Cointegration, long-run structural modelling and weak exogeneity: Two models of the UK economy

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ABSTRACT

Cointegration ideas as introduced by Granger in 1981 are commonly embodied in empirical macroeconomic modelling through the vector error correction model (VECM). It has become common practice in these models to treat some variables as weakly exogenous, resulting in conditional VECMs. This paper studies the consequences of different approaches to weak exogeneity for the dynamic properties of such models, in the context of two models of the UK economy, one a national-economy model, the other the UK submodel of a global model. Impulse response and common trend analyses are shown to be sensitive to these assumptions and other specification choices.

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1. Introduction

Cointegration ideas as introduced by Granger (1981) are commonly embodied in empirical macroeconomic modelling through the vector error correction model (VECM). The VECM representation of a dynamic system is obtained as a simple rearrangement of the vector autoregressive (VAR) model advocated by Sims (1980), once the variables in the VAR are cointegrated. Sims had argued that the structural identification of the then-existing simultaneous-equation macroeconometric models was incredible, and he proposed the alternative strategy of estimating the unrestricted reduced form, treating all variables as endogenous, namely the VAR. Having initially been banished from the scene, however, ideas of exogeneity and structural identification have gradually reappeared on stage in various guises. Thus it has become common practice in cointegrated VAR models to treat some variables as weakly exogenous, resulting in partial or conditional VECMs. And the recognition that, for policy analysis, VAR models still require identifying assumptions has resulted in a variety of ways of formulating "structural VAR" (SVAR) models. In a similar vein,

the identification of multiple cointegrating relationships by restrictions drawn from economic theory, leaving the short-run dynamic and stochastic specification unrestricted, is called "long-run structural modelling" by Pesaran and Shin (2002). This approach is applied in the construction of a small quarterly model of the UK economy by Garratt et al. (2000; 2003; 2006, henceforth GLPS). Extended to a multi-country context, the same approach is applied in the construction of the global VAR (GVAR) model of Pesaran et al. (2004), further developed by Dees et al. (2007a,b).

The GLPS model features in our previous model comparison exercise (Jacobs and Wallis, 2005). It is used as an example of the SVAR style of modelling, for comparison with a modern example of the more traditional simultaneous-equation macroeconometric model (SEM). The two models under consideration differ appreciably in size, also in their treatment of exogeneity questions. The original VAR models were noticeably distinct from SEMs in abandoning the classification of variables as endogenous or exogenous, as noted above. In the closed economy context of much of the early empirical VAR analysis – the US economy, that is – this meant treating policy variables as endogenous, but here SEMs have followed suit, now containing policy reaction functions in place of their previous treatment of policy instruments as exogenous variables. In an open economy context, however, the distinction remains. The GLPS model treats variables describing the overseas economy as variables to be modelled in the same way as those describing the domestic economy, whereas in the UK SEM considered in our previous study the effect of the UK economy on the rest of the world is

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assumed to be negligible and "world" variables are treated as exogenous and are mostly unmodelled. The GVAR model, however, takes an intermediate position. Each national-economy or regional block of the GVAR model is a conditional VECM of similar dimension to the GLPS model of the UK economy, with some differences in the menu of variables. Unlike GLPS, however, the foreign variables in each separately estimated country submodel are treated as weakly exogenous. This difference between the GLPS model and the UK block of the GVAR model (henceforth GVAR(UK)) is noted in our previous article as a subject for future comparative research, which we undertake in the present paper. Different approaches to weak exogeneity questions have developed in the cointegration literature, and many associated econometric–theoretical issues have been addressed. However the impact of different weak exogeneity assumptions on the dynamic properties of the system appears not to have been studied hitherto. This paper presents such a study, in the context of two models of the UK economy which, while both representative of the VECM style of modelling, are rather different in their approach. We work with the published versions of the models, as estimated and tested by the respective modelling teams, varying only their treatment of exogeneity.

The remainder of the paper is organised as follows. Section 2 briefly reviews the formalities of the VAR–VECM modelling framework, the role of weak exogeneity assumptions and the conditional model, and different approaches to the specification of the associated marginal model. Section 3 uses the GLPS model of the UK economy to illustrate the effects of different weak exogeneity assumptions on the dynamic properties of a model, as revealed by its long-run multipliers and its impulse responses in a simple simulation exercise. Section 4 contrasts the treatment of weak exogeneity in the estimation and solution of the global VAR model, with special reference to its UK block, and reproduces a further simulation exercise. Section 5 compares the two models' common trends, where differences reflect specification choices other than the treatment of weak exogeneity. Section 6 concludes.

2. Cointegrated VARs and conditional VECMs

The VAR system is written

$$A(L)z_t = e_t, \tag{1}$$

where the matrix polynomial $A(L)$ has degree k and leading matrix equal to the identity matrix, reflecting the reduced-form nature of the system. Once the n variables in the vector z_t have been selected, with reference to the problem at hand, there is no prior classification as endogenous or exogenous; all are treated equally as variables of interest to be modelled. Impulse responses are calculated from the vector moving average representation

$$z_t = A(L)^{-1}e_t = C(L)e_t, \tag{2}$$

where the leading matrix in $C(L)$ is again the identity matrix. The elements of e_t are correlated, that is, $E(e_t e_t') = \Omega$ is not diagonal, and Sims (1980) argued that it is useful to transform them to orthogonal form to be able to see the "distinct patterns of movement" of the system.

The VAR system (1) can be rearranged as

$$A^*(L)\Delta z_t = -\Pi z_{t-1} + e_t$$

where $\Pi = A(1)$ and the degree of $A^*(L)$ is $k-1$. If the elements of z_t are I(1) and cointegrated with rank $(\Pi) = r$, $0 < r < n$, then $\Pi = \alpha\beta'$ where α and β are $n \times r$ matrices of rank r, giving the VECM representation

$$A^*(L)\Delta z_t = -\alpha\beta' z_{t-1} + e_t. \tag{3}$$

Exact identification of β requires r restrictions on each of the r cointegrating vectors (columns of β), of which one is a normalization restriction and the other $r-1$ restrictions satisfy the identification rank condition. In the Wold representation of the differenced (stationary) variables

$$\Delta z_t = D(L)e_t \tag{4}$$

the matrix $D(1)$ of long-run multipliers corresponds to C_∞ in representation (2). It has rank $n-r$ and is given in Johansen's (1991) presentation of the Granger Representation Theorem as

$$D(1) = \beta_\perp[\alpha_\perp' A^*(1)\beta_\perp]^{-1}\alpha_\perp' \tag{5}$$

where the orthogonal complements $\alpha_\perp$ and $\beta_\perp$ are $n \times (n-r)$ matrices of rank $n-r$ such that $\alpha'\alpha_\perp = 0$ and $\beta'\beta_\perp = 0$.

Various permanent-transitory decompositions follow from this representation. Stock and Watson (1988) show that, with r stationary linear combinations $\beta' z_t$, the I(1) characteristics of z_t may be expressed in terms of $n-r$ "common trends" $\beta_\perp' z_t$. This formulation of the common trends as functions of the variables in the system has advantages for some purposes, although other formulations in terms of cumulated shocks are also available. The shocks that drive the common stochastic trends are the shocks $\alpha_\perp' e_t$, called permanent shocks, leaving r transitory shocks: since $\beta' D(1) = 0$, shocks to the cointegrating vectors have no permanent effects. Writing the Wold representation (4) as

$$\Delta z_t = D(L)H^{-1}He_t.$$

Levtchenkova et al. (1998) define a basic permanent-transitory decomposition as He_t, with the first $n-r$ elements permanent and the last r elements transitory, that is, $D(1)H^{-1}$ has its last r columns equal to zero. Then H has the form

$$H = \begin{bmatrix} \alpha_\perp' \\ \rho' \end{bmatrix}$$

for any $n \times r$ matrix ρ such that H is invertible, and Levtchenkova et al. discuss various possible choices of ρ. For example, Gonzalo and Granger (1995) take $\rho = \beta$.

Despite identification of the cointegrating vectors by restrictions on β, permanent-transitory decompositions require further structural identifying restrictions, or stories. Given β and an initial choice of $\beta_\perp$, note that $\beta' \beta_\perp P = 0$ for any nonsingular $(n-r) \times (n-r)$ matrix P. If $n-r > 1$, then identification of individual common trends $\beta_\perp' z_t$ requires restrictions on $\beta_\perp$ that make transformations $P'\beta_\perp' y_t$ inadmissible, whereas if $n-r = 1$, only a normalization restriction is required. Likewise, identifying individual permanent shocks requires further restrictions.

The conditional VECM model of a p-element subset y_t of the $n \times 1$ vector z_t is obtained if the remaining $q = n-p$ variables x_t can be treated as weakly exogenous. For this purpose it is convenient to rewrite the VECM representation (3) as

$$\Delta z_t = -\alpha\beta' z_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta z_{t-i} + e_t, \tag{6}$$

place the variables x_t in the last q positions of the vector z_t, and introduce conformable partitionings of relevant vectors and matrices as

$$z_t = \begin{bmatrix} y_t \\ x_t \end{bmatrix}, \qquad \alpha = \begin{bmatrix} \alpha_y \\ \alpha_x \end{bmatrix}, \qquad \Gamma_i = \begin{bmatrix} \Gamma_{yi} \\ \Gamma_{xi} \end{bmatrix},$$

$$e_t = \begin{bmatrix} e_{yt} \\ e_{xt} \end{bmatrix}, \qquad \Omega = \begin{bmatrix} \Omega_{yy} & \Omega_{yx} \\ \Omega_{xy} & \Omega_{xx} \end{bmatrix}.$$

If $\alpha_x = 0$ then x_t is weakly exogenous and valid inference can proceed in the conditional model of y_t given x_t and the past, namely

$$\Delta y_t = \Lambda\Delta x_t - \alpha_y\beta' z_{t-1} + \sum_{i=1}^{k-1} \tilde{\Gamma}_{yi}\Delta z_{t-i} + \tilde{e}_{yt} \tag{7}$$

where $\Lambda = \Omega_{yx}\Omega_{xx}^{-1}$, $\tilde{\Gamma}_{yi} = \Gamma_{yi} - \Lambda\Gamma_{xi}$ and $\tilde{e}_{yt} = e_{yt} - \Lambda e_{xt}$ (Johansen, 1995, ch. 8).

Consideration of the conditional model was motivated by the search for conditions under which statistical analysis of a partial model might be efficient. However, while conditioning on *current* values of the variables which are not affected by the error-correction terms is a convenient device for statistical inference, Eq. (7) implies different dynamics from those of the underlying system. To study such dynamics, for example, by impulse response analysis, it is generally necessary to retain the full VAR–VECM system (3) or (6), in which there are no contemporaneous relations among variables. Reference to the full VECM system is also essential in a comparative study of different weak exogeneity assumptions, as undertaken below. If the estimated model is reported in conditional VECM form (for example, GLPS, 2003, Table 4; 2006, Table 9.4), then this needs to be rewritten as the first (Δy_t) block of Eq. (6) using the above equivalences, while retaining the second block of (6) subject to $\alpha_x = 0$, namely the marginal model.

The theoretical development of the conditional model assumed that the possibility that it might sustain efficient inference on β would be checked by first estimating the full system and testing $\alpha_x = 0$. On the other hand, study of a partial system might be preferred, as "sometimes it is easier to model satisfactorily the conditional model of the endogenous variables given the exogenous variables, and the marginal distribution of the exogenous variables show an irregular behaviour which is difficult to model using a VAR" (Johansen, 1995, p. 121). Moreover, "it is sometimes *a priori* very likely that weak exogeneity holds [and] testing may not always be necessary", quoting Juselius (2006, p. 198), who gives an example of a foreign variable having an impact on a model of a small open economy, but experiencing negligible feedback from that economy. This is close to the traditional treatment of "world" variables in national-economy SEMs, as noted above, although in that context such variables are usually unmodelled. Juselius also remarks that it can be useful to impose weak exogeneity restrictions from the outset if the number of potentially relevant variables to be included in the initial VAR is large. The possibility that the assumed exogenous variables are cointegrated between themselves should nevertheless be borne in mind, as this would affect the statistical properties of tests and estimators.

The marginal model for the weakly exogenous variables in the full system is the second block of the partitioned equation (6), as discussed above. If the full system is not estimated from the beginning, for reasons given in the previous paragraph, then dynamic analyses require the estimated conditional model to be augmented with a separately specified marginal model. The matrices Γ_{xi} that appear in the marginal model clearly form part of the system dynamics. However partitioning these once more, in an obvious notation, as $\Gamma_{xi} = \left[\Gamma_{xy,i} \vdots \Gamma_{xx,i} \right]$, we note that it is not uncommon for a separately specified marginal model to assume that $\Gamma_{xy,i} = 0$ and include only the *x*-variables, further strengthening their exogeneity. An extreme assumption sometimes found in the literature is that a conditional VECM can be closed by assuming that the weakly exogenous variables follow random walks: this is not innocuous, as illustrated in Section 3.2.

A special case which has received attention in recent studies of the identification and estimation of permanent and transitory shocks in cointegrating VARs with exogenous variables (Fisher and Huh, 2007; Pagan and Pesaran, 2008) is that in which $p = r$, that is, there are exactly as many cointegrating vectors as endogenous variables. This requirement is clearly very limiting, but might be of use in the current context of an open economy with exogenous $I(1)$ variables. In this case, the matrix $D(1)$ of long-run multipliers defined in Eq. (5) has the form

$$D(1) = \left[\mathbf{0}_{n \times p} \vdots \tilde{D}(1)_{n \times q} \right],$$

which is the relevant permanent-transitory decomposition. Shocks to endogenous variables then have no permanent effects, also as illustrated below. Under the additional "extreme" assumption of random walk *x*-variables we obtain, with $\beta'_\perp$ partitioned as $\left[\beta'_{y_\perp} \vdots \beta'_{x_\perp} \right]$,

$$D(1) = \begin{bmatrix} \mathbf{0}_{p \times p} & \beta_{y_\perp} \beta_{x_\perp}^{-1} \\ \mathbf{0}_{q \times p} & \mathbf{I}_q \end{bmatrix},$$

thus the long-run multipliers of the *y*-variables with respect to the weakly exogenous variables depend only on the cointegration coefficients.

Some practical implications of these different approaches to weak exogeneity questions are explored in the context of two empirical VECM models of the UK economy in the next two sections. We present estimates of the $D(1)$ matrices of long-run multipliers, which typically have blocks of zeros as in the simple examples above. In the empirical examples these are the result of weak exogeneity restrictions and the assumption of no feedback between domestic and foreign variables, which in some cases are the result of pretesting. Although Johansen's (1995) discussion of inference includes the asymptotic distribution of the estimator of the unrestricted $D(1)$ matrix (Theorem 13.7; see also Paruolo, 1997), extensions to the case of matrices incorporating pretested restrictions are not available, and we do not report standard errors.

3. The GLPS model of the UK economy

3.1. The GLPS model in outline

The GLPS model incorporates long-run structural relationships suggested by economic theory as the cointegrating relations of a VECM. The UK economy is a small open economy, subject to economic developments in the rest of the world, hence in the VAR approach both domestic and foreign variables are treated as variables of interest to be modelled. Data limitations constrain the number of variables to include, as always. The present version of the model (GLPS, 2003, 2006) contains six domestic and four foreign variables, namely domestic and foreign real per capita outputs (y, y^*), producer prices (p, p^*) and nominal short-term interest rates (r, r^*), the UK Retail Prices Index inflation rate $(\Delta \tilde{p})$, the nominal effective exchange rate (e), the price of oil (p^o) and the domestic real per capita money stock (h), all modelled in logarithms. Of the three starred variables, y^* and p^* refer to the OECD's member countries, while r^* is a weighted average of the interest rates of the US, Japan, Germany and France, with weights based on IMF Special Deposits Rights weights.

The underlying economic theory delivers five long-run relations or equilibrium conditions among these variables, based on production, arbitrage, solvency and portfolio balance conditions, together with stock-flow and accounting identities. First is a purchasing power parity relation, based on international goods market arbitrage. Next, a nominal interest rate parity relation, based on arbitrage between domestic and foreign bonds. Then a relation between domestic and foreign output derived from the neoclassical growth model, assuming common technological progress in production. Also a real money balance relation, based on long-run solvency conditions and assumptions about the determinants of the demand for domestic and foreign assets. Finally, a Fisher interest parity relation.

Many more variables than those listed above appear in the theoretical framework, but are solved out. Expectations of several variables also appear, but are replaced by their actual values, assuming that expectational errors are stationary processes subsumed into the disturbance terms. The economic theory says

$$\begin{bmatrix} 2.067 & 0.129 & 1.199 & 10.938 & 0.706 & 0.642 & 0.588 & 1.264 & 0.033 \\ 2.067 & 0.129 & 1.199 & 10.938 & 0.706 & 0.642 & 0.588 & 1.264 & 0.033 \\ 0.021 & -0.012 & 0.127 & 0.208 & 0.008 & 0.020 & -0.005 & 0.038 & 0.001 \\ 0.021 & -0.012 & 0.127 & 0.208 & 0.008 & 0.020 & -0.005 & 0.038 & 0.001 \\ 0.320 & -0.037 & -2.098 & -4.680 & 0.344 & 0.439 & -0.103 & -0.127 & 0.012 \\ 0.320 & -0.037 & -2.098 & -4.680 & 0.344 & 0.439 & -0.103 & -0.127 & 0.012 \\ -1.169 & 0.658 & -7.416 & -11.664 & -0.450 & -1.096 & 0.268 & -2.127 & -0.045 \\ 0.021 & -0.012 & 0.127 & 0.208 & 0.008 & 0.020 & -0.005 & 0.038 & 0.001 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} p - p^* \\ e \\ r \\ r^* \\ y \\ y^* \\ h - y \\ \Delta \tilde{p} \\ p^o \end{matrix}$$

Box I.

nothing about the statistical characteristics of the variables, but once it is assumed that they are difference-stationary these equilibrium relations become candidate cointegrating relations in the VECM representation. It is not a necessary condition in the VECM approach that long-run relations include only integrated variables, nevertheless the authors choose to require this.

The GLPS model is estimated from quarterly, seasonally adjusted data over the period 1965q1–1999q4 (140 observations). The number of variables is reduced to nine by working with the relative price variable $p - p^*$ rather than the two separate price levels p and p^*, and the money variable appears as inverse velocity $h - y$. All nine variables are treated as approximately $I(1)$ on the basis of unit root test statistics, as reported and discussed by GLPS (2006, Section 9.2). The cointegration rank in a VAR(2) representation is estimated as five, in agreement with the number of long-run relations of the theoretical model. With the variables ordered

$$(p - p^*, e, r, r^*, y, y^*, h - y, \Delta \tilde{p}, p^o)$$

the estimate of the 5×9 matrix β' is

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 56.1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

All the cointegrating relations include an estimated intercept term, and the fourth also contains a time trend; none include the oil price. This matrix is highly overidentified, and the overidentifying restrictions are not rejected at the 5% level once bootstrapped rather than asymptotic critical values are compared to the likelihood ratio statistic. The oil price is then treated as weakly exogenous in an eight-equation conditional VECM for the remaining variables, as in Eq. (7). With $k = 2$ in the VAR, the VECM has a single lag in all variables, except for the oil price equation, which is a random walk with drift.

From the model files we calculate the matrix $D(1)$ of long-run multipliers in the full system via Eq. (5), and obtain the matrix given in Box I.

The variables are ordered as above, and repeated in the right-hand column for convenience. Since there are five cointegrating relations among the nine variables, this matrix has rank four. Rows 1 and 2 are equal, as are rows 3, 4 and 8, likewise rows 5 and 6, while rows 3 and 7 are proportional to one another (although rounding may obscure this). These restrictions are the practical consequences of the result that $\beta' D(1) = 0$, with the implication that shocks to the system have no long-run impact on the cointegrating combinations of variables. The weak exogeneity of the oil price, its absence from the cointegrating relations, and the absence of lagged variables in the ninth equation of the model deliver the unit vector in the final row of $D(1)$.

3.2. The effects of different weak exogeneity assumptions

In the first variant specification to be explored, termed GLPSX, we treat all four foreign variables as weakly exogenous. This assumption is adopted *a priori* in the specification of each national-economy block of the GVAR model, perhaps in the spirit of the quotation from Juselius (2006) cited above. We estimate a five-equation conditional VECM as in Eq. (7), taking the nominal effective exchange rate, e, and the two starred variables, r^* and y^*, to be weakly exogenous, in addition to the oil price. For the marginal model we retain the original VAR(2) specification, together with a random walk with drift for the oil price. In this subsystem the coefficients of the domestic variables are jointly insignificant, and so these variables are deleted, taking us a further step towards the traditional treatment of foreign variables in national-economy SEMs, with $\Gamma_{xy,i} = 0$. The matrix $D(1)$ of long-run multipliers in GLPSX is then obtained as the matrix given in Box II.

The variables are reordered with endogenous variables first, as shown in the right-hand column. Taking account of the reordering of the rows, the same relationships among them hold as in GLPS: the rank of the matrix remains at four. A striking property of GLPSX is that shocks to endogenous variables have no long-run impact. This variant of the original model is an example of the special case discussed in the closing paragraph of Section 2, having as many weakly exogenous variables as common trends. As also noted in that paragraph, if our estimated marginal model were to be replaced by the assumption that the weakly exogenous variables follow random walks, then the right-hand 9×4 block of the above matrix would be replaced by the expression given in the final equation in Section 2, in which the long-run multipliers depend only on the elements of the cointegrating vectors.

An intermediate variant between GLPS and GLPSX is obtained if we test for weak exogeneity, in the spirit of the original development of the conditional VECM. On applying a joint F-test to the elements of each row of the matrix α in Eq. (3) or (6) we find that the hypothesis of zero coefficients can be rejected at the 5% level for seven variables, but not rejected for the nominal effective exchange rate and the oil price. The resulting "statistical" weak exogeneity variant, termed GLPS(E), has a seven-equation conditional VECM, and with the corresponding specification for the marginal model (AR(2) for e, random walk with drift for p^o) the matrix $D(1)$ of long-run multipliers becomes the matrix given in Box III.

Again this matrix retains rank four, along with the restrictions that follow from $\beta' D(1) = 0$. Relative to the original GLPS specification, exogenising the nominal exchange rate implies that relative prices have zero long-run response to shocks to other variables, given the purchasing power parity cointegrating relationship.

To describe the effect of different weak exogeneity assumptions on dynamic adjustment processes we use the simulation of a shock to the foreign output equation, as undertaken on both published versions of the UK model by the authors (GLPS, 2000, 2006).

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1.183 & 0.103 & -0.377 & -0.012 \\
0 & 0 & 0 & 0 & 0 & -0.004 & 1.517 & 0.193 & 0.000 \\
0 & 0 & 0 & 0 & 0 & 0.020 & -0.970 & 1.737 & 0.002 \\
0 & 0 & 0 & 0 & 0 & 0.246 & -85.089 & -10.846 & -0.027 \\
0 & 0 & 0 & 0 & 0 & -0.004 & 1.517 & 0.193 & 0.000 \\
0 & 0 & 0 & 0 & 0 & 1.183 & 0.103 & -0.377 & -0.012 \\
0 & 0 & 0 & 0 & 0 & -0.004 & 1.517 & 0.193 & 0.000 \\
0 & 0 & 0 & 0 & 0 & 0.020 & -0.970 & 1.737 & 0.002 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{matrix}
p - p^* \\ r \\ y \\ h - y \\ \Delta \tilde{p} \\ e \\ r^* \\ y^* \\ p^o
\end{matrix}
$$

Box II.

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.188 & -0.011 \\
0.089 & 0.167 & 0.571 & 0.031 & 0.041 & 0.015 & 0.080 & -0.047 & 0.002 \\
0.089 & 0.167 & 0.571 & 0.031 & 0.041 & 0.015 & 0.080 & -0.047 & 0.002 \\
1.220 & -1.576 & 0.082 & 0.651 & 0.718 & 0.153 & 0.424 & -0.498 & 0.031 \\
1.220 & -1.576 & 0.082 & 0.651 & 0.718 & 0.153 & 0.424 & -0.498 & 0.031 \\
-5.018 & -9.379 & -32.036 & -1.766 & -2.291 & -0.828 & -4.482 & 2.630 & -0.127 \\
0.089 & 0.167 & 0.571 & 0.031 & 0.041 & 0.015 & 0.080 & -0.047 & 0.002 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.188 & -0.011 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{matrix}
p - p^* \\ r \\ r^* \\ y \\ y^* \\ h - y \\ \Delta \tilde{p} \\ e \\ p^o
\end{matrix}
$$

Box III.

Rather than their generalised impulse responses, however, we calculate traditional shock-one-thing-at-a-time impulse responses, as advocated in our previous article (Jacobs and Wallis, 2005). We standardise the comparison on the long-run response of foreign output, with shocks designed to give an increase of 1% in the long run. To calibrate the required initial shocks to foreign output we refer to the relevant $D(1)$ matrices given above: in the GLPS model an impulse of $0.439^{-1} = 2.28\%$ is required; in the GLPSX variant the required impulse is $1.737^{-1} = 0.58\%$; and in GLPS(E) $0.718^{-1} = 1.39\%$.

The impulse responses for six variables are plotted in Fig. 1: domestic and foreign output, domestic and foreign interest rates, domestic inflation, and the real exchange rate, constructed as $e - (p - p^*)$. The long-run response of domestic output is also 1% in all three versions of the model, since y and y^* are cointegrated. Likewise the cointegration of r and r^* implies that their long-run responses in each version are equal, while differing across versions. The long-run response of the real exchange rate is zero in all cases by virtue of the purchasing power parity cointegrating relationship. The upper right and centre right panels of Fig. 1 show that the adjustment of y^* and r^* is quickest when they are treated as weakly exogenous, and left to get on with it without feedbacks from the UK economy. On the other hand the remaining panels in general show that, the more exogenisation, the slower the domestic response. The upper left panel shows that exogenising only the exchange rate explains most of the reduction in the domestic output response, and in neither variant has the response converged to its long-run level after 50 quarters. The difference in dynamics is most striking in the lower right panel, which shows that the real exchange rate response in the GLPSX variant is still some distance from zero after 50 quarters: this represents some qualification to the concept of a "transitory" shock.

Overall, Fig. 1 shows that there is considerable similarity in the dynamic responses of GLPS and GLPS(E), whereas each variant has rather less in common with GLPSX. Thus the imposition of statistically acceptable weak exogeneity restrictions causes little distortion to system dynamics, whereas the *a priori* imposition of restrictions that turn out to be statistically unacceptable causes greater distortion. Hence in general, these results indicate that, whenever one is tempted to assume weak exogeneity *a priori*, it is important not only to test this assumption using standard statistical procedures but also to check its effects on system dynamics. In particular, the feedback from the "small open" UK economy to foreign output and interest rates appears less negligible than that in Juselius' example, the former reflecting the globalised treatment of production technology in the GLPS model and the latter the importance of the City of London as an international financial centre.

4. The UK block of the GVAR model of the global economy

4.1. The GVAR(UK) model in outline

We consider the GVAR model of Dees et al. (2007b), which is an updated and extended version of the original GVAR model of Pesaran et al. (2004), with sample period 1979q2–2003q4. It covers 33 countries, eight of which are members of the euro area and are combined into a euro-area block, so the model has 26 country or regional submodels. Each individual country model includes domestic and foreign variables, the country-specific foreign variables being obtained by aggregating data on the foreign economies, using their shares in the home country's trade as weights. All countries are other countries' trading partners, and in solving the model a globally consistent solution for the country-specific variables is obtained via the "link" matrix introduced by Pesaran, Schuermann and Weiner.

The GVAR(UK) country model contains twelve variables, namely domestic and foreign real outputs (y, y^*), inflation rates (Δp, Δp^*), real equity prices (q, q^*), short-term interest rates (rs, rs^*) and long-term interest rates (rl, rl^*), the real effective exchange rate (fx) and the price of oil (p^o). This menu of variables is common to all country models except the US model. A desire to model global financial interactions motivates the inclusion of more financial variables than are modelled in GLPS. The model specification is a conditional VECM for the six domestic variables, treating the starred variables and the oil price as weakly exogenous, these variables entering the conditional VECM only in unlagged form (that is, further partitioning the coefficient matrices in Eq. (7), $\tilde{\Gamma}_{yx,i} = 0$). There are three cointegrating vectors, corresponding to three of the five long-run relations that appear in GLPS. First, the purchasing power parity relation implies in the present menu of variables that the real effective exchange rate is
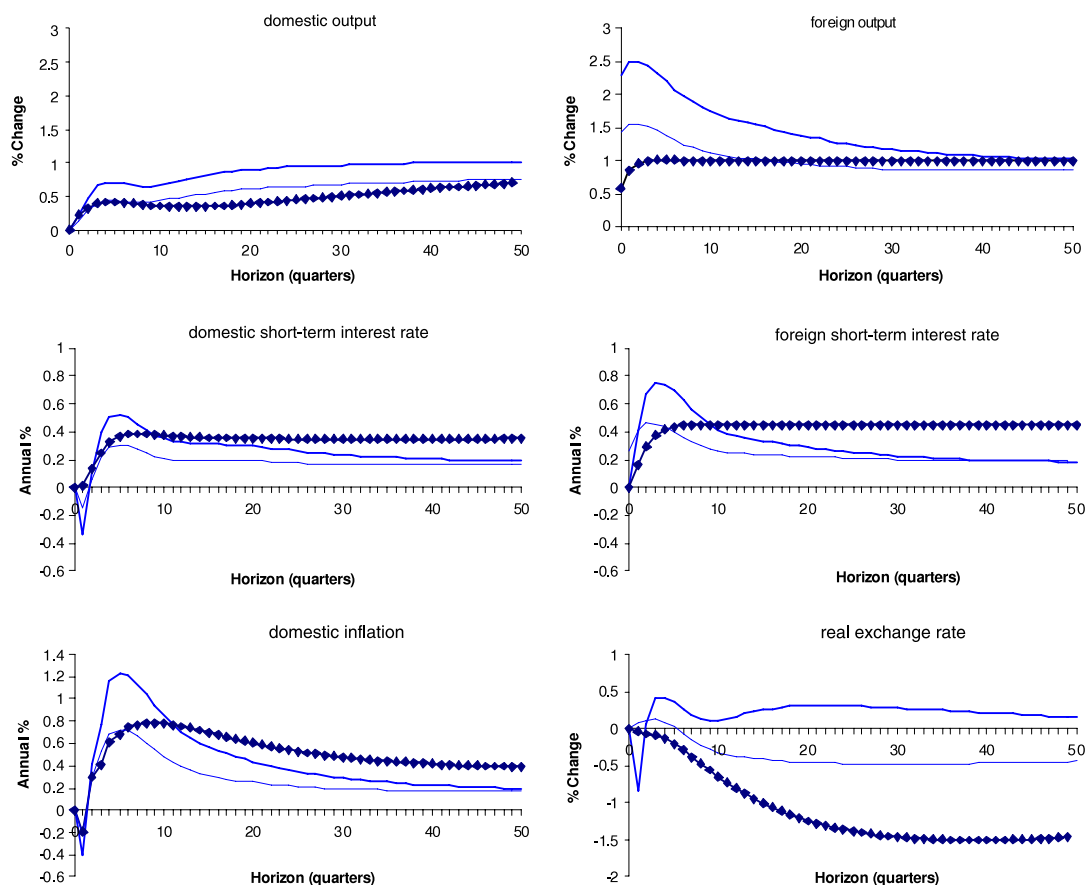
Fig. 1. Impact of weak exogeneity in GLPS: impulse responses to a permanent 1% foreign output shock (thick solid line: GLPS; solid line GLPS(E); line with markers: GLPSX).

stationary. Next, the nominal interest parity relation refers to long-term interest rates. Finally the Fisher relationship appears with an estimated non-unit coefficient on inflation. With the variables ordered

$$\left(y, \Delta p, q, rs, rl, fx, y^*, \Delta p^*, q^*, rs^*, rl^*, p^o\right)$$

the estimate of the 3×12 matrix β' is

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1.62 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Again this matrix is highly overidentified, although in this case the overidentifying restrictions are rejected ($LR = 153$, bootstrapped $1\%CV = 112$). Nevertheless the effects of shocks on these long-run relations are found to be transitory, and so the authors retain this specification.

No marginal model for the starred variables accompanies the UK's conditional VECM. Instead, in simulations with the full GVAR model a globally consistent solution for the country-specific domestic and foreign variables is obtained via the link matrix, as noted above. Each national component of an aggregate foreign variable appearing in the UK block is modelled in that country's conditional VECM, and so on for all blocks. All variables are endogenous in full system solution, as in a VAR, and the implicit marginal model for the variables treated as weakly exogenous in block-by-block estimation is the rest of the system. Thus a $D(1)$ matrix for GVAR(UK) comparable to those calculated in the previous section is not available; as far as we are aware, an equivalent calculation for the full 134-variable system has not been attempted. Long-run properties of the model are studied by running simulations until convergence, with interpretation aided, as usual, by the knowledge that cointegrating combinations of variables exhibit zero long-run multipliers.

4.2. Comparative dynamics

Our comparison of GVAR(UK) and GLPS is based on the oil price experiment of GLPS (2003), also implemented by Jacobs and Wallis (2005), except that those exercises consider only the conditional VECM model; here we refer to the full nine-equation VECM form of GLPS. We also replicate the experiment on GVAR, where we calculate the responses of the six domestic variables of GVAR(UK) to a global oil price shock, the oil price being the 134th and last variable of the full GVAR system. The shock is an increase of 16.485%, equal to one standard error of the GLPS estimated equation for this variable. As in Section 3.2, we calculate traditional shock-one-thing-at-a-time impulse responses.

Impulse responses for the four domestic variables considered in the previous illustration are presented in Fig. 2. In general GLPS exhibits greater initial fluctuations than GVAR, reflecting its inclusion of more lagged terms, many of whose coefficients are not well-determined. For the first three variables the long-run outcome in GLPS is given with reference to the corresponding elements of the final column of the $D(1)$ matrix, and the long-run response of the real exchange rate is zero; this last also applies to GVAR. Both models are close to these positions after 40 quarters. In respect of the domestic output responses, in absolute terms the reduced sensitivity of the GVAR model is notable, reflecting its later sample period (which excludes the major oil price shocks of the 1970s, as shown in the upper right panel of Fig. 3).

Possibly more interest attaches to the disagreement about the sign of the output response. First we note that the previous exercise referred to above, based on the conditional VECM GLPS model, produced a negative output response, and the simple switch to the full nine-equation model, in which the oil price no longer enters the system unlagged, produces a positive response. This
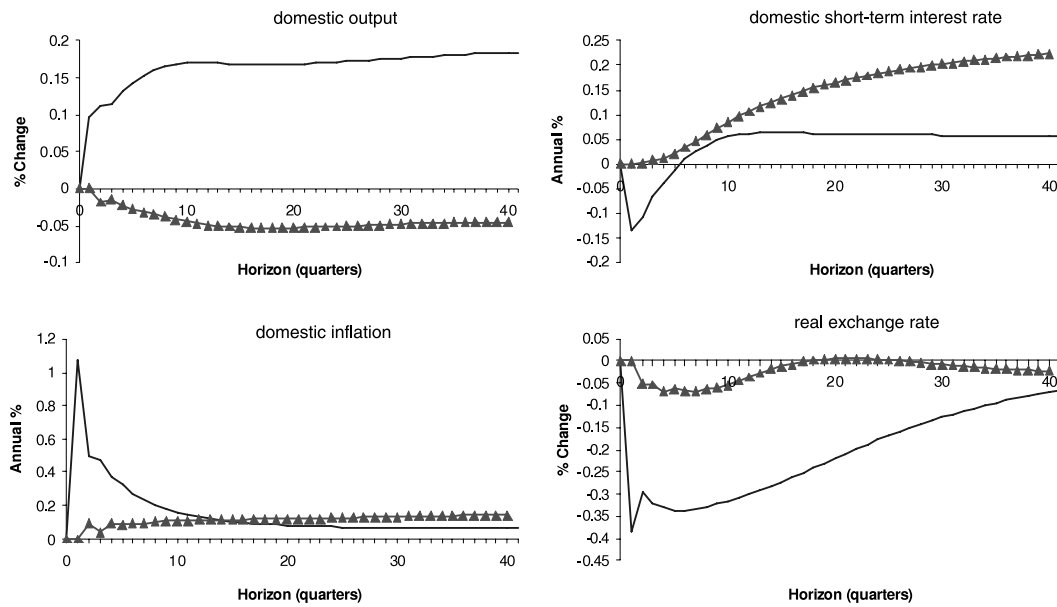
Fig. 2. Impulse responses to a permanent 16.485% oil price shock (thick solid line: GLPS; solid line with markers: GVAR).

could be seen as reflecting the UK's position as a net oil exporter, however the shift from net importer to net exporter only occurred midway through the GLPS sample period, and it is not clear why this should dominate. In both models the UK's trading partners forming the "foreign" aggregate are predominantly oil-importing countries, and in GVAR the effect on the UK of the global recession induced by the oil price rise can be interpreted as dominating the beneficial direct effect. Finally, however, we note that GLPS (2003, Fig. 2) report 95% confidence intervals for their impulse responses. For the two output variables these intervals cover zero at all lags, and the statistical significance of the differences noted above may be similarly small.

5. Common trends

We compare the common trends and long-run forcing variables in GLPS and GVAR(UK) that, according to each model, deliver the $I(1)$ characteristics of the included variables. In principle these are not affected by weak exogeneity assumptions, which in estimation of β influence its efficiency, not its consistency. Some data series that are relevant to the discussion are plotted in Fig. 3.

Beginning with the GLPS model and the estimate of its β matrix given on page 9, for comparative purposes we delete its second column, equivalently combining the first two variables into the real exchange rate, denoted fx in GVAR; both modelling teams find that this can be treated as a stationary variable. With variables

$$(fx, r, r^*, y, y^*, h - y, \Delta\tilde{p}, p^o)$$

a corresponding orthogonal complement is the 4×8 matrix

$$\beta'_{\perp} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -56.1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

This implies that two common trends and a long-run forcing variable, the oil price, deliver the $I(1)$ characteristics of the remaining variables. The first common trend is a linear combination of the domestic and foreign output variables, plotted in the upper left panel of Fig. 3. Although the functional form implies that this does not yield a simple aggregate global output measure, we nevertheless interpret it as a composite index representing global technical progress, in the spirit of the framework of the stochastic Solow growth model adopted by the model's authors.

The second linear combination involves the two interest rates, inflation and the velocity of money. This is overwhelmingly dominated by the last variable, given not only its coefficient but also the scale of measurement of the series, so it virtually becomes a long-run forcing variable in its own right, plotted in the centre left panel of Fig. 3. However the inclusion of a linear trend in the corresponding cointegrating vector raises difficulties for a common trends interpretation, although the need for such a variable in order to obtain cointegration in UK demand for money studies is well established. See, for example, Doornik et al. (1998), who also discuss the difficulties in the empirical analysis caused by structural breaks, including financial liberalisation, and the appropriate treatment of indicator variables. These lead to caution in the adoption of the obvious interpretation of this common trend, however attractive it might be to a monetarist.

Turning to the GVAR(UK) model, we note that there are several weakly exogenous variables that do not enter the cointegrating relations and are immediately identified as their own long-run forcing variables. A simple orthogonal complement of the matrix β on page 14 then contains only two combinations of variables, respectively $rl+rl^*$ and $\Delta p+1.62rs$. Again the first might be loosely interpreted as a composite global interest rate measure; as for the second, in a closing reflection GLPS "recognise the difficulties in the view that price inflation and nominal interest rate series are $I(1)$" (2006, p. 223). Compared to the GLPS model, the main missing element in GVAR(UK) is a story about output growth or, more generally, long-run linkages between the domestic and foreign real economies. Plots of paired variables are shown in Fig. 3: according to the authors' tests, rl and rl^* do cointegrate; y and y^*, q and q^* do not. The sensitivity of the cointegration of domestic and foreign output to the use of different sample periods and different foreign aggregate variables in the two models merits further investigation, given the central role of this relation in the theoretical specification of the GLPS model.

A final caveat is suggested by the preceding reference to Doornik et al. (1998), which is but one of a host of studies that find structural breaks in univariate and multivariate models of macroeconomic aggregates over this period. The question of parameter constancy is not addressed by the GLPS model's authors. In contrast, Castle and Hendry (2008) present error correction equations for UK inflation with the same sample start date as GLPS,
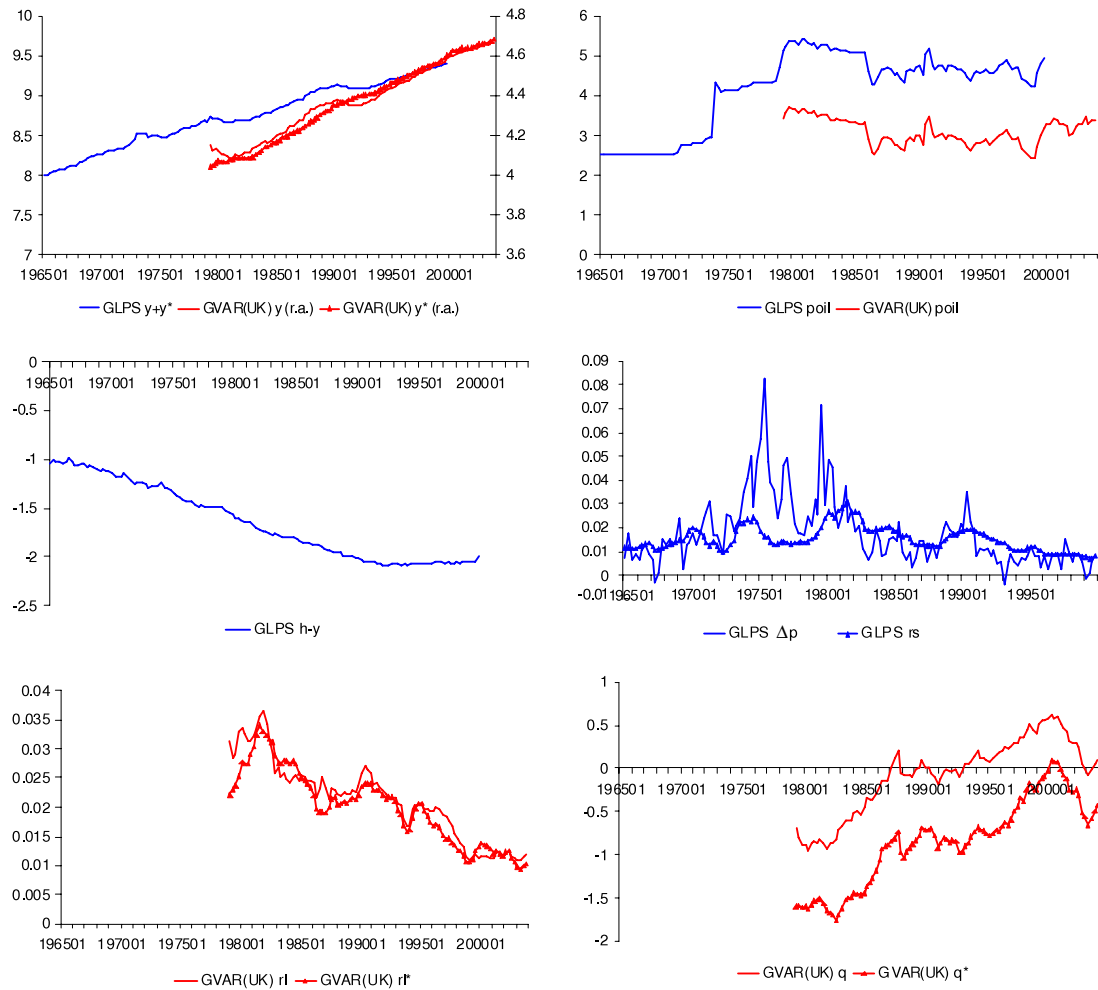
Fig. 3. Some relevant data series.

treating inflation as $I(0)$, but subject to structural breaks which give the impression that the series is $I(1)$. Similarly, over a longer period, Boero et al. (2010) develop a simple autoregressive model of inflation with structural breaks in mean and variance, constant within subperiods and with no unit roots.

6. Conclusion

The VECM model is a convenient alternative form of the VAR model when variables are cointegrated, providing easy interpretation of and differentiation between the short-run and long-run implications of the model. Treating some variables as weakly exogenous is often convenient, and may allow efficient inference on the cointegration coefficients to be conducted in a smaller, "partial" system. However, we first argue that the resulting conditional VECM representation is not appropriate by itself for study of the dynamic properties of the model, since it distorts dynamic interrelationships with the weakly exogenous variables, and neglects all the dynamic relationships among them. It is necessary to conduct dynamic analysis in the "full" VECM form of the model, which may require reconstruction from the conditional VECM form when it is the latter that is reported.

We then show that the dynamic properties of the system are sensitive to the treatment of weak exogeneity questions. In a small macroeconometric model containing domestic and foreign variables, it is seen that taking more foreign variables as weakly exogenous slows the response of domestic variables to foreign shocks. Adjustment may become so slow that the

theoretical distinction between permanent and transitory shocks is almost immaterial for practical policy analysis. Our general recommendation is that the effects on system dynamics of weak exogeneity assumptions should always be checked. Assumptions that cause serious distortions of dynamic properties of relevance to the problem at hand should not be adopted without statistical testing. Even when it is "*a priori* very likely that weak exogeneity holds", it should be tested.

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