Sunk Costs and the Inefficiency of Relationship-Specific Investment

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It is well known that relationship-specific investment will be inefficient when the parties are unable to write (binding) long-term contracts. In this note I exploit some of the recent developments in strategic bargaining theory in order to explore the relationship between the degree of inefficiency in the level of such relationship-specific investment and the degree of sunkness in the cost of investment. One result obtained is that the underinvestment result is not sensitive to the degree of sunkness in the cost of investment. Another result is that each player may actually overinvest when the cost of investment is not sunk.

INTRODUCTION

Consider the following situation. At date 0, an investor (e.g. a Japanese firm) has to choose the level of some investment (e.g. the size of a car plant in Birmingham), with the cost of investment incurred by the investor at date 0. At a subsequent date 1, when the cost of investment is sunk, the investor and some second (non-investing) party (e.g. a union) bargain over the partition of a pie (or surplus), the size of which depends on the chosen investment level. It is assumed that it is impossible to write any kind of binding (long-term) contract at date 0. In particular, the payment to be made by the investor to the union for the latter’s cooperation in generating the pie cannot be contracted upon at date 0. It has been argued by many authors (see e.g. Grout 1984 and Crawford 1982, or Hart and Holmstrom 1988, pp. 128–31) that in this situation the equilibrium investment level will be Pareto-inefficient. In particular, the investor will underinvest relative to the first-best level of investment. This arises because at date 1 the investor does not obtain the full marginal return on her date 0 investment.

Recently several authors (e.g. Hart and Moore 1988, Chung 1991, Aghion et al. 1994) have shown that it may be possible to obtain efficient investment levels if the parties (i) can at date 0 write particular kinds of binding (long-term) incomplete contracts, and (ii) are allowed to renegotiate the terms of the contract at date 1, where the structure of the renegotiation game is determined by the date 0 contract. In the situation described above, it was assumed that no binding contracts of any kind can be written at date 0. I shall maintain this extreme assumption in this note, in order to present my main ideas and results in the simplest framework. It would, of course, be interesting to embed these ideas in the richer ‘incomplete contracting with renegotiation’ framework, as first developed in Hart and Moore (1988).

The basic idea of this note is that, at date 1, the investor may be able to sell her investment and recover, if not all, at least some fraction of, her cost of investment. Hence one must allow the investor, in the bargaining game at date
1, the choice of taking up this 'outside option'. A key issue, then, is to investigate whether the threat of opting out can be used by the investor to increase her marginal return on her investment, which would then increase her ex ante investment level.

I parameterize the degree to which the costs of investment are recoverable at date 1 in the following simple manner. I define a parameter $\mu$ (where $0 \leq \mu \leq 1$) to denote the fraction of the cost of investment that is recoverable at date 1, if the investor chooses not to consummate the gains from trade that will exist between her and her partner. Our main objective is to explore the relationship between the equilibrium investment level and this parameter.

This relationship turns out to be extremely sensitive to the timings at which, during the date 1 negotiation process, the investor can opt out (and sell her investment). One can interpret the differences in the timings of opting out in terms of the different types of communication modes through which bargaining can be conducted. We consider two modes of communication: face-to-face and via a telephone. In both the face-to-face and the telephone bargaining models, players make offers and counter-offers alternately, à la Rubinstein (1982). However, the timings at which the investor can opt out are different in the two models. In the face-to-face model, the investor can opt out (and sell her investment) only after rejecting the other player's offer, while in the telephone model she can opt out even after her offer is rejected. The latter makes sense if she is bargaining via a telephone, since she can always hang up if her offer is rejected; whereas if she is bargaining face-to-face, the channel of communication cannot be closed so abruptly. It was in Shaked (1994) that the importance of the timings of opting out was first made. Indeed, the interpretation above is taken from that paper.

If the date 1 bargaining takes place according to the face-to-face model, then the classic underinvestment result holds for any value of $\mu$ (Proposition 2). Indeed, contrary to conventional wisdom, the inefficiency of the relationship-specific investment is independent of the degree of sunkness in the costs of investment. However, the results are rather different if the players bargain via a telephone (Proposition 3). If the costs of investment are sunk to some large extent, then the underinvestment result holds. On the other hand, if the costs of investment are sunk to some small extent, then the investor overinvests, relative to the first-best level. Only for some cost and benefit functions will investment be at the efficient level when the costs are fully recoverable.

Overall, then, our results seem to challenge the conventional wisdom on two grounds. First, it seems that to some extent investment is independent of the degree of sunkness in the costs of investment. Second, investment may in fact be above (rather than below) the efficient level.

The note is organized as follows. The next section lays down the basic model in a simple setting. Sections II and III study two extensions of this model, both which parameterize the degree of sunkness in the costs of investment. The former section is based on the face-to-face bargaining model, and the latter section on the telephone bargaining model. Section IV concludes.

I. THE BASIC MODEL

At date 0, player $I$ has to choose the level $\alpha$ of some investment, where $\alpha \geq 0$. The cost of investing $\alpha$ is denoted by $C(\alpha)$, where $C:\ [0, \infty) \to [0, \infty)$ is assumed...
to be a convex, strictly increasing and twice continuously differentiable function. This cost of investment is incurred by player \( I \) at date 0.

Then, at date 1, player \( I \) and a second player \( F \) bargain over the partition of a pie of size \( V(\alpha) \) according to Rubinstein’s (1982) alternating-offers bargaining model, where \( V: [0, \infty) \rightarrow [0, \infty) \) is assumed to be a concave, strictly increasing and twice continuously differentiable function. The utility to player \( i (i = I, F) \) from receiving a share \( x_i \) of the pie at time \( i \) is \( x_i \delta^i \), where \( 0 < \delta < 1 \) denotes the common discount factor.

We shall assume that the function \( V - C \) is strictly concave in \( \alpha \). Furthermore, in order to keep the problem from becoming trivial, we assume that \( V'(0)/2 > C'(0) \) and that \( V(0) = C(0) = 0 \).

The first best (or, efficient) level of investment \( \alpha^e \) maximizes total surplus \( V(\alpha) - C(\alpha) \). Given our assumptions, there exists a unique such \( \alpha^e > 0 \). Of course,

\[
(1) \quad V'(\alpha^e) = C'(\alpha^e).
\]

The following proposition describes the classic underinvestment result. As Grout (1984) was one of the early papers to establish this kind of result, we shall refer to it as the Grout equilibrium.

**Proposition 1.** In the game described above, the investment level \( \alpha^g \) chosen by player \( I \) at date 0 in the unique (limiting, as \( \delta \rightarrow 1 \)) subgame-perfect equilibrium is the unique solution, \( \alpha^g > 0 \), to:

\[
(2) \quad \frac{V'(\alpha^g)}{2} = C'(\alpha^g).
\]

**Proof.** Trivial; hence omitted. \( \square \)

Indeed, for any \( \alpha \geq 0 \) chosen at date 0, in the date 1 Rubinstein (limiting) bargaining equilibrium the pie \( V(\alpha) \) will be split equally between the two players. Hence player \( I \) will receive only one-half of the marginal return \( V'(\alpha) \) on her date 0 investment. Consequently, given our assumptions on \( V \) and \( C \), there is underinvestment relative to the first-best level of investment: \( \alpha^g < \alpha^e \).

**Partially recoverable cost of investment**

I now extend the basic model by allowing player \( I \) the option of selling her investment, at date 1, during the process of negotiation with player \( F \). Thus, in the alternating-offers bargaining game at date 1, player \( I \) has an ‘outside option’. We shall assume that, by exercising this outside option, player \( I \) is able to recover some fraction \( \mu \) \( (0 \leq \mu \leq 1) \) of the costs of investment incurred at date 0. Hence, \( I \)'s payoff from taking up her outside option at time \( t \) is \( \mu C(\alpha) \delta^t \). The parameter \( \mu \) captures the degree to which the costs of investment are sunk.

Two extensions to Rubinstein’s bargaining model will be considered. One extension is due to Binmore (1985), the other to Shaked (1994). The former is the basis of the results in Section II, the latter for the results in Section III. The differences between these two models lie in the timings at which player \( I \) can choose to opt out. As mentioned in the Introduction, the differences can be interpreted in terms of differing modes of communication.
II. Investment under Face-to-Face Bargaining

Here it is assumed that the date 1 bargaining takes place according to Binmore's (1985) extension of the Rubinstein model. In this bargaining game, player I can choose to take up her outside option only after rejecting her opponent's offer; that is, the players will make offers alternately, and whenever player I has to react to her opponent's offer she can either accept the offer, or reject it and continue bargaining, or opt out and sell her investment. Notice, therefore, that she cannot choose to opt out after her offer is rejected.

The following result describes, for each investment level \( \alpha \geq 0 \) chosen at date 0, the unique (limiting, as \( \delta \to 1 \)) subgame-perfect equilibrium payoff to player I at date 1. The result is an application of Binmore's 'outside option principle' (cf. Binmore 1985; alternatively, see Osborne and Rubinstein 1990, section 3.12.1).

**Lemma 1** Set an arbitrary investment level \( \alpha \geq 0 \) and an arbitrary \( \mu \in [0, 1] \). Then, the unique (limiting, as \( \delta \to 1 \)) subgame-perfect equilibrium payoff to player I at date 1 in the Binmore bargaining game is:

\[
P^b(\alpha, \mu) = \begin{cases} 
\frac{V(\alpha)}{2} & \text{if } \mu C(\alpha) \leq V(\alpha)/2 \\
\mu C(\alpha) & \text{if } \mu C(\alpha) \geq V(\alpha)/2.
\end{cases}
\]

The intuition behind this result is as follows. In the absence of any outside option, we know that the pie \( V(\alpha) \) will be split equally between the two players. Now, if the outside option payoff \( \mu C(\alpha) \) is less than \( V(\alpha)/2 \), then player I's threat to take up her outside option is not credible, and hence the outside option has no influence on the equilibrium partition of the pie. However, if the outside option payoff is greater than \( V(\alpha)/2 \), then her threat to take it up becomes credible, and she has to receive a share that is at least as big as the outside option payoff. However, player I will not receive a share that is strictly greater than the outside option payoff, since she would always prefer to accept a share that is greater than her outside option payoff than to take up her outside option.

The next result describes, for each \( \mu \), the unique equilibrium investment level \( \alpha^b(\mu) \) which maximizes \( P^b(\alpha, \mu) - C(\alpha) \), where \( P^b(\alpha, \mu) \) is defined in (3).

**Proposition 2** For each \( \mu \in [0, 1] \), the investment level \( \alpha^b(\mu) \) that will be chosen by player I at date 0 in the unique (limiting, as \( \delta \to 1 \)) subgame-perfect equilibrium is the unique solution, \( \alpha^b > 0 \), to:

\[
\frac{V'(\alpha^b)}{2} = C'(\alpha^b).
\]

**Proof** Straightforward; hence omitted. \( \square \)

It follows from Propositions 1 and 2 that, for any \( 0 \leq \mu \leq 1 \), \( \alpha^b(\mu) = \alpha^g \) (the Grout equilibrium). Indeed, the degree of inefficiency of the equilibrium investment is independent of the degree of sunkness in the cost of investment. Contrary to conventional wisdom (e.g. Hart and Holmstrom 1988, pp. 128–31), the classic underinvestment result would appear to have nothing to do with the sunkness in the costs of investment.

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III. INVESTMENT UNDER TELEPHONE BARGAINING

We now assume that the date 1 bargaining takes place according to Shaked’s (1994) extension of the Rubinstein model. In this bargaining game, player I can choose to take up her outside option at any point during the bargaining process. More precisely, (i) whenever player I has to react to her opponent’s offer, she can either accept the offer, or reject it and continue bargaining, or opt out and sell her investment; and (ii) whenever player I’s offer is rejected, she can choose either to opt out, or to let her opponent make an offer.

The following result describes, for each investment level \( \alpha \geq 0 \) chosen at date 0, the set of (limiting, as \( \delta \to 1 \)) subgame-perfect equilibrium payoffs to player I at date 1. The result is an application of Lemmas 1 and 2 in Shaked (1994) (alternatively, see Osborne and Rubinstein 1990, section 3.12.2).

**Lemma 2** Set an arbitrary investment level \( \alpha \geq 0 \) and an arbitrary \( \mu \in [0,1] \).

(i) If \( \alpha \) and \( \mu \) are such that \( V(\alpha)/2 > \mu C(\alpha) \), then the unique (limiting, as \( \delta \to 1 \)) subgame-perfect equilibrium payoff to player I is \( P^*(\alpha, \mu) = V(\alpha)/2 \).

(ii) If \( \alpha \) and \( \mu \) are such that \( V(\alpha)/2 \leq \mu C(\alpha) \leq V(\alpha) \), then for every \( k \in [\mu C(\alpha), V(\alpha)] \) there exists a (limiting, as \( \delta \to 1 \)) subgame-perfect equilibrium such that the payoff to player I is \( P^*(\alpha, \mu) = k \).

(iii) If \( \alpha \) and \( \mu \) are such that \( V(\alpha) < \mu C(\alpha) \), then the unique (limiting, as \( \delta \to 1 \)) subgame-perfect equilibrium payoff to player I is \( P^*(\alpha, \mu) = \mu C(\alpha) \).

The intuition for this result is as follows. As in the Binmore bargaining equilibrium (cf. Lemma 1), if the outside option payoff \( \mu C(\alpha) \) is less than \( V(\alpha)/2 \), then player I’s threat to take up her outside option is not credible, and hence the outside option has no influence on the equilibrium partition of the pie. If the outside option payoff exceeds the size of the pie \( V(\alpha) \), then there are no gains from trade and hence in the unique equilibrium player I will indeed take up her outside option.

Now consider the interesting case when the magnitude of the outside option payoff lies between \( V(\alpha)/2 \) and \( V(\alpha) \). As in the Binmore model, the threat to take up the outside option is credible, and hence will influence the equilibrium partition. However, the exact impact is quite different in the Shaked model. The basic reason is that, unlike in the Binmore model, player I is able to make ‘take-it-or-leave-it’ offers. She can credibly threaten her opponent that if her offer is not accepted then she will opt out. And, since her opponent’s payoff, if I opts out, is zero, player I could ask for the whole pie and get her opponent to accept that offer. The detailed intuition for the source of the multiplicity of equilibria can be found in Shaked (1994), but roughly speaking it has to do with the fact that player I could, in equilibrium, credibly threaten to opt out at any specified time in the future.

In characterizing the equilibrium investment, we shall assume that for any \( \alpha \) and \( \mu \) such that \( V(\alpha)/2 \leq \mu C(\alpha) \leq V(\alpha) \), player I’s equilibrium payoff at date 1 bargaining is indeed equal to \( V(\alpha) \). This particular bargaining equilibrium is selected for two reasons. First, since it gives player I the full benefit from her investment, the equilibrium investment level chosen at date 0 will possess the
least possible amount of inefficiency. Second, arguments similar to those contained in Hendon et al. (1994) could in fact be used to select this particular bargaining equilibrium.

The next proposition characterizes, for each \( \mu \), the unique equilibrium investment level \( \alpha^*(\mu) \) that maximizes \( P(\alpha, \mu) - C(\alpha) \), where from Lemma 2 (and given our bargaining equilibrium selection)

\[
P^e(\alpha, \mu) = \begin{cases} 
V(\alpha)/2 & \text{if } \mu C(\alpha) < V(\alpha)/2 \\
V(\alpha) & \text{if } V(\alpha)/2 \leq \mu C(\alpha) \leq V(\alpha) \\
\mu C(\alpha) & \text{if } \mu C(\alpha) > V(\alpha).
\end{cases}
\]

**Proposition 3** There exist numbers \( \mu_1 \) and \( \mu_2 \), where \( \frac{1}{2} < \mu_1 \leq \mu_2 \leq 1 \), such that, for each \( \mu \in [0, 1] \), the unique limiting (as \( \delta \to 1 \)) subgame-perfect equilibrium investment level is

\[
\alpha^*(\mu) = \begin{cases} 
\alpha^g & \text{if } 0 \leq \mu \leq \mu_1 \\
\hat{\alpha}(\mu) & \text{if } \mu_1 < \mu \leq \mu_2 \\
\alpha^e & \text{if } \mu_2 < \mu \leq 1
\end{cases}
\]

where \( \alpha^g \) and \( \alpha^e \), respectively, are the Grout equilibrium and the efficient investment levels, and \( \hat{\alpha}(\mu) \), \( \mu_1 \) and \( \mu_2 \) are defined as follows:

(i) For each \( 0 < \mu \leq 1 \), \( \hat{\alpha}(\mu) \) denotes the unique strictly positive solution to \( V(\alpha)/2 = \mu C(\alpha) \).

(ii) If there exists a solution \( 0 < \mu^* \leq 1 \) to \( V(\hat{\alpha}(\mu)) - C(\hat{\alpha}(\mu)) = V(\alpha^g)/2 - C(\alpha^g) \), then \( \mu_1 = \mu^* \); otherwise, \( \mu_1 = 1 \).

(iii) If \( \alpha^e < \hat{\alpha}(1) \), then \( \mu_2 = 1 \); otherwise, \( \mu_2 \) is the unique solution to \( \hat{\alpha}(\mu) = \alpha^e \).

Moreover, \( \hat{\alpha}(\mu) > \alpha^e \) and the function \( \hat{\alpha} \) is continuous and strictly decreasing in \( \mu \).

**Proof.** See Appendix. \( \square \)

The intuition for this result is as follows. First, it is straightforward to notice (e.g. by drawing a figure depicting the graphs of the functions \( V, \frac{1}{2} V, C \) and \( \mu C \)) that, for each \( \mu \in [0, 1] \), the equilibrium investment level must be either \( \alpha^g \), or \( \hat{\alpha}(\mu) \), or \( \alpha^e \). Figure 1 illustrates this information for some fixed value of \( \mu \). Now if \( \mu \) is sufficiently small (i.e. if \( \mu < \mu_2 \)) that \( \hat{\alpha}(\mu) > \alpha^e \), then the chosen investment level cannot be equal to \( \alpha^e \). On the other hand, if \( \mu \) is large enough that \( \hat{\alpha}(\mu) \leq \alpha^e \), then the investor will set investment at the efficient level. In the former case, the choice between \( \alpha^g \) and \( \hat{\alpha}(\mu) \) is settled by comparing the payoffs from these two choices. If \( \mu \) is sufficiently small, then \( \hat{\alpha}(\mu) \) will be sufficiently large that (owing to the strict concavity of \( V - C \)) \( V(\hat{\alpha}(\mu)) - C(\hat{\alpha}(\mu)) \) will be sufficiently small.

Figure 2 illustrates the equilibrium investment for the case in which \( \mu_1 < \mu_2 < 1 \). Here, for any \( \mu \in [0, \mu_1] \), there is underinvestment; for any \( \mu \in (\mu_1, \mu_2) \), there is overinvestment; and for any \( \mu \in [\mu_2, 1] \), the investment is at the efficient level. Let me provide some intuition for why the investor may overinvest relative to the efficient level of investment. Suppose that \( \mu \) is sufficiently small so that \( \hat{\alpha}(\mu) > \alpha^e \) (as, for example, illustrated in Figure 1). If the investor chooses an investment level \( \alpha < \hat{\alpha}(\mu) \), then (since \( V(\alpha)/2 > \mu C(\alpha) \))
threat to opt out is not credible, and thus, in the bargaining equilibrium, she will obtain only one-half of the return \( V(\alpha) \) from her investment. On the other hand, if she chooses the investment level \( \alpha = \tilde{\alpha}(\mu) \), then her threat to opt out is credible and, in the Shaked bargaining equilibrium that I have selected, she will obtain the full return \( V(\alpha) \) from her investment. Indeed, owing to this dramatic increase in what she can obtain in the ex post bargaining game, it is quite possible that her net gain \( V(\tilde{\alpha}(\mu)) - C(\tilde{\alpha}(\mu)) \) by choosing \( \alpha = \tilde{\alpha}(\mu) \) strictly exceeds the maximal net gain \( V(\alpha^e)/2 - C(\alpha^e) \) that she could obtain by choosing some \( \alpha < \tilde{\alpha}(\mu) \). Hence the investor may overinvest, i.e. set \( \alpha = \tilde{\alpha}(\mu) > \alpha^e \).

**Example.** It is easy to verify that, if \( V(\alpha) = \alpha \) and \( C(\alpha) = \alpha^2 \), then \( \tilde{\alpha}(\mu) = 1/(2\mu) \), \( \mu_1 = 4 - \sqrt{12} \) and \( \mu_2 = 1 \). Hence in this example the equilibrium investment level (as a function of \( \mu \)) is as depicted in Figure 3.
IV. CONCLUDING REMARKS

This note has shown that the classic underinvestment result may not be robust to reasonable specifications of the \textit{ex post} bargaining game when the investor may, in addition, be able strategically to terminate the negotiations in order to sell her investment and recover, if not all, at least some fraction of the cost of investment. I showed that, under some circumstances, investment may in fact be above (rather than below) the efficient level. It was also shown that the \textit{ex ante} level of investment may be independent of the degree to which costs of investment are sunk at the time investments are undertaken.

The main ideas and results have been developed in very simple environments. In particular, I have assumed that no binding contract whatsoever can be written before investments are chosen. Indeed, it would be interesting to embed these ideas in a ‘incomplete contracting with renegotiation’ framework as first developed in Hart and Moore (1988). They showed, and more recently Chung (1991) and Aghion et al. (1994) have shown, that if the initial incomplete contract can influence the structure of the \textit{ex post} bargaining game in a particular manner, efficient investments can be sustained in equilibrium.

APPENDIX: PROOF TO PROPOSITION 3

The proposition is proven through a series of simple claims. These claims are straightforward, and thus their proofs are omitted.

\textit{Claim 1.} If $\mu = 0$, then $\alpha^*(\mu) = \alpha^s$.

\textit{Claim 2.} For any $\mu \in (0, 1]$ there exists a unique strictly positive solution $\bar{\alpha}$ that solves $V(\alpha)/2 = \mu C(\alpha)$. The solution $\bar{\alpha}$ is a continuous and strictly decreasing function of $\mu$. Moreover, as $\mu \to 0$, $\bar{\alpha}(\mu) \to \infty$.

\textit{Claim 3.} For any $\mu \in (0, 1]$ there exists a unique strictly positive solution $\alpha^*$ that solves $V(\alpha) = \mu C(\alpha)$. This solution $\alpha^*$ is a continuous and strictly decreasing function of $\mu$. Moreover, as $\mu \to 0$, $\alpha^*(\mu) \to \infty$. Furthermore, for any $\mu \in (0, 1]$, $\alpha^*(\mu) > \bar{\alpha}(\mu)$. 

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Claims 2 and 3 imply:

Claim 4. For any $\mu \in (0, 1]$,

(i) $\alpha \preceq \bar{\alpha}(\mu)$ if and only if $\mu C(\alpha) \preceq V(\alpha)/2$, and

(ii) $\alpha \preceq \bar{\alpha}^*(\mu)$ if and only if $\mu C(\alpha) \preceq V(\alpha)$.

From Claim 4, it follows that, for any $\mu \in (0, 1]$,

$$ P'(\alpha, \mu) = \begin{cases} 
V(\alpha)/2 & \text{if } \alpha < \bar{\alpha}(\mu) \\
V(\alpha) & \text{if } \bar{\alpha}(\mu) \leq \alpha \leq \bar{\alpha}^*(\mu) \\
\mu C(\alpha) & \text{if } \bar{\alpha}^*(\mu) < \alpha.
\end{cases} $$

Claim 5. For any $\mu \in (0, 1]$, $\alpha^c < \bar{\alpha}^*(\mu)$ and $\alpha^s < \bar{\alpha}(\mu)$.

It thus follows that:

Claim 6. For any $\mu \in (0, 1]$ such that $\alpha^e \geq \bar{\alpha}(\mu)$, $\alpha^4(\mu) = \alpha^e$.

Claim 7. For any $\mu \in (0, 1]$ such that $\alpha^e < \bar{\alpha}(\mu)$,

$$ \alpha^4(\mu) = \begin{cases} 
\alpha^s & \text{if } V(\alpha^s)/2 - C(\alpha^s) \geq V(\bar{\alpha}(\mu)) - C(\bar{\alpha}(\mu)) \\
\bar{\alpha}(\mu) & \text{if } V(\alpha^s)/2 - C(\alpha^s) < V(\bar{\alpha}(\mu)) - C(\bar{\alpha}(\mu)).
\end{cases} $$

First, suppose $\alpha^e < \bar{\alpha}(1)$. This implies that, for any $\mu \in (0, 1]$, $\alpha^e < \bar{\alpha}(\mu)$. Thus, the hypothesis of Claim 7 holds. Consequently, it is straightforward to show that there exists a number $\mu_1$, where $\frac{1}{2} \leq \mu_1 \leq 1$, such that

$$ \alpha^4(\mu) = \begin{cases} 
\alpha^s & \text{if } 0 \leq \mu \leq \mu_1 \\
\bar{\alpha}(\mu) & \text{if } \mu_1 < \mu \leq 1
\end{cases} $$

where, if there exists a solution $\mu^* \in (0, 1]$ to $V(\bar{\alpha}(\mu)) - C(\bar{\alpha}(\mu)) = V(\alpha^s)/2 - C(\alpha^s)$, then $\mu_1 = \mu^*$; otherwise, define $\mu_1 = 1$.

Now suppose $\alpha^e \geq \bar{\alpha}(1)$. Define $\mu_2$, where $\mu_2 \in (0, 1]$, to be the unique solution to $\bar{\alpha}(\mu) = \alpha^e$.

The proposition will follow in a straightforward manner, using these claims.

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