Supplementary Material

Let $c_i(\xi,\alpha)$ and $w_i'(\xi,\alpha,\mu)(\xi^{'})$ be the maximisers in problem (6) - (10) and let $\lambda_i(\xi,\alpha,\mu)$ be the Lagrange multiplier associated to constraint (8). Let

$$\tilde{u}_i(\xi,\alpha,\mu) = u_i(c_i(\xi,\alpha)) + \beta(\xi,\mu_i)\sum_{\xi'}\pi_{\mu_i}(\xi'\,|\xi)\; w_i'(\xi,\alpha,\mu)(\xi').$$

Claim 1. *$\tilde{u}_i(\xi,\alpha,\mu)$ is nondecreasing in α_i for all $\alpha\in\mathbb{R}^I_+$.*

Proof. Let $\tilde{\alpha},\alpha\in\mathbb{R}^I_+$ be such that $\tilde{\alpha}_i>\alpha_i$ and $\tilde{\alpha}_j=\alpha_j$ for every $j\neq i$. To get a contradiction, suppose $\tilde{u}_i(\xi,\tilde{\alpha},\mu)<\tilde{u}_i\,(\xi,\alpha,\mu)$. Since the constrained set is independent of the welfare weights, then

$$\sum_h\tilde{\alpha}_h\left(\tilde{u}_h\left(\xi,\tilde{\alpha},\mu\right)-\tilde{u}_h\left(\xi,\alpha,\mu\right)\right)\geq 0 \text{ and } \sum_h\alpha_h\left(\tilde{u}_h\left(\xi,\alpha,\mu\right)-\tilde{u}_h\left(\xi,\tilde{\alpha},\mu\right)\right)\geq 0$$

and so, on the one hand,

$$\sum_h(\tilde{\alpha}_h-\alpha_h)\left(\tilde{u}_h\left(\xi,\tilde{\alpha},\mu\right)-\tilde{u}_h\left(\xi,\alpha,\mu\right)\right)\geq 0$$

But, on the other hand,

$$\sum_h(\tilde{\alpha}_h-\alpha_h)\left(\tilde{u}_h\left(\xi,\tilde{\alpha},\mu\right)-\tilde{u}_h\left(\xi,\alpha,\mu\right)\right)=(\tilde{\alpha}_i-\alpha_i)\left(\tilde{u}_i\left(\xi,\tilde{\alpha},\mu\right)-\tilde{u}_i\left(\xi,\alpha,\mu\right)\right)<0$$

a contradiction. □

Let $\overline{c}_i(\xi,\alpha)$ and $\overline{w}_i'(\xi,\alpha,\mu)(\xi^{'})$ be the maximisers of the relaxed problem where (8) is ignored. Let

$$\overline{u}(\xi,\alpha,\mu) = u_i(\overline{c}_i(\xi,\alpha)) + \beta(\xi,\mu_i)\sum_{\xi'}\pi_{\mu_i}(\xi'\,|\xi)\;\overline{w}_i'(\xi,\alpha,\mu)(\xi').$$

Claim 2. *Let $\alpha\in\mathbb{R}^I_+$. If $\alpha_i<\tilde{\alpha}_i$ and $\alpha_h=\tilde{\alpha}_h$ for all $h\neq i$, then $\overline{u}_i\left(\xi,\alpha,\mu\right)<\overline{u}_i\left(\xi,\tilde{\alpha},\mu\right).$*

Proof. Note that $\overline{c}_i(\xi,\alpha)$ is the unique solution to

$$c_i+\sum_{h\neq i}\left(\frac{\partial u_h}{\partial c_h}\right)^{-1}\left(\frac{\alpha_i}{\alpha_h}\;\frac{\partial u_i(c_i)}{\partial c_i}\right)=y(\xi).$$

and so it is strictly increasing in α_i. Therefore, $\overline{c}_i(\xi,\tilde{\alpha})>\overline{c}_i(\xi,\alpha)$. Note that

$$\overline{\alpha}_i'(\xi,\alpha,\mu)(\xi')=\frac{\alpha_i\int\pi(\xi'\,|\xi)\mu_i'(\xi,\mu)\,(\xi')\,(d\pi)}{\sum_h\alpha_h\int\pi(\xi'\,|\xi)\mu_h'(\xi,\mu)\,(\xi')\,(d\pi)}$$

Thus, $\overline{\alpha}_i'(\xi,\alpha,\mu)(\xi')$ is nondecreasing in α_i. Since $\overline{w}_i'(\xi,\alpha,\mu)(\xi^{'})$ satisfies (9) and (10), it follows by Lemma A.1 and Theorem 1 that $\overline{w}_i'(\xi,\alpha,\mu)(\xi^{'})=\overline{u}_i(\xi',\overline{\alpha}'(\xi,\alpha,\mu)(\xi'),\mu'(\xi,\mu)(\xi'))$. Thus, Claim 1 implies that $\overline{w}_i'(\xi,\tilde{\alpha},\mu)(\xi^{'})\geq\overline{w}_i'(\xi,\alpha,\mu)(\xi^{'})$ for all ξ'. We conclude that $\overline{u}_i\left(\xi,\alpha,\mu\right)<\overline{u}_i\left(\xi,\tilde{\alpha},\mu\right)$, as desired. □

Proof of Proposition 2 . (i) Suppose $\alpha \in \Delta(\xi, \mu)$. Consider first the case where $\alpha_i > \underline{\alpha}_i(\xi, \mu)$ for all i. By the definition of $\tilde{u}_i(\xi, \alpha, \mu)$, we have that $\tilde{u}_i(\xi, \alpha, \mu) \geq U_i(\xi, \mu)$ and $\sum_i^I \alpha_i \tilde{u}_i(\xi, \alpha, \mu) = v^*(\xi, \alpha, \mu)$. It follows by Lemma A.1, that $(\tilde{u}_1(\xi, \alpha, \mu)...\tilde{u}_I(\xi, \alpha, \mu)) \in \mathcal{U}^{\mathrm{E}}(\xi, \mu)$. Since $\sum_i^I \alpha_i \tilde{u}_i(\xi, \alpha, \mu) = v^*(\xi, \alpha, \mu)$, it is easy to see that $(u_1(\xi, \alpha, \mu)...u_I(\xi, \alpha, \mu)) \in \overline{\mathcal{U}}^E(\xi, \mu)$. Then, $\tilde{u}_i(\xi, \alpha, \mu) > U_i(\xi, \mu_i)$ for all i by definition of $\underline{\alpha}_i(\xi, \mu)$. Thus, $\lambda_i(\xi, \alpha, \mu) = 0$. Let $\alpha \in \Delta(\xi, \mu)$ be such that $\alpha_i = \underline{\alpha}_i(\xi, \mu)$ for some i. Then there is a sequence $\{\alpha^n\}_{n=1}^{\infty}$ such that $\alpha_i^n > \underline{\alpha}_i(\xi, \mu)$ for all i and n and $\alpha^n \to \alpha$. It follows that

$$\lambda_i(\xi, \alpha, \mu) = \lambda_i(\xi, \lim_{n\to\infty} \alpha^n, \mu) = \lim_{n\to\infty} \lambda_i(\xi, \alpha^n, \mu) = 0,$$

where the second equality follows by continuity of $\lambda_i(\xi, \alpha, \mu)$ in α and the last one because weak inequalities are preserved under limits. It follows that, $\tilde{u}_i(\xi, \alpha, \mu) = \overline{u}_i(\xi, \alpha, \mu)$ and so $c_i(\xi, \alpha) = \overline{c}_i(\xi, \alpha)$, i.e. $c_i(\xi, \alpha)$ solves the relaxed problem.

(ii) Let $\alpha \in \mathbb{R}_+^I$ and $\alpha^* \equiv \left(\frac{\alpha_1}{\sum_{i=1}^I \alpha_i} \cdots \frac{\alpha_I}{\sum_{i=1}^I \alpha_i}\right)$. If $\alpha^* \in \Delta(\xi, \mu)$, then $c_i(\xi, \alpha) = c_i(\xi, \alpha^*)$ because $\tilde{u}_i(\xi, \alpha, \mu)$ is homogeneous of degree zero in α. If $\alpha^* \notin \Delta(\xi, \mu)$, there is i such that $\alpha_i^* < \underline{\alpha}_i(\xi, \mu)\left(\alpha_{-i}^*\right)$.

- First, we show that $\lambda_i(\xi, \alpha, \mu) > 0$. To get a contradiction, suppose $\lambda_i(\xi, \alpha, \mu) = 0$. It follows that

$$\begin{aligned}
\tilde{u}_i\left(\xi, (\alpha_i, \alpha_{-i}), \mu\right) \quad &= \tilde{u}_i\left(\xi, \left(\alpha_i^*, \alpha_{-i}^*\right), \mu\right) \\
&= \overline{u}_i\left(\xi, \left(\alpha_i^*, \alpha_{-i}^*\right), \mu\right) \\
&= \overline{u}_i\left(\xi, \left(\frac{\alpha_i^*}{\underline{\alpha}_i(\xi, \mu)\left(\alpha_{-i}^*\right)}, \frac{\alpha_{-i}^*}{\underline{\alpha}_i(\xi, \mu)\left(\alpha_{-i}^*\right)}\right), \mu\right) \\
&< \overline{u}_i\left(\xi, \left(1, \frac{\alpha_{-i}^*}{\underline{\alpha}_i(\xi, \mu)\left(\alpha_{-i}^*\right)}\right), \mu\right) \\
&= \overline{u}_i\left(\xi, \left(\underline{\alpha}_i(\xi, \mu)\left(\alpha_{-i}^*\right), \alpha_{-i}^*\right), \mu\right) \\
&= U_i(\xi, \mu),
\end{aligned}$$

  where the first equality follows because $\tilde{u}_i$ is homogeneous of degree zero in α, the second one is due to the assumption that $\lambda_i(\xi, \alpha, \mu) = 0$ and the homogeneity of degree zero of $\lambda_i(\xi, \alpha, \mu)$ in α, the third and fifth follows by homogeneity of degree zero of $\overline{u}_i(\cdot)$ in α, the inequality follows by Claim 2 and the last equality follows by definition of the minimum enforceable weights. But then, $\tilde{u}_i\left(\xi, (\alpha_i, \alpha_{-i}), \mu\right) < U_i(\xi, \mu)$ which contradicts constraint (8).

- Second, note that problem (6) - (10) is equivalent to maximising

$$\sum_{i=1}^I (\alpha_i + \lambda_i)\left\{u_i(c_i) + \beta(\xi, \mu_i) \sum_{\xi'} \pi_{\mu_i}(\xi' \,|\xi)\; w_i'(\xi')\right\},$$

  subject to constraints (7), (9) and (10).

- Finally, the latter is equivalent to the relaxed problem with welfare weights $\tilde{\alpha}$ given by

$$\tilde{\alpha}_i = \frac{\alpha_i + \lambda_i(\xi, \alpha, \mu)}{\sum_{h=1}^I (\alpha_h + \lambda_h(\xi, \alpha, \mu))},$$

Thus, $\overline{u}_i\left(\xi, \tilde{\alpha}, \mu\right) = \tilde{u}_i\left(\xi, \alpha, \mu\right) \geq U_i(\xi, \mu) = \overline{u}_i\left(\xi, \underline{\alpha}_i, \mu\right)$. It follows by Claim 2 that $\tilde{\alpha}_i \geq \underline{\alpha}_i$. Therefore, $\tilde{\alpha} \in \Delta(\xi, \mu)$ and $c_i(\xi, \alpha) = \overline{c}_i(\xi, \tilde{\alpha}) = c_i(\xi, \tilde{\alpha})$ as desired. □

Now we prove Theorem 11. We begin with some results on Markov Processes.

Lemma 7.1. *Let $\{z_t\}_{t=0}^{\infty}$ be a two-state time homogeneous Markov process with transition function F on $(Z, \mathcal{Z})$ and invariant distribution $\psi : \mathcal{Z} \to [0,1]$, P^F be the probability measure on $(Z^\infty, \mathcal{Z}^\infty)$ uniquely induced by F and ψ and let $R : Z \times Z \to \Re$. Suppose there exists $z_+ \in Z$ such that*
(a) $E^{P^F}(R(z_1, z_2)) = 0$.
(b) $R(z, z_+) > 0$ for all z.
(c) $E^{P^F}(R(z_0, z_1)\ R(z_1, z_2)) > 0$.
Then $E^{P^F}(R(z_2, z_3)|\, z_1 = z_+) < 0 < E^{P^F}(R(z_2, z_3)|\, z_1 = z_-)$ iff $F(z_+|\, z_+) < \psi(z_+)$.

Proof. Hypothesis (a) and the Markov property implies that $E^{P^F}(R(z_k, z_{k+1})) = 0$ for any k. Thus,

$$\psi(z-)\ E^{P^F}(R(z_{k'}, z_{k'+1})|\, z_k = z_-) = -\psi(z_+)\ E^{P^F}(R(z_{k'}, z_{k'+1})|\, z_k = z_+) \tag{34}$$

where $z_- \neq z_+$. Note also that

$$\begin{aligned} & E^{P^F}(R(z_0, z_1)\ R(z_1, z_2)) = E^{P^F}\Big(R(z_0, z_1)\ E^{P^F}(R(z_1, z_2)|\, z_1)\Big) \\ = \ & \left[P^F(z_+, z_+) R(z_+, z_+) + P^F(z_-, z_+) R(z_-, z_+)\right]\ E^{P^F}(R(z_1, z_2)|\, z_1 = z_+) \\ & + \left[P^F(z_+, z_-) R(z_+, z_-) + P^F(z_-, z_-) R_e(z_-, z_-)\right]\ E^{P^F}(R(z_1, z_2)|\, z_1 = z_-). \end{aligned} \tag{35}$$

By hypothesis (a) and (b), $R(z, z_-) < 0$ for all z. Therefore,

$$\begin{aligned} P^F(z_+, z_-) R(z_+, z_-) + P^F(z_-, z_-) R(z_-, z_-) &< 0, \\ P^F(z_+, z_+) R(z_+, z_+) + P^F(z_-, z_+) R(z_-, z_+) &> 0. \end{aligned}$$

It follows from (34) evaluated at $k = 1$ and $k' = 1$, hypothesis (c) and (35) that

$$E^{P^F}(R(z_1, z_2)|\, z_1 = z_-) < 0 < E^{P^F}(R(z_1, z_2)|\, z_1 = z_+)$$

and the Markov Property implies

$$E^{P^F}(R(z_2, z_3)|\, z_2 = z_-) < 0 < E^{P^F}(R(z_2, z_3)|\, z_2 = z_+). \tag{36}$$

Condition (34), evaluated at $k = 1$ and $k' = 2$, implies that

$$E^{P^F}(R(z_2, z_3)|\, z_1 = z_-) < 0 < E^{P^F}(R(z_2, z_3)|\, z_1 = z_+) \Leftrightarrow E^{P^F}(R(z_2, z_3)|\, z_1 = z_+) > 0.$$

In addition,

$$\begin{aligned} E^{P^F}(R(z_2, z_3)|\, z_1 = z_+) &= E^{P^F}(R(z_2, z_3)|\, z_1 = z_+) - E^{P^F}(R(z_2, z_3)) \\ &= (F(z_2 = z_+|\, z_1 = z_+) - \psi(z_+))\, E^{P^F}(R(z_2, z_3)|\, z_2 = z_+) + \\ &\quad\ (F(z_2 = z_-|\, z_1 = z_+) - \psi(z_-))\, E^{P^F}(R(z_2, z_3)|\, z_2 = z_-) \\ &= (F(z_2 = z_+|\, z_1 = z_+) - \psi(z_+)) \times \\ &\quad\ \Big(E^{P^F}(R(z_2, z_3)|\, z_2 = z_+) - E^{P^F}(R(z_2, z_3)|\, z_2 = z_-)\Big). \end{aligned}$$

where the first line follows by the definition of unconditional expectation and (a). (36) implies that

$$E^{P^F}(R(z_2, z_3)|\, z_1 = z_+) < 0 \Leftrightarrow F(z_2 = z_+|\, z_1 = z_+) - \psi(z_+) < 0.$$

□

Proof of Theorem 11(a). Consider any CE of an arbitrary baseline growth economy. Since the allocation is PO, it follows by Theorem 8 that (15) holds and the marginal distribution of ψ_{po} over welfare weights is a point mass on α_∞. By standard arguments, there exists $\overline{R}_{po} : \{l,h\} \times \{l,h\} \to \Re$ such that for any $\tau \in \{1,2\}$ and $\omega \in \Omega$

$$\overline{R}_{\tau,po}(\omega) = \begin{cases} \overline{R}_{po}(l,l) & \text{if } \xi_{\tau-1}(\omega) \in \{1,3\} \text{ and } \xi_\tau(\omega) \in \{1,3\} \\ \overline{R}_{po}(l,h) & \text{if } \xi_{\tau-1}(\omega) \in \{1,3\} \text{ and } \xi_\tau(\omega) \in \{2,4\} \\ \overline{R}_{po}(h,l) & \text{if } \xi_{\tau-1}(\omega) \in \{2,4\} \text{ and } \xi_\tau(\omega) \in \{1,3\} \\ \overline{R}_{po}(h,h) & \text{if } \xi_{\tau-1}(\omega) \in \{2,4\} \text{ and } \xi_\tau(\omega) \in \{2,4\} \end{cases}$$

and

$$\overline{R}_{po}(\xi,l) < 0 < \overline{R}_{po}(\xi,h) \text{ for all } \xi \in \{l,h\}. \tag{37}$$

Let $Z = \{l,h\}$, $\mathcal{Z}$ be its finest partition, $\tilde{\pi}^*$ be the transition function on $(Z,\mathcal{Z})$ defined as the restriction of π^* to $(Z,\mathcal{Z})$ and let $\tilde{\psi}_{po}$ be the restriction of the invariant measure ψ_{po} to $(Z,\mathcal{Z})$. Let Z^∞ be the set of infinite sequences with elements in Z and $\mathcal{Z}_0 \subset \mathcal{Z}_1 \subset ... \subset \mathcal{Z}_t \subset ...\mathcal{Z}^\infty$ be the standard filtration. $P^{\tilde{\pi}^*}$ is the probability measure over $(Z^\infty, \mathcal{Z}^\infty)$ uniquely induced by $\tilde{\pi}^*$ and $\tilde{\psi}_{po}$. Let $z_t : Z^\infty \to Z$ be $\mathcal{Z}_t-$measurable. The collection $\{z_t\}_{t=0}^\infty$ on the probability space $\left(Z^\infty, \mathcal{Z}^\infty, P^{\tilde{\pi}^*}\right)$ is a two state time-homogeneous Markov process with transition function $\tilde{\pi}^*$ on $(Z,\mathcal{Z})$ and invariant distribution $\tilde{\psi}_{po} : \mathcal{Z} \times \mathcal{Z} \to [0,1]$ satisfying

$$E^{P^{\tilde{\pi}^*}}\left(\overline{R}_{po}(z_1,z_2)\right) = 0. \tag{38}$$

First note that (38) and (37) are conditions (a) and (b), respectively, in Lemma 7.1. Second, since the asset displays short-term momentum,

$$0 < E^{P_{po}}\left(\overline{R}_{1,po}\ \overline{R}_{2,po}\right) = E^{P^{\tilde{\pi}^*}}\left(\overline{R}_{po}(z_0,z_1)\ \overline{R}_{po}(z_1,z_2)\right)$$

and so condition (c) in Lemma 7.1 also holds. By Lemma 7.1, we conclude that

$$E^{P^{\tilde{\pi}^*}}\left(\overline{R}_{po}(z_2,z_3)\middle|\, z_1 = h\right) < 0 < E^{P^{\tilde{\pi}^*}}\left(\overline{R}_{po}(z_2,z_3)\middle|\, z_1 = l\right) \Leftrightarrow \tilde{\pi}^*(h|\, h) < \tilde{\psi}_{po}(h). \tag{39}$$

Let ω^+ and ω^- be such that $\overline{R}_{1,po}(\omega^+) > 0$ and $\overline{R}_{1,po}(\omega^-) < 0$. Then,

$$\begin{aligned} E^{P_{po}}\left(\overline{R}_{3,po}\middle|\,\overline{R}_{1,po}\right)(\omega^+) &= E^{P^{\tilde{\pi}^*}}\left(\overline{R}_{po}(z_2,z_3)\middle|\, z_1 = h\right), \\ E^{P_{po}}\left(\overline{R}_{3,po}\middle|\,\overline{R}_{1,po}\right)(\omega^-) &= E^{P^{\tilde{\pi}^*}}\left(\overline{R}_{po}(z_2,z_3)\middle|\, z_1 = l\right). \end{aligned}$$

It follows from (39), $\tilde{\pi}^*(h|\, h) = \pi^*(2|\, 2) + \pi^*(4|\, 2)$ and $\tilde{\psi}_{po}(h) = \psi_{po}(2) + \psi_{po}(4)$ that

$$E^{P_{po}}\left(\overline{R}_{3,po}\middle|\,\overline{R}_{1,po}\right)(\omega^+) < 0 < E^{P_{po}}\left(\overline{R}_{3,po}\middle|\,\overline{R}_{1,po}\right)(\omega^-) \Leftrightarrow \pi^*(2|\, 2) + \pi^*(4|\, 2) < \psi_{po}(2) + \psi_{po}(4)$$

that is, $E^{P_{po}}\left(\overline{R}_{3,po}\middle|\,\overline{R}_{1,po}\right)$ reverts to the mean if and only if $\pi^*(2|\, 2) + \pi^*(4|\, 2) < \psi_{po}(2) + \psi_{po}(4)$. By Proposition 9, the asset displays long-term reversal if $\pi^*(2|\, 2) + \pi^*(4|\, 2) < \psi_{po}(2) + \psi_{po}(4)$. To show the converse, suppose that $\pi^*(2|\, 2) + \pi^*(4|\, 2) \geq \psi_{po}(2) + \psi_{po}(4)$. Then by the argument above, $E^{P_e}\left(R_{3,po}\middle|\,\overline{R}_{1,e}\right)$ trends and it follows by Proposition 9 that the 2nd-order autocorrelation is positive and so long-run reversal fails. □

Proof of Theorem 11(b). Consider any CESC of an arbitrary baseline growth economy. The price of an asset at state (ξ, α) must satisfy the Bellman equation:

$$p(\xi,\alpha) = \sum_{\xi'} Q(\xi,\alpha)(\xi')\left(p(\xi', \alpha'(\xi,\alpha)(\xi')) + d(\xi')\right) \qquad \psi_{cpo} - a.s.$$

It is easy to see that the invariant distribution places positive mass only on points (ξ, α) such that $\alpha \in \underline{\Delta} \cap \Delta\left(\xi, \mu^{\pi^*}\right)$ where $\underline{\Delta} = \{(\alpha_1, \alpha_2) \in \Delta : \exists \xi \in S \text{ such that } \alpha_1 = \underline{\alpha}_1(\xi) \text{ or } \alpha_2 = \underline{\alpha}_2(\xi)\}$. The hypothesis $\underline{\alpha}_1(1) = \underline{\alpha}_1(2)$ and symmetry implies that $\underline{\alpha}_2(3) = \underline{\alpha}_2(4)$. If p_ξ, $q_{\xi\xi'}$ and d_ξ denotes $p(\xi, \underline{\alpha}(\xi))$, $Q(\xi, \underline{\alpha}(\xi))(\xi')$ and $d(\xi)$, respectively, then the Bellman equation becomes

$$p_\xi = \sum_{\xi'} q_{\xi\xi'}\left(p_{\xi'} + d_{\xi'}\right) \quad \text{for all } \xi$$

which can be written as $(I - Q)P = QD$ where Q is the 4×4 matrix with entries $q_{\xi\xi'}$, P is the 4×1 vector with entries p_ξ and D is the 4×1 vector with entries p_ξ. Note that

$$c_1(1, \underline{\alpha}(1)) = c_2(3, \underline{\alpha}(3)) \quad \text{and} \quad c_1(2, \underline{\alpha}(2)) = c_2(4, \underline{\alpha}(4))$$

and so

$$\begin{aligned} q_{\xi 1} &= \beta(\xi,\mu)\,\pi(1\,|\xi)\,\tfrac{\partial u(c_1(1,\underline{\alpha}(1))/\partial c_1}{\partial u(c_1(\xi,\underline{\alpha}(\xi)))/\partial c_1} = \beta(\xi,\mu)\,\pi(3\,|\xi)\,\tfrac{\partial u(c_2(3,\underline{\alpha}(3))/\partial c_1}{\partial u(c_2(\xi,\underline{\alpha}(\xi)))/\partial c_1} = q_{\xi 3}, \\ q_{\xi 2} &= \beta(\xi,\mu)\,\pi(2\,|\xi)\,\tfrac{\partial u(c_1(2,\underline{\alpha}(2))/\partial c_1}{\partial u(c_1(\xi,\underline{\alpha}(\xi)))/\partial c_1} = \beta(\xi,\mu)\,\pi(4\,|\xi)\,\tfrac{\partial u(c_2(4,\underline{\alpha}(4))/\partial c_1}{\partial u(c_2(\xi,\underline{\alpha}(\xi)))/\partial c_1} = q_{\xi 4}. \end{aligned}$$

It follows that Q has rank 2. Therefore, $p_1 = p_3$ and $p_2 = p_4$.

Let $\tilde{\pi}^*$ and $(Z^\infty, \mathcal{Z}^\infty)$ be the transition matrix and the measurable space, respectively, introduced in the proof of Theorem 11(a). $P^{\tilde{\pi}^*}$ is the probability measure over $(Z^\infty, \mathcal{Z}^\infty)$ uniquely induced by $\tilde{\pi}^*$ and $\tilde{\psi}_{cpo}$. Let $z_t : Z^\infty \to Z$ be $\mathcal{Z}_t-$measurable. The collection $\{z_t\}_{t=0}^\infty$ on the probability space $\left(Z^\infty, \mathcal{Z}^\infty, P^{\tilde{\pi}^*}\right)$ is a two state time-homogeneous Markov process with transition function $\tilde{\pi}^*$ on $(Z, \mathcal{Z})$ and invariant distribution $\tilde{\psi}_{cpo} : \mathcal{Z} \times \mathcal{Z} \to [0,1]$.

Let $p(l) \equiv p_1$, $p(h) \equiv p_2$, $R_{cpo}(z, z') \equiv \frac{p_{z'} + d_{z'}}{p_z}$ for all $z \in \{l, h\}$ and $\overline{R}_{cpo} : \{l, h\} \times \{l, h\} \to \Re$ be such that

$$\overline{R}_{\tau,cpo}(\omega) = \begin{cases} \overline{R}_{cpo}(l,l) & \text{if } \xi_{\tau-1}(\omega) \in \{1,3\} \text{ and } \xi_\tau(\omega) \in \{1,3\} \\ \overline{R}_{cpo}(l,h) & \text{if } \xi_{\tau-1}(\omega) \in \{1,3\} \text{ and } \xi_\tau(\omega) \in \{2,4\} \\ \overline{R}_{cpo}(h,l) & \text{if } \xi_{\tau-1}(\omega) \in \{2,4\} \text{ and } \xi_\tau(\omega) \in \{1,3\} \\ \overline{R}_{cpo}(h,h) & \text{if } \xi_{\tau-1}(\omega) \in \{2,4\} \text{ and } \xi_\tau(\omega) \in \{2,4\} \end{cases} \tag{40}$$

Moreover,

$$\overline{R}_{cpo}(z,l) < 0 < \overline{R}_{cpo}(z,h) \text{ for all } z \in \{l,h\} \tag{41}$$

and

$$E^{P^{\tilde{\pi}^*}}\left(\overline{R}_{cpo}(z_1, z_2)\right) = 0. \tag{42}$$

It follows from (40) that for any $k \in \{2, 3\}$

$$E^{P_{cpo}}\left(\overline{R}_{1,cpo}\ \overline{R}_{k,cpo}\right) = E^{P^{\tilde{\pi}^*}}\left(\overline{R}_{cpo}(z_0, z_1)\ \overline{R}_{cpo}(z_1, z_k)\right.$$

Note that (42) and (41) are conditions (a) and (b) in Lemma 7.1. Since the asset displays short-term momentum,

$$E^{P^{\tilde{\pi}^*}}\left(\overline{R}_{cpo}(z_0, z_1)\ \overline{R}_{cpo}(z_1, z_2)\right) = E^{P_{cpo}}\left(\overline{R}_{1,po}\ \overline{R}_{2,po}\right) > 0,$$

and so (c) in Lemma 7.1 also holds. The rest of the proof is identical to that in Theorem 11(a). $\square$