Efficiency and Core Properties of Valuation Equilibrium with Increasing Returns*

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March 27, 2002

Abstract

This paper considers market economies involving the choice of a public environment when there are nonconvexities in production. It discusses the decentralization of efficient allocations by means of *valuation equilibrium*, adapting to many private goods the notion due to Mas-Colell (1980) which extends the Lindahlian approach to the pure theory of public goods. It is shown that a valuation equilibrium satisfies the two welfare theorems and is in the core.

* Thanks are due to Dimitros Diamantaras for very helpful comments. Financial support from the Spanish Ministerio de Ciencia y Technología is gratefully acknowledged.

1 Introduction

This paper is concerned with the market decentralization of optimal allocations in a modern mixed economy. By this we mean a market economy with both private and public sectors. The private sector consists of households and firms who make decisions within their own private feasible sets. The public sector chooses actions which may affect overall economic activity. Private agents' feasible sets are determined by their individual characteristics (technology and private endowments), the market conditions (including the actions of other private agents), and a set of public policy variables that are chosen by the public sector. These public policy variables may include the realization of public projects (the provision of public goods together with the ways of financing them), the allocation of property rights (and in particular the operation of a tax-subsidy system), the regulation of economic activities (setting quotas, price controls or quality standards, among many others), etc. A specification of these policy variables will be called a *public environment,* or simply an *environment*.

In this type of economy we consider a form of decentralization which makes use of the notion of *valuation equilibrium* in a suitable extended sense. A valuation equilibrium is usually defined as a commodity price vector, a valuation function (or a non-linear tax system) for pricing the public environment, and a feasible allocation that includes the public environment, such that: (a) Each consumer's equilibrium combination of private consumption together with the public environment is weakly preferred to any other which is affordable given budget constraint defined by the price vector and the valuation function; (b) Each firm's equilibrium combination of commodity net supplies and public environment maximizes net profits over its production set.

The notion of valuation equilibrium was introduced in Mas-Colell (1980) to study the choice of public projects in an economy with a single private good, which was taken as the *numéraire*. He showed that valuation equilibria satisfy the standard welfare properties of competitive equilibria (the two welfare theorems). Mas-Colell's work was subsequently extended by several different authors to allow for the presence of several private and public goods in the economy. See, for example, the contributions by Mas-Colell & Silvestre (1989), Diamantaras & Wilkie (1994), Diamantaras & Gilles (1996), Diamantaras, Gilles & Scotchmer (1996) and Hammond & Villar (1998), (1999). These extensions are usually obtained under the assumption that, given the public environment, conditional production sets are all convex. In other words, for each fixed choice of public environment, the resulting conditional private economy is a classical competitive one, as in standard general equilibrium theory. Or at least, the production of public goods can be fully described by a cost function.

This paper extends the analysis further, allowing for more general non-convex production sets, while keeping the efficiency properties of the equilibrium notion. The interest of this extension is threefold. First and foremost, it allows to deal with economies whose private production sector exhibits increasing returns to scale, or fixed costs, or uses some indivisible commodities as inputs. Second, note that in some cases increasing returns to scale arise precisely from the public environment itself, as in the case of transport or information networks. So we relax the restrictive assumption that the production of public goods takes place within a standard convex technology. Third, our framework permits one clearly to identify the set of public environments with the public sector "control variables", as opposed to other models in which the environment may include also private projects precisely in order to ensure convex conditional production sets, given the public environment [e.g. Hammond & Villar (1998)].

Extending the notion of valuation equilibria to this more general setting requires taking into account two different, yet related, complications. One has to do with the fact that profit maximization might be an inadequate description of firms' behaviour. This is because when there are increasing returns to scale or other non-convexities, there may be an empty set of profit-maximizing production plans. Another complication concerns the necessary conditions for efficiency in an economy with non-convex production sets.

There are several different ways to deal with these complications —e.g., modelling the behaviour of firms in terms of "marginal" pricing rules and imposing restrictions on the admissible characteristics of the agents.[1] Here, however, we shall approach those problems by giving households an active role as co-owners of the private firms. In particular, it will be assumed that households instruct the firms to choose those production plans that maximize their owners' aggregate expenditure capacity [an idea formerly introduced in Villar (2001)].

This approach has two main consequences. First, firms do not choose their own policies independently. Rather, they try to implement the owners' objectives. Second, each household's feasible set and preference relation may

[1] The reader is referred to Bonnisseau & Cornet (1988), Brown (1991), Quinzii (1992) and Villar (2000a) for a detailed discussion of general equilibrium models with nonconvexities.

be defined on a more extensive domain than in the classical Arrow-Debreu model. In fact this formulation will require households' income to be specified as general income functions that can be regarded as including a system of taxes and transfers.

Accordingly, a *valuation equilibrium* (VE, for short) is defined here as a combination consisting of a public environment, a price vector, and a feasible allocation such that: (a) No household prefers an alternative allocation that is affordable, given the maximum income achievable; (b) Firms are in equilibrium according to the households' unanimous prescriptions; and (c) Total income equals the worth of total resources plus total profits (aggregate budget balance).

The reference model is presented in section 2. Following the approach in Mas-Colell (1980), where no linear structure is imposed on the set of public projects, we shall consider an economy with an abstract set of public environments lacking any formal structure. This setup includes many different familiar models as special cases. Private agents' feasible sets and preferences will be defined conditional on the public environment, the latter being chosen by the public sector. For each given value of the public environment, the households' conditional feasible sets are standard, whereas no such a restriction will be imposed on the conditional production sets. The equilibrium notion proposed aims at ensuring that the two welfare theorems hold in this context.

With several private goods, two different ways of extending the notion of valuation equilibrium have been considered. First, agents can be *myopic* and compare different environments taking market prices as given [Hammond & Villar (1999)]. Second, agents can be *far-sighted*, and compare different public environments after taking into account the induced changes in the relative prices of private commodities [e.g. Diamantaras & Giles (1996), Hammond & Villar (1998)]. Interestingly enough, whether agents are myopic or far sighted makes no difference to our results, which are valid under the same assumptions in either case.

Section 3 contains the results for the case in which agents are myopic. These results can be summarized as follows: (a) A VE is Pareto optimal if consumers are locally non-satiated and the income schedule is balanced; (b) Every Pareto optimal allocation can be decentralized as VE; and (c) A VE yields a core allocation, when the income schedule satisfies balancedness within coalitions.

Next, Section 4 briefly discusses the case in which consumers are far-sighted, under the simplifying assumption that their forecast of a change in the environment is a common point estimation that depends on the *status quo* prices and public environment, as well as on the new environment. It is straightforward to adapt the results of Section 3 to this context.

A few final comments are gathered in section 5.

2 The model

Consider a mixed economy consisting private agents and a public sector. The private agents in the economy are assumed to be m households, indexed by $i \in I := \{1, 2, ..., m\}$, together with n private firms, indexed by $j \in J := \{1, 2, ..., n\}$. The commodity space is made of ℓ goods and services.

Households are assumed to own both the initial endowments and shares in the private firms. They also play an active role in the economy as both consumers and shareholders. More precisely, in our model all decisions regarding consumption and production will be ultimately made by the households. That will become apparent in the way of modelling their demand mappings and in the equilibrium notion. Let $\omega \in \mathbb{R}^\ell$ represent the aggregate initial endowment vector of private goods.

The public sector makes choices in an abstract set $\mathbf{Z}$ whose elements, denoted by $\mathbf{z}$, include those variables defining the *public environment*. No particular structure will be postulated on the set $\mathbf{Z}$, except those derived from the fact that each point of $\mathbf{Z}$ must describe a consistent set of parameter values (environmental feasibility constraints). Each agent's feasible set and preferences may be affected by $\mathbf{z} \in \mathbf{Z}$. In particular, if the public environment $\mathbf{z}$ involves public goods, the firms that produce them will be restricted to meet output targets for those public goods.

For each firm $j \in J$, let Y_j be the jth firm's production set, that we take as a subset of $\mathbb{R}^\ell \times \mathbf{Z}$, for the sake of generality. Denote by $Y_j(\mathbf{z}) \subset \mathbb{R}^\ell$ firm j's restricted *conditional production set* when the public environment takes the value $\mathbf{z} \in \mathbf{Z}$. That is,

$$Y_j(\mathbf{z}) \equiv \{\mathbf{y}_j \in \mathbb{R}^\ell \ / \ (\mathbf{y}_j, \mathbf{z}) \in Y_j\}$$

The restrictions derived from the public environment may refer to the use or production of some public goods, but may also include quantity restrictions, quality specifications, etc. We shall assume that production sets are topologically closed and comprehensive. More precisely: For all $\mathbf{z} \in \mathbf{Z}$, $Y_j(\mathbf{z})$ is closed in $\mathbb{R}^\ell$, with $Y_j(\mathbf{z}) - \mathbb{R}^\ell_+ \subset Y_j(\mathbf{z})$. Comprehensiveness justifies taking the set $\mathcal{P} = \mathbb{R}^\ell_+ \setminus \{\mathbf{0}\}$ as the price space for private goods, as we actually do.

For each $\mathbf{z} \in \mathbf{Z}$, $\mathbb{Y}(\mathbf{z})$ stands for the Cartesian product of the n conditional production sets. That is, $\mathbb{Y}(\mathbf{z}) \equiv \prod_{j \in J} Y_j(\mathbf{z})$. Points in $\mathbb{Y}(\mathbf{z})$ will be denoted by $\widetilde{\mathbf{y}} = (\mathbf{y}_j)_{j \in J}$.

Given a price vector $\mathbf{p} \in \mathcal{P}$, a public environment $\mathbf{z} \in \mathbf{Z}$, and a collection $\widetilde{\mathbf{y}} \in \mathbb{Y}(\mathbf{z})$, the scalar $\pi_j(\mathbf{z}, \mathbf{p}, \widetilde{\mathbf{y}})$ denotes firm j's profits relative to $(\mathbf{z}, \mathbf{p}, \widetilde{\mathbf{y}})$. In the case when prices are linear, when the firm does not use public goods as inputs, and when there are no taxes or subsidies, we have the usual formula $\pi_j(\mathbf{z}, \mathbf{p}, \widetilde{\mathbf{y}}) = \mathbf{p}\mathbf{y}_j$. But our model allows for the presence of taxes on profits and non-linear prices (e.g., two-part tariffs on some inputs produced by firms with increasing returns to scale, or Lindahl prices paid for the use of public inputs). For this reason we use the more general notation $\pi_j(\mathbf{z}, \mathbf{p}, \widetilde{\mathbf{y}})$.

Our model does not regard firms as independent agents. Instead, they merely implement the decisions of the households who own them. Thus, given a price vector and a collection $(\mathbf{y}_j^i)_{i\in I}$ of (usually inconsistent) consumers' instructions to firm j, its "supply" is some function of the households' proposals —for example, the weighted average $\mathbf{y}_j = \sum_{i\in I} \theta_{ij}\mathbf{y}_j^i$ of the households' proposals, with weights equal to their shares θ_{ij}.

For each household $i \in I$, let $X_i \subset \mathbb{R}^\ell \times \mathbf{Z}$ denote i's consumption set, and let $X_i(\mathbf{z})$ stand for consumer i's conditional consumption set, defined by:

$$X_i(\mathbf{z}) = \{\mathbf{x}_i \in \mathbb{R}^\ell \;\; / \;\; (\mathbf{x}_i, \mathbf{z}) \in X_i\}$$

That is, $X_i(\mathbf{z})$ consists of vectors of private goods that are feasible when the public environment is $\mathbf{z}$. Assume that each household $i \in I$ has a preference ordering that can be represented by the ordinal utility function $u_i : X_i \to \mathbb{R}$. Given $\mathbf{z} \in \mathbf{Z}$, we can write the conditional utility function as $u_i^{\mathbf{z}} : X_i(\mathbf{z}) \to \mathbb{R}$, with values given by $u_i^{\mathbf{z}}(\mathbf{x}_i) := u_i(\mathbf{x}_i, \mathbf{z})$. Note that household i's preferences are allowed to depend on all the variables in each element $\mathbf{z}$ of $\mathbf{Z}$, and not only on the commodity vectors.

For each $\mathbf{z} \in \mathbf{Z}$, $\mathbb{X}(\mathbf{z})$ stands for the Cartesian product of the m conditional consumption sets. That is, $\mathbb{X}(\mathbf{z}) \equiv \prod_{i\in I} X_i(\mathbf{z})$. Points in $\mathbb{X}(\mathbf{z})$ will be denoted by $\widetilde{\mathbf{x}} = (\mathbf{x}_i)_{i\in I}$.

For each given $\mathbf{z} \in \mathbf{Z}$, each household $i \in I$ is assumed to have a *conditional income function* $r_i^{\mathbf{z}} : \mathcal{P} \times \mathbb{Y}(\mathbf{z}) \to \mathbb{R}$. Given a price vector $\mathbf{p} \in \mathcal{P}$ and a collection $\widetilde{\mathbf{y}} \in \mathbb{Y}(\mathbf{z})$, the value $r_i^{\mathbf{z}}(\mathbf{p}, \widetilde{\mathbf{y}})$ of this mapping is the income that household i obtains in environment $\mathbf{z}$ at prices $\mathbf{p}$ when it takes $\widetilde{\mathbf{y}}$ as the relevant vector of production plans. The conditional income function may include not only private income from endowments and firms' dividends, but also government net transfers in the form of taxes and subsidies, including (possibly non-linear) prices for the public goods. To be more concrete, we can think of the ith consumer's conditional income function as a mapping of the form:

$$r_i^{\mathbf{z}}(\mathbf{p}, \widetilde{\mathbf{y}}) = \mathbf{p}\omega_i + \sum_{j\in J} \theta_{ij}\pi_j(\mathbf{z}, \mathbf{p}, \widetilde{\mathbf{y}}) + \tau_i(\mathbf{z}, \mathbf{p})$$

where ω_i describes the household i's initial endowments, and θ_{ij} its share of firm j (with $\sum_{i\in I}\omega_i = \omega$, $0 \le \theta_{ij} \le 1$ for all i, j, and $\sum_{i\in I}\theta_{ij} = 1$, for all j), while $\tau_i(.)$ is a tax-subsidy mapping that depends on the public environment and the commodity price vector. This expression shows how consumer i's income can be regarded as the sum of the market value of its private assets, plus a public sector net transfer τ_i.

Consider now the following definition:

Definition 1 *An* ***income schedule*** *is a collection $R := (r_i^{\mathbf{z}})_{i\in I}$ of mappings, such that, for every $\mathbf{z} \in \mathbf{Z}$, and all $(\mathbf{p},\widetilde{\mathbf{y}})$ in $\mathcal{P}\times\mathbb{Y}(\mathbf{z})$, we have: $\sum_{i\in I} r_i^{\mathbf{z}}(\mathbf{p},\widetilde{\mathbf{y}}) \le \mathbf{p}\omega + \sum_{j\in J}\mathbf{p}\mathbf{y}_j$.*

An income schedule is a collection of functions that generate a *self-financing* income distribution, for every given $\mathbf{z} \in \mathbf{Z}$. That is, the aggregate income does not exceed the value of initial endowments plus the aggregate profits (or losses, if negative).

Remark 1 *We assume that the income schedule indirectly includes the possible taxes on the firms via the profit components.*

Given a point $(\mathbf{p}, \mathbf{z}) \in \mathcal{P} \times \mathbf{Z}$, household i's demand for private commodities is obtained as a solution to the following program:

$$\left.\begin{array}{c} \max_{(\mathbf{x}_i,\widetilde{\mathbf{y}})} \ u_i^{\mathbf{z}}(\mathbf{x}_i) \\ s.t. : \mathbf{x}_i \in X_i(\mathbf{z}) \\ \widetilde{\mathbf{y}} \in \mathbb{Y}(\mathbf{z}) \\ \mathbf{p}\mathbf{x}_i \le r_i^{\mathbf{z}}(\mathbf{p},\widetilde{\mathbf{y}}) \end{array}\right\} \qquad [1]$$

Note that the variables which household i is choosing include both the consumption plan $\mathbf{x}_i$ and all the private firms' production plans (actually, those corresponding to firms in which household i has some positive share).

Let $\mathcal{Y}^i(\mathbf{p}, \mathbf{z})$ denote the set of points $\widetilde{\mathbf{y}}^i \in \mathbb{Y}(\mathbf{z})$ such that $(\mathbf{x}_i, \widetilde{\mathbf{y}}^i)$ solves program [1], for some $\mathbf{x}_i \in X_i(\mathbf{z})$. Obviously this set, which may well vary from consumer to consumer, consists of production plans that maximize consumer i's income, given the price vector and the public environment.. Accordingly, define

$$r_i(\mathbf{p}, \mathbf{z}) := \max_{\widetilde{\mathbf{y}}} \left\{ r_i^{\mathbf{z}}(\mathbf{p},\widetilde{\mathbf{y}}^i) \ / \ \widetilde{\mathbf{y}} \in \mathbb{Y}(\mathbf{z}) \right\}$$

as the income that consumer i "demands" when facing the price vector $\mathbf{p}$ and public environment $\mathbf{z}$. This is the relevant analogy to the usual competitive budget set, in the sense that without taxes or transfers, the amount $r_i(\mathbf{p}, \mathbf{z})$

corresponds to the income that consumer i obtains when firms maximize profits at given prices. Clearly, $r_i(\mathbf{p}, \mathbf{z}) = r_i^{\mathbf{z}}(\mathbf{p}, \widetilde{\mathbf{y}})$ whenever $\widetilde{\mathbf{y}} \in \mathcal{Y}^i(\mathbf{p}, \mathbf{z})$. Also observe that, for an arbitrary $\widetilde{\mathbf{y}} \in \mathbb{Y}(\mathbf{z})$, one has

$$\sum_{i \in I} r_i(\mathbf{p}, \mathbf{z}) = \sum_{i \in I} r_i^{\mathbf{z}}(\mathbf{p}, \widetilde{\mathbf{y}}^i) \geq \sum_{i \in I} r_i^{\mathbf{z}}(\mathbf{p}, \widetilde{\mathbf{y}})$$

with the equality holding if and only if $\widetilde{\mathbf{y}} \in \bigcap_{i \in I} \mathcal{Y}^i(\mathbf{p}, \mathbf{z})$.

To summarize, an **economy** is a collection of: (a) m households, each characterized by a consumption set and an ordinal utility function; (b) n firms, each characterized by a production set; (c) a domain $\mathbf{Z}$ of public environments; (d) a vector ω of aggregate initial endowments; (e) an income schedule R, which involves the distribution of private assets as well as a tax-subsidy system. An economy can be written concisely in the form:

$$\mathcal{E} = [(X_i, u_i)_{i \in I}, (Y_j)_{j \in J}, \mathbf{Z}, \omega, R]$$

Consider now the following:

Definition 2 *An **allocation** for an economy $\mathcal{E}$ is a point $[\mathbf{z}, \widetilde{\mathbf{x}}, \widetilde{\mathbf{y}}]$ such that $(\widetilde{\mathbf{x}}, \widetilde{\mathbf{y}}) \in \mathbb{X}(\mathbf{z}) \times \mathbb{Y}(\mathbf{z})$. If, furthermore, $\sum_{i \in I} \mathbf{x}_i \leq \omega + \sum_{j \in J} \mathbf{y}_j$, then $[\mathbf{z}, \widetilde{\mathbf{x}}, \widetilde{\mathbf{y}}]$ is an **attainable allocation** for $\mathcal{E}$.*

An allocation is a public environment and a collection of actions for consumers and firms in the private commodity space that are compatible with such environment. An allocation is attainable when the aggregate consumption does not exceed the aggregate production plus the available resources. The **set of attainable allocations** is denoted by $\mathcal{A}(\mathcal{E})$.

The following definition makes precise the equilibrium notion. We assume that the net revenues derived from the activity of public firms, if any, are distributed through the tax-subsidy system.

Definition 3 *A **valuation equilibrium** (VE for short) for an economy $\mathcal{E}$ is a price vector $\mathbf{p}^* \in \mathcal{P}$, a public environment $\mathbf{z}^* \in \mathbf{Z}$, and an allocation $(\widetilde{\mathbf{x}}^*, \widetilde{\mathbf{y}}^*) \in \mathbb{X}(\mathbf{z}^*) \times \mathbb{Y}(\mathbf{z}^*)$ such that:*

(a) For all $i \in I$, one has $u_i(\mathbf{x}_i^*, \mathbf{z}^*) \geq u_i(\mathbf{x}_i, \mathbf{z})$ for all $(\mathbf{x}_i, \mathbf{z}) \in X_i$ such that $\mathbf{p}^* \mathbf{x}_i \leq r_i(\mathbf{p}^*, \mathbf{z})$.

(b) $\widetilde{\mathbf{y}}^* \in \bigcap_{i \in I} \mathcal{Y}^i(\mathbf{p}^*, \mathbf{z}^*)$.

(c) $\sum_{i \in I} \mathbf{x}_i^* = \omega + \sum_{j \in J} \mathbf{y}_j^*$.

(d) $\sum_{i \in I} r_i(\mathbf{p}^*, \mathbf{z}^*) = \mathbf{p}^* \omega + \mathbf{p}^* \sum_{j \in J} \mathbf{y}_j^*$.

That is, a valuation equilibrium is a price vector, a public environment, and an allocation such that: (a) no household $i \in I$ finds it individually beneficial to choose an alternative pair $(\mathbf{x}_i, \mathbf{z})$ that is affordable at given prices, even given the highest income achievable; (b) all households unanimously agree on the production schedule and the firms implement such a decision; (c) all markets clear; and (d) total income equals the worth of total resources plus total profits. Clearly part (d) of the definition is redundant when consumers are locally non-satiated, because then it is implied by parts (a) and (c) together.

If the public environment is fixed, a valuation equilibrium reduces to a competitive equilibrium in the corresponding private good classical economy. To see this, simply set $\mathbf{Z} = \{\overline{\mathbf{z}}\}$, and suppose that all production sets given $\overline{\mathbf{z}}$ are non-empty, closed, convex and comprehensive subsets of $\mathbb{R}^\ell$. Note that, with $\overline{\mathbf{z}}$ fixed, each consumer's income is maximized at given prices if and only if each individual firm maximizes profits at these prices. That is, $r_i(\mathbf{p},\overline{\mathbf{z}}) = \mathbf{p}\omega_i + \sum_{j \in J} \theta_{ij} \mathbf{p}\mathbf{y}_j(\mathbf{p})$, where $\mathbf{y}_j(\mathbf{p})$ is a point in $Y_j(\overline{\mathbf{z}})$ such that $\mathbf{p}\mathbf{y}_j \geq \mathbf{p}\mathbf{y}'_j$ for all $\mathbf{y}'_j \in Y_j(\overline{\mathbf{z}})$, and all $j \in J$.

If one keeps the assumption that $\mathbf{Z} = \{\overline{\mathbf{z}}\}$, but allows firms to have non-convex production sets, then a valuation equilibrium corresponds to a *market equilibrium with active consumers,* as defined in Villar (2001).

When $\mathbf{Z}$ has more than one element and the private firms have convex conditional production sets, a valuation equilibrium as defined here coincides with the concept considered in Hammond & Villar (1999), where some specific examples are discussed in detail.

3 The results for the myopic case

We now proceed to the main results, focusing first on the myopic case in which consumers evaluate alternatives taking commodity prices as given. The extension to the case of far-sighted consumers is formally straightforward —see Section 4 below.

Our first result says that a valuation equilibrium is Pareto optimal, no matter what pricing policy the firms follow. The second establishes that any interior Pareto efficient allocation can be decentralized as a valuation equilibrium. Finally, it will be shown that a valuation equilibrium is in the core, provided that inter-coalitional transfers are excluded.

The following definition makes precise the notion of "balancedness" for the income schedule, a restriction that plays a relevant role in the results presented below. Let $\mathbb{Y}^{\mathcal{A}}(\mathbf{z}) = \mathbb{Y}(\mathbf{z}) \bigcap \mathcal{A}(\mathcal{E})$ denote the set of production plans that correspond to some attainable allocation for a given environment $\mathbf{z} \in \mathbf{Z}$. Then:

Definition 4 *An income schedule R is* ***balanced*** *if, for every $\mathbf{z} \in \mathbf{Z}$, and all $(\mathbf{p},\widetilde{\mathbf{y}})$ in $\mathcal{P} \times \mathbb{Y}^{\mathcal{A}}(\mathbf{z})$, we have: $\sum_{i \in I} r_i^{\mathbf{z}}(\mathbf{p},\widetilde{\mathbf{y}}) = \mathbf{p}\omega + \sum_{j \in J} \mathbf{p}\mathbf{y}_j$.*

An income schedule is balanced if it ensures the equality between aggregate income and total profits plus the market value of the initial endowments, for all possible attainable states. This implies that Walras Law holds when consumers' preferences are locally non-satiated and the involved allocation is attainable.

In the myopic case, consumers evaluate alternatives at given prices. The following result is obtained:

Theorem 1 *Let $[\mathbf{p}^*, \mathbf{z}^*, \widetilde{\mathbf{x}}^*, \widetilde{\mathbf{y}}^*]$ be a valuation equilibrium. Suppose that consumers are locally non-satiated and that the income schedule is balanced. Then the resulting allocation is Pareto optimal.*

Proof.

Let $[\mathbf{p}^*, \mathbf{z}^*, \widetilde{\mathbf{x}}^*, \widetilde{\mathbf{y}}^*]$ be a valuation equilibrium. Suppose that $[\mathbf{z},\widetilde{\mathbf{x}}, \widetilde{\mathbf{y}}]$ is a feasible allocation such that $u_i(\mathbf{x}_i, \mathbf{z}) \geq u_i(\mathbf{x}_i^*, \mathbf{z}^*)$ for all consumers $i \in I$. Because this allocation is feasible, it follows that $\sum_{i \in I} \mathbf{x}_i \leq \omega + \sum_{j \in J} \mathbf{y}_j$. But $\mathbf{p}^* > \mathbf{0}$ and so

$$\sum_{i \in I} \mathbf{p}^*\mathbf{x}_i \leq \mathbf{p}^*\omega + \sum_{j \in J} \mathbf{p}^*\mathbf{y}_j \qquad [2]$$

Next, because feasibility implies that $\widetilde{\mathbf{y}} \in \mathbb{Y}(\mathbf{z})$, it follows from the local non-satiation hypothesis and the definition of valuation equilibrium that

$$\mathbf{p}^*\mathbf{x}_i \geq r_i(\mathbf{p}^*, \mathbf{z}) \geq r_i^{\mathbf{z}}(\mathbf{p}^*, \widetilde{\mathbf{y}}) \qquad [3]$$

for all $i \in I$. Moreover, the balancedness condition $\sum_{i \in I} r_i^{\mathbf{z}}(\mathbf{p}^*, \widetilde{\mathbf{y}}) = \mathbf{p}^*\omega + \sum_{j \in J} \mathbf{p}^*\mathbf{y}_j$ must be satisfied, so that [3] implies

$$\mathbf{p}^*\mathbf{x}_i = r_i^{\mathbf{z}}(\mathbf{p}^*, \widetilde{\mathbf{y}}) \qquad [4]$$

for all $i \in I$. But this implies that $u_i(\mathbf{x}_i, \mathbf{z}) \leq u_i(\mathbf{x}_i^*, \mathbf{z}^*)$ and so $u_i(\mathbf{x}_i, \mathbf{z}) = u_i(\mathbf{x}_i^*, \mathbf{z}^*)$ for all consumers $i \in I$. So a Pareto superior allocation cannot exist. ■

Note that the proof would not work without the balancedness condition. This is because we can not exclude the case in which $\sum_{i \in I} r_i(\mathbf{p}, \mathbf{z}) <$

$\sum_{j\in J} \mathbf{p}\mathbf{y}_j + \mathbf{p}\omega$, for some $\widetilde{\mathbf{y}} \in \mathbb{Y}(\mathbf{z})$, so equation [4] may not hold.[2] The use of $r_i(\mathbf{p}, \mathbf{z})$ instead of $r_i^{\mathbf{z}}(\mathbf{p},\widetilde{\mathbf{y}})$ in the definition of equilibrium is also necessary in order to derive [3]. If instead one took $r_i^{\mathbf{z}}(\mathbf{p},\widetilde{\mathbf{y}})$ as the relevant income mapping when increasing returns to scale are permitted, one could not deduce that $\mathbf{p}^*\mathbf{x}_i \geq r_i^{\mathbf{z}}(\mathbf{p}^*, \widetilde{\mathbf{y}})$ when consumer i weakly prefers $(\mathbf{x}_i, \mathbf{z})$ to $(\mathbf{x}_i^*, \mathbf{z}^*)$. Example 6.1 in Villar (2000) serves to illustrate this point further.

Let us move on to the decentralizability of Pareto efficient allocations by way of valuation equilibria. We can prove:

Theorem 2 *Let $[\mathbf{z}^*, \widetilde{\mathbf{x}}^*, \widetilde{\mathbf{y}}^*]$ be an interior Pareto optimal allocation —i.e., one for which each consumer $i \in I$ has $\mathbf{x}_i^*$ in the interior of $X_i(\mathbf{z}^*)$. Suppose that, for all $i \in I$ and all $\mathbf{z} \in \mathbf{Z}$, the set $X_i(\mathbf{z})$ is a convex subset of $\mathbb{R}_+^\ell$, and the conditional utility function $u_i^{\mathbf{z}} : X_i \to \mathbb{R}$ is continuous, quasi-concave, and satisfies local non-satiation. Then, there exist a price vector $\mathbf{p}^* \in \mathcal{P}$ and an income schedule R such that $[\mathbf{p}^*, \mathbf{z}^*, \widetilde{\mathbf{x}}^*, \widetilde{\mathbf{y}}^*]$ is a valuation equilibrium.*

Proof.

Take $\mathbf{z}^*$ as given. For each $i \in I$ let U_i^* denote the upper contour set of points in $X_i(\mathbf{z}^*)$ such that $u_i^{\mathbf{z}^*}(\mathbf{x}_i) \geq u_i^{\mathbf{z}^*}(\mathbf{x}_i^*)$. This is a convex set by assumption. Now let $U^* = \sum_{i\in I} U_i^*$ be the aggregate upper contour set, which is also convex. By local non-satiation, $\mathbf{x}^* := \sum_{i\in I} \mathbf{x}_i^*$ must be a point on its boundary. Therefore, the standard supporting hyperplane theorem for convex sets in finite-dimensional Euclidean space ensures that there exists $\mathbf{p}^* \neq \mathbf{0}$ such that $\mathbf{p}^*\mathbf{x}^*$ minimizes $\mathbf{p}^*\mathbf{x}$ over U^*. From this it follows that $\mathbf{x}_i^*$ minimizes $\mathbf{p}^*\mathbf{x}_i$ on U_i^*, for all $i \in I$. A standard argument shows that, because $\mathbf{x}_i^*$ is an interior point of the convex set $X_i(\mathbf{z}^*)$, this implies that $u_i^{\mathbf{z}^*}(\mathbf{x}_i^*) \geq u_i^{\mathbf{z}^*}(\mathbf{x}_i)$ for all $\mathbf{x}_i \in X_i(\mathbf{z}^*)$ such that $\mathbf{p}^*\mathbf{x}_i \leq \mathbf{p}^*\mathbf{x}_i^*$.

Let us show now that $[\mathbf{p}^*, \mathbf{z}^*, \widetilde{\mathbf{x}}^*, \widetilde{\mathbf{y}}^*]$ is a valuation equilibrium for some income schedule R. That requires constructing a suitable income schedule and then checking that parts (a) and (d) of definition 3 are satisfied (parts (b) and (c) being satisfied by construction). But as remarked after Definition 3, in fact part (d) is an implication of parts (a) and (c), so it will be enough merely to confirm part (a).

Consider first the compensation function, relative to $(\mathbf{p}^*, \mathbf{x}_i^*, \mathbf{z}^*)$, defined for each $i \in I$ by

$$E_i(\mathbf{z}) = \inf_{\mathbf{x}_i}\{\mathbf{p}^*\mathbf{x}_i \;/\; \mathbf{x}_i \in X_i(\mathbf{z}),\;\; u_i(\mathbf{x}_i, \mathbf{z}) \geq u_i(\mathbf{x}_i^*, \mathbf{z}^*)\}$$

[2] This would happen when the income rule wastes part of the resources out of equilibrium.

This is the income that consumer i needs to spend on private goods in order to be no worse off than at $(\mathbf{x}_i^*, \mathbf{z}^*)$ when the environment changes to $\mathbf{z}$. We take $E_i(\mathbf{z}) = +\infty$ if there is no $\mathbf{x}_i \in X_i(\mathbf{z})$ that allows consumer i to achieve utility level $u_i(\mathbf{x}_i^*, \mathbf{z}^*)$ when environment is $\mathbf{z}$.

Now define, for each $i \in I$

$$\alpha_i := \frac{\mathbf{p}^*\mathbf{x}_i^*}{\sum_{h \in I} \mathbf{p}^*\mathbf{x}_h^*} \ ,$$

that is, α_i is the percentage of the aggregate expenditure which consumer i needs to make $\mathbf{x}_i^*$ affordable. Consider now the income schedule R defined so that, for every given $\mathbf{z} \in \mathbf{Z}$, $\widetilde{\mathbf{y}} \in \mathbb{Y}(\mathbf{z})$ and $i \in I$, we have:

$$r_i^{\mathbf{z}}(\mathbf{p}^*, \widetilde{\mathbf{y}}) = \min \left\{ E_i(\mathbf{z}), \quad \alpha_i \left[\sum_{j \in J} \mathbf{p}^*\mathbf{y}_j + \mathbf{p}^*\omega \right] \right\}.$$

Obviously, $\sum_{i \in I} r_i^{\mathbf{z}}(\mathbf{p}^*, \widetilde{\mathbf{y}}) \leq \sum_{j \in J} \mathbf{p}^*\mathbf{y}_j + \mathbf{p}^*\omega$ —that is, R is an income schedule.

Therefore, part (a) of the definition is satisfied provided $r_i^{\mathbf{z}^*}(\mathbf{p}^*, \widetilde{\mathbf{y}}^*) = r_i(\mathbf{p}^*, \mathbf{z}^*)$. To see this, suppose that $\widetilde{\mathbf{y}}^* \notin \bigcap_{i=1}^m \mathcal{Y}^i(\mathbf{p}^*, \mathbf{z}^*)$. This implies that there exists some consumer i and a collection of production plans $\widetilde{\mathbf{y}}^i$ such that

$$r_i^{\mathbf{z}^*}(\mathbf{p}^*, \widetilde{\mathbf{y}}^i) = \min \left\{ E_i(\mathbf{z}^*), \quad \alpha_i \left[\sum_{j \in J} \mathbf{p}^*\mathbf{y}_j^i + \mathbf{p}^*\omega \right] \right\} > r_i^{\mathbf{z}^*}(\mathbf{p}^*, \widetilde{\mathbf{y}}^*) = \mathbf{p}^*\mathbf{x}_i^*$$

But this is not possible because $E_i(\mathbf{z}^*) = \mathbf{p}^*\mathbf{x}_i^*$. Therefore, $r_i^{\mathbf{z}^*}(\mathbf{p}^*, \widetilde{\mathbf{y}}^*) = r_i(\mathbf{p}^*, \mathbf{z}^*)$ for all i.

Finally, take a consumer i and any consumption plan $(\mathbf{x}_i, \mathbf{z}) \in X_i \times \mathbf{Z}$ such that $u_i(\mathbf{x}_i, \mathbf{z}) > u_i(\mathbf{x}_i^*, \mathbf{z}^*)$. We have to show that this consumption plan is not affordable given the prices and the income schedule. Indeed, suppose it were true that $\mathbf{p}^*\mathbf{x}_i \leq r_i(\mathbf{p}^*, \mathbf{z})$. But this gives a contradiction because

$$r_i(\mathbf{p}^*, \mathbf{z}) = \min \left\{ E_i(\mathbf{z}), \quad \alpha_i \left[\sum_{j=1}^m \mathbf{p}^*\mathbf{y}_j + \mathbf{p}^*\omega \right] \right\} \leq E_i(\mathbf{z}).$$

Thus part (a) of the definition is satisfied and the proof is complete. ■

Let us briefly comment on the assumptions of this theorem. The interiority condition, requiring that $\mathbf{x}_i^*$ is in the interior of $X_i(\mathbf{z}^*)$ for all i, is clearly too strong. It could be dispensed with if we used the notion of *compensated* VE and then applied a standard argument to show that, under assumptions such as non-oligarchy [see Hammond (1998)], the compensated valuation equilibrium is a valuation equilibrium.

We have assumed $X_i(\mathbf{z}) \subset \mathbb{R}^\ell_+$ for the sake of simplicity in exposition. This together with the interiority condition ensures that $\sum_{h\in I} \mathbf{p}^*\mathbf{x}^*_h > 0$ so that the share α_i is well defined.

The conditions imposed on conditional utility functions are standard and need no further comment.

Theorem 2 can be somewhat refined under our assumption about conditional production sets. In fact, when conditional production sets are closed and comprehensive in $\mathbb{R}^\ell$, then every efficient allocation can be decentralized as a valuation equilibrium in which all firms follow the *marginal pricing rule.* That is to say, $\mathbf{p}^* \in \bigcap_{j=1}^n \mathbb{N}_{Y_j(\mathbf{z}^*)}(\mathbf{y}^*_j)$, where $\mathbb{N}_{Y_j(\mathbf{z})}(\mathbf{y}_j)$ stands for Clarke normal cone of $Y_j(\mathbf{z})$ at $\mathbf{y}_j$.[3]

Observe that the first welfare theorem requires balancedness of the income schedule whereas the second welfare theorem does not. The lack of balancedness of the income schedule out of equilibrium, which is permitted by our definitions as long as then there is an unspent surplus, turns out to be essential in order to get the result in Theorem 2. Indeed, one can deduce from example 2.3 in Diamantaras, Gilles & Scotchmer (1996) that there is a fundamental trade-off between complexity and out-of-equilibrium balancedness.[4] To be more precise, one cannot get both balancedness and decentralizability when consumers are myopic. Decentralizing a Pareto efficient allocation as a VE, when consumers take as given the prices of private commodities, requires the Government to exercise an expropriation power that "punishes" the consumers who deviate from the chosen allocation.

Our last result in this section refers to the social stability of VE, as implied by the notion of core allocations. The definition of the core presented below is relative to a given pair of parameter vectors, which are to be interpreted as an assignment of property rights. One set of parameters tells us how total profits (or aggregate production shares) are distributed among consumers. The other expresses the entitlements of individuals on initial endowments.

Definition 5 *Let $\theta = (\theta_i)_{i\in I}$ and $\alpha = (\alpha_i)_{i\in I}$ be two m-dimensional vectors with $\theta_i \geq 0$, $\alpha_i \geq 0$ for all i, and $\sum_{i\in I} \theta_i = \sum_{i\in I} \alpha_i = 1$. A feasible alloca-*

[3] Let us recall here that the Clarke Normal Cone at a point $\mathbf{y}^* \in Y$ is the convex cone generated by the vectors perpendicular to Y at $\mathbf{y}^*$, and by the limits of vectors which are perpendicular to Y in a neighbourhood of $\mathbf{y}^*$ [Cf. Quinzii (1992, p. 19)]. It can be shown that if $\mathbf{p}\mathbf{y}'_j \geq \mathbf{p}\mathbf{y}_j$, for all $\mathbf{y}_j \in Y_j(\mathbf{z})$, then $\mathbf{p}$ is a marginal price system relative to $Y_j(\mathbf{z})$ at $\mathbf{y}'_j$. When $Y_j(\mathbf{z})$ is convex the converse is also true. Thus marginal pricing coincides with profit maximization when production sets are convex.

For a refinement of the Clarke normal cone due to Mordukhovich, see Khan (1999).

[4] We thank Dimitrios Diamantaras and Rob Gilles for drawing our attention to this aspect of the model.

tion $[\mathbf{z},(\mathbf{x}_i)_{i\in I},\widetilde{\mathbf{y}}]$ *is in* ***the core relative to*** (θ,α) *if there is no "blocking" coalition* $S\subset I$ *with an allocation* $[\mathbf{z}',\widetilde{\mathbf{x}}',\widetilde{\mathbf{y}}']$ *such that:*

(i) $\sum_{i\in S}\left(\mathbf{x}_i'-\alpha_i\omega-\theta_i\sum_{j\in J}\mathbf{y}_j'\right)=\mathbf{0}$.

(ii) $u_i(\mathbf{x}_i',\mathbf{z}')\geq u_i(\mathbf{x}_i,\mathbf{z})$, $\forall\ i\in S$, *with strict inequality for at least one* $i\in S$.

Thus, a core allocation is one in which no coalition can re-arrange the economy, using only its own resources as defined by (θ,α), so that the resulting allocation is weakly preferred by all its members, and strictly preferred by some.

The next definition extends the notion of balanced income schedules to individuals:

Definition 6 *An income schedule* R *is* ***individually balanced****, relative to* (θ,α) *if for any* $i\in I$ *and each* $(\mathbf{p},\mathbf{z},\widetilde{\mathbf{y}})$*, one has*

$$r_i^{\mathbf{z}}(\mathbf{p},\widetilde{\mathbf{y}})=\mathbf{p}\alpha_i\omega+\theta_i\sum_{j\in J}\mathbf{p}\mathbf{y}_j$$

These definitions imply the following result:

Theorem 3 *Let* $[\mathbf{p}^*,\mathbf{z}^*,\widetilde{\mathbf{x}}^*,\widetilde{\mathbf{y}}^*]$ *be a valuation equilibrium. Suppose that consumers are locally non-satiated and that* R *is individually balanced relative to* (θ,α)*. Then, the resulting allocation is in the core.*

Proof.

Suppose that there exist $S\subset I$ and $[\mathbf{z},\widetilde{\mathbf{x}},\widetilde{\mathbf{y}}]$ such that

(i) $\sum_{i\in S}\left(\mathbf{x}_i-\alpha_i\omega-\theta_i\sum_{j\in J}\mathbf{y}_j\right)=\mathbf{0}$.

(ii) $u_i(\mathbf{x}_i,\mathbf{z})\geq u_i(\mathbf{x}_i^*,\mathbf{z}^*)$, $\forall\ i\in S$.

Because of local non-satiation, the definition of VE and individual balancedness imply that

$$\mathbf{p}^*\mathbf{x}_i>r_i^{\mathbf{z}}(\mathbf{p}^*,\widetilde{\mathbf{y}})=\mathbf{p}^*\alpha_i\omega+\theta_i\sum_{j\in J}\mathbf{p}^*\mathbf{y}_j$$

for all $i\in I$. But then (i) above implies that $\mathbf{p}^*\mathbf{x}_i=r_i^{\mathbf{z}}(\mathbf{p}^*,\widetilde{\mathbf{y}})$ for all $i\in I$. Because the allocation is a VE, it follows that $u_i(\mathbf{x}_i^*,\mathbf{z}^*)\geq u_i(\mathbf{x}_i,\mathbf{z})$ and so $u_i(\mathbf{x}_i,\mathbf{z})=u_i(\mathbf{x}_i^*,\mathbf{z}^*)$ for all $i\in S$. This proves that a blocking coalition cannot exist. ■

Theorem 3 ensures that valuation equilibria generate allocations in the core. The core here is defined with respect to a series of parameters that describe the distribution of property rights. In our model this is absolutely essential, because the core is always dependent on the structure of individuals' entitlements and also, as opposed to the usual core theorems, production sets are not assumed to be convex cones. It is easy to see that a richer description of these property rights is well compatible with our result (e.g. specifying consumer i's shares in individual firms).

Remark 2 *Theorem 3 requires, in addition to local non-satiation, to extend the balancedeness condition to all possible coalitions. This is formally equivalent to require individual balancedness.*

4 The results for the far-sighted case

It has been assumed in the discussion so far that consumers are myopic, in the sense that they evaluate alternative actions taking the prices of private commodities as given. As in Hammond & Villar (1998), one can argue that when there are non-convexities, any change in the public environment may well induce substantial changes in market prices, and so have a large influence on individuals' budget sets. This reasoning seems even more compelling when conditional production sets are not convex.

So now we suppose instead that consumers are *far sighted*. Specifically, consumers believe they can compute the changes in the prices of private commodities that result from changes in the environment. To make things simple, define the consumers' common price forecast as a single-valued mapping $\rho : \mathcal{P} \times \mathbf{Z} \times \mathbf{Z} \to \mathcal{P}$, satisfying $\rho(\mathbf{p}, \mathbf{z}, \mathbf{z}) = \mathbf{p}$. The mapping is to be interpreted as follows. For each given $(\mathbf{p}, \mathbf{z}, \mathbf{z}') \in \mathcal{P} \times \mathbf{Z} \times \mathbf{Z}$, the point $\rho(\mathbf{p}, \mathbf{z}, \mathbf{z}')$ describes the consumers' common forecast of the price vector that results when the *status quo* $(\mathbf{p}, \mathbf{z})$ changes to a new environment $\mathbf{z}'$. Moreover, the price forecast associated with maintaining the status quo $(\mathbf{p}, \mathbf{z})$ is assumed to be precisely $\mathbf{p}$.

Function ρ becomes now part of the description of the economy. Therefore, we shall denote by $\mathcal{E}_\rho$ an economy in which agents calculate the change of prices associated with a change in the environment according to function ρ.

Now we can introduce the following definition:

Definition 7 *A **far-sighted valuation equilibrium** (FSVE for short) for an economy $\mathcal{E}_\rho$, is a price vector $\mathbf{p}^* \in \mathcal{P}$, a public environment $\mathbf{z}^* \in \mathbf{Z}$, and an allocation $(\widetilde{\mathbf{x}}^*, \widetilde{\mathbf{y}}^*)$ such that:*

(i) For every $i \in I$, the pair $(\mathbf{x}_i^*, \mathbf{z}^*)$ maximizes u_i with respect to $(\mathbf{x}_i, \mathbf{z})$ subject to the constraints $(\mathbf{x}_i, \mathbf{z}) \in X_i$ and $\rho(\mathbf{p}^*, \mathbf{z}^*, \mathbf{z})\mathbf{x}_i \leq r_i \left[\rho(\mathbf{p}^*, \mathbf{z}^*, \mathbf{z}), \mathbf{z}\right]$.

(ii) $\widetilde{\mathbf{y}}^* \in \bigcap_{i=1}^m \mathcal{Y}^i(\mathbf{p}^*, \mathbf{z}^*)$.
(iii) $\sum_{i\in I} \mathbf{x}_i^* = \omega + \sum_{j\in J} \mathbf{y}_j^*$.
(iv) $\sum_{i\in I} r_i(\mathbf{p}^*, \mathbf{z}^*) = \mathbf{p}^*\omega + \sum_{j\in J} \mathbf{p}^*\mathbf{y}_j^*$.

This definition recognizes the ability which consumers are assumed to possess of evaluating alternatives at prices that are dependent on the environment. Parts (ii), (iii) and (iv) in the definition of FSVE are already familiar. Part (i) says that no consumer finds it individually beneficial to choose an alternative allocation that is affordable with the highest income achievable, according to the associated price forecast. Note that the very definition of ρ implies that $\mathbf{p}^* = \rho(\mathbf{p}^*, \mathbf{z}^*, \mathbf{z}^*)$.

The following results are immediate extensions of those above, thus the proofs are omitted.

Theorem 4 *Let* $[\mathbf{p}^*, \mathbf{z}^*, \widetilde{\mathbf{x}}^*, \widetilde{\mathbf{y}}^*]$ *be a FSVE and suppose that consumers' preferences are locally non-satiated and that the income schedule is balanced. Then the resulting allocation is Pareto optimal.*

Theorem 5 *Let* $[\mathbf{z}^*, \widetilde{\mathbf{x}}^*, \widetilde{\mathbf{y}}^*]$ *be a Pareto optimal allocation satisfying the condition that* $\mathbf{x}_i^*$ *in the interior of* $X_i(\mathbf{z}^*)$ *for all* i. *Suppose that, for all* $i \in I$ *and all* $\mathbf{z} \in \mathbf{Z}$, *the set* $X_i(\mathbf{z})$ *is convex and the conditional utility function* $u_i^{\mathbf{z}} : X_i(\mathbf{z}) \to \mathbb{R}$ *is continuous, quasi-concave, and satisfies local non-satiation. Then, there exist a price vector* $\mathbf{p}^* \in \mathcal{P}$ *and an income schedule* R *such that* $[\mathbf{p}^*, \mathbf{z}^*, \widetilde{\mathbf{x}}^*, \widetilde{\mathbf{y}}^*]$ *is a far-sighted valuation equilibrium.*

Theorem 6 *Let* $[\mathbf{p}^*, \mathbf{z}^*, \widetilde{\mathbf{x}}^*, \widetilde{\mathbf{y}}^*]$ *be a FVE. Suppose that the consumers' preferences are locally non-satiated and that the income schedule* R *is individually balanced relative to* (θ, α). *Then, the resulting allocation is in the core.*

5 Final comments

We have presented a notion of valuation equilibrium whose usual efficiency properties extend to those economies in which the conditional production sets need not be convex. This extension requires households to compute their demands adjusting the production schedule in order to maximize their income (besides allowing for changes in the public environment). Under fairly general assumptions, it has been shown that a VE (resp. a FSVE) satisfies the two welfare theorems and yields core allocations. In particular, the model allows for the presence of increasing returns to scale, or more general forms of non-convexities in production sets, in the conditional economies. It is worth stressing that the model works the same way whether we assume that consumers are *myopic* or *far-sighted.*

Given the efficiency properties of valuation equilibria, it is not surprising that the existence of an equilibrium cannot be ensured in this general framework.

It follows from the first welfare theorem that the notion of valuation equilibrium imposes restrictions on the admissible behaviour of the firms, even though no specific policy is assumed *a priori.* Indeed, the firms must adhere to some form of marginal pricing, since efficiency requires equating the marginal rates of transformation to the relative prices. For instance, when firms are standard (closed convex and comprehensive production sets), a valuation equilibrium corresponds to a competitive equilibrium. When conditional production sets are not convex there are some degrees of freedom because marginal pricing is not uniquely determined (e.g. there are different systems of two part tariffs, some forms of constrained profit maximization and some perfect price discrimination policies that may be compatible with the marginal pricing principle).

The efficiency of valuation equilibria has also consequences on the admissible behaviour of the public sector, even though the tax system is hidden behind the abstract income schedule used in the model. On the one hand, it requires that the production of public goods must be financed by some kind of (possibly non-linear) Lindahl pricing. On the other hand, when conditional production sets are not convex the system of transfers will imply positive transfers for some consumers and negative transfers for some others *in equilibrium* [on this see the discussion in Vohra (1991)]. This implicitly says that in most cases the use of cost-sharing methods or two-part tariffs will not be enough to ensure efficiency.

The public sector's choice space has been modelled as a given fixed set, for the sake of simplicity in exposition. Yet it may be affected by the actions of some other agents and by the market prices of private goods. The type of variables included in $\mathbf{Z}$ implicitly express our assumptions about the influence that the public sector has on the private economy. It seems natural to identify the elements of the public environment with those variables which are usually decided upon by the public sector (e.g. decisions that respect the prevailing constitutional rights). Hence, $\mathbf{Z}$ typically includes aspects related to the public expenditure and in particular the provision of public goods, the tax system, the regulation of some instances of the economic activity such as imports, migration, energy prices, production of genetically modified organisms, etc. All these are "control variables" whose values can be chosen by some central agency.

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