The Design of Information Acquisition and Sharing*

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Abstract

This paper investigates the optimal design of linear compensation schemes that incentivize information acquisition and sharing in multi-divisional organizations. When the information shared between divisions is highly correlated and the information acquisition costs are not too high, the optimal strategy for the headquarters manager is to implement a compensation scheme that links the remuneration of each division to the performance of the other division. However, if the information is weakly correlated or the cost of acquisition is prohibitively high, the most effective incentive is to tie each manager's remuneration solely to the performance of their own division.

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1 Introduction

Efficient decision-making in organizations relies on high-quality information, which is not always readily available and may require acquisition and sharing among organizational participants. For instance, in multi-divisional organizations, division managers operating in local markets can obtain local information that may be useful for other divisions supplying similar products in different markets. We investigate how such organizations should design transfers to incentivize information acquisition and sharing.

In particular, we examine linear transfer schemes that allow for remuneration of individual division performance, team performance (joint bonuses and penalties), and relative performance (tournaments).[1] An organization has to consider trade-offs when choosing which transfer scheme to adopt. For instance, remunerating division managers based on their relative performance fosters information acquisition but may harm incentives to share information. On the other hand, remunerating managers as a team may facilitate communication but harm individual incentives to acquire information. Alternatively, remunerating managers solely based on their own division performance may not be the optimal way to reduce managers' rents in the context of correlated information.

To study the optimal incentive scheme, we develop a model that features a principal (company headquarters manager) and two agents (local division managers). Each agent takes a decision in their division where the outcome depends both on the agent's decision and an unobserved local state. The two local states are correlated. Prior to making their decisions, each agent can obtain a costly private signal about their local state, and the agents can inform each other about their signals using cheap talk communication. The profit of the company is separable across the divisions' choices and increasing in how closely each agent's decision matches their division's local state.

The principal offers and commits to a linear transfer scheme that remunerates each agent for their own performance and/or the other agent's performance. The contracts do not allow for negative transfers as the agents are protected by limited liability. We investigate the optimal pattern of communication and signal acquisition from the principal's perspective and identify the cheapest incentives to achieve it.

We use a cheap talk model to capture the information exchange between agents, reflecting the fact that managers in many organizations cannot commit to choices based on agents' reports. However, we also allow for contracting based on the performance of local divisions, which is often observable by headquarters management and verifiable in court. The principal in our model has intermediate commitment power between the incomplete contracts approach, where transfers are ruled out, and the mechanism approach, which assumes full commitment

[1] Multiple studies in organizational economics capture organizational design by incentive schemes that facilitate information acquisition and sharing. However, existing theories typically disregard transfers and instead focus on organizing communication and assignments of decision rights. See, e.g., Aghion and Tirole (1997); Alonso, Dessein, and Matouschek (2008, 2015); Dessein (2002); Rantakari (2008).

power.

To explain our key results, consider an environment where a division manager j expects the other manager i to acquire and provide information about their own market. If market characteristics, such as consumer preferences, are highly correlated, then the information provided has a significant impact on manager j's decision. If manager i shirks and does not acquire information, the distortion in manager j's decision can be substantial. This distortion is greater than the distortion for the division whose manager did not acquire information. To ensure local information is acquired and shared, the headquarters manager optimally links each manager's pay to the performance of the other manager.

In contrast, if the organization operates in highly heterogeneous markets where each agent's signal has limited value for the other agent's action, then the headquarters manager optimally remunerates each manager only for their own performance.

To understand the first result, suppose that local states are perfectly correlated, so that each agent's researched information is equally valuable for both agents' decisions. Suppose agent i shirks and does not exert effort in collecting information. In the principal's optimal equilibrium, agent j exerts effort researching information, reports it to i, and expects i to do the same. To fulfill this expectation, agent i has to put together something to report to j, which ends up being just noise since it is not based on research. Agent j takes i's report seriously and uses it to make decisions, alongside his own costly and valuable research. Agent j would make a better decision if he ignored agent i's report and based his decision only on his own research. Agent i bases his decision on j's report only. The shirking agent i does more damage to j's performance than his own, as he not only fails to provide useful information but also biases j's decision. As a result, the most effective incentive to prevent agent i from shirking information acquisition is to make his payment sensitive not to his own performance, but to the performance of his peer j.

This finding holds true only when the states are sufficiently correlated. When the states are only weakly correlated, the value of the non-shirking agent's (agent j) information for the shirking agent's (agent i) decision decreases, and the bias induced on agent j's decision by the noise in agent i's report is also low. In such cases, if an agent i shirks information acquisition, it harms his own performance more than agent j's performance. Therefore, the most effective incentive to prevent shirking is to make each agent i's payment dependent on their own performance.

As our second result, we show that the above characterization is not limited to the case of low information acquisition costs. It is valid as long as the research cost is not prohibitively high to prevent any information acquisition by the division managers. The only difference is that, for such intermediate research costs, the profit maximizing contract is such that only one division manager acquires and shares information. But, again, if the states are sufficiently correlated, this is optimally achieved by making that division manager's remuneration depend on the other agent's performance.

The results described so far refer to the setup in which division managers would be unwilling to reveal that they shirked their assignments, were they to deviate and not acquire information. However, we also studied a setup in which managers would inform each other that they shirked if they did not acquire information.

Our third set of results shows that in this case, the optimal contract always links each manager's remuneration only to the performance of their own division. As long as the states are imperfectly correlated, a shirking manager who reveals himself to be uninformed does more damage to his own division's performance than to the performance of the other division. Therefore, the best way to prevent shirking of information acquisition is to link the manager's remuneration to his own division's performance.

Importantly, we demonstrate that *preventing* reporting of shirking behavior results in higher profits for the organization as a whole compared to when the design allows managers to reveal their lack of information. This impedes collusive off-path behavior of shirking managers and fosters on-path cooperation in information acquisition and sharing, to the benefit of the entire organization.

Our findings demonstrate that incentivizing information acquisition and sharing among multiple division managers promotes joint performance remuneration, which can increase productivity and profits compared to fixed wage structures and individual performance evaluations. Empirical evidence supports this, as studies such as Kruse (1993), Kandel and Lazear (1992), Che and Yoo (2001) and Alonso et al. (2008) have shown productivity increases in companies that adopt profit sharing plans with joint performance evaluation.

Related literature: Firstly, we relate to the literature on organizational design in the presence of strategic communication (Alonso et al., 2008; Rantakari, 2008). In the economics of organizations, the optimal design of incentives for a team of agents is an important topic. The literature typically assumes that agents attach more weight to the profits of their own division relative to the other division(s). We endogenize those weights and show that a profit-maximizing principal might optimally link agents' incentives to the entire organizational profits instead of just their own division.

Secondly, we relate to the literature on contract design with multiple agents, and explicitly posit that individual effort is placed in costly information acquisition, and that the uncooperative behavior consists of not sharing the acquired information. This setting is regarded as significant when studying cooperation in teams (Che and Yoo, 2001; Lazear, 1989).

A contract can foster competitive or cooperative behavior, or both. Tournaments are typically associated with a competitive contract structure. With risk-neutral agents, tournaments are optimal and result in the same outcomes as piece rates (Lazear and Rosen, 1981). However, in our model, there is no common shock in the agents' performances, so there is no role for tournaments in optimal contracts. The cooperation element in contracts is studied in Holmström and Milgrom (1990); Itoh (1991); Kandel and Lazear (1992); Lazear (1989), among others. In those cases, rewards only contingent on individual performances or on relative performances can harm cooperation and the principal's objectives. Our model of information acquisition and sharing in teams of agents identifies a precise case in which joint performance evaluations improve workers' productivity.

Empirical studies show a strong positive relationship between the adoption of profit-sharing schemes and productivity increase (Ichniowski and Shaw, 2003; Kruse, 1993). For instance, Kruse (1993) presents evidence based on a survey of 500 U.S. companies with publicly traded stock. He documents a productivity increase of about 4.5%-5.5% in companies that adopt profit-sharing plans in the form of cash transfers. The productivity increase is more pronounced in smaller firms and under a larger profit-sharing plan. In the analysis of an apparel factory by Hamilton, Nickerson, and Owan (2003), a move from individual piece rates to team production and team-based incentive pay raised productivity substantially. Interestingly, the high-productivity workers were the first to voluntarily join the newly forming teams. Several case studies compare the effectiveness of different incentive schemes and show the value of joint bonuses. For example, Alonso et al. (2008) discuss the case of the management restructuring of BPX, the oil and gas exploration division of British Petroleum, in the early '90s by the then-head of BPX and future CEO of BP, John Browne.[2]

Our setting relates to a large literature studying environments with cheap talk communication (Crawford and Sobel (1982)) and endogenous information acquisition. In a setting with a single sender and a single receiver Di Pei (2015) shows that when a sender can choose a partitional information structure at a cost, she reveals all her acquired information to the receiver; Argenziano, Severinov, and Squintani (2016) and Deimen and Szalay (2019) study the implications of information acquisition on organizational design. These papers show that communication results in better incentives to acquire signals, compared to delegation. Organizational design in a multi-agent setting is studied in the context of coordinated adaptation by Alonso et al. (2008) and Rantakari (2008) who focus on whether a multi-divisional organization should implement decentralization with horizontal communication, or a centralized architecture. In a two-agent setting where the headquarters can either choose prices or quantities, Alonso, Matouschek, and Dessein (2010) show how different headquarters' choices affect quality of communicated information. In these papers decision-relevant information is exogenous. In contrast, Angelucci (2017) studies a model with two agents and endogenous acquisition of costly information. Different to our setting, in the above papers contractual transfers based on information are absent and so the principal has to rely on different instruments than monetary

[2]Browne decentralized BPX in the early 1990s, creating almost 50 semi-autonomous business units. Initially, since "business unit leaders were personally accountable for their units' performance, they focused primarily on the success of their own businesses rather than on the success of BPX as a whole." (Hansen and Von Oetinger (2001)) To encourage coordination between the business units, BPX established changes in the implicit and explicit incentives of business unit leaders to reward and promote them, not just based on the success of their own division, but also for contributing to the successes of other business units. As a result, "lone stars who deliver outstanding business unit performance but engage in little cross-unit collaboration can survive within BP, but their careers typically plateau." (Hansen and Von Oetinger (2001))." Alonso et al. (2008), page 164-165.

transfers to incentivize her agents. Callander and Harstad (2015) show that it can be optimal for the principal to constrain agents' actions in order to maximize informational sharing. In our setup in contrast, the principal solves the incentive problem of information acquisition and sharing while letting the agents choose their respective actions.

2 Model

An organization comprises a headquarters manager and two division managers. For simplicity, we refer to the division managers as agents 1 and 2 (he/him) and to the headquarters manager simply as the principal (she/her). Each agent i is assigned to a division, in which he chooses an action $y_i \in [0, 1]$. There are two unobserved local states, $\theta_1 \in [0, 1]$ and $\theta_2 \in [0, 1]$. The profit of a division i is $\pi_i(y_i, \theta_i) = \bar{\pi} - \ell_i(y_i, \theta_i)$, where $\ell_i(y_i, \theta_i) = (y_i - \theta_i)^2$ for each agent i. The decision y_i of each agent i generates a higher profit $\pi_i(y_i, \theta_i)$ the more precisely it matches the local state θ_i. The loss $\ell_i(y_i, \theta_i)$ is expressed in a simple quadratic form that is standard in organization design (Dessein, 2002). The principal wants to maximize the separable profit function $\pi = \pi_1(y_1, \theta_1) + \pi_2(y_2, \theta_2)$, which depends on the agent's performances. Since $\ell_i \in [0, 1]$ for each $i = 1, 2$, we set $\bar{\pi} = 1$.

Our results generalize qualitatively to the case in which the profits dependence on performances is asymmetric: $\pi = \lambda_1 \pi_1(y_i, \theta_i) + \lambda_2 \pi_2(y_i, \theta_i)$. Such asymmetries can arise, for example, due to different sizes of the organizational divisions (Rantakari (2008)), the leniency bias of the principal (Bol (2011); Breuer, Nieken, and Sliwka (2013)) or the different prospects of markets where the corresponding divisions operate (Liu and Migrow (2022)).

The local states θ_1, θ_2 are correlated: with probability r, they are identical and randomly drawn from the uniform distribution $U[0, 1]$; with probability $1 - r$, each state is drawn independently from $U[0, 1]$. Before choosing y_i, each agent i can exert a costly effort $c > 0$ that enables him to observe a private signal $s_i \in \{0, 1\}$ about the local state θ_i, such that $\Pr(s_i = 1|\theta_i) = \theta_i$. If an agent exerts no effort, he cannot obtain an informative signal. After the signals are received and before the decisions (y_1, y_2) are taken, the agents can simultaneously communicate with each other using cheap talk messages. We assume that each agent i has an arbitrary large set of messages M_i with a typical element $m_i \in M_i$.

The principal does not observe local states, information acquisition, and communication choices of the agents. She observes the divisional performances. Thus, contracting is based on the performances π_1 and π_2. Specifically, at the beginning of the game the principal offers (and commits to) a linear contract of the form

$$t_i(\ell_i, \ell_j) := w_i - a_i \ell_i - b_i \ell_j = w_i - a_i(y_i - \theta_i)^2 - b_i(y_j - \theta_j)^2, \tag{1}$$

where $w_i \in \mathbb{R}$, $a_i \in \mathbb{R}$, $b_i \in \mathbb{R}$, and $j \neq i$ denotes the other agent.

In the literature, agents are typically protected by limited liability, meaning they cannot be paid negative transfers: $t_i(\ell_i, \ell_j) \geq 0$ for all possible performances $\pi_i \in [0,1]$ and $\pi_j \in [0,1]$. Additionally, normalizing the value of their outside option to zero, each agent i must be willing to accept the contract t_i ex-ante, before deciding whether to acquire signal s_i, and before sending signal m_i and making choice y_i.

Linear contracts are a common tool in organizational design literature. The above linear specification allows for multiple contracts found in organizations. For instance, the contract t_i is a piece-wise linear contract composed of a fixed wage $w_i = \bar{w}_i - a_i - b_i$, a bonus payment $a_i(1-\ell_i)$ that depends on agent i's performance, and a payment $b_i(1-\ell_j)$ based on the other agent j's performance, where $\ell_i = (y_i - \theta_i)^2$ is the loss determined by agent i's imprecise matching of y_i with θ_i. The contract t_i can also be interpreted as a mixture of relative performance evaluation and joint performance evaluation based transfers by letting $\overline{a}_i = (a_i - b_i)/2$ and $\overline{b}_i = (a_i + b_i)/2$ and obtaining: $t_i(\ell_i, \ell_j) = w_i - \overline{a}_i(\ell_i - \ell_j) - \overline{b}_i(\ell_i + \ell_j)$. The parameter $\overline{a}_i$ is a weighting factor for agent i's relative performance, and $\overline{b}_i$ is a weighting factor for the team performance. The higher $\overline{a}_i$ is, the more sensitive agent i's payment $t_i(\ell_i, \ell_j)$ is to the relative loss $(\ell_i - \ell_j)$, and the higher $\overline{b}_i$ is, the more sensitive the payment is to the aggregate loss $(\ell_i + \ell_j)$.

It's worth noting that our communication protocol is the same as communication under decentralized decision-making in Alonso et al. (2008) and Rantakari (2008). However, in Section 5, we also study decentralized sequential communication, where one agent's signal acquisition decision happens after they receive signal-contingent information from the other agent.

The timing of our game proceeds as follows: First, nature privately chooses (θ_1, θ_2), and the principal offers and commits to contracts (t_1, t_2). Second, the agents decide whether to acquire signals (s_1, s_2). Then, they send simultaneous cheap talk messages m_i to each other. Finally, each agent i chooses y_i, performances π_1 and π_2 are publicly observed, and transfers are paid as specified in the contracts. The structure of the model is common knowledge.

We note that, given the contracts (t_1, t_2), multiple equilibria may exist in the agents' game. For example, there is always an equilibrium in which agents do not communicate with each other. As customary, we only consider equilibria with meaningful flow of information.

3 Conditional optimal transfers

We begin by analyzing subgames that follow every possible strategy profile by the agents, where each agent's strategy specifies their information acquisition and communication decision, as well as their optimal action y_i. We assume that, given the principal's choice of contracts t_1 and t_2, the agents coordinate on the equilibrium preferred by the principal. This analysis enables us to characterize the most cost-effective way for the principal to implement a targeted allocation. Towards the end of this section, we also consider other equilibria and their plausibility.

Let us first consider the case where the principal seeks to implement full signal acquisition and complete sharing. The principal's optimization problem is as follows:

$$\max_{t_1,t_2} E[\pi_1(\ell_1(y_1(s_1,s_2),\theta_1)) + \pi_2(\ell_2((y_2(s_2,s_1),\theta_2)) - \sum_{i=1}^{2} t_i(\ell_1(y_1(s_1,s_2),\theta_1),\ell_2(y_2(s_2,s_1),\theta_2))]$$

where the expectation is taken with respect to the tuple $(\theta_1,\theta_2,s_1,s_2)$, subject to the following constraints. First, the contract space is defined by (1). Second, given information $s_i \in \{0,1\}$ and $s_j \in \{0,1\}$, and the common belief that signals are exchanged truthfully, each agent i chooses $y_i\left(s_i,s_j\right)$ to solve:

$$y_i\left(s_i,s_j\right) = \arg\max_{y_i} E[t_i(\ell_i(y_i(s_i,s_j),\theta_i),\ell_j(y_j\left(s_j,s_i\right),\theta_j)|s_i,s_j],$$

where the expectation is taken with respect to θ_i,θ_j (since each agent i knows his own signal and believes that s_j is known to both agents after i receives the message m_j). Third, the communication constraint specifies that i communicates $s_i \in \{0,1\}$ truthfully to j:

$$E[t_i(\ell_i(y_i(s_i,s_j),\theta_i),\ell_j(y_j\left(s_j,s_i\right),\theta_j)|s_i] \geq E[t_i(\ell_i(y_i(s_i,s_j),\theta_i),\ell_j(y_j\left(s_j,1-s_i\right),\theta_j)|s_i],$$

where the expectation is taken with respect to θ_i,θ_j and s_j. Fourth, the information acquisition constraint is:

$$E_{s_i,s_j}[E_{\theta_i,\theta_j}[t_i(\ell_i(y_i(s_i,s_j),\theta_i),\ell_j(y_j(s_j,s_i),\theta_j))|s_i,s_j]] - c \geq$$
$$E_{s_j}[E_{\theta_i,\theta_j}[t_i(\ell_i(y_i(s_j),\theta_i),\ell_j(y_j(s_j,\hat{s}_i),\theta_j))|s_j]]$$

where $\hat{s}_i$ is arbitrary, and can take either value 0 or 1. Fifth, the exogenous limited liability constraint specifies that for all $\ell_i \in [0,1]$, $\ell_j \in [0,1]$,

$$t_i\left(\ell_i,\ell_j\right) \geq 0.$$

Finally, the joint feasibility constraint is:

$$t_1\left(\ell_1,\ell_2\right) + t_2\left(\ell_2,\ell_1\right) \leq 2 - \ell_1 - \ell_2.$$

We begin by noting that each agent $i = 1,2$ aims to minimize the loss $\ell_i = (y_i - \theta_i)^2$ for any $a_i \geq 0$ when choosing the action y_i. It is a dominant strategy for the principal to choose $a_i \geq 0$, as even a small positive a_i would motivate agent i to minimize ℓ_i. At the decision stage, agent i matches y_i to $E_i(\theta_i|s_i,m_j)$, the posterior expectation of θ given his signal s_i and the message m_j that is *presumed* by i to be truthful. We observe here that the expected loss $E(\ell_i|s_i,m_j) = E[(E(\theta_i|s_i,m_j) - \theta_i)^2|s_i,m_j]$, for a given (s_i,m_j), is the residual variance of the state θ_i given the estimator $y_i(s_i,m_j) = E(\theta_i|s_i,m_j)$.

Moving backwards, we consider the incentives for motivating agents to share acquired signals. The following lemma formalizes the result that, given $a_i \geq 0$ for both $i = 1,2$, each agent

i is motivated to truthfully report $m_i = s_i$ by setting $b_i \geq 0$.

Lemma 1. *For the existence of an equilibrium in which each agent $i = 1, 2$ acquires signal s_i, truthfully communicates $m_i = s_i$ to the other agent j, and chooses $y_i = E(\theta_i|s_i, s_j)$, a necessary condition is that $b_i \geq 0$. This condition ensures that agent i does not deviate from truthtelling.*

The result is intuitive: to incentivize truthful communication by agent i, the principal must make i's payoff contingent on j's performance. Since $a_j \geq 0$, agent j chooses y_j to match her expectation of θ_j given her signal s_i and the equilibrium belief that $m_i = s_i$. As communication is costless, an arbitrarily small b_i ensures that an informed agent i sends a truthful message to agent j.

Moving on to discussing incentives for acquiring signals, let u_i represent the expected equilibrium payoff for agent i before observing signal s_i if they choose to acquire it. The expected on-path payoff can be expressed as:

$$u_i(s_i) = \bar{w}_i - (a_i + b_i)E[E(\ell_i|s_i, s_j)|s_i] - c, \tag{2}$$

where the expectation in $E[E(\ell_i|s_i, s_j)|s_i]$ is taken with respect to the signal s_j of the other agent, which is unknown to i when deciding whether to acquire signal s_i.

We prove in Appendix B that due to the model's symmetry across agents and signal realizations, $E[E(\ell_i|s_i, s_j)|s_i]$ remains the same regardless of whether s_i is 0 or 1, and specifically, $E[E(\ell_i|s_i, s_j)|s_i] = \frac{3-r^2}{6(9-r^2)}$. Thus, the unconditional expected loss is also $E[E(\ell_i|s_i, s_j)] = \frac{3-r^2}{6(9-r^2)}$, with the expectation taken with respect to both s_i and s_j. Therefore, we can rewrite the expected on-path payoff for agent i before observing s_i as:

$$u_i = \bar{w}_i - (a_i + b_i)\frac{3 - r^2}{6(9 - r^2)} - c. \tag{3}$$

As the correlation between states θ_1 and θ_2 decreases, s_j becomes less informative about θ_i, causing the decision $y_i(s_i, s_j) = E(\theta_i|s_i, s_j)$ to become less precise about θ_i. This leads to a larger expected loss $E(\ell_i|s_i, s_j)$ for both $s_j = 0$ and $s_j = 1$.

Now suppose that agent i deviates during the signal acquisition stage and obtains no signal about θ_i. Agent j is unaware of this deviation and continues to believe that i acquired s_i, interpreting any message realization $m_i \in M_i$ as meaning that $s_i = 0$ or $s_i = 1$. Similar to Argenziano et al. (2016), the equilibrium language is fixed by the on-path communication strategy.

Agent i's decision is based only on the truthful message $m_j = s_j$ from agent j. His optimal decision is therefore $y_i(s_j) = E(\theta_i|s_j)$, resulting in an expected loss of $E[E(\ell_i|s_j)]$. However, agent j makes a biased decision $y_j(s_j, m_i) = E(\theta_j|s_j, m_i)$, assuming that agent i acquired a signal $s_i \in \{0, 1\}$ and truthfully reported $m_i = s_i$.

If agent i shirks at the information acquisition stage, the resulting misled decision y_j by agent j can lead to a larger expected loss $E[(E(\theta_j|s_j, m_i) - \theta_j)^2]$, compared to the scenario where agent j knows that agent i did not acquire s_i.

In Appendix B, we show that agent i's expected off-path payoff when shirking, is given by:

$$u_i^D = w_i - a_i E[E(\ell_i|s_j)] - b_i E[(E(\theta_j|s_j, m_i) - \theta_j)^2] = w_i - a_i \frac{3 - r^2}{36} - b_i \frac{27 + r^4}{6(9 - r^2)}. \quad (4)$$

Whether agent i's shirking at the information acquisition stage results in a larger or smaller expected loss for division i than for division j, depends on the correlation r between the states θ_i and θ_j. In Appendix B, we demonstrate that there exists a threshold $r_1 \in (0, 1)$ (characterized precisely in the Appendix B), such that the expected loss is smaller for division i than for division j if and only if $r < r_1$. That is, when the states θ_1 and θ_2 are highly correlated, the dominant effect of agent i not acquiring their signal s_i is in misleading the choice y_j of agent j, and not in choosing y_i with less information.

We will now discuss the principal's cost minimization program, which is given by

$$\min_{t_1, t_2} E[t_1(\pi_1, \pi_2) + t_2(\pi_2, \pi_1)],$$

subject to the constraints outlined earlier. To recap, each agent $i = 1, 2$ selects the optimal decision $y_i(s_i, s_j) = E[\theta_i|s_i, s_j]$ only if $a_i \geq 0$, communicates truthfully $m_i = s_i$ only if $b_i \geq 0$, and acquires his signal s_i only if $u_i \geq u_i^D$.

We will focus on a symmetric pair of linear contracts $t_1 = t_2$. This is without loss of generality, as the principal's cost minimization program is linear.[3] By imposing symmetry across agents, the principal's cost minimization program is reduced to the following program, for either $i = 1, 2$, and $j \neq i$:

$$\min_{\substack{a_i, b_i \geq 0 \\ w_i \geq a_i + b_i}} \left\{ w_i - (a_i + b_i) E[E(\ell_i|s_i, s_j)] = w_i - (a_i + b_i) \frac{3 - r^2}{6(9 - r^2)} \right\} \qquad \text{s.t. } u_i \geq u_i^D, \quad (5)$$

Here, $w_i \geq a_i + b_i$ comes from the limited liability constraint, and we use the result obtained earlier that $E[E(\ell_i|s_i, s_j)] = \frac{3 - r^2}{6(9 - r^2)}$.

Proposition 1 (proved in Appendix B) shows that the solution to this program is such that if the agents' states θ_1 and θ_2 are sufficiently uncorrelated (specifically, r is smaller than the threshold r_1), then each agent i is rewarded only for their own performance π_i. For $r > r_1$, each agent i's pay is optimally based on the performance of the other agent, π_j. When the states θ_1 and θ_2 are sufficiently correlated, each agent i's signal s_i is so informative about the other

[3] Each agent i's constraints are linear in the maximization arguments w_i, a_i, and b_i. Hence, the constraint set is convex. Suppose that an asymmetric pair of linear contracts $t_1 \neq t_2$ minimized the sum of expected transfers to the agents. Because the model is symmetric, the antisymmetric pair of contracts $t_1' = t_2$, $t_2' = t_1$ is also optimal. The constraint set being convex, it contains the symmetric pair of contracts obtained by averaging these two pairs. Since the objective is linear, this symmetric pair of contracts is also optimal.

agent's state θ_j that it is optimal to base agent i's remuneration mainly on the performance of the other agent.

Proposition 1. *The optimal contract t_1, t_2 for each agent $i = 1, 2$ to acquire his signal s_i, truthfully transmit $m_i = s_i$ to the other agent j, and optimally choose $y_i = E(\theta_i | s_i, m_j)$ in the principal-preferred equilibrium is as follows:*[4]

For $r < r_1$, every optimal contract t_i is such that $w_i = a_i > 0$ and $b_i = 0$. Each agent i's remuneration is based only on their own performance π_i.

For $r > r_1$, the optimal contract t_i is such that $a_i = 0$ and $w = b_i > 0$. Each agent i's remuneration is based only on the performance of the other agent, π_j.

We assume information acquisition in a standard framework, where an agent who receives a signal forms a noisy estimate of the underlying state. We use a beta-binomial setup, which is widely used in the literature on information acquisition due to the tractability of the summary statistics. In Section **??**, we show that Proposition 1 holds under fairly weak conditions on the general statistical model. In essence, in a setting where the local states are sufficiently correlated, each signal realization must contain some information about the underlying state.

An alternative way to model information acquisition, following Aghion and Tirole (1997), allows for the possibility that agents observe entirely uninformative signals with some probability. We can show that our main result, Proposition 1, holds in a variety of models that allow agents to receive uninformative signal realizations, as long as the probability of being uninformed is sufficiently small. For example, consider a framework similar to ours, but where agents may fail to observe the outcome of the trial with some probability. In the principal's preferred equilibrium, each agent either reports observing one of the trial outcomes (say, $s_i = 0$), or they report not observing that outcome. In the latter case, the agent pools the outcome $s_i = 1$ with the outcome of not observing s_i. In practice, when not observing s_i, agent i pretends that $s_i = 1$.

Having determined the optimal contracts for both agents i to acquire and share signal s_i with the other agent j, we now move on to characterizing the optimal contracts for the two remaining cases where both agents acquire information. In the first case, only one agent shares their information with the other agent, and in the second case, neither agent shares their information. The principal's optimization approach for the first case is similar to that of the previous case, and is stated in the proof of Proposition 2.

For an agent i who is not intended to share signal s_i with the other agent j, the optimal contract t_i is such that $w_i = a_i > 0$ and $b_i = 0$ for all values of the correlation coefficient r. Since the principal does not want agent i to share their information with agent j, the optimal contract is based solely on i's performance.

However, if agent i is incentivized to share signal s_i, the optimal contract involves a trade-off

[4] The precise value of the solutions a_1 and b_1, the threshold r_1, and the analogous solutions and thresholds in the following results are expressed in Appendix B.

due to information acquisition constraints. For low values of correlation among the states θ_1 and θ_2, the optimal contract requires that $w_i = a_i > 0$ and $b_i = 0$. When θ_1 and θ_2 are highly correlated, the optimal contract is such that $a_i = 0$ and $w_i = b_i > 0$.

Proposition 2. *To induce each agent to acquire a signal, and only one of them, say agent j, to transmit $m_j = s_j$ to the other agent $-j$, while ensuring that both agents choose (y_1, y_2) optimally in the principal's preferred equilibrium, the optimal contract must satisfy the following:*

(i) For agent j (who shares his signal), the optimal contract specifies $w_j = a_j > 0$ and $b_j = 0$ for $r < r_2$, and $a_j = 0$, $w_j = b_j > 0$ for $r > r_2$.

(ii) For agent $-j$ (who does not share his signal), the optimal contract specifies $w_{-j} = a_{-j} > 0$ and $b_{-j} = 0$ for all r.

If, instead, the principal wants to incentivize both agents to acquire their signals and not share them, while ensuring that both agents choose their actions y_1, y_2 optimally, then the optimal contract t_1, t_2 must satisfy $w_i = a_i > 0$ and $b_i = 0$ for each agent $i = 1, 2$, for all values of r.

This result indicates that the optimal contract, in situations where the principal wants both agents to acquire information without sharing it, links each agent's performance to their own output.

The next result characterizes the optimal contracts that induce a single agent i to acquire information and either share it with agent j or not. Agent j is not required to acquire information. The characterization in Proposition 3 is similar to the case for two agents (Propositions 1 and 2), but the optimal contract for agent j is independent of the profits π_i and π_j. That is, it has $w_j = a_j = b_j = 0$.

Proposition 3. *To induce agent j to acquire signal s_j and share it with agent $-j$, without agent $-j$ acquiring information, and to ensure that both agents choose their actions (y_1, y_2) optimally in the principal's preferred equilibrium, the optimal contract must satisfy the following:*

(i) For agent j (who shares his signal), the optimal contract specifies $w_j = a_j > 0$ and $b_j = 0$ for $r < r_3$ and $a_j = 0$, $w_j = b_j > 0$ for $r > r_3$.

(ii) For agent $-j$ (who does not share his signal), the optimal contract specifies $w_{-j} = a_{-j} = b_{-j} = 0$ for all r.

If, instead, the principal wants to incentivize only agent j to acquire and <u>not</u> share his signal, and for agent $-j$ not to acquire information, while ensuring that both agents choose their actions (y_1, y_2) optimally in the principal's preferred equilibrium, then the optimal contract must satisfy $w_j = a_j > 0$, $b_j = 0$, and $w_{-j} = a_{-j} = b_{-j} = 0$.

Finally, the optimal contracts t_1, t_2 in the case where both agents $i = 1, 2$ are not supposed to acquire information are such that $w_i = a_i = b_i = 0$. The only role played by contracts is to ensure that each agent i matches y_i with the state θ_i to the best of the shared knowledge that θ_i is uniformly distributed on $[0, 1]$. Because neither agent i derives any (dis-)utility from the decision y_i and state θ_i, this objective can be achieved with any $w_i = a_i \geq 0$ and $b_i = 0$.

Equilibrium selection

The preceding analysis assumes that agents coordinate on the principal's preferred equilibrium, given the principal's choice of contracts t_1 and t_2. Under this assumption, if an agent i shirks and does not acquire their signal s_i, they will not reveal to the other agent j that they shirked and have no useful information. Even if such cheating is not contractually sanctioned, hiding it is part of an equilibrium. Agent j expects i to acquire the signal $s_i \in \{0, 1\}$, and interprets any possible message m_i as either meaning that $s_i = 0$ or that $s_i = 1$. This is the equilibrium play preferred by the principal because it makes the information acquisition constraint $u_i \geq u_i^D$ less demanding.

We now consider an alternative equilibrium, called the "agent collusive" equilibrium, in which agents who deviate from the equilibrium path and do not acquire their signals are willing to reveal to each other that they shirked.

The following proposition shows that in the optimal contract resulting from the agent collusive equilibrium, the principal links an agent's remuneration to their own performance π_i, rather than to the performance of the other agent j.

Proposition 4. *Suppose that the agents $i = 1, 2$ play the collusive equilibrium, given the contracts t_1, t_2. Then, the optimal contract to make any agent i acquire their signal s_i, possibly transmit $m_i = s_i$, and then play y_i optimally is such that $w_i = a_i > 0$ and $b_i = 0$. If an agent i is not supposed to acquire their signal s_i, then $a_i = b_i = w_i = 0$. Each agent i's remuneration is only linked to their own performance π_i.*

To understand this result, note that an agent i who shirks and reveals that they did not acquire their signal s_i does more damage to their own division's profit π_i than to the profit of the other division j. This is because agent j is alerted that i has no useful information, and optimally chooses y_j based on their signal s_j only. Even if j is supposed to share s_j with i, the expected loss $E[E(y_j|s_j)]$ of j's division is smaller than the expected loss $E[E(y_i|s_j)]$ of i's division. This is because the states θ_1 and θ_2 are only imperfectly correlated, and hence signal s_j is more informative about θ_j than about θ_i. (Of course, the argument holds even more strongly in case j is not supposed to transmit s_j to i.) Because i's shirking information acquisition damages their own performance π_i more than the other agent j's, the optimal contract t_i links agent i's remuneration to their own performance.

The plausibility of the principal-preferred equilibrium versus the agent collusive equilibrium ultimately depends on the context in which they are applied. When the principal has the means to monitor information transmission across divisions, it is clear that agents would not reveal any shirking behavior as the messages are contractible, and the principal can sanction any sender who admits to shirking. The agent collusive equilibrium is not a viable option in this case due to the organization's well-designed communication channels.

However, if the agents' communications take forms that are harder or illegal to monitor, it may seem attractive for an agent to "collude" by informing agent j that he does not possess any useful information for j's decision y_j. Nevertheless, basic career concerns make it highly unlikely that manager i would admit to j that he shirked his assignments. Reporting a lack of research findings has significant costs for the sender's career prospects, both within internal and external markets. The latter can place a high value on the player's expertise, where a history of information acquisition acts as a proxy for the agent's informativeness (or abilities), and hence can have a considerable impact on the agent's wages.[5]

In internal labor markets, competition for better jobs or wages is likely to lead agent i to never admit to being uninformed, even if the principal cannot contract future job opportunities based on today's performance. This is because many organizations have implicit contracts where future wages and promotions are influenced by the current actions of workers (Holmström, 1999).

Apart from career concerns, agent i may hesitate to confess to agent j that they did not acquire information for another reason. Agent j, who works hard, may feel deceived by agent i's lack of cooperation since it directly reduces agent j's expected compensation. This insight, at least since Lazear (1989), suggests that such deceitful behavior may provoke retaliation among colleagues.

4 Optimal contracts

The optimal information acquisition and communication choices, along with the associated optimal contracts, depend on the values of the information acquisition cost c and state correlation r parameters, as they determine the principal's expected profit. We show that some of the agents' choices are dominated for *all* parameter values of c and r. Specifically, we consider the optimal contracts that make only one agent i acquire and share his signal s_i with the other agent j, while j does not acquire his signal s_j. We demonstrate that these contracts lead to a higher profit than the optimal contracts that make i acquire his signal s_i without sharing it with j, while j does not acquire s_j. For brevity, we will henceforth omit reference to the agents $i = 1, 2$ choosing y_i optimally given their information when describing optimal contracts.

Lemma 2. *For all values of c and r, the expected profit $E\pi_{11}$ of the optimal contracts t_1, t_2 that induce only one agent i to acquire s_i and share it with the other agent j is strictly larger than $E\pi_{10}$, the optimal profit obtained when only one agent i acquires s_i and does not share it with j.*

This result is intuitive because if the optimal contracts t_1 and t_2 only make one agent i acquire his signal s_i, there is no reason not to make him share it with the other agent j. Sharing the signal increases the precision of agent j's decision y_j and thus improves division j's

[5]There is an extensive literature on players' market value in "experts markets" where they attempt to influence market beliefs through cheap talk communication, signaling, or selective disclosure of biographies (Gow, Wahid, and Yu, 2018; Meloso, Nunnari, and Ottaviani, 2018; Ottaviani and Sørensen, 2006).

expected profit. Moreover, it is inexpensive to make agent i share s_i, as this can be achieved with any $b_i \geq 0$.

However, this simple logic does not extend to the optimal contracts that make both agents acquire information. For some c and r, it is not true that the expected profit $E\pi_{22}$ of the optimal contracts that make both agents i acquire s_i and share it with the other agent j is larger than $E\pi_{21}$, the expected profit of optimally inducing both agents i to acquire s_i but only one of them to share it. Nor is it true that $E\pi_{22}$ is larger than $E\pi_{20}$, the expected profit of optimally making both agents i acquire s_i without sharing it.

The reason for this result is as follows. Suppose that both agents $i = 1, 2$ are asked to acquire their signals s_i by the principal. Consider an agent j, and suppose that he expects to receive signal s_i from agent i. Then, the informational benefit of acquiring signal s_j is smaller than when he does not expect to receive s_i. As a result, the contractual transfer needed to make agent j acquire s_j has to reward the precision of agent j's action y_j more than when j does not receive s_i. When the cost of information acquisition c is sufficiently high, it becomes so expensive to simultaneously make agent i send s_i to agent j and agent j acquire s_j that the principal is better off not asking agent i to share s_i with agent j.

Note that this intuition does not apply to the comparison between $E\pi_{11}$ and $E\pi_{10}$ because, in this case, agent j is not asked to acquire s_j by the principal. Further, this intuition does not entirely invalidate the possibility of comparing expected profit in the three cases in which both agents are asked to acquire their signals by the principal. It turns out that for every information cost value c and every correlation value r, the choice of asking both agents i to acquire s_i and only one of them to share s_i with the other agent j is either dominated by asking both agents i to acquire and also share their signals s_i, or by asking both agents i to acquire s_i without sharing it.

Lemma 3. *For all cost c and correlation values r, the expected profit $E\pi_{21}$ of the optimal contracts t_1, t_2 inducing both agents i to acquire s_i and only one of them to share it with the other agent j is (generically strictly) smaller than either $E\pi_{22}$, the optimal profit obtained when both agents $i = 1, 2$ acquire and share s_i, or $E\pi_{20}$, the optimal profit obtained when both agents $i = 1, 2$ acquire s_i without sharing it with the other agent j.*

Given that both agents are asked to acquire their signals, it is either more advantageous to incentivize both agents i to share s_i to improve j's decision precision or not to incentivize any sharing. Since the players are symmetric, it cannot be optimal to ask one agent to share their signal and the other not to.

The optimal contracts inducing the remaining four possible actions (both agents $i = 1, 2$ acquiring and sharing signals s_i, both agents $i = 1, 2$ acquiring s_i without sharing, only one agent i acquiring and sharing s_i, and neither agent $i = 1, 2$ acquiring s_i) maximize the organization's profit in different areas of the parameter space defined by the information acquisition cost c and state correlation r.

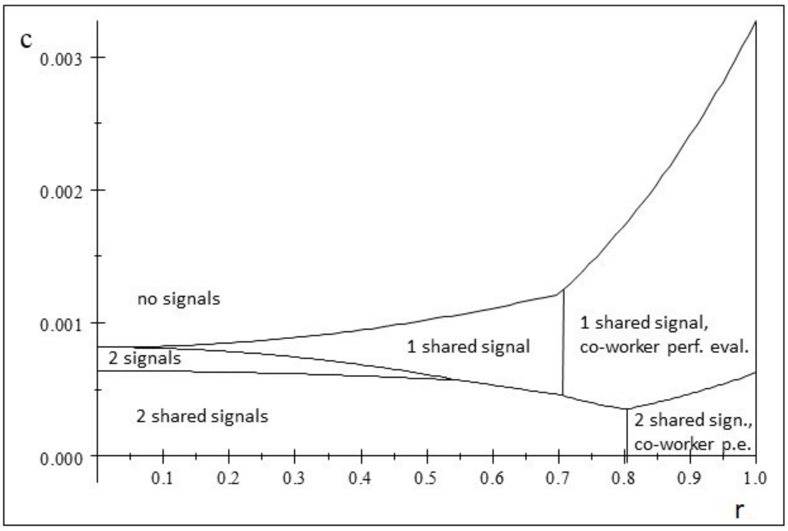

Figure 1: Optimal Linear Contracts

Figure 1: Optimal linear contracts for different signal acquisition and sharing scenarios. Contracts linking agent pay to co-worker performance are represented in regions with 1 or 2 shared signals with co-worker performance evaluation; the other signal sharing regions link agent pay to their own output. The "2 signals" region indicates two-sided signal acquisition with contracts based on agent own performance

The complete characterization is summarized in the following proposition and depicted in Figure 1, which identifies the areas where optimal contracts t_1, t_2 reward information acquisition and sharing by agent i by making i's remuneration dependent on the performance of the other agent.

Proposition 5. *The profit maximizing agents' actions achieved through the optimal linear contracts are as follows.*

1. For research cost $c < c_{22-20}(r)$ and correlation $r < \tilde{r}$, and for $r > \tilde{r}$ and $c < c_{22-11}(r)$, both agents $i = 1, 2$ collect signal s_i and share it with the other agent j.

2. When $r < \tilde{r}$ and $c_{22-20}(r) < c < c_{20-11}(r)$, both agents $i = 1, 2$ collect signal s_i but do not share it with j.

3. When $c_{22-11}(r) < c < c_{11-00}(r)$, for all r, only one agent i collects signal s_i and shares it with j.

4. When $c > c_{11-00}(r)$, for all r, neither agent i collects signal s_i.

We conclude this section by combining Propositions 1-3 and 5 to present the main result of our analysis. We describe the optimal contracts t_1, t_2 that induce agent(s) i to acquire and share information by making i's payment depend on the performance of the other agent.

Corollary 1. *If the states are sufficiently correlated ($r > r'$) and signal acquisition is cheap enough ($c < c(r)_{22-11}$), then $a_i = 0$ and $w_i = b_i > 0$ for both $i = 1, 2$. Each agent $i = 1, 2$ is* *induced to collect signal s_i and share it with the other agent j with a reward based on the other agent j's performance.*

For sufficient state correlation ($r > \sqrt{\frac{33}{67}}$) and intermediate signal costs ($c(r)_{22-11} < c < c(r)_{11-00}$), only one agent i is induced to collect s_i and share with j with a reward based on j's performance. (The other agent receives a flat payment.)

For all other values of r and c, each agent i is induced to collect s_i (and possibly share s_i with j) only with rewards based on agent i's own performance.

This section has determined the company's profit-maximizing contracts and has uncovered an important role for joint performance evaluations. Making one division manager's remuneration depend on the performance of the other division manager may be a very potent incentive for information acquisition and sharing. In the extreme case in which each manager's information is equally useful for both divisions, we have shown that such an incentive is always more potent than remunerating the manager for their own performance.

5 Discussion and conclusions

The notion that division managers should primarily prioritize their own divisions is commonly taken for granted. To the best of our knowledge, this paper is the first to propose endogeneizing the weights that division managers should attach to the different organizational divisions. We propose that the most effective managerial incentives may involve entirely different structures than what is typically assumed. It could be optimal for an organization to base a division manager's rewards primarily on the performance of other divisions, given that the local conditions associated with these divisions are sufficiently interconnected.

Our analysis is relevant within the context of personnel economics as we explore the effectiveness of different incentive schemes in motivating teams of agents involved in information acquisition. Specifically, we examine the impact of rewarding individual performance (piecemeal rates), joint performance, and relative performance (tournaments). One novel conclusion is that the optimal remuneration may require actively discouraging agents from "collusion" that takes the form of revealing each other when they shirk information acquisition assignments.

We have kept the model simple in order to present our findings in the clearest possible manner. For the sake of clarity, we have made the assumption of linear contracts. As a result, the solution to the principal's problem of minimizing costs generically follows a "bang-bang" pattern. This means that either the coefficient a_i is set to its lower bound and b_i is set to its upper bound, or vice versa. The former option is optimal when agent i determines that, by abstaining from acquiring information, they would incur a higher expected loss of profit in division j compared to their own division i. That is, if the agents operate in sufficiently similar environments, it is optimal to set $a_i = 0$ and $b_i > 0$. This choice ensures that all of agent i's remuneration in contract t_i depends solely on the performance of their peer j, denoted as π_j.

In contrast to the clear-cut solution observed in the case of linear contracts, a general non-linear contract scheme is expected to lack such a stark solution. However, we propose that our main result still holds in the following manner: when the states are sufficiently correlated, the principal will primarily connect each agent's performance to the performance of the other division, although not exclusively. This type of arrangement would align with a team-based remuneration system within organizations operating with multiple divisions.

In our working paper version, we explore several extensions to the baseline model. Firstly, we consider an organization where the principal aims to maximize the profit function $\pi = \lambda_1\pi_1 + \lambda_2\pi_2$, with $\lambda_1 \neq \lambda_2$. This distinction allows for the incorporation of the principal's idiosyncratic preferences, such as a leniency bias towards a specific division or an asymmetric organization with divisions of different sizes. We demonstrate that an asymmetric contract, featuring two-sided information acquisition and one-sided communication, can be optimal even when the costs of information acquisition are low.

Furthermore, we analyze a general statistical model characterized by symmetric distributions of the states. We establish conditions under which our main result, concerning low information acquisition costs, extends to more general statistical models. In essence, three conditions must be met. Firstly, both s_i and s_j must provide information about a state θ_i for each $i = 1, 2$. Secondly, in expectation, s_i should offer more information about θ_i compared to s_j. The third condition is more stringent, requiring that for sufficiently correlated states, *each* signal realization s_i should also provide information about the other state θ_j, in addition to the signal s_j. Essentially, there should be no signal realization believed to be uninformative. If such a signal realization were to exist, a deviating agent i would be unable to negatively impact the performance of the other division.

Lastly, we examine decentralized sequential communication, where one of the agents possesses superior knowledge about their local state compared to the other agent, when deciding whether to engage in costly research. We demonstrate that the principal favors decentralized simultaneous communication, akin to our baseline scenario, over the decentralized sequential communication setup.

Our current framework offers multiple avenues for further extension. One potential direction is to introduce skill diversity into the model. Empirical studies have demonstrated that implementing team-based compensation in heterogeneous teams can significantly enhance organizational outcomes.[6] However, the exact mechanism behind this improvement remains an open question, and it can be explored through the lens of optimal contract design in an environment with costless information transmission.

Finally, our analysis assumes transfers as a viable instrument for aligning incentives within an organization. A pertinent question that can be addressed within the current framework is how a principal should design an organization when they cannot commit to transfers. In such cases, the principal must rely on alternative instruments. For example, if the principal

[6] See, e.g., Hamilton et al. (2003) and Boning, Ichniowski, and Shaw (2007).

possesses the power to delegate decision rights, it would be interesting to examine incentives for information acquisition and sharing within a centralized decision rights architecture. Another approach to align incentives within an organization is to design communication structures where the principal can choose which agents communicate with each other and possibly determine the order of communication.

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Appendix A: Beliefs Updating

It is useful to see how the players update their beliefs based on obtained signals and received messages. Suppose, first, that only agent 1 obtains a signal. The posterior density of θ_1 given s_1 is obtained via Bayes rule:

$$f(\theta_1|s_1) = \frac{f(\theta_1)f(s_1|\theta_1)}{\int_0^1 f(s_1|\theta_1)d\theta_1}, \quad f(s_1|\theta_1) = \theta_1^{s_1}(1-\theta_1)^{1-s_1}.$$

Thus, for $s_1 = 0$ the density is $f(\theta_1|s_1) = 2(1-\theta_1)$ with the expected value $E[\theta_1|s_1] = \frac{1}{3}$ and for $s_1 = 1$ the density is $f(\theta_1|s_1) = 2\theta_1$ with the expected value $E[\theta_1|s_1] = \frac{2}{3}$.

Next, suppose that only agent 2 obtains a signal and truthfully communicates it to agent 1. The posterior density of agent 1 is

$$f(\theta_1|s_2) = \frac{f(\theta_1)f(s_2|\theta_1)}{\int_0^1 f(s_2|\theta_1)d\theta_1}, \quad f(s_2|\theta_1) = r\,\underbrace{\theta_1^{s_2}(1-\theta_1)^{1-s_2}}_{Pr(s_2|\theta_1)|\theta_1=\theta_2} + (1-r)\underbrace{(1/2)}_{Pr(s_2|\theta_1)|\theta_1\neq\theta_2}.$$

The densities and the expected values of θ_1 depending on the realization of $s_2 \in \{0,1\}$ are

$$f(\theta_1|s_2=0) = 1 + r(1-2\theta_1), \quad E(\theta_1|s_2=0) = \frac{3-r}{6},$$
$$f(\theta_1|s_2=1) = 1 - r(1-2\theta_1), \quad E(\theta_1|s_2=1) = \frac{3+r}{6}.$$

Naturally, if $r = 0$ then the posterior $f(\theta_1|s_2)$ is equal to the prior. For $r > 0$ and $s_2 = 0$ ($s_2 = 1$) the posterior puts a larger mass to the left (right) of $\frac{1}{2}$. As r increases, the expected value converges to $\frac{1}{3}$ ($\frac{2}{3}$).

The conditional distributions that agent 1 assigns to the signal realization of agent 2 are

$$\begin{aligned}\Pr(s_2=1|s_1=1) &= r\Pr(s_2=1|s_1=1,\theta_1=\theta_2) + (1-r)\Pr(s_2=1|s_1=1,\theta_1\neq\theta_2)\\ &= r\frac{2}{3} + (1-r)\frac{1}{2} = \frac{3+r}{6},\\ \Pr(s_2=0|s_1=1) &= r\frac{1}{3} + (1-r)\frac{1}{2} = \frac{3-r}{6}.\end{aligned}$$

Suppose that both agents acquire and truthfully communicate their signals. We consider θ_1, the case for θ_2 is symmetric. The density of θ_1 after obtaining s_1 and receiving $m_2 = s_2$ is

$$f(\theta_1|s_1,s_2) = \frac{f(\theta_1,s_1,s_2)}{f(s_1,s_2)} = \frac{f(s_1,s_2|\theta_1)f(\theta_1)}{\int_0^1 f(s_1,s_2|\theta_1)f(\theta_1)d\theta_1}.$$

To derive $f(s_1,s_2|\theta_1)$ notice that the following. First, the ex ante probability of $s_1+s_2=0$ and $s_1+s_2=2$ (it means when $s_1=s_2$) is $\frac{1}{3}$ each, whereas the ex ante probability of both signals being different is $\frac{1}{6}$. To see this notice that $\Pr(l|n=2) = \int_0^1 \Pr(l|\theta_1,n=2)d\theta_1 = \frac{1}{n+1}$ and that *conditional* on a particular l all sequences of signals which result in the same sum of signals l are equiprobable.

Second, if both states are correlated which happens with probability r, the probability of $l = s_1 + s_2$ is $\frac{n!}{l!(n-l)!}\theta_1^l(1-\theta_1)^{n-l}$. With the converse probability $1-r$ the probability of s_1 is $\theta_1^{s_1}(1-\theta_1)^{1-s_1}$ and the realization of s_2 is independent of θ_1 (and so of s_1) and $E[s_2|s_1]$ is equal to $\frac{1}{2}$.

Therefore, for $s_1 + s_2 = l \in \{0, 2\}$ we have

$$f(s_1, s_2|\theta_1) = r \underbrace{[\theta_1^l(1-\theta_1)^{2-l}]}_{Pr(s_1,s_2|(\theta_1,\theta_1=\theta_2))} + (1-r)\underbrace{[\theta_1^{s_1}(1-\theta_1)^{1-s_1}]\frac{1}{2}}_{Pr(s_1,s_2|(\theta_1,\theta_1\neq\theta_2))}$$

and for $s_1 + s_2 = 1$ we have

$$f(s_1, s_2|\theta_1) = r \underbrace{\frac{1}{2}[2\theta_1(1-\theta_1)]}_{Pr(s_1,s_2|(\theta_1,\theta_1=\theta_2))} + (1-r)\underbrace{[\theta_1^{s_1}(1-\theta_1)^{1-s_1}]\frac{1}{2}}_{Pr(s_1,s_2|(\theta_1,\theta_1\neq\theta_2))}$$

The corresponding densities of the posterior are, for $s_1 + s_2 = l \in \{0, 2\}$

$$\begin{aligned}
f(\theta_1|s_1, s_2) &= \frac{r\theta_1^l(1-\theta_1)^{2-l} + (1-r)\theta_1^{s_1}(1-\theta_1)^{1-s_1}\frac{1}{2}}{\int_0^1 r\theta_1^l(1-\theta_1)^{2-l} + (1-r)\theta_1^{s_1}(1-\theta_1)^{1-s_1}\frac{1}{2}d\theta_1} \\
f(\theta_1|s_1, s_2) &= \frac{r\frac{1}{2}2\theta_1(1-\theta_1) + (1-r)\theta_1^{s_1}(1-\theta_1)^{1-s_1}\frac{1}{2}}{\int_0^1[r\frac{1}{2}2\theta_1(1-\theta_1) + (1-r)\theta_1^{s_1}(1-\theta_1)^{1-s_1}\frac{1}{2}]d\theta_1}.
\end{aligned}$$

The calculations for θ_2 are symmetric.

Assume two efforts and truthful communication. The corresponding posteriors, and the expected values for agent 1 (the analysis for agent 2 is analogous) are:

$$\begin{aligned}
f(\theta_1|s_1 = s_2 = 0) &= \frac{r[\frac{2!}{0!(2-0)!}\theta_1^0(1-\theta_1)^{2-0}] + (1-r)[\frac{1!}{0!(1-0)!}\theta_1^0(1-\theta_1)^{1-0}]\frac{1}{2}}{\frac{3+r}{12}} \\
&= \frac{6(1-\theta_1)(1+r-2r\theta_1)}{3+r}, \\
E(\theta_1|s_1 = s_2 = 0) &= \int_0^1 \theta_1\frac{6(1-\theta_1)(1+r-r2\theta_1)}{3+r}d\theta_1 = \frac{1}{3+r}.
\end{aligned}$$

Further,

$$\begin{aligned}
f(\theta_1|s_1 = 0, s_2 = 1) &= \frac{6(1-\theta_1)(1-r+r2\theta_1)}{3-r}, \quad E(\theta_1|s_1 = 0, s_2 = 1) = \frac{1}{3-r}. \\
f(\theta_1|s_1 = 1, s_2 = 0) &= \frac{6\theta_1(1+r-2r\theta_1)}{3-r}, \quad E(\theta_1|s_1 = 1, s_2 = 0) = \frac{2-r}{3-r}. \\
f(\theta_1|s_1 = s_2 = 1) &= \frac{6\theta_1(1-r+2r\theta_1)}{3+r}, \quad E(\theta_1|s_1 = s_2 = 1) = \frac{2+r}{3+r}.
\end{aligned}$$

Appendix B: Proofs

Proof of Lemma 1 and Derivation of Equation 3 Recall that under $a_1 \geq 0, a_2 \geq 0$, the optimal actions are $y_i(s_i, s_j) = E(\theta_i|s_i, s_j)$. Given the optimal choices (y_1, y_2) consider the incentives of agent $i = 1, 2$ at the communication stage if he holds a signal s_i. For a common belief that both agents are truthful each agent i chooses an action that matches his posterior of θ_i, given his own private signal s_i and the message from agent j, m_j. The expected payoff is then

$$\begin{aligned} u_i(s_i) = w_i - a_i \sum_{s_j=0,1} \Pr(s_j|s_i) E[(E(\theta_i|s_i, s_j) - \theta_i)^2|s_i, s_j] \\ -b_i \sum_{s_j=0,1} \Pr(s_j|s_i) E[(E(\theta_j|s_j, s_i) - \theta_j)^2|s_i, s_j] - c, \end{aligned}$$

where the expected losses are conditioned on truthful communication. Because of the symmetry across the agents, $E(\ell_j|s_i, s_j) = E[(E(\theta_j|s_i, s_j) - \theta_j)^2|s_i, s_j] = E[(E(\theta_i|s_i, s_j) - \theta_i)^2|s_i, s_j]$ when $s_i = s_j$. Adding symmetry across signal realizations, it is also the case that $E[(E(\theta_j|s_j, s_i) - \theta_j)^2|s_j, s_i] = E[(E(\theta_i|s_i, s_j) - \theta_i)^2|s_i, s_j]$ when $s_i \neq s_j$. By letting

$$E[E(\ell_i|s_i, s_j)|s_i] = \sum_{s_j=0,1} \Pr(s_j|s_i) E(\ell_i|s_i, s_j)$$

we obtain Equation 3.

We now calculate $E[E(\ell_i|s_i, s_j)|s_i]$, taking the case $s_i = 0$. The case $s_i = 1$ is symmetric. We begin by calculating $E(\ell_i|s_i, s_j)$, for $s_j = 0, 1$. As shown in Appendix A,

$$\begin{aligned} E(\theta_i|s_i = 0, s_j = 0) = \frac{1}{3+r}, \quad f(\theta_i|s_i = 0, s_j = 0) = \frac{6(1-\theta_i)(1+r-2r\theta_i)}{3+r}, \\ E(\theta_i|s_i = 0, s_j = 1) = \frac{1}{3-r}, \quad f(\theta_i|s_i = 0, s_j = 1) = \frac{6(1-\theta_i)(1+r-2r\theta_i)}{3-r}. \end{aligned}$$

Substituting in the expected losses definitions and simplifying, we obtain,

$$\begin{aligned} E(\ell_i|s_i = 0, s_j = 0) = \int_0^1 (E(\theta_i|s_i = 0, s_j = 0) - \theta_i)^2 f(\theta_i|s_i = 0, s_j = 0) d\theta_i = \frac{5+2r-r^2}{10(3+r)^2} \\ E(\ell_i|s_i = 0, s_j = 1) = \frac{5-2r-r^2}{10(3-r)^2}. \end{aligned}$$

Substituting the expected losses into the agent's payoff function, together with the conditional posteriors assigned to signal s_j (see Appendix A),

$$\Pr(s_j = 1|s_i = 1) = \frac{3+r}{6}, \ \Pr(s_j = 0|s_i = 1) = \frac{3-r}{6},$$

and simplifying, we obtain

$$E[E(\ell_i|s_i,s_j)|s_i] = \sum_{s_j=0,1} \Pr(s_j|s_i)E(\ell_i|s_i,s_j) = \frac{3-r^2}{6(9-r^2)}.$$

If agent i deviates at the communication stage and informs agent j that his signal is $1-s_i$ instead of the true signal s_i he expects the payoff

$$\begin{aligned} u_i^L(s_i) &= w_i - a_i \sum_{s_j=0,1} \Pr(s_j|s_i)E[(y_i(s_i,s_j)-\theta_i)^2|s_i,s_j] \\ & -b_i \sum_{s_j=0,1} \Pr(s_j|s_i)E[(y_j(s_j,1-s_i)-\theta_j)^2|s_j,s_i]. \end{aligned}$$

Of course, the expression for $\sum_{s_j=0,1} \Pr(s_j|s_i)E[(y_i(s_i,s_j)-\theta_i)^2|s_i,s_j]$ is unchanged. We calculate $\sum_{s_j=0,1} \Pr(s_j|s_i)E[(y_j(s_j,1-s_i)-\theta_j)^2|s_j,s_i]$ assuming that $s_i=0$ as the case $s_i=1$ is symmetric. The agents' decisions and densities for $s_j=1$, and $s_j=0$, are, respectively:

$$\begin{aligned} y_j(s_j=1,m_i=1) &= E(\theta_j|s_j=1,s_i=1) = \frac{2+r}{3+r}, \quad f(\theta_j|s_j=1,s_j=0) = \frac{6\theta_j(1+r-2r\theta_j)}{3-r}, \\ y_j(s_j=0,m_i=1) &= \frac{1}{3-r}, \quad f(\theta_j|s_j=s_i=0) = \frac{6(1-\theta_j)(1+r-2r\theta_j)}{3+r}. \end{aligned}$$

Hence, we obtain:

$$\begin{aligned} E[(y_j(s_j=1,1-s_i)-\theta_j)^2|s_j=1,s_i] &= \int_0^1 E(\theta_j|s_j=1,s_i=1)f(\theta_j|s_j=1,s_j=0)d\theta_j \\ &= \frac{15+9r+11r^2+r^3}{10(3-r)(3+r)^2}. \\ E[(y_j(s_j=0,1-s_i)-\theta_j)^2|s_i,s_j] &= \frac{15-9r+11r^2-r^3}{10(3-r)^2(3+r)} \end{aligned}$$

Wrapping up:

$$\begin{aligned} \sum_{s_j=0,1} \Pr(s_j|s_i)E[(y_j(s_j,1-s_i)-\theta_j)^2|s_j,s_i] &= \frac{3-r}{6}\cdot\frac{15+9r+11r^2+r^3}{10(3-r)(3+r)^2} + \frac{3+r}{6}\cdot\frac{15-9r+11r^2-r^3}{10(3-r)^2(3+r)} \\ & \frac{(9+r^2)(3+r^2)}{6(9-r^2)^2}. \end{aligned}$$

The expected deviation payoff can be written as

$$u_i^L(s_i) = w_i - a_i\frac{3-r^2}{6(9-r^2)} - b_i\frac{(9+r^2)(3+r^2)}{6(9-r^2)^2}$$

so that agent 1 does not deviate at the communication stage if

$$w_i - (a_i+b_i)\frac{3-r^2}{6(9-r^2)} \geq w_i - a_i\frac{3-r^2}{6(9-r^2)} - b_i\frac{(9+r^2)(3+r^2)}{6(9-r^2)^2}$$

which implies $b_i \frac{4r^2}{(9-r^2)^2} \geq 0$, or $b_i \geq 0$.
Q.E.D.

Derivation of Equation 4. First, we calculate the expected loss $E(\ell_i|s_j) = E[(E(\theta_i|s_j) - \theta_i)^2|s_j]$, supposing $s_j = 0$ (the case $s_j = 1$ is symmetric). From Appendix A,

$$f(\theta_i|s_j = 0) = 1 + r(1 - 2\theta_i), \quad E(\theta_i|s_j = 0) = \frac{3-r}{6}.$$

Substituting in the expected loss definition and simplifying, we obtain:

$$E[(E(\theta_i|s_j) - \theta_i)^2|s_j] = \int_0^1 (E(\theta_i|s_j) - \theta_i)^2 f(\theta_i|s_j) d\theta_i = \frac{3-r^2}{36}.$$

Because, $E(\ell_i|s_j) = \frac{3-r^2}{36}$ is the same regardless of whether $s_j = 0$ or $s_j = 1$, we also obtain that $E[E(\ell_i|s_j)] = \frac{3-r^2}{36}$.

To calculate the expected loss $E[(E(\theta_j|s_j, m_i) - \theta_j)^2|s_j]$, we assume w.l.o.g. that $m_i = 0$. From Appendix A, the conditional densities and expected values are:

$$f(\theta_j|s_j = 0) = 2(1 - \theta_j) \quad E(\theta_j|s_j = 0, m_i = 0) = \frac{1}{3+r},$$
$$f(\theta_j|s_j = 1) = 2\theta_j \quad E(\theta_j|s_j = 1, m_i = 0) = \frac{2-r}{3-r},$$

the consequent expected losses are:

$$\begin{aligned} E[(E(\theta_j|s_j = 0, m_i = 0) - \theta_j)^2|s_j = 0] &= \int_0^1 (E(\theta_j|s_j = 0, m_i = 0) - \theta_j)^2 f(\theta_j|s_j = 0) d\theta_j \\ &= \frac{3 + 2r + r^2}{6(3+r)^2}. \\ E[(E(\theta_j|s_j = 1, m_i = 0) - \theta_j)^2|s_j = 1] &= \frac{3 - 2r + r^2}{6(3-r)^2}. \end{aligned}$$

Plugging the expected loss formulas into the unconditional loss formula, we obtain:

$$\sum_{s_j=0,1} \Pr(s_j) E[(E(\theta_j|s_j = 0, m_i = 0) - \theta_j)^2|s_j = 0] = \frac{27 + r^4}{6(9-r^2)^2}.$$

Appropriate rearranging yields Equation 4.
Q.E.D.

Proof of Proposition 1. Program 5 is obtained through the same process of simplification used to obtain equation 3, and based on the model's symmetry across agents and signal realizations. By linearity of the objective function and the information acquisition constraint $u_i \geq u_i^D$, the solution to the program 5 involves either $a_i > 0,\ b_i = 0$, or $a_i = 0, b_i > 0$. In each case $w_i = a_i + b_i$ and the constraint $u_i \geq u_i^D$ binds.

If $w_i = a_i > 0$, and $b_i = 0$, then the constraint $u_i = u_i^D$ becomes $a_i = \frac{36c(9-r^2)}{(3-r^2)^2}$ and the expected transfer to agent i results in

$$w_i - (a_i + b_i)E[E(\ell_i|s_i, s_j)] = \frac{36c(9-r^2)}{(3-r^2)^2}\left(1 - \frac{3-r^2}{6(9-r^2)}\right) = \frac{6c(51-5r^2)}{(3-r^2)^2},$$

and we verify the ex-ante agent i's participation constraint, because: $w_i - (a_i+b_i)E[E(\ell_i|s_i,s_j)] - c = c\frac{(9-r^2)(r^2+33)}{(3-r^2)^2} \geq 0$ for all $c \geq 0$.

If $a_i = 0, w_i = b_i > 0$, then the constraint $u_i = u_i^D$ becomes $b_i \frac{2r^2}{(9-r^2)^2} = c$. The expected transfer to agent i becomes:

$$w_i - (a_i + b_i)E[E(\ell_i|s_i, s_j)] = \frac{(9-r^2)^2 c}{2r^2}(1 - \frac{3-r^2}{6(9-r^2)}) = \frac{c(9-r^2)(51-5r^2)}{12r^2},$$

and again, we verify $w_i - (a_i + b_i)E[E(\ell_i|s_i, s_j)] - c \geq 0$ for all $c \geq 0$.

By comparing the two cases, we have $\frac{36c(9-r^2)}{(3-r^2)^2} < (>) \frac{(9-r^2)^2 c}{2r^2}$ for $r < (>) r_1$ where

$$r_1 \equiv \sqrt{5 - 10\sqrt[3]{\frac{2}{9\sqrt{29}-43}} + 2^{2/3}\sqrt[3]{9\sqrt{29}-43}} \approx 0.803.$$

As a result, in the case of $r > r_1$ the expected principal's payoff is:

$$\begin{aligned} E\pi_{22} &= E[2\,(1 - E(\ell_i|s_i, s_j))] - 2\bar{w}_i - 2(a_i + b_i)E[E(\ell_i|s_i, s_j)] \\ &= 2 - \frac{3-r^2}{3(9-r^2)} - \frac{(51-5r^2)(9-r^2)}{6r^2}c. \end{aligned}$$

When $r < r_1$, the expected principal's payoff is:

$$E\pi_{22} = 2 - \frac{3-r^2}{3(9-r^2)} - \frac{12(51-5r^2)}{(3-r^2)^2}c.$$

Q.E.D.

Proof of Proposition 2. We distinguish 2 cases.

Case '21': Two-sided acquisition and one-sided sharing.
We calculate the optimal linear contracts t_1, t_2 to induce both agents $i = 1, 2$ to acquire information and only agent, say 1, to transmit it to the other agent. First note that, again, each agent $i = 1, 2$ is motivated to choose decision y_i so as to minimize the loss $\ell_i = (y_i - \theta_i)^2$ by setting $a_i \geq 0$. Likewise, $b_1 \geq 0$ is needed so that 1 reports s_1 truthfully to 2.

The principal's cost minimization problem is:

$$\min_{\substack{a_i \geq 0, \\ b_1 \geq 0,\ w_i \geq a_i + b_i}} \quad w_1 - (a_1 + b_1)E[E(\ell_1|s_1)] + w_2 - (a_2 + b_2)E[E(\ell_2|s_2, s_1)], \text{ s.t. } u_i \geq u_i^D. \tag{6}$$

Let us consider agent 2's information acquisition stage constraint $u_2 \geq u_2^D$. The equilibrium payoff of agent 2 is, using $m_1 = s_1$,

$$\begin{aligned} u_2 &= w_2 - a_2 E[(y_2(s_2, s_1) - \theta_2)^2] - b_2 E[(y_1(s_1) - \theta_1)^2] - c \\ &= w_2 - a_2 \frac{3 - r^2}{6(9 - r^2)} - b_2 \frac{1}{18} - c. \end{aligned}$$

If agent 2 deviates at the information acquisition stage, her payoff is

$$\begin{aligned} u_2^D &= w_2 - a_2 E[(y_2(s_1) - \theta_2)^2] - b_2 E[(y_1(s_1) - \theta_1)^2] \\ &= w_2 - a_2 \frac{3 - r^2}{36} - b_2 \frac{1}{18}. \end{aligned}$$

using $$\begin{aligned} E[(y_2(s_1) - \theta_2)^2] &= \sum_{s_1 = 0,1} \frac{1}{2} \int_0^1 (E[\theta_2|s_1] - \theta_2)^2 f(\theta_2|s_1) d\theta_2 \\ &= \int_0^1 (E[\theta_2|s_1 = 0] - \theta_2)^2 f(\theta_2|s_1 = 0) d\theta_2 = \frac{3 - r^2}{36}, \end{aligned}$$

because $f(\theta_2|s_1 = 0) = 1 + r(1 - 2\theta_2)$ and $E(\theta_2|s_1 = 0) = \frac{3-r}{6}$ (see Appendix A).

The constraint $u_2 \geq u_2^D$ is thus: $a_2 \frac{1}{36} \frac{(3-r^2)^2}{9-r^2} \geq c$. This yields the optimal contract for agent 2: $w_2 = a_2 = 36 \frac{9-r^2}{(3-r^2)^2} c$ and $b_2 = 0$. The agent's ex-ante participation constraint is satisfied as an equality.

Then, we consider the optimal contract of agent 1. Again, we note that he does not deviate from truthtelling if and only if $b_1 \geq 0$. Turning to the information acquisition constraint, we note that the equilibrium payoff of agent 1 is:

$$\begin{aligned} u_1 &= w_1 - a_1 E[(y_1(s_1) - \theta_1)^2] - b_1 E[(y_2(s_2, m_1) - \theta_2)^2] - c \\ &= w_1 - a_1 \frac{1}{18} - b_1 \frac{3 - r^2}{6(9 - r^2)} - c, \end{aligned}$$

using $$\begin{aligned} E[(y_1(s_1) - \theta_1)^2] &= \sum_{s_1 = 0,1} \frac{1}{2} \int_0^1 (E(\theta_1|s_1) - \theta_1)^2 f(\theta_1|s_1) d\theta_1 \\ &= \int_0^1 (E(\theta_1|s_1 = 0) - \theta_1)^2 f(\theta_1|s_1 = 0) d\theta_1 = \frac{1}{18}. \end{aligned}$$

If agent 1 deviates and does not acquire information, then his payoff is:

$$u_1^D = w_1 - a_1 E[(y_1 - \theta_1)^2] - b_1 E[E[(y_2(s_2, m_1) - \theta_2)^2|s_2]]$$

$$w_1 - a_1\frac{1}{12} - b_1\frac{27+r^4}{6(9-r^2)^2},$$

using $E[(y_1-\theta_1)^2] = \int_0^1 (E(\theta_1)-\theta_1)^2 f(\theta_1)d\theta_1 = \frac{1}{12}$.

This yields the information acquisition constraint: $a_1\frac{1}{36} + b_1\frac{2r^2}{(9-r^2)^2} \geq c$.

Because the principal objective function is linear in w_1, a_1 and b_1, and such that $w_1 \geq 0, a_1 \geq 0, b_1 \geq 0$, there are two possibilities: either $w_1 = a_1 > 0$ and $b_1 = 0$, or $a_1 = 0, w_1 = b_1 > 0$.

In the first case, $w_1 = a_1 > 0$ and $b_1 = 0$, the constraint $u_1 \geq u_1^D$ becomes $a_1 \geq 36c$. Using $w_1 = a_1 = 36c$ the expected transfer to agent 1 becomes:

$$a_1\left[1 - E[(y_1(s_1)-\theta_1)^2]\right] = 34c.$$

In the second case, $a_1 = 0, w_1 = b_1 > 0$, and the constraint $u_1 \geq u_1^D$ becomes $b_1 \geq \frac{(9-r^2)^2c}{2r^2}$. Using $w_1 = b_1 = \frac{(9-r^2)^2c}{2r^2}$ and $a_1 = 0$, the expected transfer to agent 1 becomes:

$$b_1\left(1 - E[(y_2(s_2,s_1)-\theta_2)^2]\right) = \frac{(9-r^2)(51-5r^2)c}{12r^2}.$$

In either case, the ex-ante participation constraint is satisfied.

Now, we have

$$34 < (>)\frac{(9-r^2)(51-5r^2)c}{12r^2} \text{ if and only if } r < (>) r_2 = \sqrt{\frac{3}{5}(84-\sqrt{6801})}$$

As a result, we conclude that for $r < r_2$ the principal optimally chooses $w_1 = a_1 > 0$ and $b_1 = 0$. Otherwise, for $r \geq r_2$ the principal optimally chooses $a_1 = 0, w_1 = b_1 > 0$. The principal's payoff is

$$E\pi_{21} = 2 - \frac{9-2r^2}{9(9-r^2)} - \min\left\{36\left(1-\frac{1}{18}\right), \frac{(9-r^2)^2}{2r^2}\left(1-\frac{3-r^2}{6(9-r^2)}\right)\right\}c - \frac{6(51-5r^2)}{(3-r^2)^2}c.$$

Case '20': Two-sided acquisition and no sharing.

We calculate the optimal contract when the principal wants to incentivize both agents $i = 1, 2$ to acquire and not to share their signal s_i. The principal's cost minimization problem is:

$$\min_{\substack{a_i \geq 0,\\ w_i \geq a_i + b_i}} \bar{w}_i - (a_i+b_i)E[(y_i(s_1)-\theta_i)^2] = \min_{\substack{a_i \geq 0,\\ \bar{w}_i \geq a_i + b_i}} \bar{w}_i - (a_i+b_i)\frac{1}{18}, \text{ s.t. } u_i \geq u_i^D.$$

For each agent i the information acquisition stage constraint $u_i \geq u_i^D$ becomes

$$w_i - a_i\frac{1}{18} - b_i\frac{1}{18} - c \geq w_i - a_i\frac{1}{12} - b_i\frac{1}{18}$$

resulting in $w_i = a_i \geq 36c$. The transfer paid to each agent i is $36c(1-\frac{1}{18})$. The agents' ex-ante participation constraint is satisfied as an equality. The principal's payoff is:

$$E\pi_{20} = 2 - \frac{1}{9} - 2a_i(1 - \frac{1}{18}) = 2 - \frac{1}{9} - 2 \cdot 36c(1 - \frac{1}{18}).$$

Q.E.D.

Proof of Proposition 3 and case '00': We distinguish 3 cases.

Case '11': One-sided acquisition and sharing.

We calculate the optimal linear contracts t_1, t_2 to induce agent 1 to acquire signal s_1 and share with the other agent 2, and agent 2 to not acquire information. The optimal contract of agent 2 is, trivially, $t_2(\ell_1, \ell_2) = 0$ for all ℓ_1 and ℓ_2 (the optimal linear contract is such that $w_2 = a_2 = b_2 = 0$). The principal's cost minimization problem for agent 1 is:

$$\min_{\substack{a_1 \geq 0, \geq b_1 \geq 0, \\ w_1 \geq a_1 + b_1}} w_1 - a_1 E[(y_1(s_1) - \theta_1)^2] - b_1 E[(y_2(s_1) - \theta_2)^2], \text{ s.t. } u_1 \geq u_1^D.$$

The equilibrium payoff of agent 1 is:

$$\begin{aligned} u_1 &= w_1 - a_1 E[(y_1(s_1) - \theta_1)^2] - b_1 E[(y_2(s_1) - \theta_2)^2] - c \\ &= w_1 - a_1 \frac{1}{18} - b_1 \frac{3 - r^2}{36} - c \end{aligned}$$

If agent 1 does not acquire information, he still sends a message m_1 to agent 2 who mistakenly believes that $m_1 = s_1$. The equilibrium payoff of agent 1 is:

$$\begin{aligned} u_1^D &= w_1 - a_1 E[(y_1 - \theta_1)^2] - b_1 E[(y_2(m_1) - \theta_2)^2] \\ &= w_1 - a_1 \frac{1}{12} - b_1 \frac{3 + r^2}{36}, \end{aligned}$$

using $E[(y_2(m_1) - \theta_2)^2] = \int_0^1 [(E[\theta_2|m = 0] - \theta_2)^2] f(\theta_2) d\theta_2 = \frac{3 + r^2}{36}$.

Here, the information acquisitions constraint is:

$$-a_1 \frac{1}{18} - b_1 \frac{3 - r^2}{36} - c \geq -a_1 \frac{1}{12} - b_1 \frac{3 + r^2}{36}$$

So, the optimal contract is either $w_1 = a_1 = 36c$ and $b_1 = 0$, or $w_1 = b_1 = \frac{18c}{r^2}$ and $a_1 = 0$.

In case $w_1 = a_1 = 36c$ and $b_1 = 0$, the expected transfer to agent 1 is

$$a_1 \left[1 - E[(y_1(s_1) - \theta_1)^2]\right] = 34c,$$

and the participation constraint is met with equality.

In case, $a_1 = 0, w_1 = b_1 = \frac{18c}{r^2}$, the expected transfer to agent 1 is

$$(a_1 + b_1) \left[1 - E[(y_1(s_1) - \theta_1)^2]\right] = \frac{18c}{r^2} \left(1 - \frac{3 - r^2}{36}\right).$$

As a result, the principal's payoff is

$$E\pi_{11} = 2 - \frac{1}{18} - \frac{1}{36}(3 - r^2) - \min\left\{34, \frac{18}{r^2}(1 - \frac{1}{36}(3 - r^2))\right\} c.$$

Equating $34 = \frac{18}{r^2}(1 - \frac{1}{36}(3 - r^2))$, we obtain the admissible solution $r_3 := \sqrt{\frac{33}{67}}$.

Case '10': One-sided acquisition and no sharing.
We calculate the optimal linear contract to induce agent 1 to acquire signal s_1 and not to share it, and agent 2 not to acquire information. The optimal contract for agent 2 is $t_2(\ell_1, \ell_2) = 0$. The principal's cost minimization problem for agent 1 is:

$$\min_{\substack{a_1 \geq 0 \\ w_1 \geq a_1 + b_1}} w_1 - a_1 E[(y_1(s_1) - \theta_1)^2] - b_1 E[(y_2 - \theta_2)^2], \quad \text{s.t. } u_1 \geq u_1^D.$$

The equilibrium payoff of agent 1 is:

$$u_1 = w_1 - a_1 E[(y_1(s_1) - \theta_1)^2] - b_1 E[(y_2 - \theta_2)^2] = w_1 - a_1 \frac{1}{18} - b_1 \frac{1}{12} - c,$$

his deviation payoff at the information acquisition stage is:

$$u_1^D = w_1 - a_1 E[(y_1 - \theta_1)^2] - b_1 E[(y_2 - \theta_2)^2] = w_1 - a_1 \frac{1}{12} - b_1 \frac{1}{12},$$

so that the incentive compatibility constraint is: $a_1 \geq 36c$, and the optimal linear contract is $w_1 = a_1 = 36c$ and $b_1 = 0$. The principal's expected profit is:

$$E\pi_{10} = 2 - \frac{1}{18} - \frac{1}{12} - 36(1 - \frac{1}{18})c.$$

Case '00': No acquisition.
The optimal linear contracts t_1, t_2 in the case that both agents $i = 1, 2$ are not supposed to acquire information are such that $w_i = a_i = b_i = 0$. This leads to expected principal's profit:

$$E\pi_{00} = 2 - 2\frac{1}{12}.$$

Q.E.D.

Proof of Proposition 4. By the same logic as in Propositions 1 - 3, an agent i who is not expected to acquire a signal s_i, gets zero transfer: $w_i = 0, a_i = 0, b_i = 0$. An agent i expected to acquire a signal s_i, but not to share it, has a transfer linked only to his own performance: $w_i = a_i > 0, b_i = 0$. In the following we study the optimal transfer to any agent i who is expected to acquire and share his signal s_i.

Case '22': Two-sided acquisition and sharing.

The principal's cost minimization program is still (5): it needs to be that $a_i \geq 0$ for either agent to optimally choose $y_i(s_i, m_j)$ and that $b_i \geq 0$ to ensure truthful communication. The expected payoff of agent i if not acquiring signal s_i is:

$$u_i^D = w_i - a_i E[E(\ell_i|s_j)] - b_i E(\ell_i) = w_i - a_i \frac{3-r^2}{36} - b_i \frac{1}{18},$$

because if i does not acquire signal s_i, then she reveals that to j. The agent's equilibrium expected payoff u_i is unchanged, and hence the information acquisition constraint $u_i \geq u_i^D$ takes the following form: $a_i \frac{(3-r^2)^2}{36(9-r^2)} + b_i \frac{r^2}{81-9r^2} \geq c$. The principal's program has a linear objective function and a linear constraint. Because the coefficient $\frac{(3-r^2)^2}{36(9-r^2)}$ of a_i is larger than the coefficient $\frac{r^2}{81-9r^2}$ of b_i in the constraint $u_i \geq u_i^D$, and the two choice variables have the same coefficient in the objective function (5), the optimal contract is such that $b_i = 0$ and $a_i > 0$. Solving out $u_i = u_i^D$, we obtain $a_i = \frac{36c(9-r^2)}{(3-r^2)^2}$.

Case '21:' Two-sided acquisition and one-sided sharing.

The principal's cost minimization problem is still (6). The incentive constraint that prevents agent 1 from deviating at the information acquisition stage and reporting the lack of the signal to agent 2, is

$$w_1 - a_1 \frac{1}{18} - b_1 \frac{3-r^2}{6(9-r^2)} - c \geq w_1 - a_1 \frac{1}{12} - b_1 \frac{1}{18}$$

that can be rewritten as $a_1 \frac{1}{36} + b_1 \frac{r^2}{81-9r^2} \geq c$. The principal's program has a linear objective function and a linear constraint. The ratio between the coefficients of a_1 and b_1 in the (binding) constraint $u_1 = u_1^D$ is $\rho_{ab} = \frac{81-9r^2}{36r^2}$, whereas the same ratio in the objective function is $\bar{\rho}_{ab} = \frac{6(9-r^2)}{18(3-r^2)}$. Because $\bar{\rho}_{ab} < \rho_{ab}$, the optimal contract is such that $b_i = 0$ and $a_i > 0$. Solving out $u_1 = u_1^D$, we obtain: $a_1 = 36c$.

Case '10:' One-sided acquisition and sharing.

The principal's cost minimization problem is still (6). The incentive constraint $u_1 = u_1^D$ that prevents agent 1 from deviating at the information acquisition stage and reporting the lack of the signal to agent 2, is

$$\bar{w}_1 - a_1 \frac{1}{18} - b_1 \frac{3-r^2}{36} - c \geq \bar{w}_1 - a_1 \frac{1}{12} - b_1 \frac{1}{12}$$

that can be expressed as $a_1 \frac{1}{36} + b_1 \frac{r^2}{36} \geq c$. The ratio between the coefficients of a_1 and b_1 in the (binding) constraint $u_1 = u_1^D$ is $\rho_{ab} = \frac{1}{r^2}$, whereas the same ratio in the objective function is $\bar{\rho}_{ab} = \frac{2}{3-r^2}$. Because $\bar{\rho}_{ab} < \rho_{ab}$, the optimal contract is such that $b_i = 0$ and $a_i > 0$. Solving out $u_1 = u_1^D$, we obtain: $a_1 = 36c$.

Q.E.D.

Proof of Lemma 2: Subtracting the formulas of $E\pi_{11}(r,c)$ and $E\pi_{10}(r,c)$ and rearranging, we obtain

$$E\pi_{11} - E\pi_{10} = \frac{5}{36} - \frac{1}{36}(5-r^2) + 34c - \min\left\{34, \frac{1}{2}\frac{r^2+33}{r^2}\right\} c,$$

which is obviously strictly positive.
Q.E.D.

Proof of Lemma 3: We first compare $E\pi_{22}(r,c)$ and $E\pi_{21}(r,c)$ and consider

$$\begin{aligned} E\pi_{22}(r,c) - E\pi_{21}(r,c) &= \frac{9-2r^2}{9(9-r^2)} - \frac{3-r^2}{3(9-r^2)} - D_{22-21}(q)c \\ &= \frac{q^2}{9(9-r^2)} - D_{22-21}(r)c \geq -D_{22-21}(r)c. \end{aligned}$$

where

$$D_{22-21}(r) = \min\left\{\frac{12}{(3-r^2)^2}, \frac{(9-r^2)}{6q^2}\right\}(51-5r^2) - \min\left\{34, (51-5r^2)\frac{(9-r^2)}{12r^2}\right\} - 6\frac{51-5r^2}{(3-r^2)^2}.$$

Calculations omitted for brevity show that $D_{22-21}(r) > 0$ for $0 \leq r < r_1$ and $D_{22-21}(r) < 0$ for $r_1 < r \leq 1$. We obtain that for $0 \leq r < r_1$, whether $E\pi_{22}(r,c)$ is larger or smaller than $E\pi_{21}(r,c)$ depends on whether c is smaller or larger than a strictly positive threshold $c_{22-21}(r)$ implicitly defined by the equation $E\pi_{22}(r,c) = E\pi_{21}(r,c)$, whereas for $r_1 \leq r \leq 1$ it is the case that $E\pi_{22}(r,c) > E\pi_{21}(r,c)$ for all c.

To complete the proof we show that, for almost all c and $0 \leq r \leq r_1$ it is either the case that $E\pi_{22}(r,c) > E\pi_{21}(r,c)$ or that $E\pi_{20}(r,c) > E\pi_{21}(r,c)$. We begin by noting that the functions $E\pi_{22}(r,c)$, $E\pi_{21}(r,c)$ and $E\pi_{20}(r,c)$ are all linear in c, and that $E\pi_{22}(r,c) > E\pi_{21}(r,c) > E\pi_{20}(r,c)$ for $c = 0$. As a result, we can proceed by comparing the threshold functions

$$c_{22-21}(r) = \frac{\frac{1}{18} - \frac{3-r^2}{3(9-r^2)} + \frac{3-r^2}{6(9-r^2)}}{\min\left\{\frac{12}{(3-r^2)^2}, \frac{9-r^2}{6r^2}\right\}(51-5r^2) - \min\left\{34, \frac{9-r^2}{12r^2}(51-5r^2)\right\} - 6\frac{51-5r^2}{(r^2-3)^2}}$$

$$c_{21-20}(r) = \frac{\frac{1}{18} - \frac{3-r^2}{6(9-r^2)}}{\min\left\{34, \frac{9-r^2}{12r^2}(51-5r^2)\right\} + 6\frac{51-5r^2}{(r^2-3)^2} - 68}$$

implicitly defined by the equations $E\pi_{22}(r,c) = E\pi_{21}(r,c)$ and $E\pi_{21}(r,c) = E\pi_{20}(r,c)$, respectively. In fact, for any (r,c) such that $c < c_{22-21}(r)$, it is the case that $E\pi_{22}(r,c) > E\pi_{21}(r,c)$, and for any (r,c) such that $c > c_{21-20}(r)$, it is the case that $E\pi_{21}(r,c) < E\pi_{20}(r,c)$.

Calculations omitted for brevity show that $c_{22-21}(r) \geq c_{21-20}(r)$ for all $0 \leq r \leq r_1$. This completes the proof of the Lemma, because it implies that for almost all c and $0 \leq r \leq r_1$, it is either the case that $E\pi_{22}(r,c) > E\pi_{21}(r,c)$ or that $E\pi_{20}(r,c) > E\pi_{21}(r,c)$.

Q.E.D.

Proof of Proposition 5: We need to compare the profit functions $E\pi_{22}(r,c)$, $E\pi_{20}(r,c)$, $E\pi_{11}(r,c)$ and $E\pi_{00}(r,c)$. To determine the area in which $E\pi_{22}(r,c)$ is the largest, we note that all the profit functions are linear in c, and that $E\pi_{22}(r,c) > E\pi_{20}(r,c) > E\pi_{11}(r,c) > E\pi_{00}(r,c)$ for $c = 0$ and all r. As a result, we can proceed by comparing the threshold functions

$$c_{22-20}(r) = \frac{\frac{1}{9} - \frac{3-r^2}{3(9-r^2)}}{\min\left\{\frac{12}{(r^2-3)^2}, \frac{9-r^2}{6q^2}\right\}(51-5r^2) - 68}$$

$$c_{22-11}(r) = \frac{\frac{5-r^2}{36} - \frac{3-r^2}{3(9-r^2)}}{\min\left\{\frac{12}{(3-r^2)^2}, \frac{9-r^2}{6r^2}\right\}(51-5r^2) - \min\left\{34, \frac{1}{2}\frac{r^2+33}{r^2}\right\}}$$

$$c_{22-00}(r) = \frac{\frac{1}{6} - \frac{3-r^2}{3(9-r^2)}}{\min\left\{\frac{12}{(3-r^2)^2}, \frac{9-r^2}{6r^2}\right\}(51-5r^2)}$$

implicitly defined by the equations $E\pi_{22}(r,c) = E\pi_{20}(r,c)$, $E\pi_{22}(r,c) = E\pi_{11}(r,c)$ and $E\pi_{22}(r,c) = E\pi_{00}(r,c)$. For any such a threshold function $c_{22-(\cdot)}(r)$, and any value $r \in [0,1]$ for which $c_{22-(\cdot)}(r)$ is positive, it is the case that $E\pi_{22}(r,c) > E\pi_{(\cdot)}(r,c)$ if and only if $c < c_{22-(\cdot)}(r)$. Instead, for all r such that $c_{22-(\cdot)}(r) < 0$, it is the case that $E\pi_{22}(r,c) > E\pi_{(\cdot)}(r,c)$ for all c.

Calculations omitted for brevity prove that $c_{22-20}(r) > 0$ if and only if $r < \sqrt{\frac{252}{5} - \frac{3\sqrt{3}}{5}\sqrt{2267}}$, and that $c_{22-11}(r) > 0$ and $c_{22-00}(r) > 0$ for all $r \in [0,1]$. Further, comparing $c_{22-11}(r)$ and $c_{22-00}(r)$, omitted calculations show that $c_{22-11}(r) < c_{22-00}(r)$ for all $r \in [0,1]$, and that $c_{22-20}(r) < c_{22-11}(r)$ if and only if $r < \tilde{r} \approx 0.553$ on the relevant range $r \in [0, \sqrt{\frac{(28\sqrt{3}-\sqrt{2267})3\sqrt{3}}{5}}]$. The implication is that $E\pi_{22}(r,c) > \max\{E\pi_{20}(r,c),\ E\pi_{11}(r,c),\ E\pi_{00}(r,c)\}$ for every $c < c_{22-20}(r)$ for $r < \tilde{r}$ and for every $c < c_{22-11}(r)$ for $r > \tilde{r}$.

Likewise, to determine the area in which $E\pi_{00}(r,c)$ is larger than $E\pi_{22}(r,c)$, $E\pi_{20}(r,c)$ and $E\pi_{11}(r,c)$, we note that $E\pi_{00}(r,c) > \max\{E\pi_{22}(r,c), E\pi_{20}(r,c), E\pi_{11}(r,c)\}$ for $c \to \infty$ and all r. As a result, we can proceed by comparing the threshold function $c_{22-00}(r)$ reported above with the threshold functions

$$c_{11-00}(r) = \frac{r^2+1}{36\min\left\{34, \frac{1}{2r^2}(r^2+33)\right\}} \text{ and } c_{20-00}(r) = \frac{1}{1224},$$

implicitly defined by the equations $E\pi_{20}(r,c) = E\pi_{00}(r,c)$ and $E\pi_{11}(r,c) = E\pi_{00}(r,c)$. Omitted calculations show that, for all $r \in [0,1]$ all the functions $c_{22-20}(r)$, $c_{22-11}(r)$ and $c_{22-00}(r)$ are strictly positive. Hence, for every $r \in [0,1]$, it is the case that $E\pi_{00}(r,c) > \max\{E\pi_{22}(r,c),\ E\pi_{20}(r,c),\ E\pi_{11}(r,c)\}$ for every $c > \max\{c_{22-00}(r), c_{20-00}(r), c_{11-00}(r)\}$. Comparing $c_{22-00}(r)$, $c_{20-00}(r)$ and $c_{11-00}(r)$, omitted calculations show that $c_{11-00}(r) > c_{22-00}(r)$, and $c_{11-00}(r) > c_{20-00}(r)$ for all $r \in (0,1]$. The implication is that $E\pi_{00}(r,c) > \max\{E\pi_{22}(r,c),\ E\pi_{20}(r,c),\ E\pi_{11}(r,c)\}$ for every $r > 0$

and $c > c_{11-00}(r)$.

For any r and cost c values that are below $c_{11-00}(r)$ and either above $c_{22-20}(r)$, for $r < \tilde{r}$, or above $c_{22-11}(r)$, for $r > \tilde{r}$, it is either the case that $E\pi_{20}(r,c)$ or $E\pi_{11}(r,c)$ is the highest profit function. Because $E\pi_{20}(r,c) > E\pi_{11}(r,c)$ for $c = 0$ and all r, this is once more determined by considering a threshold function:

$$c_{20-11}(r) = \frac{1-r^2}{2448 - 36\min\left\{34, \frac{r^2+33}{2r^2}\right\}},$$

implicitly defined by the equation $E\pi_{20}(r,c) = E\pi_{11}(r,c)$. Because $2448 - 36 \cdot 34 = 1224$, the threshold function $c_{20-11}(r)$ is strictly positive for all $r \in [0,1)$. Hence, for all r it is the case that $E\pi_{20}(r,c) > E\pi_{11}(r,c)$ if and only if $c < c_{20-11}(r)$.

Comparing $c_{20-11}(r)$ with $c_{22-20}(r)$, $c_{22-11}(r)$ and $c_{11-00}(r)$, omitted calculations show that $c_{20-11}(r) = c_{11-00}(r)$ for $r = 0$, that $c_{20-11}(r) < c_{11-00}(r)$ for all $r > 0$, that $c_{20-11}(r) > c_{22-20}(r)$ for $0 \le r < \tilde{r}$, that $c_{20-11}(r) = c_{22-20}(r) = c_{22-11}(r)$ for $r = \tilde{r}$ and that $c_{20-11}(r) < c_{22-11}(r)$ for $\tilde{r} < r \le 1$.

This concludes the proof of the Proposition. We have derived the result depicted in Figure 1: For $0 < r < \tilde{r}$ and $c < c(r)_{22-20}$, and for $\tilde{r} < r \le 1$ and $r < c(r)_{20-11}$, it is the case that $E\pi_{22}(r,c) > \max\{E\pi_{20}(r,c),\ E\pi_{11}(r,c),\ E\pi_{00}(r,c)\}$. For $0 < r < \tilde{r}$ and $c(r)_{22-20} < c < c(r)_{20-11}$, $E\pi_{20}(r,c) > \max\{E\pi_{22}(r,c),\ E\pi_{11}(r,c),\ E\pi_{00(r,c)}\}$. For $c(r)_{22-11} < c < c(r)_{11-00}$, $E\pi_{11}(r,c) > \max\{E\pi_{22}(r,c),\ E\pi_{20}(r,c),\ E\pi_{00}(r,c)\}$. For $c > c(r)_{11-00}$, $E\pi_{00}(r,c) > \max\{E\pi_{22}(r,c),\ E\pi_{20}(r,c),\ E\pi_{11}(r,c)\}$.
Q.E.D.