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Assignment 1
EC9D3 Advanced Microeconomics

1. Show that the utility function

$$u(x_1, x_2) = k\ (x_1 - a)^\alpha\ (x_2 - b)^\beta,$$

where k, a, b, α, $\beta > 0$, represents the same preferences as the utility function

$$u(x_1, x_2) = \delta\ \ln(x_1 - a) + (1 - \delta)\ \ln(x_2 - b) \tag{1}$$

where $\delta = \frac{\alpha}{\alpha+\beta}$.

2. Assume that the consumer's preferences over the consumption bundles (x_1, x_2) are represented by the utility function $u(x_1, x_2)$ given in (1). Let prices be (p_1, p_2) and let income be m. Assume that $m > p_1\ a + p_2\ b$.

(i) By maximizing utility subject to the budget constraint

$$p_1\ x_1 + p_2\ x_2 \leq m,$$

find the Marshallian demands $x_i(p, m)$, $i = 1, 2$.

(ii) From your results in (i), show that the *indirect utility* function is given by

$$\begin{aligned} v(p, m) \ &= \ \ln(m - ap_1 - bp_2)\ +\ \delta \ln \delta\ + \\ &\quad + (1 - \delta) \ln(1 - \delta)\ -\ \delta \ln p_1 - (1 - \delta) \ln p_2 \end{aligned}$$

(iii) By minimizing expenditure subject to the utility constraint $u(x_1, x_2) \geq U$, find the Hicksian demands $h_i(p, U)$, $i = 1, 2$.

(iv) From your results in (iii), show that the expenditure function is given by

$$e(p, U) = ap_1 + bp_2 + e^U \, \delta^{-\delta}(1-\delta)^{-(1-\delta)} p_1^{\delta} p_2^{(1-\delta)}. \tag{2}$$

(v) Interpret a and b. Why do we need to assume that $m > (p_1 a + p_2 b)$?

3. A consumer lives in a three-goods economy (goods A, B and C), faces prices (p_A, p_B, p_C) and has income m. The consumer's demand functions for A and B are given by:

$$x_A = \alpha_0 + \alpha_1 \frac{p_A}{p_C} + \alpha_2 \frac{p_B}{p_C} + \alpha_3 \frac{m}{p_C} \tag{3}$$
$$x_B = \beta_0 + \beta_1 \frac{p_A}{p_C} + \beta_2 \frac{p_B}{p_C} + \beta_3 \frac{m}{p_C} \tag{4}$$

(i) Indicate how to find the demand function for C.

(ii) Are (3) and (4) appropriately homogeneous?

At one set of prices and income, $x_A = 1$ and $x_B = 2$. At another set of prices $x_A = 1$ and $x_B = 1$.

(iii) What is the income elasticity of demand for A?

4. A consumer with income m buys three goods A, B and C, whose prices are, respectively, p_A, p_B, and p_C. In the relevant range for this problem, the consumer's Marshallian demand for A is given by:

$$x_A = \alpha + \beta \frac{p_A}{p_C} + \gamma \frac{p_B}{p_C} + 10 \frac{m}{p_C}$$

When income is $m = 10$ and prices $p_A = p_B = p_C = 1$, the consumer buys 2 units of A, 3 units of B and 5 units of C.

(i) What restrictions, if any, can be put on β?

(ii) If B and C are complements, what restriction, if any, can be put on γ?