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EC9D3 Advanced Microeconomics
Additional Questions - Set 2

1. Ms. A's monthly budget is entirely spent on apples and oranges. Here are her consumption patterns for two months:

	September	October
apple price	3	8
orange price	4	6
apple consumption	4	3
orange consumption	3	4

Is the consumption behaviour consistent with the utility maximization model?

2. A consumer in a three-commodity environment (x, y, z) behaves as follows.

- when prices are $p_x = 1$, $p_y = 1$ and $p_z = 1$ the consumer buys $x = 1$, $y = 2$ and $z = 3$;
- when prices are $p_x = 4$, $p_y = 6$ and $p_z = 4$ the consumer buys $x = 3$, $y = 2$ and $z = 1$.

Does the consumer maximize a strictly quasi-concave utility function? Why?

3. Does the input requirement set

$$V(y) = \{(x_1, x_2, x_3) \mid x_1 + \min\{x_2, x_3\} \geq 3y, x_i \geq 0 \forall i = 1, 2, 3\}$$

corresponds to a regular (closed and non-empty) input requirement set? Does the technology satisfies free disposal? Is the technology convex?

4. Let $c(w, y) = (aw_1 + bw_2)y^{\frac{1}{2}}$ be a cost function. Derive its production function and draw a representative family of isoquants.

Answers

1. The consumption behaviour is indeed consistent with the utility maximization model.

First, observe that if does *not* contradicts the Weak Axiom of Revealed Preferences. In fact, let $t = 1 =$ September and $t = 2 =$ October and denote $p^1 = (3, 4)$, $p^2 = (8, 6)$, $x^1 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, $x^2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, $m^1 = p^1 x^1 = 24$ and $m^2 = p^2 x^2 = 48$. Then we get:

$$p^1 x^2 = 25 > m^1$$

and

$$p^2 x^1 = 50 > m^2.$$

This observation does not prove the consistency with consumption behaviour.

However, a firm proof exists here. In fact, notice that Ms. A's consumption behaviour of both September and October could be obtained from preferences represented by the Cobb-Douglas utility function $u(x_a, x_o) = \ln x_a + \ln x_o$.

2. The consumption behaviour is not consistent with the utility maximization of a quasi-concave utility function subject to budget constraint.

In fact, let $t = 1 =$ September and $t = 2 =$ October and denote $p^1 = (1, 1, 1)$, $p^2 = (4, 6, 4)$, $x^1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $x^2 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$, $m^1 = p^1 x^1 = 6$ and $m^2 = p^2 x^2 = 28$. We get:

$$p^1 x^2 = 6 = m^1$$

and

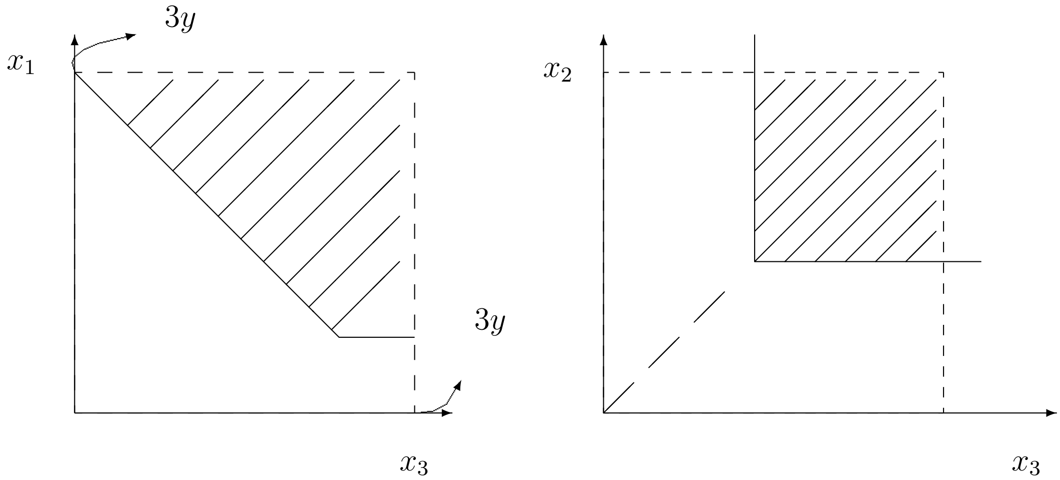
$$p^2 x^1 = 28 = m^2.$$

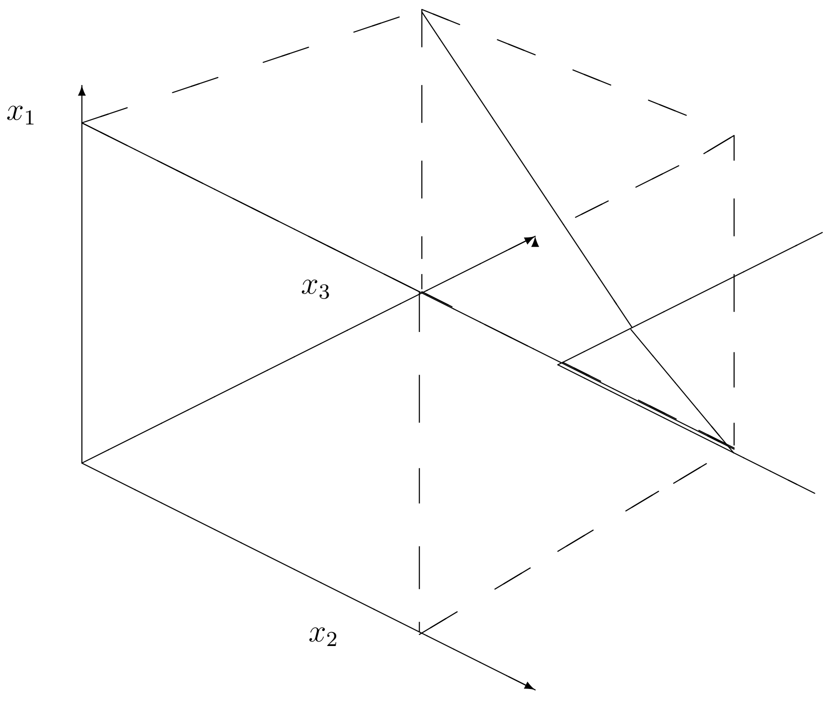
These two equality represent a violation of the Weak Axiom for a quasi concave utility function.

3. The input requirement set

$$V(y) = \{(x_1, x_2, x_3) \mid x_1 + \min\{x_2, x_3\} \geq 3y, x_i \geq 0 \forall i = 1, 2, 3\}$$

has the following graphical representation:





which shows that it is clearly closed, non-empty. As for convexity consider two input vectors, $(x'_1, x'_2, x'_3) \in V(y)$ and $(x_1, x_2, x_3) \in V(y)$, by definition of $V(y)$ we have: $x'_1 + \min\{x'_2, x'_3\} \geq 3y$ and $x_1 + \min\{x_2, x_3\} \geq 3y$. Consider now the input vector $(z_1, z_2, z_3) = \lambda(x'_1, x'_2, x'_3) + (1-\lambda)(x_1, x_2, x_3)$ and $z_1 + \min\{z_2, z_3\}$. Clearly

$$z_1 + \min\{z_2, z_3\} = \lambda x'_1 + (1-\lambda)x_1 + \min\{\lambda x'_2 + (1-\lambda)x_2, \lambda x'_3 + (1-\lambda)x_3\}$$

Consider first the case $\lambda x'_2 + (1-\lambda)x_2 \geq \lambda x'_3 + (1-\lambda)x_3$ then

$$z_1 + \min\{z_2, z_3\} = \lambda x'_1 + (1-\lambda)x_1 + \lambda x'_2 + (1-\lambda)x_2 = \lambda(x'_1 + x'_2) + (1-\lambda)(x_1 + x_2)$$

$$\geq \lambda(x'_1 + \min\{x'_2, x'_3\}) + (1-\lambda)(x_1 + \min\{x_2, x_3\}) \geq 3y$$

A symmetric argument applies for the case $\lambda x'_3 + (1-\lambda)x_3 \geq \lambda x'_2 + (1-\lambda)x_2$.

For what it concern free disposal this property is equivalent to the monotonicity of the production function:

$$F(x_1, x_2, x_3) = x_1 + \min\{x_2, x_3\}.$$

Consider an input vector $(x'_1, x'_2, x'_3) \geq (x_1, x_2, x_3)$. By definition of inequality between vectors: $x'_i \geq x_i$ for every $i \in \{1, 2, 3\}$. It then follows that $f(x'_1, x'_2, x'_3) \geq f(x_1, x_2, x_3)$.

4. By Shephard's Lemma we obtain:

$$\frac{\partial c}{\partial w_1} = a y^{\frac{1}{2}} = x_1(w, y)$$

and

$$\frac{\partial c}{\partial w_2} = b y^{\frac{1}{2}} = x_2(w, y)$$

then

$$y = f(x_1, x_2) = \min\left\{\left(\frac{x_1}{a}\right)^2, \left(\frac{x_2}{b}\right)^2\right\}$$

and the family of isoquants is represented in the following figure: