A Simple Method to Distinguish Between Liquidity Constraints and Partial Risk-sharing

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Abstract

Liquidity constraints and partial risk-sharing are often indistinguishable. However, I illustrate using artificial data from two different model economies how band spectrum regression can be used to distinguish between lack of consumption insurance and liquidity constraints.

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1   Introduction

Traditional tests of full consumption insurance (see Mace, 1991) do not distinguish between the predictions of the full risk-sharing model and the ones of the Life-cycle/PIH model. In this paper I illustrate using simulated longitudinal data from two different model economies how band spectrum regression can be used to distinguish between lack of consumption insurance (consumption smoothing across states of nature) and liquidity constraints (consumption smoothing across time).

Band spectral regression allows models to be evaluated over particular frequencies, such as business cycles or long horizons (see Appendix). To illustrate the purpose of using frequency domain methods, I simulate artificial longitudinal data from two model economies, one which allows for full risk-sharing but which includes liquidity constraints and one where liquidity constraints are absent but full risk-sharing is not possible. Standard time domain tests of full insurance are unable to tell apart the liquidity constraints model and the partial risk-sharing model but band spectrum regression is.

The remainder of the paper is organized as follows. In section 2 I describe the three model economies considered. Section 3 describes the parameter values used in the simulations. In section 4 I describe the results and section 5 concludes.

2   The Model Economies

I am interested in the partial equilibrium dynamics of consumption, saving and wealth given a constant interest rate and wage rate. There are no aggregate shocks and I assume a stationary competitive equilibrium.

2.1   Demographic Structure

I consider an overlapping-generations (OLG) framework. Consumers live T periods and are part of the working age population for $Q < T$ periods. The population size is constant, equal to N. Each period new agents are born to replace the old ones. The population structure is thus stable in the sense that the relative size of each

cohort does not change (Rios-Rull, 1996). Consumer maximize life-time utility

$$U_t = E_t \left\{ \sum_{t=0}^{T} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \right\} \tag{2.1}$$

where c is individual consumption and β is a discount factor. Consumers have no bequest motive. Agents (indexed by i) are born with zero non-human wealth and are endowed with ℓ_t^i units of time which they supply inelastically in the labor market until retirement age $Q + 1$.

The labor endowment shock ℓ_t^i is stochastic, $i.i.d$, and takes value 0 with probability p or 1 with probability $(1-p)$. Each agent aged s is endowed with $\exp(\epsilon_{t+s}^i)h_s$ units of human capital, where h_s is the cohort specific average human capital stock and ϵ^i is an individual specific shock to the human capital stock. I allow for persistent idiosyncratic shocks by specifying an $AR(1)$ process for ϵ^i

$$\epsilon_{t+1}^i = -\frac{\sigma^2}{2}\left(\frac{1-\rho}{1-\rho^2}\right) + \rho\,\epsilon_t^i + \nu_{t+1}^i \quad , \quad \nu_{t+1}^i \sim \mathcal{N}\left(0, \sigma^2\right) \tag{2.2}$$

where $\rho \in (0,1)$. The size of each cohort co_s aged s, $n_s = \frac{N}{T}$, is large enough for the idiosyncratic shocks to be washed out when aggregation is made within a cohort

$$\sum_{i \in co_s} \frac{h_s^i}{n_s} = \sum_{i \in c_s} \frac{\exp(\epsilon_{t+s}^i)h_s}{n_s} = h_s \tag{2.3}$$

The average labor income of consumers aged s is $y_s = (1-p)\,wh_s$. The average human capital stock of working age consumers aged $s \leq Q$ grows at rate g_s, implying that each cohort faces the labor income profile

$$y_s = (1 + g_s)\,y_{s-1} \tag{2.4}$$

2.2 Financial Markets

I consider three financial market structures. Each financial structure gives rise to a different model economy:

ME1 Full consumption insurance and no liquidity constraints

ME1 is the benchmark complete markets economy. There is full consumption insurance and there are no liquidity constraints. The representative consumer, endowed with perfect foresight, born in period t solves the problem

$$\max_{c} \sum_{j=0}^{T-1} \beta^{t+j} \frac{c_{t+j}^{1-\gamma}}{1-\gamma} \tag{2.5}$$

subject to

$$a_{s+1} = (1+r)(a_s + y_s - c_s)$$
$$a_{t+T} \geq 0$$

$$y_{s+1} = \begin{cases} (1+g_{s+1})y_s & if \ \ s < t+Q-1 \\ 0 & \text{elsewhere} \end{cases}$$

ME2 Full consumption insurance + liquidity constraints

As in ME1 there is full consumption insurance and therefore it is possible to model each generation of consumers with a representative agent. However each cohort's representative consumer is liquidity constrained. The representative consumer born in period t solves the problem

$$\max_{c} \sum_{j=0}^{T-1} \beta^{t+j} \frac{c_{t+j}^{1-\gamma}}{1-\gamma} \tag{2.6}$$

subject to

$$a_{s+1} = (1+r)(a_s + y_s - c_s)$$
$$a_s \geq 0 \ \forall \ s$$

$$y_{s+1} = \begin{cases} (1+g_{s+1})y_s & if \ \ s < t+Q-1 \\ 0 & \text{elsewhere} \end{cases}$$

ME3 Uninsurable idiosyncratic risk but no liquidity constraints

In ME3 risk-sharing within cohort is absent. However, there are no liquidity constraints. However, it turns out that the possibility of labor income being zero for the remainder of the agents working life is sufficient to prevent consumers ever to borrow. The problem solved by each consumer (indexed by i) born in period t is

$$\max_c E_t \left\{ \sum_{j=0}^{T-1} \beta^{t+j} \frac{c_{t+j}^{i}{}^{1-\gamma}}{1-\gamma} \right\} \tag{2.7}$$

subject to

$$a_{s+1}^i = (1+r)\left(a_s^i + wh_s^i \ell_s^i - c_s^i\right)$$
$$a_{t+T} \geq 0$$

$$h_{s+1}^i = \begin{cases} \exp(\epsilon_{s+1}^i)\left(1+g_{s+1}\right)h_s & if \;\; s < t+Q-1 \\ 0 & \text{elsewhere} \end{cases}$$
$$\ell_s^i = \begin{cases} 0 & \text{with prob } p \\ 1 & \text{with prob } 1-p \end{cases}$$

3 Calibration and Model Solution

For calibration, one period corresponds to one calendar year. Agents are born aged 21, retire at age 65 and live until age 80. The parameter values used for the numerical simulations are derived from Gourinchas and Parker (2002), Heaton and Lucas (1996)[1] and from individual labor income data obtained from the Panel Survey of Income Dynamics (PSID).

From the PSID, I use the labor income of head of households aged 21 to 65 to estimate agents' labor income profile over the life-cycle. Figure 1 shows the labor

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[1]Gourinchas and Parker (2002) carry out a structural estimation of a life cycle model of consumption incorporating precautionary saving, using household level data from the consumer expenditure survey (CEX). Heaton and Lucas (1996) use PSID data to calibrate a heterogeneous agent asset pricing model.
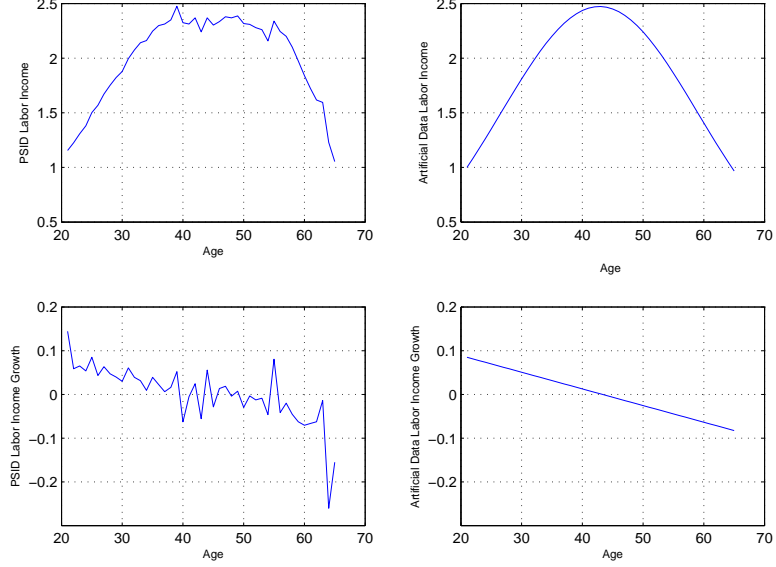
Figure 1: Labor income profile of Consumers in the PSID (left-hand panels) and labor income profile used in the simulation (right-hand panels). Labor income at age 21 is normalized to one.

income profile for the sample of consumers in the PSID and the estimated labor income profile used to generate the artificial data.

Replacing in (2.4) the expected income of consumers aged s with the corresponding sample average $\bar{y}_s$ and taking log differences yields an estimate of the expected growth rate of income

$$\hat{g}_s = \exp\left[\Delta \ln\left(\bar{y}_s\right)\right] - 1 \tag{3.1}$$

The consumption of individuals aged $s \leq Q$ consistent with the Euler equation and the budget constraint is

$$c_s = \kappa_s\left(a_s + h_s\right) \tag{3.2}$$

$$\kappa_s = \frac{1 - \beta^{1/\gamma}\left(1+r\right)^{1/\gamma-1}}{1 - \left[\beta^{1/\gamma}\left(1+r\right)^{1/\gamma-1}\right]^{T-s+1}}$$

Table 1: Parameter Values

Parameter	Value
Discount factor, β	0.9569
Risk aversion coefficient, γ	1.3969
Real interest rate, r	0.0344

Labor income stochastic process

$$\epsilon_{t+1}^i = -\frac{\sigma^2}{2}\left(\frac{1-\rho}{1-\rho^2}\right) + \rho\,\epsilon_t^i + \nu_{t+1}^i \quad,\quad \nu_{t+1}^i \sim \mathcal{N}\left(0,\sigma^2\right)$$

Autocorrelation coefficient, ρ	0.7
Standard deviation of innovations, σ	0.251
Probability of zero labor endowment, p	0.003

Note: Except for the income stochastic process, the parameter values are derived from the estimates reported in Gourinchas and Parker (2002). The parameters ρ and σ are derived from the estimates reported in Heaton and Lucas (1996).

and h_s is the present value of current and future labor income:

$$\sum_{t=s}^{Q} (1+r)^{t-Q}\, y_t$$

The Euler equation, determining the rate of change of consumption, yields

$$\ln c_{s+1} - \ln c_s = \frac{1}{\gamma}\ln(1+r) + \frac{1}{\gamma}\ln\beta \tag{3.3}$$

Full insurance implies consumption does not vary across individuals in response to idiosyncratic shocks and absence of liquidity constraints implies that consumption does not vary over time in response to life-cycle movements.

The method to obtain the policy functions solving the consumer's problem corresponding to ME2 and ME3 is available in the appendix.
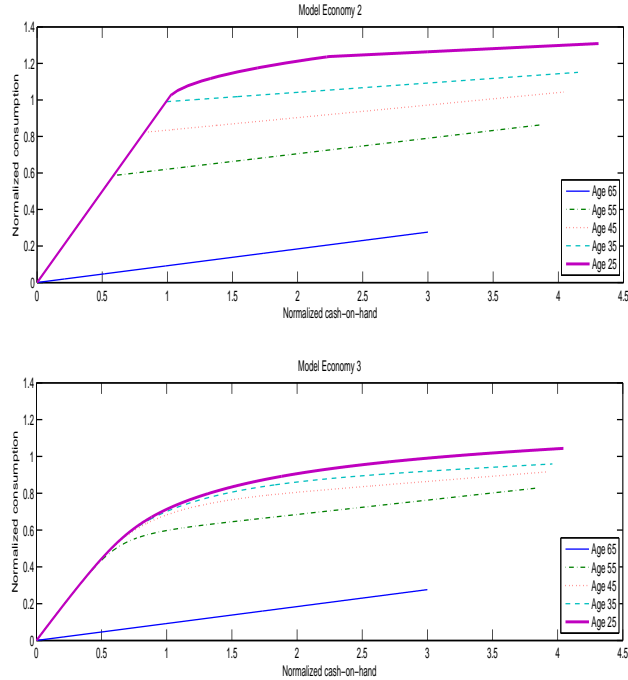
Figure 2: Consumption Function.

Figure 2 shows in the upper panel the consumption functions of the representative consumer in ME2 and in the lower panel the corresponding ME3 consumption functions. Consumers in both economies behave in a similar way. In ME2, the representative consumer is liquidity constrained and is therefore unable to smooth consumption across time. In ME3, the consumer is free to borrow but optimally chooses not to because of the possibility of receiving zero labor income. This is illustrated in figure 3.

4 Results

In a seminal contribution, Mace (1991), suggests estimating the following model to test full consumption insurance[2]

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[2]Mace (1991) includes an additional regressor, aggregate consumption, to control for aggregate shocks. Because I have assumed away aggregate uncertainty, I do not include aggregate consumption in the model specification.

Table 2: Time Domain Regressions

Variable	One Year Time Differences	Five Years Time Differences	Ten Years Time Differences
		Model Economy 2	
const	-0.0015	-0.0104**	-0.0223**
	(-0.74)	(-3.85)	(-4.71)
$\Delta \ln Y$	0.1606**	0.7676**	0.8581**
	(6.87)	(57.77)	(68.34)
		Model Economy 3	
const	0.0114**	0.0567**	0.1165**
	(7.77)	(22.10)	(20.11)
$\Delta \ln Y$	0.0958**	0.1834**	0.3523**
	(26.93)	(40.77)	(39.79)
Significance levels :	† : 10%	∗ : 5%	∗∗ : 1%

Note: The dependent variable is the growth rate of individual consumption, $\Delta \log C$.

In parenthesis are t-stats.

$$\Delta \ln C_t^i = \alpha + \beta \Delta \ln Y_t^i + e_t^i \tag{4.1}$$

where Δ is the difference operator. The prediction of the full risk-sharing model is $\beta = 0$.

In what follows, I estimate model (4.1) on simulated series. To account for measurement error, a pervasive feature of the type of data used in empirical tests of full insurance, I contaminate the artificial data and assume the observed series are

Table 3: Band Spectrum Regressions

Variable	Very Short Run 2 years - 2⅖ years	Business Cycle 3 years - 6 years	Long Run 12 years - ∞
		Model Economy 2	
$\Delta \ln Y$	0.0171	0.0256	0.7732**
	(0.33)	(0.71)	(33.87)
		Model Economy 3	
$\Delta \ln Y$	0.0788**	0.0942**	0.1646**
	(10.41)	(19.58)	(23.97)

| Significance levels : | † : 10% | ∗ : 5% | ∗∗ : 1% |

Note: The dependent variable is the growth rate of individual consumption, $\Delta \log C$.

In parenthesis are t-stats.

$$C_t^i = c_t^i + e_{ct}^i \tag{4.2}$$
$$Y_t^i = y_t^i + e_{yt}^i \tag{4.3}$$

where c_t^i and y_t^i are the true series and e_c and e_y are measurement errors. Consistent with estimates reported by Heathcote, Storesletten and Violante (2007), I choose σ_c and σ_y, the variance of the measurement error in consumption and income respectively, such that measurement error accounts for 5% and 30% of the cross-sectional variances for these variables. To generate the artificial samples I sample a panel of observations on individual consumption and income including N=900 consumers aged 21 to 65, and S=13 years. The panel dimensions where chosen to be equivalent to the ones available to econometricians testing the full consumption insurance hypothesis.

Table 2 describes the results from standard OLS regressions, for three different specifications: one year first differences; five year first differences; and ten year first differences. The results confirm that ME2 and ME3 are difficult to tell apart. The null hypothesis that $\beta = 0$, corresponding to full consumption insurance, is easily rejected for both model economies, no matter the specification used.
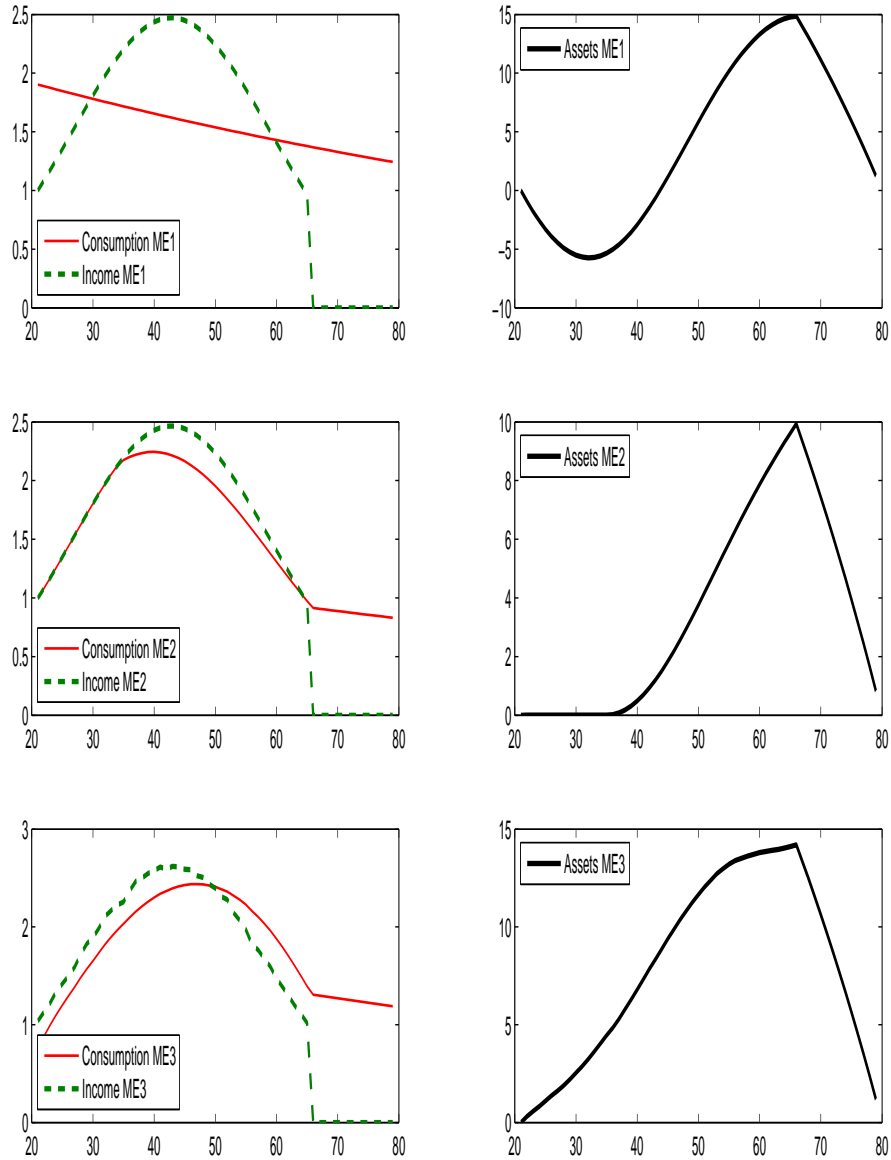
Figure 3: Life Cycle Profiles. The upper panels correspond to ME1, the middle panels to ME2 and the bottom panels to ME3

Result 1 *An econometrician who has access to data generated by ME2 and tests (in the time domain) full consumption insurance assuming absence of liquidity constraints will wrongly reject full insurance.*

Table 3 describes the findings from band spectrum regression. The central result is that when the simulated data is generated by ME2, I am unable to reject the full insurance hypothesis either at very high frequencies or at business cycle frequencies for any conventional level of significance. However, at low frequencies, capturing the life-cycle features of the data, the full insurance hypothesis is overwhelmingly rejected. This is because liquidity constraints generate a parallel between income and consumption at low frequencies. However, turning to the simulated data generated by ME3, the full insurance hypothesis is soundly rejected at all frequencies, including business cycle frequencies.

Result 2 *Using band spectrum regression at business cycle frequencies, full insurance is correctly rejected for ME3. It is not rejected (again correctly) for ME2.*

Interestingly, the results obtained using the data generated by ME3, are consistent with previous full consumption insurance tests using true household level data.

5   Conclusion

As pointed out by Carroll (2001) the effects of precautionary saving and of liquidity constraints are virtually indistinguishable. However, I have shown using simulated data that band spectrum regression is a useful method to distinguish between partial risk-sharing and liquidity constraints: liquidity constraints, which prevent consumption smoothing across time, translate into a low frequency parallel between consumption and income growth; partial risk-sharing prevents consumption smoothing across states of nature and translates into a high frequency parallel between consumption and income growth. Band spectrum regression allows to uncover features of the data at different frequencies and thus to tell apart the two models.

6   Appendix

6.1   Solving For the Optimal Consumer Rule

Defining the value function for the consumer problem in model economy 3 at time τ by V_τ, the problem of the consumer can be written as

$$V_\tau \left(x_\tau \right) = \max_{c_\tau, \dots, c_T} E_\tau \left\{ \sum_{t=\tau}^{T} \beta^{t-\tau} \frac{c_{t+j}^{1-\gamma}}{1-\gamma} \right\} \qquad (6.1)$$

where $x_{t+1} = \left(1 + r \right) \left(x_t - c_t \right) + y_{t+1}$. The Bellman equation for this problem is

$$V_\tau \left(x_\tau \right) = \max_{c_\tau} \left\{ \frac{c_\tau^{1-\gamma}}{1-\gamma} + \beta E_\tau V_{\tau+1} \left(x_\tau \right) \right\} \qquad (6.2)$$

The Euler equation can be written in terms of normalized variables $\hat{x} = \frac{x}{p}$, yielding

$$\hat{c}_\tau (\hat{x}_\tau)^{-\gamma} = \beta \left(1 + r \right) E_\tau \hat{c}_{\tau+1} (\hat{x}_{\tau+1})^{-\gamma} \qquad (6.3)$$

To solve numerically for the household policy function the following procedure is used:

- discretization of the income stochastic process.

- the solution to the problem of the consumer aged Q is known. This solution is used to start the algorithm which solves through backward recursion the consumer problem.

- solve the problem of the household for the previous periods using the method of endogenous grid points (Carroll [2006]).

For model economy 2, the same procedure is used however the endogenous grid point method is adapted to incorporate the presence of liquidity constraints.

6.2   Band Spectrum Regression

Let the data set contain T observations on each household. The complex finite Fourier transform is based on the $T \times T$ matrix W, in which each element (k, s) is

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given by

$$w_{k,s} = \frac{1}{\sqrt{T}} e^{i\,s\,\theta_k} \qquad s = 0, 1, ..., T-1 \tag{6.4}$$

where $\theta_k = \frac{2\pi k}{T}$, $k = 0, 1, ..., T-1$, and $i = \sqrt{-1}$. Pre-multiplying the vector of observations in the regression equation (4.1) by W, produces a finite fourier transform of the time domain vectors, which yields the model

$$\tilde{y}_t^j = \alpha' \tilde{z}_t^j + \delta' \tilde{v}_t^j + \tilde{\epsilon}_t^j \tag{6.5}$$

where $\tilde{y}_t^j = W y_t^j$, $\tilde{z}_t^j = W z_t^j$, $\tilde{v}_t^j = W v_t^j$ and $\tilde{\epsilon}_t^j = W \epsilon_t^j$.

Model (6.5) is a standard linear regression model made of T independent observations on $\tilde{y}$ conditioned on $\tilde{x}$, each of which corresponds to a different frequency. If the disturbance vector in (4.1) are spherical and zero mean, that is $E[\epsilon] = 0$ and $E[\epsilon\epsilon'] = \sigma^2 I_T$, then the transformed disturbance vector, $\tilde{\epsilon}$, will have identical properties.

When the relationship implied by (4.1) is only assumed to hold for certain frequencies, band spectrum regression allows to test a restricted version of the model in which some frequencies are ignored.

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