Advanced Economic Theory
Models of Elections
Lecture 3

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Aggregate uncertainty and policy-motivated candidates

. I consider a probabilistic voting model with aggregate uncertainty and policy motivated candidates.

. In unique symmetric equilibrium, candidates' platforms diverge.

. If voters update their preferences during campaigns, they are all ex ante better off when parties diverge to some extent.

. Voters are better off with moderate policy-motivated candidates than with office-motivated candidates.

. This is in contrast with models where voters preferences are fixed.

Value of platform divergence

. Each voter j with bliss point $b_j \in \mathbb{R}$ has utility $L(|b_j - x|)$, with $L' < 0$, $L'' < 0$, and $\lim_{z \downarrow 0} L'(z) = 0$, $\lim_{z \uparrow \infty} L'(z) = -\infty$.

. The ideal point b_j is decomposed as: $b_j = m + \delta_j + \varepsilon_j$:

  . δ_j is the fixed j's bias relative to the median platform m, the distribution of δ_j has compact support and zero median,

  . ε_j is i.i.d. with $E[\varepsilon_j] = 0$, symm. density on compact support.

  . m is the random median platform, with c.d.f. F and median μ.

. Assume that F is symmetric and $\mu = 0$.

. Consider divergent platforms $x_L = -x$ and $x_R = x$, with $x \geq 0$.

. Platform x_L wins if and only if $m < \frac{x_L + x_R}{2} = 0$.

. The expected welfare of voter j is:

$$W_j(x) = \int_{-\infty}^{0} L(|m+\delta_j+\varepsilon_j - x_L|) f(m) dm$$
$$+ \int_0^{\infty} L(|m+\delta_j+\varepsilon_j - x_R|) f(m) dm$$
$$= \int_0^{\infty} [L(|-m-\delta_j-\varepsilon_j+x|) + L(|m+\delta_j+\varepsilon_j-x|)] f(m) dm.$$

$W_j(x)$ is concave as it is the sum of integrals of concave functions.

Proposition There exists a welfare-improving threshold $\overline{x} > 0$ such that $W_j(x) > W_j(0)$ for all voters j whenever $0 < x < \overline{x}$.

Proof: Compare the difference one m at a time:

$$L(|\delta_j+\varepsilon_j-(m-x)|) + L(|\delta_j+\varepsilon_j-(-m+x)|)$$
$$\text{vs. } L(|\delta_j+\varepsilon_j-m|) + L(|\delta_j+\varepsilon_j-(-m)|)$$

. This is equivalent to comparing two lotteries with fixed $\delta_j+\varepsilon_j$:

even chance on $-m+x$, $m-x$ and even chance on $-m$, m.

. Clearly, when $x < m$, policy convergence is a mean-preserving spread of divergence at $-x$ and x... and voter j is better off.

. For all δ_j, ε_j in the (compact) supports, $\frac{\partial W_j}{\partial x}(x)|_{x=0} > 0$.

. By strict concavity, there is unique $x(\delta, \varepsilon) > 0$ such that $W_j(0) = W_j(x)$ and by continuity $\overline{x} = \min_{\delta,\varepsilon}\{x(\delta, \varepsilon)\} > 0$.

. The aggregate voter welfare W^* is strictly concave:

$$W^*(x) = \int_{\delta,\varepsilon} \int_0^\infty [L(|-m-\delta_j-\varepsilon_j+x|) + L(|m+\delta_j+\varepsilon_j-x|)] dF(m) dH(\delta,\varepsilon).$$

Proposition A first-order stochastic increase in $f(\cdot|m > 0)$ induces an increase in the welfare-maximizing platform x^*.

Sketch of proof: For a greater spread in f, welfare is maximized by reducing payoff of moderate m and increasing payoff of extreme m.

Quadratic-normal case

- Assume L is quadratic, i.e., $L(z) = -z^2$.
- Say m is distributed normally with mean zero and variance σ^2.
- For each voter δ, ε, simplification yields:

$W_{\delta,\varepsilon}(x) = -2\int_0^\infty (x-m)^2 dF(m) - (\delta+\varepsilon)^2 = W_{0,0}(x) - (\delta+\varepsilon)^2$.

- By mean-variance analysis, $W^*(x)$ is a quadratic fcn:

$$\begin{aligned} W^*(x) &= -\int_{\delta,\varepsilon}[2\int_0^\infty (x-m)^2 dF(m) + (\delta+\varepsilon)^2]dH(\delta,\varepsilon) \\ &= -2E[(x-m)^2|m>0] - E[(\delta+\varepsilon)^2] \\ &= -2(x - E[m|m>0])^2 - V[m|m>0] - V[\delta] - V[\varepsilon]. \end{aligned}$$

- The social optimum is then $x^* = E[m|m>0] = \sigma\sqrt{2/\pi}$.
- As W^* is symmetric around x^*, $\overline{x} = 2E[m|m>0] = 2\sigma\sqrt{\frac{2}{\pi}}$.

Model and equilibrium

- Candidates L and R have ideal points $-b$ and $b > 0$.
- Office benefit $w \in \mathbb{R}_+ \cup \{\infty\}$.
- Pure policy motivation is $w = 0$, pure office is $w = \infty$.
- Candidate R's payoff from (x_L, x_R) is
$$\Pr(L \text{ wins}) L(|b - x_L|) + \Pr(R \text{ wins})(L(|b - x_R| + w).$$
- We focus on symmetric, pure strategy equilibria.
- We assume the hazard rate $\frac{f(m)}{1-F(m)}$ is weakly decreasing.
- Let $\overline{b}$ be the unique solution to $L'(b) = -wf(0)$.

Proposition There is a unique symmetric equilibrium, $(-x^e, x^e)$, and this equilibrium satisfies $0 \leq x^e < b$. If $b \leq \overline{b}$, then $x^e = 0$; and if $b > \overline{b}$, then x^e is the unique solution of the f.o.c.:

$$-L'(b-x) = [L(b-x) + w - L(x+b)]f(0).$$

Proof: Suppose $x_L = -x$. Candidate R's payoff for $x_R \geq 0$ is:

$$F(\tfrac{x_R - x}{2})L(b+x) + [1 - F(\tfrac{x_R - x}{2})](L(b - x_R) + w).$$

. Differentiating w.r.t. x_R and setting $x_R = x$ we obtain the f.o.c.

. The s.o.c. is satisfied as $\frac{f(m)}{1-F(m)}$ is weakly decreasing.

. Rearranging the f.o.c., I obtain: $\frac{L'(b-x)}{L(b+x)-L(b-x)-w} = f(0)$.

. LHS is strictly decreasing in $x \in [0, b)$ by strict concavity of L: by intermediate value theorem, the solution $x^e \in (0, b)$.

Proposition Say L is a power function $L(z) = -z^{\alpha}$ with $\alpha > 1$. If $b > \overline{b}$, then $\frac{\partial x^e}{\partial b} > 0$, $\frac{\partial x^e}{\partial f(0)} < 0$, $\frac{\partial x^e}{\partial w} < 0$.

. Platform divergence increases as parties are more polarized, likelihood of electoral tie decreases, office benefits decrease.

. The limiting properties of equilibria are as follows:

. If $w = 0$, then x^e is a solution of $\frac{L'(b-x)}{L(b+x)-L(b-x)} = f(0)$.

. If $w \geq -\frac{L'(b)}{f(0)}$, then $x^e = 0$

. If $f(0) \to 0$, then $x^e \to$ solution of $\frac{L'(b-x)}{L(b+x)-L(b-x)-b} = 0$

. If $f(0) \to \infty$, then $x^e \to 0$

. If $b \to 0$, then $x^e \to 0$

. If L is a power function, then as $b \to \infty$, we have $x^e \to \frac{1}{2f(0)}$.

. We now turn to relating voter welfare to candidates' ideologies.

. Let $\overline{\overline{b}}$ be the ideology such that the equilibrium platform $x^e = \overline{x}$

. If $0 \leq b \leq \overline{b}$, then platforms converge at zero.

. If $\overline{b} < b < \overline{\overline{b}}$, then the ex ante welfare of all voters is higher with policy-motivates candidates than with platforms convergence.

. If $b > \overline{\overline{b}}$, then ex ante welfare of some voters is strictly lower.

Proposition In the quadratic-normal model, $\overline{\overline{b}} = \infty$:

$$\lim_{b\to\infty} x^e = \frac{1}{2f(0)} = \sigma\sqrt{\frac{\pi}{2}} < 2\sigma\sqrt{\frac{2}{\pi}} = 2E[m|m>0] = \overline{x}.$$

. All voters are always better off with policy-motivates candidates.

Policy preferences and effort (Callander, 2008)

. There is no aggregate voter preference uncertainty.

. All voters benefit from policy-makers' effort, regardless of their ideology.

. Policy-motivated candidates care more about policies than opportunistic ones.

. Opportunistic candidates converge to the median policy.

. Policy-motivated candidates commit to their ideal policies.

. They exert effort when in office to implement their ideal policies.

. Voters anticipate this, and elect policy-motivated politicians despite their divergent platforms because they benefit from effort.

The model

. There are n voters, n odd, and two candidates, L and R.

. Each candidate i commits to a platform $x_i \in \mathbb{R}$.

. The winner of the election, W, receives a benefit w and chooses a level of effort, $e_W \in [0, 1]$ at cost ce_W^2.

. Voters and candidates' payoffs depend on policy (x_W, e_W).

. All voters payoffs increase in effort e_W, but the payoffs of x_W differ because of ideological preferences.

. Each voter j's bliss point is b_j, the median voter's is $b_m = 0$.

. Each voter $j \in \{1, 2, ..., n\}$ utility is given by:

$$U_j(x_W, e_W) = -t_W[(b_j - x_W)^2 + (1 - e_W)^2].$$

. The candidates' bliss points are $b_L = -b$ and $b_R = b > 0$.

. Each candidate i's effort type $t_i \in \{\ell, h\}$ is private information, with $0 < \ell < h$ and $\Pr\{t_i = \ell\} = q$.

. The utility of candidate j is given by:

$$U_j(x_W, e_W | t_i) = -t_i[(b_j - x_W)^2 + (1 - e_W)^2] + [w - c(e_W)^2] \Pr(W = j).$$

. Voters' equilibrium beliefs on the candidates' types coincide, based on the observed platform x_L, x_R.

. Each votes for the candidate who maximizes her expected utility.

Equilibrium Analysis

Lemma In equilibrium, the effort level of the elected candidate is $t_W/(t_W + c)$ for all t_W.

Lemma If $q \in \{0, 1\}$, a unique equilibrium exists in which both candidates L and R locate at the median voter's ideal point.

Lemma In every equilibrium: office-motivated candidates win with weakly higher probability, policy-motivated candidates locate weakly closer to their ideal point.

Lemma For $q \in (0, 1)$: if a pooling equilibrium exists, then L locates at $-b$ and R locates at b.

. Let $b_1 > 0$ solve $(1 - \frac{\ell}{\ell + c})^2 = \frac{b_1^2}{1-q} + (1 - \frac{h}{h+c})^2$.

. Median voter is indifferent between $x_R = b_1$ with $\Pr(t_i | x_R) = q$ and $x_L = 0$ knowing L is office motivated and exerts low effort.

Theorem Suppose $q \in (0, 1)$. For all $b \in [0, b_1]$, a unique equilibrium exists and is pooling: candidate L locates at $-b$ and candidate R locates at b irrespective of their types. A pooling equilibrium does not exist if $b > b_1$.

Proof: For $b < b_1$, median voter prefers a high-effort candidate with platform b, to a low-effort candidate with platform 0.

. Policy-motivated candidates locate at bliss point b and then exert high effort.

. Office-motivated candidates mimic their platform not to lose the election, but then provide low effort.

. Let b_2 solve $(1 - \frac{\ell}{\ell+c})^2 = b_2^2 + (1 - \frac{h}{h+c})^2$.

. Median voter is indifferent between $x_R = b_2$ knowing R is policy motivated and exerts high effort, and $x_L = 0$ knowing L is office motivated and exerts low effort.

Theorem Suppose $q \in (0,1)$. For all $b \in (b_1, b_2)$, a unique equilibrium exists and is semi-separating: policy-motivated candidates L and R locate at $-b$ and b, and office motivated candidates mix over $-b$ and 0, and over b and 0 respectively.

Theorem Suppose $q \in (0,1)$. For all $b \geq b_2$, a unique equilibrium exists and is separating: policy-motivated candidates L and R locate at $-b$ and b, and office motivated candidate at 0.

Proofs: For $b > b_2$, the median voter prefers a low-effort candidate with platform 0 to a high-effort candidate with platform b.

. Office-motivated candidates locate at platform 0 to win the election and then provide low effort.

. Policy-motivated candidates still locate at bliss point b. They care about policy too much to mimic office-motivated candidates.

. For $b_1 < b < b_2$, neither pooling nor separating equilibrium exist. Equilibrium requires office-motivated candidates to mix.

. In conclusion:

- . policy motivated candidates choose divergent platforms;
- . but they may still get elected as platform divergence signal that they care about policy,
- . and so that they intend to exert effort when in office.

Summary

. I have introduced candidates with policy preferences in the aggregate uncertainty model.

. Because of uncertainty, equilibrium platforms diverge.

. If voters' preferences may change during campaigns, then platform divergence improves electorate welfare.

. I have presented a model without voter preference uncertainty, in which policy-motivated candidates diverge from median.

. By diverging, candidates signal they care about policy and will exert effort if elected.

Next Lecture

. I will consider how well elections aggregate information.

. I present a model where voters have different information about candidates' valence.

. I show that there exists an equilibrium in which informed non-partisan voters are pivotal, and the "best" candidate is elected.

. I present a model with candidates more informed than voters.

. Electoral competition induces candidates to convey some information to voters, but fails to achieve informational efficiency.

. The electorate welfare loss is as severe as if only one candidate's information were efficiently revealed.