Towards a quantitative theory of automatic stabilizers: the role of demographics[*]

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Abstract

Employment volatility is larger for young workers than for prime aged. At the same time, in economies with high tax rates the share of total market hours supplied by the young workers is smaller. These two observations imply a negative correlation between government size (measured by the share of taxes in total output) and aggregate hours volatility. This paper assesses in a calibrated model the quantitative importance of these empirical facts to account for the relationship between government size and macroeconomic stability. The model accounts for at least 23% and as much as 55% of the relationship between output volatility and government size.

JEL classification codes: E32; E62; H30; J10; J21.

Keywords: Automatic Stabilizers; Distortionary Taxes; Demographics.

1 Introduction

The motivation for this paper consists of two simple observations. The first motivating observation is that there is substantial evidence that countries or regions with large governments (measured by the share of taxes in total output) display business cycle fluctuations that are less volatile, as shown in Galí (1994), Rodrick (1998) and Fatás and Mihov (2001). The second motivating observation, which is documented by Clark and Summers (1981), Ríos-Rull (1996), and Gomme et al. (2004) is that fluctuations in hours of market work over the business cycle vary quite dramatically across different demographic groups of the population. In particular, the young experience much greater volatility of employment and hours worked than the prime aged over the business cycle. Moreover, in a recent paper Jaimovich and Siu (2009) find that changes in the age composition of the labor force account for a significant fraction of the variation in business cycle volatility observed in the U.S. and other G7 economies. Hence, in this article we pose the following question: can the relationship between government size and macroeconomic stability be explained by changes in the demographic composition of the workforce resulting from distortionary taxation?

The hypothesis we put forward is that large governments stabilize output fluctuations because the share of total market hours supplied by the young workers is smaller in economies with high tax rates. In turn, these differences in the demographic composition of the workforce reduce the *aggregate* labor supply elasticity. Thus, in the tax-distorted real business cycle model we analyze, a relationship emerges between the size of the government (measured by the share of taxes in total output) and the volatility of the cyclical component of aggregate output, consistent with the notion of automatic stabilizers.[1]

The suggestion that time devoted to market work is affected by changes in tax and transfer policies is one which has received considerable attention. For instance, recent work by Prescott (2004), Rogerson (2006, 2008), Krusell et al. (2008, 2010), Ohanian et al. (2008) and Berger and Heylen (2011) argue that differences in tax and transfer policies can account for a large share of the difference in the amount of hours spent working in Europe and in the U.S. Moreover, Rogerson and Wallenius (2009) document that the differences in employment rates

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[1]So-called 'built-in stabilizers' are features of the tax structure that make tax liabilities respond automatically to current economic conditions (for instance, distortionary labor and capital income taxes) and reduce aggregate volatility. The stabilizing effect of the income tax is traditionally thought to operate via an assumed sensitivity of consumption demand to changes in current tax liabilities. But, according to the Ricardian proposition, this sensitivity is zero. Thus, Christiano (1984) concludes that under a strict version of the Ricardian proposition, the income tax cannot play a role as an automatic stabilizer. Nonetheless, distortionary taxes may affect macroeconomic stability by affecting the aggregate supply and, in particular, the aggregate labor supply elasticity.

between Europe and the U.S. are due almost exclusively to differences in the employment rates for young and old workers. Thus, these authors argue that differences in market hours which result from variation in tax and transfer policies are dominated by differences among young and old individuals. This observation offers further motivation for the work we develop in this paper.

Our paper aims at providing a quantitative evaluation of the strength of the automatic stabilizers in an equilibrium business cycle model, based on the relationship between the aggregate labor supply elasticity and the tax system.[2] We incorporate labor force heterogeneity within the real business cycle framework, along the lines of Kydland (1984) and Jaimovich et al. (2011). The stand-in household is composed of different types of individuals, which we interpret as different demographic groups. Heterogeneity arises from differences in labor supply elasticities across demographic groups. These differences are calibrated to match the differences in the volatility of market work across age groups that have been documented in previous literature. We represent preferences using the Greenwood, Hercowitz and Huffman (GHH) utility function which eliminates wealth effects in the labor supply choices.[3] Although the use of GHH preferences has the drawback of being inconsistent with a balanced growth path, it offers two important advantages: first, it has the attractive implication that changes in the equilibrium levels of employment resulting from distortionary taxes are robust to changes in the assumptions concerning the use of the tax revenue and the nature of transfer programs;[4] second, the use of these preferences together with the calibration attributing a large labor supply elasticity to young workers relative to prime aged, implies that employment differences resulting from distortionary taxes are largely due to differences in hours worked by the young, consistent with empirical evidence.[5]

An important aspect that differentiates this paper from the literature examining the relationship between government size and aggregate volatility is that we study if the model

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[2]To be sure, our paper is not suitable to study the welfare impact of automatic stabilizers, which in certain contexts relates to income stabilization (Blanchard, 1984). Taxation distorts the consumption-savings decisions and the labor supply choices. Optimal taxation must balance distortions versus insurance. However, in our framework there is already perfect insurance and, hence, no gain from automatic stabilizers. For the government to have a potential insurance role, agents must be unable to enter private insurance contracts, either by assuming incomplete risk sharing because of private information and moral hazard considerations, or by assuming an overlapping generation structure in which insurance contracts are infeasible.

[3]See Greenwood et al. (1988).

[4]See Ljungqvist and Sargent (2006) for a discussion of the implications of changing the explicit details of tax and transfer programs in the context of the balanced growth path representative agent model.

[5]Also, since the focus of our paper concerns business cycle fluctuations, excluding intertemporal substitution in labor supply is consistent with the findings of Jaimovich and Rebelo (2009) that over the business cycle wealth effects are weak.

is *quantitatively* consistent with the observed strength of the automatic stabilizers. Earlier contributions mostly focus on the sign of the relationship between government size and macroeconomic stability.[6] To do so, we first calibrate the model to the U.S. economy by matching cross-sectional information on the wage profile and on the relative level and volatility of market hours across age groups. We then follow standard practice in development accounting. We feed the theoretical economy with different fiscal policy parameters that mimic the fiscal profile of OECD countries. This allows us to generate a sample of simulated OECD economies. These economies differ from the benchmark calibrated economy only in their fiscal policy parameters. The quantitative assessment of the model requires comparing the responsiveness of aggregate volatility to changes in government size implied by the model and observed in the data. The model-implied government size emerges as an endogenous outcome resulting from mimicking the fiscal profile of the OECD countries in our sample.[7]

Our model accounts for at least 23% and as much as 55% of the relationship between output volatility and government size, and between 22% and 61% of the relationship between hours volatility and government size. Two mechanisms explain the model's success. First, distortionary taxation changes the composition of the workforce so that higher tax rates are associated with higher aggregate labor supply elasticity. Second, in the baseline model government spending is a function of the previous period level of public debt. This feature of the model implies that the Ricardian equivalence is not satisfied and budget deficits affect the competitive equilibrium allocation. We calibrate the joint dynamics of government debt and spending to those of the U.S. economy. Under the baseline calibration government spending is countercyclical implying a negative relationship between the share of government spending in total output and aggregate volatility.[8] About two-thirds of the strength of the automatic stabilizers is explained by the changes in the aggregate labor supply elasticity implied by the worforce demographic composition.

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[6]Galí (1994) examines whether income taxes and government purchases behave as automatic stabilizers in the basic, technology shock-driven, real business cycle (RBC) model. He finds that the relationship between government size and macroeconomic stability implied by the standard model is qualitatively counterfactual. The model in Greenwood and Huffman (1991) also generates a positive correlation between aggregate volatility and taxes. Guo and Harrison (2006) discuss the robustness of the results in Galí (1994). Andrés et al. (2008) extend the analysis in Galí (1994) and study how alternative models of the business cycle can replicate the relationship between government size and macroeconomic stability. Their analysis shows that adding nominal rigidities and costs of capital adjustment to the standard model can generate a negative correlation between government size and the volatility of output.

[7]A fiscal profile is a set of taxes (labor income tax, capital income tax and consumption tax) and the share of government spending in GDP.

[8]There is a large body of literature describing the countercyclicality/acyclicality of fiscal policy in developed countries, as documented by e.g. Kaminsky et al. (2005).

The remainder of the paper is organized as follows. In Section 2 we provide empirical evidence about the relationship between the workforce demographic composition, government size and macroeconomic stability. We introduce the model in Section 3. In Section 4 we establish three results concerning the relationship between government size and the composition of the workforce implied by the model. In Section 5 we describe our calibration procedure and in Section 6 we examine the quantitative implications of the baseline economy. In Section 7 we study the relationship between government size and macroeconomic stability implied by the model and compare it to the data. In Section 8 we consider two additional quantitative experiments and Section 9 offers concluding remarks.

2   Motivating evidence

The hypothesis put forward in this paper is that large governments stabilize output fluctuations because they encourage the demographic groups exhibiting high labor supply volatility to work relatively fewer hours. In this section we document some empirical evidence that motivates this mechanism. We first show that the differences in cyclical volatility of employment across demographic groups are a general feature of the OECD countries: In all the countries, the cyclical volatility of employment exhibits a u-shape profile over the life cycle, with young and older workers exhibiting the highest cyclical volatility. Second, we show that the employment share of the young in total employment is lower in countries with large governments. Finally, we show that accounting for the demographic composition of the labor force is empirically relevant to explain the differences in hours and output volatility.

We begin by documenting a well established relationship between employment volatility and age: The employment volatility of young and old workers is larger than the employment volatility of prime-age workers. Jaimovich and Siu (2009) show that in all G7 countries young workers experience much greater volatility of employment and hours worked than the prime-aged over the business cycle; those closer to retirement experience volatility somewhere in between. We show that this empirical relationship is true in a large cross-section of OECD countries.[9] To illustrate this fact, we follow the approach of Gomme et al. (2004), and Jaimovich and Siu (2009), who report cyclical employment volatilities for various age groups.

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[9] Several studies have illustrated that the labor market behavior of the young and the old differs from the behavior of prime-aged workers. For instance, Pencavel (1986), Killingworth and Heckman (1986) and Blundell and MaCurdy (1999) provide microeconometric evidence that the elasticity of labor supply is larger for the younger and the older workers. Blanchard and Diamond (1990) and Janiak and Wasmer (2008) show that employment impulse responses for young and old workers are larger in magnitude than middle-age workers.

Figure 1: Volatility of Employment by Demographic Group, OECD
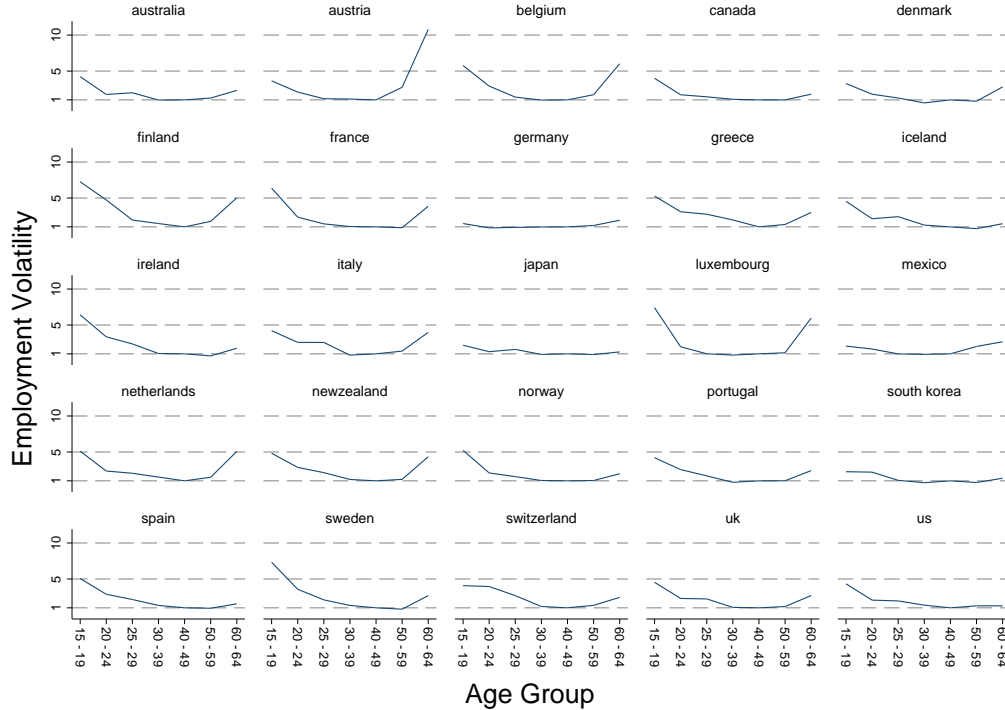
Notes: The data is annual and the source is the OECD Labour Force Statistics. All variables are reported in logs as deviations from an HP trend with smoothing parameter 6.25. The volatility is expressed relative to the 40–49 age group.

We use annual data on employment by age group from the OECD for an unbalanced panel of 25 countries from 1970 to 2009.[10]

We build seven categories: Individuals aged between 15 and 19 years old, $20-24$, $25-29$, $30-39$, $40-49$, $50-59$ and $60-64$ years old. For each of these categories, we extract the business-cycle component of employment by applying the Hodrick-Prescott (HP) filter to the logged series with smoothing parameter equal to 6.25 as suggested by Ravn and Uhlig (2002), and we calculate the standard deviation. We report the relative volatility, given by the standard deviation of each age group relative to the standard deviation of the group aged between 40 and 49. Figure 1 displays the results for a large cross-section of OECD countries.[11]

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[10]See Appendix A for details about the data used.

[11]Not reported here, we also used data at the US state level (for both employment and hours volatility), which we constructed from the Current Population Survey. Results are qualitatively similar. Quantitatively, the volatility ratio of the $15-19$ years old is lower with an average equal to 2. The $60-64$ age group displays

The figure shows an ubiquitous u-shaped relationship between age and employment volatility at business cycle frequencies. In all the countries the volatility of employment is the highest either for the workers aged 15 to 19 or for the workers aged 60 to 64. The employment volatility of the youngest workers is on average nearly five times that of the workers aged 40 to 49. The workers aged 60 to 64 also display large employment volatility, on average more than three times that of the workers aged 40 to 49. Finally, in all the countries the prime-age workers (aged 40 to 49) have the most stable labor supply. Table 9 in Appendix B shows that the differences in employment volatility over the life-cycle are statistically significant.

The second fact we document concerns the relationship between the demographic composition of the workforce and government size (measured by the ratio between total tax revenue and Gross Domestic Product). In particular, we are interested in the correlation between government size and the share of young in total employment (defined as the ratio between the employment of individuals aged 15 to 29 and the employment of individuals aged 15 to 64). The Panel (a) of Figure 2 shows the relationship between the share of young in total employment and government size. Each observation in the sample corresponds to an OECD country over one of the following time intervals: 1970 – 1979, 1980 – 1989, 1990 – 1999 and 2000 – 2009. The scatter plot shows a strong negative correlation between the share of young in employment and government size. The first column of Table 9 in Appendix B shows that the relationship is statistically significant.[12] We argue that, as a result of the negative correlation between the share of young in employment and government size, total employment should be less volatile in countries with large governments.

The hypothesis we put forward is supported by the correlations observed in the data. Panel (b) of Figure 2 shows that the correlation between the share of young in total employment and hours volatility is positive. The second column of Table 9 in Appendix B shows that the relationship is statistically significant.[13] This positive correlation follows from the life-cycle profile of employment volatility documented earlier. Finally, as Panel (c) of Figure 2 illustrates, the volatility of hours is positively associated with the volatility of

similar volatility. For the US state level data, the identity of the group displaying the lowest volatility is more heterogeneous. The lowest volatility age group is either the 30 – 39, the 40 – 49 or the 50 – 59 group.

[12]Results are similar if instead of the share of young workers we consider the *share of young and old in the labor force*, defined as the share of individuals aged 15 – 29 and 60 – 64 in total employment. We focus on the share of young workers because the differences in cyclical volatility of hours between prime-age and old workers are less substantial compared to the differences in the cyclical volatility of hours between young and prime-age workers. For instance, Table 3 (that guides the baseline calibration of the theoretical economy in Section 5) shows that if for the U.S. we consider hours instead of employment the high cyclical variation in the hours of young workers is the empirically relevant phenomenon.

[13]*Verbatim.*

Figure 2: Government Size and Aggregate Volatility (OECD countries)
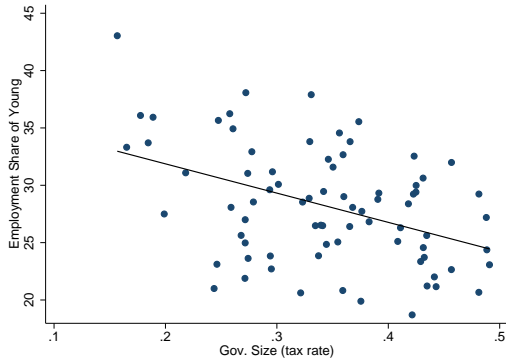
(a) Gov. Size and Share of Young
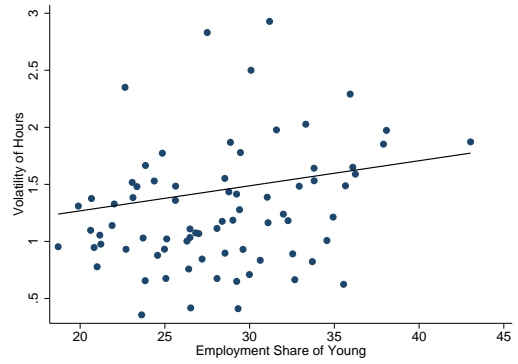
(b) Share of Young and Hours Volatility
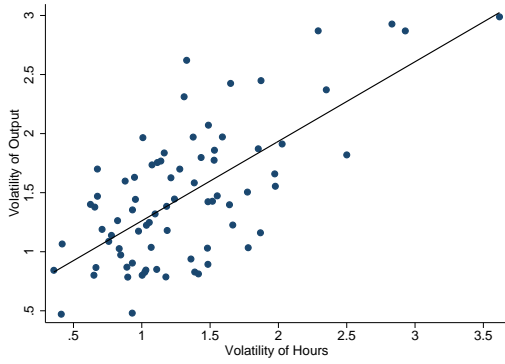
(c) Hours Volatility and Output Volatility
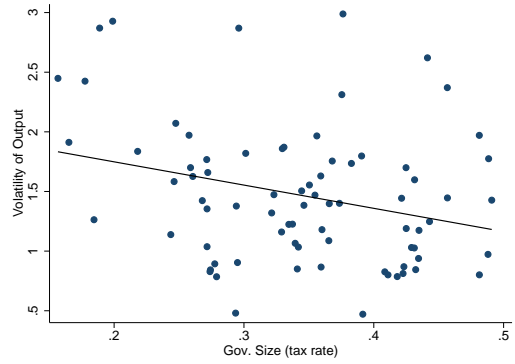
(d) Gov. Size and Output Volatility

Note: Annual data on Tax to GDP ratios and GDP are from the OECD outlook database, while data on hours worked are from the Conference Board Total Economy database. The sample includes the following countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Portugal, South Korea, Spain, Sweden, Switzerland, the United Kingdom and the United States. The Employment Share of Young corresponds to the ratio between the employment of the population aged 15 to 29 and the employment of the population aged 15 to 64. The Output and Hours Volatility corresponds to the standard deviation of the cyclical components, given by the log deviations from the HP trends with smoothing parameter 6.25. Each observation corresponds to a country and one of the following time periods: 1970 − 1979, 1980 − 1989, 1990 − 1999, and 2000 − 2009.

aggregate output. The upshot is that countries with large governments are associated with more stable output fluctuations, as illustrated in Panel (d) of Figure 2.

Finally, Table 12 in the Appendix reports results on the relationship between hours and output volatility, government size and the demographic structure of the workforce. The columns (1) and (3), show the relationship between hours volatility and government size, and between output volatility and government size, respectively. Both columns illustrate the stabilization role of the government sector: Large governments (measured by total tax revenue as a fraction of GDP) are associated with lower volatility of aggregate hours, and with lower volatility of output. In turn, columns (2) and (4) concern directly the key argument advanced in this paper. It expands the list of regressors included in the regressions (1) and (3) with the share of young workers in employment as a control variable. In both regressions, once the demographic control variable is included, the magnitude of the coefficient associated with government size is smaller and it is no longer statistically significant. In the regression concerning the volatility of hours the slope coefficient falls by 54%, while in the regression concerning output volatility the slope coefficient falls by 36%. This findings support the hypothesis that the stabilization role of the government is in part explained by the changes in the demographic structure of the workforce associated with changes in tax rates.[14]

In summary, in this section we have documented the three following facts: i) the employment of young and older individuals fluctuates much more over the business cycle than that of prime-age individuals; ii) across OECD countries, the share of young workers in the labor force declines as the size of the government increases; iii) there is a negative relationship between the size of the government and the cyclical volatility of aggregate hours and output, but controlling for the demographic structure of the population attenuates substantially this relationship. In what follows we propose a theoretical model based on these three facts. Our objective is to investigate if a real business cycle model that accomodates workforce heterogeneity is consistent with the stabilizing role of the government sector in a way that is qualitatively and quantitatively consistent with the data.

3 The model

In this section, we present a model that features age-specific differences in the cyclical volatility of hours worked. The framework is otherwise that of the standard RBC model featuring capital adjustment costs and variable capital utilization, with competitive labor and capital markets. The economy is inhabited by a large number (unit measure) of identical and in-

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[14] *Verbatim.*

finitely lived families. Each family is composed of a unit mass of individuals of different ages with each individual living a maximum of Q periods. Ages are denoted by $i \in \mathcal{I} \equiv \{1, \ldots, Q\}$. Within each family the mass of individuals aged i is a_i, with $\sum_{i=1}^{Q} a_i = 1$. All individuals are endowed with one unit of time each period. An age i individual's unit of time can be transformed into e_i efficient units of labor.

3.1 Stand-in family

Within the representative family, individuals' period utility function is age dependent and we assume that it has the form introduced by Greenwood et al. (1988):

$$
v(c, n; i) = \begin{cases} \left(\frac{c - \lambda_i n^{1+\theta_i}}{1-\sigma}\right)^{1-\sigma} & \text{if } \sigma \neq 1 \\[2ex] \ln\left(c - \lambda_i n^{1+\theta_i}\right) & \text{if } \sigma = 1, \end{cases} \tag{1}
$$

where $\sigma > 0$, and c and n are, respectively, consumption and time spent working. The parameter θ_i is the inverse of the Frisch elasticity of labor supply and is age dependent. Notice that the choice of utility function excludes intertemporal substitution in labor supply choices. Rather than being a drawback, this implication of the utility function has the advantage of emphasizing the importance of age-specific labor supply elasticities and is instrumental in the calibration exercise. The stand-in family seeks to maximize

$$
E_0 \left[\sum_{t=0}^{\infty} \beta^t \sum_{i=1}^{Q} a_i v\left(c_{it}, n_{it}; i\right) \right], \tag{2}
$$

subject to the feasibility constraints $c_{it} \geq 0$ and $0 \leq n_{it} \leq 1$ for all i and each period, and the sequence of budget constraints

$$
I_t + d_t B_{t+1} + \sum_{i=1}^{Q} (1 + \tau_c) a_i c_{it} = B_t + (1 - \tau_k) r_t u_t K_t + (1 - \tau_h) w_{it} a_i h_{it} + L_t, \tag{3}
$$

where c_{it} and $h_{it} \equiv e_i n_{it}$ are consumption and effective units of work supplied by family members aged i, and u_t is the capital utilization rate in period t; K_t is the capital stock owned by the stand-in family and I_t is investment in period t.

Each period t the stand-in family purchases bonds B_{t+1} at discount price d_t. The real wage rate and the rate of return on capital services are w_t and r_t. Labor income, capital

income and consumption are taxed at rates τ_h, τ_k and τ_c, respectively. The earnings of the stand-in family include after-tax capital income $(1 - \tau_k) r_t u_t K_t$, the payment from its holdings of government bonds B_t, and lump-sum transfers L_t. Finally, each family member aged i earns after-tax labor income $(1 - \tau_h) w_{it} n_{it}$. We assume that increases in the utilization rate of capital are costly because higher utilization rates imply faster depreciation rates; the depreciation function is $\delta(u_t) = \bar{\delta} u_t^{1+\varsigma}$, with $\bar{\delta}$ and ς strictly positive. The capital accumulation equation is given by

$$K_{t+1} - K_t = \Phi \left(\frac{I_t}{K_t} \right) K_t - \delta(u_t) K_t, \tag{4}$$

where, as in King and Watson (1996) and Basu and Kimball (1997), the capital adjustment cost function $\Phi(\,.\,)$ is increasing, concave, and satisfies $\Phi\left(\bar{\delta}\right) = \bar{\delta}$ and $\Phi'\left(\bar{\delta}\right) = 1$.

The optimal allocation within the extended family requires that the marginal utility of consumption of each family member be the same irrespectively of their age, implying the following condition[15]

$$\frac{\partial v\left(c_{it}, n_{it}; i\right)}{\partial c_{it}} = v_t', \qquad \text{for all } i. \tag{5}$$

The optimality condition for bond holdings is given by

$$q_t = E_t \left[\beta \frac{v_{t+1}'}{v_t'} \right], \tag{6}$$

and the optimality condition for investment is

$$q_t = E_t \left\{ \beta \frac{v_{t+1}'}{v_t'} \left[(1 - \tau_k) r_{t+1} u_{t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1} \left(1 - \delta(u_{t+1}) + \Phi_{t+1}\right) \right] \right\}, \tag{7}$$

where $\Phi_t \equiv \Phi\left(I_t/K_t\right)$ and $q_t \equiv \Phi'\left(I_t/K_t\right)^{-1}$ corresponds to the shadow value of capital (Tobin's q). Finally, the labor supply and the capital utilization choices must satisfy the following

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[15]Equation (5) has two implications concerning the relationship between labor supply and consumption *within* and *between* demographic groups. First, the form of the period utility function implies substitutability between leisure and consumption *within* each demographic group, entailing that an increase in the labor supply of a group raises its consumption too. Second, there is complementarity between leisure and consumption *between* each demographic group, implying that an increase in the labor supply for a particular group (holding consumption constant for that group) reduces the consumption of the other groups. In Section 6 we refer to these two effects to explain the evolution of consumption volatility over the life-cycle.

11

list of intratemporal optimality conditions

$$-\frac{\partial v\left(c_{it}, n_{it}; i\right)}{\partial n_{it}} = \left(\frac{1-\tau_h}{1+\tau_c}\right) w_t e_i v_t', \qquad \text{for all } i \tag{8}$$

$$\left(1-\tau_k\right) r_t = \delta'(u_t) q_t. \tag{9}$$

3.2 Firms

We consider a one-sector model economy where the single good produced serves two purposes: consumption and investment. Output is produced by a representative firm that combines capital and labor services via a constant returns to scale Cobb-Douglas production function

$$Y_t = e^{z_t} \left(u_t K_t\right)^\alpha H_t^{1-\alpha}, \tag{10}$$

where capital services are the product of the stock of capital and the rate of capital utilization, and $H_t \equiv \sum_{i=1}^{Q} e_i N_{it}$ are the efficiency units of labor services in period t (where $N_{it} \equiv a_i n_{it}$ is the aggregate labor effort by individuals aged i in period t). Fluctuations are driven by random transitory movements in total factor productivity

$$z_t = \rho z_{t-1} + \sigma_z \epsilon_t^z, \tag{11}$$

where ϵ_t is identically and independently standard normal distributed and $\rho \in (0,1)$. The first-order conditions for the firm's profit maximization yield the following functions for the wage rate and rental rate of capital

$$w_t = (1-\alpha) e^{z_t} \left(u_t K_t\right)^\alpha H_t^{-\alpha}, \tag{12}$$

$$r_t = \alpha e^{z_t} \left(u_t K_t\right)^{\alpha-1} H_t^{1-\alpha}. \tag{13}$$

3.3 Government

The government taxes capital income, labor income and consumption expenditure, at the rates τ_k, τ_h and τ_c, respectively. From the expenditure side, the government spends G_t as government consumption, provides lump-sum transfers denoted L_t and services its debt

obligations. Hence, the government budget constraint reads

$$d_t B_{t+1} = G_t + L_t + B_t - \tau_k r_t K_t - \tau_h w_t H_t - \tau_c \sum_{i=1}^{Q} a_i c_{it}. \tag{14}$$

There is a simple feedback rule relating lump-sum transfers to the level of debt, while government spending in log-deviation from steady state $\widetilde{G}_t$ is the sum of two components, a stochastic disturbance and a predetermined component. The dynamics of L_t and G_t are described by the following two equations

$$l_t = -\varphi_L b_t \tag{15}$$

and

$$\widetilde{G}_t = \rho_G \widetilde{G}_{t-1} - \varphi_G b_{t-1} + \sigma_g \epsilon_t^g, \tag{16}$$

where $l_t \equiv \left(L_t - \bar{L}\right) \bar{Y}^{-1}$ and $b_t \equiv \left(B_t - \bar{B}\right) \bar{Y}^{-1}$ are, respectively, lump-sum transfers and debt in deviation from steady-state as percentage of the steady state output, and ϵ_t^g is a serially independent standard-normal disturbance, mutually independent. The parameters φ_L, ρ_G and φ_G are positive constants, consistent with the transversality condition of the government sector, namely

$$E_t \left[\lim_{T \to \infty} \left(\Pi_{i=t}^T d_i \right) b_{T+1} \right] = 0. \tag{17}$$

The purpose of the fiscal rules introduced in equations (15) and (16) is quantitative.[16] They allow to replicate the volatility of government spending as well as its countercyclicality.[17] Moreover, they are empirically motivated by the fact that in many OECD countries successful fiscal consolidation is ensured through expenditure adjustments.[18]

In our baseline calibration, we choose values for the parameters φ_L, ρ_G, φ_G and σ_g based on estimates from a vector autoregression in reduced form that determine the joint dynamics of government spending, public debt and lump-sum transfers. A feature of the baseline calibration is that debt and government spending shocks affect the equilibrium allocations. For this reason, we also consider an alternative calibration that sets φ_G and σ_g equal to zero,

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[16]The introduction of these fiscal rules also allow us in Section 8 to compare the quantitative importance of the demographic channel as opposed to countercyclical government spending to generate a negative correlation between government size and aggregate volatility.

[17]There is large body of literature that describes the countercyclicality/acyclicality of fiscal policy in developed countries. See e.g. Kaminsky et al. (2005).

[18]See e.g. Alesina and Perotti (1995), McDermott and Wescott (1996), Andrés and Doménech (2006).

implying that

$$G_t = \bar{G}, \qquad \text{for all } t. \tag{18}$$

Under this alternative specification, the debt consolidation is made only through lump-sum transfers and the Ricardian equivalence holds. When the fiscal regime is described by equations (15) and (16) we call the regime *non-Ricardian*, and we call the regime *Ricardian* when equations (15) and (18) apply instead. The latter framework is appropriate if we want to study the impact that changes in the tax-rate parameters have on macroeconomic stability without examining the potential automatic-stabilizing role of budget deficits.

3.4 Market clearing

Finally, turning to the market clearing conditions, equilibrium in the labor market and in the good's market requires

$$H_t = \sum_{i=1}^{Q} a_i e_i n_{it}, \tag{19}$$

$$Y_t = C_t + I_t + G_t, \tag{20}$$

where $C_t = \sum_{i=1}^{Q} a_i c_{it}$ is aggregate consumption.

3.5 Equilibrium

A *competitive equilibrium* is an allocation $\left\{ (c_t^i)_{i=1}^{Q}, (n_t^i)_{i=1}^{Q}, B_{t+1}, u_t, K_{t+1}, G_t \right\}_{t=0}^{\infty}$, and a sequence of prices $\{w_t, r_t, d_t\}_{t=0}^{\infty}$, such that for given initial conditions $K_0 > 0$, B_0 and stochastic processes for technology and government spending shocks: given prices, the allocation solves both the representative household's problem, equations (3)—(9), and the representative firm's problem, equations (12) and (13), for all t; the equations (14)—(17), describing the government sector, are satisfied for all t; and the market clearing equations (19) and (20) are satisfied for all t.

Following standard steps, the firm's and the stand-in family's optimality conditions, and the market clearing conditions are log-linearized and combined so as to characterize the equilibrium dynamics. We represent a variable X in log-deviation from steady state by $\widetilde{X}$, and we denote the steady state of X by $\bar{X}$ and the expectation at t of $\widetilde{X}_{t+1}$ by $\widetilde{X}_{t+1}^t$. By making use of the various market clearing conditions and of condition (5) that relates the between age groups relative consumption levels and labor supplies, the log-linear model can

be reduced to a system of difference equations that includes only the aggregate variables. The system of equations describing the model aggregate dynamics is given by

$$\mathbf{A}\mathbf{Z}_{t+1}^t = \mathbf{B}\mathbf{Z}_t + \mathcal{E}_t \tag{21}$$

where the vector of aggregate variables is $\mathbf{Z}_t = \left(\widetilde{C}_t, \widetilde{H}_t, \widetilde{u}_t, \widetilde{K}_t, b_t, \widetilde{G}_{t-1}, z_{t-1} \right)'$ and $\mathcal{E}_t$ is a vector of stochastic disturbances. The detailed derivation of the system is collected in Appendix C.4.

4 Government size and aggregate labor supply elasticity

In this section, we examine three important aspects of the model. First, we illustrate the differences in the cyclical volatility of hours across the different demographic groups in the model. Second, we focus on the steady-state of the economy and ask how the share of hours worked by each demographic group varies as the size of the government is changed. Third, we show that the aggregate labor supply elasticity is increasing in either τ_h, τ_k or τ_c, justifying the stabilizing role of distortionary taxation. Moreover, the sensitivity of the aggregate labor supply elasticity to tax rates is larger, the larger the cross-sectional dispersion of Frisch elasticities.

We start by considering the cyclical properties of hours worked by the different demographic groups. The result that follows compares hours volatility across each demographic group.

Lemma 1. *Denote by σ_i the standard deviation of the logarithm of hours worked by individuals aged i and σ_w the standard deviation of the logarithm of the wage rate. It follows that*

$$\sigma_i = \eta_i \sigma_w, \tag{22}$$

where $\eta_i \equiv 1/\theta_i$ is the Frisch labor supply elasticity.

Lemma 1 follows immediately from equation (8). It implies that demographic groups with large labor supply elasticity display more volatile labor effort over the business cycle. This simple result is the main element of the mechanism explaining the relation between the government size and macroeconomic stability in the model that we study. If the share of hours worked by the high volatility group decreases, the volatility of aggregate hours worked also decreases because of the change in the composition of the labor force. As larger tax rates raise the share of hours worked by the more stable demographic groups, the cyclical volatility of aggregate hours worked decreases.

The next result concerns the relationship between the workforce composition and government size. To show how the share of hours worked by each demographic group varies as the size of the government is changed, we characterize the steady state of the model. Making use of equations (6) and (7) we obtain the steady state rental cost of capital, given by

$$\bar{r} = \frac{1/\beta + (1 - \bar{\delta})}{1 - \tau_k}.$$ (23)

Also, in steady state the rental cost of capital is equal to $\alpha(\bar{Y}/\bar{K})$. The upshot is that the capital-output ratio in steady state is given by

$$\frac{\bar{K}}{\bar{Y}} = \frac{(1 - \tau_k)\,\alpha}{1/\beta + (1 - \bar{\delta})}.$$ (24)

By combining the above equation with conditions (8), (12) and (13), the amount of time spent working in steady state by individuals aged i is found to satisfy

$$\bar{n}_i = \left[\frac{(1 - \tau_h)(1 - \alpha)\,e_i}{(1 + \tau_c)\,\lambda_i}\right]^{\eta_i} \left[\frac{(1 - \tau_k)\,\alpha}{1/\beta + (1 - \bar{\delta})}\right]^{\eta_i \alpha/(1-\alpha)},$$ (25)

where $\eta_i \equiv 1/\theta_i$ is the Frisch labor supply elasticity for individuals aged i. Notice that, because of the form chosen for the utility function, each family member's labor effort is determined independently of the intertemporal consumption/saving choice. Thus, as the size of the government increases, the time spent working by individuals with high labor supply elasticity (high η_i) falls relatively to the time spent working by individuals with low labor supply elasticity (low η_i). These relative changes alter the workforce composition toward individuals with less elastic labor supplies. When analyzing how changes in the size of the government, as controlled by τ_h, τ_k and τ_c, affect labor supply volatility, our framework stresses changes in the workforce composition brought about by differences in the elasticity of labor supply across individuals in different stages of their life-cycle.

Lemma 2. *Consider the steady-state equilibrium of alternative economies that have different fiscal policy profiles as captured by differences in the tax rates τ_j, with $j \in \{h, c, k\}$. The elasticity of labor effort to changes in each tax rate, for individuals aged i, is*

$$\frac{d\bar{n}_i}{d\tau_j}\frac{\tau_j}{\bar{n}_i} = -\beth_j \eta_i,$$ (26)

where

$$
\daleth_j \equiv \begin{cases} \frac{\tau_h}{1-\tau_h} & \text{if } j = h, \\ \frac{\tau_c}{1+\tau_c} & \text{if } j = c \text{ and} \\ \frac{\alpha}{1-\alpha}\frac{\tau_k}{1-\tau_k} & \text{if } j = k. \end{cases}
$$

The proof of Lemma 2 follows immediately from the inspection of equation (25). The upshot is that increases in any of the three tax rates lead to changes in the composition of the aggregate labor supply toward the less volatile individuals and, from Lemma 1, a decrease in the aggregate labor supply volatility.

The third result we obtain concerns the relationship between the aggregate labor supply elasticity and taxes. In Appendix C we show that around the steady state equilibrium the aggregate labor supply elasticity is given by the following expression[19]

$$
\frac{d \ln N_t}{d \ln w_t} \equiv \mathcal{E}_n = \sum_{i=1}^{Q} \bar{s}_{hi} \eta_i, \tag{27}
$$

where $\bar{s}_{hi} \equiv a_i e_i \bar{n}_i / \bar{H}$ is the share of efficient units of labor supplied by individuals aged i in steady state, and $N_t = \sum_{i=1}^{Q} N_{it}$ is the aggregate labor supply. The share supplied by the stable demographic groups increases, as taxes increases lowering the aggregate labor supply elasticity. Moreover, this effect is stronger, the larger the cross-sectional dispersion of Frisch elasticities. Thus, we establish the following proposition:

Proposition 1. *The aggregate labor supply elasticity $\mathcal{E}_n$ is decreasing in each tax rate τ_j, with $j \in \{h, c, k\}$. Moreover,*

$$
\frac{d \mathcal{E}_n}{d \tau_j} = -\frac{\daleth_j}{\tau_j} \sigma_\eta, \qquad \forall j \in \{h, c, k\}, \tag{28}
$$

where

$$
\sigma_\eta \equiv \sum_{i=1}^{Q} \bar{s}_{hi} \eta_i^2 - \mathcal{E}_n^2 \geq 0
$$

is the cross-sectional variance of Frisch elasticities. Thus, the sensitivity of the aggregate labor supply elasticity to changes in taxes is increasing in the dispersion of the individual elasticities η_i across demographic groups.

---

[19]See equation (C.41).

The proof of Proposition 1 is in Appendix D. In what follows, we examine the quantitative properties of the model and, in particular, we investigate whether the model is capable of replicating the stabilizing role of the government that features in the empirical data.

5 Calibration

We set a period length to be one year to match the frequency of the OECD data on hours fluctuations. We calibrate the model to the U.S. economy for the period $1970 - 2009$, by making use of three types of data: i) data on the fiscal structure of the economy, ii) cross-sectional information on the wage profile and on the relative level and volatility of market hours across age groups, and iii) aggregate annual time-series.

5.1 Government sector

We choose the tax rates on capital income, labor income and consumption based on evidence documented in Carey and Rabesona (2002), who have produced series for the average effective tax rates on capital income, labor income and consumption for the OECD countries based on the methodology proposed by Mendoza et al. (1994). In Section 7 we make use of these cross-country data for examining the relation between government size and aggregate volatility across OECD economies. For the purpose of the calibration, we use the tax rates which are reported by these authors for the U.S. economy. The values chosen for each tax rate are $\tau_k = 0.371$, $\tau_c = 0.053$ and $\tau_h = 0.256$, as reported in Table 1.

We set values for the parameters φ_L, ρ_G, φ_G and σ_g based on the estimates of a vector autoregression (VAR) in reduced form that models the joint dynamics of government spending, public debt and lump-sum transfers. To measure G_t we use data on real government consumption expenditures and gross investment from the Bureau of Economic Analysis, and to measure public debt in percentage of steady state output we use the ratio between gross federal debt held by the public from the Council of Economic Advisors and the Congressional Budget Office's estimate of potential output. The system of equations to be estimated is

$$\begin{pmatrix} \log G_t \\ b_t \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{pmatrix} \log G_{t-1} \\ b_{t-1} \end{pmatrix} + \Gamma_t + \begin{pmatrix} e_t^g \\ e_t^b \end{pmatrix}, \qquad (29)$$

where the same notation for G_t and b_t are used to refer to their empirical counterparts, the matrix A is the AR(1) coefficients of the VAR, Γ_t represents a linear time trend and e_t^g and e_t^b are residuals.
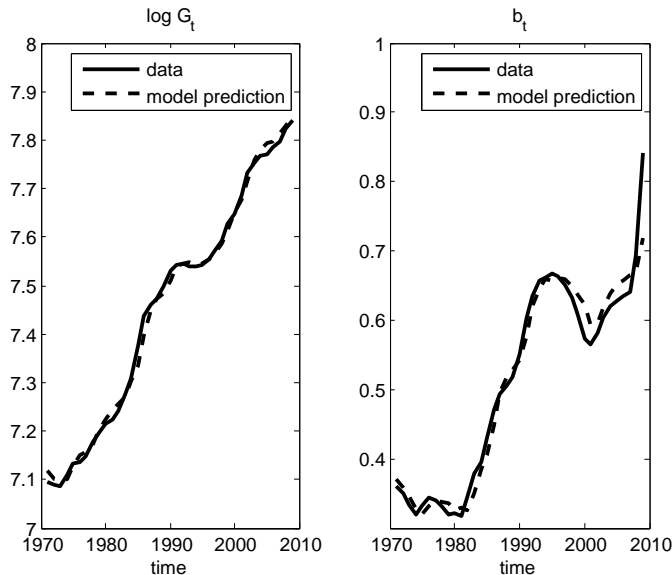
Table 1: Baseline Calibration: Summary

Parameter		Target/Source
β	0.974	Investment/GDP ratio of 14%
σ	2	Greenwood et al. (1988)
α	0.283	Capital income share
$\bar{\delta}$	0.010	10% capital depreciation
ξ	2.5	Estimation by Basu and Kimball (1997)
ς	0.271	Investment/GDP ratio of 14%
ρ	0.812	Solow residuals autocorrelation
σ_z	0.015	Innovation to Solow residuals, standard deviation
$\bar{g}_y$	0.22	Government spending as a fraction of GDP of 22%
ρ_G	0.913	VAR estimation
σ_g	0.015	VAR estimation, standard deviation of residuals
φ_G	0.110	VAR estimation
φ_L	0.180	VAR estimation
τ_h	0.256	Tax rate on labor income, Carey and Rabesona (2002)
τ_k	0.371	Tax rate on capital income, Carey and Rabesona (2002)
τ_c	0.053	Tax rate on consumption, Carey and Rabesona (2002)

η_{15-19}	η_{20-24}	η_{25-29}	η_{30-39}	η_{40-49}	η_{50-59}	η_{60-64}
5.144	2.529	1.753	1.348	1.000	0.938	0.942

λ_{15-19}	λ_{20-24}	λ_{25-29}	λ_{30-39}	λ_{40-49}	λ_{50-59}	λ_{60-64}
0.939	1.465	2.123	3.127	4.543	5.016	6.789

Note: Target/Sourse indicates either the target informing the choice or parameter value or the source that providing the chosen parameter value. The targets guiding the choice of values for the η_i and the λ_i are the, respectively, the relative volatility of hours worked by each age group and the hours worked by each age group as a share of total hours worked in the U.S., as reported in Table 3.

Figure 3: VAR Estimation of the G and b Processes

Table 2 reports the estimation results. We find that both the government spending and the public debt are persistent processes, with partial autocorrelation coefficients equal to 0.91 and 0.84 respectively. The results indicate that increases in past debt tend to reduce current spending (the estimate of A_{12} is -0.110). This finding is consistent with the prevalence of expenditure based fiscal consolidation. Increases in past spending raise current debt (the estimate for A_{21} is 0.278), implying deficit-financed expenditure. The R^2's in Table 2, together with the displayed prediction of the estimated VAR in Figure 3, indicate a good fit of the estimation. This is mostly a consequence of the large persistence of the process and of the presence of a deterministic time trend.

In the Appendix C, we show that in the theoretical economy and under the non-Ricardian regime (our baseline calibration), the joint dynamics of government spending and the debt in percentage of steady-state output are described by the following system of equations

$$\begin{pmatrix} \tilde{G}_t \\ b_t \end{pmatrix} = \begin{bmatrix} \rho_G & -\varphi_G \\ \beta^{-1}\bar{g}_y & \beta^{-1}(1-\varphi_L) \end{bmatrix} \begin{pmatrix} \tilde{G}_{t-1} \\ b_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_t^g \\ \epsilon_t^b \end{pmatrix}, \tag{30}$$

where $\epsilon_t^b \equiv -\beta^{-1}\left[\tau_y \widetilde{Y}_t + \bar{c}_y \tau_c \widetilde{C}_t\right]$ and $\bar{g}_y$ and $\bar{c}_y$ are the steady-state shares of government spending and aggregate consumption in output. Hence, we use the estimates of A_{11}, A_{12} and

20

Table 2: VAR Estimation for G and b

	$\log G_{t-1}$	b_{t-1}	R^2	Linear Trend
$\log G_t$	0.9129***	−0.1101***	0.999	Yes
	(0.0709)	(0.0443)		
b_t	0.2777**	0.8423***	0.979	Yes
	(0.0981)	(0.0613)		

Standard errors in parentheses.

** significant at 5%, *** significant at 1%.

A_{22} to deduce the implied values of ρ_G, φ_G and φ_L, while the estimate of A_{21} can be used as a non-restricted moment to evaluate the model. This exercise produces values for $\rho_G = 0.913$, $\varphi_G = 0.110$ and $\varphi_L = 0.180$. We obtain a value for σ_g by calculating the standard deviation of the estimated residuals e_t^g. This gives $\sigma_g = 0.015$.

Using data from the Bureau of Economic Analysis, the steady-state ratio of government consumption to output $\bar{g}_y$ is calculated to be 22%, which corresponds to the average share of government spending in output over the period 1970 – 2009. Given the calibrated value for the discount factor (see below), we can compare the estimation of the coefficient A_{21} with the calibrated value for $\beta^{-1}\bar{g}_y$. The latter is equal to 0.226, while the estimate of A_{21} is 0.278, with the difference not statistically significant. Finally, the value of $\bar{g}_y$ implies a steady-state value for L which is 10.2% of output.

5.2   Demographic structure

We now describe the aspects of the calibration which have to do with the demographic structure of the workforce. This is an important part of the calibration because it determines the relation between the demographic composition of the workforce and aggregate volatility. We assume that the stand-in family is composed of seven distinct demographic groups, whose members have ages comprised between 15 and 64. The partition into the seven demographic groups is as illustrated in Table 3. The targets which are used for the purpose of calibration are the share of total hours worked by each age group and the relative volatility of hours worked by each age group. We take as the reference age group, the group composed of individuals aged between 40 and 49.

From Lemma 1 it follows that the standard deviation of the logarithm of hours worked by individuals in the age group i relative to the volatility of the logarithm of hours worked

Table 3: Distribution of Hours and Relative Volatilities by Age

	$15-19$	$20-24$	$25-29$	$30-39$	$40-49$	$50-59$	$60-64$
Share of hours	0.032	0.103	0.129	0.254	0.233	0.172	0.048
Relative volatility	5.144	2.529	1.753	1.348	1.000	0.938	0.942

Note: The relative volatilities represent the relative standard deviation of the logarithm of hours worked and are computed based on HP filtered data as reported in Jaimovich and Siu (2009). The distribution of hours by age is obtained from the same source.

by individuals aged between 40 and 49, is given by

$$\frac{\sigma_i}{\sigma_{40-49}} = \frac{\eta_i}{\eta_{40-49}}. \tag{31}$$

Therefore, given a value for the Frisch labor supply elasticity of the reference group, η_{40-49}, the Frisch elasticities of the other age groups are chosen so that for each age group i, the ratio η_i/η_{40-49} equals the relative volatility of that group as shown in Table 3. We are left with only the reference age group labor supply elasticity undetermined. There is a large literature that has estimated the Frisch elasticities for prime aged workers (e.g., see Blundell and MaCurdy, 1999). For instance, for adult males, MaCurdy (1981) obtained estimates of about 0.3. From Heckman and MaCurdy (1980) the corresponding value for females is about 2.2. We choose to set $\eta_{40-49} = 1$ which is in the middle range of the existing estimates.

Making use of equation (25), it follows that the hours worked in steady state by the individuals in the age group i are given by

$$\bar{N}_i = a_i \left[\frac{e_i \left(1 - \tau_h\right) \left(1 - \alpha\right)}{\left(1 + \tau_c\right) \lambda_i} \right]^{\eta_i} \left[\frac{\left(1 - \tau_k\right) \alpha}{\frac{1}{\beta} - 1 + \bar{\delta} \left(1 - \tau_k\right)} \right]^{\frac{\alpha}{1-\alpha} \eta_i}, \tag{32}$$

The shares of individuals in each age group i, a_i, are derived from the OECD population statistics. The efficient labor units for each age group, e_i, are set to match the life-cycle profile of hourly earnings implied by the Panel Survey of Income Dynamics (PSID), which collects household level earnings data from a representative sample of the U.S. population. The resulting profile of efficiency units by age group and the shares of individuals in each age group are both shown in Figure 4. The only remaining parameters from equation (32) are the λ_i, which control the disutility of work for individuals in each age group i. Given a value for the reference's age group disutility parameter, λ_{40-49}, the remaining λ_i's are chosen to match the relative shares of total hours worked by each age group shown in Table 3. Finally, λ_{40-49}

Figure 4: Efficiency Units and Population Shares
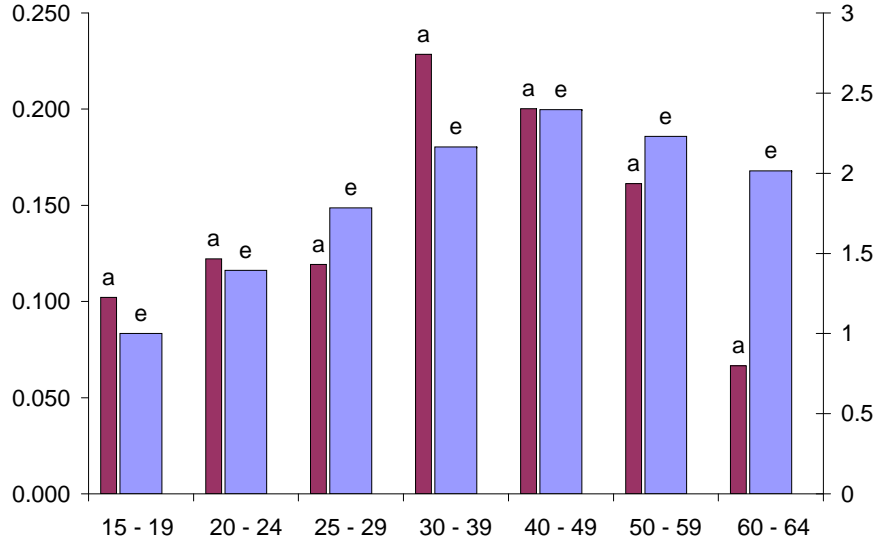
Note: The population shares of each age group i, a_i, are derived from the OECD population statistics. The efficient labor units for each age group, e_i, are set to match the life-cycle profile of hourly earnings implied by the Panel Survey of Income Dynamics (PSID). The left-hand vertical axis show the population shares and the right-hand vertical axis the efficiency units.

is chosen so that in steady state the stand-in family spends 25.5% of its endowment of time working, based on Gomme and Rupert (2007), who interpret evidence from the American Time-use Survey.

5.3 Technology and preferences

The calibration of the technology parameters requires setting values for the parameters of the capital adjustment costs function and the capital depreciation function (ξ, $\bar{\delta}$ and ς) and the stochastic process for the technology shock (ρ and σ_z). Our methodology here follows ideas developed in Basu et al. (2006), King and Rebelo (2000) and Basu and Kimball (1997). Moreover, two preference parameters remain to be fixed: The discount factor β and the inverse of the elasticity of intertemporal substitution σ.

Basu and Kimball (1997) estimate Solow residuals in a model characterized by variable capital utilization and convex adjustment costs for capital. They use annual data for a panel of U.S. firms from 21 manufacturing industries for the period 1949 – 1985. Our calibration of ξ considers their estimate of convex adjustment costs, and allows us to replicate the volatility of investment. The fixed value for ξ is 2.5. We set $\bar{\delta} = 0.1$, implying a steady-state annual

depreciation rate of 10%, which is consistent with evidence in Gomme and Rupert (2007). The capital income share α, is set equal to 0.283 based on the value implied by the National Income and Product Accounts (NIPA). The investment to output ratio is measured at 14% using the NIPA. In Appendix C we show that the steady-state investment-output ratio can be expressed as

$$\frac{\bar{X}}{\bar{Y}} = \frac{(1 - \tau_k)\alpha}{1 + \varsigma}. \tag{33}$$

Given the values of τ_k and α, we use this ratio as a target to fix a value for $\varsigma = 0.271$. This value falls in the range of estimates of Basu and Kimball (1997). It is also close to the value that Burnside and Eichenbaum (1996) estimate with aggregate data. Our parameter choices for $\bar{\delta}$ and ς imply a value for the discount factor β equal to 0.974. In addition we set the inverse elasticity of intertemporal substitution $\sigma = 2$ as in Greenwood et al. (1988).

We use data from the Multifactor Productivity tables of the Bureau of Labor Statistics (BLS) to determine the process for z_t. The BLS database provides information on capital services, labor input and real value added output. However, because the rate of capital utilization is not observable, Solow residuals cannot be directly calculated. For this reason, we use model-based proxies for utilization as in Basu et al. (2006) and King and Rebelo (2000). Specifically, we use the first-order conditions of the stand-in family and the representative firm to substitute out u_t in the production function and then calculate residuals. In the Appendix E, we show that this exercise allows to express z_t as follows

$$z_t + \Gamma_t^z \approx \left(1 - \frac{\alpha \bar{\delta} \xi}{\nu}\right) \log Y_t - \alpha \left(1 + \frac{1 - \bar{\delta} \xi}{\nu}\right) \log K_t + \frac{\alpha}{\nu} \log K_{t+1} - (1 - \alpha) \log N_t,$$

where Γ_t^z is a trend component and $\nu = (1 + \bar{\delta}\xi)(1 + \varsigma)$. We calculate the residuals from this equation and use them to estimate an AR(1) process with a linear trend and obtain values for $\sigma_z = 0.015$ and $\rho = 0.812$. The resulting correlation between the estimated ϵ_t^z and ϵ_t^g is -0.11, which is not significantly different from zero, justifying the assumption that the two shocks are uncorrelated.

6   Properties of the baseline economy

In this section we study the behavior of the model under the benchmark calibration, before analyzing how aggregate volatilities are affected by changes in the size of the government.

Table 4: U.S. Business Cycle Statistics, 1970 – 2009: Model vs. Data

Variable	Std. Dev.		Correlation		Output Share	
	Data	Model	Data	Model	Data	Model
Output	1.45	1.46	1.00	1.00	–	–
Consumption	1.20	1.18	0.90	0.98	0.67	0.64
Investment	5.13	5.17	0.94	0.98	0.14	0.14
Government spending	0.93	0.98	-0.13	-0.04	0.22	0.22
Hours	1.28	0.73	0.92	1.00	–	–

Note: Data on GDP, consumption, investment and government spending is from the NIPA tables. Inventories are excluded from the measure of investment. Data on hours worked is from the Conference Board Total Economy Database. Cyclical component is the log in deviations from an HP trend with smoothing parameter 6.25. The model's reported statistics are calculated from the simulation of the theoretical economy under the U.S. fiscal profile over 10,000 periods.

6.1 Aggregate business cycle statistics

Table 4 displays relevant aggregate statistics for the theoretical economy under the baseline calibration. The fiscal profile of the theoretical economy corresponds to that of the U.S., so that we may compare the U.S. business cycle statistics with those implied by the model. The table shows the properties of output, consumption, investment, government spending and hours worked in both the data and the model, as described by the volatility of their cyclical components and the correlation of the cyclical components with the cyclical component of output. The annual data on hours worked is from the Conference Board Total Economy Database for the sample period 1970 – 2007, while Output, Private Consumption and Private Investment (fixed capital formation) are taken from the NIPA Tables. The cyclical components are found by applying the HP filter to the logged series with smoothing parameter equal to 6.25, as recommended for annual data in Ravn and Uhlig (2002).

The baseline model matches the volatility of aggregate variables at least as well as the standard RBC model. The output volatility is the same as in the data. Volatilities of consumption and investment are also comparable to their empirical counterparts. The model suffers from the same drawback as the standard RBC model: The volatility of hours is about half that of output but in the data the relative volatility of aggregate hours is close to 90%.[20]

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[20]See Hansen (1985), Rogerson (1988) and King and Rebelo (2000).

Interestingly, the model is able to generate a volatility of government spending that is in line with U.S. business cycle statistics. Moreover, the weak countercyclicality of government expenditures observed in the data is reproduced under the baseline calibration. This success of the model is a result of the fiscal rules introduced in (15) and (16). In particular, the fiscal consolidation feedback parameter φ_G generates the negative correlation between the cyclical components of output and government spending.[21]

Hours and output in the theoretical economy are perfectly correlated because of the choice of utility function that excludes intertemporal substitution in the individual's labor supply decisions. The high correlations between output and the private components of aggregate expenditure are the result of the RBC structure of the model.[22] Finally, the shares of consumption, investment and government spending in output are the same as the ones found in the data because of the restrictions imposed by our calibration strategy.

6.2 Life-cycle implications

We now examine the evolution of consumption and hours over the life-cycle in the benchmark economy. The purpose of the calibration exercise is to exactly replicate the way labor supply volatility varies across age groups as well as the level of hours worked by each demographic group. This allows us to study whether this dimension of heterogeneity in the labor force contributes to explain the relation between government size and macroeconomic stability.

Table 5 reports the levels and the volatilities of consumption and hours for each age group relative to the reference group aged between 40 and 49. The model reproduces well the life-cycle profile of the first and second moments of hours worked. The level of hours worked follows a hump-shaped pattern, and the volatility of hours worked by the young is substantially larger than that of the prime-aged.[23] The ability of the model to reproduce these features is not a surprise since our calibration strategy sets these moments as targets for the determination of the λ_i's and the η_i's. For instance, a comparison of Tables 3 and 5 reveals that the life-cycle profile of the relative volatility of hours is exactly reproduced.

The moments characterizing the evolution of consumption have been left unrestricted. Table 5 shows that the life-cycle profile of the first and second moments of consumption is qualitatively similar to that of hours: The level is hump-shaped and the volatility is u-shaped over the life-cycle. The hump-shape profile of consumption over the life-cycle is consistent with the data, as documented by Carroll and Summers (1991), Attanasio and Weber (1995),

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[21]See Appendix F for details.

[22]See King and Rebelo (2000).

[23]See e.g. Clark and Summers (1981), Gomme et al. (2004) and Jaimovich and Siu (2009).

Table 5: Consumption and hours over the life cycle: benchmark economy

	$15-19$	$20-24$	$25-29$	$30-39$	$40-49$	$50-59$	$60-64$
Relative levels							
Consumption	0.60	0.80	0.94	1.00	1.00	0.91	0.76
Hours	0.27	0.73	0.93	0.96	1.00	0.92	0.62
Relative volatilities							
Consumption	1.41	1.28	1.15	1.07	1.00	1.01	1.07
Hours	5.14	2.53	1.75	1.35	1.00	0.94	0.94

Note: The relative levels are calculated based on the stead-state of the model. The relative volatilities are based on the standard deviation of the logs in deviation from an HP trend with smoothing parameter 6.25, calculated from the simulation of the theoretical economy over 10,000 periods.

Gourinchas and Parker (2002) and Fernandez-Villaverde and Krueger (2008), among others. For instance, Gourinchas and Parker (2002) report that household life-cycle consumption profiles, adjusted for family composition and cohort effects, have a distinct and statistically significant hump, with actual consumption on the increase during ages $20-40$ and falling off during ages $50-70$. These findings are consistent with the implications of our model reported in Table 5, both qualitatively and quantitatively. This success has to do with the chosen utility function implying that leisure and consumption are substitutes—see equation (8). The upshot is that the hump-shape profile of hours worked generates a hump-shape profile of consumption over the life-cycle.[24]

Finally, we consider the life-cycle profile of the cyclical volatility of consumption. There are two opposing forces that determine this profile. First, the substitutability between leisure and consumption implies that a large labor supply wage-elasticity gives rise to a large consumption wage-elasticity. The upshot is that hours volatility implies consumption volatility. We call this first effect the "within-group substitutability" effect. Second, there is a "between-group complementarity" effect whereby, when a group raises its supply of labor—hence, raising its marginal utility of consumption—the marginal utility of consumption of the other demographic groups increases too. This second effect reduces the within-group wage-elasticity of consumption offsetting the "within-group substitutability" effect. If the "within-group substitutability" effect dominates, high relative volatility of hours worked is associated with a high relative volatility of consumption. If instead, the "between-group complementarity" effect dominates, high relative volatility of hours worked is associated with a

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[24]Bullard and Feigenbaumb (2007) explore in detail the implications of consumption and leisure substitutability in household preferences to explain the life-cycle consumption data.

low relative volatility of consumption. The results in Table 5 indicate that the "within-group substitutability" is the most important as the volatility of consumption is highest for the young individuals with elastic labor supply.

Empirically, we do not know as much about the differences in cyclical volatility of consumption across demographic groups as we do about the differences in cyclical volatility of hours. Blundell et al. (2008) find evidence that the response of consumption to persistent income shocks falls with age and interpret this as evidence of partial insurance increasing with age. This finding is consistent with a declining cyclical volatility of consumption over the life-cycle.

7   Government size and automatic stabilization

We now address the question of whether our model is able to reproduce a negative correlation between the size of the government (measured by the tax-revenue to output ratio) and aggregate volatility. Beyond the simple qualitative relation between the fiscal policy variables and macroeconomic stability, we are also interested in the quantitative implications. In particular, we compare the strength of the automatic stabilization in the model and in the data. We proceed in two steps: first we study the model comparative statics—i.e., we examine what happens to the volatility of the macroeconomic aggregates as we change each fiscal policy parameter individually; second, we feed into the model different combinations of values for the tax rates and for the government spending as a share of GDP, with each combination chosen to mimic the fiscal policy parameters of a particular OECD country. By following this procedure, we make sure that the size of the government is varied endogenously, in a way which is dictated by the changes in fiscal policy parameters across OECD countries. This allows us to investigate whether we are able to replicate quantitatively the relation between government size and macroeconomic stability across OECD countries.

7.1   Comparative statics

In this section we examine the automatic-stabilization role of each tax rate in isolation, leaving all other fiscal parameters equal at their baseline level. In doing this exercise, we consider the theoretical economy under the Ricardian fiscal-regime described by equations (15) and (18). We examine the Ricardian regime because we want to measure the strength of each tax rate's automatic-stabilization effect in isolation, without allowing for budget deficits to affect the equilibrium dynamics. The exercise consists of changing each of the three tax rates, τ_h, τ_k and τ_c, one at a time (leaving all other fiscal parameters unchanged) and ob-

Figure 5: Automatic Stabilization: Comparative Static Analysis

(a) Output Fluctuations   (b) Hours Fluctuations
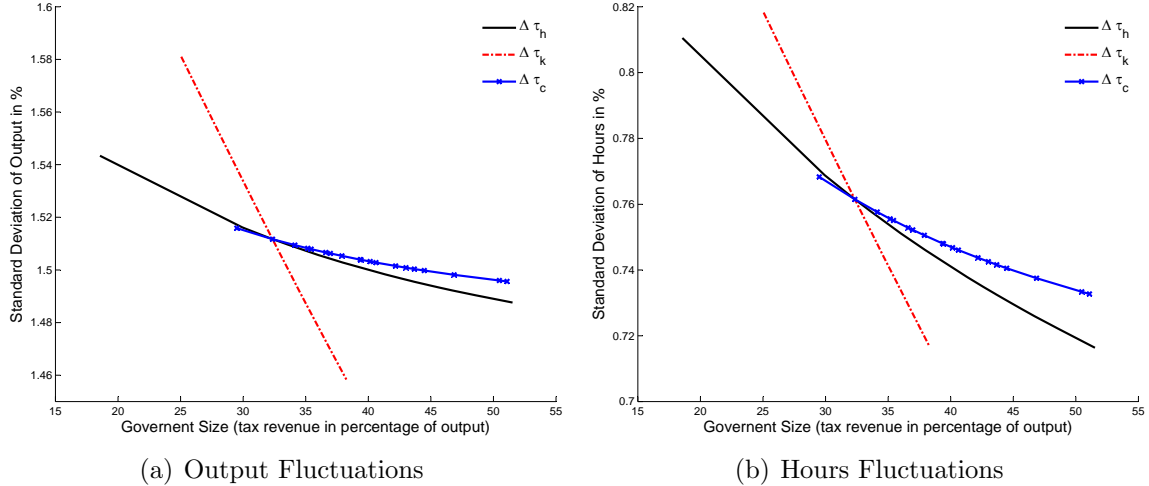
Notes: The horizontal axis show the implied steady-state size of the government measured by the tax to output ratio; the vertical axis shows the standard deviation of the HP filtered output and hours resulting from the simulation of the model over 10,000 periods, under the Ricardian fiscal-regime calibration. Each line corresponds to changes in one of the tax rates, leaving all other tax rates and government spending equal to the U.S. levels.

serve how aggregate volatilities change. In particular, the Figures 6(a) and 6(b) show results concerning the stabilization of output and hours, respectively.

These figures show in the horizontal axis the size of the government in steady-state (measured by the ratio between total tax revenue and output) implied by the fiscal parameters considered. In the vertical axis, Figure 6(a) shows the resulting standard deviation of output and Figure 6(b) shows the resulting standard deviation of hours. In both figures, each curve that is traced represents changes in the government size obtained by changing only one of the tax rates and leaving at their baseline levels all other fiscal parameters.[25] The first finding is that as each tax-rate is raised aggregate volatility falls. Under the Ricardian regime, all three tax-rates behave as automatic-stabilizers because of the mechanism outlined in Proposition 1: Hours and output volatility decrease as the tax-rates are raised because the change in the age composition of the workforce lowers the aggregate labor supply elasticity.

But the main finding that emerges is that, for an implied change in government size of the same magnitude, the output and hours automatic-stabilization effect resulting from changes

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[25]The range of tax-rate changes we consider for each fiscal instrument (τ_h, τ_k and τ_c) corresponds exactly to the range that is observed in the cross-section of OECD countries. The elasticity of the tax to output ratio to changes in each tax rate is always positive in the range of tax changes considered: As we raise each tax rate leaving fixed at their baseline levels all other fiscal parameters, the size of the government increases.

in the capital income tax τ_k is much stronger than the one resulting from changes in the labor or consumption income taxes. This finding is explained by the fact that the capital income share α is roughly half the labor income share $(1 - \alpha)$ and also roughly half the consumption-output ratio, equal to 67%. Therefore, raising the capital income tax τ_k by one percent, implies an increase in the government size much lower than the increase implied by raising the labor income tax or the consumption income tax by one percent. In other words, the elasticity of the ratio between total tax revenue and output (our measure of government size) to changes in the capital income tax rate is much lower than the corresponding elasticity to changes in the labor or consumption tax rates.

This has important implications for how we should quantify the strength of automatic stabilization implied by our model. We define the strength of automatic stabilization as the change in output volatility that is implied by a given change in government size. However, the result illustrated in Figures 6(a) and 6(b) implies that the same government size is associated with a different output volatility depending on the profile of taxes. Therefore, when we compare the relationship between government size and macroeconomic stability implied by the model and in the data, it is important to treat government size as an endogenous outcome resulting from differences in fiscal profiles that mimic the differences observed in the cross-section of OECD countries. In the next section we examine quantitatively if the model reproduces the association between government size and macroeconomic stability observed in the data, following exactly this strategy.

7.2   Automatic stabilization: quantitative evaluation

In this section we examine if the theoretical economy is able to reproduce quantitatively the negative correlation between the size of the government and the volatility of aggregate hours and output. For this purpose, we feed into the model different combinations of fiscal parameters—each combination mimicking the fiscal profile of an OECD country. The fiscal parameters that need to be chosen for each artificial economy are the three tax rates, and the steady-state government spending to output ratio. Each set of tax rates are chosen based on the tax rates of a given country, as estimated by Carey and Rabesona (2002). In addition, the steady state government spending to output ratio is chosen to match the historical average for the same country as reported in the national accounts. For each fiscal profile mimicking an OECD country, we simulate time series of 10,000 periods length and calculate the implied size of the government and the standard deviation of the HP filtered output and aggregate hours. The implied size of the government, an endogenous outcome, is measured by the steady-state ratio of total tax-revenue to output. We reproduce the

Figure 6: Government Size and Aggregate Volatility (Model)
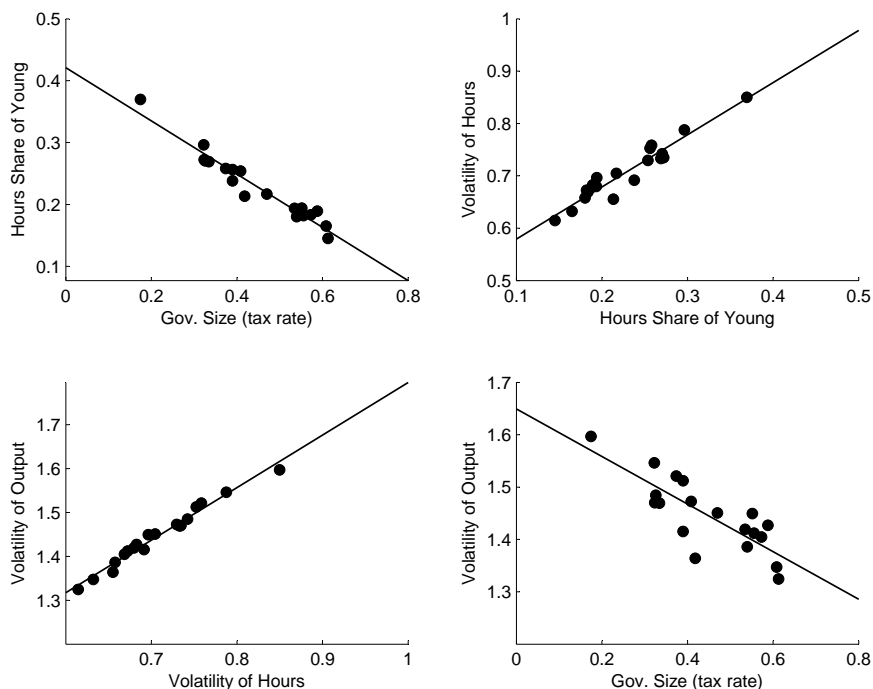
Notes: Volatility of output and hours from the model are the standard deviation of the HP filtered output and hours resulting from the simulation of the model over 10,000 periods. Each observation corresponds to an economy whose fiscal policy parameters are chosen to mimic the fiscal profile of a specific OECD economy.

cross-country regressions which are performed in Fatás and Mihov (2001) by regressing the volatility of output and aggregate hours on government size.

We turn first to Figure 6. In this figure we reproduce the scatter plots in Figure 2 but this time with the simulated data from 19 artificial economies, each parameterized to reproduce the fiscal profile of an OECD economy. The variables defined in Figure 6 correspond exactly to the ones defined in Figure 2, except for the variable Hours Share of Young. The reason is that in the theoretical economy hours fluctuate in the intensive margin, while the empirical proxy we use to measure the share of young workers in the workforce is the Employment Share of Young that only accounts for the extensive margin. For this reason, it is likely that the variable Employment Share of Young underestimates the cross-section dispersion in the share of hours worked by the young.[26] Nonetheless, it is apparent from comparing Figures 2

[26]Shimer (2009) illustrates the importance of the intensive margin with the following example: the number

and 6 that the association between government size, the demographic composition of the workforce and macroeconomic stability implied by the theoretical economy is qualitatively consistent with the data: Higher taxes imply a low share of young workers in the workforce; The smaller the share of young workers in the workforce, the lower the volatility of aggregate hours worked and the lower the volatility of output.

In Table 6 we investigate to what extend the model reproduces quantitatively the negative relationship between government size and the volatility of aggregate output and hours fluctuations. The table reports the estimates from a linear regression between aggregate volatility and government size using the empirical OECD data and the simulated data from the theoretical economy (both for hours and output volatility). The slope coefficient measures the strength of the automatic stabilizers. This exercise allows us to interpret our results from a quantitative perspective: the last row of Table 6 shows the ratio between the slope coefficient estimated using the simulated data and the slope coefficient estimated using the empirical data.

As in the data, the strength of the automatic stabilizers for output fluctuations and hours fluctuations is almost the same. This indicates the importance of the labor market to understand the role of automatic stabilizers. The baseline model implies a slope coefficient in the regression of hours volatility on government size that is 22% of the coefficient resulting from the empirical regression. The findings concerning the relationship between output volatility and government size are similar: the slope associated with the regression of output volatility on government size corresponds to 23% of the slope observed in the data. Hence, the baseline model explains about one quarter of the strength of the automatic stabilizers. Previous literature has suggested that cross-country differences in taxes are important to explain differences in hours worked. For instance, Prescott (2004), Rogerson (2006, 2008) and Ohanian et al. (2008) have shown that differences in taxes on labor income over time and across countries in the OECD account for a large share of differences in hours worked. Rogerson and Wallenius (2008) emphasize the impact taxes have on cross-country variations in hours worked along the life-cycle. Our results suggest that cross-country differences in taxes also are important to explain differences in the volatility of hours worked across OECD countries.

The main mechanism that explains the strength of the automatic stabilizers in the theoretical economy is the change in the composition of the workforce as tax rates change. This is well illustrated in Figure 7. Each panel (except the final panel) shows in the vertical

of hours worked per adult in both France and Germany was 32 log points below the level in the United States in 2006; the employment-population ratio accounted for 18 log points of the difference in France and 9 log points of the difference in Germany, with the number of hours per worker accounting for the remainder.

Figure 7: Automatic Stabilizers: the role of demographics
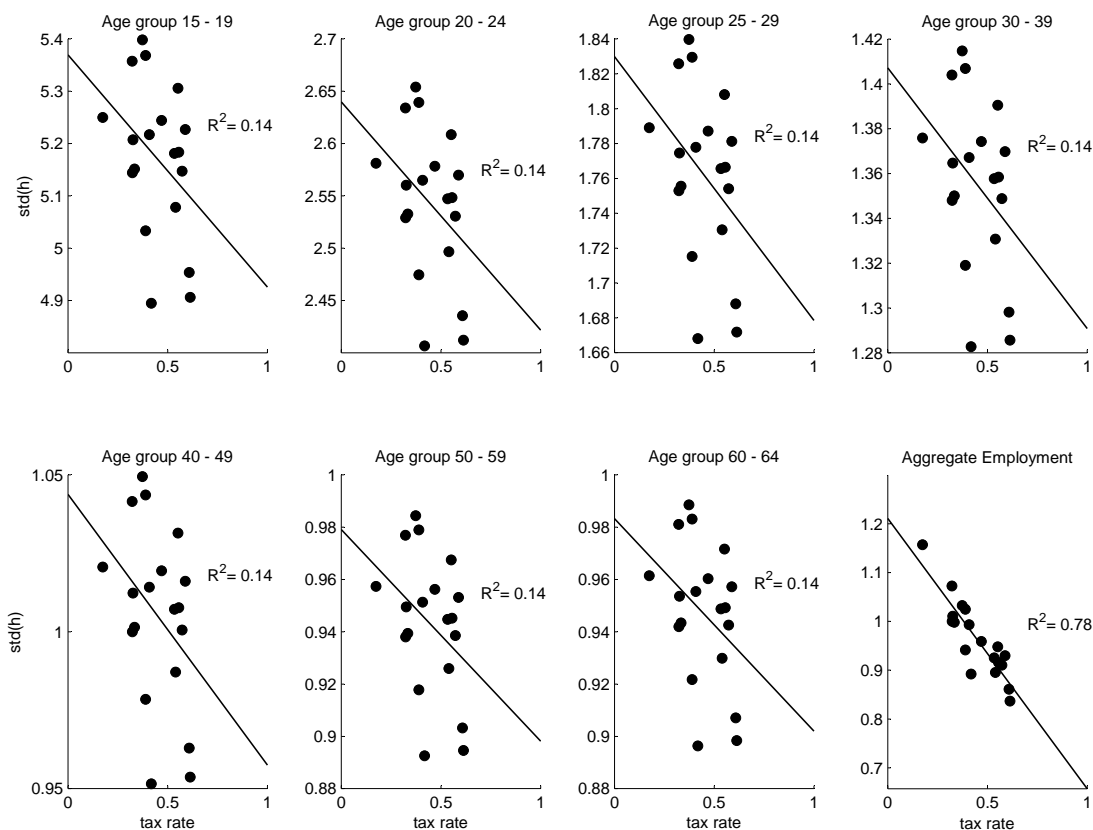
Notes: Each panel (except the final panel) shows in the vertical axis the standard deviation of the HP filtered total hours worked by a given age group relative to the standard deviation of hours worked by the reference group under the baseline calibration. The final panel shows in the vertical axis the standard deviation of aggregate hours worked relative to the standard deviation of aggregate hours under the baseline calibration. The horizontal axis shows the ratio of total taxes to output. Each observation results from the simulation of the model over 10,000 periods and setting the fiscal parameters to mimic the fiscal profile of a specific OECD economy. Each panel shows the R^2 statistic for the fitted regression line.

Table 6: Regressions of volatility over government size: model vs. data

		Empirical regression	Simulation regression	Ratio of slopes
Output volatility	Slope	−1.984	−0.460	23%
	Constant	2.334	1.651	
Hours volatility	Slope	−1.831	−0.409	22%
	Constant	1.993	0.890	

Note: This table gives results for OLS regressions where the dependent variables are, respectively, output volatility and hours worked volatility and the explanatory variable is the tax-revenue to output ratio. The volatility of output and hours is given by the standard deviation of the series in log deviations from an HP trend with smoothing parameter 6.25. The effective tax rates used to calibrate the fiscal profile of each economy in the simulations are from Carey and Rabesona (2002). Concerning the empirical regressions, each observation corresponds to a country and one of the following time periods: 1970 – 1979, 1980 – 1989, 1990 – 1999, and 2000 – 2009. Time dummies are included but not listed.

axis the volatility of hours worked by a specific age group relative to the volatility of hours worked by the reference group under the baseline calibration and on the horizontal axis the government size. The final panel shows in the vertical axis the volatility of aggregate hours worked resulting from each fiscal profile relative to that under the baseline calibration. For each demographic group, the correlation between government size and hours volatility is only weakly negative: for all age groups, the R^2 of the linear regression of volatility on government size is only 14%. Instead, there is a strong negative correlation between the government size and the volatility of aggregate hours worked (the R^2 of the regression line is 78%). Hence, the automatic stabilizing effect of taxes happens mostly because as the composition of the workforce changes towards the prime-aged workers, the aggregate labor supply elasticity is reduced. These results indicate that the changes in the composition of the workforce are indispensable for the model to be consistent with the automatic stabilization effect of taxes.

8   Additional experiments

In the theoretical economy, two channels contribute to the strength of the automatic stabilizers: the change in the aggregate labor supply elasticity resulting from distortionary taxes and the countercyclicality of government spending. In this section, we report on two additional experiments. The first experiment separates the two channels through which automatic stabilizers work in the model and quantifies the relative importance of each channel in isolation.

The second experiment is designed to investigate if an alternative calibration, bringing the volatility of aggregate hours relative to that of output closer to the data, improves the model's ability to explain the relationship between government size and macroeconomic stability.

8.1 Automatic stabilization: channels

As documented earlier, the baseline model predicts roughly 1/4 of the relationship observed in the data between government size and macroeconomic stability. But the baseline calibration considers the *non-Ricardian* fiscal regime described by equations (15) and (16). Under the *non-Ricardian* regime, budget deficits affect the equilibrium allocation and, in particular, they imply that government spending is countercyclical. Thus, the negative correlation between government size and output volatility results in part from the change in the aggregate labor supply elasticity implied by distortionary taxes and in part from the countercyclicality of government spending. Table 7 considers the strength of the relationship between government size and aggregate volatility under alternative model parameterizations to isolate the contribution of these two channels. The first row of the table reports the volatility of output and hours under each parametrization for an economy having the same fiscal profile as the United States.[27] The second row shows the slope coefficient from the linear regression of, respectively, output and hours volatility on government size implied by the theoretical economy. The third row reports the ratio between the slope coefficient implied by the model and the slope coefficient obtained from the empirical OECD data.

The first alternative parametrization we consider examines the *Ricardian* fiscal regime instead of the *non-Ricardian* regime. Under the latter regime, the dynamics of government spending are described by equation (16), where the φ_g and σ_g are parameters taking non-zero values, and budget deficits affect the equilibrium allocation. Instead, under the *Ricardian* fiscal regime φ_g and σ_g are set equal to zero, and G_t satisfies equation (18). Under the *Ricardian* fiscal regime budget deficits do not affect the equilibrium allocation and the strength of the automatic stabilizers is entirely attributable to the changes in the aggregate labor supply elasticity implied by distortionary taxation. The results in columns (2a) and (2b) indicate that under this parametrization the slope coefficient of the regression of, respectively, output volatility and hours volatility on government size are 15% and 18% of their empirical counterparts. Thus, about 2/3 of the automatic stabilization strength implied by the model is attributable to changes in the aggregate labor supply elasticity. Finally, contrasting the first row of columns (2a) and (2b) to the first row of columns (1a) and (1b) (the first row

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[27]A fiscal profile corresponds to a choice of values for τ_k, τ_c and τ_h, and a value for the share of government spending in output $\bar{g}_y$ (see Table 1).

Table 7: Alternative Parameterizations

	Baseline calibration		Ricardian regime		Labor force homogeneity		Homogeneity + Ricardian regime	
	(1a)	(1b)	(2a)	(2b)	(3a)	(3b)	(4a)	(4b)
	Output	Hours	Output	Hours	Output	Hours	Output	Hours
U.S. fiscal profile Volatility (model)	1.46	0.73	1.51	0.76	1.46	0.73	1.51	0.76
Gov. size regressions Estimated slope	−0.460	−0.409	−0.293	−0.321	−0.236	−0.135	−0.050	−0.029
Ratio of slopes	23%	22%	15%	18%	12%	7%	3%	2%

Note: The first row reports the volatility of output and hours under each parametrization for an economy having the U.S. fiscal profile. The second row reports the slope coefficient from the linear regression of, respectively, output and hours volatility on government size implied by the theoretical economy. The third row reports the ratio between the slope coefficient implied by the model and the slope coefficient obtained from the empirical OECD data (reported in the first column of Table 6). The volatilities are based on the standard deviation of the logs in deviation from an HP trend with smoothing parameter 6.25, calculated from the simulation of the theoretical economy over 10,000 periods.

reports the implied business cycle statistics for an economy with the fiscal profile of the U.S.) reveals that the volatility of output and hours are somewhat higher under the *Ricardian* regime. This finding indicates that the countercyclical government spending increases macroeconomic stability under the baseline calibration.[28]

The second alternative parametrization we consider eliminates changes in the aggregate labor supply elasticity resulting from distortionary taxation by removing any source of labor force heterogeneity. To do this we restrict all demographic groups to have the same Frisch elasticity. The value chosen for this elasticity is 1.35, which corresponds to the average elasticity under the baseline calibration.[29] Hence, the labor supply elasticity is the same as the average labor supply under the baseline calibration, and the volatility of output and hours given the U.S. fiscal profile—first row of columns (3a) and (3b)—is the same as in the baseline calibration. Under this alternative parametrization, the stabilizing role of the government results only from the countercyclicality of government spending. In the OECD economies with large tax rates the government spending as a share of total output, $\bar{g}_y$, is large. Since government spending is countercyclical, large governments increase macroeconomic stability. The results in the third row of columns (3a) and (3b) indicate that countercyclical budget deficits alone are able to reproduce only 12% and 7% of the strength of the automatic

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[28]This result is consistent with empirical evidence in Fatás and Mihov (2006) indicating that the adoption of fiscal feedback rules reduces output volatility.

[29]We consider the average across demographic groups weighted by the share of hours worked supplied by each age group. This corresponds to the inverse of the parameter ψ defined in the Appendix.

Table 8: Government Size and Aggregate Volatility: alternative parametrization

	Business Cycle Statistics			Gov. Size Regressions		
Var.	Std. Dev.	Corr.			Y Vol	Ratio of Slopes
Y	1.45	–		Gov. Size	−1.08	55%
C	1.26	0.99		Constant	1.90	
I	4.50	0.98			H Vol	
G	0.97	−0.03		Gov. Size	−1.11	61%
N	1.02	1.00		Constant	1.48	

Note: The volatilities are based on the standard deviation of the logs in deviation from an HP trend with smoothing parameter 6.25, calculated from the simulation of the theoretical economy over 10,000 periods. The ratio of slopes is the ratio between the slope coefficient implied by the model and the slope coefficient obtained from the empirical OECD data (reported in the first column of Table 6).

stabilizers of, respectively, output and hours fluctuations observed in the data. We conclude that to explain the observed stabilization of aggregate hours fluctuations, changes in the aggregate labor supply elasticity caused by distortionary taxes play a predominant role.

Finally, columns (4a) and (4b) considers a parametrization that nullifies both automatic stabilization channels: *non-Ricardian* budget deficits and labor supply heterogeneity. The association between government size and macroeconomic stability disappears under this alternative parametrization, indicating that these two channels are the only relevant mechanisms in the theoretical economy.

8.2 Labor supply elasticity: an alternative parametrization

The main shortcoming of the model baseline calibration is that the volatility of hours relative to the volatility of output is only 50% but in the data the relative volatility of hours is 88% that of output. This feature of the baseline model is pervasive in the RBC literature: to obtain volatile fluctuations in aggregate hours requires a large labor supply elasticity. In the baseline model we set the labor supply elasticity of prime aged workers to 1, which is often considered an upper-bound admissible from the micro level evidence (see Pencavel, 1986).

In a recent paper, Rogerson and Wallenius (2009) argue that micro and macro elasticities are unrelated and that the macro elasticities are large, in the range between 2.3 and 3.

The strength of the automatic stabilizers in the theoretical economy is undermined by the fact that the relative volatility of aggregate hours is counterfactually low under the baseline calibration. In particular, the main mechanism at work in the model is the reduction in the volatility of hours implied by the distortionary taxation. Thus, raising the importance of fluctuations in the labor supply raises the strength of the stabilization implied by the distortionary taxes. For this reason, we consider an alternative parametrization for the labor supply elasticity of prime aged workers, that is consistent with large macro elasticities. We set the Frisch labor supply elasticity of prime aged workers η_{40-49} equal to 2.6, consistent with the macro elasticities found in Rogerson and Wallenius (2009).[30] The purpose of the experiment is to investigate if an alternative parametrization, bringing the relative volatility of hours closer to the data, improves the model ability to explain the relationship between government size and macroeconomic stability.

The findings from this additional experiment are reported in Table 8. The first result concerns the business cycle statistics implied by the alternative calibration when the theoretical economy has the U.S. fiscal profile. The comparison with the results in Table 4 reveals that this alternative parametrization also matches very well the U.S. business cycle statistics and improves upon the baseline calibration in one important aspect: the standard deviation of hours relative to the standard deviation of output is 70% (much closer to the 88% value observed in the data). This result is important because it implies that the fluctuations in hours are responsible for a larger share of the total fluctuations in output.

As the main mechanism explaining the relationship between government size and macroeconomic stability is the change in the aggregate labor supply elasticity implied by distortionary taxation, we expect the strength of the automatic stabilizers to increase under the alternative parametrization. And indeed we obtain a substantial increase in the percentage explained of the total strength of the automatic stabilizers. The slope coefficient in the relationship between government size and output volatility implied by the model is 55% the size of its empirical counterpart. As for the relationship between government size and aggregate hours volatility, the model explains 61% of the strength of automatic stabilizers.[31]

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[30]To preserve the volatility of output under the U.S. fiscal profile in line with the business cycle statistics documented in Table 4 we decrease the standard deviation of the technology shock by 25%.

[31]Under the *Ricardian* fiscal regime, the theoretical economy implies a slope coefficient of 46% and 55% of the slope of the relationship between, respectively, government size and output volatility, and government size and hours volatility. Thus as the relative volatility of aggregate hours increases the importance of the demographic channel to explain the strength of the automatic stabilizers increases too.

9 Conclusion

Two empirical facts serve as the principal motivation for this paper. The first is that there is a strong negative correlation between government size and the volatility of business cycles across OECD countries. This feature of the data is difficult to explain using the standard real business cycle model. The second empirical fact is that there is substantial heterogeneity across demographic groups in terms of the cyclical volatility of employment and hours worked. Taken together, these two empirical facts suggest a mechanism whereby changes in the size of the government are associated with changes in the demographic composition of the workforce. An increase in tax rates tilts the workforce composition towards the prime aged workers lowering the aggregate labor supply elasticity and business cycle volatility.

We calibrate the model using data about the cross section of volatility of market hours across demographic groups and data on the joint dynamics of debt and government spending. We use the theoretical economy to investigate the relationship between the size of the government and the volatility over the business cycle of hours worked and output. We find that the model is able to explain a substantial part of the negative correlation between government size and business cycle volatility. Our results suggest that modeling labor force heterogeneity and, in particular, differences across demographic groups is important to explain quantitatively some important features of the business cycle.

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Appendix

A Data

In Section 2 we consider an unbalanced panel of 25 OECD countries composed of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Portugal, South Korea, Spain, Sweden, Switzerland, United Kingdom and United States, over the period 1970–2009. The variable definitions and data sources are as follows:

Employment Volatility by Age Group: This variable is constructed from annual data on the employment by age group obtained from the OECD Labour Force Statistics by Sex and Age. The age groups categories are $15-19$, $20-24$, $25-29$, $30-39$, $40-49$, $50-59$ and $60-64$ years old. For each country we obtain the cyclical component by removing the HP trend using the smoothing parameter 6.25 and the volatility measures correspond to the standard deviations of the cyclical components.

Employment Share of Young: This variable is obtained from the same OECD labor force statistics. We split the sample period in four subperiods: Each observation corresponds to a country and one of the following time periods: $1970-1979$, $1980-1989$, $1990-1999$, and $2000-2009$. For a country/subperiod to be included we require to have data for that country at least in 5 occasions in that subperiod (yielding a sample of 77 country/subperiod observations). The variable is defined as the ratio between the employment of individuals aged 15 to 29 and the employment of individuals aged 15 to 64, averaged over the subperiod and in percentage.

Government Size: This variable corresponds to total tax revenue as percentage of GDP from the OECD Revenue Statistics, an annual database that presents detailed and internationally comparable tax data.

Hours Volatility: The data on total annual hours comes from the Conference Board Total Economy Database. From this database we obtain a balanced panel for total hours worked in each OECD country between 1970 and 2009. The variable Hours Volatility is the standard deviation (in each subperiod) of the cyclical component obtained using the HP filter over the entire sample period.

Output Volatility: This variable is the standard deviation (in each subperiod) of the cyclical component of real output. Real output is obtained from the OECD national accounts.

The U.S. business cycle statistics concerning GDP, consumption, investment and government spending is from the NIPA tables. Inventories are excluded from the measure of investment.

Public Debt in Percentage of Output is the ratio between gross federal debt held by the public from the Council of Economic Advisors and the Congressional Budget Offices estimate of potential output.

Finally, concerning the fiscal policy variables, we choose the tax rates on capital income, labor income and consumption based on evidence documented in Carey and Rabesona (2002) who have produced series for the average effective tax rates on capital income, labor income and consumption for the OECD countries based on the methodology proposed by Mendoza et al. (1994).

B Regression tables

Table 9: Employment Volatility Over the Life Cycle

	(1) Employment Volatility
$15-19$	3.609***
	(12.47)
$20-24$	1.448***
	(5.00)
$25-29$	0.798***
	(2.76)
$30-39$	0.101
	(0.35)
$50-59$	0.228
	(0.79)
$60-64$	2.223***
	(7.68)
Observations	175
R^2	0.680

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: Each observation corresponds to a country and one of the following age groups: 15–19, 20–24, 25–29, 30–39, 40–49, 50–59 and 60–64. The listed regressors are dummy variables for each age group except the reference group (40–49). Country dummies and the intercept are included but not listed. See Appendix A for details about the data.

Table 10: Government Size, Demographic Structure and Hours Volatility

	(1) Share of Young	(2) Volatility of Hours
Gov. Size (tax rate)	−20.232***	
	(5.302)	
Share of Young		0.056***
		(0.015)
Constant	31.553***	−0.029
	(2.042)	(0.392)
Observations	77	77
R^2	0.469	0.185

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: Each observation corresponds to a country and one of the following time periods: 1970 − 1979, 1980 − 1989, 1990 − 1999, and 2000 − 2009. Time dummies are included but not listed. Volatility of Hours is the standard deviation of the respective cyclical component (calculated using the HP filter with smoothing parameter 6.25). The Share of Young corresponds to the share of employment of the population aged 15 to 29 in the total employment of the population aged 15 to 64. Gov Size is the ratio between total tax revenue and GDP. See Appendix A for details about the data.

Table 11: Government Size and Aggregate Volatility

	(1) Volatility of Output	(2) Volatility of Output
Volatility of Hours	0.716***	
	(0.078)	
Gov. Size (tax rate)		−1.984**
		(0.775)
Constant	0.666***	2.334***
	(0.131)	(0.299)
Observations	77	77
R^2	0.566	0.138

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: Each observation corresponds to a country and one of the following time periods: 1970 − 1979, 1980 − 1989, 1990 − 1999, and 2000 − 2009. Time dummies are included but not listed. Volatility of Output and Volatility of Hours are the standard deviation of the respective cyclical component (calculated using the HP filter with smoothing parameter 6.25). Gov Size is the ratio between total tax revenue and GDP. See Appendix A for details about the data.

Table 12: Demographic Structure and Aggregate Volatility

	(1) Volatility Hours	(2) Volatility Hours	(3) Volatility Output	(4) Volatility Output
Gov. Size (tax rate)	−1.831**	−0.834	−1.984**	−1.274
	(0.802)	(0.837)	(0.775)	(0.831)
Share of Young		0.049***		0.035**
		(0.017)		(0.017)
Constant	1.993***	0.437	2.334***	1.226**
	(0.309)	(0.611)	(0.299)	(0.607)
% Change in Fiscal Coefficient		−54%		−36%
Observations	77	77	77	77
Adjusted R^2	0.050	0.139	0.091	0.131

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: Each observation corresponds to a country and one of the following time periods: 1970 − 1979, 1980 − 1989, 1990 − 1999, and 2000 − 2009. Time dummies are included but not listed. Volatility Hours and Volatility Output are, respectively, the standard deviation of the cyclical component of aggregate hours and of output (calculated using the HP filter with smoothing parameter 6.25). The Share of Young corresponds to the share of employment of the population aged 15 to 29 in the total employment of the population aged 15 to 64. Gov Size is the ratio between total tax revenue and GDP. See Appendix A for details about the data.

C   Steady state and equilibrium dynamics

C.1   Equilibrium conditions

The equilibrium of the model is characterized by the following conditions

$$f_t = c_{it} - \lambda_i n_{it}^{1+\theta_i} \qquad \text{for all } i \qquad \left(\text{where } f_t \equiv v_t'^{-1/\sigma}\right) \tag{C.1}$$

$$d_t = E_t\left[\beta\left(\frac{f_{t+1}}{f_t}\right)^{-\sigma}\right] \tag{C.2}$$

$$K_{t+1} = \phi\left(\frac{I_t}{K_t}\right)K_t + (1 - \delta_t)K_t \tag{C.3}$$

$$\delta_t = \delta u_t^{1+\varsigma} \tag{C.4}$$

$$q_t = E_t\left\{\beta\left(\frac{f_{t+1}}{f_t}\right)^{-\sigma}\left[(1-\tau_k)r_{t+1}u_{t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}(1 - \delta_{t+1} + \Phi_{t+1})\right]\right\} \tag{C.5}$$

$$q_t = \phi'(I_t/K_t)^{-1} \tag{C.6}$$

$$r_t = \frac{\delta(1+\varsigma)u_t^\varsigma q_t}{1 - \tau_k} \tag{C.7}$$

$$n_i = \left[\frac{(1-\tau_h)w_t e_i}{(1+\tau_c)(1+\theta_i)\lambda_i}\right]^{\frac{1}{\theta_i}} \tag{C.8}$$

$$Y_t = e^{zt}(u_t K_t)^\alpha H_t^{1-\alpha} \tag{C.9}$$

$$w_t = (1 - \alpha) \frac{Y_t}{H_t} \tag{C.10}$$

$$r_t = \alpha e^{z_t} (u_t K_t)^{\alpha - 1} H_t^{1 - \alpha} \tag{C.11}$$

$$H_t = \sum_{i=1}^{Q} a_i h_{it} \tag{C.12}$$

$$C_t = \sum_{i=1}^{Q} a_i c_{it} \tag{C.13}$$

$$Y_t = C_t + I_t + G_t \tag{C.14}$$

where the capital adjustment cost function $\phi(.)$ is increasing and concave, and with z_t that follows a first order autoregressive process. In addition, the following parameter restrictions are assumed

$$\delta = \bar{\delta} \tag{C.15}$$

$$\phi(\delta) = \delta \tag{C.16}$$

$$\phi'(\delta) = 1 \tag{C.17}$$

$$-\frac{1}{\Phi''(\delta)\,\delta} = \xi \tag{C.18}$$

Finally, the government sector is characterized by a budget constraint and a fiscal rule

$$G_t + L_t + B_t \;\; = \;\; q_t B_{t+1} + \tau_k r_t u_t K_t + \tau_h w_t \sum_{i=1}^{Q} a_i h_{it} + \tau_c \sum_{i=1}^{Q} a_i c_{it}. \tag{C.19}$$

$$\begin{cases} l_t = -\varphi_L b_t, \quad \text{and} \quad \widetilde{G}_t = 0, & \text{if the fiscal rule is Ricardian} \\[2mm] l_t = -\varphi_L b_t, \quad \text{and} \quad \widetilde{G}_t = \rho_G \widetilde{G}_{t-1} - \varphi_G b_t, & \text{if the fiscal rule is non-Ricardian,} \end{cases} \tag{C.20}$$

where $l_t \equiv \left(L_t - \bar{L}\right)/\bar{Y}$ and $b_t \equiv \left(B_t - \bar{B}\right)/\bar{Y}$.

In what follows we consider first the steady state equilibrium and next we characterize the equilibrium dynamics of the log-linearized model. We represent a variable X in log-deviation from steady state by $\widetilde{X}$, and we denote the steady state of X by $\bar{X}$.

C.2   Steady state equilibrium

In steady state there are the following conditions that are satisfied

$$\bar{d} = \beta \tag{C.21}$$

$$\bar{q} = 1 \tag{C.22}$$

$$\delta = \frac{(1 - \tau_k)\,\bar{r}\bar{u}}{1 + \varsigma} \tag{C.23}$$

$$(1 - \tau_k)\,\bar{r}\bar{u} = \frac{1}{\beta} - (1 - \delta) \tag{C.24}$$

$$\bar{u} = 1 \tag{C.25}$$

$$\bar{k}_y = \frac{\bar{K}}{\bar{Y}} = \frac{(1 - \tau_k)\,\alpha}{1/\beta - (1 - \delta)} \tag{C.26}$$

$$\bar{C} = \left(1 - \delta \bar{k}_y\right)\bar{Y} - \bar{G} \tag{C.27}$$

$$\bar{Y} = \bar{k}_y^{\alpha/(1-\alpha)} \sum_{i=1}^{Q} a_i e_i \bar{n}_i \tag{C.28}$$

$$\bar{w} = (1 - \alpha)\,\bar{k}_y^{\alpha/(1-\alpha)} \tag{C.29}$$

$$\bar{n}_i = \left[\frac{(1 - \tau_h)(1 - \alpha) e_i}{(1 + \tau_c)(1 + \theta_i) \lambda_i} \right]^{\eta_i} \bar{k}_y^{\eta_i \alpha/(1-\alpha)} \tag{C.30}$$

$$\gamma \equiv \frac{\bar{f}}{\bar{C}} = 1 - \left(\sum_{i=1}^{Q} a_i \lambda_i \bar{n}_i^{1+\theta_i} \right) \bar{C}^{-1} \tag{C.31}$$

$$\frac{\bar{I}}{\bar{K}} = \delta \tag{C.32}$$

$$\bar{B} = 0 \tag{C.33}$$

with $\eta_i \equiv 1/\theta_i$.

C.3 Equilibrium conditions in log-linear form

Following standard steps, the firm and the stand-in family's optimality conditions, and the market clearing conditions are log-linearized and combined so as to characterize the equilibrium.

From equations (C.4), (C.5) and (C.6) follows

$$\widetilde{q}_t = \beta E_t (\widetilde{q}_{t+1}) + \left[1 - \beta (1 - \delta) \right] E_t (\widetilde{r}_{t+1}) + \widetilde{d}_t, \tag{C.34}$$

$$\widetilde{I}_t - \widetilde{K}_t = \xi \widetilde{q}_t, \tag{C.35}$$

where we have made use of the parameter restrictions (C.15)—(C.18) and result (C.23).

From (C.2) the stand-in family's Euler equation in log-linear form is given by

$$E_t \left(\widetilde{f}_{t+1} - \widetilde{f}_t \right) = -\frac{\widetilde{d}_t}{\sigma}. \tag{C.36}$$

From (C.3) and (C.4) the motion equation for capital is

$$\widetilde{K}_{t+1} = (1 - \delta) \widetilde{K}_t + \delta \left[\widetilde{I}_t - (1 + \varsigma) \widetilde{u}_t \right]. \tag{C.37}$$

The first order condition concerning capital utilization, equation (C.7), in log-linear form is

$$\widetilde{u}_t = \frac{1}{\varsigma}\left(\widetilde{r}_t - \widetilde{q}_t\right). \tag{C.38}$$

From equation (C.8) follows that the labor supply of individuals aged i in log-linear form is

$$\widetilde{h}_{it} = \eta_i \widetilde{w}_t, \tag{C.39}$$

while from the market clearing condition (C.12) it follows that

$$\widetilde{H}_t = \sum_{i=1}^{Q} \bar{s}_{hi} \widetilde{h}_{it}, \tag{C.40}$$

where $\bar{s}_{hi} \equiv a_i \bar{h}_i / \bar{H}$, and by combining (C.39) and (C.40) we obtain

$$\widetilde{w}_t = \psi \widetilde{H}_t, \tag{C.41}$$

where $\psi \equiv \left(\sum_{i=1}^{Q} \bar{s}_{hi}\eta_i\right)^{-1}$.

The factor demand equations (C.10) and (C.11) in log-linear form are

$$\widetilde{w}_t = \widetilde{Y}_t - \widetilde{H}_t, \tag{C.42}$$

$$\widetilde{r}_t = \widetilde{Y}_t - \widetilde{K}_t - \widetilde{u}_t, \tag{C.43}$$

while the log-linear production function is

$$\widetilde{Y}_t = \alpha\left(\widetilde{u}_t + \widetilde{K}_t\right) + (1-\alpha)\widetilde{H}_t + z_t. \tag{C.44}$$

The market clearing conditions in the goods market, equations (C.13) and (C.14), take the following log-linear form

$$\widetilde{C}_t = \sum_{i=1}^{Q} \bar{s}_{ci} \widetilde{c}_{it}, \tag{C.45}$$

$$\widetilde{Y}_t = \bar{c}_y \widetilde{C}_t + \delta \bar{k}_y \widetilde{I}_t + \bar{g}_y \widetilde{G}_t, \tag{C.46}$$

where $\bar{c}_y$, $\bar{k}_y$ and $\bar{g}_y$ are, respectively, the consumption to output ratio, the capital to output ratio and the government's consumption to output ratio in steady state.

Equation (C.1) log-linearized takes the form

$$\gamma \widetilde{f}_t = \widetilde{C}_t - \vartheta \widetilde{H}_t, \tag{C.47}$$

where $\gamma \equiv \bar{f}/\bar{C}$ and $\vartheta = \left(\psi \sum_i^Q a_i \lambda_i \bar{n}_i^{1+\theta_i}/\theta_i \right) \bar{C}^{-1}$.

Finally, by linearizing the government budget constraint (C.19) around a steady state with zero debt and a balanced primary budget, and making use of the fiscal feedback rule (C.20) we obtain the following description of the debt dynamics

$$b_{t+1} = \begin{cases} \beta^{-1} \left(1 - \varphi_L\right) b_t - \beta^{-1} \left(\tau_y \widetilde{Y}_t + \bar{c}_y \tau_c \widetilde{C}_t \right) & \text{if Ricardian} \\ \\ \beta^{-1} \left(1 - \varphi_L\right) b_t - \beta^{-1} \left(\tau_y \widetilde{Y}_t + \bar{c}_y \tau_c \widetilde{C}_t - \bar{g}_y \widetilde{G}_t \right) & \text{if non-Ricardian}, \end{cases} \tag{C.48}$$

where $\tau_y \equiv \alpha \tau_k + (1 - \alpha) \tau_h$.

C.4 Equilibrium dynamics

By combining equations (C.34)—(C.48) we find that the equilibrium dynamics are fully characterized by a system of difference equations in $\left(\widetilde{C}_t, \widetilde{H}_t, \widetilde{u}_t, \widetilde{K}_t, \widetilde{G}_t, b_t, z_t \right)$.

Combining (C.41) and (C.42) implies

$$\widetilde{Y}_t = (1 + \psi) \widetilde{H}_t$$

and replacing in (C.46) yields

$$\bar{c}_y \widetilde{C}_t + \delta \bar{k}_y \widetilde{I}_t + \bar{g}_y \widetilde{G}_t = (1 + \psi) \widetilde{H}_t.$$

Finally, using (C.37) to substitute out investment implies

$$\bar{k}_y \widetilde{K}_{t+1} = \bar{k}_y \left(1 - \delta\right) \widetilde{K}_t + (1 + \psi) \widetilde{H}_t - \delta \bar{k}_y \left(1 + \varsigma\right) \widetilde{u}_t - \bar{c}_y \widetilde{C}_t - \bar{g}_y \widetilde{G}_t, \tag{C.49}$$

where

$$
\widetilde{G}_t =
\begin{cases}
0, & \text{if the fiscal rule is Ricardian,} \\
\rho_G \widetilde{G}_{t-1} - \varphi_G b_{t-1} + \sigma_g \epsilon_t^g, & \text{if the fiscal rule is non-Ricardian.}
\end{cases}
\tag{C.50}
$$

Next, combining (C.35), (C.38), (C.43) and (C.44) we obtain

$$
\xi (1 + \varsigma - \alpha) \widetilde{u}_t = \xi (1 - \alpha) \widetilde{H}_t + [1 - \xi (1 - \alpha)] \widetilde{K}_t + \xi z_t - \widetilde{I}_t
$$

and once again making use of (C.37) to substitute out investment yields

$$
\widetilde{K}_{t+1} - \delta \xi z_t = [1 - \delta \xi (1 - \alpha)] \widetilde{K}_t + \delta \xi (1 - \alpha) \widetilde{H}_t - \delta [(1 + \varsigma)(1 + \xi) - \alpha \xi] \widetilde{u}_t.
\tag{C.51}
$$

Turning now to equations (C.34) and (C.36) and combining them together with (C.38), we obtain

$$
\widetilde{r}_t - \varsigma \widetilde{u}_t = \zeta \widetilde{r}_{t+1}^t - \beta \varsigma \widetilde{u}_{t+1}^t - \sigma \widetilde{f}_{t+1}^t + \sigma \widetilde{f}_t,
$$

where $\zeta \equiv (1 + \beta \delta)$ and for notational simplicity X_{t+1}^t denotes the expectation at t of the variable X_{t+1}. Making use of (C.41)—(C.43) to substitute $\widetilde{r}$ yields

$$
(1 + \psi) \widetilde{H}_t - \widetilde{K}_t - (1 + \varsigma) \widetilde{u}_t = \zeta (1 + \psi) \widetilde{H}_{t+1}^t - \zeta \widetilde{K}_{t+1} - (\zeta + \beta \varsigma) \widetilde{u}_{t+1}^t - \sigma \widetilde{f}_{t+1}^t + \sigma \widetilde{f}_t
$$

Finally, making use of equation (C.47) to substitute out f yields

$$
\begin{aligned}
\zeta \widetilde{K}_{t+1} + \frac{\sigma}{\gamma} \widetilde{C}_{t+1}^t \quad &- \left[\zeta (1 + \psi) + \frac{\sigma \vartheta}{\gamma} \right] \widetilde{H}_{t+1}^t + (\zeta + \beta \varsigma) \widetilde{u}_{t+1}^t \\
&= \widetilde{K}_t + \frac{\sigma}{\gamma} \widetilde{C}_t - \left(1 + \psi + \frac{\sigma \vartheta}{\gamma} \right) \widetilde{H}_t + (1 + \varsigma) \widetilde{u}_t
\end{aligned}
\tag{C.52}
$$

Combining equations (C.41)—(C.43) with the log-linear production function (C.44) yields

$$
z_t = (\alpha + \psi) \widetilde{H}_t - \alpha \widetilde{u}_t - \alpha \widetilde{K}_t.
\tag{C.53}
$$

Substituting out $\widetilde{Y}_t$ from (C.48) yields

$$
b_{t+1} =
\begin{cases}
\beta^{-1} (1 - \varphi_L) b_t - \beta^{-1} \left[\tau_y \alpha \widetilde{u}_t + \tau_y \alpha \widetilde{K}_t + \tau_y (1 - \alpha) \widetilde{H}_t + \tau_y z_t + \bar{c}_y \tau_c \widetilde{C}_t \right], \\
\\
\beta^{-1} (1 - \varphi_L) b_t - \beta^{-1} \left[\tau_y \alpha \widetilde{u}_t + \tau_y \alpha \widetilde{K}_t + \tau_y (1 - \alpha) \widetilde{H}_t + \tau_y z_t + \bar{c}_y \tau_c \widetilde{C}_t - \bar{g}_y \widetilde{G}_t \right],
\end{cases}
\tag{C.54}
$$

for the cases of the Ricardian and non-Ricardian rules, respectively, and where we have made

use of (C.44).

Finally, the stochastic technology shock follows z_t a first-order autoregressive process given by

$$z_t = \rho z_{t-1} + \sigma_z \epsilon_t^z. \tag{C.55}$$

The system of equations (C.49)—(C.55) characterize the equilibrium dynamics of the model. In more compact notation, this system is expressed as follows

$$\mathbf{A} \begin{bmatrix} \widetilde{C}_{t+1}^t \\ \widetilde{H}_{t+1}^t \\ \widetilde{u}_{t+1}^t \\ \widetilde{K}_{t+1} \\ b_{t+1} \\ \widetilde{G}_t \\ z_t \end{bmatrix} = \mathbf{B} \begin{bmatrix} \widetilde{C}_t \\ \widetilde{H}_t \\ \widetilde{u}_t \\ \widetilde{K}_t \\ b_t \\ \widetilde{G}_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \sigma_g \epsilon_t^g \\ \sigma_z \epsilon_t^z \end{bmatrix} \tag{C.56}$$

the elements of $\mathbf{A}$ and $\mathbf{B}$ are square matrices whose elements are functions of the structural parameters and ε_t is a standard normal random variable.

D   Proof of Propostion 1

From equation (C.41) the aggregate labor supply elasticity is given by the following expression

$$\mathcal{E}_n \equiv \frac{d \ln N}{d \ln w} = \sum_{i=1}^{Q} \bar{s}_{hi} \eta_i, \tag{D.1}$$

where $\bar{s}_{hi} \equiv a_i e_i \bar{n}_i / \bar{H}$ is the share of efficient units of labor supplied by individuals aged i in steady state. Differentiating equation (D.1) with respect to the tax rate τ_j (for $j = h, c, k$) we obtain

$$\frac{d \mathcal{E}_n}{d \tau_j} = \sum_{i=1}^{Q} \frac{\bar{s}_{hi} \eta_i}{\tau_j} \left(\frac{d\bar{n}_i}{d\tau_j} \frac{\tau_j}{\bar{n}_i} - \frac{d\bar{H}}{d\tau_j} \frac{\tau_j}{\bar{H}} \right)$$

$$= \sum_{i=1}^{Q} \left(\frac{\bar{s}_{hi} \eta_i}{\tau_j} \right) \left(\frac{d\bar{n}_i}{d\tau_j} \frac{\tau_j}{\bar{n}_i} \right) - \frac{\mathcal{E}_n}{\tau_j} \frac{d\bar{N}}{d\tau_j} \frac{\tau_j}{\bar{N}}$$

If follows that

$$\frac{d\,\mathcal{E}_n}{d\,\tau_j}\frac{\tau_j}{\mathcal{E}_n} = \sum_{i=1}^{Q}\left(\frac{\bar{s}_{hi}\eta_i}{\mathcal{E}_n}\right)\left(\frac{d\bar{n}_i}{d\tau_j}\frac{\tau_j}{\bar{n}_i}\right) - \frac{d\bar{N}}{d\tau_j}\frac{\tau_j}{\bar{N}},$$

which yields

$$\frac{d\,\mathcal{E}_n}{d\,\tau_j}\frac{\tau_j}{\mathcal{E}_n} = \sum_{i=1}^{Q}\left(\frac{\bar{s}_{hi}\eta_i}{\mathcal{E}_n}\right)\left(\frac{d\bar{n}_i}{d\tau_j}\frac{\tau_j}{\bar{n}_i}\right) - \sum_{i=1}^{Q}\bar{s}_{hi}\left(\frac{d\bar{n}_i}{d\tau_j}\frac{\tau_j}{\bar{n}_i}\right).$$

This equation can be rewritten as

$$\frac{d\,\mathcal{E}_n}{d\,\tau_j}\frac{\tau_j}{\mathcal{E}_n} = \sum_{i=1}^{Q}\bar{s}_{hi}\left(\frac{\eta_i - \mathcal{E}_n}{\mathcal{E}_n}\right)\left(\frac{d\bar{n}_i}{d\tau_j}\frac{\tau_j}{\bar{n}_i}\right).$$

By making use of Lemma 2, we have

$$\frac{d\,\mathcal{E}_n}{d\,\tau_j}\frac{\tau_j}{\mathcal{E}_n} = -\mathbf{1}_j\sum_{i=1}^{Q}\bar{s}_{hi}\left(\frac{\eta_i - \mathcal{E}_n}{\mathcal{E}_n}\right)\eta_i,$$

or equivalently,

$$\frac{d\,\mathcal{E}_n}{d\,\tau_j}\frac{\tau_j}{\mathcal{E}_n} = -\mathbf{1}_j\left[\sum_{i=1}^{Q}\left(\frac{\bar{s}_{hi}}{\mathcal{E}_n}\right)\eta_i^2 - \sum_{i=1}^{Q}\bar{s}_{hi}\eta_i\right].$$

Finally, by multiplying the equation by $\frac{\mathcal{E}_n}{\tau_j}$, we get

$$\frac{d\,\mathcal{E}_n}{d\,\tau_j} = -\frac{\mathbf{1}_j}{\tau_j}\left[\sum_{i=1}^{Q}\bar{s}_{hi}\eta_i^2 - \mathcal{E}_n^2\right].$$

By noticing that $\sum_{i=1}^{Q}\bar{s}_{hi}\eta_i^2 \geq \mathcal{E}_n^2$ (with equality only when $\eta_i = \eta$ for all i) it follows that the aggregate labor supply elasticity decreases as τ_j raises ($j = h, k, c$). It decreases by more, the more dispersed are the η_i across demographic groups.

E   Calculating the Solow residuals

An expression for capital utilization in terms of output and capital, in deviation from steady state, can be obtained from equations (C.35), (C.38) and (C.43)

$$(1+\varsigma)\widetilde{u}_t = \widetilde{Y}_t - \frac{\xi-1}{\xi}\widetilde{K}_t - \frac{1}{\xi}\widetilde{I}_t.$$

After replacing investment, one obtains

$$(1+\bar{\delta}\xi)(1+\varsigma)\widetilde{u}_t = \bar{\delta}\xi\widetilde{Y}_t + (1-\bar{\delta}\xi)\widetilde{K}_t - \widetilde{K}_{t+1}.$$

Moreover, notice that

$$\widetilde{H}_t = \widetilde{N}_t.$$

By replacing the expressions for $\widetilde{H}_t$ and $\widetilde{u}_t$ in the production function in log-linear form,

$$\log Y_t = z_t + \Gamma_t^z + \alpha \log u_t + \alpha \log K_t + (1-\alpha)\log H_t,$$

where Γ_t^z is a trend component for TFP. We obtain the following approximation

$$z_t + \Gamma_t^z \approx \left(1 - \frac{\alpha\bar{\delta}\xi}{\nu}\right)\log Y_t - \alpha\left(1 + \frac{1-\bar{\delta}\xi}{\nu}\right)\log K_t + \frac{\alpha}{\nu}\log K_{t+1} - (1-\alpha)\log N_t,$$

where $\nu = (1+\bar{\delta}\xi)(1+\varsigma)$.

F   Impulse response functions

Figure 8 shows the responses of output, consumption, investment, hours worked and government debt and spending to a technology shock. The responses of output, consumption, investment and hours worked are standard (see e.g. King and Rebelo, 2000), with the observation that our choice of preferences implies no wealth effect for the response of labor supply. The decrease in government debt following the shock is the consequence of higher collected taxes. This is turn leads to a increase in government spending because the fiscal rule (16) induces the government to spend more when $\varphi_g > 0$. Notice that the decrease in debt and the increase in spending occur several periods after the shock, while the response of output, consumption, investment and hours worked is immediate, a consequence of the structure in (30). These features of the model generate a negative correlation between the

Figure 8: Impulse response functions: technology shock
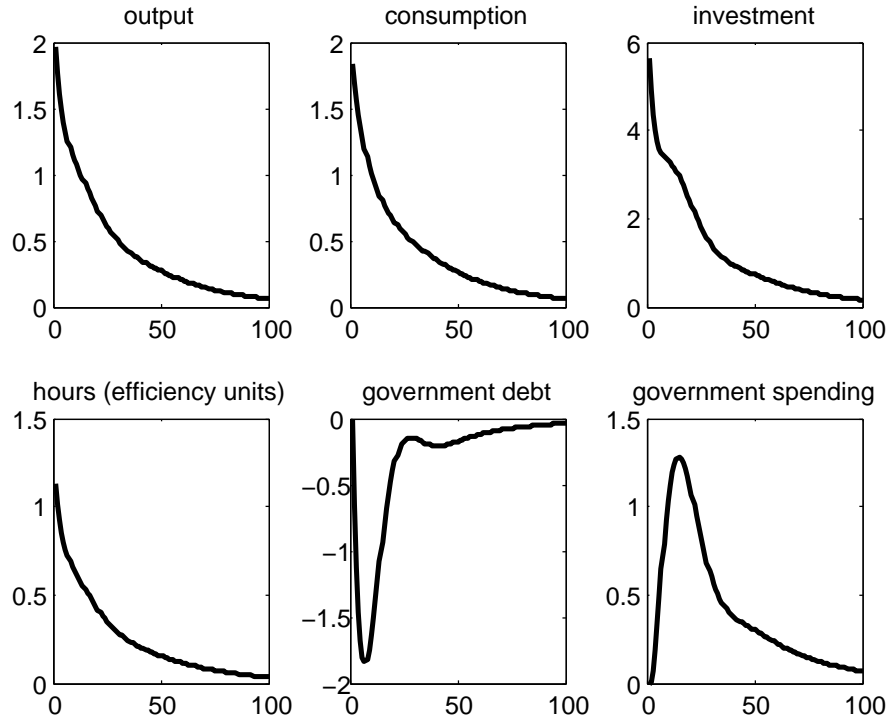
cyclical components of government spending and output.

Figure 9 shows the responses to a government spending shock. Government debt goes up in order to finance the increase in spending. The response of hours is explained by the change in the marginal product of labor. The latter increases upon impact because of the increase in capital utilization. In the transition path the capital stock depreciates, dampening wages and hours. Output follows the evolution of hours. The dynamics of consumption and investment are the same as in models with adjustment cost of capital (Burnside et al. 2004).

Figure 9: Impulse response functions: government spending shock