

# Information Transmission in Political Networks\*

Francesco Squintani  
University of Warwick<sup>†</sup>

December, 2020  
(First draft, July 2018)

## Abstract

Motivated by political economy applications such as networks of policy-makers, interest groups, or judges, I formulate and study a model of strategic information transmission in networks of ideologically differentiated decision makers. When agents' preferences are sufficiently diverse, the optimal network is the line in which the agents are ordered according to their ideologies. When agents are partitioned in ideologically diverse groups, each composed of agents with similar views, it is optimal for all agents that the groups segregate into factions. Such optimal networks obtain as Nash equilibria of a game in which each link requires sponsorship by both connected agents, and are the unique strongly pairwise stable networks. These results suggest positive and normative rationales for “horizontal” links between like-minded agents in political networks, as opposed to hierarchical networks, that have been shown to be optimal in organizations where agents' preferences are more closely aligned.

---

\*This paper previously circulated with the title “Networks and Ideology” (2018). For helpful comments and discussions I thank Torun Dewan, Andrea Galeotti, Christian Ghiglino, Pablo Montagnes, Vasiliki Skreta, Pierre Yared, as well as audiences at seminars/conferences, including UCLA (2016), University of Chicago (2016), Università di Verona (2016), University of Nottingham (2016), Erasmus University Rotterdam (2016), University of St Andrews (2016), Harvard University (2017), Stanford University (2017), Columbia University (2017), UCSD (2017), Duke University (2017), University of Queensland (2017), Oxford University (2018), Yale University (2018), Paris School of Economics (2019), University of Utah (2019), EIEF - Bank of Italy (2017), University of Bonn (2022), University of Urbana-Champaign (2022), and Northwestern University (2023).

<sup>†</sup>Department of Economics, Gibbett Hill Road, Coventry CV47AL, UK; f.squintani@warwick.ac.uk

# 1 Introduction

Social connections and networks are understood to be an important, if not fundamental, feature of political economy.<sup>1</sup> One main established function of political networks is to facilitate the flow of information. Communication is usually strategic, and agents often have diverse views. As explained later, existing network analysis based on reduced-form information transmission models does not cover strategic communication among agents with misaligned preferences. This paper studies an explicit model of information transmission in networks of political decision makers, and investigates network optimality and stability. My results recover stylized facts established by the empirical literature, and suggest a normative rationale for often observed political network patterns.

The model comprises a set of agents linked in a network. One agent is called at random to make a policy decision, identified as a point on the real line representing the left-right spectrum. (The main results generalize qualitatively to the case in which multiple agents are called to make decisions, as I explain in Section 3.) All agents care about every agent's decision, possibly to a different extent, i.e., agents' authorities may differ. Each agent would like a decision closer to her ideal policy, which depends on an unknown state of the world and on an individual ideal preference. Hence, all agents prefer policies informed about the state. But given information, each would like to push policy towards her ideological preference.

Agents may hold information correlated with the state of the world, and hence relevant for the policy choice, that they can transmit through the network. At least some of this information is verifiable. An agent's choice is whether to relay it to her linked peers, or withhold it from some or all of them. With small probability, information may decay in transmission. There are as many rounds of communication as needed for all information to have the possibility to reach every agent in the network. Then the policy choice is made. The solution concept is Perfect Bayesian Equilibrium. Network links are costly, and formed ex-ante (i.e., before agents receive any information from nature). I calculate both the networks that maximize the sum of the agents' ex-ante equilibrium payoffs, and the networks that form endogenously when the agents sponsor links ex-ante.

To make this model concrete, we may think about a network of policy makers in different jurisdictions. (A summary of the vast empirical literature on policy networks is in Section 2.) At any moment in time, one of them may face a policy challenge on a specific public administration issue. She would gain from the experience gathered by her peers, who may have a specific expertise

---

<sup>1</sup>An exhaustive review of political networks is provided by the handbook edited by Victor, Montgomery and Lubell (2017).

in the area, or have faced similar policy challenges in the past. The most valuable information often takes the form of pilot studies or other policy experiments, whose results cannot be falsified, but can be easily concealed in the abundance of bureaucratic documents. In order to make better decisions, the policy maker consults her network of peers, but these connections are costly to maintain.

Other examples include networks of lobbyists, interest groups, political experts, advocates, consultants, investigative journalists, bloggers, or academics. One of them may be called to campaign or provide advice on a specific matter. In order to deliver more effectively, she will try and consult her peers to collect information, but again, these connections are costly to maintain. An intriguing example concerns informal networks of judges. At any moment in time, one of them may face a novel legal challenge. She may then seek advice from her peers (although she ought to abstain from delving into the facts of specific case). And there is ample evidence that informal networks of judges play an important role in judicial sentences. Section 2 discusses empirical studies on networks of interest groups and networks of judges.

The first and most important question I pose within my model is how to efficiently organize networks of political decision makers ex-ante. When the agents' ideological preferences are sufficiently aligned, I show that there is no strategic withholding of information. Hence, several results extend from the extant network literature based on reduced-form models of communication. Suppose that agents are homogeneous with respect to authority, and to the likelihood to be informed or making decisions. Then, as Jackson and Wolinsky (1996) and Bala and Goyal (2000), every ex-ante optimal network is a star, i.e., there is a unique "center" agent to which all other agents are linked, and such "periphery" agents are not linked with each other. This result holds unless the link cost is very small or very large, and for nearly all link cost values as the decay probability vanishes.<sup>2</sup>

My main results concern the case in which agents' ideologies are sufficiently diverse. Then, the agents only relay information that conforms to their individual biases relative to the decision maker. As a result, the shape of the optimal network changes dramatically. The optimal network is now the line in which the agents are ordered according to their ideal preferences. It is optimal that links only form between the ideologically closest agents. But I prove that this does not lead to communication breakdown. And when agents are very likely to be informed, any agent's choice in the ordered line is almost as informed as if all information was shared. These results hold for all generic authority parameters and probabilities of being informed or making decisions.

Of special interest is the case in which agents are partitioned in ideologically diverse groups,

---

<sup>2</sup>Of course, for any fixed positive decay probability, the optimal network is complete when the link cost is small enough, and it is empty when the cost is too large.

each composed of agents with similar views. As posited by Hume (1741), and documented since at least Rose (1964), these distributions of ideologies arise naturally in several political environments. I show that it is then optimal *for all agents* that each group organizes as a separate minimally connected subnetwork with a unique link to the ideologically closest groups. Factionalization is a common organizational structure in political networks, but it is usually perceived as detrimental (e.g., Hume, 1741). My analysis provides an efficiency rationalization based on information transmission.

This result is refined further when agents are homogeneous with respect to authority and expertise. Then, each group should organize as a star, whose center forms a link with the centers of the ideologically closest stars. This suggests an efficiency rationale for the often observed hierarchical organization of political factions (cf. the discussion in Section 2). All agents in a faction report to the unique faction leader, who communicates with the leaders of ideologically nearest factions. Hierarchies may be optimal in factions even in absence of authority and expertise differences among political agents.

It is important that the optimality of factionalization is shown here in a set up where link costs are the same across agents. In reality, agents likely bear a smaller cost when connecting within ideological groups than across groups, and may even enjoy a benefit. This fact provides a further justification for factionalization. Further, when within-group link costs are sufficiently small or negative, groups should optimally organize as complete networks, instead of stars. Heterogeneous within-group connection costs would lead to the optimality of intermediate structures with a center agent to which periphery agents are linked, and periphery agents may form additional links or not depending on the costs.

I then turn to the question of whether these optimal networks form endogenously and whether they are stable. I focus on the case in which agents sponsor links primarily to access other agents' information, as they do not care much about their decisions. Except for extreme link costs, I show that the earlier characterized optimal networks obtain as Nash equilibria of a network formation game, in which the agents invest *ex-ante* in the links they wish to form, and each link requires sponsorship by both connected agents. Further, the optimal networks are pairwise stable in the sense of Jackson and Wolinsky (1996), and they are the unique "strongly" pairwise stable networks. That is to say, the optimal networks include all links that are beneficial to both connected agents individually, and are the unique Nash equilibrium networks that include all links whose cost is smaller than their aggregate value to the connected agents.

To summarize: A possibly quintessential feature of politics is that agents hold diverse ideal views. My analysis highlights the importance of ideology for political network optimality and stability. This study of information transmission in political networks provides a normative and

positive rationale for political agents to form “horizontal” links with like-minded peers, beyond their possibly intrinsic preference for doing so. In contrast, hierarchical networks such as the star are more likely prevalent, and useful, in organizations such as armies or companies in which the agents’ preferences are closely aligned.<sup>3</sup>

My results recover stylized facts established by the empirical literature on networks of political decision makers, and especially on connections built to facilitate the flow of information. As I detail in Section 2, this literature documents factionalization based on political preferences as the main characteristic of such networks. Modal network structures can be described as a collection of politically homogeneous subnetworks loosely connected among each other. Individual subnetworks are often structured hierarchically with a small core of leaders to which a large set of less prominent agents are connected.

## 2 Literature Review

This section is divided in two parts. The first one briefly surveys the empirical literature on networks of political decision makers discussed in the introduction. (Appendix C presents an extended review.) The second part discusses theoretical literature on information transmission in networks and applications in organizational economics.

**Networks of political decision makers.** There is a vast empirical/empirical literature on networks of political decision makers such as, for example, local policy makers, interest groups and judges. It is established that a main function of such networks is to facilitate the flow of information. The evidence is overwhelming that ideological and policy preference similarity is a main driver of political connections.

These features have been observed in several studies of policy makers networks. Among them, Zafonte and Sabatier (1998) investigate the San Francisco Bay area government agencies’ network; Henry, Lubell, and McCoy (2011) trace the connections among transportation and land planning agencies in California; Matti and Sandström (2011) study the network of policy makers responsible for wildlife management; Henry (2011) and Gerber, Henry and Lubell (2013) analyse the network of planning and management agencies in the main California regions; and Desmarais, Harden and Boehmke (2015) trace connections among U.S. States governments by tracking timing of policies adoptions.

---

<sup>3</sup>Organization design studies model hierarchical organizations as trees, where the (maximal) distance from terminal nodes identifies the agent’s level in the hierarchy. Hierarchies with more than one level are regarded as optimal for minimizing information processing costs together with the risk of information decay. This literature is briefly discussed in Section 2.

Likewise, policy preference based homophily has been documented in studies on networks of interest groups. Among them, Laumann and Knoke (1987) and Carpenter, Esterling and Lazer (2004) trace the connections among lobbyists, government agencies, and congressional staff in the 1970's health and energy policy domains; König and Bräuninger (1998) investigate the ties among interest groups, trade unions, governmental agencies and legislators in the events that shaped 1980's German labour policies; Schneider et al. (2003) analyze the connections of policy makers and interest groups in U.S. estuary areas; Weible and Sabatier (2005) investigate networks of policy makers in marine protected areas; Koger, Masket and Noel (2009) study the network of US party candidates, activists, interest groups and media outlets using donor and subscriber names data; and Box-Steffensmeier and Christenson (2014) build a network of interest groups coalitions using *amicus curiae* briefs to the US Supreme Court.

Several studies demonstrate the influence on sentences of judicial networks constructed with various methodologies, such as, by tracking precedent citation across Courts (e.g., Caldeira, 1985; Choi and Gulati, 2008), or by following the flow of clerks across judges (e.g., Katz and Stafford, 2010; Baum and Ditslear, 2001; and Baum, 2014). The evidence is that political preference similarity is one of the main drivers of such connections. Weiser (2015) interviewed some judges in Manhattan's Federal District Court and reports that almost all have consulted colleagues when puzzled by legal or other issues.

As well as segregation into separate subnetworks based on political preferences, another stylized fact that emerges from the empirical literature is that such subnetworks are often organized in a centralized fashion. Each is hierarchically differentiated between a small core of leaders to which a large set of periphery agents are connected. Evidence of policy-preference based segregation into hierarchical subnetworks has been found, for example, by Henry (2011) and by Gerber, Henry and Lubell (2013) in their studies on regional cooperation among policy-makers in California: they uncover separate politically homogeneous subnetworks, each differentiated among a small set centroids and periphery agents connected to the centroids.<sup>4</sup>

The interest groups network constructed by Box-Steffensmeier and Christenson (2014) "appears to resemble a host of tightly grouped factions" with homogeneous ideology, in which a few central agents are connected to a larger set of less prominent agents, who may also form connections amongst each others. Evidence of policy-preference based segregation into hierarchical subnetworks has also been found by Koger et al. (2009) in their map of links between formal

---

<sup>4</sup>Similar network structures are found by Feiock, Lee and Park (2012) in their survey of the local governments' connections in the four-county Orlando, Florida, metropolitan area, although there is also evidence of multiple links across and within subnetworks. Relatedly, Fischer and Sciarini (2016) find that preference similarity and perceived power are the main drivers of connection among the entities involved in the most important political processes in Switzerland over the years 2001-06.

US party organizations and interest groups, and by Ingold (2011) in her study of the networks of interest groups and policy makers that involved in the formulation of the 2000 Swiss CO2 law. The judicial network constructed by Caldeira (1985) can be approximated as a collection of star-shaped subnetworks, in which “Courts cite the most prestigious among the culturally closest Courts.”

Political preference based segregation into separate hierarchically organized subnetworks may be a more or less predominant feature of political networks. Loose associations may also display connections across ideological groups, and multiple centroids or links among periphery agents, within groups. These features are less common in organized groups, such as, for example, party factions. In a large cross-national survey of over 100 parties, Janda (1980) found strong evidence of ideological factionalization. Such factions are usually led by distinct leaders, to whom members of the faction relate exclusively (see also, Janda, 1993, and Ceron, 2019). Similarly, Persico, Rodriguez-Pueblita, and Silverman (2011) report evidence of hierarchical organization and exclusiveness in factions of interest. Indeed, textbook definitions characterize factions as organized groups that work for the advancement of their leaders and of specific policies (e.g., Lasswell, 1931).

**Information transmission in networks.** An important motivation of theoretical network economics is information transmission. The general, reduced-form approach usually adopted in that literature applies to communication among agents that do not have any incentive to strategically mislead each other, for example because their preferences are aligned. Seminal papers such as Jackson and Wolinsky (1996) and Bala and Goyal (2000) identify the maximally centralized architecture of the star as socially optimal, and determine conditions under which this optimal architecture arises in equilibrium, and is a stable network. These results, generalized in a number of subsequent studies, suggest that agents with aligned preferences organize in a centralized, hierarchical, manner, and that this organization is efficient.<sup>5</sup>

Later work considers networks with heterogeneous agents (e.g., Galeotti, Goyal and Kamphorst, 2006). These papers customarily assume that the value of connecting a pair of agents  $i$  and  $j$  is independent of the identities of the other players on the connecting path. This assumption is shown not to hold in this paper’s full fledged analysis of strategic communication.<sup>6</sup> And while this is shown here for the case of transmission of verifiable information, Ambrus, Azevedo and Kamada (2013) prove an analogous result for the case of cheap talk. These results clarify why existing reduced-form network models do not cover strategic communication among agents with

---

<sup>5</sup>A detailed review of network economics literature is in Jackson (2010), for example.

<sup>6</sup>Galeotti et al. (2006) also rule out connection externalities: they assume that whether or not an agent  $i$  is connected to another agent  $j$  does not matter for any other agents’ payoffs. Here instead, every agent  $i$  would like that every agent  $j$  is informed as possible if called to making decisions, i.e., that  $j$  is connected to, and receives information from, as many agents as possible.

misaligned preferences.

The design of organizations whose members' preferences are aligned is an important topic in business science. In organization economics and theories of the firm, hierarchical networks are regarded as the optimal structure for reducing the costs of information processing (Radner, 1993; Bolton and Dewatripont, 1994), and for preventing conflicts between subordinates and their superiors (Friebel and Raith, 2004). Instead, cliques emerge among subgroups of agents who wish to coordinate their actions (Calvó, de Martí and Prat, 2015).<sup>7</sup>

Unlike the papers above, I study networks of political agents, who may wish to mislead each other because of their ideological preference differences. Factions form and are optimal in my model because agents with close ideologies are less likely to mislead each other, regardless of how much they care about each others' actions, and of how costly it is to form their links. By contrast, for example, cliques in Calvó, de Martí and Prat (2015) form among agents who have the highest interest in coordinating their actions, and in Galeotti et al. (2006) interconnected stars form because of heterogeneous link costs.

Models of information transmission in political networks include Chwe (2000), who characterizes minimal networks that lead to coordinated collective action, Stokman and Berveling (1998) who structurally estimate influence network formation within local organizations in Amsterdam, Larson (2017), who investigates how networks form in a context of interethnic cooperation, and Larson and Lewis (2018), who structurally estimate how networks influence cooperation among rebel groups in Uganda. These papers consider environments where there is no role for strategic manipulation/withholding of information. Instead, I study strategic communication and determine optimal networks.<sup>8</sup>

Further, Patty and Penn (2014) explore sequential decision making in networks of privately informed agents. There is no explicit communication in their model, but each agent's action may signal information to subsequent decision makers in the network. While I consider transmission of multi-agent (verifiable) information in networks, Bloch, Demange and Kranton (2018) study the spreading of possibly false information, originated by a single agent, in a network where agents may either wish correct decisions are made, or have a private agenda in favor of one alternative. Unlike my paper, they do not consider network optimality. Instead, Migrow (2019) studies optimal hierarchies of possibly biased agents who strategically report unverifiable information to a single decision maker.

---

<sup>7</sup>For a detailed exposition of the networks literature within organization design, see the handbook by Gibbons and Roberts (2012), for example.

<sup>8</sup>More distantly related, Battaglini and Patacchini (2018) study how interest groups allocate campaign contributions when congressmen are connected by social ties.



A recent paper by Gieczewski (2020) studies a model of learning in networks with transmission of verifiable information among agents with misaligned preferences. Unlike my paper, he does not consider network formation, stability nor optimality. His analysis shows that full learning requires sufficiently dense networks. Signals closer to the mean are more likely to propagate, because agents tend to block signals contrary to their bias, as is the case in my paper. Relatedly, Acemoglu, Bimpikis and Ozdaglar (2014) study network formation and learning in a model of information transmission among agents with aligned preferences. Instead, Egorov and Sonin (2020) analyze a network model of communication á-la Kamenica and Gentzkow (2011). A sender controls the informativeness of her signal but cannot falsify it. Each of several decision makers may either costly access the signal or rely on her network connections.

In a broad sense, my paper belongs to the recent surge of studies of multi-agent communication in political economy, developed within the framework of Galeotti, Ghiglino and Squintani (2013) and, more distantly, Hagenbach and Koessler (2010). Patty (2013) determines the optimal exclusion and inclusion policies to maximize information sharing in meetings. Dewan et al. (2015) investigate the optimal assignment of decision-making power in the executive of a parliamentary democracy. Penn (2016) studies the engagement and association of societal groups and subcultures. Dewan and Squintani (2016) consider the role of factions in shaping the formulation of a party manifesto. Patty and Penn (2020) study how organizational identity influences communication and action.

All these papers study transmission of unverifiable information (cheap talk). Instead, I here suppose that at least some of the information transmitted is verifiable. As is well known, this distinction is crucial for equilibrium communication. As I show in Appendix B, there is no information transmission whatsoever when ideologies are sufficiently diverse, within the framework of Galeotti et al. (2013). Hence, the optimal network is empty (no agent is linked with any other agent) regardless of link costs. In stark contrast, the optimal network is the ordered line when some information is verifiable, and information transmission is never fully blocked in equilibrium.

### 3 The model

My analysis of information transmission in political networks is based on the set up that I describe below. After presenting it, I will discuss the motivation and implications of its main assumptions, so as to assess the robustness of the analysis.

A set  $\mathcal{N}$  of  $n$  agents is connected in a network  $N$ , that describes who can transmit information to whom. Formally,  $N$  is a  $n \times n$  matrix with entries  $n_{ij} \in \{0, 1\}$ , for  $i = 1, \dots, n$  and  $j = 1, \dots, n$ . A link connects agent  $i$  to  $j$  whenever  $n_{ij} = 1$ , in which case  $j$  is called a neighbor of  $i$ . The network

$N$  is symmetric,  $n_{ij} = n_{ji}$  for all  $i, j$ : each agent  $i$  is linked to  $j$  if and only if  $j$  is linked to  $i$ . For future reference, let me denote a network  $N$  as connected if for every pair of agent  $i$  and  $j$ , there exists at least a path  $p$  that connects  $i$  and  $j$ , and as minimally connected, or a tree, if every pair of agents  $i$  and  $j$  is linked by a unique path.<sup>9</sup>

A state of the world  $\theta$  is uniformly distributed on  $[0, 1]$ . There is a random non-empty set  $E \subseteq \mathcal{N}$  of expert agents who may have information on  $\theta$ . The random set of actually informed agents is  $I \subseteq E$ , and it can be empty. Every agent  $i \in I$  receives from nature a signal  $s_i \in \{0, 1\}$  informative of  $\theta$  according to the probability  $\Pr(s_i = 1|\theta) = \theta$ . The signals  $s_i$  are independent across agents. A random agent  $d \in \mathcal{N}$  is called to make a decision  $\hat{y}_d \in \mathbb{R}$ . (The main results generalize qualitatively to the case in which multiple agents are called to make decisions, as I discuss later in this section.) Every agent  $i$  can be the decision maker and can be an expert agent. Denoting by  $\xi$  the joint distribution of  $(d, E, I)$ , I stipulate that for all  $d \in \mathcal{N}$ ,  $\xi(d, E, I) > 0$  only if  $I \subseteq E \neq \emptyset$ , and further that for all  $d \in \mathcal{N}$  and  $e \in \mathcal{N}$ ,  $\xi(d, E, I) > 0$  for some  $E, I$  with  $e \in I \subseteq E$ .

When the state is  $\theta$ , the decision maker is  $d$ , and the decision  $\hat{y}_d$ , the utility of each agent  $i$  is  $u_i(\hat{y}_d; \theta) = -\alpha_{id}(\hat{y}_d - \theta - b_i)^2$ . For every agent  $i$ , the authority weights  $\alpha_{id}$  are all strictly positive, and  $\sum_{d \in \mathcal{N}} \alpha_{id} = 1$ . Every agent  $i$  would like that every decision  $\hat{y}_j$  is close to her bliss point  $\theta + b_i$ , which is composed of the “unbiased optimum”  $\theta$  and the idiosyncratic ideological bias  $b_i \in \mathbb{R}$ . Yet, agents need not care equally about any agents’ decisions. The preference biases  $b_i$  are common knowledge. Without loss of genericity I stipulate that  $b_1 < \dots < b_n$ , with the notation  $\mathbf{b} = (b_1, \dots, b_n)$ . The case in which agents are *homogeneous* in terms of authority and expertise is of special interest, and defined as when  $\alpha_{ij}$  and  $\xi(d, E, I)$  are the same across every permutation  $\pi : \mathcal{N} \rightarrow \mathcal{N}$  of the agents’ identities.

At the beginning of the game, nature selects the state  $\theta$ , the decision maker  $d$ , the sets  $E$  and  $I$  of expert and informed agents, and the realizations of the signals  $s_i$  for the agents  $i \in I$ . Each agent  $i \in I$  is then informed of her private signal  $s_i$ , and every agent learns the identity of the decision maker  $d$ . Agent  $d$  also learns who is in the set  $E$  of expert agents. (Whether or not any other agent observes  $E$  is irrelevant.) Agents do not observe the set of actually informed agents  $I$ , nor signal realizations other than their own.

The agents may transmit signals to the decision maker along the network  $N$ . There are  $T$  rounds of information transmission. I assume that  $T \geq n - 1$  so that every signal has the possibility to reach every agent in any connected network  $N$ . If informed of a signal  $s_i$  at any round  $t$ , any agent  $j$  chooses whether to relay  $s_i$  or not to some or all of her neighbors  $k$  in the network  $N$ . Neither

---

<sup>9</sup>Two agents  $i$  and  $j$  are linked by the path  $p = (i, h_1, \dots, h_{\ell-1}, j)$  of length  $\ell$  in network  $N$ , if  $i$  is linked to  $h_1$ ,  $h_k$  is linked to  $h_{k+1}$  for every  $k = 1, \dots, \ell - 2$ , and  $h_{\ell-1}$  is linked to  $j$ .

the content of signal  $s_i$  nor the identity of its “source”, agent  $i$ , can be falsified.

Formally, a strategy choice of agent  $j$  of reporting signal  $s_i$  to a neighbor  $k$  at a history  $h^t$  is denoted by  $r_{jkdi}(\omega_{ij}; h^t)$  as a function of the information  $\omega_{ij} \in \{0, \phi, 1\}$  held by agent  $j$  on signal  $s_i$  at history  $h^t$ . If  $j$  is not informed of  $s_i$  then  $\omega_{ij} = \phi$ , and else  $\omega_{ij} = s_i$ . The signals’ verifiability entails that  $r_{jkdi}(\omega_{ij}; h^t) \in \{\omega_{ij}, \phi\}$  for all  $\omega_{ij} \in \{0, \phi, 1\}$ . If  $j$  is informed of  $s_i$ ,  $\omega_{ij} = s_i$ , the notation  $r_{jkdi}(s_i; h^t) = s_i$  identifies the choice of relaying  $i$ ’s signal  $s_i$  to  $j$ . Instead,  $r_{jkdi}(s_i; h^t) = \phi$  represents the choice of withholding  $s_i$ . Once agent  $j$  has observed a signal  $s_i$ , further disclosure of  $s_i$  to  $j$  is immaterial, because  $s_i$  cannot be confused with any other signal.

As in network papers cited earlier, information can be lost or decay in transmission. If  $j$  plays  $r_{jkdi}(s_i; h^t) = s_i$ , then  $k$  observes  $s_i$  with probability  $\delta \leq 1$ . With probability  $1 - \delta$ , agent  $k$  does not receive  $s_i$  and observes the report  $\phi$ .<sup>10</sup> I focus on the case of small decay, in which  $\delta$  is close to one.<sup>11</sup> For future reference, let me define as  $q(\tau, \ell)$  the probability that a signal reaches an agent  $d$  in network  $N$  along a path  $p$  of length  $\ell$  in  $\tau$  periods if it moves one step in  $p$  with probability  $\delta$  in every period. The probability  $q(\tau, \ell)$  is defined recursively as  $q(\tau, \ell) = \delta q(\tau - 1, \ell - 1) + (1 - \delta)q(\tau - 1, \ell)$  for  $\tau \geq \ell$ , together with  $q(\tau, \ell) = 0$  if  $\tau < \ell$ , and  $q(\tau, 0) = 1$  for all  $\tau \geq 0$ . For  $\delta$  close to one,  $q(\tau, \ell)$  is approximately equal to  $\delta^\ell$  for all  $\tau \geq \ell$ .

In the final stage of the game, agent  $d$  chooses  $\hat{y}_d$  and the payoffs  $u_i(\hat{y}_d; \theta)$  are realized by every agent  $i$ . The solution concept is (pure and mixed strategy) perfect Bayesian equilibrium. For any given network  $N$ , there may be multiple equilibria. But I will show that each agent  $i$  ranks them in the same order ex-ante (that is, before agents receive any information from nature). For the parameter values such that multiple equilibria exist, as customary in the literature on communication games since Crawford and Sobel (1982), I focus on the equilibria (called here *optimal*) that maximize each agent’s ex-ante payoffs. As I show in the following sections, the equilibrium outcome is unique in the parameter range where the main results hold.

Network formation is costly, and each link costs  $c > 0$ . I am interested in the networks that are ex-ante optimal in the utilitarian sense, that is, networks that maximize the sum of the agents’ payoffs ex-ante, including links costs. As I will explain later in details, I also study endogenous equilibrium networks when link formation requires ex-ante sponsorship by both linked agents (they each must pay  $c/2$  to sponsor the link), and when connections can be ex-ante unilaterally sponsored by one of the linked agents, who pays the whole cost  $c$ .

The model is kept as simple as possible, in order to deliver insights in the clearest fashion.

---

<sup>10</sup>It is immaterial for the analysis whether agent  $j$  observes that the signal reached agent  $k$  or not.

<sup>11</sup>The possibility of signal decay is not needed for my main results, it is included in the model to allow comparison with early network theoretical results. The model does not include information processing costs, but it can be shown that their addition would reinforce main results.

Before proceeding with the analysis, I now discuss motivation and implications of each major assumption, so as to assess robustness of the analysis.

The representation of agents' utilities as quadratic loss functions with diverse bliss points is predominant in communication games since Crawford and Sobel (1982), and this paper's formulation is a simple generalization of this standard representation. It can be proved that main results generalize qualitatively to utility representations that satisfy the single-peakedness and single-crossing assumptions standard in the communication games literature since Crawford and Sobel (1982). Likewise, the assumptions that the state is uniformly distributed and signals binary are made only for simplicity. Appendix B proves that main results generalize qualitatively as long as the signal space is bounded (and measurable, of course), regardless of whether it is discrete or continuous. Further, the assumption that link costs are the same across pairs of agents is also not essential for main results, as I explain in the following sections.

Decentralized decision making is an integral part of my model. I assume that one random agent is called to make decisions. This assumption is based on the earlier motivating examples. At any moment in time, judges in different Courts face different legal questions, policy makers in different jurisdictions confront with different policy challenges and emergencies, and different interest groups and policy stake-holders campaign or provide advice on different topics. A decision-making problem in my model represents one of these different matters, and it is then natural to assume a random decision maker.<sup>12</sup> And if allowing multiple decision makers, this paper's results generalize qualitatively as long as expert agents are sufficiently likely to be informed, and every agent can be the unique decision maker with positive probability (details available upon request).

It is here assumed that the decision maker knows who the expert agents are and their ideological biases. In the motivating examples, they are the policy-makers, lobbyists and judges known to have expertise in the area of the decision maker's problem, for example because of educational background or past experience. While such expert agents need not be actually informed, it is reasonable to assume that their identities would be public. And this assumption is unrestrictive if every agent can be expert, as is stipulated in the second part of the analysis. Just like these agents' education and past experience, it is reasonable to assume that also political views and biases are known to the decision maker.

This paper's model considers verifiable information transmission. To be clear, I do not claim that political agents do not engage in cheap talk. However, the possibility of cheap talk communication is irrelevant for my main results, as is shown in Appendix B. What is assumed here is that *some* of the exchanged information (modelled as signals  $s_i$ ) is verifiable, and I also stipulate

---

<sup>12</sup>As is common in studies of information aggregation, I abstract from the possibility of interaction across decision-making problems and game repetition effects. Their investigation would be a nice extension of this paper's analysis.

that the “source,” agent  $i$ , of each signal  $s_i$  is identifiable.<sup>13</sup> These assumptions are hardly contestable within the motivating examples presented in the introduction. The briefings and reports of interest groups and policy advocates are filled with verifiable data and facts, whose source is identifiable. Likewise, policy makers largely base decisions on policy trials data, and judges base rulings on careful interpretation of selected legislation, legal doctrine and precedents.

## 4 Optimal networks

This section calculates the optimal network  $N$  in my model, as a function of the cost and decay parameters  $c$  and  $\delta$ , as well as the bias differences  $|b_i - b_j|$  across agents  $i$  and  $j$ . I define optimal the network  $N$  that maximizes the sum of the agents  $i$  ex-ante equilibrium payoffs  $U_i(N)$  given  $N$ , minus  $c(N) \equiv c \sum_{i=1}^n \sum_{j < i} n_{ij}$ , the sum of the costs of the links in  $N$ .

In the interest of a smoother presentation, I begin the exposition with the case in which there is a unique random expert agent  $e$ , informed with probability  $\rho \in (0, 1]$ . I assume that every pair of agents  $(e, d)$  can be drawn as expert and decision maker with positive probability. Thus, I restrict attention to distributions  $\xi$  such that  $\xi(d, E, I) > 0$  if and only if  $E$  is a singleton set  $\{e\}$ , and that the probability that the expert  $e$  is actually informed  $\frac{\xi(d, \{e\}, \{e\})}{\xi(d, \{e\})}$  is equal to  $\rho$ , where  $\xi(d, \{e\}) \equiv \xi(d, \{e\}, \{e\}) + \xi(d, \{e\}, \emptyset)$ .

**The case of a single expert agent: Communication strategies.** In order to determine optimal networks, one needs to calculate equilibrium communication strategies given network  $N$ , expert  $e$  and decision maker  $d$ . Suppose that  $e$  and  $d$  are linked via a single path  $p$  of length  $\ell$ . The agents’ order of biases along  $p$  is arbitrary. As shown in the proofs of Propositions 2 - 4, there is no need to calculate equilibrium communication strategies when  $e$  and  $d$  are linked through multiple paths to determine optimal networks for this paper’s case of interest, in which the link cost  $c$  is not too low.

Consider an agent  $j$  on the path  $p$  and say that  $b_j > b_d$ . Agent  $j$  would like to bias agent  $d$ ’s decision  $\hat{y}$  to the right. When the signal  $s$  of agent  $e$  is equal to 1, agent  $j$  has no incentive to withhold  $s$ , as it conforms with her bias relative to  $d$ . By relaying  $s = 1$  to the next agent on the path  $p$ , agent  $j$  can only (weakly) increase the probability that  $d$  receives signal  $s$ , and move her decision  $\hat{y}$  to the right, closer to  $j$ ’s bliss point  $b_j + E[\theta|s]$ . When  $j$  is informed of  $s = 0$ , the signal

---

<sup>13</sup>Main results generalize qualitatively to the case in which the source  $i$  of any signal  $s_i$  is not identifiable, as long as the realization of signal  $s_i$  is verifiable, regardless of whether or not agents can distinguish different signals  $s_i$  and  $s_j$ . It is also immaterial whether or not any agent  $j$  can certify to any agent  $k$  that any signal  $s_i$  has arrived to her. (Details available upon request.) Instead, it matters that  $j$  cannot certify to the decision maker  $d$  that she has *not* received  $s_i$ , but this assumption seems realistic, at least in the applications that motivate the analysis.

$s$  is contrary to her bias  $b_j - b_d > 0$  relative to  $d$ . By relaying  $s = 0$  to the next agent on  $p$ , agent  $j$  can only move the decision  $\hat{y}$  to the left, farther from her bliss point  $b_j + E[\theta|s]$ . Unless the bias  $b_j - b_d$  is smaller than the threshold calculated in Lemma 1 below, agent  $j$  prefers to withhold  $s = 0$ . And by symmetry, any agent  $j$  with  $b_j - b_d < 0$  prefers to withhold  $s = 1$  in equilibrium unless  $|b_j - b_d|$  is smaller than the same threshold.<sup>14</sup>

**Lemma 1** *Suppose  $\ell + 1$  agents  $j$  are linked along a path  $p$  from agent  $e$  informed of a signal  $s$  with probability  $\rho \in (0, 1]$  to decision-maker  $d$ . In every equilibrium, every such agent  $j$  with  $b_j \geq b_d$  (respectively,  $b_j \leq b_d$ ) prefers to relay the signal  $s = 1$  (resp.,  $s = 0$ ) to the next agent on  $p$ , but every agent  $j$  such that  $b_j - b_d > \frac{1}{6[2-\rho q(T,\ell)]}$ , (respectively,  $b_j - b_d < -\frac{1}{6[2-\rho q(T,\ell)]}$ ) prefers to withhold signal  $s = 0$  (resp.,  $s = 1$ ).*

When agents  $j$  are sufficiently biased relative to  $b_d$ ,  $|b_j - b_d| > \frac{1}{6[2-\rho q(T,\ell)]}$ , they withhold signals contrary to their bias. Because strategic withholding of information is the research subject of this paper, let me momentarily suppose that  $|b_j - b_d| > \frac{1}{6[2-\rho q(T,\ell)]}$  for every agent  $j$  on the path  $p$  from  $e$  to  $d$ . I now describe a simple necessary and sufficient condition for informed decision making.

If every agent  $j$ 's ideological bias  $b_j - b_d$  has the same sign, then the signal  $s$  reaches agent  $d$  in equilibrium only if it conforms to the agents' biases relative to agent  $d$ . (I call this *one-sided information transmission*.) As in the simpler case of verifiable information transmission between two agents studied by Milgrom (1981), this leads to informed decision making. Both receiving and not receiving signal  $s$  are informative to agent  $d$  in equilibrium. If not observing  $s$ , she believes that  $s$  is contrary to the agents' biases with probability  $\frac{1}{2-\rho q(T,\ell)} > \frac{1}{2}$ .

Instead, there is *full communication breakdown* to agent  $d$  when  $b_j > b_d$  for some agents  $j$  on the path  $p$ , and  $b_k < b_d$  for some other agents  $k$  on  $p$ . The former relay the signal  $s = 1$  but block  $s = 0$ , whereas the latter block  $s = 1$  and relay  $s = 0$ . As a result, the signal  $s$  never reaches agent  $d$ , who acts fully uninformed. Unlike in the simpler case of two agents, equilibrium disclosure of verifiable information need not be informative. For future reference, I call *bias reversal path* any path  $p$  from an agent  $e$  to an agent  $d$  in which there are both agents  $j$  with significant positive bias,  $b_j - b_d > \frac{1}{6[2-\rho q(T,\ell)]}$ , and agents  $k$  with negative bias  $b_k - b_d < -\frac{1}{6[2-\rho q(T,\ell)]}$ .

The above analysis is summarized in the following proposition.

**Lemma 2** *Suppose  $\ell + 1$  agents  $j$  are linked along a path  $p$  from agent  $e$  informed of a signal  $s$  with probability  $\rho \in (0, 1]$  to decision-maker  $d$ . Suppose that  $|b_j - b_d| > \frac{1}{6[2-\rho q(T,\ell)]}$  for all  $j$  on  $p$ .*

1. *If  $b_j > b_d$  (respectively,  $b_j < b_d$ ) for every agent  $j$  on  $p$ , then every agent relays  $s = 1$  (resp.  $s = 0$ ) along  $p$  in every equilibrium, but  $s = 0$  (resp.  $s = 1$ ) never reaches agent  $d$ .*

---

<sup>14</sup>The formal proofs of Lemma 1 and of the subsequent results are in Appendix A.

2. If there exists agents  $j, k$  on  $p$  such that  $b_j > b_d$  and  $b_k < b_d$ , then neither  $s = 0$  nor  $s = 1$  ever reaches agent  $d$  in any equilibrium.

While for expositional reasons, this result is here presented within the simple model of Section 3, it is much more general. Appendix B generalizes Lemma 2 qualitatively to any non-trivial statistical information model and utility function that satisfies monotonicity and concavity assumptions, which are standard in the communication games literature at least since Crawford and Sobel (1982). It shows that neither a quadratic loss function, nor a uniform distribution state, nor binary signals are needed for the result. In particular, Lemma 2 generalizes qualitatively to the case of continuous signals. Further, Lemma 2 clarifies that the study of strategic information transmission is not covered by earlier network models (e.g., Galeotti et al., 2006), in which the value of connecting a pair of agents is independent of the identity of the other agents on the connecting paths.

The agents' equilibrium utility depends on whether the path  $p$  from  $e$  to  $d$  has bias reversals or not. If it does not, then equilibrium communication is one sided. Regardless of whether signal realization  $s = 0$  or  $s = 1$  is the one withheld in equilibrium, and I prove in Appendix A that each agent  $i$ 's ex-ante expected payoffs are:

$$U_{ied}(p) = -\alpha_{id}(b_i - b_d)^2 - \alpha_{id} \frac{1}{18} \left(1 + \frac{1 - \rho q(T, \ell)}{2 - \rho q(T, \ell)}\right). \quad (1)$$

If the path  $p$  includes bias reversals, then communication breaks down, and  $d$ 's decision is not informed. Each agent  $i$ 's ex-ante payoffs are:

$$U_{ied}(p) = -\alpha_{id}(b_i - b_d)^2 - \alpha_{id} \frac{1}{12}. \quad (2)$$

The case opposite to communication break down is full information transmission: neither signal realization  $s = 0, 1$  is withheld on the path  $p$  to decision maker  $d$ . Because the length of the path  $p$  is  $\ell$ , signal  $s$  reaches  $d$  with probability  $\rho q(T, \ell)$ , and each agent  $i$ 's ex-ante expected equilibrium payoffs are:

$$U_{ied}(p) = -\alpha_{id}(b_i - b_d)^2 - \alpha_{id} \frac{1}{18} \left(1 + \frac{1 - \rho q(T, \ell)}{2}\right). \quad (3)$$

It is immediate by comparing the expressions (1) - (3) that every agent  $i$  ranks ex-ante payoffs  $U_{ied}(p)$  in the same way. They all prefer full communication to one-sided information transmission, which they all prefer to communication breakdown. As usual in communication games, each agent's equilibrium ex-ante payoffs are higher when the decision maker is more informed. Hence, the optimal equilibrium maximizes the probability that signal  $s$  reaches agent  $d$ .

The next result shows that full communication takes place in equilibrium if and only if all the agents' ideologies are sufficiently close. Hence, when ideological differences are small, they do not entail any strategic withholding of information (at least, in the equilibrium that maximizes each agent's payoffs). This establishes the generalization for sufficiently aligned preferences of all results from the extant literature on information transmission in networks, which abstracts from strategic communication.

**Lemma 3** *Suppose  $\ell + 1$  agents  $j$  are linked along a path  $p$  from agent  $e$  informed of a signal  $s$  with probability  $\rho \in (0, 1]$  to decision-maker  $d$ . An equilibrium exists in which every agent relays both  $s = 0$  and  $s = 1$  to the next agent on  $p$  if and only if  $|b_j - b_d| \leq \frac{1}{12}$  for every agent  $j$  on  $p$ .*

The result is intuitive. Suppose that  $|b_j - b_d| < \frac{1}{12}$  for every  $j$  on  $p$ . Consider an agent  $j$  with  $b_j - b_d > 0$  who is informed that  $s = 0$ , in contrast with her bias. By blocking the transmission of signal  $s$  along the path  $p$ , agent  $j$  increases the probability that  $s$  does not reach  $d$ , thus moving the equilibrium decision  $\hat{y}_d$  to the right from  $b_d + E[\theta|s = 0] = b_d + 1/3$  to  $b_d + E[\theta|\phi] = b_d + \frac{1}{2}$ .<sup>15</sup> When  $2(b_j - b_d) \leq \frac{1}{2} - 1/3 = 1/6$ ; this movement of  $\hat{y}_d$  “leapfrogs” the bliss-point  $b_j + E[\theta|s = 0]$  of agent  $j$  by so much that it makes  $j$  worse off. Agent  $j$  prefers to not deviate from equilibrium and to relay  $s = 0$  to the next agent on  $p$ . A symmetric argument holds for any agent  $j$  with  $b_j < b_d$  who is informed that  $s = 1$ .

The equilibrium characterization beyond the cases in which  $|b_j - b_d| > \frac{1}{6[2-\rho q(T,\ell)]}$  for every agent  $j$  on  $p$ , or  $|b_j - b_d| > \frac{1}{12}$  for every  $j$  on  $p$  is completed in Appendix A. As well as characterizing under which conditions there exists an equilibrium with either full, one-sided, or no communication, Proposition A1 calculates optimal equilibrium as a function of the parameters values.

When all the biases  $|b_j - b_d|$  of agents  $j$  on  $p$  are smaller than  $1/12$ , the optimal equilibrium has full communication. As biases increase, the optimal equilibrium is initially in mixed strategy. All agents relay both  $s = 0$  and  $s = 1$ , except the agent  $j$  whose bias  $|b_j - b_d|$  is highest, who randomizes disclosure of the signal realization  $s$  contrary to her bias. As biases increase further, the optimal equilibrium has one-sided communication. Notably, if  $|b_j - b_d| < \frac{1}{6[2-\rho q(T,\ell)]}$  for all agents  $j$  on  $p$ , then both one sided equilibria exist: one in which  $s = 0$  is relayed and  $s = 1$  blocked, and one in which  $s = 1$  is relayed and  $s = 0$  blocked. Finally, there is no equilibrium communication when the path  $p$  has bias reversals, i.e., when  $b_j - b_d < -\frac{1}{6[2-\rho q(T,\ell)]}$  and  $b_k - b_d > \frac{1}{6[2-\rho q(T,\ell)]}$ , for some agents  $j, k$  on  $p$ .

---

<sup>15</sup>Agent  $d$ 's expectation about  $\theta$  if not observing  $s$  equals  $\frac{1}{2}$  in this equilibrium, because the event that  $s$  is lost in transmission is independent of whether  $s = 0$  or  $s = 1$ .



**The case of a single expert agent: Optimal networks.** The conditions identified above that determine whether ideological differences are too small or sufficiently large to entail strategic information withholding, also determine as a result the shape of the optimal network. If ideologies are close,  $|b_i - b_j| \leq \frac{1}{12}$  for all pairs of agents  $i, j$  (i.e.,  $b_n - b_1 \leq \frac{1}{12}$ ), then earlier results based on reduced-form models of information transmission hold. In the case agents are homogeneous, in particular, every optimal network  $N$  is a star. There is a unique agent  $i$  (the center) to which all other agents  $j$  are linked, and there are no other links across agents. The optimal network is *maximally centralized*.

When instead agents are ideologically distant,  $|b_i - b_j| > \frac{1}{6(2-\rho)}$  for all pairs  $i, j$  (i.e.,  $b_{i+1} - b_i > \frac{1}{6(2-\rho)}$ , for all  $i < n$ ), then the optimal network  $N$  is the ordered line. Every agent  $i = 2, \dots, n - 1$  is linked to both agents  $i - 1$  and  $i + 1$ , and there are no other links across agents. The optimal network is *minimally centralized* (among minimally connected networks). This result holds regardless of whether agents are homogeneous or not.

The first result, Proposition 1, concerns the case of small ideological differences and homogeneous agents. As in earlier network papers, the optimality of stars hinges on the link cost  $c$  being neither too small nor too large. If  $c$  is sufficiently small, then the optimal network  $N$  is complete, i.e., every pair of agents  $i, j$  is connected. Likewise, if  $c$  is too large, then the optimal network  $N$  is not connected: there exist agents  $i$  and  $j$  that are not linked by any path. Because of small decay, Proposition 1 covers nearly all cost values  $c$  in the relevant cost range  $[0, \bar{c}_0]$ . The upper endpoint  $\bar{c}_0$  is defined as the maximum cost for which the optimal network is connected, in the hypothetical case that there is full communication without decay between every pair of agents.

**Proposition 1** *Suppose that agents are homogeneous and there is one randomly drawn expert  $e$ , informed with probability  $\rho > 0$ . When  $b_n - b_1 \leq \frac{1}{12}$ , there exist thresholds  $\bar{\delta} < 1$  and  $\underline{c}(\delta) < \bar{c}(\delta)$ , with  $\lim_{\delta \rightarrow 1} \underline{c}(\delta) = 0$  and  $\lim_{\delta \rightarrow 1} \bar{c}(\delta) = \bar{c}_0$ , such that for all  $\delta \in (\bar{\delta}, 1)$  and  $c \in (\underline{c}, \bar{c})$ , every optimal network is a star  $S$ .<sup>16</sup>*

This result confirms earlier findings on the optimality of star networks that date back at least to Jackson and Wolinsky (1996). Ideology differences are small, and so there is no strategic withholding of information, regardless of the identities of the decision maker  $d$  and the expert  $e$ . The only obstacle to information transmission is decay. Because agents are homogeneous, all stars are optimal networks: they connect all agents with the shortest sum of paths, and hence minimize decay.

The most important result of this section, Proposition 2, concerns the case in which ideological differences are sufficiently large, according to the threshold calculated in Lemma 2. In the case of

---

<sup>16</sup>When  $\delta = 1$ , there exists  $\bar{c}(\delta)$  with  $\lim_{\delta \rightarrow 1} \bar{c}(\delta) = \bar{c}_0$ , such that every tree is optimal for all  $c \in (0, \bar{c})$ .

interest with small decay and excluding extreme link cost, the optimal network is the ordered line  $L$ . It is optimal for the whole group of political agents that each one of them forms connections only with the ideologically closest agents. Importantly, this does not lead to information transmission breakdown. Equilibrium communication from the expert agent to the decision maker is one sided. And when the expert is very likely to be informed, the decision maker's choice in the ordered line is almost as informed as if all information was shared.

Analogously to Proposition 1, also Proposition 2 covers nearly all cost values  $c$  in the relevant range, defined here as  $[0, \bar{c}_1]$ . The endpoint  $\bar{c}_1$  corresponds to the maximum cost for which the optimal network is connected, in the hypothetical case that one-sided information transmission with no decay takes place between every pair of agents  $i$  and  $j$ .

**Proposition 2** *Suppose there is one randomly drawn expert  $e$ , informed with probability  $\rho > 0$ . When  $b_{i+1} - b_i > \frac{1}{6(2-\rho)}$  for all  $i < n$ , there exist thresholds  $\bar{\delta} < 1$  and  $\underline{c}(\delta) < \bar{c}(\delta)$ , with  $\lim_{\delta \rightarrow 1} \underline{c}(\delta) = 0$  and  $\lim_{\delta \rightarrow 1} \bar{c}(\delta) = \bar{c}_1$ , such that for all  $\delta \in (\bar{\delta}, 1]$  and  $c \in (\underline{c}, \bar{c})$ , the optimal network is the ordered line  $L$ .<sup>17</sup>*

The above result is based on the equilibrium communication strategies determined in Lemma 2. Just like that result, Proposition 2 generalizes well beyond the simple model presented in Section 3. It holds qualitatively for any non-trivial information model and utility function that satisfies standard assumptions. Further, it can be shown that Proposition 2 also generalizes qualitatively to the case that link costs are heterogeneous across pairs of players, although it need not then be the case that  $\lim_{\delta \rightarrow 1} \underline{c}(\delta) = 0$  and  $\lim_{\delta \rightarrow 1} \bar{c}(\delta) = \bar{c}_1$ .

The intuition for Proposition 2 is as follows. Suppose momentarily that the optimal network is a tree (i.e., it is minimally connected). There is only one path connecting any pair of agents  $e$  and  $d$ , and the information flow is fully blocked when that path has bias reversals. Evidently, there are no bias reversal paths in ordered line  $L$ : If  $b_e < b_d$ , then  $b_k < b_d$  for all agents on the path from  $e$  to  $d$ , and vice versa. With the ordered line  $L$ , information is never fully blocked.

Moving on to consider other trees, I first focus on the case of 4 agents. Interchanging agents' identities, there are two classes of trees: the line, and the star. It can be seen that every star has at least a bias reversal path,<sup>18</sup> and of course, non-ordered lines contain bias reversal paths.

---

<sup>17</sup>The concept of optimality is expressed in terms of the sum of agents  $i$  ex-ante payoffs  $U_i$  minus the aggregate link costs. The same result holds qualitatively if considering any weighted sum of agents' payoffs and link costs. Fixing any weight distribution, there exist thresholds  $\bar{\delta}$ ,  $\underline{c}$ , and  $\bar{c}$  such that exactly the same stated results hold. However, the specific threshold values depend on the weight distribution considered.

<sup>18</sup>When agent 1 is the center, then the path from agent 3 to 2 has a bias reversal, and so does the path from 4 to 3 when agent 2 is the center. The cases when 4 and 3 are the star centers are analogous to when the center is 1 and 2, respectively, by interchanging 1 with 4 and 2 with 3.

Then, I note that for any number of agents  $n \geq 4$ , the only tree that does not contain a star or a non-ordered line as a sub-tree is the  $n$ -agent ordered line  $L$ . Hence, the unique minimally connected network in which information is never fully blocked is the ordered line. When decay is small ( $\delta$  close to one), the first concern is to ensure that information is not blocked, as opposed to the length of the communication paths. The ordered line  $L$  is thus the (unique) optimal tree.

Let me then consider networks  $N$  that are not minimally connected. When the link cost  $c$  is not too small, adding links to minimally connected networks is wasteful. When  $c$  is not too high, it is optimal that there is a path between each pair of agents, because each agents' information is useful to every agent: the optimal network must be connected. I prove in Appendix A that for small decay, there exist a non-empty "intermediate" range of costs  $(\underline{c}, \bar{c})$ , function of  $\delta$ , such that for all  $c \in (\underline{c}, \bar{c})$  the unique optimal network is the ordered line. As the decay factor  $\delta$  converges to one, the cost range  $(\underline{c}(\delta), \bar{c}(\delta))$  grows to cover the whole range  $(0, \bar{c})$ . This last result holds because, as decay becomes negligible, the penalty borne by the ordered line relative to networks with shorter paths (the fact that signals are more likely lost, as the total length of paths is larger) also becomes negligible.

I conclude this part of the analysis with a short digression that considers cheap talk communication (as in Galeotti et al., 2013), instead of transmission of verifiable information.<sup>19</sup> I first consider cheap talk from an expert agent  $e$  who holds a signal  $s$  informative of  $\theta$  with probability  $\rho$ , to a decision maker  $d$  along a path  $p$  of length  $\ell$ . In the case of interest of small decay, Appendix B proves that if  $|b_j - b_d| > \frac{1}{6}$  for all agents  $j$  on  $p$ , then communication fully breaks down in equilibrium.<sup>20</sup> This is in stark contrast with the case of verifiable information transmission, where even if  $b_{i+1} - b_i > \frac{1}{6}$  for all agents  $i < n$ , equilibrium supports (one-sided) information transmission along any path  $p$  without bias reversals (Lemma 2).

The implications for network optimality are also stark. For small decay and excluding extreme link cost, if the agents ideological biases are sufficiently diverse,  $b_{i+1} - b_i > \frac{1}{6}$  for all  $i < n$ , the optimal network  $N$  is empty, when the signal  $s$  is not verifiable.<sup>21</sup> Instead, if the signal  $s$  is verifiable, then the optimal network is the ordered line (Proposition 2).

The same reasoning shows that adding cheap talk on top of transmission of verifiable infor-

---

<sup>19</sup>Formally, I lift the verifiability restriction on reporting strategies that  $r_{jkdi}(\omega_{ij}; h^t) \in \{\omega_{ij}, \phi\}$  given  $j$ 's information  $\omega_{ij} \in \{0, \phi, 1\}$  on signal  $s_i$ , for any agents  $d, i, j, k$ , and history  $h^t$ . Regardless of the realization of signal  $s_i$ , the messages  $r_{jkdi}(s_i; h^t)$  may here take any value out of possibly large message sets  $M_{jkdi}^t$ , that I assume finite for simplicity. The remainder of the model of Section 3 is unchanged.

<sup>20</sup>In every period  $t$ , the decision maker  $d$  receives the same report  $r_{jd}(\omega_j; h^t)$  from the last agent  $j$  on  $p$ , regardless of the signal realization  $s$  and of the history  $h^t$ . Appendix B also shows that, if there is any agent  $j$  on  $p$  with  $|b_j - b_d| > \frac{1}{6(2-\rho)}$ , then communication breaks down in the unique equilibrium that is outcome equivalent to any equilibrium of the game with verifiable communication.

<sup>21</sup>The empty network  $N$  is such that  $n_{ij} = 0$  for all  $i, j$ .

mation would be irrelevant, if the agents' ideologies are sufficiently diverse. If together with the verifiable signal  $s$ , there were also unverifiable signals  $\tilde{s} \in \{0, 1\}$  informative of  $\theta$  with probability  $\Pr(\tilde{s} = 1|\theta) = \theta$ , then the decision maker would not be informed of any such signals  $\tilde{s}$  in equilibrium, when  $b_{i+1} - b_i > \frac{1}{6}$  for all agents  $i < n$ . And for any general distribution of unverifiable signals  $\tilde{s}$  informative of  $\theta$ , it can be shown that the decision maker would not be informed of  $\tilde{s}$  in equilibrium, when the biases  $b_{i+1} - b_i$  are sufficiently large.

**The case of multiple expert agents.** The logic behind Proposition 2 extends to the case in which there is more than one expert agent, i.e., the random set  $E$  is not a singleton set. The condition that identifies “sufficiently large” ideological differences for the ordered line  $L$  to be the optimal network is that  $b_{i+1} - b_i > \frac{1}{2(n+2)}$  for all  $i < n$ . And note that this implies that  $b_n - b_1 > \frac{n-1}{2(n+2)} \rightarrow \frac{1}{2}$  for  $n \rightarrow \infty$ , a requirement seemingly not very demanding. As Proposition 2, the next result concerns the case of small delay and excludes extreme link costs.

**Proposition 3** *Suppose that every agent may be informed on  $\theta$ , and that ideologies are sufficiently diverse,  $b_{i+1} - b_i > \frac{1}{2(n+2)}$  for all  $i < n$ . Then, there exists thresholds  $\bar{\delta} < 1$  and  $\underline{c}(\delta) < \bar{c}(\delta)$ , with  $\lim_{\delta \rightarrow 1} \underline{c}(\delta) = 0$  and  $\lim_{\delta \rightarrow 1} \bar{c}(\delta) = \bar{c}_2$ , such that for all  $\delta \in (\bar{\delta}, 1]$  and  $c \in (\underline{c}, \bar{c})$ , the optimal network is the ordered line  $L$ .<sup>22</sup>*

Just like Proposition 2, also Proposition 3 generalizes qualitatively for any non-trivial information model and utility function that satisfies standard assumptions, as well as with heterogeneous link costs. The proof of Proposition 3 is an extension of the proof of Proposition 2. Again, it is shown that the ordered line  $L$  is the only tree for which the transmission of none of signals  $s_i$  is blocked to any possible decision maker  $d$ . Hence,  $L$  is the optimal minimally connected network for small decay,  $\delta$  close to 1; and  $L$  is the optimal network unless the link cost  $c$  is too small (when a connected network with loops may dominate  $L$ ) or  $c$  is too high (when a not connected network may dominate  $L$ ).

Now however, there are potentially  $n$  different signals, and hence there is an informational loss if the decision maker  $d$  does not receive all of them. I prove in Appendix A that when  $b_{i+1} - b_i > \frac{1}{2(n+2)}$  for all  $i < n$ , any agent  $j$  wants to block a signal  $s_i$  contrary to her bias  $b_j - b_d$  relative to agent  $d$ , upon knowing that none of the other  $n - 1$  signals is withheld from  $d$  in equilibrium. As in Proposition 2, the ordered line  $L$  is the only tree that makes such blocking impossible, and hence it dominates all other trees, as long as it is not too much penalized by decay.

---

<sup>22</sup>The threshold  $\bar{c}_2$  is again defined as the maximum cost for which the optimal network is connected if one-sided information transmission with no decay were to take place between every pair of agents  $i$  and  $j$ .

**The optimality of factionalization.** To conclude the analysis of optimal networks, suppose that agents are partitioned into ideologically diverse groups of agents with similar ideology. As also pointed out earlier, such distributions of ideologies are prevalent in politics. An early study by Rose (1964) identifies such ideological groups as “stable set[s] of attitudes,” as opposed to organized party factions, described as groups “self-consciously organized as a body, with a measure of discipline and cohesion.” Ideologically diverse groups of agents with similar ideology arise when agents form their views as students of different party schools or political organizations.<sup>23</sup> Divisive issues such as abortion, civil rights, or environmental protection also tend to polarize views and create ideologically coherent groups. Further, ideological polarization among policy makers may be the result of increasing inequalities and diversity among the voters they represent. Indeed, polarization and segmentation into ideologically distant groups has been a dominant trend in American politics in recent decades (see, for example, McCarty, Poole and Rosenthal, 2016).

The results presented so far show that if the agents are all ideologically diverse, then they should optimally only form links with their immediate ideological networks, whereas if agents are homogeneous and ideologically close, then they should optimally organize as a star network. These results bear intuitive implications for ideological groups of agents. In the case of interest of small decay and excluding extreme link costs, it is optimal for all agents that each group organizes as a (minimally) connected subnetwork, and that only ideologically adjacent groups are connected, and by a unique link. In the case agents are homogeneous with respect to authority and expertise, each group should organize as a star, whose center is connected to the centers of the ideologically closest stars.<sup>24</sup>

**Proposition 4** *Suppose that every agent may be informed on  $\theta$ . Say that there are  $M \leq N$  ideological groups of agents  $G_1, \dots, G_M$  such that for all  $m = 1, \dots, M$ , and all  $i \in G_m$ ,  $|b_i - b_j| \leq \frac{1}{2(n+2)(n+1)}$  for all  $j \in G_m$ , and  $b_i - b_k > \frac{1}{2(n+2)}$  for all  $k \in G_{m-1}$ . Then, there exist  $\bar{\delta} < 1$  and  $\underline{c}(\delta) < \bar{c}(\delta)$ , with  $\lim_{\delta \rightarrow 1} \underline{c}(\delta) = 0$  and  $\lim_{\delta \rightarrow 1} \bar{c}(\delta) = \bar{c}_3$ , such that for all  $\delta \in (\bar{\delta}, 1)$  and  $c \in (\underline{c}, \bar{c})$ ,*

1. *every optimal network is minimally connected with a unique link across ideologically adjacent groups, and there are no other links across groups;*
2. *if agents are homogeneous, then it is optimal that each group  $G_m$  organizes as a star  $S_m$ , whose center is linked with the centers of  $S_{m-1}$  and  $S_{m+1}$  (and there are no other links across groups).*

Proposition 4 provides an efficiency rationale for *ideological factionalization*. Because agents

---

<sup>23</sup>For example, Argentinian policy makers and economists often divide themselves among those who studied in national universities and favor socially oriented policy making, and “fresh-water” US university trained purponents of free-market measures. Similar divisions are present in other countries.

<sup>24</sup>For brevity, the next results (Propositions 4 - 6) are presented for the case in which all agents may be informed on  $\theta$ , analogous results hold for the case of a single expert agent.

are partitioned in separate ideological groups, it is optimal *for the whole set of agents* that each group organizes as a separate faction. Specifically, the optimal network is connected, but there is only one link across adjacent ideological groups, and no other links across groups. Just like Lemma 2, and Propositions 2 and 3, the optimality of factionalization is a very robust result that holds qualitatively for general utility specifications and information models, as well as heterogeneous agents and link costs.

Besides rationalizing factionalization, Proposition 4 also provides an efficiency justification for the hierarchical organization often observed within political factions. When agents are homogeneous, each ideological group  $m$  optimally forms a faction organized as a star  $S_m$ , whose center is naturally interpreted as the faction leader, and every other member of the faction reports only to her. Communication across factions occurs only through the leaders, and each faction leader communicates only with the leaders of the ideologically closest factions. The reason why such stars are connected through their centers is that this minimizes the overall length of paths that signals need to travel across stars. Such a centralized organization of factions, here derived for the case of homogeneous agents, would be optimal also in the case that the central agent has a higher authority.

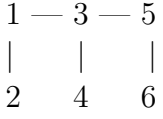
It is important that Proposition 4 predicts the optimality of factionalization, despite the assumption that link costs are unrelated to the agents' ideological differences. In many environments, the cost of forming a link is likely smaller within an ideological group than across groups, and may even be negative. Evidently, optimality of factionalization would hold a-fortiori when including this observation in the model. The implications of link cost heterogeneity for the optimal internal organization of factions are more intricate, and a full description of results is outside the boundaries of this paper. The insights of the extant literature on information transmission in networks carry over (see, for example, Galeotti et al., 2006), because there is no strategic withholding of information within factions here, under the conditions of Proposition 4.

While political agents are often partitioned into separate ideologically cohesive groups, other distributions of ideologies are (at least theoretically) possible. For example, it may be that all pairs of agents are ideologically close, in the sense that  $b_{i+1} - b_i \leq \frac{1}{2} \frac{1}{(n+2)(n+1)}$  for all  $i < n$ , and yet agents are not grouped, because  $b_{i+2} - b_i > \frac{1}{2} \frac{1}{(n+2)(n+1)}$  for all  $i < n - 1$ . Beyond the case of separate ideological groups, the optimal network can take forms different from the structure characterized in Proposition 4. This is demonstrated in the following example, which is presented for the case of a single random expert agent, to make matters simpler.

**Example 1.** Suppose there are  $n = 6$  homogeneous agents. A decision maker  $d$  and an expert  $e$ , informed with probability  $\rho \leq 1$ , are randomly drawn. The biases are as follows:

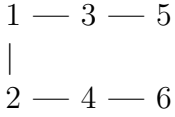
$$\frac{1}{12} < b_2 - b_1 \leq \frac{1}{6(2-\rho)}, b_3 - b_2 > \frac{1}{6(2-\rho)}, \frac{1}{12} < b_4 - b_3 \leq \frac{1}{6(2-\rho)}, b_5 - b_4 > \frac{1}{6(2-\rho)}, \text{ and } \frac{1}{12} < b_6 - b_5 \leq \frac{1}{6(2-\rho)}.$$

Let me first consider the following network  $N$ :



In line with the optimal network characterization of Proposition 4, every pair of agents whose ideology difference is smaller than  $\frac{1}{6(2-\rho)}$  is linked, and these pairs (stars) are then linked according to their ideology order. Because  $b_{i+1} - b_i > \frac{1}{12}$  for all  $i < n$ , there cannot be any path  $p$  along which both signals  $s = 0$  and  $s = 1$  reach any agent  $d$ . By Proposition 4, there is one sided information transmission across all pairs of agents  $e$  and  $d$ . However, there is significant signal decay: the sum of the lengths of the paths among pairs of agents is 72.

Let me compare network  $N$  with the following network  $N'$ :



While network  $N'$  can be represented as two linked stars,  $1 - 3 - 5$  and  $2 - 4 - 6$ , the connections within these stars are among ideologically distant agents, whereas the connection across the stars is among ideologically close agents. This is the opposite of the optimal networks characterized in Proposition 4.

Nevertheless, one-sided communication takes place among every pair of agents  $e$  and  $d$  also in network  $N'$ . In fact, there are no bias reversals in any paths between  $e \leq 3$  and  $d \geq 4$  (or  $e \geq 4$  and  $d \leq 3$ ). And there is one-sided communication also along any other path with  $d = 3$  or  $d = 4$ . This is because (i) agents 3 and 4 are so ideologically close that they may send each other either  $s = 0$  or  $s = 1$  in equilibrium (cf. the discussion after Lemma 2, and Proposition A1 in Appendix A), and (ii) there is a path without bias reversal from any agent  $e$  to either  $d = 3$  or  $d = 4$ . An analogous argument shows that there is also one-sided communication along any path with  $e = 3$  and  $e = 4$ .

The case in which  $e = 6$  and  $d = 5$ , or  $e = 5$  and  $d = 6$ , is more subtle. The path  $p$  from 6 to 5 goes through 3 and 4. Because  $b_4 - b_5 < -\frac{1}{6(2-\rho)}$ , the signal  $s = 1$  is blocked. Nevertheless, because agent 6 is sufficiently close to 5,  $b_6 - b_5 \leq \frac{1}{6(2-\rho)}$ , she is willing to relay signal  $s = 0$  along  $p$ , and this signal is not blocked by either 3 or 4. The converse argument holds when  $e = 5$  and  $d = 6$ . By symmetry, the cases in which  $e = 0$  and  $d = 1$ , or  $e = 1$  and  $d = 0$ , are addressed in the same way.

Network  $N'$  dominates network  $N$  because it achieves the same information transmission with

less decay. The sum of the lengths of the paths in  $N'$  is 58, whereas in network  $N$  it is 72. It can be further proved that  $N'$  is the efficient network for the above profile of biases. This is because it achieves the maximum possible information transmission, with the shape closest to a star.<sup>25</sup>  $\diamond$

Having concluded my analysis of optimal networks, the next section studies endogenous network formation and stability both in the case that links require the sponsorship of both connected agents, and when links can be individually sponsored.

## 5 Endogenous network formation and stability

Beyond network efficiency, an important question is how networks form endogenously and whether they are stable. This question has been addressed by Jackson and Wolinsky (1996) in terms of *pairwise stability*. The idea is that every link needs the cooperation of both linked agents to form and be maintained, and that agents do not forego the opportunity of forming mutually advantageous links. As is the case for optimality, also stability is assessed here *ex-ante*, i.e., before the agents are informed by nature.

**Pairwise stable networks.** Pairwise stability can be described as a refinement of the set of Nash equilibria of a “bilateral sponsorship” endogenous network formation game, in which the agents simultaneously submit lists of links they sponsor, and each link forms if it is in the lists of both connected agents.

In the context of this paper suppose that, before nature discloses any information, all agents  $i$  simultaneously submit a profile  $l_i \in \{0, 1\}^n$  that identifies by  $l_{ij} = 1$  the agents  $j \neq i$  with whom  $i$  sponsors a link by paying the upfront cost  $c/2$ . The link between any agents  $i$  and  $j$  forms,  $n_{ij} = 1$ , if and only if  $l_{ij} = 1 = l_{ji}$ , else  $n_{ij} = 0$ . Each agent  $i$  pays  $c/2$  for every  $j$  such that  $l_{ij} = 1$  also in the event that the link with  $j$  does not form. Given the endogenous network  $N = \{n_{ij}\}_{(i,j) \in \mathcal{N}^2}$ , the game proceeds as before, with nature disclosing  $d$  and  $E$  publicly and  $s_i$  privately to each agent  $i \in I$ , followed by  $T$  rounds of information transmission along  $N$ , and by the decision  $\hat{y}_d$ .

As in Jackson and Wolinsky (1996), I consider two different pairwise stability concept. A network  $N$  is said (weakly) pairwise stable if two conditions are met. First,  $N$  obtains as a Nash equilibrium of the bilateral sponsorship game, i.e., for every link  $n_{ij} = 1$ , both agents  $i$  and  $j$  are better off with the network  $N$ , than with the network  $\hat{N}$  in which  $\hat{n}_{ij} = \hat{n}_{ji} = 0$  and  $\hat{n}_{hk} = n_{hk}$  for every other pair  $h, k$ , and not paying the cost  $c/2$  for the severed link between  $i$  and  $j$ . Second,

---

<sup>25</sup>Example 1 shows an ideology profile for which the optimal network, while representable as interlinked stars, is not composed of stars of agents of similar ideology. Example 3 in the Appendix takes this one step further, in identifying a bias profile for which the optimal network is a tree that is not even representable as interlinked stars.



for every link  $n_{ij} = 0$ , either agent  $i$  or  $j$ , or both, are better off with the network  $N$ , than with the network  $\hat{N}$  in which  $\hat{n}_{ij} = \hat{n}_{ji} = 1$  and  $\hat{n}_{hk} = n_{hk}$  for every other pair  $h, k$ , and paying the cost  $c/2$  of the added link between  $i$  and  $j$ .<sup>26</sup>

In the second pairwise stability concept I consider, the second requirement is strengthened to require that there are no “joint incentives” to form extra links in the network  $N$ . It is demanded that, for every link  $n_{ij} = 0$ , the sum of the equilibrium payoffs of the agents  $i$  and  $j$  with the network  $N$  is larger than the sum of these agents’ payoffs with the network  $\hat{N}$  in which  $\hat{n}_{ij} = \hat{n}_{ji} = 1$  and  $\hat{n}_{hk} = n_{hk}$  for every other pair  $h, k$ , including the cost  $c$  of the added link between  $i$  and  $j$ . Under this “strong” pairwise stability concept, a network  $N$  is unstable if there is an agent  $i$  who is so interested in forming a link with another agent  $j$ , to be willing to subsidize  $j$  in case the value of the extra link to  $j$  falls short of the cost  $c/2$ .

The analysis is here conducted under the assumption that, for every agent  $i$ , the weight  $\alpha_{ii}$  is sufficiently close to one that the main concern of agent  $i$  is to obtain information from the other agents  $j$  for her decision  $\hat{y}_i$ , rather than improving their information. I prove that the optimal networks of Propositions 2 - 4 obtain as Nash equilibria of the bilateral sponsorship game, and are the unique strongly stable networks, under conditions that are qualitatively analogous.<sup>27</sup>

**Proposition 5** *Suppose the agents simultaneously choose whether or not to sponsor links with each other in the bilateral sponsorship game. After the network is formed, every agent may receive information on  $\theta$ .*

1. *For diverse ideologies,  $b_{i+1} - b_i \geq \frac{1}{2(n+2)}$  for all  $i < n$ , there exist  $\bar{\alpha} < 1$ ,  $\bar{\delta}' < 1$  and  $\underline{c}'(\delta) < \bar{c}'(\delta)$  with  $\lim_{\delta \rightarrow 1} \underline{c}'(\delta) = 0$  and  $\lim_{\delta \rightarrow 1} \bar{c}'(\delta) = \bar{c}_2$ , such that when  $\alpha_{ii} > \bar{\alpha}$  for all  $i$ ,  $\delta > \bar{\delta}'$  and  $c \in (\underline{c}', \bar{c}')$ , the ordered line  $L$  obtains as Nash equilibrium, and  $L$  is the unique strongly pairwise stable network.*
2. *Suppose there are  $M \leq n$  groups of agents  $G_1, \dots, G_M$  such that for all  $m = 1, \dots, M$  and all  $i \in G_m$ ,  $|b_i - b_j| \leq \frac{1}{2(n+2)(n+1)}$  for all  $j \in G_m$  and  $b_i - b_k > \frac{1}{2(n+2)}$  for all  $k \in G_{m-1}$ . Then, there exist  $\bar{\alpha} < 1$ ,  $\bar{\delta}' < 1$  and  $\underline{c}'(\delta) < \bar{c}'(\delta)$  with  $\lim_{\delta \rightarrow 1} \underline{c}'(\delta) = 0$  and  $\lim_{\delta \rightarrow 1} \bar{c}'(\delta) = \bar{c}_3$ , such that, when  $\alpha_{ii} > \bar{\alpha}$  for all  $i$ ,  $\delta > \bar{\delta}'$  and  $c \in (\underline{c}', \bar{c}')$ , every strongly pairwise stable network is minimally connected with a unique link across ideologically adjacent groups, and no other links across groups; further, if agents are homogeneous, then every group  $G_m$ ,  $m = 2, \dots, M - 1$ , is organized as a star  $S_m$  whose center is linked with the centers of the stars  $S_{m-1}$  and  $S_{m+1}$ .*

These results are intuitive. For small decay and excluding extreme link costs, the ordered line  $L$  obtains as Nash equilibrium of the bilateral sponsorship game because no agent would profit by

<sup>26</sup>Multiple equilibria are unavoidable in the bilateral sponsorship game, as a best response of any agent  $i$  to  $\ell_{ji} = 0$  is  $\ell_{ij} = 0$  regardless of whether the link with  $j$  would be beneficial or not.

<sup>27</sup>The thresholds  $\delta'$ ,  $\underline{c}'$  and  $\bar{c}'$  of Proposition 5 below need not coincide with the analogous thresholds of Propositions 2 - 4.

breaking any link, and cannot unilaterally establish alternative links. And the same reasons that make  $L$  the optimal network imply that  $L$  is also the unique strongly pairwise stable network. In fact,  $L$  is the unique minimally connected network in which the path between every pair of agents has no bias reversals. It is then the unique strongly pairwise stable network, when the link cost is not so low that agents wish to pay for duplicate paths, nor it is so high that pairs of agents would forego the opportunity to exchange information by co-sponsoring a link. Just like Propositions 2 and 3, Proposition 5 extends qualitatively to general utility functions and information models.

However, I also show in Appendix A that the ordered line  $L$  is not uniquely selected by weak pairwise stability, even when every pair of agents is ideologically distant. Other trees such as all stars are weakly pairwise stable networks. While in every star there exists pairs of agents  $i$  and  $j$  such that the path from  $i$  and  $j$  has a bias reversal, it is never the case that also the path from  $j$  to  $i$  has bias reversals. The star is thus weakly pairwise stable because, although agent  $i$  would benefit by forming a link with  $j$ , agent  $j$  would not and prefers not sponsor the link. That said, not all networks are weakly pairwise stable. I show in Appendix A that weak pairwise stability selects minimally connected networks and may rule out non-ordered lines.

The same reasoning as above demonstrates that, when the agents are partitioned in ideological groups, every strongly pairwise stable network is minimally connected with a unique link across ideologically adjacent groups, and no other links across groups; and further if agents are homogeneous with respect to expertise, then the groups organize as stars whose centers are connected along the ideological order.

Having concluded that the optimal networks of Propositions 2 - 4 obtain as Nash equilibria of a bilateral network sponsorship game, and are the unique strongly pairwise stable networks, I now turn to consider endogenous network formation in a game of unilateral sponsorship.

**Unilateral sponsorship game.** Unlike in the game of bilateral sponsorship, suppose that each link is formed and maintained by one of the two connected agents (as in Bala and Goyal, 2000). Again, before nature discloses any information, all agents  $i$  simultaneously submit a link proposal profile  $l_i \in \{0, 1\}^n$ . Here, a link between any pair of agents  $i$  and  $j$  forms as long as one of them pays for the link:  $n_{ij} = 1$  if and only if  $l_{ij} + l_{ji} > 0$ . For brevity, I study this game focusing on the case of diverse ideologies,  $b_{i+1} - b_i \geq \frac{1}{2(n+2)}$  for all  $i < n$ . Again, I focus on the case that  $\alpha_{ii}$  is close to one for every agent  $i$ .

I show that the ordered line  $L$  obtains as Nash equilibrium of the unilateral sponsorship game, with small decay and excluding extreme link cost, if and only if there are at most five agents. In the equilibrium implementing  $L$ , I further establish that every link is sponsored by the most ideologically moderate agent, i.e., by the agent closer to the median agent(s) in  $\mathcal{N}$ .

**Proposition 6** *Suppose that  $n$  ideologically diverse agents,  $b_{i+1} - b_i \geq \frac{1}{2(n+2)}$  for all  $i < n$ , simultaneously choose whether or not to unilaterally sponsor links with each other. After the network is formed, every agent may receive information on  $\theta$ .*

1. *For  $n \leq 5$ , there exist  $\bar{\alpha} < 1$ ,  $\bar{\delta}'' < 1$  and  $\underline{c}''(\delta) < \bar{c}''(\delta)$  with  $\lim_{\delta \rightarrow 1} \underline{c}''(\delta) = 0$ ,  $\lim_{\delta \rightarrow 1} \bar{c}''(\delta) = \bar{c}_2$ , such that when  $\alpha_{ii} > \bar{\alpha}$  for all  $i$ ,  $\delta > \bar{\delta}''$  and  $c \in (\underline{c}'', \bar{c}'')$ , the ordered line  $L$  obtains as Nash equilibrium of unilateral sponsorship game. For  $n = 4, 5$  and  $\delta < 1$ , every link  $n_{ij} = 1$  is sponsored by the most moderate among  $i$  and  $j$ .<sup>28</sup>*

2. *For  $n \geq 6$ , the ordered line  $L$  cannot obtain as Nash equilibrium of the unilateral sponsorship game, when  $\delta < 1$  and  $\alpha_{ii}$  is sufficiently close to one for all  $i$ .*

I prove this result first by showing that the ordered line  $L$  obtains as an equilibrium for  $\delta < 1$  only if all links are paid by the most moderate agent. This is best explained taking the case of 5 agents as an example. The link between agents 2 and 3 in  $L$  cannot be sponsored by 2 in equilibrium, because she would prefer to deviate and connect to 4, instead. Through agent 4, agent 2 has access to the same set of agents, 3, 4 and 5, but with shorter paths (and none of these paths has bias reversals, as is the case when linking to 3). Instead, agent 3 has no incentive to deviate when sponsoring the link to 2, because switching to 1 gives her exactly the same payoffs. This “shortcut argument” holds for any  $n$  and for any pair of ideological neighbors, to conclude that  $L$  obtains as a Nash equilibrium only if all links are paid by the most moderate agent.

As a consequence, whether the ordered line  $L$  is implemented as an equilibrium depends on whether or not the most moderate agent of every pair of ideological neighbors would benefit by bypassing the link with her neighbor, instead of sponsoring it. In turn, this depends on the length of the line of agents she reaches through the link with her neighbor. As long as  $n \leq 5$ , there are not enough agents for any shortcut to be a profitable deviation, and  $L$  obtains as a Nash equilibrium. But for any  $n \geq 6$ , the ordered line  $L$  cannot be an equilibrium. In this case, neither agent 3 nor 4 are willing to sponsor a link between each other, as each of them prefers to bypass that link and find a shortcut.

Having established that the optimal network  $L$  cannot obtain as a Nash equilibrium of the unilateral sponsorship game for  $n \geq 6$  agents, I conclude this section by explicitly constructing the optimal Nash equilibrium for  $n = 6$ .

**Example 2.** Suppose there are 6 agents with diverse ideologies,  $b_{i+1} - b_i > \frac{1}{2(n+2)}$  for all  $i < n$ , who simultaneously choose whether or not to unilaterally sponsor links of cost  $c$ . After the network  $N$  is formed, a decision-maker  $d$  is randomly drawn, and every agent may receive information on

---

<sup>28</sup>In the case of  $n = 3$ , every link in  $L$  can be sponsored by either of the two linked agents.

$\theta$ . For every agent  $i$ , the weight  $\alpha_{ii}$  is sufficiently close to one that  $i$  form links just to maximize the information she receives from the other agents. The probability of decay is small. The link cost  $c$  is not so high that an agent  $i$  would rather having signals blocked from her than paying for a link, and  $c$  is not so low that  $i$  would want to form duplicate links to avoid signal decay.

In light of Proposition 6, there is no Nash equilibrium that implements the ordered line  $L$ . Consider the strategy profile  $l$  such that  $l_{21} = 1$ ,  $l_{32} = 1$ ,  $l_{35} = 1$ ,  $l_{42} = 1$ ,  $l_{45} = 1$ ,  $l_{56} = 1$ , and  $l_{ij} = 0$  for all other  $i, j$ . Agent 2 sponsors a link with 1, agent 3 sponsors a link with 2 and 5, and symmetrically agent 5 sponsors a link with 6, and agent 4 sponsors a link with 5 and 2.

The profile  $l$  yields a network  $N$  that is not minimally connected. As in the ordered line  $L$ , no signal is blocked from any agent. To establish that  $l$  is a Nash equilibrium, I check that no agent prefers to deviate. Evidently, this is not the case for the agents 1 and 6 who do not pay for any link. Agents 2 and 5 do not gain by severing their links to 1 and 6 as they would lose information; nor do 3 and 4 by severing the links with 2 and 5 and sponsoring links with 1 and 6 instead. Finally, the link preferred by 3 to reach the agents 4, 5 and 6 is the link with 5; and symmetrically, the best link for 4 to access 1, 2 and 3 is the link with 2. Hence,  $l$  is a Nash equilibrium. Because the network  $N$  induced by  $l$  is as close as possible to the optimal network  $L$ , I conclude that  $l$  is the optimal Nash equilibrium.  $\diamond$

## 6 Conclusion

Motivated by political economy applications such as networks of policy-makers, interest groups, or judges, this paper solves an explicit model of strategic information transmission in networks of ideological agents. When agents' preferences are sufficiently close, earlier theoretical network analyses based on reduced-form communication models apply. In particular, if agents are homogeneous with respect to authority and expertise, then every optimal and stable network is maximally centralized among minimally connected networks, i.e., it is a star. But when all agents' ideologies are sufficiently diverse, I have found that the optimal network is minimally centralized, and specifically, it is the line in which each agent connects only with the ideologically closest agents. This network obtains as Nash equilibrium of a game in which each link requires sponsorship by both connected agents, and it is the unique strongly pairwise stable network.

The case in which agents are partitioned in ideologically diverse groups, each composed of agents with similar views, is especially important in politics. In this case, I have proved that it is optimal for all agents that the groups organize as distinct subnetworks, with a unique link across ideologically nearest groups. When agents are homogeneous in authority and expertise, the groups should organize as stars, whose leaders communicate with the leaders of the ideologically nearest

star. Again, these optimal networks obtain as Nash equilibria of a bilateral link sponsorship game, and are uniquely strongly pairwise stable.

My results suggest positive and normative rationales for why factionalization and connections between ideologically close agents are so common in political networks. Instead, such “horizontal” connections are less likely useful in organizations where the agents’ preferences are more closely aligned, such as the army or some commercial companies. This paper may be useful for future theoretical and empirical research on political networks. The analysis is based on a simple model and functional forms, it should be easy to test results in lab experiments.<sup>29</sup> By and large, my findings confirm and explain empirical stylized facts. Networks of political decision makers can be described as a collection of subnetworks, differentiated by political preferences, and loosely connected among each other. Each subnetwork is structured hierarchically with a few leaders to which less prominent agents are connected. As well as stable, my analysis suggests that these network structures are efficient for the purposes of information transmission.

I have kept my model as simple as possible, in order to deliver insights in the clearest fashion. This paper’s analysis can be generalized and modified in several directions. Ostensibly, introducing information processing costs into the model would lead to the optimality and stability of hierarchies with more layers than stars, within groups of ideologically close-minded agents. Such congestion costs may also yield optimality and stability of multiple centroids in hierarchies within ideologically groups. Allowing for negative connection costs within groups would make connections among periphery agents optimal and stable, and this would also likely be the implication of taking decay probability to be large relative to link cost.

In general, because there is no strategic information withholding within groups, earlier results in network and organizational economics based on reduced-form models of communication can be applied here to characterize optimal within faction organisation. More ambitious elaborations of the analysis would also be interesting. For example, it would be possible to study the robustness of a political network when an antagonist attempts to remove agents from the network. This could happen within a party’s network of elected officials when politicians are up for re-election. The opposing parties may try to disrupt the party’s network by strategically allocating campaign resources. This question could then be modelled as a variation of the Colonel Blotto game (Borel, 1921).

It would be especially valuable to formulate and solve an explicit model of strategic communication in networks of legislators in Congress. Unlike the local policy makers, interest groups, advisers and judges considered here, legislators are not individual decision makers, because their

---

<sup>29</sup>As well survey-based analysis, empirical work on information transmission in political networks includes also experimental studies. Among them, Ahn et al. (2013) investigate citizens political participation.

votes translate into bills only if they achieve majority. Embedding a model of collective decision making and voting into the framework developed here would be an interesting and non-trivial task. In the large empirical literature that has developed since Roubin (1938), information transmission is understood to be probably the most important functions of legislator networks.<sup>30</sup> The evidence is strong that ideological and policy preference proximity is a main driver of connections among legislators (e.g., Fiellin, 1962, Zhang et al., 2008, and Bratton and Rouse, 2011).<sup>31</sup>

Likewise, it would be very interesting to develop an explicit model of strategic information transmission in citizen networks. Unlike here, it would there be plausible to model communication as cheap talk. Although voting in elections is one of the main forms of political participation, a citizen's actions may still be modelled as individual decisions, when the expressive motive is the main driver of her choices. As is the case for networks of political decision makers considered here, there is strong evidence of ideology and policy preference based homophily in citizens' connections (e.g., Huckfeldt and Sprague, 1987, and Eveland and Kleinman, 2013).

## References

- Acemoglu, Daron, Kostas Bimpikis, and Asuman Ozdaglar (2014): "Dynamics of information exchange in endogenous social networks," *Theoretical Economics*, 9: 41-97
- Ahn, T.K., Robert Huckfeldt, Alexander K. Mayer and John Barry Ryan. 2013. "Expertise and Bias in Political Communication Networks." *American Journal of Political Science* 57: 357-373.
- Ambrus, Attila, Eduardo Azevedo, and Yuichiro Kamada. 2013. "Hierarchical cheap talk," *Theoretical Economics*, 8: 223-261.
- Bala, Venkatesh and Sanjeev Goyal. 2000. "A non-cooperative model of network formation," *Econometrica*, 68: 1181-1230.
- Battaglini, Marco, and Eleonora Patacchini. 2018. "Influencing connected legislators," *Journal of Political Economy*, 126(6): 2277-2322.
- Baum, Lawrence. 2014. "Hiring supreme court law clerks: probing the ideological linkage between judges and justices." *Marquette Law Review*, 98: 333.

---

<sup>30</sup>In a classical piece, Fiellin (1962) surveys 1961 Democrat representatives about their network of friends in the New York State House, and documents that "probably the most important functions of informal groups and relationships result from their use as communication networks. [...] Representatives receive information from many sources. [...] This profusion of information probably complicates the problem of decision-making rather than facilitating its solution. It is for this reason that communication within informal groups may be particularly valuable to the individual member."

<sup>31</sup>Canen, Trebbi and Jackson (2019) structurally estimate a model of endogenous network formation and legislative activity. Unlike my paper, where network homophily is micro-founded, legislators are assumed to form links according to ideological proximity.

- Baum, Lawrence, and Corey Ditslear. 2001. "Selection of law clerks and polarization in the U.S. Supreme Court," *Journal of Politics*, 63(3): 869 - 885.
- Bloch, Francis, Gabrielle Demange, and Rachel Kranton. 2018. "Rumors and social networks," *International Economic Review*, 59(2): 421-448.
- Bolton, Patrick, and Matthias Dewatripont. 1994. "The firm as a communication network," *Quarterly Journal of Economics*, 109: 809-39.
- Borel, Émile. 1921. "La théorie du jeu et les équations intégrales à noyau symétrique gauche," *Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences*, 19 December 1921, 1304-1308.
- Box-Steffensmeier, Janet M., and Dino P. Christenson. 2014. "The evolution and formation of *amicus curiae* networks," *Social Networks*, 36: 82-96.
- Bratton, Kathleen A., and Stella M. Rouse. 2011. "Networks in the legislative arena: how group dynamics affect cosponsorship," *Legislative Studies Quarterly*, 34(3): 423-460.
- Caldeira, Gregory A. 1985. "The transmission of legal precedent: A study of State Supreme Courts," *American Political Science Review*, 79(1): 178-194.
- Calvó-Armengol, Antoni, Joan de Martí, and Andrea Prat. 2015. "Communication and influence," *Theoretical Economics*, 10: 649-690.
- Canen, Nathan, Matthew O. Jackson, and Francesco Trebbi. 2019. "Endogenous networks and legislative activity," mimeo, Berkeley University.
- Carpenter, Daniel P., Kevin M. Esterling, and David M. J. Lazer. 2004. "Friends, brokers, and transitivity: Who informs whom in Washington politics?" *Journal of Politics*, 66(1): 22-246.
- Ceron, Andrea (2019): *Leaders, Factions and the Game of Intra-Party Politics*, Routledge, London.
- Choi, Stephen J., and G. Mitu Gulati. 2008. "Bias in judicial citations: a window into the behavior of judges?," *Journal of Legal Studies*, 37(1): 87-130.
- Chwe, Michael. 2000. "Communication and coordination in social networks," *Review of Economic Studies*, 67: 1-16.
- Crawford, Vincent, and Joel Sobel. 1982. "Strategic information transmission," *Econometrica*, 50: 1431-145.
- Desmarais, Bruce A., Jeffrey J. Harden, and Frederick J. Boehmke. 2015. "Persistent policy pathways: Inferring diffusion networks in the American States," *American Political Science Review*, 109(2): 392-406.
- Dewan, Torun, Andrea Galeotti, Christian Ghiglino, and Francesco Squintani. 2015. "Information aggregation and optimal executive structure," *American Journal of Political Science*, 59(2): 475-494.
- Dewan, Torun, and Francesco Squintani. 2016. "In defense of factions," *American Journal of Political Science*, 60(4): 860-881.

- Egorov, Georgy, and Konstantin Sonin (2020): “Persuasion on networks,” mimeo.
- Eveland, William P. Jr. and Steven B. Kleinman. 2013. “Comparing general and political discussion networks within voluntary organizations using social network analysis,” *Political Behavior* 35: 65 - 87.
- Feiock, Richard C., In Won Lee, and Hyung Jun Park. 2012. “Administrators and elected officials collaboration networks: Selecting partners to reduce risk in economic development.” *Public Administration Review* 72(S1): 58-68.
- Fiellin, Alan. 1962. “The functions of informal groups in legislative institutions,” *Journal of Politics*, 24(1): 72-91.
- Fischer, Manuel, and Pascal Sciarini. 2016. “Collaborative Tie Formation in Policy Networks: A Cross-Sector Perspective,” *Journal of Politics*, 78(1): 63-74.
- Friebel, Guido and Michael Raith. 2004. “Abuse of authority and hierarchical communication,” *RAND Journal of Economics*, 35(2): 224–244.
- Galeotti, Andrea, Christian Ghiglino and Francesco Squintani. 2013. “Strategic communication networks,” *Journal of Economic Theory*, 148(5): 1751–1769.
- Galeotti, Andrea, Sanjeev Goyal, and Jurjen Kamphorst. 2006. “Network formation with heterogeneous players,” *Games and Economic Behavior*, 54(2): 353–372.
- Gerber, Elisabeth R., Adam D. Henry, and Mark Lubell. 2013. “Political homophily and collaboration in regional planning networks,” *American Journal of Political Science*, 57(3): 598-610.
- Gieczewski, German. 2020. “Verifiable communication on networks,” mimeo, *Princeton University*.
- Gilligan, Thomas W. and Keith Krehbiel. 1987. “Collective decision-making and standing committees: An informational rationale for restrictive amendment procedures,” *Journal of Law, Economics, and Organization*, 3: 287–335.
- Hagenbach, Jeanne, and Frédéric Koessler. 2010. “Strategic communication networks,” *Review of Economic Studies*, 77(3): 1072-99.
- Henry, Adam D. 2011. “Ideology, power, and the structure of policy networks,” *Policy Studies Journal*, 39(3): 361-383.
- Henry, Adam D., Mark Lubell, and Michael McCoy. 2011. “Belief systems and social capital as drivers of policy network structure: The case of California regional planning.” *Journal of Public Administration Research and Theory*, 21(3): 419-444.
- Huckfeldt, Robert, and John Sprague. 1987. ”Networks in Context: The Social Flow of Political Information,” *American Political Science Review*, 81: 1197 - 1216.
- Hume, David. 1741. “Part I, Essay VIII: Of parties in general,” in *Essays, Moral and Political*, by David Hume, Edinburgh: A. Kincaid.
- Ingold, Karin. 2011. “Network structures within policy processes: Coalitions, power, and brokerage in Swiss climate policy,” *Policy Studies Journal*, 39(3): 435- 459.



- Jackson, Matthew O. 2010. *Social and Economic Networks*, New York: Princeton University Press.
- Jackson, Matthew O., and Asher Wolinsky. 1996. "A strategic model of social and economic networks," *Journal of Economic Theory*, 71(1): 44–74.
- Janda, Kenneth. 1980. *Political Parties: A Cross-National Survey*, New York: Free Press.
- Janda, Kenneth. 1993. "Comparative Political Parties: Research and Theory," in *Political Science: The State of the Discipline II*, ch. 7. American Political Science Association.
- Kamenica, Emir, and Matthew Gentzkow. 2011. "Bayesian persuasion," *American Economic Review*, 101(6): 2590-2615.
- Katz, Martin D., and Derek K. Stafford. 2008. "Hustle and flow: A social network analysis of the American federal judiciary," *Ohio State Law Journal*, 71(3): 457-507.
- Koger, Gregory, Seth Masket, and Hans Noel. 2009. "Partisan webs: Information exchange and party networks," *British Journal Political Science*, 39: 633-653.
- König, Thomas, and Thomas Bräuninger. 1998. "The formation of policy networks: Preferences, institutions and actors' choice of information and exchange relations," *Journal of Theoretical Politics*, 10(4): 445-71.
- Larson, Jennifer M. 2017. "Networks and interethnic cooperation," *Journal of Politics*, 79(2): 546 - 59.
- Larson, Jennifer M., and Janet I. Lewis. 2018. "Rumors, kinship networks, and rebel group formation," *International Organization*, 72(4): 871-903.
- Lasswell, Harold D. (1931): "Social conflict," in *Encyclopaedia of the Social Sciences*, Vol. 4, Eds: Edwin R. Seligman and Alvin S. Johnson, London: Macmillan.
- Laumann, Edward O., and David Knoke. 1987. *The Organizational State: Social Choice in National Policy Domains*, Madison WI: The University of Wisconsin Press.
- Matti, Simon, and Annica Sandström. 2011. "The rationale determining advocacy coalitions: examining coordination networks and corresponding beliefs," *Policy Studies Journal*, 39(3): 385-410.
- McCarty, Nolan, Keith T. Poole and Howard Rosenthal. 2016. *Polarized America: The Dance of Ideology and Unequal Riches*, 2nd Ed., MIT Press, Cambridge, MA.
- Migrow, Dimitri. 2019. "Designing communication hierarchies," mimeo, *University of Calgary*.
- Milgrom, Paul R. 1981. "Good news and bad news: Representation theorems and applications," *Bell Journal of Economics*, 12(2): 380–391.
- Patty, John W. 2013. "A theory of cabinet-making: The politics of inclusion, exclusion, and information," mimeo, *Washington University at Saint Louis*.
- Patty, John W. and Elizabeth M. Penn. 2014. "Sequential decision making and information aggregation in small networks." *Political Science Research and Methods*, 2(2): 243-271.

- Patty, John W. and Elizabeth M. Penn. 2020. "Identity and information in organizations," mimeo, *Emory University*.
- Penn, Elizabeth M. 2016. "Engagement, disengagement and exit," *American Journal of Political Science*, 60(2): 322–336.
- Persico, Nicola, Josè C. R. Pueblita, and Dan Silverman. 2011. "Factions and political competition," *Journal of Political Economy*, 119(2): 242-288.
- Ringe, Nils and Jennifer N. Victor, with Christopher J. Carman. 2013. *Bridging the Information Gap: Legislative Member Organizations as Social Networks in the United States and the European Union*, Ann Arbor: University of Michigan Press.
- Radner, Roy. 1993. "The organization of decentralized information processing," *Econometrica*, 61: 1109–1146.
- Routt, Garland C. 1938. "Interpersonal relationships and the legislative process," *Annals of the American Academy of Political and Social Science*, 195(1): 129–136.
- Rose, Richard. 1964. "Parties, factions and tendencies in Britain," *Political Studies*, 12(1): 33-46.
- Schneider, Mark, John Scholz, Mark Lubell, Denisa Mindruta, and Matthew Edwardsen. 2003. "Building consensual institutions: Networks and the national estuary program," *American Journal of Political Science*, 47(1): 143 - 158.
- Victor, Jennifer N., Alexander H. Montgomery, and Mark Lubell. 2017. *The Oxford Handbook of Political Networks*, Oxford University Press, Oxford, UK.
- Weible, Christopher M., and Paul A. Sabatier (2005). "Comparing policy networks: Marine protected areas in California." *Policy Studies Journal*, 33(2): 181-201.
- Weiser, Benjamin. 2015. "Faced with legal puzzles, judges often turn to fellow jurists," *The New York Times*, May 19, 2015.
- Zafonte, Mathew, and Paul A. Sabatier. 1998. "Shared beliefs and imposed interdependencies as determinants of ally networks in overlapping subsystems," *Journal of Theoretical Politics*, 10(4): 473-505.
- Zhang, Yan, A. J. Friend, Amanda L. Traud, Mason A. Porter, James H. Fowler, and Peter J. Mucha. 2008. "Community structure in Congressional cosponsorship networks." *Physica A: Statistical Mechanics and Its Applications*, 387: 1–8.

## Appendix A: Omitted Proofs

**Proof of Lemma 1.** In every equilibrium, the final decision  $\hat{y}_d$  of agent  $d$  maximizes  $E[u_d(\hat{y}_d; \theta) | \omega_d] = -E[(\hat{y}_d - \theta - b_d)^2 | \omega_d]$ , where  $\omega_d \in \{0, 1, \phi\}$  denotes agent  $d$ 's equilibrium information. Hence the strategy of agent  $d$  is  $y_d = b_d + E[\theta | \omega_d]$ . If the signal  $s$  reaches agent  $d$ , she plays

$$y_d = b_d + E[\theta | s] = \begin{cases} b_d + 1/3 & \text{if } s = 0 \\ b_d + 2/3 & \text{if } s = 1. \end{cases}$$

If  $s$  does not reach agent  $d$ , she plays  $y_d = b_d + E[\theta | \phi]$ .

Consider an arbitrary agent  $j$  informed of signal  $s$  at an arbitrary history  $h^t$ . Agent  $j$  chooses whether or not to relay  $s$  to the next agent  $k$  on the path  $p$ . Omitting its dependence on  $h^t$  for brevity, let  $Q_{jd}(s)$  be the equilibrium probability that  $s$  reaches agent  $d$  if agent  $j$  plays  $r_{jkdi}(s; h^t) = s$ , thus relaying  $s$  to  $k$ . With probability  $1 - Q_{jd}(s)$ , agent  $d$ 's information is  $\omega_d = \phi$ . Likewise, let  $Q_{jd}(\phi)$  be the equilibrium probability that the signal  $s$  reaches  $d$  if agent  $j$  plays  $r_{jkdi}(s; h^t) = \phi$  and withholds  $s$  from  $k$ .

Agent  $j$ 's expected payoffs if relaying signal  $s$  to the next agent  $k$  on the path  $p$  at history  $h^t$  are:

$$u_{jd}(s; h^t) = -\alpha_{jd}E[(b_d + E[\theta | s] - b_j - \theta)^2 | s]Q_{jd}(s) - \alpha_{jd}E[(b_d + E[\theta | \phi] - b_j - \theta)^2 | s][1 - Q_{jd}(s)],$$

whereas agent  $j$ 's payoffs if not relaying  $s$  are:

$$u_{jd}(\phi; h^t) = -\alpha_{jd}E[(b_d + E[\theta | s] - b_j - \theta)^2 | s]Q_{jd}(\phi) - \alpha_{jd}E[(b_d + E[\theta | \phi] - b_j - \theta)^2 | s][1 - Q_{jd}(\phi)].$$

Hence, agent  $j$  strictly prefers to relay  $s$  if and only if:

$$\begin{aligned} u_{jd}(s; h^t) - u_{jd}(\phi; h^t) &= -\alpha_{jd}[Q_{jd}(s) - Q_{jd}(\phi)] \{E[(b_d + E[\theta | s] - b_j - \theta)^2 | s] - E[(b_d + E[\theta | \phi] - b_j - \theta)^2 | s]\} \\ &\equiv -\alpha_{jd}\Delta Q_{jd}(s; h^t)\Delta \mathcal{L}_{jd}(s) > 0. \end{aligned} \quad (4)$$

At time  $t$ , there are  $T - t + 1$  periods left to deliver  $s$  to  $d$ , including the current period. Suppose that agent  $j$  is  $\ell_j$  steps away from  $d$  on the path  $p$ . Say that each successor of  $j$  on  $p$  relays signal  $s$  to the next agent on  $p$  at every history. Then, if  $j$  relays  $s$  to the next agent  $k$  on  $p$ , the signal  $s$  reaches  $d$  with probability  $q(T - t + 1, \ell_j)$ , whereas if  $j$  withholds it,  $s$  reaches  $d$  with probability at most  $q(T - t, \ell_j)$ . Because  $q(T - t + 1, \ell_j) > q(T - t, \ell_j)$ , it follows that  $\Delta Q_{jd}(s; h^t) > 0$ . If  $\Delta \mathcal{L}_{jd}(s) < 0$ , then  $j$  relays  $s$  to her next agent  $k$  on  $p$ , whereas if  $\Delta \mathcal{L}_{jd}(s) > 0$ , then  $j$  withholds  $s$ . Because  $\Delta \mathcal{L}_{jd}(s)$  is independent of  $t$  and  $h^t$ , I have concluded that if every  $j$ 's successor on the path  $p$  relays  $s$  to her next agent on  $p$  at every history, then either  $j$  relays  $s$  to her next agent on  $p$  at every history  $h^t$ , or  $j$  withholds  $s$  at every history  $h^t$ . I also note that the argument (trivially) covers the last agent on the path  $p$  before  $d$ , as she does not have any successors other than  $d$ .

By induction, I then conclude that in every equilibrium:

1. if  $\Delta \mathcal{L}_{jd}(s) < 0$  for all agents  $j$  on the path  $p$ , then each agent  $j$  relays  $s$  to the next agent  $k$  on  $p$  at every history  $h^t$ , so that  $s$  reaches  $d$  with probability  $q(T, \ell)$ .
2. if there exists an agent  $j$  such that  $\Delta \mathcal{L}_{jd}(s) > 0$ , then signal  $s$  does not ever reach  $d$ , as on every path it is blocked by at least one such agent.

Next, I simplify  $\Delta\mathcal{L}_{jd}(s)$  as follows:

$$\begin{aligned}
\Delta\mathcal{L}_{jd}(s) &= E[(b_d + E[\theta|s] - \theta - b_j)^2 - (b_d + E[\theta|\phi] - \theta - b_j)^2 | s] \\
&= E[(b_d - b_j - \theta)^2 | s] + E[\theta|s]^2 + 2E[b_d - b_j - \theta|s]E[\theta|s] \\
&\quad - E[(b_d - b_j - \theta)^2 | s] - E[\theta|\phi]^2 - 2E[b_d - b_j - \theta|s]E[\theta|\phi] \\
&= E[\theta|s][E[\theta|s] + 2(b_d - b_j) - 2E[\theta|s]] + E[\theta|s]E[\theta|\phi] \\
&\quad - E[\theta|\phi][E[\theta|\phi] + 2(b_d - b_j) - 2E[\theta|s]] - E[\theta|s]E[\theta|\phi] \\
&= E[\theta|s][E[\theta|\phi] + 2(b_d - b_j) - E[\theta|s]] - E[\theta|\phi][E[\theta|\phi] + 2(b_d - b_j) - E[\theta|s]] \\
&= [E[\theta|\phi] - E[\theta|s]][2(b_j - b_d) - (E[\theta|\phi] - E[\theta|s])] \\
&\propto \begin{cases} 2(b_j - b_d) - (E[\theta|\phi] - E[\theta|s]) & \text{if } s = 0 \\ -2(b_j - b_d) + (E[\theta|\phi] - E[\theta|s]) & \text{if } s = 1. \end{cases} \tag{5}
\end{aligned}$$

Hence, there are two cases in which  $\Delta\mathcal{L}_{jd}(s) < 0$  and agent  $j$  strictly prefers to relay signal  $s$  (unless it is blocked by her successors in equilibrium):

$$\begin{aligned}
s = 0 \text{ and } b_j - b_d &< \frac{E[\theta|\phi] - E[\theta|s]}{2} = \frac{E[\theta|\phi] - 1/3}{2}, \\
s = 1 \text{ and } b_j - b_d &> \frac{E[\theta|\phi] - E[\theta|s]}{2} = \frac{E[\theta|\phi] - 2/3}{2}, \tag{6}
\end{aligned}$$

whereas  $\Delta\mathcal{L}_{jd}(s) > 0$  in the converse, complementary cases, and agent  $k$  strictly prefers to withhold signal  $s$ .

As a consequence, I show that every agent  $j$  on the path  $p$  such that  $b_j > b_d$  (respectively,  $b_j < b_d$ ) always relays the signal  $s = 1$  (resp.,  $s = 0$ ) to the next agent  $k$  on  $p$  in equilibrium, unless  $s$  is blocked by  $j$ 's successors.

To prove it, I first note that the event that  $s$  does not reach agent  $d$  is on the path of play of every possible equilibrium, because when an agent  $j$  relays  $s$  to the next agent  $k$  on  $p$ , the signal  $s$  is lost with probability  $1 - \delta > 0$ . As a result, agent  $d$ 's equilibrium beliefs  $E[\theta|\phi]$  are pinned down by the Bayes rule, and are therefore bounded strictly between  $E[\theta|s = 0] = 1/3$  and  $E[\theta|s = 1] = 2/3$ . When  $s = 0$ , the inequality  $E[\theta|\phi] > E[\theta|s = 0]$  implies that conditions (6) holds when  $b_j < b_d$ ; and conversely when  $s = 1$ , the inequality  $E[\theta|\phi] < E[\theta|s = 1]$  implies that conditions (6) holds when  $b_j > b_d$ . In these two cases, agent  $j$  prefers to relay  $s$  to the next agent  $k$  on  $p$  at every history  $h^t$  and in every equilibrium.

Conversely, every agent  $j$  on the path  $p$  such that  $b_j - b_d > \frac{1}{6[2 - \rho q(T, \ell)]}$  (resp.,  $b_j - b_d < -\frac{1}{6[2 - \rho q(T, \ell)]}$ ) withholds signal  $s = 0$  (resp.  $s = 1$ ) from the next agent  $k$  on  $p$  at every history  $h^t$  in every equilibrium, unless  $s$  is blocked by  $j$ 's successors.

In every equilibrium in fact, the expected value  $E[\theta|\phi]$  is weakly smaller than in the equilibrium in which every agent  $j$  on  $p$  relays signal  $s = 0$  with probability one, and signal  $s = 1$  is blocked from agent  $d$ . In this "boundary" equilibrium,

$$E[\theta|\phi] = \int_0^1 \theta f(\theta|\phi) d\theta = \int_0^1 \theta \frac{\theta + (1 - \theta)[1 - \rho + \rho(1 - q(T, \ell))]}{\int_0^1 [\theta + (1 - \theta)[1 - \rho + \rho(1 - q(T, \ell))]] d\theta} d\theta = \frac{1 - \rho q(T, \ell)/3}{2 - \rho q(T, \ell)}, \tag{7}$$

because with probability  $\theta$ , it is the case that  $s = 1$ , and then  $s$  does not ever reach  $d$ ; whereas with probability  $1 - \theta$ , it is the case that  $s = 0$ , so that  $s$  does not reach  $d$  either when not observed by the

expert agent  $e$ , with probability  $1 - \rho$ , or when lost in transmission, with probability  $1 - q(T, \ell)$ . The inequality  $E[\theta|\phi] \leq \frac{1-\rho q(T, \ell)/3}{2-\rho q(T, \ell)}$  implies that when  $s = 0$ ,  $E[\theta|\phi] - E[\theta|s] = \frac{1-\rho q(T, \ell)/3}{2-\rho q(T, \ell)} - 1/3 \leq \frac{1}{3[2-\rho q(T, \ell)]}$ , so that conditions (6) cannot hold when  $b_j - b_d > \frac{1}{6[2-\rho q(T, \ell)]}$ , and as a result  $s = 0$  is blocked in equilibrium. Conversely, when  $b_j - b_d < -\frac{1}{6(2-\rho q(T, \ell))}$  and  $s = 1$ , conditions (6) cannot hold and  $s = 1$  is blocked. ■

**Proof of Lemma 2.** This result is proved as points 2 and 4 of Proposition A1 below. ■

**Proof of Lemma 3.** This result is proved as point 1 of Proposition A1 below. ■

Let  $\underline{b} = \min_j b_j$ ,  $\bar{b} = \max_j b_j$ , and  $\bar{j} = \arg \max_j |b_j - b_d|$ , denote respectively the extreme left and right ideologies  $b_j$  on  $p$ , and the agent  $j$  with the largest absolute bias  $|b_j - b_d|$  relative to  $d$ .

**Proposition A1** Suppose  $\ell + 1$  agents  $j$  are linked along a path  $p$  from agent  $e$  informed of a signal  $s$  with probability  $\rho \in (0, 1]$  to decision-maker  $d$ .

1. If  $|b_j - b_d| \leq \frac{1}{12}$  for every agent  $j$  on  $p$ , then in the optimal equilibrium every agent relays both  $s = 0$  and  $s = 1$  to the next agent on  $p$ .

2. If  $\frac{1}{12} < \bar{b} - b_d \leq \frac{1}{6(2-\rho q(T, \ell))}$  and  $\underline{b} \geq \bar{b} - \frac{1}{6}$ , then the optimal equilibrium is in mixed strategies: every agent on  $p$  relays  $s = 1$ , and  $s = 0$  is relayed by every agent except  $\bar{j}$  who randomizes.

A symmetric characterization holds when  $-\frac{1}{6[2-\rho q(T, \ell)]} \leq \underline{b} - b_d < -\frac{1}{12}$  and  $\bar{b} \leq \frac{1}{6} + \underline{b}$ .

3. If there exists agents  $j, k$  on  $p$  such that  $b_j - b_d \geq -\frac{1}{6[2-\rho q(T, \ell)]}$  and  $b_k - b_d \leq \frac{1}{6[2-\rho q(T, \ell)]}$ , then neither  $s = 0$  nor  $s = 1$  ever reaches agent  $d$  in any equilibrium.

4. Else, information transmission is one-sided in the optimal equilibrium. If  $b_j - b_d > -\frac{1}{6[2-\rho q(T, \ell)]}$  (respectively,  $b_j - b_d < \frac{1}{6[2-\rho q(T, \ell)]}$ ) for every agent  $j$  on  $p$ , then every agent relays  $s = 1$  (resp.  $s = 0$ ) along  $p$ , but  $s = 0$  (resp.  $s = 1$ ) never reaches agent  $d$ .<sup>32</sup>

**Proof of Proposition A1.1: Full communication.**

I prove that there exists an equilibrium in which every agent  $j$  on the path  $p$  relays both  $s = 0$  and  $s = 1$  at every history  $h^t$  if and only if  $|b_j - b_d| \leq \frac{1}{12}$  for all  $j$ . I note that, when the agents conform to these strategies, it is the case that  $E[\theta|\phi] = \frac{1}{2}$ , because the event that signal  $s$  does not reach agent  $d$  is independent of whether  $s = 0$  or  $s = 1$ . Hence, conditions (6) hold if and only if either  $s = 0$  and  $b_j - b_d < \frac{E[\theta|\phi] - 1/3}{2} = \frac{\frac{1}{2} - 1/3}{2} = \frac{1}{12}$ , or if  $s = 1$  and  $b_j - b_d > \frac{\frac{1}{2} - 2/3}{2} = -\frac{1}{12}$ . As a result, each agent  $j$  has no incentive to deviate from the equilibrium strategies if and only if  $|b_j - b_d| \leq \frac{1}{12}$ .

Each agent's  $i$  expected payoffs  $U_{ied}(p)$  are as calculated follows. Because neither  $s = 0$  nor  $s = 1$  is ever withheld by any agent  $j$  on  $p$ , they both reach agent  $d$  with probability  $\rho q(T, \ell)$ . In either case,  $d$  chooses  $\hat{y}_d = E[\theta|s] + b_d$ , and each agent  $i$ 's expected payoffs are, for both  $s = 0$  and  $s = 1$ ,

$$\begin{aligned} -\alpha_{id}E[(E[\theta|s] + b_d - \theta - b_i)^2|s] &= -\alpha_{id}E[(E[\theta|s] - \theta)^2 + (b_d - b_i)^2 + (E[\theta|s] - \theta)(b_d - b_i)|s] \\ &= -\alpha_{id}(b_d - b_i)^2 - \alpha_{id}E[(E[\theta|s] - \theta)^2|s] \\ &= -\alpha_{id}(b_d - b_i)^2 - \alpha_{id}\frac{1}{18}. \end{aligned}$$

<sup>32</sup>It is immediate that case 4 includes the case in which  $|b_j - b_d| > \frac{1}{6[2-\rho q(T, \ell)]}$  for all  $j$  on  $p$ , and there are no bias reversals.

With complementary probability  $1 - \rho q(T, \ell)$ , agent  $d$  acts uninformed and chooses  $y_d = \frac{1}{2} + b_d$ . Each agent  $i$ 's ex-ante payoffs are:

$$\begin{aligned} -\alpha_{id}E[(\frac{1}{2} + b_d - \theta - b_i)^2] &= -\alpha_{id}E[(\frac{1}{2} - \theta)^2 + (b_d - b_i)^2 + (\frac{1}{2} - \theta)(b_d - b_i)] \\ &= -\alpha_{id}(b_d - b_i)^2 - \alpha_{id}E[(\frac{1}{2} - \theta)^2] \\ &= -\alpha_{id}(b_d - b_i)^2 - \alpha_{id}\frac{1}{12}. \end{aligned}$$

Because the signals  $s = 0$  and  $s = 1$  are equally likely ex-ante, I can wrap up as follows:

$$\begin{aligned} U_{ied}(p) &= -\alpha_{id}(b_d - b_i)^2 - \alpha_{id}\rho q(T, \ell)\frac{1}{18} - \alpha_{id}[1 - \rho q(T, \ell)]\frac{1}{12} \\ &= -\alpha_{id}(b_i - b_d)^2 - \alpha_{id}\frac{1}{18}(1 + \frac{1 - \rho q(T, \ell)}{2}). \end{aligned}$$

**Proof of Proposition A1.2: Communication breakdown.** Suppose that there exists agents  $j, k$  on the path  $p$  such that  $b_j - b_d > \frac{1}{6[2 - \rho q(T, \ell)]}$  and  $b_k - b_d < -\frac{1}{6[2 - \rho q(T, \ell)]}$ . By Lemma 1, both signals  $s = 0$  and  $s = 1$  are blocked in equilibrium. Agent  $d$  acts uninformed and chooses  $y_d = \frac{1}{2} + b_d$ . Each agent  $i$ 's ex-ante payoffs are:

$$\begin{aligned} U_{ied}(p) &= -\alpha_{id}E[(\frac{1}{2} + b_d - \theta - b_i)^2] \\ &= -\alpha_{id}(b_d - b_i)^2 - \alpha_{id}\frac{1}{12}. \end{aligned}$$

**Proof of Proposition A1.4: One sided equilibrium.** I first consider equilibrium in which every agent  $j$  on the path  $p$  relays  $s = 0$  and at least one  $j$  blocks  $s = 1$ . With these strategies, agent  $d$ 's expectation of  $\theta$  upon not observing  $s$  is  $E[\theta|\phi] = \frac{1 - \rho q(T, \ell)/3}{2 - \rho q(T, \ell)}$ , cf. expression (7). Hence, each agent  $j$  on  $p$  prefers not to deviate from relaying  $s = 0$  if and only if  $b_j - b_d \leq \frac{E[\theta|\phi] - 1/3}{2} = \frac{1 - \rho q(T, \ell)/3}{2[2 - \rho q(T, \ell)]} - \frac{1}{6} = \frac{1}{6[2 - \rho q(T, \ell)]}$ . Further,  $s = 1$  is blocked if and only if  $b_j - b_d < \frac{E[\theta|\phi] - 2/3}{2} = \frac{1 - \rho q(T, \ell)/3}{2[2 - \rho q(T, \ell)]} - \frac{1}{3} = -\frac{1 - \rho q(T, \ell)}{6[2 - \rho q(T, \ell)]}$  for some  $j$  on  $p$ .

In sum, the one sided equilibrium in which  $s = 0$  is relayed and  $s = 1$  is blocked exists if and only if  $b_j - b_d \leq \frac{1}{6[2 - \rho q(T, \ell)]}$  for all  $j$  on  $p$ , and  $b_j - b_d < -\frac{1 - \rho q(T, \ell)}{6[2 - \rho q(T, \ell)]}$  for some  $j$  on  $p$ . A symmetric argument shows that the one sided equilibrium in which  $s = 1$  is relayed and  $s = 0$  is blocked exists if and only if  $b_j - b_d \geq -\frac{1}{6[2 - \rho q(T, \ell)]}$  for all  $j$  on  $p$ , and  $b_j - b_d > \frac{1 - \rho q(T, \ell)}{6[2 - \rho q(T, \ell)]}$  for some  $j$  on  $p$ .

Because  $\rho q(T, \ell) \in (0, 1]$ , it is the case that  $\frac{1 - \rho q(T, \ell)}{6[2 - \rho q(T, \ell)]} < \frac{1}{12} < \frac{1}{6[2 - \rho q(T, \ell)]}$ . Thus the area of existence of the one sided equilibria overlaps with the area of existence of the full communication equilibrium ( $|b_j - b_d| \leq \frac{1}{12}$  for all  $j$  on  $p$ ), when  $|b_j - b_d| \leq \frac{1}{6[2 - \rho q(T, \ell)]}$  for all  $j$  on  $p$ , and  $b_j - b_d < -\frac{1 - \rho q(T, \ell)}{6[2 - \rho q(T, \ell)]}$  or  $b_j - b_d > \frac{1 - \rho q(T, \ell)}{6[2 - \rho q(T, \ell)]}$  for some  $j$  on  $p$ . If instead  $|b_j - b_d| > \frac{1}{12}$  for some  $j$  on  $p$ , then the full communication equilibrium does not exist and the one-sided equilibria is optimal. And note that the analysis conducted so far covers the whole parameter space. If the path  $p$  has bias reversals (there exist  $j, k$  such that  $b_j - b_d < -\frac{1}{6[2 - \rho q(T, \ell)]}$  and  $b_j - b_d > \frac{1}{6[2 - \rho q(T, \ell)]}$ ), then communication breaks down in equilibrium. Else, if  $|b_j - b_d| \leq \frac{1 - \rho q(T, \ell)}{6[2 - \rho q(T, \ell)]}$  for all  $j$  on  $p$ , so that one sided equilibria do not exist, then  $|b_j - b_d| < \frac{1}{12}$  for all

$j$  on  $p$ , so that full communication equilibrium exists. And for all other bias profiles  $\mathbf{b}$ , there exist one sided communication equilibria.

The calculation of the agents' ex-ante payoffs in one-sided equilibria is postponed to the next part of the proof, as a special case of the ex-ante payoffs in the optimal mixed strategy equilibria.

**Proof of Proposition A1.4: Optimal mixed strategy equilibrium.** Let me first note that it is enough that one agent  $j$  on  $p$  blocks a signal realization  $s$  for it not to reach  $d$ . Second, conditions (6) imply that if any agent  $j$  such that  $b_j > b_d$  is indifferent on whether or not  $s = 0$  reaches the decision maker  $d$ , then any agent  $k$  such that  $b_k > b_j$  strictly prefers that  $s = 0$  is blocked. And a converse argument holds for  $s = 1$  and agents  $j, k$  such that  $b_k < b_j < b_d$ . Third I note that, generically, there may be at most one agent who is indifferent on whether or not the signal contrary to her bias reaches  $d$ . Hence, I do not consider the possibility of randomization on both signal realizations  $s = 0$  and  $s = 1$ . Fourth, I note that for the parameter range in which the one-sided (pure strategy) equilibrium characterized above does not exist, there is full communication breakdown in every equilibrium.

These considerations imply that the only fighting chance for any mixed strategy equilibrium to maximize the agents' payoffs are that the most biased agent  $\bar{j}$ , the agent  $j$  whose  $|b_j - b_d|$  is maximal, randomizes on the signal realization  $s$  contrary to her bias, and all other agents relay both  $s = 0$  and  $s = 1$  at every history. This is the only case in which more information is released than with one-sided information transmission.

Suppose that the most biased agent  $\bar{j}$  is such that  $b_{\bar{j}} > b_d$ . Agent  $\bar{j}$  relays  $s = 1$  at every history, and randomizes (at least) at some histories when  $s = 0$ . For this to be the case,  $\bar{j}$  must be indifferent between relaying and withholding  $s = 0$ . Hence it must be the case that  $b_{\bar{j}} - b_d = \frac{E[\theta|\phi] - 1/3}{2}$ , cf. condition (6). As a consequence, agent  $\bar{j}$  is indifferent between relaying and withholding  $s = 0$  in *every* history  $h^t$ , and I may focus on the mixed strategy equilibrium in which  $\bar{j}$  always randomizes and always with the same probability  $\sigma$  of relaying  $s = 0$ .

For this strategy profile, agent  $d$ 's expectation of  $\theta$  if not receiving  $s$  is:

$$\begin{aligned} E[\theta|\phi] &= \int_0^1 \theta \frac{\theta[1 - \rho + \rho(1 - \sigma) + \rho\sigma(1 - q(T, \ell))] + (1 - \theta)[1 - \rho + \rho(1 - q(T, \ell))]}{\int_0^1 [\theta[1 - \rho + \rho(1 - \sigma) + \rho\sigma(1 - q(T, \ell))] + (1 - \theta)[1 - \rho + \rho(1 - q(T, \ell))]} d\theta \\ &= \frac{1 - \rho q(T, \ell)(1 + 2\sigma)/3}{2 - \rho q(T, \ell)(1 + \sigma)}. \end{aligned}$$

For agent  $\bar{j}$  to be indifferent between relaying and withholding  $s = 0$ , the indifference condition simplifies to:

$$b_{\bar{j}} - b_d = \frac{E[\theta|\phi] - 1/3}{2} = \frac{\frac{1 - \rho q(T, \ell)(1 + 2\sigma)/3}{2 - \rho q(T, \ell)(1 + \sigma)} - 1/3}{2} = \frac{1 - \sigma \rho q(T, \ell)}{6[2 - \rho q(T, \ell)(1 + \sigma)]}.$$

Hence, the mixed strategy  $\sigma$  takes the form:

$$\sigma = \frac{1 - 12(b_{\bar{j}} - b_d) + 6\rho q(T, \ell)(b_{\bar{j}} - b_d)}{\rho q(T, \ell)[1 - 6(b_{\bar{j}} - b_d)]} \quad (8)$$

which is easily shown to be admissible, i.e.,  $\sigma \in [0, 1]$ , if and only if  $\frac{1}{12} \leq b_{\bar{j}} - b_d \leq \frac{1}{6[2 - \rho q(T, \ell)]}$ .

Further, note that for all agents  $k \neq \bar{j}$  such that  $b_k > b_d$ , it is the case that  $b_k - b_d < \frac{1 - \sigma \rho q(T, \ell)}{6[2 - \rho q(T, \ell)(1 + \sigma)]}$ . They all strictly prefer to relay signal realization  $s = 0$  despite it being contrary to their biases.

Now, consider all agents  $k$  such that  $b_k < b_d$ . They relay  $s = 0$  at every history  $h^t$ , and they relay  $s = 1$  if and only if

$$b_k - b_d > \frac{E[\theta|\phi] - 2/3}{2} = \frac{\frac{1 - \rho q(T, \ell)(1 + 2\sigma)/3}{2 - \rho q(T, \ell)(1 + \sigma)} - 2/3}{2} = -\frac{1 - \rho q(T, \ell)}{6[2 - \rho q(T, \ell)(1 + \sigma)]} = -\frac{1}{6} + b_{\bar{j}} - b_d,$$

where the last equality is obtained by plugging in (8). And I note that this condition implies that  $b_k - b_d \geq -\frac{1}{12}$  when  $b_{\bar{j}} - b_d \geq \frac{1}{12}$ , which implies that  $|b_k - b_d| < |b_{\bar{j}} - b_d|$  for all agents  $k$  with  $b_k < b_d$ .

In sum, I have proved that when  $\frac{1}{12} < \bar{b} - b_k \leq \frac{1}{6(2 - \rho q(T, \ell))}$  and  $\underline{b} \geq \bar{b} - \frac{1}{6}$ , then the optimal equilibrium is in mixed strategies: Every agent  $j$  relays  $s = 1$ , and  $s = 0$  is relayed by every agent except  $\bar{j}$  who relays  $s = 0$  with probability  $\sigma$  at every history  $h^t$ . A symmetric characterization holds when  $-\frac{1}{6(2 - \rho q(T, \ell))} \leq \underline{b} - b_k < -1/12$  and  $\bar{b} \leq \frac{1}{6} + \underline{b}$ .

I now calculate the agents' ex-ante payoffs. Without loss of generality, I again consider the mixed-strategy equilibrium in which the disclosure of  $s = 0$  is randomized. Hence,  $s = 1$  reaches  $d$  with probability  $\rho q(T, \ell)$ , whereas  $s = 0$  reaches  $d$  with probability  $\sigma \rho q(T, \ell)$ . Because  $s = 0$  and  $s = 1$  are equally likely ex-ante, each agent  $i$ 's expected payoffs ex-ante can be written as:

$$\begin{aligned} U_{ied}(p) &= -\alpha_{id}(b_d - b_i)^2 - \alpha_{id} \frac{\sigma \rho q(T, \ell)}{2} E[(1/3 - \theta)^2 | s = 0] - \alpha_{id} \frac{\rho q(T, \ell)}{2} E[(2/3 - \theta)^2 | s = 1] \\ &\quad - \alpha_{id} \left(1 - \frac{1 + \sigma}{2} \rho q(T, \ell)\right) E[(E[\theta|\phi] - \theta)^2 | \phi]. \end{aligned}$$

The last term is calculated as follows:

$$\begin{aligned} E[(E[\theta|\phi] - \theta)^2 | \phi] &= E[\theta^2 | \phi] - (E[\theta | \phi])^2 = \frac{4 - \rho q(T, \ell)(1 + \sigma) - 2\sigma \rho q(T, \ell)}{12 - 6\rho q(T, \ell)(1 + \sigma)} - \left[ \frac{1 - \rho q(T, \ell)(1 + 2\sigma)/3}{2 - \rho q(T, \ell)(1 + \sigma)} \right]^2 \\ &= \frac{\rho^2 q(T, \ell)^2 [(1 + \sigma)^2 + 2\sigma] + 6[1 - \rho q(T, \ell)(1 + \sigma)]}{18[2 - \rho q(T, \ell)(1 + \sigma)]^2}. \end{aligned}$$

Plugging this term into the formula for  $U_{ied}$  and simplifying, I obtain:

$$\begin{aligned} U_{ied}(p) &= -\alpha_{id}(b_d - b_i)^2 - \alpha_{id} \frac{\sigma \rho q(T, \ell)}{2} \frac{1}{18} - \alpha_{id} \frac{\rho q(T, \ell)}{2} \frac{1}{18} \\ &\quad - \alpha_{id} \left[1 - \frac{1 + \sigma}{2} \rho q(T, \ell)\right] \left[ \frac{\rho^2 q(T, \ell)^2 [(1 + \sigma)^2 + 2\sigma] + 6[1 - \rho q(T, \ell)(1 + \sigma)]}{18[2 - \rho q(T, \ell)(1 + \sigma)]^2} \right] \\ &= -\alpha_{id}(b_d - b_i)^2 - \alpha_{id} \frac{1}{18} \left[1 + \frac{1 - \rho q(T, \ell)(1 + \sigma) + \sigma \rho^2 q(T, \ell)^2}{2 - \rho q(T, \ell)(1 + \sigma)}\right] \\ &= -\alpha_{id}(b_i - b_d)^2 - \alpha_{id} \frac{1}{18} (1 + 6|b_{\bar{j}} - b_d|[1 - \rho q(T, \ell)]). \end{aligned}$$

One sided communication equilibria can be understood as the extreme case of the above mixed strategy equilibria in which  $\sigma = 0$ . The most extreme agent always withholds the signal realization contrary to her bias. Hence, each agent  $i$ 's ex-ante payoffs in the one sided communication equilibria are:

$$U_{ied} = -\alpha_{id}(b_d - b_i)^2 - \alpha_{id} \frac{1}{18} \left[1 + \frac{1 - \rho q(T, \ell)}{2 - \rho q(T, \ell)}\right].$$

■



**Proof of Proposition 2.** I start by proving the second statement. Suppose that ideologies are divergent, i.e., that  $b_{i+1} - b_i > \frac{1}{6(2-\rho)}$  for all  $i < n$ , so that  $b_{i+1} - b_i > \frac{1}{6[2-\rho q(T,\ell)]}$  for all  $\rho > 0$ ,  $T \geq \ell > 0$ ,  $\delta \leq 1$ , and all  $i < n$ .

Proceeding in two steps, I first restrict attention to minimally connected networks (i.e., to trees), and show that the optimal tree is the ordered line  $L$ .

To prove this result, I start by noting that for any  $n$ , the ordered line  $L$  has a path  $p$  without bias reversals between every pair of agents  $e$  and  $d$ . If it is the case that  $e < d$ , then every agent  $k = e, \dots, d-1$  on the path  $p$  is such that  $b_k < b_d$ ; whereas if  $e > d$ , then every agent  $k = d+1, \dots, e$  on  $p$  is such that  $b_k > b_d$ . By Lemma 2, this implies that whenever  $e < d$ , signal  $s = 0$  reaches the decision maker  $d$  with probability  $\rho q(T, |d-e|)$ , and  $s = 1$  does not reach  $d$ . When  $e > d$ , signal  $s = 1$  reaches agent  $d$  with probability  $\rho q(T, |d-e|)$ , and  $s = 0$  does not. Thus, simplifying the notation  $\xi(d, \{e\})$ , each agent  $i$ 's payoffs are:

$$\begin{aligned} U_i(L) &= - \sum_{(e,d) \in \mathcal{N}^2} \xi_{de} \alpha_{id} \left\{ (b_d - b_i)^2 + \frac{1}{18} \left[ 1 + \frac{1 - \rho q(T, |d-e|)}{2 - \rho q(T, |d-e|)} \right] \right\} \\ &\rightarrow - \sum_{(e,d) \in \mathcal{N}^2} \xi_{de} \alpha_{id} \left\{ (b_d - b_i)^2 + \frac{1}{18} \left( 1 + \frac{1 - \rho}{2 - \rho} \right) \right\} \text{ as } \delta \rightarrow 1. \end{aligned}$$

Consider any tree  $N$  with at least a pair of agents  $e$  and  $d$  linked by a bias reversal path  $p$ . Because  $N$  is a tree, there are no other paths connecting  $e$  to  $d$ . By Lemma 2, both signals  $s_e = 0$  and  $s_e = 1$  are blocked from agent  $d$  along the path  $p$ . Let  $P(N) = \{(e, d) : (b_k - b_d)(b_j - b_d) > 0 \text{ for all } k, j \text{ on the path between } e \text{ and } d\}$  be the set of agent pairs  $(e, d)$  linked by a path  $p$  without bias reversals (note that  $(e, d) \in P(N)$  if  $e = d$ , trivially). I let  $\ell_N(e, d)$  be the length of the path between any pair  $e$  and  $d$  of agents in the tree  $N$ . Then, each agent  $i$ 's payoffs are:

$$\begin{aligned} U_i(N) &= - \sum_{(e,d) \in P(N)} \xi_{de} \alpha_{id} \left\{ (b_d - b_i)^2 + \frac{1}{18} \left[ 1 + \frac{1 - \rho q(T, \ell_N(e, d))}{2 - \rho q(T, \ell_N(e, d))} \right] \right\} - \sum_{(e,d) \notin P(N)} \xi_{de} \alpha_{id} \left\{ (b_d - b_i)^2 + \frac{1}{12} \right\} \\ &\rightarrow - \sum_{(e,d) \in P(N)} \xi_{de} \alpha_{id} \left\{ (b_d - b_i)^2 + \frac{1}{18} \left( 1 + \frac{1 - \rho}{2 - \rho} \right) \right\} - \sum_{(e,d) \notin P(N)} \xi_{de} \alpha_{id} \left\{ (b_d - b_i)^2 + \frac{1}{12} \right\} \text{ as } \delta \rightarrow 1. \end{aligned}$$

Hence, for  $\delta$  close to one, the payoff difference between the ordered line  $L$  and tree  $N$  is:

$$U_i(L) - U_i(N) \approx \sum_{(d,e) \notin P(N)} \xi_{de} \alpha_{id} \left[ \frac{1}{12} - \frac{1}{18} \left( 1 + \frac{1 - \rho}{2 - \rho} \right) \right] = \frac{1}{36} \frac{\rho}{2 - \rho} \sum_{(d,e) \notin P(N)} \xi_{de} \alpha_{id} > 0.$$

Because every tree  $N$  has the same number of links,  $n - 1$ , and hence the same aggregate link cost  $c(N) = (n - 1)c$ , it then follows that for  $\delta$  close to 1, any tree  $N$  with a bias reversal path between any pair of agents  $e$  and  $d$  is dominated by the ordered line  $L$ .

It is obvious that every line network  $N$  that does not conform with the agents' bias order includes pairs of agents  $e$  and  $d$  connected only by a bias reversal path.

Turning to consider trees that are not lines, I first notice that every such a tree  $N$  must contain a 4-agent star  $S$  as a sub-tree. Hence, if I show that every 4-agent star  $S$  includes at least a bias reversal path, then I have concluded that every tree  $N$  that is not a line has at least a bias reversal path, and the proof that  $L$  is the optimal tree is complete. In fact, if the "extreme-bias" agent 1 is the center of  $S$ ,

then there is a bias reversal path, for example, from agent  $e = 3$  to agent  $d = 2$ . When the “moderate” agent 2 is the center of  $S$ , there is a bias reversal path from  $e = 4$  to  $d = 3$ . The cases when 4 or 3 are the centers of  $S$  are analogous to when the centers are 1 or 2 respectively, by interchanging 1 with 4 and 2 with 3.

I have proved that the optimal minimally connected network is the ordered line  $L$ , for  $\delta$  close to one.

The second step of the proof consists in showing the existence of thresholds  $\underline{c}(\delta)$  and  $\bar{c}(\delta)$  such that  $\lim_{\delta \rightarrow 1} \underline{c}(\delta) = 0$  and  $\lim_{\delta \rightarrow 1} \bar{c}(\delta) = \bar{c}$ , and that for  $\delta$  close to one and  $c \in (\underline{c}, \bar{c})$ , the sum of the agents’ ex-ante payoffs  $U_i(L)$  with the ordered line  $L$  minus the sum of link costs  $c(L) = (n - 1)c$  is larger than with any connected network  $N$  with more links than  $L$ , and than with any network  $N$  that is not connected.

Consider any connected network  $N$  that is not minimally connected. Because  $b_{i+1} - b_i > \frac{1}{6[2 - \rho q(T, \ell)]}$  for all  $\ell > 0$ , all  $i = 1, \dots, n - 1$ , and all  $\delta \leq 1$ , full communication and mixed strategy communication are impossible in equilibrium between any pair of agents  $e$  and  $d$ . Consider the hypothetical (and counterfactual) worst case in which there is one sided communication along every path in network  $N$  (including bias reversal paths). Even in this case, each agent  $i$ ’s ex-ante payoffs  $U_i(N)$  are approximately the same as with the ordered line  $L$ , for  $\delta$  close to one. In fact, let  $q_N(T; e, d)$  be the probability that one signal realization  $s$  reaches  $d$  along one of the paths from  $e$ . Note that  $q_N(T; e, d) \geq q(T, \underline{\ell}(e, d))$ , where  $\underline{\ell}(e, d)$  is the length of the shortest path from  $e$  to  $d$ . Then, because  $\lim_{\delta \rightarrow 1} q_N(T; e, d) \geq \lim_{\delta \rightarrow 1} q(T, \underline{\ell}(e, d)) = 1$ ,

$$\begin{aligned} U_i(N) &= - \sum_{(e,d) \in \mathcal{N}^2} \xi_{de} \alpha_{id} \left\{ (b_d - b_i)^2 + \frac{1}{18} \left[ 1 + \frac{1 - \rho q_N(T; e, d)}{2 - \rho q_N(T; e, d)} \right] \right\} \\ &\rightarrow - \sum_{(e,d) \in \mathcal{N}^2} \xi_{de} \alpha_{id} \left\{ (b_d - b_i)^2 + \frac{1}{18} \left( 1 + \frac{1 - \rho}{2 - \rho} \right) \right\} = U_i(L) \text{ as } \delta \rightarrow 1. \end{aligned}$$

By construction, network  $N$  has more than the  $n - 1$  links of  $L$ . Hence for  $\delta = 1$  and  $c > 0$ , the ordered line  $L$  dominates every connected network  $N$  with loops, in terms of the sum of agents’ ex-ante payoffs  $U_i(N)$  minus the sum of link costs  $c(N)$ . By continuity, for  $\delta$  close to one, there exists a threshold  $\underline{c}$  function of  $\delta$  with  $\lim_{\delta \rightarrow 1} \underline{c}(\delta) = 0$ , such that  $L$  dominates every connected network  $N$  with loops when  $c > \underline{c}(\delta)$ .

Now I consider networks  $N$  that are not connected, again taking the hypothetical worst case in which there is one sided information transmission even along bias reversal paths in  $N$ . Letting the set of pairs of agents linked via a path be  $\bar{P}(N)$ , the ex-ante payoffs of each agent  $i$  are then:

$$\begin{aligned} U_i(N) &= - \sum_{(e,d) \in \bar{P}(N)} \xi_{de} \alpha_{id} \left\{ (b_d - b_i)^2 + \frac{1}{18} \left[ 1 + \frac{1 - \rho q_N(T; e, d)}{2 - \rho q_N(T; e, d)} \right] \right\} - \sum_{(e,d) \notin \bar{P}(N)} \xi_{de} \alpha_{id} \left\{ (b_d - b_i)^2 + \frac{1}{12} \right\} \\ &\rightarrow - \sum_{(e,d) \in \mathcal{N}^2} (b_d - b_i)^2 - \frac{1}{18} \left( 1 + \frac{1 - \rho}{2 - \rho} \right) \sum_{(e,d) \in \bar{P}(N)} \xi_{de} \alpha_{id} - \frac{1}{12} \sum_{(e,d) \notin \bar{P}(N)} \xi_{de} \alpha_{id} \text{ as } \delta \rightarrow 1. \end{aligned}$$

Hence, for  $\delta = 1$ ,

$$\sum_{i \in \mathcal{N}} U_i(L) - \sum_{i \in \mathcal{N}} U_i(N) = \frac{1}{36} \frac{\rho}{2 - \rho} \sum_{(e,d) \notin \bar{P}(N)} \xi_{de} \sum_{i \in \mathcal{N}} \alpha_{id} > 0,$$

and any not connected network  $N$  that dominates  $L$  must have sum of link costs  $c(N) < c(L) = (n-1)c$ . Hence for  $\delta = 1$ , the ordered line  $L$  may be dominated by a not connected network  $N$  in terms of the sum of agents' ex-ante payoffs  $U_i(N)$  minus the sum of link costs  $c(N)$  only if  $N$  has less than  $n-1$  links. By definition, the individual link cost  $c$  cannot be below the threshold  $\bar{c}$ . By continuity, for  $\delta$  close to one, there exists a threshold  $\bar{c}$  function of  $\delta$  with  $\lim_{\delta \rightarrow 1} \bar{c}(\delta) = \bar{c}$ , such that for  $c < \bar{c}$ ,  $L$  dominates every network  $N$  that is not connected.

I have proved the second statement of Proposition 2. When  $\delta$  is sufficiently close to one, there exist thresholds  $\underline{c}(\delta) < \bar{c}(\delta)$ , with  $\lim_{\delta \rightarrow 1} \underline{c}(\delta) = 0$  and  $\lim_{\delta \rightarrow 1} \bar{c}(\delta) = \bar{c}$ , such that the ordered line  $L$  is the optimal network for  $c \in (\underline{c}, \bar{c})$ .

Turning to prove the first statement, suppose that  $b_n - b_1 \leq \frac{1}{12}$ . Consider any minimally connected network  $N$ , and any pair of agents  $e$  and  $d$ . By Lemma 2, there is full communication along any path from any informed agent  $e$  to any decision maker  $d$ . Hence, each agent  $i$ 's ex-ante payoffs are:

$$U_{ied} = -\alpha_{id}(b_i - b_d)^2 - \alpha_{id} \frac{1}{18} \left[ 1 + \frac{1 - \rho q(T, \ell_N(e, d))}{2} \right].$$

which for  $\delta$  close to one can be approximated with the Taylor expansion:

$$U_{ied} \approx -\alpha_{id}(b_i - b_d)^2 - \alpha_{id} \frac{1}{18} \left[ 1 + \frac{1 - \rho}{2} + (1 - \delta)\rho \ell_N(e, d) \frac{1}{2} \right],$$

because for  $\delta \approx 1$ , it is the case that  $q(T, \ell) \approx \delta^\ell$ .

The above functional form is covered by Proposition 1 of Jackson and Wolinsky (1996). It thus follows as a corollary that every optimal minimally connected network is a star  $S$ .

The demonstration that for  $\delta$  sufficiently close to one, there exist thresholds  $\underline{c}(\delta) < \bar{c}(\delta)$  such that also every optimal network is a star for  $c \in (\underline{c}, \bar{c})$  is analogous to the first part of the proof.  $\blacksquare$

**Proof of Proposition 3.** The first part of the proof extends the analysis of Lemma 1, that determined conditions for equilibrium disclosure of a unique signal  $s$  to a decision maker  $d$ , to the case in which every agent  $i$  may hold a signal  $s_i$  informative on  $\theta$ . I note that here, because the decision maker  $d$  can receive up to  $n$  signals, information transmission is inefficient even if only one of them is blocked.

Let me consider a path  $p$  from an agent  $i$  to agent  $d$ , and an agent  $j$  on  $p$  who holds  $i$ 's signal  $s_i$  at a history  $h^t$ . Say that  $b_j > b_d$ . Arguments analogous to the ones leading to conditions (6) in the proof of Lemma 1 show that, unless  $s_i$  is blocked by  $j$ 's successors on  $p$ , agent  $j$  relays  $s_i = 1$  at every history  $h^t$  in equilibrium, and relays  $s_i = 0$  if and only if:

$$b_k - b_d \leq E_{l,r} \left[ \frac{E_\theta[\theta | \omega_{id} = \phi; l, r] - E_\theta[\theta | \omega_{id} = 0; l, r]}{2} \middle| \Omega_j(h^t) \right], \quad (9)$$

where  $\omega_{id}$  is agent  $d$ 's information on  $s_i$  at period  $T$ ,  $r$  is the random number of signals  $s_k$  other than  $s_i$  received by  $d$  in equilibrium,  $l$  is the random number of signals  $s_k$  equal to 1, and  $\Omega_j(h^t)$  is the information held by agent  $j$  at history  $h^t$  used to make equilibrium inferences on  $r$  and  $l$ . The right-hand side of inequality (9) can be bounded as follows:

$$\begin{aligned} E \left[ \frac{E[\theta | \omega_{id} = \phi; l, r] - E[\theta | \omega_{id} = 0; l, r]}{2} \middle| \Omega_j(h^t) \right] &\leq E \left[ \frac{E[\theta | \omega_{id} = 1; l, r] - E[\theta | \omega_{id} = 0; l, r]}{2} \middle| \Omega_j(h^t) \right] \\ &= E \left[ \frac{1}{2} \left( \frac{l+2}{r+3} - \frac{l+1}{r+3} \right) \middle| \Omega_j(h^t) \right]. \end{aligned} \quad (10)$$

In any efficient network and equilibrium, there is no signal blocking. At least one of the realizations  $s_k = 0$  or  $s_k = 1$  of the signal  $s_k$  of every agent  $k$  is relayed to any decision maker  $d$ . The event  $\mathcal{E}$  that agent  $j$  is informed of all  $n - 1$  signals  $s_k$  different from  $s_i$  has positive probability. In this event, for  $\delta$  close to one, agent  $j$  expects also agent  $d$  to be informed of all these  $n - 1$  signals with probability close to one, in the efficient equilibrium. As a consequence, the right-hand side of inequality (10) can be approximated as  $\frac{1}{2^{(n+2)}}$ , for  $\Omega_j(h^t) = \mathcal{E}$ .

Hence, if  $b_j - b_d > \frac{1}{2^{(n+2)}}$ , then agent  $j$  blocks  $s_i = 0$  at least when  $\Omega_j(h^t) = \mathcal{E}$ , unless  $s_i = 0$  is blocked from  $d$  by  $j$ 's successors on  $p$ . Likewise, if  $b_j - b_d < -\frac{1}{2^{(n+2)}}$ , then  $j$  blocks  $s_i = 1$  at least when  $\Omega_j(h^t) = \mathcal{E}$ .

Moving on to show Proposition 3, as in the proof of Proposition 2, the first step consists in restricting attention to minimally connected networks, and in showing that the optimal tree is the ordered line  $L$ , for  $\delta$  sufficiently close to one.

I begin by noting that although  $b_{i+1} - b_i \geq \frac{1}{2^{(n+2)}}$  for all  $i < n$ , none of the signals is fully blocked along the ordered line  $L$ , regardless of the decision maker  $d$ . The first part of this proof implies that given decision maker  $d$ , each agent  $j < d$  relays every signal  $s_i = 0$  to her neighbor  $j + 1$  along the path to  $d$ , and each agent  $j > d$  relays every signal  $s_i = 1$  to her neighbor  $j - 1$  along the path to  $d$ , in equilibrium.

Instead, every other tree  $N$  entails the blocking of some signal  $s_i$  to some possible decision maker  $d$  with positive probability. Proceeding as in the proof of Proposition 2, I see that if  $N$  is a tree that contains a 4-agent star  $S$ , then there is a decision maker  $d$  in  $S$  from whom some signals  $s_i$  originating in  $S$  are blocked in equilibrium with positive probability. Likewise, I see that if  $N$  is a tree that contains a 4-agent line that is not ordered, there are decision makers  $d$  in the line from whom some signals  $s_i$  originating in the line are blocked with positive probability.

The second step of the proof consists in showing the existence of thresholds  $\underline{c}(\delta) < \bar{c}(\delta)$  for  $\delta$  close to one, such that  $\lim_{\delta \rightarrow 1} \underline{c}(\delta) = 0$  and  $\lim_{\delta \rightarrow 1} \bar{c}(\delta) = \bar{c}$ , and that for  $c \in (\underline{c}, \bar{c})$ , the sum of the agents' ex-ante payoffs  $U_i(L)$  with the ordered line  $L$  minus the sum of link costs  $c(L)$  is larger than with any connected network  $N$  with more links than  $L$ , and than with any network  $N$  that is not connected. The proof of this step is omitted as it is an extension of the second step of the proof of Proposition 2 analogous to the above extension of the first step.  $\blacksquare$

**Proof of Proposition 4.** The first part of the proof extends the analysis of part 1 of Lemma 2, that determined conditions for full communication between a unique informed agent  $e$  and a decision maker  $d$ , to the case in which every agent  $i$  may hold a signal  $s_i$  informative of  $\theta$ .

Proceeding as in the proof of Proposition 3, consider an agent  $j$  such that  $b_j > b_d$  who is informed of a signal  $s_i$ . Unless  $s_i$  is blocked by  $j$ 's successors on the path  $p$  to agent  $d$ , agent  $j$  relays  $s_i = 1$  to the next agent on  $p$  at every history  $h^t$  in equilibrium, and relays  $s_i = 0$  if and only if condition (9) is satisfied. In the full communication equilibrium, agent  $d$ 's expectation of  $\theta$  if receiving  $r$  signals of which  $l$  equal to one is  $E[\theta|l, r] = \frac{l+1}{r+2}$ . Hence, inequality (9) simplifies to:

$$\begin{aligned} b_j - b_d &\leq E_{l,r} \left[ \frac{E_\theta[\theta|\omega_{id} = \phi; l, r] - E_\theta[\theta|\omega_{id} = 0; l, r]}{2} \middle| \Omega_j(h^t) \right] = E \left[ \frac{1}{2} \left( \frac{l+1}{r+1} - \frac{l+1}{r+2} \right) \middle| \Omega_j(h^t) \right] \\ &= E \left[ \frac{1}{2} \frac{l+1}{(r+2)(r+1)} \middle| \Omega_j(h^t) \right] \leq \frac{1}{2(n+2)(n+1)}, \end{aligned}$$

where the latter bound is approximately achieved when  $\Omega_j(h^t)$  is a vector of  $n - 1$  signals all equal to zero,

so that  $r = n$  and  $l = 0$ . This is an event with positive probability in the full communication equilibrium.

Conversely, any agent  $j$  such that  $b_j < b_d$  relays the signal  $s_i = 1$  contrary to her bias if and only if:

$$\begin{aligned} b_j - b_d &\geq E_{l,r} \left[ \frac{E_\theta[\theta | \omega_d = \phi; l, r] - E_\theta[\theta | s_i = 1; l, r]}{2} \middle| \Omega_j(h^t) \right] = E \left[ \frac{1}{2} \left( \frac{l+1}{r+1} - \frac{l+2}{r+2} \right) \middle| \Omega_j(h^t) \right] \\ &= -E \left[ \frac{1}{2} \frac{r+1-l}{(r+2)(r+1)} \middle| \Omega_k \right] \geq -\frac{1}{2(n+2)(n+1)}, \end{aligned}$$

where the latter bound is approximately achieved when  $\Omega_j(h^t)$  is a vector of  $n - 1$  signals all equal to one, so that  $r = l = n$ .

Wrapping up, there is equilibrium full communication on path  $p$  if and only if  $|b_j - b_d| \leq \frac{1}{2(n+2)(n+1)}$  for all agents  $j$  on  $p$ .

Let me now turn to prove Proposition 4. Again, the first step is to restrict attention to minimally connected networks, to then consider general networks.

For any group  $G_m$ ,  $m = 1, \dots, M$ , and any possible decision maker  $d \in G_m$ , consider any signal  $s_i$  such that  $i \in G_m$ . The first part of this proof implies that neither  $s_i = 0$  and  $s_i = 1$  is blocked from  $d$  along any path of agents  $j$  in  $G_m$ , because  $|b_j - b_d| \leq \frac{1}{2(n+2)(n+1)}$  for all such agents. Hence, the same arguments that prove the first part of Proposition 2 apply to each group  $G_m$ . For  $\delta$  close to one, there exist thresholds  $\underline{c}(\delta) < \bar{c}(\delta)$  such that  $\lim_{\delta \rightarrow 1} \underline{c}(\delta) = 0$  and  $\lim_{\delta \rightarrow 1} \bar{c}(\delta) = \bar{c}$ , and that if  $c \in (\underline{c}, \bar{c})$ , then each group  $G_m$  is optimally organized as a star  $S_m$ , as stated in the hypothesis.

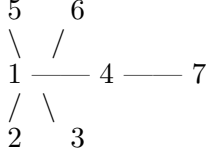
I now consider communication across groups. Take any signal  $s_i$  such that  $i \in G_{m'}$ , with  $m' < m$ . Let me show that in any tree with the properties stated in the hypothesis, the signal realization  $s_i = 0$  reaches every decision maker  $d \in G_m$ , whereas  $s_i = 1$  is blocked. In fact, for every agent  $j$  on the path  $p$  between  $i$  and  $d$ , it must be either the case that  $b_j < b_d$  or that  $|b_j - b_d| \leq \frac{1}{2(n+1)(n+2)}$ . Hence, every  $j$  on  $p$  has bias  $b_j - b_d < \frac{1}{2(n+1)}$ , and the results in the first part of the proof of Proposition 3 imply that  $j$  relays  $s_i = 0$  along the path  $p$  in equilibrium. Instead, because  $d \in G_m$  and  $i \in G_{m'}$  with  $m' < m$ , it follows that  $b_i - b_d < -\frac{1}{2(n+1)}$  and signal  $s_i = 1$  is blocked from  $d$ . The case in which  $m' > m$  is handled symmetrically.

Consider any tree  $N$  in which the groups  $G_m$  are organized as stars  $S_m$ . If the stars  $S_m$  are not connected along the ideological order, then arguments analogous to those in the second part of the proof of Proposition 3 conclude that some signal  $s_i$  is blocked in equilibrium from some decision maker  $d$ . Finally, I note that connecting every pair stars  $S_m$  and  $S_{m+1}$  for  $m < M$  in the tree  $N$  through the star centers minimizes the sum of the paths among pairs of agents  $i$  and  $d$  with  $i \in S_m$  and  $d \in S_{m'}$  and  $m \neq m'$ .

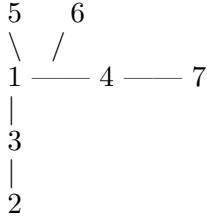
For  $\delta$  close to one, I have concluded that the set of optimal minimally connected networks consists of the trees  $N$  with the characteristics stated in the hypothesis. The proof of Proposition 4 is concluded by showing that for  $\delta$  close to one, there exist thresholds  $\underline{c}(\delta) < \bar{c}(\delta)$  such that  $\lim_{\delta \rightarrow 1} \underline{c}(\delta) = 0$  and  $\lim_{\delta \rightarrow 1} \bar{c}(\delta) = \bar{c}$ , and that such trees  $N$  are also the optimal networks, for  $c \in (\underline{c}, \bar{c})$ . Again, the proof of this step is analogous to the proof of the same step in Proposition 2, and hence is omitted.  $\blacksquare$

**Example 3.** Suppose that there are 7 agents. The identities of a unique decision maker  $d$  and of an agent  $e$ , informed with probability  $\rho < 1$ , are randomly drawn. No other agent is informed. Suppose that  $b_2 - b_1 \geq \frac{1}{6(2-\rho)}$ ,  $b_3 - b_2 \leq \frac{1}{12}$ ,  $b_4 - b_3 \leq \frac{1}{12}$ ,  $\frac{1}{12} < b_4 - b_2 < \frac{1}{6(2-\rho)}$ ,  $b_6 - b_4 \leq \frac{1}{12}$  (and hence  $b_5 - b_4 \leq \frac{1}{12}$ ),

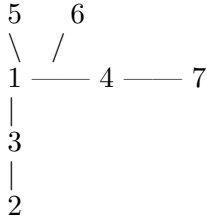
$\frac{1}{12} < b_5 - b_3 < \frac{1}{6(2-\rho)}$ , and  $b_7 - b_6 \geq \frac{1}{6(2-\rho)}$ . Because the ideology of every agent but 1 and 6 is within  $\frac{1}{6(2-\rho)}$  from agent 4, the ordered star network is the star centered on agent 4:



Evidently, the star minimizes the overall decay in communication, its sum of lengths of paths is  $2 \cdot \binom{6}{2} + 2 \cdot 6 = 42$ . Nevertheless, it makes communication from 3 to 2 possible only as one-sided information transmission. This is because the path from 3 to 2 is through 4 and  $\frac{1}{12} < b_4 - b_2 < \frac{1}{6(2-\rho)}$ . Hence, agent 4 is not willing to relay signal  $s = 0$  to agent 2. For that reason, this network structure is dominated by the following ‘quasi-star’ in which all agents but 2 are linked with 4, and 2 is linked with 3 instead:



Here, there is full communication from 3 to 2, both  $s = 0$  and  $s = 1$  reach 2, as these agents are linked directly. As in the star above, there is full information transmission between 4, 5 and 6, and between 3 and 4, and one sided communication among all remaining pairs of experts and decision makers. Because this is not the star network, the overall signal decay is larger; specifically the sum of lengths of paths is  $2 \cdot \binom{5}{2} + 2 \cdot 5 + 2 + 2 \cdot 2 + 2 \cdot 4 \cdot 3 = 60$ . For  $\delta$  sufficiently close to one, however, the loss due to the extra decay vanishes relative to the gain achieved by full communication, which does not vanish because  $\rho < 1$ . For completeness, I compare the quasi star network also with the ‘ordered star’ network below, in which all the pairs of agents of ideological distance smaller than  $\frac{1}{12}$  are linked directly:



Just like in the quasi star network above, there is full information transmission within all pairs of agents  $i$  and  $j$  of ideological distance  $|b_i - b_j|$  smaller than  $\frac{1}{12}$ , and one sided information transmission in all other pairs of agents. Nevertheless, this ordered star network has more information transmission decay: the sum of the lengths of paths is  $2 \cdot \binom{4}{2} + 2 \cdot 4 + 2(2 + 2 \cdot 2 + 2 \cdot 3 \cdot 3) = 68$ , whereas the sum of the lengths of paths in the quasi star above is 60.  $\diamond$

**Proof of Proposition 5.** The proof of the first part builds on the proof of Proposition 3.

I begin by proving that there are thresholds  $\bar{\alpha} < 1$ ,  $\bar{\delta}' < 1$  and  $\bar{c}'(\delta)$  such that  $\lim_{\delta \rightarrow 1} \bar{c}'(\delta) = \bar{c}$ , and that when  $\alpha_{ii} > \bar{\alpha}$  for all  $i$ ,  $\delta > \bar{\delta}'$  and  $c < \bar{c}'$ , the ordered line  $L$  obtains as Nash equilibrium.

Consider a profile of link strategies  $l$ , such that  $l_{i,i+1} = 1 = l_{i+1,i}$  and  $l_{i,i-1} = 1 = l_{i-1,i}$  for every  $i = 2, \dots, n - 1$ , and all other links  $l_{ij}$  equal zero. The resulting network  $N$  is the ordered line  $L$ . For any  $j$  such that  $l_{ij} = 0$ , any agent  $i$  pays  $c/2$  without changing  $N$  by deviating to  $l_{ij} = 1$ . This deviation

is clearly not profitable. By deviating from  $l_{i,i+1} = 1$  to  $l_{i,i+1} = 0$ , agent  $i$  deletes her link with  $i + 1$  and loses all signals  $s_e$  from agents  $e > i$ , instead of receiving them with positive probability. And by deviating from  $l_{i,i-1} = 1$ , she loses all signals  $s_e$  from agents  $e < i$ . Hence, for the same reasons that a connected network is optimal in the proof of Proposition 3, there exist thresholds  $\bar{\alpha} < 1$ ,  $\bar{\delta}' < 1$  and  $\bar{c}'_i > 0$  such that for all  $\alpha_{ii} > \bar{\alpha}$ ,  $\delta > \bar{\delta}'$  and  $c < \bar{c}'_i$ , agent  $i$  does not want to delete neither link  $l_{i,i+1} = 1$ , nor  $l_{i,i-1} = 1$ . The same arguments also imply that  $\lim_{\delta \rightarrow 1} \bar{c}'_i(\delta) = \bar{c}$ . Hence, for  $c < \bar{c}' \equiv \min\{\bar{c}'_i : i \in \mathcal{N}\}$ , the ordered line  $L$  obtains as Nash equilibrium, and  $\lim_{\delta \rightarrow 1} \bar{c}'(\delta) = \bar{c}$ .

Let me now turn to pairwise stability. I begin by showing that there exist thresholds  $\bar{\alpha} < 1$ ,  $\bar{\delta}' < 1$  and  $\underline{c}'(\delta)$  such that  $\lim_{\delta \rightarrow 1} \underline{c}'(\delta) = 0$ , and that when  $\alpha_{ii} > \bar{\alpha}$  for all  $i$ ,  $\delta > \bar{\delta}'$  and  $c > \underline{c}'$ , networks  $N$  with loops are not weakly pairwise stable. I consider a loop  $O$  of agents in network  $N$  and prove that the agent  $\underline{i}$  with the smallest index  $i$  (and ideology  $b_i$ ) in  $O$  gains by severing one of its two links in  $O$ , for  $\delta$  close to 1, unless the link cost  $c$  is too small. In fact, every agent  $i \in O$  has (at least) two paths without bias reversals to  $\underline{i}$ , by construction. Further, every agent  $i \notin O$  connected to  $\underline{i}$  only through agents in  $O$  has either (at least) two paths without bias reversals to  $\underline{i}$  (when  $b_i < b_{\underline{i}}$ ), or none at all (when  $b_i > b_{\underline{i}}$ ). Hence, by severing one of its two links in the loop  $O$ , agent  $\underline{i}$  does not block the transmission of any signal  $s_i$  to her. For  $\alpha_{ii}$  and  $\delta$  close to one and when the link cost  $c$  is not too small, agent  $\underline{i}$  gains by severing one of its two links in  $O$ .

I have concluded that there exist thresholds  $\bar{\alpha} < 1$ ,  $\bar{\delta}' < 1$ ,  $\underline{c}'(\delta) < \bar{c}'(\delta)$  such that for when  $\alpha_{ii} > \bar{\alpha}$  for all  $i$ ,  $\delta > \bar{\delta}'$  and  $c \in (\underline{c}', \bar{c}')$ , every weakly pairwise stable network needs to be minimally connected. Because strong pairwise stability is more demanding, also every strongly pairwise stable network needs to be minimally connected.

It is easy to show that the ordered line  $L$  is weakly pairwise stable under the parametric restrictions stated in the hypothesis. In  $L$ , no signal is blocked to any agent, and there are no duplicate links. So, for  $\delta$  close to one, each agent loses by forming a duplicate link, unless the link cost  $c$  is too low.

To prove that all stars are weakly pairwise stable under analogous parametric restrictions, I note that in any star  $S$  there does not exist any pair of agents  $i, j$  such that both the path from  $i$  to  $j$  and the path from  $j$  to  $i$  have bias reversals. In fact, suppose that  $i$  and  $j$  are periphery agents. Letting  $b_i > b_j$  without loss of generality, there is a bias reversal path connecting  $i$  and  $j$  if and only if the biases of the center agent  $c$  of  $S$  relative to  $i$  and  $j$  have the same signs,  $(b_c - b_i)(b_c - b_j) > 0$ . Hence, when both biases are positive (negative), there is a bias reversal from  $j$  to  $i$  (respectively, from  $j$  to  $i$ ) but not from  $i$  to  $j$  (respectively, not from  $j$  to  $i$ ). And of course, there cannot be bias reversal paths if either  $i$  or  $j$  is the center agent, because all these paths are composed of only two agents. Because there does not exist any pair of agents  $i, j$  such that both the path from  $i$  to  $j$  and the path from  $j$  to  $i$  have bias reversals, there does not exist any pair of agent  $i, j$  that both gain by forming a link with each other, when  $\delta$ ,  $\alpha_{ii}$  and  $\alpha_{jj}$  are close to 1, and the link cost  $c$  is not too small.

To show that there are trees that are not weakly pairwise stable under the parametric restrictions stated in the hypothesis, I consider the 4-agent line  $2 - 1 - 4 - 3$ . Agents 2 and 3 would gain by forming a link, unless the link cost  $c$  is too high, because the signal  $s_2$  is blocked from 3 and vice-versa. Indeed, the presence of 4 in the path from 2 to 3 implies a bias reversal, and the presence of 1 in the path from 3 to 2 implies a bias reversal. And if the link cost  $c$  is too high for 2 and 3 to want to form a direct link, then it is also too high for 1 to maintain a link with 2. In either case, the line  $2 - 1 - 4 - 3$  cannot be weakly pairwise stable.

To show that the ordered line  $L$  is the unique strongly pairwise stable network, first recall that every

strongly pairwise stable network needs to be minimally connected, for  $\delta$  and  $\alpha_{ii}$  close to one for all  $i$ , and excluding extreme link cost  $c$ . Then, I note that there exists thresholds  $\bar{\delta}' < 1$ ,  $\bar{\alpha}' < 1$  and  $\bar{c}'(\delta)$ , such that  $\lim_{\delta \rightarrow 1} \bar{c}'(\delta) = \bar{c}$  and that when  $\alpha_{ii} > \bar{\alpha}$  for all  $i$ ,  $\delta > \bar{\delta}'$ , and  $c < \bar{c}'$ , there is a joint incentive to form a link by a pair of agents  $i$  and  $j$  if and only if  $i$  and  $j$  are not connected by a path without bias reversal. In the proof of Proposition 2, I have shown that  $L$  is the unique minimally connected network for which there is a path without bias reversals across every pair of agents  $i, j$ . Hence, the only minimally connected network that can be strongly pairwise stable is  $L$ , for  $\delta$  close to one, unless  $c$  is too small.

The ordered line  $L$  is indeed strongly pairwise stable unless  $c$  is too large. Formally, there exists thresholds  $\bar{\delta}' < 1$ ,  $\bar{\alpha}' < 1$  and  $\underline{c}(\delta)'$  such that  $\lim_{\delta \rightarrow 1} \underline{c}(\delta)' = 0$ , and that when  $\alpha_{ii} > \bar{\alpha}$  for all  $i$ ,  $\delta > \bar{\delta}'$ , and  $c < \bar{c}$ ,  $L$  is strongly pairwise stable. Under these conditions in fact, no agent  $i$  has any incentive to sever any of her links  $n_{ij} = 1$  in  $L$  and lose the signals  $s_e$  gathered through that link.

I have concluded that there exist thresholds  $\bar{\alpha} < 1$ ,  $\bar{\delta}' < 1$ ,  $\underline{c}'(\delta) < \bar{c}'(\delta)$  such that when  $\alpha_{ii} > \bar{\alpha}$  for all  $i$ ,  $\delta > \bar{\delta}'$  and  $c \in (\underline{c}', \bar{c}')$ , the unique strongly pairwise stable network is the ordered line  $L$ .

The proof of the second part of the proposition is built on the proof of Proposition 4 in the same way as the proof of the first part is built on the proof of Proposition 3. It is omitted and available upon request. ■

**Proof of Proposition 6.** I first show that there exist a threshold  $\bar{\alpha} < 1$  such that if  $\delta < 1$  and  $\alpha_{ii} > \bar{\alpha}$  for all  $i$ , then the only network formation strategy profile  $l'$  that implements the ordered line  $L$  as a Nash equilibrium is such that every agent  $i$  with  $1 < i \leq \lfloor (n+1)/2 \rfloor$  plays  $l'_{i,i-1} = 1$ , that every  $i$  with  $\lfloor (n+1)/2 \rfloor \leq i < n$  plays  $l'_{i,i+1} = 1$ , and that  $l'_{i,j} = 0$  for all other pairs of agents  $i, j$ .

Suppose that a strategy profile  $l \neq l'$  yields the ordered line  $L$ . Hence, there must exist an agent  $i$  such that  $1 \leq i < \lfloor (n+1)/2 \rfloor$  with  $l_{i,i+1} = 1$ , or an agent  $i$  such that  $\lfloor (n+1)/2 \rfloor < i \leq n$  with  $l_{i,i-1} = 1$ . I consider the former, as the arguments for the latter are symmetric. The link with agent  $i+1$  allows agent  $i$  to access the information of the agents  $i+1, i+2, \dots, n$ . Agent  $i$  receives signal  $s_j$  of the agent  $j = i+k$  with probability  $q(T, k)$  for  $k = 1, \dots, n-i$ . Suppose that agent  $i$  deviates, and plays  $l_{i,i+1} = 0$  and  $l_{i, \lceil (n+i+1)/2 \rceil} = 1$ . In words,  $i$  bypasses  $i+1$  by forming a link with the agent  $\lceil (n+i+1)/2 \rceil$  who is in the middle of the ordered line between  $i+1$  and  $n$ . This way, agent  $i$  has still access to the information of agents  $j = i+1, i+2, \dots, n$ , but now with shorter paths, and hence smaller decay. In fact, she receives one of each signals  $s_j$  with probabilities  $q(T, 1), q(T, 2), \dots, q(T, \lfloor (n-i-1)/2 \rfloor + 1)$ , then again one signal  $s_j$  with probabilities  $q(T, 2), \dots, q(T, \lfloor (n-i-1)/2 \rfloor + 1)$ , and when  $n-i-1$  is odd, also one signal with probability  $q(T, \lceil (n-i-1)/2 \rceil)$ . Instead, by maintaining her link with agent  $i+1$ , agent  $i$  receives one of each signals  $s_j$  with probabilities  $q(T, 1), q(T, 2), \dots, q(T, \lfloor (n-i-1)/2 \rfloor + 1)$ , and then one signal  $s_j$  with probabilities  $q(T, \lfloor (n-i-1)/2 \rfloor + 2), \dots, q(T, n-i)$ . As long as  $n-i \geq 3$ , agent  $i$  is then more likely better informed by connecting to agent  $\lceil (n+i+1)/2 \rceil$  than by connecting to  $i+1$ . Because  $i < \lfloor (n+1)/2 \rfloor$ , the condition  $n-i \geq 3$  is satisfied if  $n \geq 3 + (\lfloor (n+1)/2 \rfloor - 1)$ , and this holds when  $n \geq 4$ .

I have established that every agent  $i$  such that  $1 \leq i < \lfloor (n+1)/2 \rfloor$  with  $l_{i,i+1} = 1$  strictly prefers to deviate and play  $l_{i,i+1} = 0$  and  $l_{i, \lceil (n+i+1)/2 \rceil} = 1$ . A symmetric argument holds for every agent  $i$  such that  $\lfloor (n+1)/2 \rfloor < i \leq n$  with  $l_{i,i-1} = 1$ .

This concludes that any strategy profile  $l \neq l'$  that yields the ordered line  $L$  cannot be implemented as a Nash equilibrium of the unilateral sponsorship game, when  $\delta < 1$  and  $\alpha_{ii}$  is sufficiently close to one for all  $i$ .



Now, I show that for  $n \leq 5$ , there exists thresholds  $\bar{\alpha} < 1$ ,  $\bar{\delta}'' < 1$ , and  $\underline{c}''(\delta) < \bar{c}''(\delta)$  such that if  $\alpha_{ii} > \bar{\alpha}$  for all  $i$ ,  $\delta < \bar{\delta}''$  and  $c \in (\underline{c}'', \bar{c}'')$ , the above strategy profile  $l'$  is a Nash equilibrium of the unilateral sponsorship game, and hence the ordered line  $L$  obtains as a Nash equilibrium.

To prove this, I check whether any agent  $i$  has any profitable deviation. Again, for  $\delta$  close to 1, no agent  $i$  would benefit by sponsoring extra links, i.e., by playing  $l_i > l'_i$ , if the link cost  $c$  is not too small. And unless the link cost is too high, it is not profitable to deviate from  $l'_i$  by adopting a strategy  $l_i < l'_i$  for  $\alpha$  close to one, because agent  $i$  foregoes valuable information by severing her link(s).

I now show that for any agent  $i$  such that  $1 < i \leq \lfloor (n+1)/2 \rfloor$ , it would not be profitable to deviate by setting  $l_{i,i-1} = 0$ , together with  $l_{i,j} = 1$  for some  $j < i-1$ . The cases  $n = 3$  and  $n = 4$  are trivial, and so is the case with  $n = 5$  for any agent  $i$  other than 3. When  $n = 5$ , agent 3 may deviate from  $l'_3$ , by setting  $l_{3,2} = 0$  together with  $l_{3,1} = 1$ . But if doing so, her payoffs would not change: instead of receiving  $s_2 = 0$  with probability  $q(T, 1)$  and  $s_1 = 0$  with probability  $q(T, 2)$ , agent 3 would receive  $s_1 = 0$  with probability  $q(T, 1)$  and  $s_2 = 0$  with probability  $q(T, 2)$ . Hence, agent 3 has no reasons to deviate from the strategy  $l'_3$ . The analysis for  $i$  such that  $\lfloor (n+1)/2 \rfloor \leq i < n$  is symmetric. I have shown that for  $n \leq 5$ , the strategy profile  $l'$  is a Nash equilibrium.

I now turn to the case in which  $n > 5$ . Consider the central agent  $i = \lfloor (n+1)/2 \rfloor$ . The strategy profile  $l'$  is not a Nash equilibrium for any  $\delta < 1$  and  $\alpha_{ii}$  close to one for all  $i$ , because agent  $i$  has an incentive to deviate from the strategy  $l'_i$  in which  $l'_{i,i+1} = 1$  to a strategy  $l$  such that  $l_{i, \lfloor (n+i+1)/2 \rfloor} = 1$  and  $l_{ij} = 0$  for all other agent  $j > i$ , as long as  $n \geq 6$ . In fact, the same calculations as in the first part of the proof prove that, for  $n \geq 6$ , agent  $i = \lfloor (n+1)/2 \rfloor$  has access to the signals of the same agents  $i+1, \dots, n$  by deviating from  $l'$  to  $l$ , but with shorter paths and hence higher probability. ■

# Appendix B: Generalizations

(Not Submitted for Publication)

This Appendix presents generalizations of results presented in the paper. We first consider general utility functions and statistical information models. Then, we consider cheap talk instead of transmission of verifiable information.

**General Utility Function and Statistical Model.** This part of the section generalizes the results of Lemma 2 qualitatively to general utility functions, and statistical information models. As a consequence, also Proposition 2 and subsequent results generalizes qualitatively. This shows that neither a quadratic loss function, nor a uniformly distribution state, nor binary signals are needed for this paper's main results. In particular, Lemma 2 and subsequent results can be generalized qualitatively to the case of continuous signals.

Suppose that there is a single expert  $e$  and a unique decision maker  $d$  connected on a path of length  $l$ . The state of the world  $\theta \in \Theta$ . Every agent  $i$  has a utility function  $u_i(y, \theta)$  such that  $\partial^2 u_i / \partial y^2 < 0$ ,  $\partial^2 u_i / \partial y \partial \theta > 0$  and  $\partial u_{i+1} / \partial y > \partial u_i / \partial y$ . Because  $\partial^2 u_i / \partial y^2 > 0$ , for every  $\theta$ , there is a unique  $y_i(\theta)$  that maximizes  $u_i(y, \theta)$ . Further,  $\partial^2 u_i / \partial y \partial \theta > 0$  implies that  $\partial y_i / \partial \theta > 0$ , and  $\partial u_{i+1} / \partial y > \partial u_i / \partial y$  implies that  $y_{i+1}(\theta) > y_i(\theta)$  for all  $\theta$ .

The expert may be informed of signal  $s \in S$  with probability  $\rho < 1$ . The signal  $s$  is informative of  $\theta$  in the sense that the likelihood ratio is monotonic. Let  $F(s|\theta)$  be the c.d.f. of  $s$  given  $\theta$  and  $p(s|\theta)$  denote the associated p.d.f or density. If  $s' > s$  and  $\theta' > \theta$ , then,  $p(s'|\theta') / p(s|\theta') > p(s'|\theta) / p(s|\theta)$ .

Agent  $i$ 's expected utility if knowing  $s$  is  $Eu_i(y; s) = \int_{\Theta} u_i(y, \theta) dF(\theta|s)$ . Because  $\partial^2 u_i / \partial y^2 < 0$ , the function  $Eu_i(y; s)$  is also strictly concave in  $y$ , and for every  $s$ , there is a unique  $y_i(s)$  that maximizes  $Eu_i(y; s)$ . The monotone likelihood ratio property, together with  $\partial^2 u_i / \partial y \partial \theta > 0$ , imply that this maximum  $y_i(s)$  increases in  $s$ ,  $\partial Eu_i(y; s) / \partial s > 0$ . Because  $\partial u_{i+1} / \partial y > \partial u_i / \partial y$ , the maximum  $y_i(s)$  increases in  $i$ ,  $y_{i+1}(s) > y_i(s)$ .

In line with the hypothesis of Lemma 2, I assume that ideology differences are sufficiently large, so that every agent  $k$  on the path from  $e$  to  $d$  always wants to misrepresent  $s$  to any decision maker  $d$ . A tight sufficient condition is:

$$\inf\{y_{i+1}(s) : s \in S\} > \sup\{y_i(s) : s \in S\}. \quad (11)$$

In other terms, the sets of optimal choices  $Y_i = \{y_i(s) : s \in S\}$  are strictly ordered, they do not overlap.

A slacker, easier to visualize sufficient condition is to assume that  $\Theta$  is a compact set and that  $y_{i+1}(\underline{\theta}) > y_i(\bar{\theta})$ . Of course, this condition implies:  $\inf\{y_{i+1}(\theta) : \theta \in \Theta\} > \sup\{y_i(\theta) : \theta \in \Theta\}$ , which in turns implies (11).

A natural example is the standard quadratic-utility formulation  $u_i(y, \theta) = -(y - \theta - b_i)^2$ , together with uniformly distributed state  $\theta \sim U[0, 1]$ , and signal  $s$  informative with probability  $p$  and independent of  $\theta$  with probability  $1 - p$ . That is,  $s = \theta$  with probability  $p$  and  $s \perp \theta$ ,  $s \sim U[0, 1]$  with probability  $1 - p$ . Condition (11) is then satisfied if and only if  $b_{i+1} - b_i > 1$ .

Note however that  $\Theta$  being a compact set is not necessary for condition (11) to hold. For example, suppose that  $y_i(\theta) = \arctan(\theta) + ig\pi$ , e.g.  $u_i(y, \theta) = -y^2 + 2(\arctan(\theta) + ig\pi)y$ , for  $g \geq 2$ , and  $s = \theta + \varepsilon$  with  $\varepsilon \sim U[-\pi/2, \pi/2]$ .

The following result generalizes Lemma 2. Let me define:

$$y_{d,-}(\emptyset) = \arg \max_y \frac{\int_{s \leq s^+} \int_{\Theta} u_d(y, \theta) dF(\theta|s) dF(s) + (1 - \rho) \int_{s > s^+} \int_{\Theta} u_d(y, \theta) dF(\theta|s) dF(s)}{\int_{s \leq s^+} dF(s) + (1 - \rho) \int_{s > s^+} dF(s)}, \quad (12)$$

$$y_{d,+}(\emptyset) = \arg \max_y \frac{(1 - \rho) \int_{s \leq s^-} \int_{\Theta} u_d(y, \theta) dF(\theta|s) dF(s) + \int_{s > s^-} \int_{\Theta} u_d(y, \theta) dF(\theta|s) dF(s)}{(1 - \rho) \int_{s \leq s^-} dF(s) + \int_{s > s^-} dF(s)}. \quad (13)$$

**Lemma B1** Suppose  $\ell + 1$  agents  $j$  are linked along a path  $p$  from agent  $e$  informed of a signal  $s$  with probability  $\rho \in (0, 1]$  to decision-maker  $d$ . Assume condition (11).

1. Suppose the path from  $e$  to  $d$  has no bias reversals. If  $b_k > b_d$  for all  $k$  on path from  $e$  to  $d$ , then every agent  $k$  forwards signal  $s$  if and only if  $s \geq s^+$ , with the condition  $y_d(s^+) = y_{d,-}(\emptyset)$ . Because  $\rho < 1$ , it is the case  $s^+ > \underline{s} = \inf S$ . If  $\rho \rightarrow 1$ , then  $s^+ \rightarrow \inf S$ . The converse result holds when  $b_k < b_d$  for all  $k$  on path from  $e$  to  $d$ : then, every agent  $k$  on path forwards signal  $s$  if and only if  $s \leq s^-$ , with the condition  $y_d(s^-) = y_{d,+}(\emptyset)$ , and  $s^- < \bar{s} = \sup S$ . If  $\rho \rightarrow 1$ , then  $\bar{s} \rightarrow \sup S$ .

2. Suppose the path from  $e$  to  $d$  has bias reversals. Then, the choice of the decision maker if receiving no signal is  $y_{d,0} = \arg \max_y \int_S \int_{\Theta} u_d(y, \theta) dF(\theta|s) dF(s)$ . Every player  $k$  with  $b_k > b_d$  blocks signal  $s < y_{d,0}$ , and every player  $k$  with  $b_k < b_d$  blocks signal  $s > y_{d,0}$ . Generically, signal  $s$  does not reach the decision-maker who acts fully uninformed.

**Proof.** Part 2. For any given choice  $y$  of decision maker  $d$  if she does receives signal  $s$ , because of condition (11) and  $y_d(s)$  strictly increases in  $s$ , it is a unique best response of any agent  $k$  such that  $b_k > b_d$  to forward  $s$  along the path if and only if  $y_d(s) \geq y$  (unless the signal is blocked by one of his successors). Conversely, the unique best response of any agent  $k$  such that  $b_k < b_d$  is to reveal  $s$  if and only if  $y_d(s) \leq y$ . Generically, signal  $s$  does not reach the decision-maker who acts fully uninformed, and plays  $y_{d,0} = \arg \max_y \int_S \int_{\Theta} u_d(y, \theta) dF(\theta|s) dF(s)$ .

Part 1. Suppose that  $b_k > b_d$  for all agents  $k$  on path from  $e$  to  $d$ . For any given choice  $y$  of decision maker  $d$  if she does receives signal  $s$ , because of condition (11) and  $y_d(s)$  strictly increases in  $s$ , every agent  $k$  forwards  $s$  along the path if and only if  $s \geq y_d^{-1}(y)$ . Letting  $s^+$  be the equilibrium threshold  $y_d^{-1}(y)$ , the decision maker  $d$ 's equilibrium choice  $y_{d,-}(\emptyset)$  is defined as follows:

$$y_{d,-}(\emptyset) = \arg \max_y \left\{ E[u_d(y, \theta) | \emptyset] = \int_S \int_{\Theta} u_d(y, \theta) dF(\theta|s) dF(s | \emptyset) \right\},$$

where

$$dF(s | \emptyset) = \frac{\Pr(\emptyset|s) dF(s)}{\int_S \Pr(\emptyset|s) dF(s)} = \frac{dF(s)}{\int_{s \leq s^+} dF(s) + (1 - \rho) \int_{s > s^+} dF(s)}, \quad \text{if } s \leq s^+$$

$$dF(s | \emptyset) = \frac{\Pr(\emptyset|s) dF(s)}{\int_S \Pr(\emptyset|s) dF(s)} = \frac{(1 - \rho) dF(s)}{\int_{s \leq s^+} dF(s) + (1 - \rho) \int_{s > s^+} dF(s)}, \quad \text{if } s > s^+.$$

Wrapping up, I obtain expressions (12) and (13).

To show that, because  $\rho < 1$ , it is the case  $s^+ > \underline{s} = \inf S$ , proceed by contradiction: if  $s^+ = \underline{s}$ , then

$$y_{d,-}(\emptyset) = \arg \max_y \int_S \int_{\Theta} u_d(y, \theta) dF(\theta|s) dF(s) > y_d(\underline{s}),$$

by the intermediate value theorem, thus contradicting  $y_d(s^+) = y_d(\emptyset)$ .

For  $\rho \rightarrow 1$ , the equilibrium condition becomes:

$$y_{d,-}(s^+) = \arg \max_y \frac{\int_{s \leq s^+} \int_{\Theta} u_d(y, \theta) dF(\theta|s) dF(s)}{\int_{s \leq s^+} dF(s)},$$

which holds only in the limit for  $s^+ \rightarrow \underline{s}$ , because for all  $s^+ > \underline{s}$ ,

$$y_{d,-}(s^+) < \arg \max_y \frac{\int_{s \leq s^+} \int_{\Theta} u_d(y, \theta) dF(\theta|s) dF(s)}{\int_{s \leq s^+} dF(s)}$$

by the intermediate value theorem.

The proof of the converse result, when  $b_k < b_d$  for all agents  $k$  on path from  $e$  to  $d$  is analogous.  $\blacksquare$

For example, consider the case of the standard quadratic-utility formulation  $u_i(y, \theta) = -(y - \theta - b_i)^2$ , together with uniformly distributed state  $\theta \sim U[0, 1]$ , and signal  $s$  informative with probability  $p$  and independent of  $\theta$  with probability  $1 - p$ . That is,  $s = \theta$  with probability  $p$  and  $s \perp \theta$ ,  $s \sim U[0, 1]$  with probability  $1 - p$ . Condition (11) is then satisfied if and only if  $b_{i+1} - b_i > 1$ .

Let me define:

$$\begin{aligned} y_{d,-}(\emptyset) &= E[\theta|\emptyset, -] + b_d = \frac{\int_0^{s^+} E(\theta|s) f(s) ds + (1 - \rho) \int_{s^+}^1 E(\theta|s) f(s) ds}{\int_0^{s^+} f(s) ds + (1 - \rho) \int_{s^+}^1 f(s) ds} + b_d, \\ y_{d,+}(\emptyset) &= E[\theta|\emptyset, +] + b_d = \frac{(1 - \rho) \int_0^{s^-} E(\theta|s) f(s) ds + \int_{s^-}^1 E(\theta|s) f(s) ds}{(1 - \rho) \int_0^{s^-} f(s) ds + \int_{s^-}^1 f(s) ds} + b_d. \end{aligned}$$

Because  $E(\theta|s) = ps + (1 - p)/2$  and  $f(s) = p \int_0^1 d\theta + (1 - p) = 1$  for  $s \in [0, 1]$ , the equilibrium conditions are:

$$s_+ = E[\theta|\emptyset, -] = \frac{ps_+^2/2 + (1 - \rho)p(1 - s_+^2)/2 + (2 - \rho)(1 - p)/2}{s_+ + (1 - \rho)(1 - s_+)},$$

$$s_- = E[\theta|\emptyset, +] = \frac{(1 - \rho)ps_-^2/2 + p(1 - s_-^2)/2 + (2 - \rho)(1 - p)/2}{(1 - \rho)s_- + (1 - s_-)}.$$

$$s^+ = -\frac{1}{\rho(2 - p)} \left( 1 - \rho - \sqrt{1 - 4p\rho - \rho^2 + p\rho^2 + p^2\rho + 2\rho} \right)$$

$$s^- = \frac{1 - \sqrt{1 - 4\rho + 4p\rho + 2\rho^2 - 3p\rho^2 - p^2\rho + p^2\rho^2}}{\rho(2 - p)}.$$

$$\text{When } p = 1, \quad s^+ = \frac{1}{\rho} \left( 1 - \rho - \sqrt{1 - \rho} \right), \quad s^- = \frac{1}{\rho} \left( \sqrt{1 - \rho} - 1 \right).$$

**Multiple simultaneous decision makers.**

**Cheap talk.** This part of the section introduces cheap talk (i.e., transmission of unverifiable information) in the game of transmission of verifiable information presented in Section 3. It shows that cheap talk does not allow for more informative equilibrium communication than transmission of verifiable information, when the players' ideological differences are sufficiently large, as is the case in Lemma 2 and Proposition 2. Hence, adding the possibility of cheap talk communication on top of transmission of verifiable information is irrelevant for this paper's main results.

The first part of the analysis considers the cheap talk version of the game introduced in Section 3. Such a game is defined as follows. I lift the restriction on reporting strategies that  $r_{jkdi}(\omega_{ij}; h^t) \in \{\omega_{ij}, \phi\}$  given  $j$ 's information  $\omega_{ij} \in \{0, \phi, 1\}$  on signal  $s_i$ , for any agents  $d, i, j$  and  $k$  and history  $h^t$ . Regardless of the realization of signal  $s_i$ , the reports  $r_{jkdi}(s_i; h^t)$  may here take any value out of possibly large message sets  $M_{jkdi}^t$ , that I assume finite for ease of exposition. The remainder of the model of Section 3 is unchanged, including the statistical assumptions on the state  $\theta$ , and the binary signals  $s_i$  which may be observed by the players  $i$ .

The first part of the analysis considers equilibria of the game of cheap talk that are outcome equivalent to the equilibria of the game of verifiable information transmission of Section 3. Suppose  $\ell + 1$  agents  $j$  are linked along a path  $p$  from an expert  $e$ , informed of an unverifiable signal  $s$  with probability  $\rho$ , to a decision-maker  $d$ . There are  $T$  rounds of communication. In each round  $t$ , any agent  $j$  on the path  $p$  may communicate to his successor  $k$ . All communication takes the form of cheap talk.

I begin by considering the one-sided information equilibria of the verifiable information transmission game, specifically those in which every agent  $j$  relays only signal realization  $s = 0$  to the next agent  $k$  on the path  $p$ . Regardless of the specific report strategies  $r$  adopted, any such equilibrium is strategically and outcome equivalent to the equilibrium of the cheap talk game in which  $M_{ikdi}^t = \{0, \phi, 1\}$ ,  $r_{jkdi}(0; h^t) = 0$  and  $r_{jkdi}(\phi; h^t) = r_{jkdi}(1; h^t) = \phi$ , for all  $t, i, d, j$  and  $k$ . The possibility that the message  $r_{jkdi}$  is lost with probability  $1 - \delta$  is again represented by the assumption that when  $r_{jkdi} \neq \phi$ , agent  $k$  receives report  $\phi$  with probability  $\delta$ .

DO NOT INTRODUCE THE  $q$  THING. JUST DO THIS FOR  $|b_j - b_d| > \frac{1}{6(2-\rho)}$ . DROP THE FIRST PROOF, WHICH IS IRRELEVANT, USE THE SECOND ONE TO SHOW THAT THERE CANNOT BE MORE INFORMATION TRANSMISSION THAN WITH ONLY CHEAP TALK. THE PROOF I HAVE NOW IDENTIFIES THE BOUND  $|b_j - b_d| > \frac{1}{6}$ . THIS COMES FROM THE FACT THAT

$$2(b_j - b_d) > E[\theta | r'_{jdT}] - E[\theta | \mathbf{r}_{nj}],$$

AND THAT WITH ONLY ONE SIGNAL,  $E[\theta | r'_{jdT}] \leq 2/3$  AND  $E[\theta | \mathbf{r}_{nj}] \geq 1/3$ . THE BOUNDS ARE GIVEN BY THE SIGNAL  $s_j = 1$  ARRIVING WITH PROBABILITY ONE, AND BY THE OFF PATH BELIEF ASSOCIATED WITH THE SIGNAL  $s_j = 0$  WITH PROBABILITY ONE. IN FACT, SIGNALS CANNOT ARRIVE WITH PROBABILITY ONE, SO IT WOULD BE POSSIBLE TO CLAIM THE BOUND  $|b_j - b_d| > \frac{1}{6(2-\rho q(T, \delta))}$  FOR  $\delta = 1$ . THEY CORRESPOND TO THE BELIEFS IF SIGNALS  $s_j = 1$  OR  $s_j = 0$  ARRIVE TO THE DM.

TO ACHIEVE THE BOUND  $|b_j - b_d| > \frac{1}{6(2-\rho q(T, \delta))}$  IS MORE COMPLEX BECAUSE I CANNOT REDUCE EVERYTHING TO A DECISION IN THE LAST PERIOD.

ADDING MORE THAN ONE BINARY CHEAP TALK SIGNAL, THINGS BECOME MORE COMPLEX. IT IS POSSIBLE TO HAVE SEMI-POOLING EQUILIBRIA... THE CONDITION

$$2(b_j - b_d) > E[\theta | r'_{jdT}] - E[\theta | \mathbf{r}_{nj}],$$

REMAINS, BUT NOW IT IS MORE COMPLICATED TO CALCULATE THE BOUNDS FOR  $E[\theta|r'_{jdT}]$  AND  $E[\theta|r_{nj}]$ .

The general result is that for  $|b_j - b_d| > 1/4$  and any distribution of cheap talk messages, with or without verifiable information, there is no transmission of unverifiable information in equilibrium.

BECAUSE THIS IS TRULY THE CHOICE BETWEEN TWO DIFFERENT MESSAGES, IT IS SAFE TO CONSIDER THE BINARY MESSAGE EQUILIBRIUM CASE IN WHICH THE BELIEFS ARE LINKED BY A MARTINGALE CONDITION, ALLOWING FOR MIXED STRATEGIES:

$$E[\theta|m_1] \Pr(m_1) + E[\theta|m_2] \Pr(m_2) = E[\theta]$$

$$\Pr(m_1) E[\theta|m_1] + [1 - \Pr(m_1)] E[\theta|m_2] = \frac{1}{2}$$

$$\begin{aligned} pE + (1-p)F &= \frac{1}{2} \\ Q &= F - E \end{aligned}$$

$$Q = \frac{1}{2p-2} (2E - 1)$$

$$E[\theta|m_2] - E[\theta|m_1] = \frac{1}{2} \frac{1 - 2E[\theta|m_1]}{1 - \Pr(m_1)}$$

HENCE,

$$E[\theta|m_2] - E[\theta|m_1] = \frac{E[\theta|m_2] - 1/2}{\Pr(m_1)}$$

That is:

$$\begin{aligned} 2(b_j - b_d) &> |E[\theta|m_2] - E[\theta|m_1]| \\ &= \frac{E[\theta|m_2] - 1/2}{\Pr(m_1)} \\ &= \frac{E[\theta|m_2] - 1/2}{\sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s})} \end{aligned}$$

To calculate  $E(\theta|m_2)$ , note that:

$$f(\theta|m_2) = \frac{\sum_{\mathbf{s}} [1 - \sigma_j(m_1|\mathbf{s})] \Pr(\mathbf{s}|\theta)}{\sum_{\mathbf{s}} [1 - \sigma_j(m_1|\mathbf{s})] \int_0^1 \Pr(\mathbf{s}|\theta') f(\theta') d\theta'}$$

and

$$\begin{aligned}
E(\theta|m_2) &= \int_0^1 \theta \frac{\sum_{\mathbf{s}} [1 - \sigma_j(m_1|\mathbf{s})] \Pr(\mathbf{s}|\theta) d\theta}{\sum_{\mathbf{s}} [1 - \sigma_j(m_1|\mathbf{s})] \int_0^1 \Pr(\mathbf{s}|\theta') f(\theta') d\theta'} \\
&= \frac{\sum_{\mathbf{s}} [1 - \sigma_j(m_1|\mathbf{s})] \int_0^1 \theta \Pr(\mathbf{s}|\theta) f(\theta) d\theta}{\sum_{\mathbf{s}} [1 - \sigma_j(m_1|\mathbf{s})] \int_0^1 \Pr(\mathbf{s}|\theta') f(\theta') d\theta'} \\
&= \frac{\sum_{\mathbf{s}} [1 - \sigma_j(m_1|\mathbf{s})] \Pr(\mathbf{s}) E(\theta|\mathbf{s})}{\sum_{\mathbf{s}} [1 - \sigma_j(m_1|\mathbf{s})] \Pr(\mathbf{s})} \\
&= \frac{E[\theta] - \sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s}) E(\theta|\mathbf{s})}{1 - \sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s})} \\
&= \frac{1/2 - \sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s}) E(\theta|\mathbf{s})}{1 - \sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s})}
\end{aligned}$$

Using:  $\int_0^1 \theta \Pr(\mathbf{s}, \theta) d\theta = \Pr(\mathbf{s}) \int_0^1 \theta f(\theta|\mathbf{s}) d\theta = \Pr(\mathbf{s}) E(\theta|\mathbf{s})$ , and  $\sum_{\mathbf{s}} \Pr(\mathbf{s}) E(\theta|\mathbf{s}) = E[\theta]$ .

$$\begin{aligned}
2(b_j - b_d) &> \frac{\frac{1/2 - \sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s}) E(\theta|\mathbf{s})}{1 - \sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s})} - \frac{1}{2}}{\sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s})} \\
&= \frac{1 - 2 \sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s}) E(\theta|\mathbf{s}) - [1 - \sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s})]}{2 [1 - \sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s})] \sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s})} \\
&= \frac{\sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s}) - 2 \sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s}) E(\theta|\mathbf{s})}{2 [1 - \sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s})] \sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s})} \\
&= \frac{1}{2} \frac{1 - 2 \frac{\sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s}) E(\theta|\mathbf{s})}{\sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s})}}{1 - \sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s})} \\
&= \frac{1}{2} \frac{1 - 2E[\theta|m_1]}{1 - \Pr(m_1)}
\end{aligned}$$

$$\frac{\frac{1/2 - q}{1 - p} - \frac{1}{2}}{p} = \frac{1}{2} \frac{1 - 2q/p}{1 - p}$$

is true :

$$E[\theta|m_2] - E[\theta|m_1] = \frac{1}{2} \frac{1 - 2E[\theta|m_1]}{1 - \Pr(m_1)},$$

I argue that

$$\frac{1 - 2E[\theta|m_1]}{1 - \Pr(m_1)} < 1.$$

In fact:

$$1 - 2E[\theta|m_1] < 1 - \Pr(m_1),$$

is equivalent to

$$2E[\theta|m_1] > \Pr(m_1),$$

Suppose not:

$$2E[\theta|m_1] \leq \Pr(m_1),$$

then because

$$E[\theta|m_2] - E[\theta|m_1] = \frac{1 - 2E[\theta|m_1]}{2 - \Pr(m_1)}$$

it would be the case that

$$E[\theta|m_2] - E[\theta|m_1] = \frac{1 - 2E[\theta|m_1]}{2 - \Pr(m_1)} > \frac{1 - 2E[\theta|m_1]}{2 - 2E[\theta|m_1]} = 1,$$

but that's impossible, because  $0 \leq E[\theta|m_1] \leq E[\theta|m_2] \leq 1$ .

DONE.

$$\begin{aligned} E(\theta|m_1) &= \int_0^1 \theta \frac{\sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s}|\theta) d\theta}{\sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \int_0^1 \Pr(\mathbf{s}|\theta') f(\theta') d\theta'} \\ &= \frac{\sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \int_0^1 \theta \Pr(\mathbf{s}|\theta) f(\theta) d\theta}{\sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \int_0^1 \Pr(\mathbf{s}|\theta') f(\theta') d\theta'} \\ &= \frac{\sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s}) E(\theta|\mathbf{s})}{\sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s})} \end{aligned}$$

$$\frac{\sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s}) E(\theta|\mathbf{s})}{\sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s})} > \frac{1}{2} \sum_{\mathbf{s}} \sigma_j(m_1|\mathbf{s}) \Pr(\mathbf{s}).$$

In the uniform-Bernoulli case:

$$\begin{aligned} 2(b_j - b_d) &> \frac{E[\theta|m_2] - 1/2}{\Pr(m_1)} \\ &= \frac{E[\theta|m_2] - 1/2}{\sum_{i=1}^n \Pr(\mathbf{s}_j) \sigma_j(m_1|\mathbf{s}_j)} \end{aligned}$$

where  $n$  is the number of vector signal realizations.

AND BY BAYES RULE:

$$E[\theta|m_2] = \int_0^1 \theta f(\theta|m_2) d\theta$$



$$\begin{aligned}
f(\theta|m_2) &= \frac{f(\theta, m_2)}{\int_0^1 f(\theta', m_2) d\theta'} \\
&= \frac{\Pr(m_2|\theta) f(\theta)}{\int_0^1 \Pr(m_2|\theta') f(\theta') d\theta'} \\
&= \frac{\sum_{i=1}^n [1 - \sigma_j(m_1|\mathbf{s}_j)] \Pr(\mathbf{s}_j|\theta)}{\sum_{i=1}^n [1 - \sigma_j(m_1|\mathbf{s}_j)] \int_0^1 \Pr(\mathbf{s}_j|\theta) d\theta}.
\end{aligned}$$

Using the fact that the only thing that matters in a Beta-Bernoulli environment is the number of signals equal to 1, set as  $k$ , let me rewrite this as:

$$\begin{aligned}
f(\theta|m_2) &= \frac{\sum_{k=0}^n [1 - \sigma_j(m_1|k)] \Pr(k|\theta)}{\sum_{k=0}^n [1 - \sigma_j(m_1|k)] \int_0^1 \Pr(k|\theta) d\theta} \\
&= \frac{n \sum_{k=0}^n [1 - \sigma_j(m_1|k)] \Pr(k|\theta)}{\sum_{k=0}^n [1 - \sigma_j(m_1|k)]},
\end{aligned}$$

where I am using the fact that  $\int_0^1 \Pr(k|\theta) d\theta = \Pr(k) = 1/n$  for all  $k$  in an uniform-Bernoulli environment. So,

$$\begin{aligned}
E[\theta|m_2] &= \int_0^1 \theta f(\theta|m_2) d\theta \\
&= \frac{\sum_{k=0}^n [1 - \sigma_j(m_1|k)] \int_0^1 \theta \Pr(k|\theta) d\theta}{\frac{1}{n} \sum_{k=1}^n [1 - \sigma_j(m_1|k)]} \\
&= \frac{\sum_{k=0}^n [1 - \sigma_j(m_1|k)] \int_0^1 \theta \frac{1}{n} \Pr(\theta|k) d\theta}{\frac{1}{n} \sum_{k=1}^n [1 - \sigma_j(m_1|k)]} \\
&= \frac{\sum_{k=0}^n [1 - \sigma_j(m_1|k)] \frac{k+1}{n+2}}{\sum_{k=0}^n [1 - \sigma_j(m_1|k)]} \\
&= \frac{\sum_{k=m}^n \frac{k+1}{n+2}}{n-m}
\end{aligned}$$

$$\text{Using: } \Pr(k|\theta) = \frac{\Pr(k, \theta)}{f(\theta)} = \frac{\Pr(\theta|k) \Pr(k)}{f(\theta)} = \frac{1}{n} \Pr(\theta|k)$$

So, I obtain:

$$\begin{aligned}
2(b_j - b_d) &> \frac{E[\theta|m_2] - 1/2}{\sum_{i=1}^n \Pr(\mathbf{s}_j) \sigma_j(m_1|\mathbf{s}_j)} \\
&= \frac{\frac{\sum_{k=m}^n \frac{k+1}{n+2}}{n-m} - 1/2}{\frac{1}{n} m} = \frac{1}{2} n (m+1) \frac{m-n-2}{m(n+2)(m-n)}
\end{aligned}$$

: : : In the case of a pure-strategy equilibrium, which needs to be partitional and with no gaps, letting  $m-1$ , for  $m=1, \dots, n$  be the highest  $k$  for which  $m_1$  is uttered,

$$\begin{aligned}
D_n \left( \frac{1}{2} n (m+1) \frac{m-n-2}{m(n+2)(m-n)} \right) &= (m+1) \frac{m-2n-2}{(m-n)^2 (n+2)^2} < 0 \\
D_m \left( \frac{1}{2} n (m+1) \frac{m-n-2}{m(n+2)(m-n)} \right) &= \frac{1}{2} n \frac{4m-2n+2mn+m^2-n^2}{m^2 (m-n)^2 (n+2)}
\end{aligned}$$

$$\begin{aligned} & \frac{1}{2}n(m+1) \frac{m-n-2}{m(n+2)(m-n)} - 1/3 = \frac{1}{6} \frac{-6n-mn^2+m^2n+mn-4m^2-3n^2}{m(n+2)(m-n)} \\ & D_m \left( \frac{1}{6} \frac{-6n-mn^2+m^2n+mn-4m^2-3n^2}{m(n+2)(m-n)} \right) = \frac{1}{2}n \frac{4m-2n+2mn+m^2-n^2}{m^2(m-n)^2(n+2)} \\ & : \end{aligned}$$

NOW, ADDING A BINARY CHEAP TALK SIGNAL TO THE BINARY VERIFIABLE SIGNAL WOULD WORK LIKE THIS.

THE FINAL AGENT IN THE PATH SIMULTANEOUSLY CHOOSES WHETHER TO DISCLOSE THE VERIFIABLE AND THE UNVERIFIABLE SIGNAL. FOR ANY NON-TRIVIAL DISCLOSURE STRATEGY OF THE UNVERIFIABLE SIGNAL, THE CONDITION

$$2(b_j - b_d) > E[\theta|r'_{jdT}] - E[\theta|\mathbf{r}_{nj}],$$

BECOMES

$$2(b_j - b_d) > E[\theta|r'_{jdT}, s_j] - E[\theta|\mathbf{r}_{nj}, s_j],$$

FOR AT LEAST ONE VERIFIABLE SIGNAL REALIZATION  $s_j$ . NOW, THE (SLACK) BOUNDS BECOME  $E[\theta|r'_{jdT}] \leq (1+s_j+1)/4$  AND  $E[\theta|\mathbf{r}_{nj}] \geq (s_j+1)/4$ . AND THIS IS JUST A CONSEQUENCE OF BETA-BERNOULLI. THE RESULTING

SO, OVERALL THE BEST APPROACH IS TO KEEP THINGS AS THEY ARE... CHANGING MAYBE THE BOUNDS TO  $|b_j - b_d| > \frac{1}{6(2-\rho)}$  IN THE MAIN RESULT.

Suppose momentarily that there is no decay,  $\delta = 1$ . Then in the one-sided equilibrium, if receiving the report  $r_{jkdi} = \phi$  from her predecessor  $j$  on  $p$  in every period  $t$ , then the expectation about  $\theta$  of any agent  $k$  (including the decision maker  $d$ ) is  $E[\theta|\phi] = \frac{1-\rho/3}{2-\rho}$  —cf. expression (7). If instead  $k$  receives the report  $r_{jkdi} = 0$  in any period  $t$ , her expectation about  $\theta$  is  $E[\theta|\phi] = \frac{1}{3}$  henceforth. On the equilibrium path, agent  $k$  cannot ever receive report  $r_{jkdi} = 1$ , nor she can receive report  $r_{jkdi} = \phi$  after receiving  $r_{jkdi} = 0$ . If these events, her equilibrium beliefs are not pinned down by the Bayes rule. I want to show sufficient conditions under which the one-sided equilibrium does not exist. Hence, I consider off path beliefs that do not make off path deviations from equilibrium more profitable than deviations on the equilibrium path.

Suppose that the information  $\omega_j(h^t)$  about  $s$  at history  $h^t$  of an agent  $j$  on the path  $p$  with  $b_j > b_d$  is  $\omega_j = 0$ . Then, as in the proof of Lemma 2,  $u_{jd}(0; h^t) \geq u_{jd}(\phi; h^t)$  and  $j$  sends the equilibrium message  $r_{jkdi} = 0$  to  $k$  instead of deviating to  $r_{jkdi} = \phi$  if and only if  $2(b_j - b_d) \leq E[\theta|\phi] - E[\theta|0] = \frac{1}{3(2-\rho)}$ , or  $b_j - b_d \leq \frac{1}{6(2-\rho)}$ . Now however, also any agent  $j$  on the path  $p$  with  $b_j < b_d$  has the opportunity to deviate from equilibrium and send  $r_{jkdi} = 0$  instead of  $r_{jkdi} = \phi$ , when her information about  $s$  is  $\omega_j(h^t) \in \{\phi, 1\}$ . She does not deviate if and only if  $u_{jd}(0; h^t) \leq u_{jd}(\phi; h^t)$ , that is,  $2(b_j - b_d) \geq E[\theta|0] - E[\theta|\phi] = -\frac{1}{3(2-\rho)}$ , or  $b_j - b_d \geq -\frac{1}{6(2-\rho)}$ .

Wrapping up, for  $\delta = 1$  the one-sided communication equilibrium in which agents reveal  $s = 0$  exists if and only if  $|b_j - b_d| \leq \frac{1}{6(2-\rho)}$  for all agents  $j$  on the path  $p$ . To extend the result to  $\delta$  close to one, I

note that the payoff functions  $u_{jd}(s; h^t)$  and  $u_{jd}(\phi; h^t)$  are continuous in  $\delta$ —cf. expression (4). Hence, for  $\delta$  close to one, it is still the case that  $u_{jd}(0; h^t) > u_{jd}(\phi; h^t)$  for any agent  $j$  on the path  $p$  with  $b_j - b_d < -\frac{1}{6(2-\rho)}$  and  $\omega_j(h^t) \in \{\phi, 1\}$ , if the opponents abide to the one sided equilibrium strategies. Likewise, it is still the case that  $u_{jd}(0; h^t) < u_{jd}(\phi; h^t)$  for any agent  $j$  on  $p$  with  $b_j - b_d > \frac{1}{6(2-\rho)}$ , even if  $\omega_j(h^t) = 0$ . In other terms, also if  $\delta$  is close to one it is the case that the one-sided communication equilibrium in which the agents reveal  $s = 0$  exists if and only if  $|b_j - b_d| \leq \frac{1}{6(2-\rho)}$  for all  $j$  on  $p$ . A symmetric argument leads to the same conclusion for the one-sided communication equilibrium in which the agents reveal  $s = 1$ .

Now consider full communication equilibria. The agents reveal both signal realizations  $s = 0$  and  $s = 1$ . Any such equilibrium is outcome and strategic equivalent to the equilibrium of the cheap talk game in which  $M_{jkdi}^t = \{0, \phi, 1\}$  and  $r_{jkdi}(\omega_j; h^t) = \omega_j$  for all  $t, i, d, j$  and  $k$ . The existence of this equilibrium requires the truth-telling conditions  $2(b_j - b_d) \leq E[\theta|\phi] - E[\theta|0] = \frac{1}{6}$  and  $2(b_j - b_d) \leq E[\theta|1] - E[\theta|\phi] = \frac{1}{6}$  for  $b_j > b_d$ , and  $2(b_j - b_d) \geq E[\theta|\phi] - E[\theta|0] = -\frac{1}{6}$  and  $2(b_j - b_d) \geq E[\theta|1] - E[\theta|\phi] = -\frac{1}{6}$  for  $b_j < b_d$ . These existence conditions are more demanding than the above conditions for the existence of the one-sided communication equilibrium.

The same is true for any mixed strategy equilibrium that is outcome equivalent to any equilibrium of the verifiable information transmission game. Any mixing across the messages  $r_{jkdi} = 0$  and  $r_{jkdi} = \phi$  of the one sided communication equilibrium makes  $E[\theta|0]$  larger than  $\frac{1}{3}$  and  $E[\theta|\phi]$  smaller than  $\frac{1-\rho/3}{2-\rho}$ , thus making the equilibrium conditions,  $2(b_j - b_d) \leq E[\theta|\phi] - E[\theta|0]$  for  $b_j > b_d$ , and  $2(b_j - b_d) \geq E[\theta|0] - E[\theta|\phi]$  for  $b_j < b_d$ , harder to be satisfied. And conversely, any mixing across the messages  $r_{jkdi} = \phi$  and  $r_{jkdi} = 1$  makes  $E[\theta|1]$  smaller and  $E[\theta|\phi]$  larger, thus making the equilibrium conditions harder to be satisfied. And of course, any mixing across the messages  $r_{jkdi} = 0$ ,  $r_{jkdi} = \phi$  and  $r_{jkdi} = 1$  of the full communication equilibrium makes equilibrium conditions even harder to satisfy.

For  $\delta$  close to one, I conclude that if  $|b_j - b_d| > \frac{1}{6(2-\rho)}$  for some agents  $j$  on the path  $p$ , then the babbling equilibrium is the only equilibrium of the cheap talk game that is outcome equivalent to an equilibrium of the verifiable information transmission game.

I now show that if  $b_{i+1} - b_i > \frac{1}{6}$  for all  $i < n$ , then there do not exist any perfect Bayesian equilibria other than the babbling equilibrium, regardless of whether they are equivalent to equilibria of the verifiable information transmission game, or take more complicated forms.

I study cheap talk games with message spaces  $M_{jkdi}^t$  constructed as follows. In period  $t = 1$ , let  $M_{jkdi}^1 = \{0, \phi, 1\}$  for all agents  $i, d, j, k$ ; and for any time  $t > 1$ , let  $M_{jkdi}^t = \times_{h \neq j, k} M_{hjd}^{t-1} \times M_{jkdi}^{t-1}$ . This is because I want to allow for the possibility that in each period  $t$ , each agent  $j$  can report to any one neighbor  $k$  not only the messages  $r_{hj}$  she just learnt from her neighbors  $h \neq j$  but also what she could have learnt and said at any earlier period  $\tau$ . In the message spaces constructed above, for all agents  $j$  and  $k$ , the space  $M_{jkdi}^t$  includes  $M_{jkdi}^{t-1}$  at every period  $t$ , and then by concatenation  $M_{jkdi}^t$  also includes  $M_{jkdi}^{t-2}, \dots, M_{jkdi}^1$ . The possibility that  $j$ 's report  $r_{jkdi}$  is lost in transmission at any time  $t$  is represented by the assumption that with probability  $1 - \delta$ , agent  $k$  receives a report vector whose components are all equal to  $\phi$ , when  $j$  sent a different report  $r_{jkdi}$ .

Again, I begin by considering the case of no decay,  $\delta = 1$ . For each neighbor  $j$  of the decision maker  $d$ , let me introduce the following collection of cheap talk games between agents  $j$  and  $d$ . Nature draws  $\theta$  from the uniform distribution on  $[0, 1]$ , and then  $s \in \{0, \phi, 1\}$  according to  $\Pr(s = 1|\theta) = \rho\theta$ ,  $\Pr(s = 0|\theta) = \rho(1 - \theta)$ . Then, there are  $T$  periods of communication. In each period  $t = 1, \dots, T$ , nature

sends agent  $j$  a report  $r_{njt}$  out of the message set  $M_{nj}^t = \times_{h \neq j} M_{hjd}^t$ . That is, nature can send any vector of reports  $r_{hj}$  that  $j$  would have received from her neighbors  $h$ . The report  $r_{njt}$  depends on the signal  $s$  according to the probability distribution  $\zeta(\mathbf{r}_{nj}|s)$ , where  $\mathbf{r}_{nj}$  is a profile of  $T$  reports  $r_{njt}$ . In each period  $t = 1, \dots, T$ , agent  $j$  also sends a message  $r_{jdt}$  to agent  $d$  function of the profile  $\mathbf{r}_{njt}$  of  $t$  reports  $r_{nj\tau}$  for  $\tau = 1, \dots, t$ , possibly adopting a mixed strategy  $\sigma_{jd}$ . After these  $T$  rounds of communication, agent  $d$  chooses  $\hat{y}_d$  as function of the profile of messages  $\mathbf{r}_{jd}$  received from  $j$ .

Any given nature signal disclosure distribution  $\zeta$  identifies a different game between agent  $d$  and one of her neighbors  $j$ . If no information is revealed to  $d$  in any Bayesian Nash equilibrium of any of these games, then no information can be revealed to  $d$  also in any Bayesian Nash (and hence a fortiori any perfect Bayesian) equilibrium of the game in which agent  $e$  is informed of  $s$  with probability  $\rho$  and agents may communicate for  $T$  rounds before  $d$ 's decision. Hence, I want to prove that every Bayesian Nash equilibrium of every game between agent  $d$  and one of his neighbors  $j$  is a babbling equilibrium. The message  $r_{jdt}(\mathbf{r}_{njt})$  of agent  $j$  to  $d$  is the same for every profile of nature messages  $\mathbf{r}_{njt}$  at every time  $t$ .

I first note that, because there is no decay,  $\delta = 1$ , it is without loss of generality to restrict attention to agent  $j$ 's strategies  $\sigma_{jd}$  in which  $j$  does not transmit any information to  $d$  until the last period  $T$ . (I.e., at every time  $t < T$ , agent  $j$  sends the same message  $r_{jdt}$  in every history  $h^t$ .) After collecting all the reports  $\mathbf{r}_{nj}$  from nature,  $j$  decides what to communicate to agent  $d$ , and sends a message  $r_{jdT} \in M_{jd}^T$ . This restriction is without loss of generality because by construction, every vector of nature's reports  $\mathbf{r}_{nj}$  is identified by at least a message in the space  $M_{jd}^T$ . Hence, I just want to show that if  $|b_j - b_d| > \frac{1}{6}$ , then agent  $j$  sends the same message  $r_{jdT}$  at every history  $h^T$  in every equilibrium.

Suppose by contradiction that there exists an equilibrium in which  $\sigma_{jd}(\mathbf{r}_{nj}) \neq \sigma_{jd}(\mathbf{r}'_{nj})$  for (at least) two report vectors  $\mathbf{r}_{nj}$  and  $\mathbf{r}'_{nj}$  such that  $E[\theta|\mathbf{r}_{nj}] \neq E[\theta|\mathbf{r}'_{nj}]$ . Then, there would exist messages  $r_{jdT}$  and  $r'_{jdT}$  such that  $y_d(r_{jdT}) \neq y_d(r'_{jdT})$ . For the strategy  $\sigma_{jd}$  to be an equilibrium strategy, it would then need to be the case that

$$\begin{aligned} u_{jd}(r_{jdT}; \mathbf{r}_{nj}) &= -E[(y_d(r_{jdT}) - \theta - b_j)^2 | \mathbf{r}_{nj}] \geq u_{jd}(r'_{jdT}; \mathbf{r}_{nj}) = -E[(y_d(r'_{jdT}) - \theta - b_j)^2 | \mathbf{r}_{nj}] \\ u_{jd}(r'_{jdT}; \mathbf{r}'_{nj}) &= -E[(y_d(r'_{jdT}) - \theta - b_j)^2 | \mathbf{r}'_{nj}] \geq u_{jd}(r_{jdT}; \mathbf{r}'_{nj}) = -E[(y_d(r_{jdT}) - \theta - b_j)^2 | \mathbf{r}'_{nj}] \end{aligned}$$

Because agent  $d$  is sequentially rational, it must be that  $y_d(r_{jdT}) = b_d + E[\theta|r_{jdT}]$  and  $y_d(r'_{jdT}) = b_d + E[\theta|r'_{jdT}]$ , and I can rewrite the above conditions as:

$$\begin{aligned} E[(b_d + E[\theta|r_{jdT}] - \theta - b_j)^2 | \mathbf{r}_{nj}] &\leq E[(b_d + E[\theta|r'_{jdT}] - \theta - b_j)^2 | \mathbf{r}_{nj}] \\ E[(b_d + E[\theta|r'_{jdT}] - \theta - b_j)^2 | \mathbf{r}'_{nj}] &\leq E[(b_d + E[\theta|r_{jdT}] - \theta - b_j)^2 | \mathbf{r}'_{nj}]. \end{aligned}$$

Simplifying these inequalities as in (5), I obtain:

$$\begin{aligned} [2(b_j - b_d) + 2E[\theta|\mathbf{r}_{nj}] - E[\theta|r_{jdT}] - E[\theta|r'_{jdT}]] [E[\theta|r'_{jdT}] - E[\theta|r_{jdT}]] &\leq 0 \\ [2(b_j - b_d) + 2E[\theta|\mathbf{r}'_{nj}] - E[\theta|r_{jdT}] - E[\theta|r'_{jdT}]] [E[\theta|r'_{jdT}] - E[\theta|r_{jdT}]] &\geq 0. \end{aligned} \quad (14)$$

I note that if agent  $j$ 's strategy  $\sigma_{jd}$  identifies  $\mathbf{r}_{nj}$  and  $\mathbf{r}'_{nj}$  perfectly, i.e. both the support of  $\sigma_{jd}(\mathbf{r}_{nj})$  and of  $\sigma_{jd}(\mathbf{r}'_{nj})$  are disjoint from the supports of  $\sigma_{jd}(\mathbf{r}''_{nj})$  for every other report vector  $\mathbf{r}''_{nj}$ , then  $E[\theta|\mathbf{r}_{nj}] = E[\theta|r_{jdT}]$  and  $E[\theta|\mathbf{r}'_{nj}] = E[\theta|r'_{jdT}]$ . The decision maker identifies  $\mathbf{r}_{nj}$  from the message  $r_{jdT}$  and  $\mathbf{r}'_{nj}$  from the message  $r'_{jdT}$ , and the conditions (14) take the same form as in (5):

$$2(b_j - b_d) \leq E[\theta|r'_{jdT}] - E[\theta|\mathbf{r}_{nj}], \quad 2(b_j - b_d) \geq E[\theta|r_{jdT}] - E[\theta|\mathbf{r}'_{nj}]$$

Instead, if agent  $j$ 's strategy  $\sigma_{jd}$  pools or randomizes information, it determines a Blackwell scrambling of nature's report distribution  $\zeta$ . As a consequence, agent  $j$  must have more extreme beliefs about  $\theta$  than the decision maker  $d$ , for some realizations  $\mathbf{r}_{nj}$ ,  $\mathbf{r}'_{nj}$  of  $\zeta$  and  $r_{jdT}$  and  $r'_{jdT}$  of  $\sigma_{jd}$ . For such values of  $\mathbf{r}_{nj}$ ,  $\mathbf{r}'_{nj}$ ,  $r_{jdT}$  and  $r'_{jdT}$ , it is then either the case that  $E[\theta|r_{jdT}] < E[\theta|\mathbf{r}_{nj}]$  and  $E[\theta|r'_{jdT}] > E[\theta|\mathbf{r}'_{nj}]$  when  $E[\theta|r'_{jdT}] > E[\theta|r_{jdT}]$ , or that  $E[\theta|r_{jdT}] > E[\theta|\mathbf{r}_{nj}]$  and  $E[\theta|r'_{jdT}] < E[\theta|\mathbf{r}'_{nj}]$  when  $E[\theta|r'_{jdT}] < E[\theta|r_{jdT}]$ .

Suppose without loss of generality that  $E[\theta|r'_{jdT}] > E[\theta|r_{jdT}]$ . Say that  $b_j > b_d$ . Then, the first equilibrium condition (14) fails if:

$$2(b_j - b_d) > E[\theta|r_{jdT}] + E[\theta|r'_{jdT}] - 2E[\theta|\mathbf{r}_{nj}].$$

Because  $E[\theta|r_{jdT}] \leq E[\theta|\mathbf{r}_{nj}]$ , the latter condition simplifies to:

$$2(b_j - b_d) > E[\theta|r'_{jdT}] - E[\theta|\mathbf{r}_{nj}],$$

and because  $E[\theta|r'_{jdT}] \leq 2/3$  and  $E[\theta|\mathbf{r}_{nj}] \geq 1/3$ , this simplifies to:  $2(b_j - b_d) > 1/3$ , i.e.,  $b_j - b_d > 1/6$ .

Hence, if  $b_j - b_d > 1/6$ , then the hypothesized (possibly partially) separating strategy  $\sigma_{jd}$  cannot be part of an equilibrium. A symmetric argument shows that if  $b_j - b_d < -1/6$ , then the second equilibrium condition (14) fails, and again  $\sigma_{jd}$  cannot be part of an equilibrium.

I have concluded that there is no equilibrium information transmission in the game in which agent  $e$  is informed of  $s$  with probability  $\rho$  and agents may communicate with no decay,  $\delta = 1$ , for  $T$  rounds before  $d$ 's decision. The extension of this result to when  $\delta$  is close to one is based on the same arguments used in the first part of the analysis of cheap talk. Because the payoff functions  $u_{jd}(r_{jdT}; \mathbf{r}_{nj})$  are continuous in  $\delta$  for all  $r_{jdT}$  and  $\mathbf{r}_{nj}$ , if one or both inequalities (14) fail for  $\delta = 1$ , then they must still fail for  $\delta$  close to 1 for the same strategy profiles. As a result, there there still cannot be any equilibrium information transmission.

I have concluded that if  $b_{i+1} - b_i > \frac{1}{6}$  for all  $i < n$ , then the babbling equilibrium is the only equilibrium of the cheap talk game in which an expert agent  $e$  is informed of signal  $s$  such that  $\Pr(s = 1|\theta) = \theta$ . The result can be generalized to any unverifiable signal  $s$  informative of  $\theta$ , that is, any signal  $s$  such that  $E[\theta|s]$  is not the same for all realizations of  $s$ . For any such signal  $s$ , there exist a threshold  $\beta(s)$  such that if  $b_{i+1} - b_i > \beta(s)$  for all  $i < n$ , then the unique equilibrium of the cheap talk game is the babbling equilibrium.

It is obvious that these results hold also if agents have verifiable information to communicate, together with these unverifiable signals. Hence, if together with the verifiable signal  $s$ , there were also unverifiable signals  $\tilde{s} \in \{0, 1\}$  informative of  $\theta$  with probability  $\Pr(\tilde{s} = 1|\theta) = \theta$ , any possible decision maker  $d$  would not be informed of any such signals  $\tilde{s}$  in equilibrium, when  $b_{i+1} - b_i > \frac{1}{6}$  for all agents  $i < n$ . And for any general distribution of unverifiable signals  $\tilde{s}$  informative of  $\theta$ , it can be shown that  $d$  would not be informed of  $\tilde{s}$  in equilibrium, if the biases  $b_{i+1} - b_i$  are sufficiently large.

# Appendix C: Extended Literature Survey

(Not Submitted for Publication)

This section surveys the empirical literature on the examples of networks of political decision makers discussed in the introduction.

I begin by considering networks of policy makers across different jurisdictions. Using data from a survey of California planners and government officials, Henry (2011), and Gerber, Henry and Lubell (2013) investigate the factors that lead local governments to collaborate within regional planning networks. Their analysis considers land-use planning, where regional cooperation among counties is advocated as a way of mitigating negative externalities. They construct networks of local institutions in five California regions, using data from a survey of local government officials. They construct measures of political distance, as well as demographic, socioeconomic and geographic proximity between local jurisdictions in each region. An exponential random graph model analysis is used to investigate the significance of political similarity in the regional planning networks, while controlling for other network effects.

The overall network structure consists of 5 geographical cliques organized as stars or complete networks or something intermediate. These 5 cliques are only linked through their centroids, with the exception of the San Diego and Riverside cliques. Higher political distance significantly lowers probability that jurisdictions collaborate in regional planning networks in all regression specifications, even after controlling for demographic and geographic factors. Dyads with higher demographic and socioeconomic differences are also less likely to collaborate. The main conclusion is that local governments whose constituents are similar politically are more likely to collaborate with one another in regional planning efforts than those whose constituents are politically diverse. Political homophily turns out to be important even in settings where partisan considerations should not to relevant.

Zafonte and Sabatier (1998) construct the network of governmental agencies in the San Francisco Bay-Delta region, by directly surveying their officers. They identify 4 groups. The first one includes State or local agencies concerned with or involved in exporting water from the delta. The second is made of local agencies who develop shoreline primarily in the Bay. A third one is composed of local government organizations that have counterparts who discharge into the Bay-Delta. The fourth one is composed of organizations primarily interested in environmental management. One residual group is less cohesive than these 4 groups and includes organizations with environmental protection objectives, together with organizations linked to developers who value environmental quality. The empirical analysis provides strong evidence that these groups form on the basis of preference homogeneity.

Henry, Lubell, and McCoy (2011) uses exponential random graph models to investigate the roles of policy-relevant beliefs and social capital as drivers of network structure. The following hypotheses are formulated: (1) actors with similar policy beliefs are more likely to form connections, (2) this leads to segregated ideological subnetworks, (3) social capital, defined as trust and norms of reciprocity, helps cement cooperative relationships. These hypotheses are tested using surveys of policy-makers involved in land-use and transportation planning in four California regions. The data show that coalitions of ideologically similar agents organize around policy leaders and brokers.

This article aspires to test several important hypotheses regarding the nature of coordination networks and the formation of coalitions, treating the ACF both as an inspiration and as a framework in need of further refinement. This is done in the context of a complex and conflict-ridden policy subsystem: the

Swedish carnivore-management subsystem. The results indicate, firstly, that perceived belief correspondence, and not perceived influence, is the driving mechanism behind coordination; and, secondly, that the catalog of beliefs shared by actors within a coalition is composed by policy core beliefs, in particular, with a more normative content, while no connection between deep core beliefs and coordination is found.

Matti and Sandström (2011) study the network of policy makers responsible for wildlife management in the Swedish carnivore-management subsystem. Their result identify similarity in policy preferences and beliefs the main driver for forming connections aimed at coordinating policy efforts. Interestingly, perceived influence is not found to be a significant driver for connection, nor are deep core normative beliefs.

Schneider et al. (2003) analyze the connections of policy makers and interest groups in U.S. estuary areas. They compare networks part of the National Estuary Program with networks in comparable estuaries that are not in the program. The networks in NEP areas are found to comprise more levels of government, to seek the advice of a larger set of experts within policy discussions, and to foster stronger connections among the different stakeholders. This leads to a greater trust into the fairness of local policy. While networks are denser in network areas, some common features appear across all networks, namely: policy preference similarity and perceived influence are main drivers for the establishment of connections within the network.

Weible and Sabatier (2005) investigate networks of policy makers in the areas subject to California Marine Life Protection Act, with particular interest on the overlap between networks with different functions, such as information transmission and advice, and alliance and coordination. The main findings are that alliance and coordination networks overlap slightly more than information/advice networks, and that policy core beliefs and preference similarity are good predictor of connections within these networks. The advice/information network is composed of four segregated subnetworks composed respectively of local governments and harbor masters, of commercial fishers and divers together with State and Federal government agencies, and environmental groups, of recreational fishers and commercial passenger fishing vessels, and of kelp harvesters.

Feiock, Lee and Park (2012) survey of the local governments' connections in the four-county Orlando, Florida, metropolitan area. Interestingly, they find little evidence of differences in network patterns for administrators and elected officials. Local government actors organize in hierarchical networks, but there is also evidence of transitivity and connections among agents in the subnetworks periphery. Similarity of economic problems and differences in population are the main drivers of connection.

Desmarais, Harden and Boehmke (2015) study policy diffusion across US States. They formulate a network connecting States by keeping track of when different State governments adopt 187 specific policies. They code a link from a State to another, if the second State adopts a policy similar to a pre-existing policy in the first State. In the resulting network of policy imitation, State population, median citizen ideology and geographical distance are the main drivers of connections.

Empirical analysis of networks of political decision makers is not limited to policymakers. Broader networks including political consultants, lobbyists and interest groups have also been considered. Laumann and Knoke (1987) trace over 40,000 connections among lobbyists, government agencies, and congressional staff using 1970s health and energy policy data. Because of the complexities of policies and the stakes in play the importance of information sharing among lobbyists has long been understood (e.g., Hecl 1978). The analysis of connections among lobbyists shows that lobbyists communicate mainly with lobbyists with similar preferences. More broadly, similarity in policy preferences and issue involvement are highly

correlated with communication between the political agents involved in energy policy: lobbyists, congress committees, government agencies.

Carpenter, Esterling and Lazer (2004) confirm these results and proceed to show that connections are transitive: a pair of lobbyists more likely forms if both are connected to a common third party. The amount of resources employed to monitor Washington politics makes a lobbyist firm more influential, in the sense that it informs more political agents in the network. The proportion of other firms in the network with similar interests reduces the likelihood that two lobbyists communicate.

König and Bräuninger (1998) investigate network connection determinants among interest groups, trade unions, governmental agencies and legislators in a set of 32 political events that are representative of German labour policies during the 1980s. The main explanatory variable is found to be political preference proximity. Connections are also more likely formed among actors in the same industry. Further, legislators have the highest number of connections, because of their position of power. There is evidence that the network organizes a star-like groups with homogeneous ideology, in which the most powerful actors act as star centroids.

Koger, Masket and Noel (2009) build the networks of stakeholders (interest groups, media outlets, donors, activists and candidates) in the Democratic and Republican parties. The networks are constructed by tracking down lists of donors and subscribers transferred between political organizations. Two disconnected networks are observed, a Democratic group and a Republican group. Both networks have a core-periphery structure, with a dense inner network of interconnected organizations and a fringe of groups with only one connection. The network centroids are the two formal party organizations. They receive information from a significant number of media outlets and interest groups. Such intermediate organizations also share information among each other.

Box-Steffensmeier and Christenson (2014) build a network of interest groups coalitions using data on co-signing of *amicus curiae* briefs to the US Supreme Court. Evidence is found of interlinked subnetworks. The full interest group network appears to resemble a host of tightly grouped factions and leadership hub organizations. Subnetworks are either completely connected or organized as stars, or something in between. Leaders act as hubs of star-shaped networks that are highly centralized. Some organizations connect with many organizations in their subnetworks. Other organizations do not join in with others in signing briefs. The structural empirical estimation is based on the exponential random graph model and includes several interest group characteristics that may explain the likelihood of connections, such as ideology, size, longevity, industry, policy area and region. Several of these covariates are statistically significant. The interest groups that act as hubs in star-shaped subnetworks are largest in terms of volumes, budgets, and sales. Subnetworks are differentiated in terms of ideology, preference over issues, and industry. Over time, the average network degree and betweenness centrality have increased, whereas transitivity has decreased. Interest groups have become increasingly more connected, and hierarchical structures have become more common.

Fischer and Sciarini (2016) analyze collaboration network data on the 11 most important political decision-making processes of the years 2001-06 in Switzerland, including a major pension reform, the extension of the bilateral agreement with the European Union on the free movement of people, and the bilateral agreement with the European Union on the taxation of savings. They find that preference similarity and perceived power are the main drivers of connection among the political decision-making entities involved.

In her study of the network of interest groups and policy-makers that took part in the decision-making



process that led to the 2000 Swiss CO2 law, Ingold (2011) identifies three major blocks. The first one is dominated by economic actors and right-wing parties, the second block is composed of federal agencies, and the third block comprises green NGOs and leftist parties. While especially the third block display dense connections, entities are differentiated within each block in terms of power and centrality, and this is the case especially for the first block.

The final class of networks I describe in this brief survey are networks of judges. Weiser (2015) interviewed some judges in Manhattan's Federal District Court and reports that almost all have consulted colleagues when puzzled by legal or other issues. These networks are usually constructed by linking judges that cite other judges' rulings as precedents in their decisions. For example, Caldeira (1985) constructs judicial networks as measured by precedent citation across State supreme Courts. The paper uncovers explanations for citation patterns that include geographical distance between Courts, political similarity, cultural distance (as measured by migration interchange among States), and the prestige and caseload of the cited Court. Interestingly, the main explanations for citations are cultural difference and prestige of the cited Court. The resulting citation network can be approximated as a collection of star-shaped subnetworks, in which Courts cite the most prestigious among the culturally closest Courts. Choi and Gulati (2008) trace connections among federal circuit Courts by with citations in opinions. Evidence of political preference based homophily is uncovered: outside-circuit citations are influenced by the political parties of the connected judges. This bias is more pronounced in some specific high-stakes situations.

Fowler et al. (2007) construct the network of the Court cases that cite the US Supreme Court, and formulate network measures that are shown to be within-sample correlated with future citation behaviour. Lupu and Fowler (2013) study the determinants of citations to precedent employed in the US Supreme Court. They find evidence of strategic citation and bargaining within the Court. The majority-opinion writer relies more heavily on precedent when the Court's decision is accompanied by separate opinions. The ideology of the median justice influences citation practices more than ideology of the majority-opinion writer.

Instead of using Court citations, Katz and Stafford (2010) build a network of judges by tracing the flow of clerks from a judge to another. This movement can be taken as a proxy for social and professional linkages between jurists. The degree distribution in the resulting network is centered upon a small number of socially prominent actors. There is a densely connected center composed of Supreme Court justices, to which circuit Court judges are connected. District Court judges are mostly at the periphery. Baum (2014) and Baum and Ditslear (2001) record the movement of clerks from lower Court judged to federal Courts of Appeals since the 1970s. They document a tendency for Justices to hire clerks from judges who share their ideological tendencies, especially in the period since 1990s. This is in part due to changes in the numbers of applications and the development of a practice in which applicants apply to all Justices.

In conclusion, many insights are delivered by the vast empirical literature on political networks, and in particular on the literature on the role of networks in facilitating the flow of information. By and large, the stylized facts validate my theoretical findings. Preference-based homophily is a main driver of networks of legislators, policy makers, judges, and interest groups. The modal network structures in these domains can be described as a collection of subnetworks, differentiated by political preferences, and loosely connected among each other. Each subnetwork is tightly connected and hierarchically differentiated between a core of leaders to which less prominent agents are connected. These periphery agents may or may not be connected among each other, ostensibly depending on their connection costs.

## References

- Ahn, T.K., Robert Huckfeldt, Alexander K. Mayer and John Barry Ryan. 2013. "Expertise and Bias in Political Communication Networks." *American Journal of Political Science* 57: 357-373.
- Baum, Lawrence. 2014. "Hiring supreme court law clerks: probing the ideological linkage between judges and justices." *Marquette Law Review*, 98: 333.
- Baum, Lawrence, and Corey Ditslear. 2001. "Selection of law clerks and polarization in the U.S. Supreme Court," *Journal of Politics*, 63(3): 869 - 885.
- Box-Steffensmeier, Janet M., and Dino P. Christenson. 2014. "The evolution and formation of *amicus curiae* networks," *Social Networks*, 36: 82-96.
- Bratton, Kathleen A., and Stella M. Rouse. 2011. "Networks in the legislative arena: how group dynamics affect cosponsorship," *Legislative Studies Quarterly*, 34(3): 423-460.
- Caldeira, Gregory A. 1985. "The transmission of legal precedent: A study of State Supreme Courts," *American Political Science Review*, 79(1): 178-194.
- Carpenter, Daniel P., Kevin M. Esterling, and David M. J. Lazer. 2004. "Friends, brokers, and transitivity: Who informs whom in Washington politics?" *Journal of Politics*, 66(1): 22-246.
- Choi, Stephen J., and G. Mitu Gulati. 2008. "Bias in judicial citations: a window into the behavior of judges?," *Journal of Legal Studies*, 37(1): 87-130.
- Desmarais, Bruce A., Jeffrey J. Harden, and Frederick J. Boehmke. 2015. "Persistent policy pathways: Inferring diffusion networks in the American States," *American Political Science Review*, 109(2): 392-406.
- Feiock, Richard C., In Won Lee, and Hyung Jun Park. 2012. "Administrators and elected officials collaboration networks: Selecting partners to reduce risk in economic development." *Public Administration Review* 72(S1): 58-68.
- Fiellin, Alan. 1962. "The functions of informal groups in legislative institutions," *Journal of Politics*, 24(1): 72-91.
- Fischer, Manuel, and Pascal Sciarini. 2016. "Collaborative Tie Formation in Policy Networks: A Cross-Sector Perspective," *Journal of Politics*, 78(1): 63-74.
- Gerber, Elisabeth R., Adam D. Henry, and Mark Lubell. 2013. "Political homophily and collaboration in regional planning networks," *American Journal of Political Science*, 57(3): 598-610.
- Henry, Adam D. 2011. "Ideology, power, and the structure of policy networks," *Policy Studies Journal*, 39(3): 361-383.
- Henry, Adam D., Mark Lubell, and Michael McCoy. 2011. "Belief systems and social capital as drivers of policy network structure: The case of California regional planning." *Journal of Public Administration Research and Theory*, 21(3): 419-444.
- Huckfeldt, Robert, and John Sprague. 1987. "Networks in Context: The Social Flow of Political Information," *American Political Science Review*, 81: 1197 - 1216.

- Ingold, Karin. 2011. "Network structures within policy processes: Coalitions, power, and brokerage in Swiss climate policy," *Policy Studies Journal*, 39(3): 435- 459.
- Katz, Martin D., and Derek K. Stafford. 2008. "Hustle and flow: A social network analysis of the American federal judiciary," *Ohio State Law Journal*, 71(3): 457-507.
- Koger, Gregory, Seth Masket, and Hans Noel. 2009. "Partisan webs: Information exchange and party networks," *British Journal Political Science*, 39: 633-653.
- König, Thomas, and Thomas Bräuningner. 1998. "The formation of policy networks: Preferences, institutions and actors' choice of information and exchange relations," *Journal of Theoretical Politics*, 10(4): 445-71.
- Lasswell, Harold D. (1931): "Social conflict," in *Encyclopaedia of the Social Sciences, Vol. 4*, Eds: Edwin R. Seligman and Alvin S. Johnson, London: Macmillan.
- Laumann, Edward O., and David Knoke. 1987. *The Organizational State: Social Choice in National Policy Domains*, Madison WI: The University of Wisconsin Press.
- Matti, Simon, and Annica Sandström. 2011. "The rationale determining advocacy coalitions: examining coordination networks and corresponding beliefs," *Policy Studies Journal*, 39(3): 385-410.
- Ringe, Nils and Jennifer N. Victor, with Christopher J. Carman. 2013. *Bridging the Information Gap: Legislative Member Organizations as Social Networks in the United States and the European Union*, Ann Arbor: University of Michigan Press.
- Schneider, Mark, John Scholz, Mark Lubell, Denisa Mindruta, and Matthew Edwardsen. 2003. "Building consensual institutions: Networks and the national estuary program," *American Journal of Political Science*, 47(1): 143 - 158.
- Weible, Christopher M., and Paul A. Sabatier (2005). "Comparing policy networks: Marine protected areas in California." *Policy Studies Journal*, 33(2): 181-201.
- Weiser, Benjamin. 2015. "Faced with legal puzzles, judges often turn to fellow jurists," *The New York Times*, May 19, 2015.
- Zafonte, Mathew, and Paul A. Sabatier. 1998. "Shared beliefs and imposed interdependencies as determinants of ally networks in overlapping subsystems," *Journal of Theoretical Politics*, 10(4): 473-505.