

## Problem Set 9

**Exercise 1.** Two players use the following procedure to divide a cake. Player 1 divides the cake into two pieces, and then player 2 chooses one of the pieces; player 1 obtains the remaining piece. The cake is continuously divisible (no lumps!), and each player likes all parts of it.

a. Suppose that the cake is perfectly homogeneous, so that each player cares only about the size of the piece of cake she obtains. How is the cake divided in a subgame perfect equilibrium?

b. Suppose that the cake is not homogeneous: the players evaluate different parts of it differently. Represent the cake by the set  $C$ , so that a piece of the cake is a subset  $P$  of  $C$ . Assume that if  $P'$  is a subset of  $P$  not equal to  $P$ , then each player prefers  $P$  to  $P'$ . Assume also that the players' preferences are continuous: if player  $i$  prefers  $P'$  to  $P$  then there is a subset of  $P'$  not equal to  $P'$  that player  $i$  also prefers to  $P$ . Let  $(P_1, P_2)$ , where  $P_1$  and  $P_2$  together constitute the whole cake  $C$ , be the division chosen by player 1 in a subgame perfect equilibrium of the divide-and-choose game,  $P_2$  being the piece chosen by player 2.

a. Is player 2 indifferent between  $P_1$  and  $P_2$ ?

b. Is player 1 indifferent between  $P_1$  and  $P_2$ ?

**Exercise 2.** Consider a  $T$ -period version of the Rubinstein alternating offer bargaining game. Player 1 makes offers at odd periods, and player 2 makes offer in even periods. Assume that  $T$  is even and  $\delta_1 = \delta_2 = \delta$ .

a. Calculate the unique subgame perfect equilibrium.

b. Show that the subgame perfect equilibrium converges to the solution of the Rubinstein game when  $T \rightarrow \infty$ .

**Exercise 3.** Consider the Rubinstein alternating offer bargaining game, and its unique subgame perfect equilibrium. Assume that  $\delta_1 = \delta_2 = \delta$ .

a. Describe the Nash bargaining problem and solution defined by this game and subgame perfect equilibrium.

b. Which of Nash's four axioms does this bargaining solution satisfy?