b - c_A < R_B^b - c_B$. An obvious additional component in a banking contract is that now the representative depositor can commit to make transfers to the bank at $t = 2$, contingent on the actions chosen by the bank at $t = 0$. Note that under our assumptions, the representative depositor cannot make negative transfers to the bank. This is because, by assumption, the payoffs of the bank are private and non-contractible. Therefore, any transfer made to the bank has to be non-negative.

Suppose the representative depositor commits to make a transfer, at $t = 2$, to the bank of $\tau_2^b(\gamma)$, such that $R_A^b + \tau_2^b(1) - c_A = R_B^b - c_B + \varepsilon$, where $\varepsilon > 0$ but infinitesimal, while $\tau_2^b(\gamma) = 0$ for all $\gamma \neq 1$. In this case, the bank will choose $\gamma = 1$ if banking continues to $t = 2$. The resource constraint is

(P'1) $R_A - \tau_2^b(1) \geq R_A(\lambda x_1^1 + (1 - \lambda)x_1^2) + (\lambda x_2^1 + (1 - \lambda)x_2^2)$. Let γ' , x' denote a solution to the representative depositor's maximization problem with the resource constraint (P'1). Remark that a necessary condition for efficient risk-sharing between type 1 and type 2 depositors is that the equations (1), (2) and the inequality (P'1) be simultaneously satisfied. Remark also that for depositors' participation constraints to be satisfied, any solution to the representative depositor's maximization problem must also satisfy the inequality

$$(5) \quad x_1'^1 \geq 1 \text{ and } x_1'^2 + x_2'^2 \geq 1.$$

The following example demonstrates the (robust) possibility that there is no x' satisfying (1), (2), (P'1) and (5).

Example 1 Suppose $u(x) = \frac{x^{1-\beta}}{1-\beta}$, $\lambda > 0$ and $R_A - \tau_2^b(1) < 1 - \varepsilon$. Suppose to the contrary, there is some x' satisfying (1), (2), (P'1) and (5). Then, any x' that satisfies (1), (2) must also satisfy the equation $R_A^{\frac{1}{\beta}} x_1'^1 = x_2'^2$. Evaluated at $x_1'^1 = 1$, the expression on the right hand side of (P'1) is $\lambda R_A + (1 - \lambda) R_A^{\frac{1}{\beta}} > 1$ as $R_A > 1$ while the left hand side of (P'1) is strictly less than $1 - \varepsilon$, a contradiction.

The above example shows that with transfers, even with costless and perfect monitoring, there is, in general, a trade-off between (a) efficient risk-sharing between type 1 and type 2 depositors and provision of liquidity, and (b) providing the bank with appropriate incentives. In robust banking scenarios, banking contracts with transfers results in no risk-sharing between type 1 and type 2 depositors and consequently, no provision of liquidity i.e. in narrow banking.

In general, however, even if risk-sharing between type 1 and type 2 depositors and providing the bank with appropriate incentives are consistent i.e. if there is a solution to the representative depositor's problem satisfying (1), (2), (P'1) and (5), incentive compatible transfers to the bank will lower consumption for both types of depositors. To make this point, observe that when equations (1) and (2) are satisfied, we have that

$$u'(x_1'^1) = R_A u'(x_2'^2)$$

and as $u''(\cdot) < 0$, the preceding equation implicitly defines a function $f(\cdot)$ such that

$$x_2'^2 = f(x_1'^1)$$

where

$$f(x_1'^1) = u'^{-1}\left(\frac{u'(x_1'^1)}{R_A}\right).$$

Consider the inequality

$$(6) \quad R_A - \tau_2^b(1) \geq \lambda R_A + (1 - \lambda) f(1).$$

By computation, it is easily checked that when (6) holds, an interior solution to the representative depositor's problem is possible. Note that (6) is equivalent to

$$R_A u' \left(\frac{R_A - \tau_2^b(1) - \lambda R_A}{1 - \lambda} \right) < u'(1)$$

which implies that as $R_A > 1$ and $u''(\cdot) < 0$, $x_2'^2 > x_1'^1$, $x_2'^2 < f(1)$ and therefore, $x_1'^1 > 1$. But we also have that $x_1'^1 < x_1^{*1}$ and $x_2'^2 < x_2^{*2}$. It follows that if (6) holds, any solution to the representative depositor's problem can be implemented by an appropriately designed banking contract, augmented with transfers and with suspension of convertibility. However, such a contract will inevitably entail a consumption loss for both types of depositors.

As, by assumption, depositors have all the bargaining power, assuming that the action chosen by the bank, at $t = 0$ can be observed costlessly at $t = 1$, can the representative depositor design a banking contract *without transfers* that implements the allocation x^* ?

The following argument shows that this is indeed possible. The main idea of the argument is that as the representative depositor can observe γ , and therefore make the terms of the banking contract contingent on γ so that if $\gamma = 1$, there is no bank run while if $\gamma < 1$, there is a bank run (equivalently, asset liquidation) with probability one. Such a banking contract would induce the bank to choose $\gamma = 1$ at $t = 0$. Therefore, in the game induced by the banking contract, although bank runs are never observed along the equilibrium path of play, the threat of a bank run off the equilibrium path of play induces the bank to choose $\gamma = 1$ along the equilibrium path of play.

The details are as follows. Let $r'(\gamma)$ be a function defined from $[0, 1]$ to \mathfrak{R}_+^2 while let $k'(\gamma)$ be a function defined from $[0, 1]$ to itself. Consider banking contract, subject to a sequential service constraint, described by a vector (γ', r', k') such that $r_1'(1) = x_1^{*1}$ per unit deposited at $t = 0$, $r_1'(\gamma) = 1$ for $\gamma < 1$, $k'(1) = \lambda$ while $k'(\gamma) = 1$ for $\gamma < 1$, and if banking continues to $t = 2$, $r_2'(1) = x_2^{*2}$ while for $\gamma < 1$, $r_2'(\gamma) = 0$ per unit deposited at $t = 0$ and not withdrawn at $t = 1$. The contract also specifies the bank's asset portfolio where $\gamma' = 1$. It follows that when $\gamma = 1$, it is a dominant action for type one depositors to withdraw and for type two depositors not to withdraw at $t = 1$, while it is a dominant action for all types of depositors to withdraw their deposits at $t = 1$ whenever $\gamma < 1$. Anticipating this behavior by depositors, the bank will choose $\gamma = \gamma' = 1$ as this yields a payoff $R_A^b - c_A > 1 - c_A$ while choosing $\gamma < 1$

yields a payoff $\gamma(1 - c_A) + (1 - \gamma)(1 - c_B) < (1 - c_A)$ (since by assumption, $c_A < c_B$ and therefore, $1 - c_A > 1 - c_B$).

2.4 Depositor control and bank runs with non-contractible actions

What happens if $R_A^b - c_A < R_B^b - c_B$ and there is no available monitoring technology for verifying and observing the investment decision of the bank at $t = 1$? In this case, allowing transfers to the bank will have no impact on the bank's incentives. A banking contract, all of whose Nash equilibria at $t = 1$ involve a zero probability of a bank run, will fail to implement any $\gamma > 0$. As $c_A < c_B$ and $1 - c_A > 1 - c_B$ and if there is enough chance of a bank run (equivalently, asset liquidation)⁸ at $t = 1$, so that technology A gets to generate a higher private utility return to the bank than technology B , one might get the bank to invest all available resources at $t = 0$ in asset technology A . So a run is clearly necessary to implement any allocation with $\gamma > 0$. That it is sufficient is proved below. However requiring $\gamma > 0$ entails a positive probability of a bank run at $t = 1$ and although efficient risk-sharing between type 1 and type 2 depositors is never implemented with probability one, it is achieved with strictly positive probability.

Consider the randomization scheme (S, π) where $S = \{s_1, \dots, s_M\}$, $M \geq 2$, is some arbitrary but finite set of states of nature and $\pi = \{\pi_1, \dots, \pi_M\}$, $\pi_m \geq 0$, $\sum_{m=1}^M \pi_m = 1$ is a probability distribution over S ⁹. The randomization scheme works as follows: at $t = 0$, no agent, including the bank, observes s_m while at $t = 1$, before any choices are made, the realized value of s_m is revealed to all agents and as before, each depositor privately observes her own type. A random allocation is a collection $(\tilde{\gamma}, \tilde{x}, \tilde{x}^b)$ where $\tilde{\gamma} \in [0, 1]$, $\tilde{x} : S \rightarrow \mathbb{R}_+^4$ and $\tilde{x}^b : S \rightarrow \mathbb{R}_+^2$. Let $\tilde{S} = \{s_m \in S : \tilde{x}_t^k(s_m) \geq 0, \lambda \tilde{x}_1^1(s_m) + (1 - \lambda) \tilde{x}_1^2(s_m) \geq 1\}$, $\tilde{M} = \{m : s_m \in \tilde{S}\}$ and let $\tilde{\pi} = \sum_{m \in \tilde{M}} \pi_m$. The interpretation is that whenever $s_m \in \tilde{S}$, the asset needs to be liquidated at $t = 1$ and therefore, $\tilde{\pi}$ is the probability of a bank run. Therefore, at $t = 1$, both the bank and the depositors can condition any choices they make on s_m .

For $\gamma \in [0, 1]$, let $R_\gamma = \gamma R_A + (1 - \gamma) R_B$. The representative depositor's maximization problem (labelled as $(\tilde{P}1)$ for later reference) is:

$$\max_{\{S, \pi, \tilde{\gamma}, \tilde{x}, \tilde{x}^b\}} \sum_{s_m \in S} \pi_m g(\tilde{x}(s_m), \tilde{x}^b(s_m))$$

subject to

$$(\tilde{P}1) \quad R_{\tilde{\gamma}}(\lambda \tilde{x}_1^1(s_m) + (1 - \lambda) \tilde{x}_1^2(s_m)) + (\lambda \tilde{x}_2^1(s_m) + (1 - \lambda) \tilde{x}_2^2(s_m)) \leq R_{\tilde{\gamma}}, s_m \in S$$

⁸By assumption, no other agent can replace the bank to take over the operation of either illiquid asset from the bank at $t = 1$ which, in turn, implies that the second-best banking contract studied below is renegotiation proof.

⁹Obviously, there are other ways of introducing randomness in the social planner's problem. We choose the randomization scheme presented here as a matter of convenience.

- ($\tilde{P}2$) $\tilde{x}_t^k(s_m) \geq 0, k = 1, 2, t = 1, 2, s_m \in S,$
($\tilde{P}3$) $u(\tilde{x}_1^1(s_m)) \geq u(\tilde{x}_1^2(s_m)), s_m \in S,$
($\tilde{P}4$) $u(\tilde{x}_1^2(s_m) + \tilde{x}_2^2(s_m)) \geq u(\tilde{x}_1^1(s_m) + \tilde{x}_2^1(s_m)), s_m \in S,$
($\tilde{P}5$) $\tilde{\gamma} \in \arg \max_{\gamma \in [0,1]} \left\{ \begin{array}{l} \bar{\pi} + (1 - \bar{\pi})(\gamma R_A^b + (1 - \gamma) R_B^b) \\ - [\gamma c_A + (1 - \gamma) c_B] \end{array} \right\}.$

Fix a pair (S, π) , $M \geq 2$, such that \tilde{S} is non-empty. At any socially optimal allocation we must have that $\tilde{\gamma} = 1$. Evaluated at $\tilde{\gamma} = 1$, the payoffs of the bank is given by the expression

$$\bar{\pi} + (1 - \bar{\pi}) R_A^b - c_A.$$

For the moral hazard constraint ($\tilde{P}5$) to be satisfied, we require that

$$\bar{\pi} + (1 - \bar{\pi}) R_A^b - c_A \geq \left\{ \begin{array}{l} \bar{\pi} + (1 - \bar{\pi})(\gamma R_A^b + (1 - \gamma) R_B^b) \\ - [\gamma c_A + (1 - \gamma) c_B] \end{array} \right\}$$

for all $\gamma \in [0, 1]$. When $\bar{\pi} = 0$, as $R_A^b < R_B^b$, ($\tilde{P}5$) will always be violated for all $\gamma \in [0, 1]$. On the other hand when $\bar{\pi} = 1$, as $1 - c_A > 1 - c_B$, ($\tilde{P}5$) will hold as a strict inequality for all $\gamma \in [0, 1]$. Further, both sides of the inequality are continuous in π and $R_A^b > 1$, the expression $\bar{\pi} + (1 - \bar{\pi}) R_A^b$ is also decreasing in $\bar{\pi}$ at the rate $1 - R_A^b$; moreover, as $R_B^b > 1$, for each $\gamma \in [0, 1]$, the expression $\bar{\pi} + (1 - \bar{\pi})(\gamma R_A^b + (1 - \gamma) R_B^b)$ is also decreasing in $\bar{\pi}$ at the rate $1 - (\gamma R_A^b + (1 - \gamma) R_B^b)$. It follows that for each $\gamma \in [0, 1]$, as $R_B^b > R_A^b > 1$,

$$\begin{aligned} & |1 - R_A^b| \\ &= |R_A^b - 1| \\ &< |(\gamma R_A^b + (1 - \gamma) R_B^b) - 1| \\ &= |1 - (\gamma R_A^b + (1 - \gamma) R_B^b)| \end{aligned}$$

and therefore, there exists a unique threshold $\tilde{\pi}$, $0 < \tilde{\pi} < 1$, such that for all $\bar{\pi} > \tilde{\pi}$, $\bar{\pi} < 1$, the moral hazard constraint ($\tilde{P}5$) holds as a strict inequality for all $\gamma \in [0, 1]$.

Production efficiency and hence, constrained efficient risk-sharing requires that $\tilde{\gamma} = 1$. Next, note that

- (1') $\tilde{x}_1^{2*}(s_m) = 0, s_m \in S,$
(3') $R_A(\lambda \tilde{x}_1^{1*}(s_m) + (1 - \lambda) \tilde{x}_1^{2*}(s_m)) + (1 - \lambda) \tilde{x}_2^{2*}(s_m) = R_A, s_m \in S,$
(4'a) $\tilde{x}_1^{b*}(s_m) = r_A^b, s_m \in \tilde{S},$
(4'b) $\tilde{x}_1^{b*}(s_m) = 0, s_m \in S \setminus \tilde{S},$
(4'c) $\tilde{x}_2^{b*}(s_m) = 0, s_m \in \tilde{S},$
(4'd) $\tilde{x}_2^{b*}(s_m) = R_A^b, s_m \in S \setminus \tilde{S}.$

By construction,

$$(2'a) \quad \tilde{x}_2^{2*}(s_m) = 0, s_m \in \tilde{S},$$

and as $u'(\cdot) > 0$, using ($\tilde{P}3$) and ($\tilde{P}4$), we obtain that

$$(2'b) \quad \tilde{x}_1^{1*}(s_m) = \tilde{x}_1^{2*}(s_m), s_m \in \tilde{S},$$

while using (3'),

$$(2'c) \quad \tilde{x}_1^{1*}(s_m) = \tilde{x}_1^{2*}(s_m) = 1, s_m \in \tilde{S},$$

as when there is a bank run, this is the only allocation consistent with the feasibility and participation constraints in \tilde{P} . It follows that

$$(2'd) \quad \tilde{x}_1^{2*}(s_m) = 0, s_m \in S \setminus \bar{S},$$

and

$$(2'e) \quad u'(\tilde{x}_1^{1*}(s_m)) = R_A u'(\tilde{x}_2^{2*}(s_m)), s_m \in S \setminus \bar{S},$$

and therefore

$$(2'f) \quad \tilde{x}_1^{1*}(s_m) = x_1^{1*}, s_m \in S \setminus \bar{S},$$

$$(2'g) \quad \tilde{x}_2^{2*}(s_m) = x_2^{2*}, s_m \in S \setminus \bar{S}.$$

It follows that for a fixed pair (S, π) , $M \geq 2$, such that \bar{S} is non-empty and $\bar{\pi} \geq \bar{\pi}$, $\bar{\pi} < 1$, there is a unique random allocation satisfying (1') – (4'). For a fixed pair (S, π) , such that either \bar{S} is empty or $\bar{\pi} < \bar{\pi}$, we have already established that there is no allocation that satisfies (1') – (4'). Finally, for a fixed pair (S, π) , such that either $S \setminus \bar{S}$ is empty or $\bar{\pi} = 1$, both $\tilde{x}_1^{1*}(s_m) = \tilde{x}_2^{2*}(s_m) = 1$, $\tilde{x}_1^{b*}(s_m) = r_A^b$ and $\tilde{x}_2^{b*}(s_m) = 0$ for all $s_m \in S$. In this case observe that though the moral hazard constraint (4') always holds, there is no state at which there is efficient risk-sharing.

Next, we examine the optimal choice of the pair (S, π) . First note that at any optimal choice of (S, π) , generating a unique random allocation satisfying (1') – (4'), both $S \setminus \bar{S}$ and \bar{S} will have to be non empty. Fix a pair (S, π) , $M \geq 2$, generating a unique random allocation satisfying (1') – (4') denoted by (\tilde{x}, \tilde{x}^b) . Then, there is a pair (S', π') , $S' = \{s'_1, s'_2\}$ and $\pi' = \{\pi'_1, \pi'_2\}$ so that (a) $\tilde{x}(s_m) = \tilde{x}(s'_1)$ and $\tilde{x}^b(s_m) = \tilde{x}^b(s'_1)$ for all $s_m \in S \setminus \bar{S}$, (b) $\tilde{x}(s_m) = \tilde{x}(s'_2)$ and $\tilde{x}^b(s_m) = \tilde{x}^b(s'_2)$ for all $s_m \in \bar{S}$, (c) $\pi'_1 = (1 - \bar{\pi})$ and $\pi'_2 = \bar{\pi}$ and therefore,

$$\sum_{s_m \in S} \pi_m g(\tilde{x}(s_m), \tilde{x}^b(s_m)) = \pi'_1 g(\tilde{x}(s'_1), \tilde{x}^b(s'_1)) + \pi'_2 g(\tilde{x}(s'_2), \tilde{x}^b(s'_2))$$

It follows that without loss of generality, we can restrict attention to S' such that $M = 2$. Finally, as the representative depositor wants to maximize the probability with which efficient risk sharing is implemented, she will choose the lowest value of π'_2 compatible with $(\tilde{P}5)$ being satisfied as a strict inequality i.e. choose $\pi'_2 = \bar{\pi} + \varepsilon < 1$, where $\varepsilon > 0$ is small but strictly positive number so that $(\tilde{P}5)$ is satisfied as a strict inequality. Setting $\pi'_2 = \bar{\pi}$ will imply that $(\tilde{P}5)$ will be satisfied as an equality in which case the representative depositor will have to rely on the bank choosing a tie-breaking rule in favour of asset technology A .

It remains to specify a random banking contract that will implement the random allocation satisfying (1') – (4'). A random banking contract¹⁰ is described by the vector $(S', \pi', \tilde{\gamma}, \tilde{r}, \tilde{k})$ where the pair (S', π') are as in the preceding paragraph, $\tilde{\gamma} = 1$ and $\tilde{r}_1(s'_1) = x_1^{1*}$, $\tilde{r}_1(s'_2) = 1$, $\tilde{r}_2(s'_1) = x_2^{2*}$, $\tilde{r}_2(s'_2) = 1$, $\tilde{k}(s'_1) = \lambda$, $\tilde{k}(s'_2) = 1$. The interpretation is that subject to a sequential service constraint and suspension of convertibility, each depositor who withdraws in period 1 obtains a random claim $\tilde{r}_1(s'_m)$, $s'_m \in S'$ per unit deposited at $t = 0$.

¹⁰As before we assume that at $t = 0$, no agent, including the bank, observes s_m while at $t = 1$, before any choices are made, the realized value of s_m is revealed to all agents. Therefore, at $t = 1$, both the bank and the depositors can condition any choices they make on s_m .

If banking continues to $t = 2$, each agent who withdraws at $t = 2$, obtains a random claim $\tilde{r}_2(s'_m)$, $s'_m \in S'$ per unit deposited at $t = 0$. With such a contract, given $s'_m \in S'$, the payoff to per unit of deposit withdrawn at $t = 1$, which depends on the fraction of deposits serviced before agent j , k_j , is given by the expression

$$\tilde{v}_1(f_j, \tilde{r}_1(s'_m), \tilde{k}(s'_m), s'_m) = \begin{cases} u(\tilde{r}_1(s'_m)), & \text{if } f_j \leq \tilde{k}(s'_m) \\ u(0), & k_j > \tilde{k}(s'_m) \end{cases}$$

while the period 2 payoff per unit deposit withdrawn at $t = 2$, which depends on total fraction of deposits withdrawn in period 1, $k(s'_m)$, is given by the expression

$$\tilde{v}_2(f, \tilde{r}_1(s'_m), s'_m) = \begin{cases} u(\tilde{r}_2(s'_m)), & \text{if } 1 > k(s'_m) \tilde{r}_1(s'_m) \\ 0, & \text{otherwise} \end{cases}$$

At $t = 1$, for each value of $s'_m \in S'$, the above contract induces a noncooperative game between depositors where each depositor chooses what fraction of their deposits to withdraw. Fix $s'_m \in S'$. Suppose depositor j withdraws a fraction $\mu_j(s'_m)$. Then, a type 1 depositor obtains a payoff $\mu_j(s'_m) \tilde{v}_1(f_j, \tilde{r}_1(s'_m), \tilde{k}(s'_m), s'_m)$ while a type 2 depositor obtains a payoff of $\mu_j(s'_m) \tilde{v}_1(f_j, \tilde{r}_1(s'_m), \tilde{k}(s'_m), s'_m) + (1 - \mu_j(s'_m)) \tilde{v}_2(f, \tilde{r}_1(s'_m), s'_m)$. Remark that for a type 1 depositor, $\mu_j(s'_m) = 0$ strictly dominates all other actions. For s'_1 , as $\tilde{k}(s'_1) = \lambda$, $\tilde{r}_1(s'_1) = x_1^{1*}$ and $\tilde{r}_2(s'_1) = x_2^{2*}$, it follows that $\tilde{v}_2(f, \tilde{r}_1(s'_1), s'_1) > \tilde{v}_1(f_j, \tilde{r}_1(s'_1), \tilde{k}(s'_1), s'_1)$ and for type 2 depositors, $\mu_j(s'_1) = 0$ strictly dominates all other actions. For s'_2 , as $\tilde{r}_1(s'_2) = 1$ while $\tilde{r}_2(s'_2) = 0$, it follows that for type 2 depositors, $\mu_j(s'_2) = 1$ strictly dominates all other actions. Therefore, (i) for s'_1 , the unique Nash equilibrium in strictly dominant actions is $\mu_j(s'_1) = 1$ if j is a type 1 depositor while $\mu_j(s'_1) = 0$ if j is a type 2 depositor and (ii) s'_2 , the unique Nash equilibrium in strictly dominant actions is $\mu_j(s'_1) = 1$ for all j . At $t = 0$, the bank's payoffs are:

$$\tilde{v}^b(\gamma) = \pi'_2 + \pi'_2 (\gamma R_A^b + (1 - \gamma) R_B^b) - [\gamma c_A + (1 - \gamma) c_B]$$

As (6') holds as a strict inequality, it follows that choosing $\gamma = \tilde{\gamma} = 1$ is the strictly dominant choice for the bank.

The above random banking contract implements the allocation satisfying (1') - (4').

We summarize the above discussion with the following proposition:

Proposition 2 *When $R_A^b - c_A < R_B^b - c_B$, the second-best allocation determined by (1') - (4') is implemented by the random banking contract $(S', \pi', \tilde{\gamma}, \tilde{r}, \tilde{k})$.*

The above result makes clear that whenever the moral hazard constraint binds, bank runs are an endemic feature of the banking contract and limit efficient risk-sharing (equivalently, efficient liquidity provision) by banks.

Remark 3 *In the preceding analysis, what is critical is that the aggregate proportion λ of type 1 depositors is commonly observed at $t = 1$. Consider a modification of the problem so that the aggregate proportion λ of type 1 depositors can be any one element from a set $\{\lambda_1, \dots, \lambda_N\}$ and at $t = 0$ there is a common probability distribution over $\{\lambda_1, \dots, \lambda_N\}$. However, at $t = 1$, the realized value of λ is commonly observed. In such a case, the efficient risk-sharing allocation will be contingent on $\lambda \in \{\lambda_1, \dots, \lambda_N\}$ and all the preceding results, after appropriate reformulation, continue to apply. In this sense, our results don't require but can be extended to scenarios with exogenous uncertainty. With exogenous uncertainty, the class of random contracts studied here, introduce noise that is independent of fundamentals in the banking process.*

2.5 Bank control and banking contracts

In this subsection, we assume that the bank has all the bargaining power. By assumption, both $R_A + R_A^b > R_B + R_B^b$ and $c_A < c_B$, and therefore, as long as the bank is able to appropriate all of the public return generated by investment in the asset A , subject to the participation constraint for depositors that $x_1^1 \geq 1$ and $x_1^2 + x_2^2 \geq 1$, the bank will choose $\gamma = 1$. As the utility of the bank is strictly increasing in consumption, the bank will set $x_1^1 = x_2^2 = 1$, and $x_1^2 = x_2^1 = 0$. The problem is that at this corner allocation, efficient risk-sharing between type 1 and type 2 depositors is not possible: there will be no provision of liquidity, resulting in narrow banking.

There is a banking contract (γ, r, k) , satisfying a sequential service constraint and with suspension of convertibility¹¹, that will implement the above allocation in dominant actions. Each depositor who withdraws in period 1 obtains a fixed claim $r_1 = 1$ per unit deposited at $t = 0$ and convertibility is suspended at $k = \lambda$. If banking continues to $t = 2$, each agent who withdraws at $t = 2$, obtains a fixed claim $r_2 = 1 + \varepsilon$ per unit deposited at $t = 0$ and not withdrawn at $t = 1$ for some strictly positive but small ε . Moreover, $\gamma = 1$. It follows that, by using arguments symmetric to ones used in the preceding analysis, the banking contract will implement narrow banking.

3 Monitoring and collaterals

In this section we extend the model of banking studied in the preceding section in two ways: by studying more general monitoring scenarios and allowing the

¹¹The sequential service constraint implies that (a) withdrawal tenders are served sequentially in random order until the bank runs out of assets and (b) the bank's payoff to any agent can depend only on the agent's place in the line and not on any future information about agents behind him in the line while suspension of convertibility implies that any agent attempting to withdraw at $t = 1$ will receive nothing at $t = 1$ if he attempts to withdraw at $t = 1$ after a fraction \hat{k} of depositors. Note that along the equilibrium path of play, neither the sequential service constraint nor the suspension of convertibility constraint ever binds in any of the banking contracts, whether random or deterministic, studied in this paper.

use of collaterals in banking. Throughout this section, we will assume that the depositors have all the bargaining power.

3.1 Costly and imperfect monitoring

We assume that at $t = 0$, before the bank has mobilized the endowments of depositors and before the bank has made any investment decision, the representative depositor invests in a monitoring technology with a resource cost M . We will assume that the resource cost is funded by a tax on the total endowments mobilized by the bank at $t = 0$. By investing in the monitoring technology, conditional on γ being chosen by the bank at $t = 0$, the representative depositor observes a signal χ , defined over subsets of $[0, 1]$ so that $\chi = \gamma$ with probability $q > 0$ while $\chi = [0, 1]$ with probability $1 - q$.

With a monitoring cost M , remark that the resource constraint is

$$(7) \quad R_A(1 - M) \geq R_A(\lambda x_1^1 + (1 - \lambda)x_1^2) + (\lambda x_2^1 + (1 - \lambda)x_2^2).$$

Consider first the case where $q = 1$. This is the case of costly but perfect monitoring. It is evident from (7) that the trade-off between efficient risk-sharing between type 1 and type 2 depositors and provision of incentive to the bank that arise in the case with costless, perfect monitoring and transfers also arise relevant with costly, perfect monitoring. Suppose, for simplicity, the representative depositor makes no transfers to banks but seeks to implement efficient risk-sharing between type one and type two depositors by designing a banking contract that holds out the threat of a bank run off the equilibrium path of play. Even in this case it is straightforward to see from (7), using arguments similar to ones already used in section 2.3, that (a) for high values of M , i.e. when $R_A(1 - M) < \lambda R_A + (1 - \lambda)f(1)$, it is possible that there is a trade-off between monitoring and efficient risk-sharing between type one and type two depositors and (b) whenever $M > 0$, monitoring by banks will lower consumption among both types of depositors.

Now, consider the case when $q > 0$ but $q < 1$. Assume that $R_A(1 - M) \geq \lambda R_A + (1 - \lambda)f(1)$. In this case, let the efficient risk sharing allocation with monitoring between type one and type two depositors be denoted by x^M . Note that (1), (2), (7) and (5) will characterize x^M and as $R_A > 1$ and $u''(\cdot) < 0$, $x_2^{M,2} > x_1^{M,1}$, $x_2^{M,2} < f(1)$ and therefore, $x_1^{M,1} > 1$. Suppose that $r_A^b > r_B^b$.

Let $S' = \{s'_1, s'_2\}$ and $\pi^M = \{\pi_1^M, \pi_2^M\}$ be a randomization scheme chosen by the representative depositor at $t = 0$. Let \mathcal{C} be the collection of subsets of $[0, 1]$ consisting of singleton sets $\{i\}$ for each $i \in [0, 1]$ and $[0, 1]$ itself. We can interpret \mathcal{C} as the set of signals generated by the monitoring technology. Let $r^M(\cdot, \cdot)$ be a function defined from $\mathcal{C} \times S'$ to \mathbb{R}_+^2 while let $k^M(\cdot, \cdot)$ be function defined from $\mathcal{C} \times S'$ to $[0, 1]$. Consider the banking contract, subject to a sequential service constraint, described by a vector $(S', \pi^M, \gamma^M, r^M, k^M)$ such that (i) for all $s \in S'$, $r_1^M(\{1\}, s) = x_1^{M,1}$ with $r_1^M(\{\gamma\}, s) = 1 - M^{12}$ for $\gamma < 1$ per unit deposited at $t = 0$, (ii) $r_1^M(\{[0, 1]\}, s'_1) = x_1^{M,1}$ $r_1^M(\{[0, 1]\}, s'_2) = 1 - M$ per

¹²We assume that $0 < M < \hat{M} < 1$ where \hat{M} is sufficiently small so that farsighted depositors always have an incentive to participate in banking.

unit deposited at $t = 0$, (iii) for all $s \in S'$, $k^M(\{1\}, s) = \lambda$ with $k^M(\{\gamma\}, s) = 1$ for $\gamma < 1$, and (iv) $k^M(\{[0, 1]\}, s'_1) = \lambda$ with $k^M(\{[0, 1]\}, s'_2) = 1$. If banking continues to $t = 2$, (i) for all $s \in S'$, $r_2^M(\{1\}, s) = x_2^{M,1}$ with $r_2^M(\{\gamma\}, s) = 0$ for $\gamma < 1$ per unit deposited at $t = 0$ and not withdrawn at $t = 1$, (ii) $r_2^M(\{[0, 1]\}, s'_1) = x_1^{M,1}$ with $r_2^M(\{[0, 1]\}, s'_2) = 0$ per unit deposited at $t = 0$ and not withdrawn at $t = 1$. The contract also specifies the bank's asset portfolio where $\gamma^M = 1$. Using arguments symmetric to the ones used in establishing proposition 2, for each $s \in S'$, when $\chi = 1$, it is a dominant action for type one depositors to withdraw and for type two depositors not to withdraw at $t = 1$ while it is a dominant action for all types of depositors to withdraw their deposits at $t = 1$ whenever $\chi < 1$. When $\chi = [0, 1]$, it is a dominant action for type one depositors to withdraw and for type two depositors not to withdraw at $t = 1$ if $s = s'_1$ while it is a dominant action for all types of depositors to withdraw their deposits at $t = 1$ if $s = s'_2$. Let $\bar{\pi}^M = (1 - q)\pi_2^M$. Anticipating this behavior by depositors, the bank will choose $\gamma = \gamma^M = 1$ if and only if

$$\bar{\pi}^M + (1 - \bar{\pi}^M) R_A^b - c_A \geq \left\{ \begin{array}{l} (q + \bar{\pi}^M) \\ + (1 - (q + \bar{\pi}^M)) (\gamma R_A^b + (1 - \gamma) R_B^b) \\ - (\gamma c_A + (1 - \gamma) c_B) \end{array} \right\}$$

It follows, using arguments symmetric to the ones used in establishing proposition 2, there exists $\tilde{\pi}_2^M > 0$, $\tilde{\pi}_2^M < 1$ such that the bank's incentive compatibility constraint is satisfied iff $\pi_2^M \geq \tilde{\pi}_2^M$. Therefore, setting $\pi_2^M = \tilde{\pi}_2^M + \varepsilon$, for some ε strictly positive but close to zero make it a dominant action for the bank to choose $\gamma = \gamma^M = 1$. In this set-up, conditional on $\gamma = 1$, the probability of termination is $(1 - q)\pi_2^M < \tilde{\pi}$: in other words, efficient risk-sharing is implemented with higher probability when there is costly but imperfect monitoring relative to the random contract studied in section 2.4. However, now the threat of bank runs off-the-equilibrium path of play and equilibrium bank runs, are both features of the banking contract.

3.2 Monitoring conditional on a bank run

So far all the monitoring scenarios we have studied have the feature that at time $t = 0$, it becomes common knowledge that the representative depositor invested in the monitoring technology. Here, in contrast, we study a monitoring scenario when all monitoring takes place conditional on there being a bank run. Intervention by central banks or government agents takes place typically after the onset of a crisis (see, for instance, OECD (2002)). The sequence of events involved in such interventions is as follows. Initially, temporary bailout measures are put in place, followed by a discovery phase when the books of the bank are examined and finally, there is a restructuring phase when the a decision is made

to either liquidate the bank or leave the bank's status unchanged^{13, 14} (see, for instance, Hoggarth and Reidhill (2003)). In what follows, we characterize the structure of second-best intervention in such sequential monitoring scenarios.

In this part of the paper, we will assume that conditional on a bank run it becomes common knowledge that the representative depositor has invested in a monitoring technology with a resource cost m . In keeping with the timing of events, we will assume that the resource cost is paid at $t = 1$. By investing in the monitoring technology, conditional on γ being chosen by the bank at $t = 0$, the representative depositor observes a signal χ , defined over subsets of $[0, 1]$ so that $\chi = \gamma$ with probability $\hat{q} > 0$ while $\chi = [0, 1]$ with probability $1 - \hat{q}$. Conditional on monitoring at $t = 1$, the resource constraint is

$$(8) \quad R_A \geq R_A (\lambda x_1^1 + (1 - \lambda) x_1^2 + m) + (\lambda x_2^1 + (1 - \lambda) x_2^2).$$

For simplicity of exposition we focus on the case when there is an allocation, denoted by x^m , to (1), (2), (8), and (5).

Let $S^m = \{s_1^m, s_2^m\}$ and $\pi^m = \{\pi_1^m, \pi_2^m\}$ be a randomization scheme, defined independently, of the randomization scheme S', π' studied in section 2.3. Let $\omega \in \{0, 1\}$ where $\omega = 0$ indicates a situation without monitoring and $\omega = 1$ indicates a situation with monitoring. Let $r^m(\cdot, \cdot, \cdot, \cdot)$ be a function defined from $S' \times \{0, 1\} \times S^m$ to \mathbb{R}_+^2 and let $k^m(\cdot, \cdot, \cdot)$ be function defined from $S' \times \{0, 1\} \times S^m$ to $[0, 1]$.

Consider the banking contract, subject to a sequential service constraint, described by a vector $(S', \pi', S^m, \pi^m, \gamma^m, r^m, k^m)$ such that:

- (i) for all $\omega \in \{0, 1\}$, $\chi \in \mathcal{C}$ and $s^m \in S^m$,

$$r_1^m(s'_1, \{\omega\}, \{\chi\}, s^m) = x_1^{1*}$$

per unit deposited at $t = 0$, and

$$\begin{aligned} k^m(s'_1, \{\omega\}, \{\chi\}, s^m) &= \lambda, \\ r_2^m(s'_1, \{\omega\}, \{\chi\}, s^m) &= x_2^{2*} \end{aligned}$$

per unit deposited at $t = 0$ and not withdrawn at $t = 1$,

- (ii) for all $\chi \in \mathcal{C}$, $s^m \in S^m$,

$$r_1^m(s'_2, \{0\}, \{\chi\}, s^m) = 1$$

per unit deposited at $t = 0$, and

$$\begin{aligned} k^m(s'_2, \{0\}, \{\chi\}, s^m) &= 1, \\ r_2^m(s'_2, \{0\}, \{\chi\}, s^m) &= 0 \end{aligned}$$

¹³This timing of events is consistent with the sequential service constraint which requires that the return obtained by a depositor depends only on her position in the queue of depositors wishing to withdraw. That a depositor can announce a desire withdraw, then await the signal and decide not to withdraw is equivalent to assuming that she can leave the queue when she changes her mind.

¹⁴Typically, there is also a third option which involves either replacing the existing management of the bank or a takeover (via merger) by a another bank. This third scenario isn't taken up in this paper as we have assumed that no other agent can replace the bank to take over the operation of either illiquid asset from the bank at $t = 1$.

per unit deposited at $t = 0$ and not withdrawn at $t = 1$,
(iii) for all $s^m \in S^m$,

$$r_1^m(s'_2, \{1\}, \{1\}, s^m) = x_1^{m,1}$$

per unit deposited at $t = 0$, and

$$\begin{aligned} k^m(s'_2, \{1\}, \{1\}, s^m) &= \lambda, \\ r_2^m(s'_2, \{1\}, \{1\}, s^m) &= x_1^{m,1} \end{aligned}$$

per unit deposited at $t = 0$ and not withdrawn at $t = 1$,
(iv) when $s' = s'^{15}$,

$$r_1^m(s'_2, \{1\}, \{\gamma\}, s^m) = 1 - m$$

per unit deposited at $t = 0$, and

$$\begin{aligned} k^m(s'_2, \{1\}, \{\gamma\}, s^m) &= 1, \\ r_2^m(s'_2, \{1\}, \{\gamma\}, s^m) &= 0 \end{aligned}$$

per unit deposited at $t = 0$ and not withdrawn at $t = 1$,
(v) when $s' = s'_2$,

$$r_1^m(s'_2, \{1\}, \{[0, 1]\}, s_1^m) = x_1^{m,1}$$

per unit deposited at $t = 0$,

$$\begin{aligned} k^m(s'_2, \{1\}, \{[0, 1]\}, s_1^m) &= \lambda, \\ r_2^m(s'_2, \{1\}, \{[0, 1]\}, s_1^m) &= x_2^{m,2} \end{aligned}$$

per unit deposited at $t = 0$ and not withdrawn at $t = 1$,
(vi) when $s' = s'_2$,

$$r_1^m(s'_2, \{1\}, \{[0, 1]\}, s_2^m) = 1 - m$$

per unit deposited at $t = 0$,

$$\begin{aligned} k^m(s'_2, \{1\}, \{[0, 1]\}, s_2^m) &= 1, \\ r_2^m(s'_2, \{1\}, \{[0, 1]\}, s_2^m) &= 0 \end{aligned}$$

per unit deposited at $t = 0$ and not withdrawn at $t = 1$. The contract also specifies the bank's asset portfolio where $\gamma^m = 1$.

The sequence of events is as follows. At $t = 0$, the representative depositor offers the contract $(S', \pi', S^m, \pi^m, \gamma^m, r^m, k^m)$. Conditional on such a contract being accepted by the bank, depositors' endowments are mobilized by the bank who, then, allocates funds across the two assets. At $t = 1$, a state $s \in S'$

¹⁵We assume that $0 < m < \hat{m} < 1$ where \hat{m} is sufficiently small so that farsighted depositors always have an incentive to participate in banking.

is selected according to the probability distribution π' and a proportion λ' of depositors choose to withdraw their deposits. If $\lambda' \leq \lambda$, there is no monitoring and each depositor j who chooses to withdraw a fraction μ_j of her deposits obtains a return of x_1^{1*} per unit deposited at $t = 0$ while a depositor j who withdraws a fraction $1 - \mu_j$ of her deposits obtains a return of x_2^{2*} per unit deposited at $t = 0$ and not withdrawn at $t = 1$. If on the other hand, $\lambda' > \lambda$, there is a temporary suspension of convertibility for all depositors, and the representative depositor operates the monitoring technology at a fixed cost m . Conditional on monitoring taking place, a state $s^m \in S^m$ is chosen according to the probability distribution π^m . Conditional on $s \in S'$, the signal χ and the state of the world s^m , the suspension of convertibility threshold is set according to k^m . At this point, each depositor who has chosen to withdraw a positive fraction her deposits can leave the queue of depositors. If a depositor j remains in the queue, and continues to choose to withdraw a fraction μ_j of her deposits, then, subject to a sequential service constraint, she obtains a return of r_1^m per unit deposited at $t = 0$ while a depositor j who leaves the queue obtains a return of r_2^m per unit deposited at $t = 0$ and not withdrawn at $t = 1$.

Using arguments symmetric to the ones used in establishing proposition 2, for each $s \in S'$, it is a dominant action for type one depositors to withdraw, while it is a dominant action for type two depositors not to withdraw at $t = 1$ if the realized state is s'_1 , and for type two depositors to withdraw at $t = 1$ if the realized state is s'_2 . Therefore, there is no monitoring at $t = 1$ if the realized state is s'_1 while monitoring is triggered if the realized state is s'_2 . Conditional on monitoring, it is a dominant action for type one depositors to not to leave the queue of depositors. If $\chi = 1$ or $\chi = [0, 1]$ and conditional on monitoring, the realized state is s_1^m , then it is a dominant action for type one depositors to not to leave the queue of depositors but if $\chi < 1$ or $\chi = [0, 1]$ and conditional on monitoring, the realized state is s_2^m , then it is a dominant action for type two depositors not to leave the queue and continue to want to withdraw their deposits at $t = 1$. Let $\bar{\pi}^m = \pi'_2(1 - \hat{q})\pi_2^m$. Anticipating this behavior by depositors, the bank will choose $\gamma = \gamma^m = 1$ if and only if

$$\{\bar{\pi}^m + (1 - \bar{\pi}^m)R_A^b\} \geq \left\{ \begin{array}{l} (\pi'_2\hat{q} + \bar{\pi}^m) \\ + (1 - (\pi'_2\hat{q} + \bar{\pi}^m))(\gamma R_A^b + (1 - \gamma)R_B^b) \\ - (\gamma c_A + (1 - \gamma)c_B) \end{array} \right\}$$

It follows, using arguments symmetric to the one used in establishing proposition 2, there exists $\tilde{\pi}^m > 0$, $\tilde{\pi}^m < 1$ such that the bank's incentive compatibility constraint is satisfied iff $\pi^m \geq \tilde{\pi}^m$. Therefore, setting $\pi^m = \tilde{\pi}^m + \varepsilon$, for some ε strictly positive but close to zero makes it a dominant action for the bank to choose $\gamma = \gamma^m = 1$. In this set-up, conditional on $\gamma = 1$, the probability of termination is $\pi'_2(1 - \hat{q})\pi_2^m < \tilde{\pi}$: in other words, efficient risk-sharing is implemented with higher probability when there is costly but imperfect monitoring relative to the random contract studied in section 2. However, now both bank runs and monitoring occur with positive probability along the equilibrium path of play.

3.3 Collaterals and bank runs

So far we have assumed that the bank's payoffs are non-contractible and cannot be attached or confiscated by an outside agent (a court). Indeed, one consequence of this assumption is that in any banking contract designed by the representative depositor, transfers to the bank have to be non-negative. We drop this assumption here. We examine the case where a proportion β of the bank's non-contractible payoffs can be seized directly by the social planner.

Assume that a proportion β , $0 \leq \beta \leq 1$, of the bank's private payoffs can be seized by the social planner. For simplicity, assume that all the other assumptions made in section 2 continue to hold. When β is large enough, i.e. when $\beta R_B^b < R_A^b$, it immediately follows that there is a banking contract which will implement the allocation x^* . Indeed, consider the banking contract $(\hat{\gamma}, \hat{r}, \hat{k})$ augmented by a commitment by the social planner to confiscate βR_B^b if banking continues to $t = 2$ and should the planner observe that the public return is consistent with $\gamma < 1$. In such case, it is easily checked that at $t = 0$, the bank will choose $\gamma = 1$ and it will be a dominant action for type one depositors to withdraw at $t = 1$ and type two depositors to withdraw at $t = 2$.

Of course, when $\beta R_B^b > R_A^b$, the positive probability of equilibrium bank runs will be required to satisfy the bank's incentives. In this case, appropriately designed random banking contracts Pareto improve on autarchy.

With monitoring and transfers, either the threat of bank runs off the equilibrium path of play and/or the positive probability of equilibrium bank runs may be required to satisfy the bank's incentives.

4 Conclusion

In a model of banking *with* moral hazard but *without* aggregate pay-off relevant uncertainty, we show that risk-sharing between depositors may no longer be implementable and therefore, even with forward looking depositors, banks runs occur with positive probability along the equilibrium path of play in any second-best banking contract. We also show that bank runs may be a feature of banking contracts even with costless but perfect monitoring and costly but perfect monitoring. Situations where the bank has all the bargaining power result in narrow banking. With imperfect monitoring, we show that both the threat of bank runs off-the-equilibrium path of play and bank runs along the equilibrium path of play, are features of banking contracts. Next, in a different monitoring scenario when all monitoring takes place conditional on a bank run, we show that both the positive probability of bank runs and monitoring along the equilibrium path of play are features of banking contracts.

We interpret the significance of our results in two distinct ways. First, our results provide a theoretical foundation for the doctrine of "creative ambiguity". Second, our result shows that in low asset economies, where productive agents like banks or firms with little or no collateral, appropriately designed random demandable debt contracts Pareto improve on autarky.

Extending our result to examine issues of financial contagion and episodes of twinned bank runs and currency crises are topics for future research.

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Appendix

Consider the banking contract $(\hat{\gamma}, \hat{r}, \hat{k})$ as specified in section 2.2. The payoff to per unit of deposit withdrawn at $t = 1$, which depends on the fraction of deposits serviced before agent j , k_j , is given by the expression

$$\hat{v}_1(f_j, \hat{r}_1, \hat{k}) = \begin{cases} u(\hat{r}_1), & \text{if } k_j \leq \hat{k}, \\ u(0), & k_j > \hat{k} \end{cases}$$

while the period 2 payoff per unit deposit withdrawn at $t = 2$, which depends on total fraction of deposits withdrawn in period 1, k , is given by the expression

$$\hat{v}_2(f, \hat{r}_1) = \begin{cases} u(\hat{r}_2), & \text{if } 1 > k\hat{r}_1, \\ 0, & \text{otherwise} \end{cases}$$

At $t = 1$, the above contract induces a noncooperative game between depositors where each depositor chooses what fraction of their deposits to withdraw. Suppose depositor j withdraws a fraction μ_j . Then, a type 1 depositor obtains a payoff $\mu_j \hat{v}_1(k_j, \hat{r}_1, \hat{k})$ while a type 2 depositor obtains a payoff of $\mu_j \hat{v}_1(k_j, \hat{r}_1, \hat{k}) + (1 - \mu_j) \hat{v}_2(k, \hat{r}_1)$. Remark that for a type 1 depositor, $\mu_j = 1$ strictly dominates all other actions. As $\hat{k} = \lambda$, $\hat{r}_1 = x_1^{1*}$ and $\hat{r}_2 = x_2^{2*}$, it follows

that $\hat{v}_2(k, \hat{r}_1) > \hat{v}_1(k_j, \hat{r}_1)$ and for type 2 depositors, $\mu_j = 0$ strictly dominates all other actions and therefore, $k = \hat{k} = \lambda$. The bank's payoffs are

$$\hat{v}^b(\gamma) = \gamma R_A^b + (1 - \gamma) R_B^b - (\gamma c_A + (1 - \gamma) c_B)$$

There is only one subgame at $t = 1$ (as depositors don't observe the bank's choice of γ). As $R_A^b \geq R_B^b$, and $c_A < c_B$, at $t = 1$, $1 - \hat{k}\hat{r}_1 > 0$, choosing $\gamma = \hat{\gamma}_A$ is a strictly dominant choice for the bank. ■