Problem Set 8

Exercise 1. A legislature with three members decides, by majority vote, the fate of three bills, A, B, and C. Each legislator's preferences are represented by the sum of the values she attaches to the bills that pass. The value attached by each legislator to each bill is indicated below.

	A	В	С
Legislator 1	2	-1	1
Legislator 2	1	2	-1
Legislator 3	-1	1	2

Each majority coalition can achieve the passage of any set of bills, whereas each minority is powerless. Find the core of the coalitional game that models this situation. Find the core of the game in which the values the legislators attach to the payoff of each bill differ from those in the figure above only in that legislator 3 values the passage of bill C at 0. Find the core of the game in which the values the legislators attach to the payoff of each bill differ from those in the figure above only in that each 1 is replaced by -1.

Exercise 2. Consider the variant of the landowner–worker game in which each additional worker produces more additional output than the previous one. (That is, f(k)/k < f(k+1)/(k+1) for all k.) Does the core of this game contain the action of the grand coalition in which each player obtains an equal share of the total output?

Exercise 3. Find the matchings produced by the deferred acceptance procedure both with proposals by members of the set X and with proposals by members of the set Y for the preferences given in the figure below.

x_1	x_2	x_3	y_1	y_2	y_3
y_1	y_1	y_1	x_1	x_1	x_1
y_2	y_2	y_3	x_2	x_3	x_2
y_3	y_3	y_2	x_3	x_2	x_3

Verify that the only matchings in the core are those generated by the procedures.