# Political Economy
# Theory and Experiments
# Lecture 4

Francesco Squintani
University of Warwick

email: f.squintani@warwick.ac.uk

## Information Aggregation

- One typical feature of democracies is voting.
- Each citizen out of a large group is asked for her opinion.
- Opinions are counted and each weighs the same.
- Is democratic voting better than letting a single competent expert decide (enlightened autocracy)?
- What are the information aggregation properties of voting?
- Condorcet supposed that voting will perform better than expert decision, as long as the number of voters is sufficiently large.
- This, even if every voter has low competence/information.

## The model

. There are 2 alternatives $d \in \{0, 1\}$. One of them is "right".

. The decision is made by majority vote.

. Voters vote independently of each other, no abstention.

. The a-priori probability (i.e. before voters get further information) of being right is the same for both alternatives.

. Each voter has the same probability $p$ to vote "correctly."

. There is an odd number of voters $n = 2m + 1$, with $n \geq 3$.

## Analysis

. The probability that in the case of $n$ voters, exactly $x$ of them make the right decision, depends on $p$ and is given by:

$$b_n(x;p) = \binom{n}{x} p^x (1-p)^{n-x}$$

with $x = 0, 1, ..., n$ and $p \in (0,1)$.

. The probability a majority vote selects the right decision is:

$$M_{2m+1}(p) \equiv \Pr(X \geq m+1) = \sum_{x=m+1}^{2m+1} b_{2m+1}(x;p)$$

. This function is symmetric: $M_{2m+1}(p) = 1 - M_{2m+1}(1-p)$.

. Thus, we have: $M_{2m+1}(1/2) = 1/2$.

- How does probability to make right decision by majority voting change with more voters?
- Suppose we add two voters to a group of size $2m-1$.
- Without additional voters, a majority of $2m-1$ (i.e. at least $m$ voters) makes the right decision with probability $M_{2m-1}(p)$.
- Note that $M_{2m-1}(p) = \sum_{m}^{2m-1} b_{2m-1}(x;p)$.
- For $2m+1$ voters, there is a majority for the right decision,
  - if at least $m+1$ of $2m-1$ voters vote correctly,
  - or if exactly $m$ of $2m-1$ voters vote correctly and at least 1 of the 2 other voters votes correctly,
  - or if exactly $m-1$ of $2m-1$ voters vote correctly and both the 2 other voters vote correctly.

. Thus, we have:

$$M_{2m+1}(p) = M_{2m-1}(p) - b_{2m-1}(m; p) \\ + b_{2m-1}(m; p)(p^2 + 2p(1-p)) + b_{2m-1}(m-1; p)p^2.$$

. Let $q \equiv 1 - p$. Since $b_{2m-1}(m-1; p) = b_{2m-1}(m; p)(q/p)$, we can simplify the above expression and get:

$$M_{2m+1}(p) = M_{2m-1}(p) + q(p-q)b_{2m-1}(m; p). \qquad (1)$$

**Condorcet Jury Theorem** For $p > (<)1/2$, the majority function $M_{2m+1}(p)$ is monotonically increasing (resp. decreasing) in $m$ and $\lim_{m\to\infty} M_{2m+1}(p) = 1$ $(= 0)$. $M_{2m+1}(1/2) = 1/2$, for all $m$. For $p \in (1/2, 1)$, we have: $M_{2m+1}(p) > p$.

. If $n = 2m + 1$ voters act independently and each one decides correctly with probability $p > 1/2$, then the probability for majority voting to get the right decision converges to 1 for $n \to \infty$.

. The convergence is fast, e.g. it is $> 0,99$ for $p = 0.8$ and $n = 13$.

. We have "vox populi, vox dei", i.e. majority voting will be almost never wrong if the number of voters is sufficiently large.

. It may be reasonable that a decision is made by a group even if every member of the group has lower competence (but $p > 1/2$) than a single "competent expert" who would decide alone.

. The proof is made by means of the recursion formula (1):

$$M_{2m+1}(p) = M_{2m-1}(p) + q(p-q)b_{2m-1}(m;p)$$

. For $p > 1/2$, $M_{2m+1}(p)$ increases monotonically in $m$, as $q(p-q)$ is positive.

. For $p < 1/2$, $M_{2m+1}(p)$ decreases monotonically in $m$, as $q(p-q)$ is negative.

. For $p = 1/2$, $M_{2m+1}(p)$ is constant in $m$, as $q(p-q) = 0$.

. The proof for $\lim_{m\to\infty} M_{2m+1}(p) = 1$ is made by expansion of the recursion formula. The calculation is very extensive.

. The property $\lim_{m\to\infty} M_{2m+1}(p) = 1$ can also be derived directly by the law of large numbers.

## Extensions

. Is there really a "right" and a "wrong" decision?
Voters may have different preferences.

. Voters differ in their competence and information.
Each voter $i$'s signal's precision may take a different value $p_i$.

. Voters observe common information: signals may be correlated.

. The model assumes that voters vote "truthfully."

  . Each voter $i$ observes a signal $s_i \in \{0, 1\}$, which is "correct" with probability $p$. Then $i$ votes $v_i = s_i$.

  . Is this behavior compatible with equilibrium?

## Strategic Voting (Austen-Smith and Banks, 1996)

. 3 voters choose an alternative $d \in \{0,1\}$ by majority.

. Given unknown state $x \in \{0,1\}$, each voter $j$'s payoff is

$$u_j(x,d) = \begin{cases} 0 & \text{if } d = x \\ -1 & \text{if } d \neq x. \end{cases}$$

. The prior is favorable to the status quo: $\Pr(x=0) = \pi > 1/2$.

. Each $j$ has a private signal $s_j \in \{0,1\}$, $\Pr(s_j = x|x) = p > 1/2$, signals are independent across voter.

. Each $j$'s voting strategy is a function $v_j : \{0,1\} \to \{0,1\}$ that maps each signal into a vote 0 or 1.

. I show that if $\pi$ is large relative to $p$, then truthful voting is not a Bayesian equilibrium.

. Consider a voter $j$ with signal $s_j = 1$. Suppose by contradiction that the other voters $k$ and $\ell$ vote truthfully.

. Let $r_j$ be probability of $x = 1$, given $j$'s equilibrium information.

. Her expected payoff is $-r_j$ for $d = 0$ and $-(1 - r_j)$ for $d = 1$. Voter $j$ prefers $d = 0$ if $r_j < 1/2$, and $d = 1$ if $r_j > 1/2$.

. $j$'s vote has no effect on $d$, unless one other voter votes $v_k = 0$ and the other one votes $v_\ell = 1$.

. Hence, voter $j$ with $s_j = 1$ votes $v_j = 1$ if and only if

$$r_j = Pr(x = 1 | s_j = s_\ell = 1, s_k = 0) = \frac{(1-p)p^2(1-\pi)}{(1-p)p^2(1-\pi)+p(1-p)^2\pi} > \frac{1}{2}.$$

. If $\pi$ is large relative to $p$, then $j$ votes $v_j = 0$ although $s_j = 1$.

. This happens even if truthful voting would be Pareto superior,

$$r_j = Pr(x = 1 | s_j = s_k = s_\ell = 1) = \frac{p^3(1-\pi)}{p^3(1-\pi)+(1-p)^3\pi} > \frac{1}{2}.$$

## Condorcet Jury Theorem (Feddersen and Pesendorfer, 1997)

**The model**

. $n+1$ voters must make a decision $d = 0, 1$.

. Each voter $i$'s payoff is $u(d, x, b_i)$,

$x \in X = [0, 1]$ is an unknown state with full support density $g$,

$b_i \in B = [-1, 1]$ is private information bias, full support density $f$.

. Let $v(x, b) \equiv u(1, x, b) - u(0, x, b)$ be the utility difference,

$v(x, b)$ is continuous and strictly increasing,

$v(-1, b) < 0$ and $v(1, b) > 0$ for all $b$.

. Each voter receives a signal $s \in \{\underline{s}, ..., \overline{s}\} \equiv S$, with full support probability $p(s|x)$, continuous in $s$ for all $x$.

. Monotone Likelihood Ratio Property: If $s > s'$ and $x > x'$, then $p(s'|x')p(s|x) > p(s|x')p(s'|x)$.

. Given quorum $q \in [1/2, 1)$, each voter $i$ votes $v_i = 0, 1$.

. The voting outcome is $d = 1$ iff $\#\{i : v_i = 1\} \geq (n+1)q$.

. A mixed strategy for voter $i$ is $\sigma_i : B \times S \to [0, 1]$.

. We consider weakly undominated symmetric Nash equilibria.

## Analysis

. A voter $i$'s vote influences the election outcome iff one vote is pivotal: exactly $qn$ of the other $n$ voters voted $v = 1$.

. The "average" probability that a voter votes 1 in state $x$ is

$$\tau(x,\sigma) = \sum_{s=\underline{s}}^{\bar{s}} p(s|x) \int_B \sigma(b,s) f(b) db.$$

. The probability that a vote is pivotal in state $x$ is:

$$\Pr(piv|x,\sigma) = \binom{n}{qn} \tau(x,\sigma)^{qn} (1 - \tau(x,\sigma))^{n-qn}.$$

. When $0 < \tau(x,\sigma) < 1$, we have $\Pr(piv|x,\sigma) > 0$ for all $x$.

. Densities of state $x$ conditional on event $piv$, and on signal $s$ are:

$$g(x|piv,\sigma) = \frac{\Pr(piv|x,\sigma)g(x)}{\int_X \Pr(piv|x,\sigma)g(x)dx},$$
$$g(x|s,piv,\sigma) = \frac{p(s|x)g(x|piv,\sigma)}{\int_X p(s|x)g(x|piv,\sigma)dx}.$$

. As $s$ satisfies MLRP, $g(x|s,piv,\sigma)$ is first-order stochastically increasing in $s$, and $E[v(b,x)|s,piv,\sigma]$ increases in $s$.

. Hence, every voting equilibrium $\sigma$ is characterized by ordered cutpoints $(b_s)_{s\in S}$ such that $-1 < b_{\overline{s}} < ... < b_{\underline{s}} < 1$ and

$$E[v(b_s,x)|s,piv,\sigma] = 0 \text{ for all } s.$$

. For all $s$, $\sigma(b,s) = 0$ if $b < b_s$ and $\sigma(b,s) = 1$ if $b > b_s$, and $0 < \tau(x,\sigma) < 1$ increases in $x$. The election is informative.

. Suppose that the number of voters grows to infinity.

. Then, the expected fraction of voters who vote informatively in equilibrium must converge to zero, and the election must be close.

**Theorem 1** Let $(\sigma^n)_{n\geq 1}$ be a sequence of voting equilibria, and let $((b_s)^n_S)_{n\geq 1}$ be the corresponding cutpoints. Then $b^n_{\underline{s}} - b^n_{\overline{s}} \to 0$.

*Sketch of Proof:* In equilibrium, each voter chooses as if she was pivotal, i.e., as if $qn$ out of $n$ voters voted $v = 1$.

. Equilibrium beliefs about $\tau(x, \sigma)$ must be concentrated around $q$.

. Beliefs about the state $x$ must concentrate on states $x'$ such that $\tau(x', \sigma)$ is close to $q$, regardless of what the true state $x$ is.

. Regardless of the state, the election must be close.

. If the fraction of voters who voted informatively did not vanish, then the election would not be close for all states.

. Large elections almost always choose the alternative that would have been chosen if the state $x$ were common knowledge.

. Let $b^* = F^{-1}(q)$ be expected bias of the "pivotal" voter.

. Let $x^*$ be the marginal state such that $v(b^*, x) = 0$.

**Theorem** Every sequence of voting equilibria $(\sigma^n)_{n \geq 1}$ is such that $\Pr(x < x^*, d^n = 1) \to 0$ and $\Pr(x > x^*, d^n = 0) \to 0$.
The probability of a decision contrary to the pivotal voter's preference vanishes as $n$ grows to infinity.

*Sketch of Proof:* Conditional on pivotal voting, the distribution over states puts almost all weight close to one state $x^n$.

. If $v(b^*, x^n) < \varepsilon < 0$, then the fraction of votes for 1 in state $x^n$ would be boundedly smaller than $q$: election would not be close.

. We conclude that $x^n \to x^*$ as $n \to \infty$.

## Swing voter's curse (Feddersen and Pesendorfer, 1996)

. Elections aggregate individual preferences and information.

. Information of common value, but some voters are not informed.

. Uninformed voters abstain, to avoid swinging the election against common interest.

. In fact, many voters do not vote, although the cost of voting is often negligible.

. Strategic abstention delivers first best.

. The winning candidate is the same as if all voters knew all voters' information.

## The model

. There are 2 states $\omega = 0, 1$, with $\pi = \Pr(\omega = 0) \geq 1/2$, and 2 party candidates $j = 0, 1$, with platforms $x_j = 0, 1$.

. There are $N + 1$ possible voters, each votes with prob. $1 - p_A$.

. With prob. $p_0$ (prob. $p_1$), a voter is partisan for party 0 (party 1).

. With probability $p_n = 1 - p_0 - p_1$ the voter is independent: her utility is $u_n(x, \omega) = -|x - \omega|$.

. Each voter receives a signal $s \in S = \{0, a, 1\}$.

. With probability $1 - q$, $s$ is uninformative and equal to $a$.

. When signal $s$ is informative, $\Pr(s = \omega|\omega) = p > 1/2$.

. Each voter chooses $v \in \{0, A, 1\}$, where $A$ is abstention.

. I focus on symmetric Nash equilibria: voters with same type and signal vote the same candidate.

. In equilibrium, type-0 (type-1) voters vote $v_0 = 0$ ($v_1 = 1$).

. All informed independents vote according to their signal: $v_n(s) = s$ if $s = 0, 1$.

. The mixed strategy of uninformed independent agents (UIAs) is $\sigma = (\sigma_0, \sigma_1, \sigma_A) \in \Delta^3$.

## Equilibrium

. Given the strategy $\sigma$, let $\rho_{\omega,j}(\sigma)$ be the probability of a vote for $j$ if the state is $\omega$ is as follows

$$\rho_{\omega,j}(\sigma) = p_j + p_n(1-q)\sigma_j + p_n q(1-p) \quad \text{if } \omega \neq x_j,$$

$$\rho_{\omega,j}(\sigma) = p_j + p_n(1-q)\sigma_j + p_n qp \quad \text{if } \omega = x_j.$$

. Let $\rho_{\omega,A}(\sigma)$ be the probability of an abstention if the state is $\omega$:

$$\rho_{0,A}(\sigma) = \rho_{1,A}(\sigma) = \rho_A(\sigma) = p_n(1-q)\sigma_A + p_A.$$

. For any voter, the probability of a tie among the other voters is:

$$P_T^{\omega,\sigma} = \sum_{\ell=0}^{N/2} \frac{N!}{\ell!\ell!(N-2\ell)!}\rho_{\omega,A}(\sigma)^{N-2\ell}\rho_{\omega,0}(\sigma)^{\ell}\rho_{\omega,1}(\sigma)^{\ell}.$$

. The probability that candidate $j$ is down by 1 vote is:

$$P_j^{\omega,\sigma} = \sum_{\ell=0}^{(N/2)-1} \frac{N!}{(\ell+1)!\ell!(N-2\ell-1)!}\rho_{\omega,A}(\sigma)^{N-2\ell-1}\rho_{\omega,1-j}(\sigma)^{\ell+1}\rho_{\omega,j}(\sigma)^{\ell}.$$

. Let $Eu_n(v,\sigma)$ be an UIA expected payoff of voting $v$, when the other voters use $\sigma$:

$$Eu_n(1,\sigma) - Eu_n(A,\sigma) = \frac{1}{2}[(1-\pi)(P_T^{1,\sigma} + P_1^{1,\sigma}) - \pi(P_T^{0,\sigma} + P_1^{0,\sigma})]$$

$$Eu_n(0,\sigma) - Eu_n(A,\sigma) = \frac{1}{2}[\pi[P_T^{0,\sigma} + P_0^{0,\sigma}] - (1-\pi)[P_T^{1,\sigma} + P_0^{1,\sigma}]].$$

$$\begin{aligned} Eu_n(1,\sigma) - Eu(0,\sigma) = (1-\pi)[P_T^{1,\sigma} + \tfrac{1}{2}(P_1^{1,\sigma} + P_0^{1,\sigma})] \\ -\pi[P_T^{0,\sigma} + \tfrac{1}{2}(P_1^{0,\sigma} + P_0^{0,\sigma})]. \end{aligned}$$

**Proposition** Suppose $p_A > 0$, $q > 0$, $N \geq 2$ and $N$ even. For any symmetric $\sigma$ s.t no voter plays a strictly dominated strategy, $Eu_n(1,\sigma) = Eu_n(0,\sigma)$ implies $Eu_n(1,\sigma) < Eu_n(A,\sigma)$.

. An UIA strictly prefers to abstain whenever indifferent between voting for 1 or 0, and no voter uses a strictly dominated strategy.

. This is the swing voter's curse.

. To consider large elections, define a sequence of games with $N+1$ voters and associated strategy profiles $\{\sigma^N\}_{N=0}^{\infty}$.

**Proposition** Suppose $q>0$, $p_n(1-q)<|p_0-p_1|$ and $p_A>0$. Let $\{\sigma^N\}_{N=0}^{\infty}$ be a sequence of equilibria.

. If $p_n(1-q)<p_0-p_1$ then $\lim_{N\to\infty}\sigma_1^N=1$, i.e., all UIAs vote for candidate 1.

. If $p_n(1-q)<p_1-p_0$ then $\lim_{N\to\infty}\sigma_0^N=1$, i.e., all UIAs vote for candidate 0.

. The swing voter's curse can lead to large scale abstention by the UIAs in large elections.

. This happens when the expected fraction of UIAs is too small to compensate for a candidate partisan advantage.

. Instead, when the fraction of UIAs is large enough to offset partisan bias, there are no pure strategy equilibria.

. UIAs mix between abstention and voting against the difference in partisan support to compensate exactly.

. The equilibrium winning candidate is approximately the same as the candidate that would win if voters had perfect information.

**Proposition** Suppose $q > 0$, $p_n(1-q) \geq |p_0 - p_1|$ and $p_A > 0$. Let $\{\sigma^N\}_{N=0}^{\infty}$ be a sequence of equilibria.

. If $p_n(1-q) \geq p_0 - p_1 > 0$ then UIAs mix between voting for candidate 1 and abstaining, with $\lim \sigma_1^N = \frac{p_0 - p_1}{p_n(1-q)}$.

. If $p_n(1-q) \geq p_1 - p_0 > 0$ then UIAs mix between voting for candidate 0 and abstaining, with $\lim \sigma_1^N = \frac{p_1 - p_0}{p_n(1-q)}$.

. If $p_0 - p_1 = 0$ then UIAs abstain: $\lim \sigma_A^N = 1$.

. For every $\epsilon$ there exists an $N$ such that for $\bar{N} > N$ the probability that equilibrium fully aggregates information is greater than $1 - \epsilon$.

## Summary

. I have considered how well elections aggregate information.

. If voters vote truthfully, then they select the “best” alternative by the law of large numbers.

. The fraction of voters who vote informatively in equilibrium converges to zero in large elections, and the election must be close.

. Nevertheless the chosen alternative is the same that would be chosen if all information became common knowledge.

. I have presented a model in which voters have different information about candidates’ valence.

. There exists an equilibrium in which informed non-partisan voters are pivotal, and the “best” candidate is elected.

## Next lecture

. We will review models of cheap talk and political advice.

. Congress may benefit from committing not to amend a committee's bill proposal, and put it to vote against the status quo.

. Unless the status quo is in line with the committee's bias, it disciplines the committee's proposal. (Gilligan and Krehbiel 1987).

. If an expert's loyalty is uncertain, repeated information transmission yields reputational concerns.

. Reputational concerns may lead to more disclosure but also to "political correctness" and conformism (Morris 2001).

. When information is verifiable, beliefs divergent from the DM act as incentives for information acquisition (Che and Kartik 2009).

. This incentive is reinforced by preference divergences, and dominates information withholding unless beliefs diverge too much.