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ABSTRACT

Inflation and Welfare in Long-Run Equilibrium with Firm Dynamics*

We analyze the welfare cost of inflation in a model with cash-in-advance constraints and an endogenous distribution of establishments' productivities. Inflation distorts aggregate productivity through firm entry dynamics. The model is calibrated to the United States economy and the long-run equilibrium properties are compared at low and high inflation. We find that, when the period over which the cash-in-advance constraint is binding is one quarter, an annual inflation rate of 10 percent leads to a decrease in the steady-state average productivity of roughly 0.5 percent compared to the optimum's steady-state. This decrease in productivity is not innocuous: it leads to a doubling of the welfare cost of inflation.

JEL Classification: E40, E50, L16, O40

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1 Introduction

Whether the adoption of monetary policy rules that reduce inflation and interest rates leads to important welfare gains is a central question in monetary economics.[1] Calculations often suggest that the effects of changes in the inflation rate on capital accumulation are modest. However, if international differences in income per capita are explained by differences in the accumulation of productive factors and by differences in the efficiency in the employment of these factors, then the welfare cost of inflation will be high if it discourages the accumulation of factors of production or if it leads to less efficiency in their use.[2] The first possibility has been extensively examined in the literature however the latter has been neglected. In this paper we begin the exploration of this second possibility.

In an influential paper, Cooley and Hansen (1989) provide estimates of the welfare costs of inflation within the framework of a neoclassical monetary economy where money is held because of cash-in-advance constraints. At moderate inflation rates, these models produce relatively modest welfare costs; for example, Cooley and Hansen (1989) report that, in steady-state, a 10 percent inflation rate results in a welfare cost of about 0.4 percent of income relative to an optimal monetary policy.

However, in these earlier models average productivity is exogenous and only the accumulation of factors of production matters to determine income. Gomme (1993), De Gregorio (1993) and Jones and Manuelli (1995) extend the work on the effects of monetary policy to models of endogenous growth and find the welfare cost of inflation to be either of the same magnitude or an order of magnitude smaller. But their work assumes a single representative firm and abstract from heterogeneity in production units. If, however, the allocation of aggregate resources across uses is important in understanding cross-country differences in per capita incomes, then it is not only the level of factor accumulation that matters, but also how these factors are allocated across heterogeneous production units.[3]

[1] See Lucas (2000).

[2] Indeed, the prevailing view in development accounting is that cross-country differences in income per capita are mostly explained by differences in Total Factor Productivity. See King and Levine (1994), Klenow and Rodriguez-Clare (1997), Prescott (1998), Hall and Jones (1999) and Caselli (2005).

[3] There is substantial evidence of the importance of capital and labor allocation across establishments as a determinant of aggregate productivity. Studies document that about half of overall productivity growth in U.S. manufacturing can be attributed to factor reallocation from low productivity to high productivity

In this paper, we investigate what is the impact of higher rates of monetary growth on the economy in a model where the productivity distribution of incumbent establishments is endogenous. For this purpose, we consider establishment heterogeneity along the lines of Hopenhayn (1992), Hopenhayn and Rogerson (1993) and Melitz (2003) to explain the endogenous selection of firms in the industry. We incorporate this framework into a monetary economy characterized with cash-in-advance constraints on consumption and investment goods, and in addition we assume that liquidity constraints also apply to the creation of new establishments. Thus, in the model individuals must use cash to create new business start-ups. This assumption is supported by substantial evidence that finance constraints are often binding constraints facing aspiring entrepreneurs.

For instance, in work using U.S. micro data, Evans and Leighton (1989) and Evans and Jovanovic (1989) have argued formally that entrepreneurs face liquidity constraints. Blanchflower and Oswald (1998) present further evidence on the barriers to entrepreneurship, this time based on the National Survey of the Self-Employed, which draws on information from a random sample of approximately 12,000 adults interviewed in Britain in the spring of 1987. Individuals who were recently self-employed were asked to name the main source of finance used to set up their business. Out of the 243 respondents who were in this special category, 42 percent reported that they used their own savings to set up the business, 15 percent used money from family or friends, while only 17 percent took a bank loan. When asked the question "What help would have been most useful to you in setting up your business?" the most commonly recorded item – by the same group of individuals – was assistance with money and finance (mentioned by a quarter of respondents).[4] All this evidence is consistent with binding cash-in-advance constraints for business start-ups and suggests that whatever goods or services need to be purchased to create new businesses, they are difficult to purchase trough credit.

establishments for different time periods. See for instance Baily et al. (1992), Bartelsman and Doms (2000) and Foster et al. (2008), among others.

[4]Blanchflower and Oswald provide another elucidating test of the finance-constraint hypothesis. The test uses data on inheritances and gifts and their results show that individuals who have received money through inheritances or gifts are more likely to run their own businesses. This finding suggests that some sort of cash constraint is a binding constraint on the creation of new businesses. Similar evidence is reported in Holtz-Eakin et al. (1994).

Our framework allows us to analyze the effect of long-run monetary growth on average productivity. In addition to discouraging investment and labor supply, we find that an increase in the long-run rate of money growth increases the cost of creating new establishments. As a result, incumbent establishments' profits must increase so as to encourage industry entry. This allows new establishments with low productivity to stay in the industry leading to a reallocation of the factors of production toward less efficient establishments. The adjustment in the size distribution of incumbents lowers the economy's average productivity.

We calibrate the model to the U.S. economy and find that an annual inflation rate of 10 percent leads to a decrease in the steady-state average productivity of about 0.5 percent, compared to the efficient steady-state. Furthermore, we estimate the welfare cost due to the inflation tax of 10 percent inflation to be about 0.9 percent of aggregate consumption, using a quarter for the period over which money must be held. As it turns out, roughly half of the welfare cost of inflation is associated with the fall in average productivity. We consider several alternative calibrations to the benchmark economy, revealing the importance of the assumptions made regarding the returns to scale and the dispersion of productivities across establishments.

In work which is related to this paper, Wu and Zhang (2001) examine the effects of anticipated inflation in a framework characterized by monopolistic competition and a well defined industry structure. In their paper, firms' mark-ups are affected by the rate of inflation. They find that at higher rates of inflation the number of firms is less and their size is smaller. The resulting welfare cost of inflation is larger than the conventional estimates. In our paper, the welfare cost of inflation is also higher than those obtained in conventional models. Moreover, as their model, our model also predicts that the number of incumbent establishments is lower at high rates of inflation. However, in our paper markets are competitive and the higher welfare cost is associated with the change in the productivity distribution of incumbent establishments.

Given the abundance of empirical evidence indicating the importance of producers' heterogeneity and selection-based productivity growth, it is hardly surprising that an influential literature has developed, which examines the reallocation effects of policy distortions. In the article mentioned earlier, Hopenhayn and Rogerson (1993) consider the effect on average productivity and welfare of employment protection in a setting characterized with firm entry and exit dynamics. They find that a tax on job destruction results in a decrease in average productivity of over 2 percent. In a related paper Veracierto (2001) extends Hopenhayn and Rogerson's analysis of firing taxes by introducing a flexible form of capital and considering transition dynamics. Veracierto finds that firing taxes equal to one year of wages have large long-run effects: they decrease steady-state output, capital, consumption, and wages by 7.84 percent and steady-state employment by 6.62 percent. With the purpose of studying the role of international trade, Melitz (2003) shows how aggregate industry productivity growth caused by reallocations across heterogeneous establishments contributes to additional welfare gains from trade liberalization.

The role of policy distortions in environments with industry dynamics has also influenced the literature on development. For instance, Restuccia and Rogerson (2008) consider policy distortions that lead to reallocation of resources across heterogeneous firms. Their aim is to examine whether policies that leave aggregate relative prices unchanged but distort the prices faced by different producers can explain cross-country differences in per capita incomes. In their benchmark model they find that the reallocation of resources implied by such policies can lead to decreases in output and productivity in the range of 30 to 50 percent, even though the underlying range of available technologies across establishments is the same in all policy configurations. Samaniego (2006) proposes a model of plant dynamics to analyze the effects of policies that affect establishments differently depending on the stage of their life-cycle, notably subsidies to failing plants. He finds that these subsidies may increase aggregate productivity. Guner et al. (2008) find that policies that distort the size-distribution of incumbent establishments may lead to substantial output and productivity falls. Finally, Alfaro et al. (2008) investigate, using plant-level data for several countries, whether differences in the allocation of resources across heterogeneous plants are a significant determinant of cross-country differences in income per worker. They find that allowing for firm heterogeneity improves the model ability to explain differences in productivity across countries. Our paper introduces firm heterogeneity and industry dynamics into a monetary growth model and considers the distortions introduced by the inflation tax, when money holdings are required to create new establishments.

The remainder of the paper is organized as follows. In section 2 we lay out the details of our model and describe the stationary competitive equilibrium. In Section 3 we investigate the qualitative effect of changes in the monetary growth rate on the endogenous real aggregates and the size distribution of productive establishments. Section 4 discusses the procedure for calibrating our model and section 5 presents our model-based quantitative findings. Finally, section 6 concludes.

2 The model

We consider a cash-in-advance production economy, which exhibits establishment level heterogeneity as studied by Hopenhayn (1992) and Hopenhayn and Rogerson (1993). Establishments have access to a decreasing returns to scale technology, pay a fixed cost to remain in operation each period and are subject to entry and exit. In what follows we first describe the problem of the household confronted with a cash-in-advance constraint, next we describe the production side in more detail and finally characterize the stationary competitive equilibrium.

2.1 The household

There is an infinitely-lived representative household with preferences over streams of consumption and leisure at each date described by the utility function

$$U = \sum_{t=0}^{\infty} \beta^t \left(\ln C_t + A \ln L_t \right),$$

where C_t is consumption at date t, L_t is leisure and $\beta \in (0,1)$ is the discount factor. The representative agent is endowed with one unit of productive time each period. She owns three types of assets: capital, cash, and production establishments. The period 0 endowment of each asset is strictly positive.

The timing of the household decision problem resembles the one in Stockman (1981). The household enters period t with nominal money balances equal to m_{t-1} that are carried over from the previous period and in addition receives a lump-sum transfer equal to gM_{t-1} (in nominal terms), where M_t is the per capita money supply in period t. Thus, the money stock follows the law of motion

$$M_t = (1+g)\, M_{t-1}.$$

Output has three purposes: (i) it can serve as a *consumption good*; (ii) as an *investment good* which increases the stock of capital owned by the household; (iii) as a *marketing good* which has to be purchased in order to create new establishments and constitutes a sunk cost. Households are required to use their previously acquired money balances to purchase goods. Because we want to compare situations when the constraint applies to some types of good but not to others, we introduce three parameters that we denote by θ_i with $i = c, k, h$. When $\theta_c = 1$ the cash-in-advance constraint applies to the consumption good, when $\theta_k = 1$ purchases of the investment good are constrained and when $\theta_h = 1$ the constraint applies to the marketing good needed to create a new establishment. When $\theta_i = 0$ $(i = c, k, h)$ the constraint does not apply to the specific good and this good is said to be a *credit good* in the Lucas and Stokey (1987) sense. Hence, the constraint reads as

$$\theta_c C_t + \theta_k X_t + \theta_h \kappa E_t \leq \frac{m_{t-1} + g M_{t-1}}{p_t}, \tag{1}$$

where p_t is the price level at time t, κ is the quantity of marketing good that has to be purchased to create each new establishment, E_t is the mass of new establishments created and X_t is investment, given by

$$X_t = K_{t+1} - (1 - \delta) K_t, \tag{2}$$

where K_t is the capital stock.

The representative household must choose consumption, investment, leisure, nominal money holdings and the mass of new establishments subject to the cash-in-advance constraint (1) and the budget constraint

$$C_t + X_t + \kappa E_t + \frac{m_t}{p_t} \leq w_t N_t + r_t K_t + \bar{z}_t H_t + (m_{t-1} + g M_{t-1}) / p_t, \tag{3}$$

where $N_t \equiv (1 - L_t)$ is time spent working and H_t is the mass of (incumbent) establishments at time t; also, w_t is the wage rate, r_t the rate of return on capital and $\bar{z}_t$ are average dividends across incumbent establishments.

We assume that the gross growth rate of money, $1 + g$, always exceeds the discount factor, β, which is a sufficient condition for (1) to always bind in equilibrium and existence of a stationary equilibrium.[5] We sometimes denote real money balances by $\mu_t = \frac{m_t}{p_t}$.

[5]It can be shown that the existence of a steady-state requires $1 + g \geq \beta$. See Abel (1985).

2.2 Production establishments

Once a new establishment is created at t, its idiosyncratic productivity $s \in S$ is revealed as drawn from a distribution $F(s)$ and remains constant over time until the establishment exits the industry. At $t+1$ the establishment starts production. Incumbent establishments produce output by renting labor and capital. The production function of an establishment with idiosyncratic productivity s at time t is

$$y_{s,t} = s n_{s,t}^{\alpha} k_{s,t}^{\nu} - \eta, \tag{4}$$

where $n_{s,t}$ and $k_{s,t}$ are labor and capital employed, η is a fixed operating cost, $\alpha \in (0,1)$, $\nu \in (0,1)$ and $\nu + \alpha < 1$. The flow profits of an incumbent establishment are given by

$$z_{s,t} = \max_{n_{s,t},k_{s,t}} \left\{ s n_{s,t}^{\alpha} k_{s,t}^{\nu} - w_t n_{s,t} - r_t k_{s,t} - \eta \right\}, \tag{5}$$

where w_t is the wage rate and r_t is the return on capital.

Establishments exit both because of exogenous exit shocks and endogenous decisions. In particular, in any given period after production takes place, each establishment faces a constant probability of death equal to λ. Moreover, an establishment decides to leave the industry if its discounted profits are negative. Given that we only analyze the stationary equilibrium of the economy and idiosyncratic productivities are constant over time, it turns out that the only moment when an establishment decides to leave the industry is upon entry. This is because profits are constant over time in the stationary equilibrium. Consequently, establishments choose to exit when

$$z_s < 0.$$

We denote by $s^\star$ the idiosyncratic productivity threshold below which establishments choose to exit. Specifically, $s^\star$ is such that $z_{s^\star} = 0$.

Given the first order conditions which solve the problem of incumbent firms (5) the labor demand by an establishment with productivity s is

$$n_{s,t} = s^{\sigma} \left(\frac{\alpha}{w_t}\right)^{(1-\nu)\sigma} \left(\frac{\nu}{r_t}\right)^{\nu\sigma} \tag{6}$$

and the demand for capital reads

$$k_{s,t} = s^{\sigma} \left(\frac{\alpha}{w_t}\right)^{\alpha\sigma} \left(\frac{\nu}{r_t}\right)^{(1-\alpha)\sigma}, \tag{7}$$

where $\sigma = (1-\alpha-\nu)^{-1}$. Replacing the factor demands into the profit function yields

$$z_{s,t} = \Omega \frac{s^{\sigma}}{w_t^{\alpha\sigma} r_t^{\nu\sigma}} - \eta, \tag{8}$$

where $\Omega = \alpha^{\alpha\sigma}\nu^{\nu\sigma} - \alpha^{(1-\nu)\sigma}\nu^{\nu\sigma} - \alpha^{\alpha\sigma}\nu^{(1-\alpha)\sigma}$.

Let $h(s;t)$ denote the mass of incumbent establishments with productivity level s at time t. The motion equation for $h(s;t)$ is given by

$$h(s;t+1) = (1-\lambda)h(s;t) + E_t dF(s) I[s \geq s_t^{\star}], \tag{9}$$

where I is an indicator function that takes value one if the expression in brackets is true and zero otherwise. With $H_t = \int_{s\in S} h(s;t)ds$ denoting the mass of incumbent establishments. Consequently, the mass of entrants reads

$$E_t = \frac{H_{t+1} - (1-\lambda) H_t}{1 - F(s_t^{\star})}. \tag{10}$$

2.3 Household optimal behavior

The Bellman equation characterizing household's optimal behavior reads as

$$V(m_{t-1}, K_t, H_t) = \max_{C_t, L_t, m_t, K_{t+1}, H_{t+1}} \{\ln C_t + A \ln L_t + \beta V(m_t, K_{t+1}, H_{t+1})\}, \tag{11}$$

and is subject to the cash-in-advance constraint (1) and the budget constraint (3).

Let ϕ_t and γ_t be the Kuhn-Tucker multipliers for the constraints (1) and (3), respectively. The first-order conditions which characterize the solution to the problem of the household are

$$\frac{1}{C_t} - \theta_c \phi_t - \gamma_t = 0, \tag{12}$$

$$\frac{A}{L_t} - \gamma_t w_t = 0 \tag{13}$$

$$\beta V_1(m_t, K_{t+1}, H_{t+1}) - \frac{\gamma_t}{p_t} = 0, \tag{14}$$

$$\beta V_2(m_t, K_{t+1}, H_{t+1}) - \theta_k \phi_t - \gamma_t = 0, \tag{15}$$

$$\beta V_3(m_t, K_{t+1}, H_{t+1}) - \frac{\kappa}{1 - F(s_t^{\star})}(\theta_h \phi_t + \gamma_t) = 0, \tag{16}$$

plus the budget constraint and the complementary slackness condition associated with the budget constraint. Moreover, by the envelope theorem, the shadow values of money, capital and the mass of establishments are respectively

$$V_1\left(m_{t-1}, K_t, H_t\right) = \frac{\phi_t + \gamma_t}{p_t}, \tag{17}$$

$$V_2\left(m_{t-1}, K_t, H_t\right) = \left(1-\delta\right)\left(\theta_k\phi_t + \gamma_t\right) + \gamma_t r_t \tag{18}$$

and

$$V_3\left(m_{t-1}, K_t, H_t\right) = \frac{1-\lambda}{1-F(s_t^\star)}\kappa\left(\theta_h\phi_t + \gamma_t\right) + \gamma_t\bar{z}_t. \tag{19}$$

Combining (17), (18) and (19) and the first-order conditions (14), (15) and (16) yields the three Euler equations

$$\beta\frac{\phi_{t+1} + \gamma_{t+1}}{p_{t+1}} - \frac{\gamma_t}{p_t} = 0, \tag{20}$$

$$\beta\left(1-\delta\right)\left(\theta_k\phi_{t+1} + \gamma_{t+1}\right) + \beta\gamma_{t+1}r_{t+1} - \theta_k\phi_t - \gamma_t = 0 \tag{21}$$

and

$$\beta\frac{1-\lambda}{1-F(s_{t+1}^\star)}\kappa\left(\theta_h\phi_{t+1} + \gamma_{t+1}\right) + \beta\gamma_{t+1}\bar{z}_{t+1} - \kappa\frac{\theta_k\phi_t + \gamma_t}{1-F(s_t^\star)} = 0. \tag{22}$$

Equations (12) and (20)-(22), combined with the intra-temporal first-order condition (13) and the budget constraint (3) characterize the solution to the household problem.

2.4 Market clearing

Market clearing conditions for labor and capital are given, respectively, by

$$N_t = \int_{s\in S} n_{s,t}h(s;t)ds \tag{23}$$

and

$$K_t = \int_{s\in S} k_{s,t}h(s;t)ds. \tag{24}$$

Market clearing in the money market requires

$$m_t = M_t. \tag{25}$$

Finally, the economy's feasibility constraint reads

$$C_t + X_t + \kappa E_t = Y_t, \tag{26}$$

where $Y_t \equiv \int_{s\in S} y_{s,t}h(s;t)ds$.

2.5 Stationary equilibrium

We consider the steady-state competitive equilibrium of the model. In a steady-state equilibrium, all rental rates and real aggregates are constant over time. Moreover, the gross rate of inflation $\Pi \equiv \frac{p_{t+1}}{p_t}$ is also constant, equal to the gross rate of monetary growth $1+g$. Thus, we henceforth ignore all time subscripts to simplify the notation. Following Melitz (2003), it is useful to define average productivity as

$$\bar{s} = \left\{ \int_{s \geq s^\star} s^\sigma \frac{dF(s)}{1-F(s^\star)} \right\}^{\frac{1}{\sigma}}. \tag{27}$$

Hence, with knowledge of $s^\star$ one can identify average productivity, $\bar{s}$. From equation (8), this implies that average dividends read as

$$\bar{z} = \int_{s \geq s^\star} z_s \frac{dF(s)}{1-F(s^\star)} ds = \Omega \frac{\bar{s}^\sigma}{w^{\alpha\sigma} r^{\nu\sigma}} - \eta. \tag{28}$$

We now illustrate three effects of inflation related to the three cash-in-advance constraints of the economy.

Since the shadow values ϕ and γ are each positive and constant in the steady-state,[6] from equations (12), (13) and (20), consumption and leisure in the steady-state equilibrium satisfy the condition

$$\frac{L}{C} = \frac{A}{w} \left[1 + \theta_c \left(\frac{1+g}{\beta} - 1 \right) \right]. \tag{29}$$

Equation (29) suggests that, when the cash-in-advance constraint applies to consumption, an increase in inflation raises the cost of consumption relative to leisure. This result corresponds to the effect examined in Cooley and Hansen (1989).

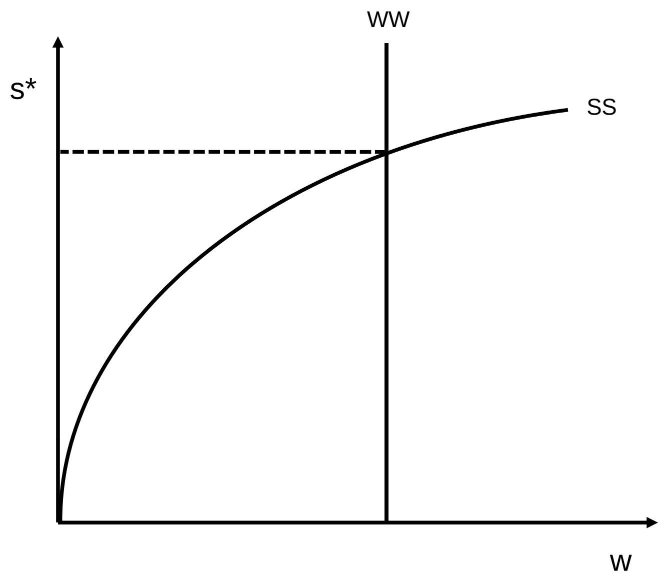
Given equations (20) and (21), the representative household problem yields the stationary equilibrium rental rate of capital, given by

$$r = \left(\frac{1}{\beta} - 1 + \delta \right) \left[1 + \theta_k \left(\frac{1+g}{\beta} - 1 \right) \right] \tag{30}$$

Equation (30) shows that the rental cost of capital is increasing in the rate of anticipated inflation when the cash-in-advance constraint applies to the investment good. It also suggests the following mechanism. When the cash-in-advance constraint applies to investment, inflation increases the cost of holding money balances, which reduces capital accumulation.

[6] See Stockman (1981).

Figure 1: Determination of $s^{\star}$ and w in the stationary economy



As a result, at higher inflation, the rental cost of capital is higher. This result is due to Stockman (1981).

Finally, from equations (20) and (22) the establishment's free-entry condition reads

$$\kappa \left[1 + \theta_h \left(\frac{1+g}{\beta} - 1\right)\right] = [1 - F(s^{\star})] \frac{\beta \bar{z}}{1 - \beta(1-\lambda)}. \tag{31}$$

Equation (31) states that in equilibrium the sunk cost that has to be paid to create a new establishment (the left-hand side of (31)) has to be equal to the expected discounted profits from creating this establishment (the right-hand side of (31)). The rate of discount of profits depends on the household discount factor β and the probability λ that the new establishment dies in future periods. The probability $[1 - F(s^{\star})]$ also appears on the right-hand side of (31) because one has to account for the probability of successful entry when evaluating discounted profits.

Equation (31) characterizes the mechanism by which money growth affects the establishments entry decision. When the cash-in-advance constraint applies to the marketing good, an increase in inflation makes entry more costly. The next Section shows that this has an effect on average productivity too.

Hence, inflation may have three effects, depending on the structure of the cash-in-advance constraint. It may affect labor supply, capital accumulation and the productivity distribution of incumbent establishments. Each effect contributes to lowering the level of output. This allows us, in the next Section, to state a Proposition on the real effects of inflation. Before doing this, we go through the remaining relations characterizing the equilibrium.

In the stationary competitive equilibrium the optimal exit rule by incumbent establishments requires $z_{s^\star} = 0$. This yields a solution for the productivity threshold, given by

$$s^\star = w^\alpha r^\nu \left(\frac{\eta}{\Omega}\right)^{1-\alpha-\nu}. \tag{32}$$

Since the equilibrium interest rate is determined by (30), the exit condition characterizes a relation between the wage rate and the productivity threshold which is represented by the SS locus in Figure 1.

In turn, the expected value of entry, i.e. the right-hand side of the free-entry condition (31) is locally independent of $s^\star$ by the envelope theorem (see Appendix A for proof). Consequently, the equilibrium wage rate is independent of $s^\star$, as illustrated by the WW locus in Figure 1. Hence, in an equilibrium with production the free-entry condition determines the wage rate.

Finally, solving for the fixed point of (9) and integrating over productivity levels yields

$$H = E \int_{s\in S} \frac{I[s \geq s^\star]}{\lambda} dF(s), \tag{33}$$

which, combined with the resource constraint (26), gives a solution for the mass of incumbent establishments, completing the characterization of the stationary competitive equilibrium. Specifically, the stationary competitive equilibrium is defined as follows:[7]

Definition 1. *A stationary competitive equilibrium is a wage rate, w, a rental rate of capital, r, an aggregate distribution of establishments, $h(s)$, a mass of entry, E, a household value function, $V(m, K, H)$, an establishment profit function, z_s, a productivity threshold, $s^\star$, policy functions for incumbent establishments, n_s and k_s, and aggregate levels of consumption, C, employment, N, capital, K and real money balances, μ, such that:*

i. The household optimizes: equations (11), (29), (30) *and* (31)*;*

ii. Establishments optimize: equations (6), (7), (8) *and* (32)*;*

[7]It is shown in the appendix B that the equilibrium exists and is unique.

iii. Markets clear: equations (23), (24), (25) *and* (26)*;*

iv. $h(s)$ *is an invariant distribution, i.e. a fixed point of* (9).

To summarize, the model is solved as follows. First, the rental cost of capital is pinned down by equation (30). Then, given the value of r, one can solve for the values of the wage rate, w, and the productivity threshold, $s^{\star}$, from (31) and (32). One can consequently characterize fully the stationary distribution of capital, employment, profits and output with equations (4), (6), (7) and (8) across incumbent firms. Finally, the feasibility constraint (26), together with the other market-clearing conditions and the first-order condition for leisure (29), allow to determine the mass of incumbents, H, and all the aggregates of the economy such as investment, consumption, output, the stock of capital and employment.[8]

3 The real effects of inflation

We now investigate the relation between inflation, the equilibrium aggregates K and N, and the size distribution of productive establishments, characterized by $s^{\star}$. Proposition 1 summarizes our main result

PROPOSITION 1. *Consider the stationary competitive equilibrium as defined earlier.*

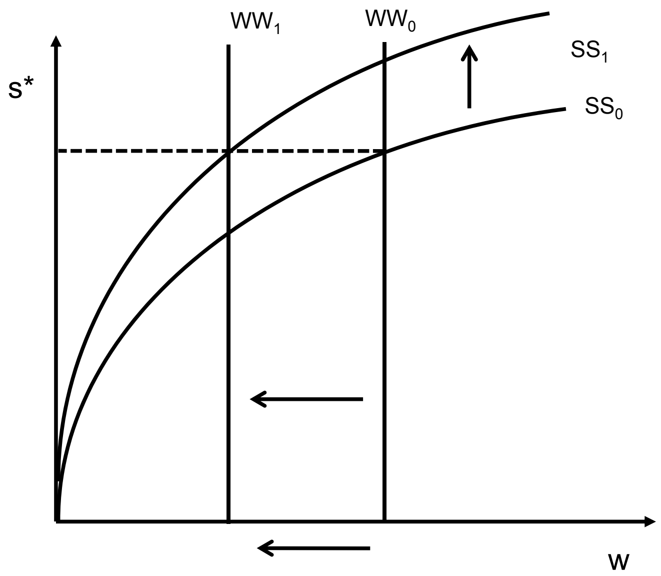
i. If $\theta_c = \theta_k = \theta_h = 0$*, an increase in the inflation rate,* Π*, has no effect on the economy.*

ii. If $\theta_c = 1$ *and* $\theta_k = \theta_h = 0$*, an increase in the inflation rate,* Π*, is associated with a fall in the equilibrium capital stock,* K*, and a fall in the employment rate,* N*. However, the productivity threshold,* $s^{\star}$*, does not change.*

iii. If $\theta_k = 1$ *and* $\theta_c = \theta_h = 0$*, an increase in the inflation rate,* Π*, is associated with a fall in the equilibrium capital stock,* K*, and a fall in the employment rate,* N*. However, the productivity threshold,* $s^{\star}$*, does not change.*

[8] In the Appendix E, we present all the equations that characterize the stationary equilibrium for the particular restriction that we impose on the distribution F. See also Section 4, where we describe the calibration procedure.

Figure 2: Effect of an increase in the monetary growth rate g on $s^\star$ and w when $\theta_k = 1$ and $\theta_h = 0$

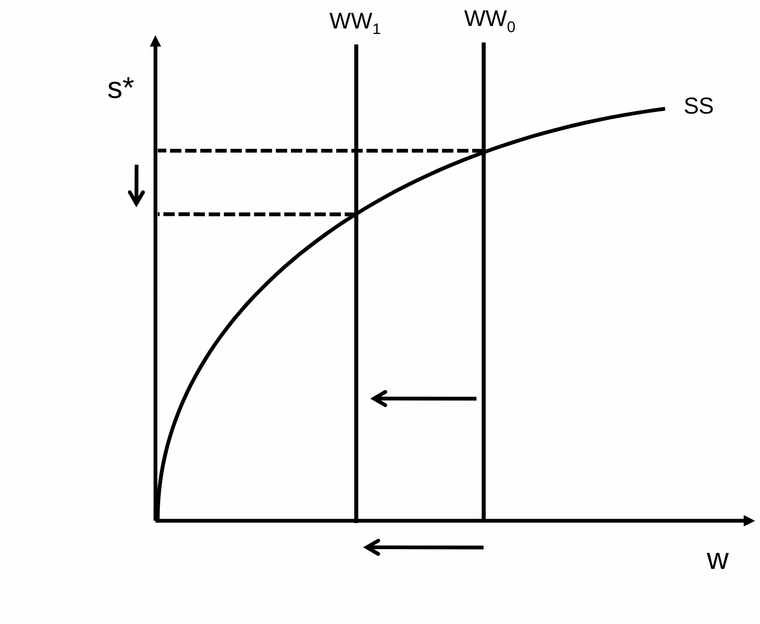


iv. If $\theta_h = 1$ and $\theta_c = \theta_k = 0$, an increase in the inflation rate, Π, is associated with a fall in the equilibrium capital stock, K, a fall in the employment rate, N, and a fall in the productivity threshold, $s^\star$.

In what follows we discuss some aspect related to Proposition 1, however, the detailed proof is developed in the Appendix D. When $\theta_i = 0$ for all i, all goods are *credit goods* and therefore money growth has no real effects. When consumption is a *cash good* condition (29) is affected by money growth. At high rates of inflation, the marginal utility of leisure must fall with respect to the product of the wage rate and the marginal utility of consumption, leading the household to supply less labor. Lower hours worked leads to lower output and therefore lower consumption and capital stock. The rental cost of capital, determined by (30), remains the same and, therefore both the SS relation and the WW relation, in Figure 1, are unaffected. Thus the wage rate and average productivity are unaffected.

When $\theta_k = 1$ – thus, investment is a *cash good* – condition (30) is affected. At high rates of inflation the return on capital must increase as individuals are less willing to invest. The increase in the rental cost of capital lowers profits for the same wage rate and therefore the probability of a successful entry decreases at each wage rate (i.e. the SS locus in Figure 2 shifts upward). However, the probability of successful entry must remain

Figure 3: Effect of an increase in the monetary growth rate g on $s^{\star}$ and w when $\theta_h = 1$ and $\theta_k = 0$



unchanged in equilibrium since the cost of creating a new establishment (the left-hand side of equation (31)) has not changed. Thus, for there to be an equilibrium with entry, the wage rate must fall sufficiently for the free entry condition to be satisfied. The WW locus in Figure 1 shifts left. At high rates of inflation the wage rate is lower but the productivity threshold is unaffected, as illustrated by Figure 2.

When the marketing good is a *cash good*, $\theta_h = 1$, the cash-in-advance constraint increases the cost of creating new establishments and the comparative static is the same as the one corresponding to an increase in the sunk cost, illustrated in Figure 3. In particular, consider the comparative statics of moving from a stationary equilibrium with a low rate of monetary growth to an equilibrium with a high rate of monetary growth. For there to be an equilibrium with entry, firms' expected value of entry must increase. Since the rental cost of capital remains unchanged, firms are not willing to enter the industry unless the wage rate falls. Accordingly the WW locus has to shift to the left which translates into a movement along the SS curve. This in turn leads to a lower productivity threshold.

4 Calibration

In this section we describe the model calibration procedure. Since we consider different model specifications – corresponding to different values for θ_i, $i = c, k, h$ – the calibration of some parameters changes across specification. When this happens, we report the values taken by the parameters for each specification (see Table 1).

In order to solve our model we need to specify a distribution for the establishments' productivity draws $F(s)$. Following Helpman et al. (2004), we assume a Pareto distribution for F with lower bound s_0 and shape parameter $\varepsilon > \sigma$, i.e. $F(s) = 1 - \left(\frac{s_0}{s}\right)^{\varepsilon}$. The shape parameter is an index of the dispersion of productivity draws: dispersion decreases as ε increases, and the productivity draws are increasingly concentrated toward the lower bound s_0. This assumption has two advantages: it generates a distribution of idiosyncratic productivities among incumbent establishments that fits microeconomic data quite well[9] and delivers close-form solutions for the endogenous aggregates.[10] Specifically, the distribution of productivities among incumbent establishments, which is the distribution F left-truncated at $s^\star$, is also Pareto with lower bound $s^\star$ and shape parameter ε.

Parameter values are selected so that the steady-state of the model economy reproduces several important features of U.S. data. Furthermore, we assume that the length of time that the cash-in-advance constraint is binding is one quarter and calibrate the model accordingly. The growth rate of the money supply, g, is chosen to be 0.006, corresponding to an annual rate of inflation of 2.43 percent, which matches the average annual rate of inflation in the U.S. between 1988 and 2007, reported in the World Economic Indicators.

For the labor and capital income shares, α and ν respectively, empirical evidence concerning establishment level returns to scale, reported by Atkeson and Kehoe (2005) suggests the relation $\alpha + \nu = 0.85$. In particular, these authors consider this choice to be consistent with the evidence in Atkenson et al. (1996). The separate identification of α and ν is done by setting the labor income share to be 64 percent, $\alpha = 0.64$, as is standard in the real business cycle literature.

The depreciation rate is chosen on the basis of estimated depreciation by the Bureau of Economic Analysis (BEA). Thus, we set $\delta = 0.0160$, implying an annual depreciation rate

[9] See Axtell (2001) and Cabral and Mata (2003).

[10] See the Appendix E for the complete description of the model solution.

Table 1: Parameters: summary

Notation	Parameter	Value
g	Monetary growth rate	0.0060
α	Labor income share	0.6400
ν	Capital income share	0.2100
δ	Depreciation rate of capital	0.0160
β	Household's discount factor	
	Model specifications for which:	
	$\theta_k = 0$	0.9902
	$\theta_k = 1$	0.9906
ε	Pareto distribution shape parameter	7.2655
λ	Failure rate of incumbent establishments	0.0179
s_0	Pareto distribution lower bound	1.0000
κ	Sunk entry cost	1.0000
η	Fixed operating cost	
	Model specifications for which:	
	$\theta_k = 1$ and $\theta_h = 1$	0.9063
	$\theta_k = 0$ and $\theta_h = 1$	0.9035
	$\theta_k = 1$ and $\theta_h = 0$	0.9092
	$\theta_k = 0$ and $\theta_h = 0$	0.9065
A	Disutility of labor	
	Model specifications for which:	
	$\theta_c = 1$ $\theta_k = 1$ and $\theta_h = 1$	2.3939
	$\theta_c = 1$ $\theta_k = 0$ and $\theta_h = 1$	2.3889
	$\theta_c = 0$ $\theta_k = 1$ and $\theta_h = 1$	2.4311
	$\theta_c = 0$ $\theta_k = 0$ and $\theta_h = 1$	2.4270
	$\theta_c = 1$ $\theta_k = 1$ and $\theta_h = 0$	2.3982
	$\theta_c = 1$ $\theta_k = 0$ and $\theta_h = 0$	2.3932
	$\theta_c = 0$ $\theta_k = 1$ and $\theta_h = 0$	2.4354
	$\theta_c = 0$ $\theta_k = 0$ and $\theta_h = 0$	2.4314

Note: The calibration of β, η and A varies according to the model specification and, in particular, according to the value taken by θ_c, θ_k and θ_c. Thus, we report the values taken by the parameters by β, η and A, for each specification.

of 6.54 percent. Given the depreciation rate, the rental cost of capital r is chosen so that the annual real interest rate is 4 percent. The implied value for the rental cost of capital, r is 0.03. In turn, this implies $\beta = 0.9906$ when investment is a *cash good*, $\theta_k = 1$, and $\beta = 0.9902$ when investment is a *credit good*, $\theta_k = 0$.

The parameter measuring the disutility of labor, A, is chosen so that individuals spend 25.5 percent of their endowment of time working, based on Gomme and Rupert (2007), who interpret evidence from the American Time-use Survey. Depending on the model specification, this yields a value for A ranging between 2.3889 and 2.4354.

Following Ghironi and Melitz (2005), we choose the shape parameter of the productivity draws' distribution in order to match the standard deviation of log U.S. plant sales, which

Table 2: Calibration: targets

Target	Value
U.S. average annual inflation rate (1988-2007)	0.0243
Production function returns to scale	0.85
Labor income share	0.64
Annual real interest rate	0.04
Annual depreciation rate of capital	0.0654
Standard deviation of log U.S. plant sales	1.67
Manufacturing establishments (6 – 10 years old) failure rates	0.303
Average establishment size (number of employees)	62
Fraction of time spent working (rate)	0.255

Note: The parameters s_0 and κ are normalized to 1.

in our case is also output and is reported to be 1.67 in Bernard et al. (2003). Since in our model, this standard deviation is $\frac{1}{\varepsilon-\sigma}$, this implies that the value for ε is 7.2655.

The establishments death rate λ is chosen based on empirical evidence reported in Dunne et al. (1989). These authors perform an empirical investigation of establishment turnover using data on plants that first began operating in the 1967, 1972, or 1977 Census of Manufacturers, a rich source of information concerning the U.S. manufacturing sector. They report five-year exit rates among plants aged 1–5 year old (39.7 percent), 6–11 year old (30.3 percent) and older (25.5 percent). As expected, plant failure rates decline with age. We choose to calibrate the exit rate of incumbent establishments by matching the exit rate of 6–11 year old firms. This yields a value for λ of 0.0179, implying that each quarter 1.79 percent of incumbent establishments exit the industry.

The remaining parameters to be calibrated are s^0, η and κ. Notice first that s_0 can be normalized to 1 without loss of generality because it has no impact on the endogenous exit-decision of new establishments. Moreover, only the ratio $\frac{\eta}{\kappa}$ is identifiable and, hence, we normalize the sunk cost, κ, to 1 and solve for the resulting fixed operating cost η. The statistic used to determine η is the establishments' average employment. In particular, Hopenhayn and Rogerson (1993), using data from the Manufacturing Establishments Longitudinal Research Panel, report the average number of employees in manufacturing establishments to be about 62 employees. Since, as reported above, individuals spend 25.5

percent of their endowment of time working, this implies that the average establishment employment in units of time is 15.81. The resulting value of the fixed operating cost η ranges between 0.9035 and 0.9092, depending on the model specification.

This completes the calibration description. Table 1 summarizes the parameter values and Table 2 the targets informing our choices.

5 Results

The Friedman rule, that is, deflating the economy at the rate of time preference is optimal in this economy.[11] We use the model economy just described to contrast the efficient steady-state to the long-run equilibria associated with alternative monetary policy rules. In particular, we describe how the macroeconomic aggregates, including output, consumption, investment and aggregate hours as well as the number of incumbent establishments and average productivity vary with respect to the Pareto optimal allocation, at various rates of monetary growth. We then use the model to measure the welfare costs of anticipated inflation under alternative model specifications. Finally, we examine the role played by firm heterogeneity in explaining our findings.

5.1 Steady-state properties

We choose as the benchmark monetary growth rate, $g = \beta - 1$, which is the policy rule yielding the Pareto optimal allocation. Accordingly, Tables 3 and 4 report the level of each macroeconomic aggregate of interest and of average productivity relative to the levels corresponding to the Pareto optimal steady-state. As shown in Tables 3 and 4, anticipated inflation has a significant impact on the long-run equilibrium of the economy. Steady-state output, consumption, investment, hours and the number of establishments in the economy are all lower whenever the monetary growth rate exceeds $\beta - 1$. We begin by interpreting the results in each table.

Table 3 corresponds to model specifications where $\theta_h = 1$ and, hence, the marketing good is a *cash good*. The Table includes four Panels, each corresponding to an alternative configuration of the cash-in-advance constraint. When the cash-in-advance constraint

[11] We show this is the case in Appendix C.

Table 3: Steady-states associated with various annual monetary growth rates relative to the benchmark when the marketing good is a *cash good*, i.e.: $\theta_h = 1$

	Panel A: $\theta_c = 1$ and $\theta_k = 1$					Panel B: $\theta_c = 1$ and $\theta_k = 0$				
Annual Inflation	100×					100×				
Rate in %	$(\beta^4 - 1)$	0.00	2.43*	10.00	15.00	$(\beta^4 - 1)$	0.00	2.43*	10.00	15.00
Output	100.00	98.60	97.71	95.14	93.56	100.00	98.98	98.36	96.55	95.43
Consumption	100.00	98.87	98.15	96.04	94.74	100.00	99.09	98.54	96.92	95.91
Investment	100.00	97.67	96.22	92.03	89.50	100.00	98.98	98.36	96.55	95.43
Hours	100.00	99.10	98.54	96.87	95.84	100.00	99.19	98.70	97.25	96.35
# Establishments	100.00	98.60	97.71	95.14	93.56	100.00	98.98	98.36	96.55	95.43
Productivity	100.00	99.87	99.79	99.54	99.39	100.00	99.87	99.78	99.54	99.39
	Panel C: $\theta_c = 0$ and $\theta_k = 1$					Panel D: $\theta_c = 0$ and $\theta_k = 0$				
Annual Inflation	100×					100×				
Rate in %	$(\beta^4 - 1)$	0.00	2.43*	10.00	15.00	$(\beta^4 - 1)$	0.00	2.43*	10.00	15.00
Output	100.00	99.29	98.84	97.52	96.71	100.00	99.71	99.53	99.00	98.67
Consumption	100.00	99.56	99.28	98.45	97.93	100.00	99.82	99.71	99.38	99.17
Investment	100.00	98.36	97.33	94.33	92.52	100.00	99.71	99.53	99.00	98.67
Hours	100.00	99.80	99.67	99.30	99.07	100.00	99.92	99.87	99.72	99.63
# Establishments	100.00	99.29	98.84	97.52	96.71	100.00	99.71	99.53	99.00	98.67
Productivity	100.00	99.87	99.79	99.54	99.39	100.00	99.87	99.78	99.54	99.39

Notes: * average U.S. inflation rate over the 1988-2007 period. The steady-states levels are reported in percentage points relative to the model which corresponds to the economy where the monetary growth rate is $g = \beta - 1$.

applies to the creation of new establishments, the size distribution of productive establishments moves toward lower productivity levels at higher monetary growth rates. Hence, the average productivity of incumbent establishments is lower at high rates of inflation. The bottom row of each Panel of Table 3 reports the level of average productivity at various rates of money growth. Inspecting each panel reveals that the money growth rule affects productivity in the same way for each possible configuration of the cash-in-advance constraint as long as $\theta_h = 1$. When the annual rate of inflation is 10 percent, productivity, relative to the optimum, is 0.46 percent lower. Thus, increasing the monetary growth rate has a negative impact on average productivity which results directly from the fact that money holdings are a requirement for the creation of new establishments.

The results regarding the other macroeconomic aggregates are of course sensitive to the model specification. Examining Panel B of both Table 3 and Table 4 illustrates the implications of anticipated inflation when consumption is a *cash good*. Agents facing high rates of inflation substitute away from consumption and toward leisure which leads to lower output and therefore lower consumption and investment. Moreover, Panel B of Table 4 reveals that, even when the liquidity constraint only applies to the consumption

Table 4: Steady-states associated with various annual monetary growth rates relative to the benchmark when the marketing good is a *credit good*, i.e.: $\theta_h = 0$

	Panel A: $\theta_c = 1$ and $\theta_k = 1$					Panel B: $\theta_c = 1$ and $\theta_k = 0$				
Annual Inflation Rate in %	$100\times(\beta^4-1)$	0.00	2.43*	10.00	15.00	$100\times(\beta^4-1)$	0.00	2.43*	10.00	15.00
Output	100.00	98.88	98.17	96.09	94.82	100.00	99.27	98.83	97.52	96.72
Consumption	100.00	99.04	98.43	96.63	95.53	100.00	99.27	98.83	97.52	96.72
Investment	100.00	97.95	96.67	92.95	90.70	100.00	99.27	98.83	97.52	96.72
Hours	100.00	99.18	98.67	97.15	96.21	100.00	99.27	98.83	97.52	96.72
# Establishments	100.00	98.88	98.17	96.09	94.82	100.00	99.27	98.83	97.52	96.72
Productivity	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	Panel C: $\theta_c = 0$ and $\theta_k = 1$					Panel D: $\theta_c = 0$ and $\theta_k = 0$				
Annual Inflation Rate in %	$100\times(\beta^4-1)$	0.00	2.43*	10.00	15.00	$100\times(\beta^4-1)$	0.00	2.43*	10.00	15.00
Output	100.00	99.57	99.30	98.50	98.01	100.00	100.00	100.00	100.00	100.00
Consumption	100.00	99.73	99.56	99.06	98.74	100.00	100.00	100.00	100.00	100.00
Investment	100.00	98.64	97.78	95.28	93.76	100.00	100.00	100.00	100.00	100.00
Hours	100.00	99.88	99.80	99.58	99.45	100.00	100.00	100.00	100.00	100.00
# Establishments	100.00	99.57	99.30	98.50	98.01	100.00	100.00	100.00	100.00	100.00
Productivity	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Notes: * average U.S. inflation rate over the 1988-2007 period. The steady-states levels are reported in percentage points relative to the model which corresponds to the economy where the monetary growth rate is $g = \beta - 1$.

good, still output and investment both fall proportionally, preserving the investment-output ratio, despite the fact that the investment good and the marketing good are *credit goods*. This result follows from the fact that the purpose of increasing the capital stock is to provide consumption in the future, which is affected by the inflation tax in the same way as consumption today.

Another implication of our model economy is that the amount of time spent working is lower at higher rates of inflation, implying an upward slopping long-run Phillips curve. This finding is robust across model specifications.

Also, both Table 3 and Table 4 show that as the monetary growth rate is increased, the number of incumbent establishments and equivalently the creation of new establishments lower substantially. There are two reasons why less establishments enter at high rates of inflation. First, since the purpose of creating new establishments is to produce consumption in the future, which is subject to exactly the same inflation tax as consumption today, the creation of new establishments is discouraged at high rates of inflation. This happens even when the marketing good is a *credit good* – Table 4. The second reason, which only intervenes when the marketing good is a *cash good* – Table 3 – has to do with the fact that the cost of creating new establishments increases as the monetary growth rate is raised. As the cost of creating new establishments is increased, the profits of incumbents must increase as well, which allows low productivity establishments to stay in the industry. This adjustment in the size distribution of productive establishments implies that labor and capital are employed less efficiently, which lowers aggregate output and, consequently, the creation of new establishments.

Finally, Panel D in Table 4 simply illustrates that the cash-in-advance constraints are the only channel through which the economy is affected by changes in the rate of growth of money. In what follows, we investigate the welfare cost of inflation and we study more carefully the role played by firm heterogeneity.

5.2 Welfare costs of inflation

To obtain a measure of the welfare cost associated with inflation we proceed in the same way as in Cooley and Hansen (1989). In particular, we compute the increase in steady-state consumption which an individual would require at a given rate of money growth, g, to be as well-off as under the optimal monetary policy rule, which achieves the Pareto optimal allocation. Thus, to compute the welfare cost associated with variations in the monetary growth rate, we solve for $\mathcal{W} \equiv \frac{\Delta C}{C}$ in the equation

$$\widehat{U} = \ln\left[(1+\mathcal{W})\,C\right] + A\ln(1-N)\,, \tag{34}$$

where $\widehat{U}$ is the level of utility attained in steady-state under the optimal monetary policy rule, $g = \beta - 1$, and C and N are the steady-state consumption and hours associated with the monetary growth rate g.

The welfare cost of inflation, $\mathcal{W}$, can be expressed in closed form as[12]

$$\mathcal{W} = \underbrace{\left[1+\theta_c\left(\frac{1+g}{\beta}-1\right)\right]}_{(i)} \underbrace{\left[1+\theta_k\left(\frac{1+g}{\beta}-1\right)\right]^{\frac{\nu}{\alpha}}}_{(ii)} \underbrace{\left(\frac{\hat{s}^\star}{s^\star}\right)^{\frac{1}{\alpha}}}_{(iii)} \underbrace{\left(\frac{1-\widehat{N}}{1-N}\right)^{1+A}}_{(iv)} -1, \tag{35}$$

where $\hat{s}^\star$ and $\widehat{N}$ are, respectively, the productivity threshold and the fraction of time spent working under the Pareto optimal allocation and $s^\star$ and N are the equivalent outcomes

[12]The solution for the welfare cost of inflation is derived in Section F of the Appendix.

Table 5: Welfare costs associated with various annual growth rates of money

	$\theta_h = 1$				$\theta_h = 0$			
	$\theta_c = 1$	$\theta_c = 1$	$\theta_c = 0$	$\theta_c = 0$	$\theta_c = 1$	$\theta_c = 1$	$\theta_c = 0$	$\theta_c = 0$
100× g	$\theta_k = 1$	$\theta_k = 0$	$\theta_k = 1$	$\theta_k = 0$	$\theta_k = 1$	$\theta_k = 0$	$\theta_k = 1$	$\theta_k = 0$
100× $(\beta^4 - 1)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.00	0.393	0.239	0.274	0.113	0.286	0.125	0.168	0.000
2.43*	0.651	0.389	0.451	0.182	0.474	0.205	0.276	0.000
10.00	1.451	0.857	0.986	0.391	1.057	0.456	0.601	0.000
15.00	1.975	1.165	1.326	0.524	1.439	0.624	0.807	0.000
20.00	2.495	1.470	1.658	0.652	1.818	0.791	1.007	0.000
40.00	4.527	2.669	2.901	1.129	3.300	1.462	1.752	0.000

Notes: * average U.S. inflation rate over the 1988-2007 period. The measure of the welfare cost of inflation is $\Delta C/C \times 100$ where ΔC is the consumption compensation needed for the representative agent to achieve the same steady-state utility associated with the optimal monetary policy rule.

under the alternative monetary growth rate. Equation (35) illustrates the various channels through which anticipated inflation affects welfare. Term (*i*) illustrates how anticipated inflation lowers welfare when consumption is a *cash good*. Term (*ii*) illustrates how anticipated inflation lowers welfare when investment is a *cash good*. Term (*iii*) illustrates how welfare is affected by changes in the threshold productivity, $s^\star$. If the marketing good is a *cash good* the productivity threshold falls as the monetary growth rate increases and the cost of anticipated inflation is amplified. Finally, term (*iv*) shows the contribution of leisure.[13]

Table 5 shows our findings. The left-hand side Panel corresponds to the specifications where the cash-in-advance constraint applies to the entry sunk cost and the right-hand side Panel considers the other cases. When the cash-in-advance constraint does not apply to the sunk cost the welfare costs of inflation we obtain are of the same order of magnitude as the ones obtained by Cooley and Hansen (1989). In particular, when only consumption is a *cash good* – the specification which corresponds more closely to the Cooley and Hansen model – the welfare cost of a 10 percent rate of inflation is 0.46 percent of steady-state consumption. This is roughly the same cost which is reported in Cooley and Hansen (1989).

However, when the cash-in-advance constraint applies to the marketing good, the welfare costs of inflation is almost doubled. For example, the welfare cost of a 10 percent rate of inflation when the consumption and marketing goods are *cash goods* is 0.86 percent of

[13]As each cash-in-advance constraint contributes to increase leisure as the monetary growth rate is increased – implying, $\widehat{N} > N$ – term (*iv*) lowers the cost of anticipated inflation.

steady state consumption. A substantial part of the welfare losses at high rates of inflation are explained by the lower efficiency in the allocation of resources across incumbent establishments and not just by less accumulation of factors of production. Contrasting the second row of the left-hand side panel and the second row of the right-hand side panel shows that when the cash-in-advance constraint applies to the creation of new establishments the welfare cost of inflation nearly doubles.

If all three goods are *cash goods*, the welfare cost of 10 percent inflation is 1.45 percent of steady-state consumption. Thus, the cost of inflation resulting from lower investment and time spent working can be substantially amplified by the fall in the wage rate implied by the distortion to the establishments' entry and exit dynamics. Finally, if money only affects firm entry and exit dynamics – when the marketing good is the only *cash good* – the welfare cost of 10 percent inflation is 0.39 percent of steady-state consumption.

5.3 The role of returns to scale

Atkeson et al. (1996) forcefully show that the choice of the returns to scale in models with industry dynamics is an important determinant of the size of the effect of policy distortions on average productivity and welfare.[14] Therefore, in this section we consider how sensitive our estimates of the welfare costs of inflation are to changes in the returns to scale. As expected, as $\alpha + \nu$ approaches one, productivity is no longer affected by changes in the monetary growth rate and the contribution of factors reallocation to the welfare cost of inflation disappears. However, this contribution increases at a high rate, as the intensity of diminishing returns increases.

Table 6 shows the average productivity associated with different degrees of diminishing returns to scale and the corresponding welfare cost of inflation. For each model specification consumption is a *cash good* but investment is a *credit good* – Cooley and Hansen's (1989) specification. This allows us to understand the role of productivity in explaining the welfare cost of inflation for different degrees of diminishing returns. Once again, we consider the welfare cost of 10 percent inflation.

[14]Moreover, it should be noted that Atkeson et al. (1996) present evidence against the hypothesis that plant production or profit functions are nearly linear. This offers support to the view that policy distortions have sizable effects.

Table 6: Welfare costs corresponding to different degrees of diminishing returns to scale

		$\theta_h = 0$	$\theta_h = 1$	Share of welfare cost
$\alpha + \nu$	$100 \times \frac{\Delta \bar{s}}{\bar{s}}$	$100 \times \mathcal{W}$	$100 \times \mathcal{W}$	explained by fall in $\bar{s}$
0.75	-0.73	0.43	1.05	0.59
0.80	-0.60	0.44	0.96	0.54
0.85	-0.46	0.46	0.86	0.47
0.90	-0.32	0.47	0.74	0.37
0.95	-0.16	0.47	0.62	0.23
0.99	-0.03	0.48	0.51	0.06

Note: The measure of the welfare cost of inflation is $\mathcal{W}$, the percentage increase in consumption required to for the representative agent to achieve the same steady-state utility associated with the Pareto optimum allocation, when the annual inflation rate is 10 percent. For each model specification consumption is a cash good – $\theta_c = 1$ – and investment is a *credit good* – $\theta_k = 0$.

Naturally, when the returns to scale are nearly constant, $\alpha+\nu = 0.99$, the productivity is almost not affected as the monetary growth rate is increased. Indeed, average productivity is only 0.03 percent lower at 10 percent inflation, compared to the level under the optimal policy. Hence, the welfare costs of inflation are roughly the same, irrespectively of whether the cash-in-advance constraint applies to the marketing good or not. The last column of Table 6 shows how distortions to the size distribution of productive establishments contribute to the welfare costs of inflation.[15] As expected, when the returns to scale are nearly constant this contribution is very small. However, the contribution increases fast, as the intensity of diminishing returns increases. Indeed, for the range of $\alpha + \nu$ between 0.75 and 0.90, which is likely to include the empirically relevant values, the contribution of distortions to the size distribution of incumbents is sizable, taking values between 37 and 59 percent of the total welfare cost of inflation.

As the intensity of diminishing returns increases, the share of welfare cost explained by a fall in average productivity increases (see the last column in Table 6). This happens for two reasons. First, as returns diminish faster, the distortions to the size distribution of establishments, resulting from the inflation tax, are more important and lead to significant falls in average productivity. Thus, when the cash-in-advance constraint applies to the marketing good, i.e. $\theta_h = 1$, the welfare cost of inflation is high. However, an additional

[15] We quantify this by computing the percentage increase in the welfare cost of inflation when the cash-in-advance constraint applies to the sunk entry cost.

Table 7: Welfare costs corresponding to different degrees of establishment heterogeneity

vol		$\theta_h = 0$	$\theta_h = 1$	Share of welfare cost
$\equiv \frac{1}{\epsilon-\sigma}$	$100 \times \frac{\Delta \bar{s}}{\bar{s}}$	$100 \times \mathcal{W}$	$100 \times \mathcal{W}$	explained by fall in $\bar{s}$
0.01	-0.03	0.33	0.35	0.07
0.50	-0.39	0.43	0.77	0.43
1.00	-0.44	0.45	0.83	0.46
1.67	-0.46	0.46	0.86	0.47
2.00	-0.47	0.46	0.87	0.47
5.00	-0.49	0.46	0.89	0.48

Note: The measure of the welfare cost of inflation is $\mathcal{W}$, the percentage increase in consumption required to for the representative agent to achieve the same steady-state utility associated with the Pareto optimum allocation, when the annual inflation rate is 10 percent. For each model specification consumption is a cash good – $\theta_c = 1$ – and investment is a *credit good* – $\theta_k = 0$.

reason why the contribution of falls in average productivity to the welfare cost of inflation increases at lower values of $\alpha + \nu$ is that when the marketing good is a *credit good*, i.e. $\theta_h = 0$, the welfare cost of inflation increases as the intensity of diminishing returns to scale decreases. This is because, when $\theta_h = 0$, the welfare cost is explained by the fall in the accumulation of factors. Thus, when $\alpha + \nu$ is low, the falls in output and welfare associated with the inflation tax are less important.

Overall, for values of $\alpha + \nu$ between 0.75 and 0.90, the empirically relevant range, the contribution of distortions to the size distribution of productive establishments is substantial and the welfare costs of 10 percent anticipated inflation, when the cash-in-advance constraint applies to the creation of new establishments, vary between 0.74 and 1.05 percent of steady-state consumption.

5.4 The role of firm heterogeneity

When the marketing good is a *cash good* the level of heterogeneity turns into an important determinant of the way changes in the monetary growth rate affect the economy: the larger the heterogeneity, the larger is the fall in productivity. Here we investigate what happens to the estimate of the welfare cost of inflation as we change the level of firm heterogeneity.

Table 7 shows different welfare cost estimates as we vary the amount of establishment heterogeneity, for two different models specifications – when the marketing good is a *credit* *good* and when it is a *cash good*.[16] As the dispersion of establishments' productivities increases, the fall in productivity associated with an increase in the rate of inflation, varies from 0.03 percent to 0.49 percent. In particular, when there is almost no heterogeneity ($\frac{1}{\epsilon-\sigma} = 0.01$), productivity is virtually not affected by the inflation tax. Moreover, as the level of heterogeneity falls to zero, productivity is not affected by changes in the monetary growth rate and, accordingly, the welfare cost of anticipated inflation is the same no matter whether the marketing good is a *cash good* or a *credit good*. This illustrates clearly that the mechanism proposed in this paper intervenes through the productivity channel.

Furthermore, we notice that as the level of heterogeneity increases toward empirically relevant values, the sensitivity of productivity to the inflation tax increases very fast. For instance, if the standard deviation of log output is 0.50 (which is about one third of our benchmark calibration), at a 10 percent monetary growth rate, productivity is lowered by 0.39 percent and the welfare cost of inflation increases substantially. Therefore, we conclude that our findings are robust to changes in the variability of establishment productivity draws over the empirically relevant range.

6 Conclusion

In this paper we set out to investigate whether it is important to model heterogeneity across productive establishments when quantifying the welfare cost of inflation. For this purpose, we study a model characterized with cash-in-advance constraints on consumption and investment goods, and in addition we assume that cash-in-advance constraints also apply to the creation of new establishments. This assumption is motivated by substantial evidence that finance constraints are often binding constraints facing aspiring entrepreneurs.

Two results come out of our analysis. First, anticipated inflation lowers aggregate productivity. This happens because an increase in the long-run rate of money growth increases the cost of creating new establishments and distorts firm entry dynamics. As a consequence, incumbent establishments' profits must increase so as to encourage industry entry, and less productive establishments choose to become incumbents, lowering average productivity. This opens a channel through which inflation may affect welfare which has

[16] Once again, for each model specification consumption is a *cash goods* but investment is a *credit good* – Cooley and Hansen's (1989) specification.

been paid little attention to in the literature. Second, the mechanism identified in the current paper is likely to be quantitatively important. In particular, our results suggest that the adjustment in the productivity distribution of incumbent establishments is responsible for about half of the welfare cost of inflation.

We have only examined the long-run benefits of implementing an optimal monetary policy associated with the reallocation of resources within an industry. The re-allocation of these resources may also entail short-run costs which could undermine our estimates of the welfare cost of inflation. Future work should examine the benefits of adopting an optimal monetary policy taking into account the adjustment path.

As was mentioned earlier, Baily et al. (1992) document that about half of overall productivity growth in U.S. manufacturing in the 1980's can be attributed to factor reallocation from low productivity to high productivity establishments. It is tempting to imagine that the sustained disinflation which occurred over the same period may have contributed to the reallocation of factors and improvements in efficiency.

Appendix

A Locally vertical WW locus

The purpose of this section is to show that the WW locus is locally vertical. Hence, equilibrium wage rate w and $s^\star$ are independent. To do this, we apply the implicit function theorem to the relation (31) with the purpose of finding $\frac{dw}{ds^\star}$. First, notice that the relation (31) can be re-written as

$$\kappa\left[1+\theta_h\left(\frac{1+g}{\beta}-1\right)\right]\frac{1-\beta(1-\lambda)}{\beta}+\left[1-F(s^\star)\right]\eta-\frac{\Omega\int_{s^\star}^{\infty}s^\sigma dF(s)}{w^{\alpha\sigma}r^{\nu\sigma}}=0, \tag{36}$$

which can simply be written as $\Phi(s^\star,w)=0$. Moreover, by the implicit function theorem $\frac{dw}{ds^\star}=-\frac{\partial\Phi(s^\star,w)}{\partial s^\star}/\frac{\partial\Phi(s^\star,w)}{\partial w}$.

Since

$$\frac{\partial\Phi(s^\star,w)}{\partial w}=\frac{\alpha\sigma\Omega}{w^{1+\alpha\sigma}r^{\nu\sigma}}\int_{s^\star}^{\infty}s^\sigma dF(s)>0,$$

a sufficient and necessary condition for $\frac{dw}{ds^\star}=0$ is simply $\frac{\partial\Phi(s^\star,w)}{\partial s^\star}=0$. In turn

$$\frac{\partial\Phi(s^\star,w)}{\partial s^\star}=f(s^\star)\left(\frac{\Omega s^{\star\sigma}}{w^{\alpha\sigma}r^{\nu\sigma}}-\eta\right)=0,$$

because relation (32) implies that in equilibrium $\frac{\Omega s^{\star\sigma}}{w^{\alpha\sigma}r^{\nu\sigma}}=\eta$. Therefore $\frac{dw}{ds^\star}=0$ and the WW locus is locally vertical.

B Existence and uniqueness of equilibrium

This Section contains a proof that the relations (31) and (32) always define a unique equilibrium[17]. The condition (31) implies a relation for average profits, given by

$$\bar{z}=\kappa\left[1+\theta_h\left(\frac{1+g}{\beta}-1\right)\right]\frac{\frac{1}{\beta}-1+\lambda}{1-F(s^\star)}. \tag{37}$$

In turn, combining the relations (28) and (32) implies that average profits must satisfy the equilibrium condition given by

$$\bar{z}=\eta\left[\left(\frac{\bar{s}}{s^\star}\right)^\sigma-1\right]. \tag{38}$$

[17] A similar argument for proving existence and uniqueness of equilibrium in this class of heterogeneous firm models can be found in Melitz (2003).

Consequently, a sufficient condition for ensuring the existence and uniqueness of $s^{\star}$ is that

$$j\left(\hat{s}\right)=\left[1-F\left(\hat{s}\right)\right]\left[\left(\frac{\bar{s}\left(\hat{s}\right)}{\hat{s}}\right)^{\sigma}-1\right]$$

be monotonically decreasing from infinity to zero on $(0,\infty)$, where

$$\bar{s}\left(\hat{s}\right)^{\sigma}=\frac{1}{1-F\left(\hat{s}\right)}\int_{\hat{s}}^{\infty}s^{\sigma}dF\left(s\right).$$

Define

$$\iota\left(\hat{s}\right)=\left(\frac{\bar{s}\left(\hat{s}\right)}{\hat{s}}\right)^{\sigma}-1.$$

By applying the Chain and Leibniz rules, the derivative of $\iota\left(\hat{s}\right)$ with respect to $\hat{s}$ is found to be

$$\begin{aligned}\iota'\left(\hat{s}\right) &= \frac{f\left(\hat{s}\right)}{1-F\left(\hat{s}\right)}\left[\left(\frac{\bar{s}\left(\hat{s}\right)}{\hat{s}}\right)^{\sigma}-1\right]-\frac{\sigma}{\hat{s}}\left(\frac{\bar{s}\left(\hat{s}\right)}{\hat{s}}\right)^{\sigma}. && (39)\\ &= \frac{\iota\left(\hat{s}\right)f\left(\hat{s}\right)}{1-F\left(\hat{s}\right)}-\frac{\sigma\iota\left(\hat{s}\right)+\sigma}{\hat{s}} && (40)\end{aligned}$$

Thus, the derivative and elasticity of $j\left(\hat{s}\right)$ are given by

$$\begin{aligned}j'\left(\hat{s}\right) &= -\frac{\sigma}{\hat{s}}\left(\iota\left(\hat{s}\right)+1\right)\left[1-F\left(\hat{s}\right)\right]<0, && (41)\\ \frac{j'\left(\hat{s}\right)\hat{s}}{j\left(\hat{s}\right)} &= -\sigma\left(1+\frac{1}{\iota\left(\hat{s}\right)}\right)<-\sigma. && (42)\end{aligned}$$

Since $j\left(\hat{s}\right)$ is non-negative and its elasticity with respect to $\hat{s}$ is strictly negative, $j\left(\hat{S}\right)$ must be decreasing to zero as $\hat{s}$ goes to infinity. Moreover, $\lim_{\hat{s}\to 0}j\left(\hat{s}\right)=\infty$ since $\lim_{\hat{s}\to 0}\iota\left(\hat{s}\right)=\infty$. Hence, $j\left(\hat{s}\right)$ is monotonically decreasing from infinity to zero on $(0,\infty)$ as needed to be proved.

C Optimal monetary policy

Here we derive the optimal rate of inflation. The proof relies on the observation that the optimal inflation rate corresponds to the case where the cash-in-advance constraint is not binding. When the cash-in-advance constraint is not binding the corresponding Lagrange multiplier is zero, i.e. $\phi_t=0$ for all t. To derive the optimal rate of inflation we start by noticing that Equation (20) can be rewritten as

$$\phi_{t+1}=\frac{\gamma_t}{\beta}\frac{p_{t+1}}{p_t}-\gamma_{t+1}. \qquad (43)$$

Hence, $\phi_{t+1} = 0$ if and only if

$$\frac{\gamma_{t+1}}{\gamma_t} = \frac{p_{t+1}/p_t}{\beta} \tag{44}$$

Given that γ_t is constant in the stationary equilibrium and positive (from equation (13)), and the growth rate of money is equal to inflation in that equilibrium, it follows that the Friedman rule applies to the stationary equilibrium of our model, that is, the optimal rate of inflation is equal to $(\beta - 1)$.

D Proof of Proposition 1

Following is a proof of Proposition 1. The case where all θ_i's are zero is trivial. In the next subsections, we analyze in more details the effect of anticipated inflation when one of the θ_i's takes value one.

D.1 Case where $\theta_c = 1$, $\theta_k = 0$ and $\theta_h = 0$

We consider first the case where $\theta_c = 1$, $\theta_k = 0$ and $\theta_h = 0$. Notice that in this context inflation does not affect the rental cost of capital in (30), nor the productivity threshold and the wage rate in (31) and (32). From (4), (6), (7) and (8), this implies that average output, employment, capital use and profits are also not affected by inflation.

To determine the effect of inflation on the other aggregates, notice that in the stationary equilibrium $X = \delta K = \delta \bar{k} H$, $\kappa E = \kappa \frac{\lambda}{1-F(s^\star)} H$ and $Y = \bar{y} H$. Replace those equations and (29) in (26) to get:

$$\frac{Lw}{A\left[1 + \theta_c \left(\frac{1+g}{\beta} - 1\right)\right]} + \delta \bar{k} H + \kappa \frac{\lambda}{1 - F(s^\star)} H = \bar{y} H \tag{45}$$

Given the labor-market clearing condition, we can write $L = 1 - N = 1 - \bar{n} H$. Replacing this relation in the above equation and rearranging terms leads:

$$H = \frac{w}{A\left[1 + \theta_c \left(\frac{1+g}{\beta} - 1\right)\right]} \left(\bar{y} - \delta \bar{k} - \kappa \frac{\lambda}{1 - F(s^\star)} + \frac{w \bar{n}}{A\left[1 + \theta_c \left(\frac{1+g}{\beta} - 1\right)\right]} \right)^{-1} \tag{46}$$

Equation(46) shows that when $\theta_c = 1$, an increase in the anticipated rate of inflation g decreases the mass of incumbent firms H. Given that average employment, capital and output are not affected, this implies that an increase in the anticipated rate of inflation g also decreases the aggregate level of capital, employment and output.

D.2 Case where $\theta_c = 0$, $\theta_k = 1$ and $\theta_h = 0$

When $\theta_k = 1$, equation (30) shows that an increase in g increases the rental cost of capital r.

To determine the effect of inflation on the productivity threshold and the wage rate in this context we use condition (38). Replacing this relation in the free-entry condition (31), we then have

$$\kappa \left[1 + \theta_h \left(\frac{1+g}{\beta} - 1\right)\right] = [1 - F(s^\star)] \frac{\beta}{1 - \beta(1-\lambda)} \eta \left[\left(\frac{\bar{s}}{s^\star}\right)^\sigma - 1\right]. \tag{47}$$

Hence, the productivity threshold does not depend on the rental cost of capital. Following an increase in g, the negative effect of the increase in r on profits cancels out with the positive effect of a decrease in wages. This latter can be seen from equations (30), (32) and (47).

Regarding the effect of inflation on average output per establishment, remark that, from equations (4), (6) and (7), average output can be written as

$$\bar{y} = \bar{s}^\sigma \left(\frac{\alpha}{w}\right)^{\alpha\sigma} \left(\frac{\nu}{r}\right)^{\nu\sigma}. \tag{48}$$

By replacing (32) in the above equation, one gets

$$\bar{y} = \frac{\eta}{\Omega} \left(\frac{\bar{s}}{s^\star}\right)^\sigma \alpha^{\alpha\sigma} \nu^{\nu\sigma}. \tag{49}$$

Hence inflation does not affect average output.

To determine the impact on average capital and employment, notice from (6) and (7) and the fact that the productivity threshold is not affected by inflation that

$$d \ln \bar{n} = -(1-\nu)\sigma d \ln w - \nu\sigma d \ln r \tag{50}$$

$$d \ln \bar{k} = -\alpha\sigma d \ln w - (1-\alpha)\sigma d \ln r \tag{51}$$

Given that

$$\alpha d \ln w = -\nu d \ln r \tag{52}$$

from equation (32) and the fact that $s^\star$ is not affected by inflation, this set of equations can be rewritten as

$$d \ln \bar{n} = \frac{\nu}{\alpha} d \ln r \tag{53}$$

$$d \ln \bar{k} = -d \ln r \tag{54}$$

Thus an increase in inflation increases the average level of employment per establishment, while it decreases average capital use.

Equation (46) is still valid if the cash-in-advance constraint only applies to investment. Consequently, if inflation increases average employment, decreases the wage rate and average capital and does not affect average output and the productivity threshold, then it decreases the mass of incumbent establishments from equation (46). Hence, aggregate output and stock of capital decrease too. But, the effect on aggregate employment is a priori ambiguous given that H decreases and $\bar{n}$ increases. To show that the effect on aggregate employment is actually negative, first notice that

$$d\ln N = d\ln \bar{n} + d\ln H. \tag{55}$$

Next, from equation (46), observe that

$$d\ln H = d\ln w - N d\ln w - N d\ln \bar{n} + \frac{\delta K A\left(1+\theta_c\left(\frac{1+g}{\beta}-1\right)\right)}{w} d\ln \bar{k}. \tag{56}$$

Replacing the above equation and (52) and (53) in (56)

$$d\ln N = \frac{\delta K A\left(1+\theta_c\left(\frac{1+g}{\beta}-1\right)\right)}{w} d\ln \bar{k}. \tag{57}$$

Thus, aggregate employment decreases following an increase in inflation.

D.3 Case where $\theta_c = 0$, $\theta_k = 0$ and $\theta_h = 1$

Here the rental cost of capital is not affected by inflation (see equation (30)).

To understand the effect on the productivity threshold and the wage rate, combine (8) and (32) with (31) to get

$$\kappa\left[1+\theta_h\left(\frac{1+g}{\beta}-1\right)\right] = [1-F(s^\star)]\frac{\beta}{1-\beta(1-\lambda)}\eta\left[\left(\frac{\bar{s}}{s^\star}\right)^\sigma - 1\right]. \tag{58}$$

Hence an increase in inflation decreases the productivity threshold $s^\star$.

From equation (32) it follows that the wage rate decreases too.

From (49), average output either increases or remains unchanged given that

$$d\ln \bar{y} = \sigma\left[d\ln \bar{s} - d\ln s^\star\right] \tag{59}$$

and $d\ln s^\star \leq d\ln \bar{s}$.

To determine the effect on average employment and capital, notice from (32) that

$$d\ln s^{\star} = \alpha d\ln w. \tag{60}$$

By replacing the above equation in (6) and (7), we have

$$d\ln \bar{n} = \sigma\left[d\ln \bar{s} - \frac{1-\nu}{\alpha}d\ln s^{\star}\right] \tag{61}$$

$$d\ln \bar{k} = \sigma\left[d\ln \bar{s} - d\ln s^{\star}\right] \tag{62}$$

Hence, average capital increases or remains unchanged following an increase in the rate of money growth and the impact of inflation on average employment is ambiguous.

We now investigate the effect of g on H. Observe that we have from (46) that

$$d\ln H = d\ln w - \frac{AY}{w}d\ln \bar{y} + \frac{AX}{w}d\ln \bar{k} - Nd\ln w - Nd\ln \bar{n} + \frac{AE\kappa}{w}\frac{f(s^{\star})s^{\star}}{1-F(s^{\star})}d\ln s^{\star}. \tag{63}$$

The above equation can be rewritten as

$$\begin{aligned} d\ln H = &\left\{\frac{AX\sigma}{w} - \frac{AY\sigma}{w} - N\sigma\right\}d\ln \bar{s} \\ &+\left\{\frac{1-N}{\alpha} + \frac{N\sigma(1-\nu)}{\alpha} + \frac{AY\sigma}{w} - \frac{AX\sigma}{w} + \frac{AE\kappa}{w}\frac{f(s^{\star})s^{\star}}{1-F(s^{\star})}\right\}d\ln s^{\star}. \end{aligned}$$

Given $d\ln \bar{s} \leq d\ln s^{\star}$, $Y \geq X$ and $\frac{1-N}{\alpha} + \frac{N\sigma(1-\nu)}{\alpha} > N\sigma$, it follows the mass of incumbents H decreases as a result of an increase in g.

The impact on aggregate employment is given by

$$\begin{aligned} d\ln N = &\left\{\frac{AX\sigma}{w} - \frac{AY\sigma}{w} + (1-N)\sigma\right\}d\ln \bar{s} \\ &+\left\{\frac{AY\sigma}{w} - \frac{AX\sigma}{w} + \frac{AE\kappa}{w}\frac{f(s^{\star})s^{\star}}{1-F(s^{\star})} - (1-N)\sigma\right\}d\ln s^{\star}. \end{aligned}$$

By use of (26) and (29), this equation simplifies as

$$\begin{aligned} d\ln N = &\left\{\frac{AC\sigma}{w}\theta_c\left(\frac{1+g}{\beta} - 1\right) - \frac{AE\kappa\sigma}{w}\right\}d\ln \bar{s} \\ &+\left\{\frac{AE\kappa}{w}\frac{f(s^{\star})s^{\star}}{1-F(s^{\star})} - \frac{AC\sigma}{w}\theta_c\left(\frac{1+g}{\beta} - 1\right) + \frac{AE\kappa\sigma}{w}\right\}d\ln s^{\star}. \end{aligned}$$

Hence, aggregate employment decreases following an increase in g if $\theta_c = 0$.

Notice that, from (59) and (62), the effect on average capital and average output are the same. Hence, to determine the effect on aggregate output and capital, it is sufficient to know only one of the two effects given that they are the same. We choose to determine the effect on aggregate output:

$$d \ln Y = d \ln \bar{y} + d \ln H \tag{64}$$

This equation can be rewritten as

$$\begin{aligned} d \ln Y = & \left\{ \frac{AX\sigma}{w} - \frac{AY\sigma}{w} + (1-N)\sigma \right\} d \ln \bar{s} \\ & + \left\{ \frac{AY\sigma}{w} - \frac{AX\sigma}{w} + \frac{AE\kappa}{w} \frac{f(s^\star)s^\star}{1-F(s^\star)} - (1-N)\sigma + \frac{1}{\alpha} \right\} d \ln s^\star. \end{aligned}$$

Given the discussion regarding the effect of g on N, by the same arguments, it follows that the effect of g on Y and K is negative as well.

E Solutions

$$r = \left(\frac{1}{\beta} - 1 + \delta \right) \left[1 + \theta_k \left(\frac{1+g}{\beta} - 1 \right) \right] \tag{65}$$

$$w = \left(\frac{\beta\sigma/(\varepsilon-\sigma)}{\kappa \left[1 + \theta_h \left(\frac{1+g}{\beta} - 1 \right) \right] [1 - \beta(1-\lambda)]} \right)^{\frac{1}{\alpha\varepsilon}} \left(\frac{s_0 \Omega^{\frac{1}{\sigma}} \eta^{\frac{\sigma-\varepsilon}{\sigma\varepsilon}}}{r^\nu} \right)^{\frac{1}{\alpha}} \tag{66}$$

$$s^\star = \left(\frac{\beta}{1-\beta(1-\lambda)} \frac{\sigma}{\varepsilon-\sigma} \frac{\eta}{\kappa} \frac{1}{1 + \theta_h \left(\frac{1+g}{\beta} - 1 \right)} \right)^{\frac{1}{\varepsilon}} s_0 \tag{67}$$

$$\bar{s} = \left(\frac{\varepsilon}{\varepsilon-\sigma} \right)^{1/\sigma} s^\star \tag{68}$$

$$\bar{k} = \frac{\varepsilon}{\varepsilon-\sigma} \left(\frac{\alpha}{w} \right)^{\alpha\sigma} \left(\frac{\nu}{r} \right)^{(1-\alpha)\sigma} s^{\star\sigma} \tag{69}$$

$$\bar{n} = \frac{\varepsilon}{\varepsilon-\sigma} \left(\frac{\alpha}{w} \right)^{(1-\nu)\sigma} \left(\frac{\nu}{r} \right)^{\nu\sigma} s^{\star\sigma} \tag{70}$$

$$\bar{y} = \frac{\varepsilon}{\varepsilon-\sigma} \left(\frac{\alpha}{w} \right)^{\alpha\sigma} \left(\frac{\nu}{r} \right)^{\nu\sigma} s^{\star\sigma} - \eta \tag{71}$$

$$\bar{z} = \Omega \frac{\varepsilon}{\varepsilon - \sigma} \frac{s^{\star\sigma}}{w^{\alpha\sigma} r^{\nu\sigma}} - \eta \tag{72}$$

$$H = \frac{w}{A\left[1 + \theta_c(\frac{1+g}{\beta} - 1)\right]} \left(\bar{y} - \delta\bar{k} - \kappa\lambda \left(\frac{s^\star}{s_0}\right)^\varepsilon + \frac{w\bar{n}}{A\left[1 + \theta_c(\frac{1+g}{\beta} - 1)\right]}\right)^{-1} \tag{73}$$

$$E = \frac{\lambda}{(s_0/s^\star)^\varepsilon} H \tag{74}$$

$$K = H\bar{k} \tag{75}$$

$$X = \delta K \tag{76}$$

$$N = H\bar{n} \tag{77}$$

$$C = \frac{(1-N)w}{A\left[1 + \theta_c(\frac{1+g}{\beta} - 1)\right]} \tag{78}$$

$$Y = H\bar{y} \tag{79}$$

F The welfare cost of inflation

The welfare cost associated with the monetary growth rate g is defined as

$$\mathcal{W} : \widehat{U} \equiv \ln \widehat{C} + A \ln\left(1 - \widehat{N}\right) = \ln\left[(1 + \mathcal{W})\, C\right] + A \ln(1 - N)\,. \tag{80}$$

where $\widehat{C}$ and $\widehat{N}$ are consumption and time spent working in the steady-state Pareto optimal equilibrium and C and N are consumption and time spent working in steady-state, in an economy where the monetary growth rate is g. Solving for $\mathcal{W}$ yields

$$\mathcal{W} = \frac{\widehat{C}}{C}\left(\frac{1 - \widehat{N}}{1 - N}\right)^A - 1. \tag{81}$$

Using the expression (78) above to substitute for each alternative consumption level, yields the solution

$$\mathcal{W} = \left[1 + \theta_c\left(\frac{1+g}{\beta} - 1\right)\right] \frac{\widehat{w}}{w}\left(\frac{1 - \widehat{N}}{1 - N}\right)^{1+A} - 1. \tag{82}$$

Moreover, the wage rates can be expressed in terms of the respective productivity threshold using (32), yielding

$$\mathcal{W} = \left[1 + \theta_c \left(\frac{1+g}{\beta} - 1\right)\right] \left[\frac{\hat{s}^\star}{s^\star} \left(\frac{r}{\widehat{r}}\right)^{\nu}\right]^{\frac{1}{\alpha}} \left(\frac{1-\widehat{N}}{1-N}\right)^{1+A} - 1, \tag{83}$$

where $\hat{s}^\star$ and $\widehat{r}$ are, respectively, the productivity threshold and the return on capital in the Pareto optimal equilibrium, and $s^\star$ and r are the productivity threshold and the return on capital under the alternative monetary policy rule. Finally, making use of equation (65) to replace for the respective rates of return on capital, yields equation (35) in the paper.

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