Monetary Policy and Wealth Effects with External Positions

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Abstract

I study how financial integration affects the international transmission of macroeconomic shocks and the stance of monetary policy in the context of a two-country DSGE model where households receive dividends from foreign firms. In this environment, exogenous disturbances cause international wealth effects that reallocate consumption across countries. The direction of these effects depends on the nature of the shocks that drive the business cycle, while their strength depends on the size of the external positions. I show that wealth transfers are countercyclical under technology shocks and cost-push shocks affecting the labour market, while they are procyclical under monetary shocks and cost-push shocks affecting the goods market. In the former case, they exert a stabilising role on consumption relative to economic activity, while in the latter case they destabilise it. Numerical work shows that this affects the balance of monetary policy between the alternative objectives of inflation and output stabilisation.

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1 Introduction

In a series of influential papers, Lane and Milesi-Ferretti (2001, 2006, 2007) documented a dramatic upsurge of gross cross-border holdings of foreign assets and liabilities over the last few decades. They interpreted this as a measure of the increase in international financial integration between countries.

Figure 1 overleaf is from Lane and Milesi-Ferretti (2007). It shows that the sum of foreign assets and liabilities to gross domestic product (GDP) grew sevenfold for developed economies and threefold for emerging and developing economies over three decades. The authors argue that these external portfolios are large enough for significant international reallocations of wealth to occur when asset prices and exchange rates move. The question then arises as to whether and how this changes the kind of stabilisation policy that monetary authorities should follow.

External positions are not explicitly modelled in the early literature on monetary policy in open economy. Clarida, Gali, and Gertler (2002) and Gali and Monacelli (2005), among others, assume complete international markets for state-contingent securities and exclude the possibility that countries hold claims on each other’s output or profits. They conclude that a regime of strict domestic inflation targeting is optimal for both closed and open economies. This “isomorphism” result is rejected in subsequent works by Benigno and Benigno (2003) and Devereux and Engel (2003) which adopt more general preference specifications and different assumptions about pricing; these studies, however, maintain the absence of financial integration.

Devereux and Sutherland (2008) and Tille (2008) are among the first attempts to introduce external positions into an open-economy dynamic stochastic general equilibrium (DSGE) setting for the explicit purpose of studying their impact on the conduct of monetary policy. The former work develops a two-country model with endogenous portfolio choice and explores how gross holdings of cross-country financial assets alter the conditions under which monetary policy operates. It shows that the case for price stability is reinforced in this environment, because such a policy enhances the risk-sharing properties of nominal bonds against country-specific technology shocks. The latter work uses a two-country model with exogenous portfolios and investigates how foreign assets create international interdependence and affect the cross-border transmission of monetary shocks. It shows that the exact composition of bonds and equity holdings matters for valuation effects and for the welfare implications of exchange rate movements. Both models are solved using linear methods.

The objective of the present work is to investigate how the international transmission of macroeconomic shocks is affected by the presence of external positions, and whether this changes the objective of monetary policy. I cast the analysis in a two-country DSGE model with monopolistic competition, nominal rigidities and incomplete international asset markets,
Figure 1: Ratio of sum of external assets and liabilities to GDP for different country groups

which I solve using nonlinear methods. Unlike Devereux and Sutherland (2008), I do not study how monetary policy affects the risk-sharing properties of nominal assets; in fact, I work in a Heathcote and Perri (2002) type environment with no international trade in bonds at all. Differently from Tille (2008), I study a cashless economy with traded goods only, and I do not formally focus on nominal exchange rate movements; on the contrary, I assume for simplicity that the two countries use the same currency\(^1\).

I model external positions and their impact on macroeconomic interdependence as follows. Households own exogenously given claims on the profits of domestic and foreign firms; the latter entitle them to receive some dividend payments from abroad in a lump-sum fashion. Because of this, exogenous disturbances that affect firms’ profits spread across countries through wealth effects on households’ labour supply; this has implications for the dynamics of output, employment and consumption.

I show that the strength of the wealth effects depends on the dimension of external positions, while their direction is tied to the dynamics of firms’ profits and depends on the nature of the shocks that are causing the business cycle. More precisely, wealth effects are countercyclical under shocks to technology and wage markups, because output and profits comove positively under these kinds of disturbances. On the contrary, they are procyclical under shocks to interest rates and price markups, because in these cases output and profits comove negatively. Countercyclical wealth effects stabilise consumption relative to economic

\(^1\)Distinct monetary policies can still exist because there is no capital mobility between the two economies.
activity, as households receive net transfers of resources from abroad when domestic output is low; procyclical wealth effects exert a destabilising role on consumption instead, since net transfers are received when domestic output is high.

Whether external positions mitigate or exacerbate the volatility of consumption matters for monetary policy. The output stabilisation objective tends to take higher priority when the volatility of output is large relative to that of consumption, i.e. when wealth effects are countercyclical; a policy of “flexible” inflation targeting is appropriate in this case. Conversely, the stabilisation of output becomes less important when the volatility of output is small relative to that of consumption, i.e. when wealth effects are procyclical; a policy of “strict” inflation targeting tends to emerge as the most desirable policy in this case. This result suggests that external positions can alter the balance between inflation and output stabilisation in either direction, depending on the prevalent type of disturbances in the economy.

The rest of this paper is organised as follows. Section 2 presents the model with frictionless labour markets and Section 3 solves for its equilibrium. Section 4 discusses international wealth effects and macroeconomic adjustment in the presence of external positions. Section 5 defines my welfare criterion and discusses monetary policy tradeoffs. Section 6 introduces nominal wage rigidities and discusses the effects of wage markup shocks. Section 7 concludes.

2 External positions in a two-country model

This is a two-country DSGE model with incomplete international markets and country-specific goods. Each country is populated by a continuum of measure one of infinitely lived households who get utility from consuming both domestic and foreign goods, and disutility from working. Since households fully share risk within each country via trade in a full set of contingent assets, attention is limited to the representative households to save on notation.

International financial markets are incomplete à la Heathcote and Perri (2002): no private asset is available for trade between the countries. Households have access to riskless one-period nominal bonds which cannot be traded across borders; their prices are controlled by the local central banks. As there is no international capital mobility, the model abstracts from different currencies: the prices of all goods are expressed in the same unit of account.

Production in each country takes place in two stages. First, monopolistically competitive producers hire labour and supply a continuum of measure one of intermediate goods, indexed by \( i \); these goods are not traded internationally. Second, perfectly competitive producers adopt a CES technology to aggregate domestically produced intermediates into final consumption goods, denoted \( y_{h,t} \) and \( y_{f,t} \) respectively; these are freely traded.

Each household owns claims to exogenously fixed portions of the profits of both domestic and foreign firms. These claims represent the external positions in the present setup.
2.1 Households

2.1.1 Intertemporal problem: utility maximisation

Households choose consumption, hours and bonds purchases to maximise their total lifetime utility over an infinite horizon. Their intertemporal problems read

\[
\max_{\{c_t, b_{t+1}, n_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)
\]

\[
\text{s.t. } p_t c_t + q_t b_{t+1} = b_t + w_t n_t + s_h p_{h,t} \Pi_{h,t} + s_f p_{f,t} \Pi_{f,t} + t_t
\]

and

\[
\max_{\{c^*_t, b^*_{t+1}, n^*_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c^*_t, n^*_t)
\]

\[
\text{s.t. } p^*_{t} c^*_t + q^*_t b^*_{t+1} = b^*_t + w^*_t n^*_t + (1 - s_h) p_{h,t} \Pi_{h,t} + (1 - s_f) p_{f,t} \Pi_{f,t} + t^*_t.
\]

\(q_t\) and \(q^*_t\) are the prices of the nominally risk-free one period bonds that are not internationally traded. \(t_t\) and \(t^*_t\) are nominal transfers to and from the respective governments.

The household of the home country is entitled to receive exogenously given shares \(s_h\) and \(s_f\) of the real profits from domestic and foreign firms, denoted respectively as \(\Pi_{h,t}\) and \(\Pi_{f,t}\). The foreign household is entitled to receive the remaining shares \(1 - s_h\) and \(1 - s_f\).

2.1.2 Intratemporal problem: consumption allocation

Within each period, households choose the bundles of goods that maximise their consumption, defined here as a standard Armington aggregator with elasticity of substitution \(\eta\) and share of imports \(\zeta\). Their static problems read

\[
\max_{c_{h,t}, c_{f,t}} c_t \equiv \left[(1 - \zeta)^{\frac{1}{\eta}} (c_{h,t})^{\frac{n-1}{\eta}} + (\zeta)^{\frac{1}{\eta}} (c_{f,t})^{\frac{n-1}{\eta}}\right]^\frac{n}{n-1}
\]

\[
\text{s.t. } p_t c_t = p_{h,t} c_{h,t} + p_{f,t} c_{f,t}
\]

and

\[
\max_{c^*_{h,t}, c^*_{f,t}} c^*_t \equiv \left[(1 - \zeta)^{\frac{1}{\eta}} (c^*_{f,t})^{\frac{n-1}{\eta}} + (\zeta)^{\frac{1}{\eta}} (c^*_{h,t})^{\frac{n-1}{\eta}}\right]^\frac{n}{n-1}
\]

\[
\text{s.t. } p^*_{t} c^*_t = p_{f,t} c^*_{f,t} + p_{h,t} c^*_{h,t}.
\]

\(p_t\) and \(p^*_t\) represent the home and foreign CPIs, i.e. the indices of the prices of all domestically consumed goods:

\[
p_t \equiv \left[(1 - \zeta) (p_{h,t})^{1-\eta} + (\zeta) (p_{f,t})^{1-\eta}\right]^{\frac{1}{1-\eta}}, \quad (1)
\]

\[
p^*_t \equiv \left[(1 - \zeta) (p_{f,t})^{1-\eta} + (\zeta) (p_{h,t})^{1-\eta}\right]^{\frac{1}{1-\eta}}. \quad (2)
\]
2.2 Firms

2.2.1 Final goods producers

Perfectly competitive producers demand local inputs, indexed by $i$, to make final goods with standard CES technologies:\(^2\):

$$\max_{y_{h,t}(i)} p_{h,t} y_{h,t} - \int_0^1 p_{h,t}(i) y_{h,t}(i) \, di \quad \text{s.t.} \quad y_{h,t} = \left( \int_0^1 y_{h,t}(i) \frac{\zeta_{i+1}}{\zeta_i} \, di \right)^{\frac{\zeta_i}{\zeta_i - 1}},$$

$$\max_{y_{f,t}(i)} p_{f,t} y_{f,t} - \int_0^1 p_{f,t}(i) y_{f,t}(i) \, di \quad \text{s.t.} \quad y_{f,t} = \left( \int_0^1 y_{f,t}(i) \frac{\zeta_{i+1}}{\zeta_i} \, di \right)^{\frac{\zeta_i}{\zeta_i - 1}}.$$  

The elasticities of substitution between varieties of intermediates are subject to exogenous innovations that cause cost-push shocks in the goods markets. These disturbances determine fluctuations in the gap between the natural (i.e. flexible-prices) allocation and the efficient one, creating a short-run monetary policy tradeoff; monetary authorities are put in the dilemma of stabilising prices or economic activity.

2.2.2 Intermediate goods producers

Monopolistically competitive producers in the home and foreign country make intermediate goods $i$ with the following technologies:

$$y_{h,t}(i) = a_t n_t(i)^{1-\alpha}, \quad (3)$$

$$y_{f,t}(i) = a^*_t n^*_t(i)^{1-\alpha}. \quad (4)$$

Labour is internationally immobile, so hours are entirely supplied by local households. The prices of intermediate products are chosen to maximise profits in a Calvo-Yun setting, subject to isoelastic demands by final goods producers. The problems faced by the home and foreign producers are respectively

$$\max_{\bar{p}_{h,t}(i)} \mathbb{E}_t \sum_{\tau=0}^\infty \theta^\tau q_{t+\tau} \left\{ y_{h,t+\tau}(i) \frac{\bar{p}_{h,t}(i)}{\bar{p}_{h,t+\tau}} - \Psi \left( y_{h,t+\tau}(i) \right) \right\}$$

$$\text{s.t.} \quad y_{h,t+\tau}(i) = \left( \frac{p_{h,t+\tau}(i)}{p_{h,t+\tau}} \right)^{-\varepsilon_t} y_{h,t+\tau}$$

$$q_{t+\tau} = \beta^\tau \mathbb{E}_t \left( \frac{\lambda_{t+\tau}}{\lambda_t} \right),$$

\(^2\)This functional form specification is fundamental to the model: since intermediate products enter them symmetrically, using identical quantities of all inputs is the most efficient way to produce any given amount of output. Price dispersion due to nominal rigidities à la Calvo, however, causes the prices of intermediates to differ. This forces final goods producers to use asymmetric quantities of inputs, which reduce the final output.
and
\[
\max_{p_{f,t}(i)} E_t \sum_{\tau=0}^{\infty} \theta_p^\tau q_{t,t+\tau} \left\{ y_{f,t+\tau} (i) \frac{p_{f,t}(i)}{p_{f,t+\tau}} - \Psi (y_{f,t+\tau} (i)) \right\}
\]

s.t.
\[
\begin{aligned}
y_{f,t+\tau} (i) &= \left( \frac{p_{f,t+\tau}(i)}{p_{f,t+\tau}} \right)^{-e_t^\tau} y_{f,t+\tau} \\
q_{t,t+\tau} &= \beta^\tau E_t \left( \frac{\lambda_{t+\tau}^t}{\lambda_t^t} \right).
\end{aligned}
\]

$q_{t,t+\tau}$ and $q_{t,t+\tau}^*$ denote the households’ stochastic discount factors for $\tau$ periods-ahead real payoffs, while $\theta_p$ is the index of price stickiness. The $\Psi (\cdot)$ functions represent the real cost of production.

### 2.3 Monetary policy

As mentioned above, the nominal returns on domestic and foreign one-period bonds are certain at the issuing date. They are defined as

\[
R_t \equiv \frac{1}{q_t}, \quad R_t^* \equiv \frac{1}{q_t^*}.
\]

These returns are the instruments of monetary policy. The objective of the central banks is to stabilise both prices and economic activity, due to the presence of cost-push shocks.\(^3\)

This objective is pursued by setting interest rates according to the following Taylor rules, where $R$, $R^*$, $\pi_h$ and $\pi_f$ represent the respective nominal interest rate and inflation targets:

\[
R_t = \left( \frac{R_{t-1}}{R} \right)^{\gamma_R} \left[ \left( \frac{\pi_{h,t}}{\pi_h} \right)^{\gamma_\pi_h} \left( \frac{y_{h,t}}{y_{h,t-1}} \right)^{\gamma_y} \right]^{1-\gamma_R} m_t, \quad (5)
\]

\[
R_t^* = \left( \frac{R_{t-1}^*}{R^*} \right)^{\gamma_R} \left[ \left( \frac{\pi_{f,t}}{\pi_f} \right)^{\gamma_\pi_f} \left( \frac{y_{f,t}}{y_{f,t-1}} \right)^{\gamma_y} \right]^{1-\gamma_R} m_t^*, \quad (6)
\]

Following Clarida et al. (2002) and Galí and Monacelli (2005), these rules have been specified in terms of **PPI inflation rates**

\[
\pi_{h,t} \equiv \frac{p_{h,t}}{p_{h,t-1}}, \quad \pi_{f,t} \equiv \frac{p_{f,t}}{p_{f,t-1}},
\]

rather than **CPI inflation rates**

\[
\pi_t \equiv \frac{p_t}{p_{t-1}}, \quad \pi_t^* \equiv \frac{p_t^*}{p_{t-1}^*}.
\]

Under these conditions, a policy of **strict** inflation targeting would replicate the flexible prices allocation, not the socially optimal one. Since central banks face the conflicting goals of preventing suboptimal fluctuations in relative prices and avoiding suboptimal fluctuations in output, they must follow **flexible** inflation targeting policies instead.
Notice two features of rules (5) and (6). First, the relevant inflation measure that monetary authorities must control is that of the goods *produced* in their respective countries, rather than those *consumed* there, because inflation and price dispersion entail a resource misallocation on the supply side of the economy in this model. Second, the Taylor rules are specified in terms of output *growth* rather than output *gap*, since the former is easily observable in actual practice while the latter is not.

### 2.4 Exogenous processes

Technologies follow a vector autoregressive process in logs:

\[
\begin{bmatrix}
\log a_t \\
\log a^*_t
\end{bmatrix} =
\begin{bmatrix}
\rho_a & \nu_a \\
\nu_a & \rho_a
\end{bmatrix}
\begin{bmatrix}
\log a_{t-1} \\
\log a^*_{t-1}
\end{bmatrix} +
\begin{bmatrix}
e_{a,t} \\
e^*_{a,t}
\end{bmatrix}.
\]

The off-diagonal elements \( \nu_a \) represent technology spillovers and introduce correlation between the two processes. The innovations \( e_{a,t} \) and \( e^*_{a,t} \) follow orthogonal i.i.d. normal processes:

\[
\text{corr} \ (e_a, e^*_a) = 0.
\]

Monetary policy shocks follow an autoregressive process in logs:

\[
\begin{bmatrix}
\log m_t \\
\log m^*_t
\end{bmatrix} =
\begin{bmatrix}
\rho_m & \nu_m \\
\nu_m & \rho_m
\end{bmatrix}
\begin{bmatrix}
\log m_{t-1} \\
\log m^*_{t-1}
\end{bmatrix} +
\begin{bmatrix}
e_{m,t} \\
e^*_{m,t}
\end{bmatrix}.
\]

The off-diagonal elements \( \nu_m \) control international spillovers of monetary policy shocks. The innovations \( e_{m,t} \) and \( e^*_{m,t} \) follow orthogonal i.i.d. normal processes:

\[
\text{corr} \ (e_m, e^*_m) = 0.
\]

The elasticities of substitution between intermediate products follow stochastic processes in logs:

\[
\begin{bmatrix}
\log \left( \frac{\bar{\epsilon}_t}{\bar{\epsilon}_e} \right) \\
\log \left( \frac{\bar{\epsilon}^*_t}{\bar{\epsilon}_e} \right)
\end{bmatrix} =
\begin{bmatrix}
\rho_\epsilon & \nu_\epsilon \\
\nu_\epsilon & \rho_\epsilon
\end{bmatrix}
\begin{bmatrix}
\log \left( \frac{\bar{\epsilon}_{t-1}}{\bar{\epsilon}_e} \right) \\
\log \left( \frac{\bar{\epsilon}^*_{t-1}}{\bar{\epsilon}_e} \right)
\end{bmatrix} +
\begin{bmatrix}
e_{\epsilon,t} \\
e^*_{\epsilon,t}
\end{bmatrix}.
\]

The off-diagonal elements \( \nu_\epsilon \) represent spillovers of cost-push shocks and introduce correlation between the two processes. The innovations \( e_{\epsilon,t} \) and \( e^*_{\epsilon,t} \) follow orthogonal i.i.d. normal processes:

\[
\text{corr} \ (e_\epsilon, e^*_\epsilon) = 0.
\]
3 Equilibrium conditions

3.1 Households

3.1.1 Intertemporal optimisation

The first-order conditions for consumption, saving and labour supply of the home country household, together with the budget constraint, are as follows:

\[ u_c(c_t, n_t) = \lambda_t, \]  
\[ q_t = \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \frac{p_t}{p_{t+1}} \right), \]  
\[ q_t - u_n(c_t, n_t) \frac{w_t}{p_t} = \lambda_t, \]  
\[ c_t + q_t b_{t+1} = \frac{b_t}{p_t} + \frac{w_t}{p_t} n_t + s_h \frac{p_{h,t}}{p_t} \Pi_{h,t} + s_f \frac{p_{f,t}}{p_t} \Pi_{f,t} + \frac{t_t}{p_t}, \]  

Analogous conditions hold for the foreign household: see Appendix E. Notice that (10) is written in real terms, so \( \lambda_t \) is the marginal utility of an additional unit of consumption.

3.1.2 Static optimisation

The final consumption demands for domestic and imported goods by home and foreign households are respectively

\[ c_{h,t} = (1 - \zeta) \left( \frac{p_{h,t}}{p_t} \right)^{-\eta} c_t, \quad c_{f,t} = (\zeta) \left( \frac{p_{f,t}}{p_t} \right)^{-\eta} c_t, \]  
\[ c_{f,t}^* = (1 - \zeta) \left( \frac{p_{f,t}}{p_t} \right)^{-\eta} c_t^*, \quad c_{h,t}^* = (\zeta) \left( \frac{p_{h,t}}{p_t} \right)^{-\eta} c_t^*. \]  

3.2 Firms

3.2.1 Final goods production

The input demand schedules that solve the problems of final goods producers are as follows:

\[ y_{h,t}(i) = \left( \frac{p_{h,t}(i)}{p_{h,t}} \right)^{-\varepsilon_t} y_{h,t}, \quad y_{f,t}(i) = \left( \frac{p_{f,t}(i)}{p_{f,t}} \right)^{-\varepsilon_t^*} y_{f,t}. \]  

The associated price indices implied by perfect competition (i.e. zero profits) are

\[ p_{h,t} = \left( \int_0^1 p_{h,t}(i)^{1-\varepsilon_t} di \right)^{\frac{1}{1-\varepsilon_t}}, \quad p_{f,t} = \left( \int_0^1 p_{f,t}(i)^{1-\varepsilon_t^*} di \right)^{\frac{1}{1-\varepsilon_t^*}}. \]
3.2.2 Productivity, employment and aggregate output

In order to facilitate the passage from individual to aggregate price dynamics, it is useful to rewrite the objective functions of all firms in terms of the economy-wide marginal cost. For this purpose, notice first that labour market clearing in each country requires that

\[ n_t = \int_0^1 n_t(i) \, di, \quad n^*_t = \int_0^1 n^*_t(i) \, di. \]  

(15)

Combining these conditions with the production functions (3) and (4) and the input demands (11) and (12), as outlined in Appendix A, we get the exact aggregate production functions:

\[ y_{h,t} = \frac{n_t}{d_{h,t}} n_{h,t}^{1-\alpha}, \quad y_{f,t} = \frac{n^*_t}{d_{f,t}} n^*_{f,t}^{1-\alpha}. \]  

(16)

The measures of home and foreign price dispersion are defined for brevity as

\[ d_{h,t} \equiv \int_0^1 \left( \frac{p_{h,t}(i)}{p_{h,t}} \right)^{-\frac{\varepsilon_t}{1-\alpha}} \, di, \quad d_{f,t} \equiv \int_0^1 \left( \frac{p_{f,t}(i)}{p_{f,t}} \right)^{-\frac{\varepsilon^*_t}{1-\alpha}} \, di. \]  

(17)

Comparing the real marginal costs of individual firms with the average marginal costs of the respective economies, as explained in Appendix B, we find

\[ mc_{h,t}(i) = mc_{h,t} \left( \frac{p_{h,t}(i)}{p_{h,t}} \right)^{-\frac{\varepsilon_t}{1-\alpha}}, \quad mc_{f,t}(i) = mc_{f,t} \left( \frac{p_{f,t}(i)}{p_{f,t}} \right)^{-\frac{\varepsilon^*_t}{1-\alpha}}. \]  

(18)

3.2.3 Intermediate goods production

Let us restrict our attention to a symmetric equilibrium where all price resetters in each country face the same problem and therefore choose the same price, respectively \( \bar{p}_{h,t} \) and \( \bar{p}_{f,t} \).

Define optimal relative prices in each country as

\[ \tilde{p}_{h,t} \equiv \frac{\bar{p}_{h,t}}{p_{h,t}}, \quad \tilde{p}_{f,t} \equiv \frac{\bar{p}_{f,t}}{p_{f,t}}. \]  

(19)

As outlined in Appendix D, the optimal price setting conditions for home producers are

\[ g^2_{h,t} = \mathcal{M}_{p,t} g^1_{h,t}, \]  

(20)

where I have defined recursive auxiliary variables

\[ g^2_{h,t} = y_{h,t} (\tilde{p}_{h,t})^{-\varepsilon_t} + \theta_p \beta_{1,\varepsilon} \left( \frac{\lambda_{t+1}}{\lambda_t} \right) g^2_{h,t+1} \left( \frac{\bar{p}_{h,t}}{\tilde{p}_{h,t+1}} \right)^{-\varepsilon_t} \left( \frac{1}{\pi_{h,t+1}} \right)^{1-\varepsilon_t}, \]  

(21)
\[
g_h^1 = y_{h,t} \frac{mc_{h,t}}{d_{h,t}} (\bar{p}_{h,t})^{\frac{\alpha-\varepsilon t}{1-\alpha}} + \theta_p \beta \mathbb{E}_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) g_{h,t+1}^1 \left( \frac{\bar{p}_{h,t}}{\bar{p}_{h,t} + 1} \right)^{\frac{\alpha-1-\varepsilon t}{1-\alpha}} \left( \frac{1}{\pi_{h,t+1}} \right)^{-\frac{\varepsilon t}{1-\alpha}}, \quad (22)
\]
and the desired “frictionless” price markup\(^4\)

\[
\mathcal{M}_{p,t} \equiv \frac{\varepsilon_t}{\varepsilon_t - 1}.
\]

Analogous pricing conditions can be written for foreign firms: see Appendix E.

### 3.3 Dynamics of aggregate price levels and price dispersion

Given Calvo-Yun pricing, we make use of two facts now: (i) all resetters in each country choose the same prices (symmetric equilibrium); (ii) the distribution of prices among non-resetters at time \(t\) corresponds to the distribution of effective prices at time \(t-1\) (law of large numbers).

These two facts imply that producer price indices evolve according to

\[
1 = \theta_p \left( \frac{p_{h,t-1}}{p_{h,t}} \right)^{1-\varepsilon t} + (1 - \theta_p) \left( \frac{\bar{p}_{h,t}}{p_{h,t}} \right)^{1-\varepsilon t},
\]

\[
1 = \theta_p \left( \frac{p_{f,t-1}}{p_{f,t}} \right)^{1-\varepsilon^{*} t} + (1 - \theta_p) \left( \frac{\bar{p}_{f,t}}{p_{f,t}} \right)^{1-\varepsilon^{*} t}.
\]

By the same Calvo-Yun logic, we can write the price dispersion indices (17) recursively:

\[
d_{h,t} = \theta_p \left( \frac{1}{\pi_{h,t}} \right)^{-\frac{\varepsilon t}{1-\alpha}} d_{h,t-1} + (1 - \theta_p) (\bar{p}_{h,t})^{-\frac{\varepsilon t}{1-\alpha}},
\]

\[
d_{f,t} = \theta_p \left( \frac{1}{\pi_{f,t}} \right)^{-\frac{\varepsilon^{*} t}{1-\alpha}} d_{f,t-1} + (1 - \theta_p) (\bar{p}_{f,t})^{-\frac{\varepsilon^{*} t}{1-\alpha}}.
\]

\(^4\)\(\mathcal{M}_{p,t}\) is called “frictionless” markup because with \(\theta = 0\) (flexible prices) the FOCs become

\[
\bar{p}_{h,t} = p_{h,t} [\mathcal{M}_{p,t} mc_{h,t}]^{\Theta},
\]

where the index is defined as in Galí (2008) and is decreasing in both \(\varepsilon\) and \(\alpha\):

\[
\Theta \equiv \frac{1 - \alpha}{\varepsilon \alpha + 1 - \alpha} \leq 1.
\]

Notice that for \(\alpha = 0\) (constant returns) we get the following standard markup pricing condition:

\[
\bar{p}_{h,t} = \mathcal{M}_{p,t} mc_{h,t} p_{h,t}.
\]
3.4 Market clearing and the terms of trade

Labour market clearing has been imposed in the calculation of the aggregate production functions. Goods market clearing requires that the following equalities hold:

\[ y_{h,t} = c_{h,t} + c_{h,t}^*, \quad y_{f,t} = c_{f,t} + c_{f,t}^*. \]

Since this is a cashless economy model, nominal variables are not uniquely defined: only real variables are. Therefore, while the absolute price levels \( p_{h,t} \) and \( p_{f,t} \) cannot be identified, the relative price that clears both markets can be. Such price is called the terms of trade:

\[ s_t \equiv \frac{p_{f,t}}{p_{h,t}}. \]

As to the asset markets, the fact that the bonds cannot be traded by the two households implies that their holdings must be zero in equilibrium:

\[ b_t = 0, \quad b_t^* = 0. \]

A summary of the whole set of equilibrium conditions, rearranged in real terms for computational convenience, can be found in Appendix E.

4 Equilibrium dynamics

In this section I investigate the dynamic responses of the main macroeconomic variables to technology, monetary and cost-push shocks, and I explore how they differ in the presence and in the absence of external positions.

A conventional CRRA specification is assigned to period utility functions:

\[ u(c_t, n_t) = \frac{c_t^{1-\sigma} - n_t^{1+\varphi}}{1 - \sigma}. \]

Table 1 overleaf displays the parameterisation adopted for the simulations that follow.

Notice that the preferences of home and foreign households over domestic goods and imports are identical in the absence of home bias, i.e. with \( \zeta = 1/2 \). In this case, the equilibrium adjustment of relative prices to movements in relative output depends directly on the elasticity of substitution between domestic and foreign goods. In the absence of external positions, the Cole and Obstfeld (1991) result applies: if \( \eta = 1 \), \( c_t \) and \( c_t^* \) always move one-to-one because terms-of-trade movements neutralise output risks. With external positions, this result does not hold any more: as we will see, \( c_t \) and \( c_t^* \) move asymmetrically even with a unit Armington elasticity, because wealth effects occur.
Table 1: Parameterisation of the model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0</td>
<td>Technology coefficient controlling returns to scale</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1</td>
<td>Elasticity of intertemporal substitution</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>1</td>
<td>Frisch elasticity of labour supply</td>
</tr>
<tr>
<td>$\bar{\varepsilon}$</td>
<td>6</td>
<td>Steady state elasticity of substitution between intermediates</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.5</td>
<td>Share of consumption allocated to imported goods</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1</td>
<td>Armington elasticity of international substitution</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>0.66</td>
<td>Price stickiness parameter</td>
</tr>
<tr>
<td>$\gamma_R$</td>
<td>0.7</td>
<td>Interest rate smoothing parameter in the Taylor rule</td>
</tr>
<tr>
<td>$\gamma_\pi$</td>
<td>1.5</td>
<td>Inflation parameter in the Taylor rule</td>
</tr>
<tr>
<td>$\gamma_y$</td>
<td>3</td>
<td>Output growth parameter in the Taylor rule</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.95</td>
<td>Serial correlation of the log of technology</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>0.95</td>
<td>Serial correlation of the log of monetary shocks</td>
</tr>
<tr>
<td>$\rho_\varepsilon$</td>
<td>0.95</td>
<td>Serial correlation of the log of cost-push shocks</td>
</tr>
<tr>
<td>$\nu_a$</td>
<td>0</td>
<td>International spillover of technology shocks</td>
</tr>
<tr>
<td>$\nu_m$</td>
<td>0</td>
<td>International spillover of monetary shocks</td>
</tr>
<tr>
<td>$\nu_\varepsilon$</td>
<td>0</td>
<td>International spillover of cost-push shocks</td>
</tr>
<tr>
<td>std($e_a$)</td>
<td>0.01</td>
<td>Standard deviation of technology shocks</td>
</tr>
<tr>
<td>std($e_m$)</td>
<td>0.01</td>
<td>Standard deviation of monetary shocks</td>
</tr>
<tr>
<td>std($e_\varepsilon$)</td>
<td>0.01</td>
<td>Standard deviation of cost-push shocks</td>
</tr>
</tbody>
</table>

The dynamical properties of the economy without external positions are consistent with those of the standard small-scale New Keynesian model discussed in Galí (2008). The impulse responses of that economy will be shown in what follows as a benchmark for comparison against the economy with external positions.

4.1 Technology shocks

Figure 2 shows that a positive technology shock in the home country causes a decrease in the marginal cost of production and an increase in domestic output. The real profits of home firms jump; as both consumption and leisure are normal goods, the home household reduces the supply of labour.

In the absence of external positions (red lines), home and foreign consumption move one-to-one thanks to the risk-sharing role played by the terms-of-trade: both households enjoy the larger supply of home goods in equal proportions. Since the foreign household is not entitled to receive any dividends from abroad, there is no wealth effect on labour supply; as a consequence, foreign output is stable too.

In the presence of external positions (blue lines), home country firms pay part of the extra dividends to the foreign household. This creates a first wealth effect: the lump-sum component
of the foreign household’s income jumps, so the household reduces the supply of labour to enjoy more leisure. As a consequence, foreign hours and output fall while the real wage rises. This triggers a second wealth effect: since the profits of foreign firms drop, the household of the home country receives smaller dividends from abroad and supplies more labour. For this reason, home output and hours are temporarily higher than in the previous case.

Notice that the combined result of these two wealth effects is that foreign consumption jumps on impact, while home consumption falls: the presence of external positions reallocates consumption across countries.

Figure 2: Impulse responses to a technology shock with and without external positions (blue vs red)

4.2 Monetary shocks

Figure 3 shows that a monetary policy shock raises the real interest rate in the home country; this exerts a contractionary effect on domestic output and pushes domestic CPI inflation down. Since nominal wages are flexible but prices are not, real wages in the home country fall after the shock, and aggregate profits jump.

In the absence of external positions, home and foreign consumption fall because the total supply of home goods has decreased. As mentioned above, the decline in $c_t$ and $c^*_t$ is symmetric because $\eta = 1$. Since the foreign household is insulated from the dynamics of home profits, there is no change in foreign hours, output and profits.

In the presence of external positions, the two wealth effects described above come into play again. The foreign household receives higher dividends from abroad and supplies less labour, so foreign output and profits decline. The home household, in turn, receives lower
dividends from foreign firms and supplies more labour; as a consequence, home output stays higher than in the previous case.

The combined impact of these effects is that consumption shows a pronounced fall in the home country and a jump in the foreign country: wealth effects again transfer consumption across the world.\(^5\)

Figure 3: Impulse responses to a monetary shock with and without external positions (blue vs red)

4.2.1 The importance of external positions

The magnitude of the output spillovers and the strength of the international comovement of consumption following a monetary shock depend heavily on the size of the external positions. To uncover this dependence, in Figure 4 overleaf I let \(s_h\) and \(s_f\) range between 0% and 100% and I keep \(s_h + s_f = 1\), so that in steady state countries pay identical dividends to each other and consumption is symmetric.

The picture shows that an asymmetric monetary shock triggers no international wealth effects when external positions are zero. As the external positions get larger, instead, international spillovers play an increasingly important role; this shows up in larger movements of foreign output and hours on impact (due to the first wealth effect) and smaller movements in home output and hours (due to the second wealth effect).

\(^5\)Notice that total world consumption (not shown here) clearly declines after a contractionary monetary shock, because the supply of home goods drops.
Figure 4: External positions and the international spillover of monetary shocks
The picture also shows that if the two countries were sufficiently exposed to each other’s dividends, the income effects would be large enough to make foreign output move more than home output does. In that extreme case, the terms of trade of the home country would actually **depreciate**.

### 4.3 Cost-push shocks

Figure 5 shows the impulse responses of selected macroeconomic variables to an adverse cost-push shock in the home country, which takes the form of an exogenous 1% decrease in \( \varepsilon_t \).

The cost-push shock temporarily boosts price markups in the home economy, expanding the wedge between the marginal product of labour and the marginal rate of substitution between consumption and leisure. Output, hours and real wages fall, while real profits rise.

**Figure 5: Impulse responses to a cost-push shock with and without external positions (blue vs red)**

In the presence of external positions (blue lines), additional dividends are paid to the foreign household, triggering a wealth effect that reduces the supply of labour; foreign output and profits fall. This triggers a second wealth effect, because the domestic household in turn receives smaller dividends from abroad and supplies more labour; this makes real wages fall even more in the home country.

In the absence of external positions (red lines), the foreign household is immune to changes in the profits of home firms, so foreign labour supply and output are unaffected. The extra profits of home firms are received entirely by the home household, who reduces labour supply relative to the previous case. For this reason, domestic real wages stay higher and output
stays lower than before.

Figure 6 shows how wealth effects get larger as external positions increase. The legend is the same as in Figure 4, with blue lines representing the economy with no external positions, and yellow lines representing the polar opposite.

Figure 6: External positions and impulse responses to a cost-push shock

5 Monetary policy and welfare

In this section I study monetary policy within the class of interest rate rules (5) and (6). In order to compare the performance of different parameterisations of these rules, I adopt a welfare-based criterion. I search for the Taylor rule parameters vector \((\gamma_R, \gamma_\pi, \gamma_y)\) that maximises the conditional expectation of the total lifetime utility of households, given the current state of the economy.

To begin with, I define the welfare of the home and foreign households respectively as

\[
V_{h,t} \equiv \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u(c_t, n_t), \quad V_{f,t} \equiv \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u(c^*_{t}, n^*_t).
\]  

(23)
These welfare measures must be rearranged recursively for computational reasons. They are then appended to the competitive equilibrium conditions that appear in Appendix E to form the so-called “expanded dynamic system” of the model. A word of caution is necessary about the order of approximation of this system.

I depart from the standard practice of combining a second-order approximation of the welfare function with a first-order approximation of the remaining equilibrium conditions. This approach would be prone to potentially large approximation errors in my incomplete markets framework, because some second-order terms of the welfare functions (23) would be ignored while others are included. As shown by Kim and Kim (2003), such a miscalculation can result in spurious welfare rankings.

Following Schmitt-Grohé and Uribe (2007), I compute a second-order accurate solution of the entire expanded system around its nonstochastic steady state. This has two main implications. First, I do not need to make the steady state efficient, so I dispense with the factor-input subsidies financed by lump-sum taxes that are often assumed to induce the perfectly competitive level of employment in steady state. Second, I adopt a recursive representation of the exact nonlinear price dynamics, rather than a New Keynesian Phillips Curve, so I must keep track of the evolution of an additional state variable that measures price dispersion across varieties of intermediate goods.

The identification of the most desirable monetary policy mix involves the comparison of expected lifetime welfare (23) across different calibrations of the Taylor rules. Expectations are taken conditional on the initial state of the economy being the competitive equilibrium nonstochastic steady state, which is constant across all monetary regimes; this ensures that the economy starts from the same point in all cases under consideration.

Welfare appears to be decreasing in the interest rate smoothing parameter $\gamma_R$, increasing in $\gamma_\pi$, and non-monotonic (i.e. concave) in the output reaction coefficient $\gamma_y$. The results of numerical work point to the desirability of a regime of flexible inflation targeting, where the stability of the prices of domestic goods is traded off against that of domestic output.

As observed above, the relevant inflation target in this environment is the rate of change of domestic goods prices only, as opposed to that of the overall consumer price index. The reason is that the law of one price always holds here, so central banks do not have to target the movements of imports prices: these are regarded as efficient.

Furthermore, central banks should engage in domestic inflation targeting in an uncoordinated way here; prices would become more unstable if both countries reacted to each other’s inflation. Interest rate feedback rules augmented to respond to foreign inflation à la Clarida, Galí, and Gertler (2002) would be welfare-reducing here, because domestic and imported goods are independent in consumption, so that no cross-border supply spillovers create potential gains from cooperation. Due to the absence of these spillovers, supply-side gains from cooperation do not arise in the present model.
In this framework, it is possible to get arbitrarily close to the level of welfare of the nonstochastic economy if central banks adopt Taylor rules with (i) arbitrarily large inflation coefficients, (ii) no inertia in interest rates, and (iii) suitable output coefficients (which depend on the size of external positions here). These facts are well-established in the literature, and can be explained as follows.

First, minimising inflation variations reduces the need to reset prices and helps the economy approach the “natural” or flexible prices allocation. Second, an inertial adjustment of interest rates is unnecessary in a cashless economy, because there is no need to stabilise the opportunity cost of holding money in such an environment. Third, a policy of “leaning against the wind” makes output less volatile (at the cost of some inflation volatility) in the presence of cost-push shocks.

If nominal rigidities were the only friction, central banks would face no monetary policy tradeoff and inflation stabilisation would emerge as the sole objective of monetary policy, regardless of external positions. In that case, the welfare level of the nonstochastic economy would be well approximated by the following “constrained” configuration of the interest rate rules:

\[
(\gamma_R, \gamma_\pi, \gamma_y) = (0, 4, 0).
\]

In the presence of cost-push shocks, instead, monetary authorities must strike a balance between different goals because the natural level of output varies while its efficient counterpart remains unchanged. In that case, the presence of external positions can tilt the balance towards one objective.

The existence of a monetary policy tradeoff would go unnoticed if the volatility of cost-push shocks was as small as it is in Table 1 and Figure 5, and strict inflation targeting would remain the welfare-maximising policy. The monetary policy dilemma becomes visible if the volatility of cost push shocks is as large as \(\text{std}(e_\epsilon) = 0.25\), for instance. A null response to output fluctuations does not maximise welfare any longer under these conditions: central banks can improve upon strict inflation targeting by adopting a policy mix that puts some emphasis on domestic output stability.

As shown in Table 2, the output coefficient that maximises welfare in this case tends to stay around one for external positions between zero and fifty percent (which is the most plausible interval), and it steadily declines to zero if the size of external positions gets larger than that. The reason is that the international wealth effects observed in Figures 5 and 6 operate to make output and hours progressively less variable in the face of cost-push shocks; this tends to tilt the balance in favour of inflation stabilisation again.

\(^6\)Notice that the welfare maximisation problem has no solution here, because the objective function is monotonically increasing in \(\gamma_\pi\) and the domain of this parameter is unbounded in principle. Following Schmitt-Grohé and Uribe (2007), I rule out Taylor rule parameters larger than 4, as they would not be realistic in practice.
Table 2: External positions and Taylor rule parameters under cost-push shocks

<table>
<thead>
<tr>
<th>External positions</th>
<th>Optimal ((\gamma_R, \gamma_\pi, \gamma_y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>(0, 4, 1)</td>
</tr>
<tr>
<td>25%</td>
<td>(0, 4, 1)</td>
</tr>
<tr>
<td>50%</td>
<td>(0, 4, 1)</td>
</tr>
<tr>
<td>75%</td>
<td>(0, 4, 0.5)</td>
</tr>
<tr>
<td>100%</td>
<td>(0, 4, 0)</td>
</tr>
</tbody>
</table>

6 Sticky wages

The last section illustrated how wealth effects tend to stabilise output and destabilise consumption in the face of cost-push shocks affecting the production of final goods, if external positions are present.

This section shows that the opposite would happen if cost-push shocks affected the labour markets as in Clarida, Galí, and Gertler (2002): wealth effects would stabilise consumption and destabilise output.

Notice that wages are flexible in Clarida, Galí, and Gertler (2002), so cost-push shocks are assumed to affect the wage markups directly. Here wages are subject to nominal rigidities as in Schmitt-Grohé and Uribe (2007), instead.

Households no longer choose hours taking the wage as given. On the contrary, they choose their wages and then supply any quantity of labour that satisfies demand. The optimal labour supply conditions (9) are replaced by the optimal wage setting conditions outlined below.

Cost-push shocks are modelled as exogenous movements in the elasticity of substitution between different types of labour, along the lines of shocks to the elasticity of substitution between varieties of intermediate goods.

6.1 Staggered wage setting and the supply of labour

The household of each country is made up of a continuum of workers indexed on the unit interval, each supplying a differentiated labour service \(j\). Since each type of labour is an imperfect substitute for the others, workers can choose their wage now.

Wages are sticky in a Calvo-Yun fashion: in each period, only a fraction \(1 - \theta_w\) of workers can reset their wage. The rest must keep their existing wage, with no indexation.

6.1.1 Labour packers

Firms demand a homogeneous labour service. Perfectly competitive labour packers (i.e. “contractors” or “employment agencies”) act as aggregators in each country: they purchase dif-
differentiated labour inputs and turn them into a composite labour service. Their goal is to maximise profits subject to a standard CES technology:

$$\max_{n_t(j)} \ w_t n_t - \int_0^1 w_t(j) n_t(j) \ dj \quad \text{s.t. } \ n_t = \left( \int_0^1 n_t(j) \frac{\psi_{t-1}}{\psi_t} \ dj \right) \frac{\psi_t}{\psi_{t-1}}. $$

The elasticities of substitution between different types of labour follow exogenous processes in logs with orthogonal i.i.d. normal innovations:

$$\log \left( \frac{\psi_t}{\psi_{t-1}} \right) = \rho \nu_t + \log \left( \frac{\psi_t}{\psi_{t-1}} \right) + e_{\psi,t},$$

$$\text{corr} \left( e_{\psi,t}, e_{\psi,t}^* \right) = 0.$$

These disturbances make wage markups time-variable and represent cost-push shocks in the labour markets. They are sometimes referred to as “labour supply shocks” in the literature, as their effect on aggregate labour supply resembles that of a shock to the preference for leisure.

They cause further fluctuations in the gap between the natural (i.e. flexible-prices and flexible-wages) allocation and its efficient counterpart, exacerbating the tradeoff faced by the monetary authorities. In fact, as argued below, the natural allocation cannot be achieved any longer if both prices and wages are sticky.

The solution to the problem of labour packers is a set of demand schedules for each type of labour:

$$n_t(j) = \left( \frac{w_t(j)}{w_t} \right)^{-\psi_t} n_t. \quad (24)$$

The aggregate nominal wage index obtains from a zero-profits condition:

$$w_t = \left( \int_0^1 w_t(j)^{1-\psi_t} \ dj \right)^{-1 \psi_t \psi_{t-1}}. \quad (25)$$

### 6.1.2 Workers

Assuming that utility is separable in labour and consumption, the relevant part of the Lagrangian for the optimal wage setting problem of the workers in the home country is as follows:

$$\max_{\bar{w}_t(j)} \mathbb{E}_t \sum_{\tau=0}^\infty (\beta \theta_w)^\tau u(c_{t+\tau}, n_{t+\tau})$$

$$\begin{cases}
p_{t+\tau} c_{t+\tau} + q_{t+\tau} b_{t+\tau+1} = b_{t+\tau} + \int_0^1 \bar{w}_t(j) n_{t+\tau}(j) \ dj \\
\quad + s_k p_{h,t+\tau} \Pi_{h,t+\tau} + s_{fp} f_{t+\tau} \Pi_{f,t+\tau} + t_{t+\tau} \\
n_{t+\tau}(j) = \left( \frac{w_{t+\tau}(j)}{w_t} \right)^{-\psi_t} n_{t+\tau}.
\end{cases}$$
As outlined in Appendix F, the optimal wage setting conditions in a symmetric equilibrium are

\[ f_{h,t}^1 = M_{w,t} f_{h,t}^2, \]

where I have defined the desired "frictionless" wage markup as

\[ M_{w,t} \equiv \left( \frac{\psi_t}{\psi_t - 1} \right) \]

and the recursive auxiliary variables as

\[ f_{h,t}^1 \equiv (\bar{w}_t)^{1-\psi_t} (w_t)^{\psi_t} \lambda_t n_t + \beta \theta_w E_t \left( \frac{1}{\pi_{t+1}} \right)^{1-\psi_t} \left( \frac{w_t}{\bar{w}_{t+1}} \right)^{1-\psi_t} f_{h,t+1}^1, \quad (26) \]

\[ f_{h,t}^2 \equiv -u_n (c_t, n_t) n_t (\bar{w}_t)^{-\psi_t} + \beta \theta_w E_t \left( \frac{1}{\pi_{t+1}} \right)^{-\psi_t} \left( \frac{w_t}{\bar{w}_{t+1}} \right)^{-\psi_t} f_{h,t+1}^2. \quad (27) \]

6.1.3 The dynamics of aggregate wages and wage dispersion

Applying the Calvo-Yun algebra to the wage index (25) and its foreign counterpart, we obtain the laws of motion of wages subject to nominal rigidities:

\[ w_t = \left[ \theta_w (w_{t-1})^{1-\psi_t} + (1 - \theta_w) (\bar{w}_t)^{1-\psi_t} \right]^{\frac{1}{1-\psi_t}}, \]

\[ w_t^* = \left[ \theta_w (w_{t-1}^*)^{1-\psi_t^*} + (1 - \theta_w) (\bar{w}_t^*)^{1-\psi_t^*} \right]^{\frac{1}{1-\psi_t^*}}. \]

Let us define domestic and foreign nominal wage dispersion respectively as

\[ d_{w,t}^d \equiv \int_0^1 \left( \frac{w_t(j)}{w_t} \right)^{-\psi_t} dj, \quad d_{w,t}^f \equiv \int_0^1 \left( \frac{w_t^*(j)}{w_t^*} \right)^{-\psi_t^*} dj. \]

These state variables evolve according to the laws of motion

\[ d_{w,t}^d = \theta_w \left( \frac{1}{\pi_{w,t}} \right)^{-\psi_t} d_{w,t-1}^d + (1 - \theta_w) (\bar{w}_t)^{-\psi_t}, \]

\[ \bar{w}_t = \text{mrs}_t \left( \frac{\psi_t}{\psi_t - 1} \right), \]

where the real cost of supplying labour is given by the marginal rate of substitution between consumption and leisure:

\[ \text{mrs}_t = \frac{u_n (c_t, n_t)}{u_c (c_t, n_t)}. \]
\[
d_{w,t} = \theta_{w} \left( \frac{1}{\pi_{w,t}} \right)^{-\psi_{t}^*} d_{f,t-1}^{w} + (1 - \theta_{w}) (\bar{w}_{t}^*)^{-\psi_{t}^*},
\]
where the domestic and foreign gross wage inflation rates have been defined as

\[
\pi_{w,t} \equiv \frac{w_{t}}{w_{t-1}}, \quad \pi_{w,t}^* \equiv \frac{w^*_{t}}{w^*_{t-1}},
\]

and the domestic and foreign optimal relative wages are

\[
\bar{w}_{t} \equiv \frac{\bar{w}_{t}}{w_{t}}, \quad \bar{w}_{t}^* \equiv \frac{\bar{w}_{t}^*}{w_{t}^*}.
\]

As outlined in Appendix G, the aggregate output levels in this environment are respectively

\[
y_{h,t} = a_t \left( \frac{n_t}{d_{w,h,t}^w d_{h,t}^p} \right)^{1-\alpha} \quad \text{and} \quad y_{f,t} = a_t^* \left( \frac{n_t^*}{d_{w,f,t}^w d_{f,t}^p} \right)^{1-\alpha}.
\]

It is a well-established fact in the literature that the natural allocation cannot be obtained under these conditions, because two distinct sources of inefficiency operate to reduce economic activity: price dispersion and wage dispersion. To restore the level of output that prevails with flexible wages and prices, zero inflation would be needed in both the labour and the goods markets. This, however, would impede the movements of the real wage that are needed to sustain the natural allocation.

Appendix H provides a full list of equilibrium conditions for the economy with sticky wages and prices. Notice that the Taylor rules must prescribe a positive reaction of interest rates to both price and wage inflation now\(^*\).

### 6.2 Equilibrium adjustment to labour supply shocks

I assume that the \(\psi\) processes do not spill over across countries, and I set their steady state to coincide with that of the \(\varepsilon\) processes. The parameters in Table (3) below complement those in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta_{w})</td>
<td>0.75</td>
<td>Wage stickiness parameter</td>
</tr>
<tr>
<td>(\psi)</td>
<td>6</td>
<td>Steady state elasticity of substitution between labour types</td>
</tr>
<tr>
<td>(\rho_{\psi})</td>
<td>0.95</td>
<td>Serial correlation of the log of (\psi) shocks</td>
</tr>
<tr>
<td>(\nu_{\psi})</td>
<td>0</td>
<td>International spillover of (\psi) shocks</td>
</tr>
</tbody>
</table>

\(^*\)The Taylor principle requires that the sum of coefficients on all nominal variables exceed one: \(\gamma_{\pi} + \gamma_{w} > 1\) if \(\gamma_{\psi} = 0\). A lower sum is sufficient for determinacy when \(\gamma_{\psi} > 0\).
The wage stickiness parameter chosen here implies a four-quarters average duration of wage spells, as opposed to a three-quarters average duration of price spells. Although quick and loose, this textbook calibration captures the fact that wages appear to adjust more sluggishly than prices.

Figure 7 displays the impulse responses of selected macroeconomic variables to an inverse cost-push shock to the home country’s labour market, taking the form of a 1% drop in $\psi_t$.

The shock raises real wages and reduces profits in the home country. If external positions are present (blue lines), wealth effects are triggered as usual. If they are absent (red lines), wealth effects do not occur.

The response of real wages is hump-shaped in this economy due to the sluggish adjustment of both nominal wages and prices. As a consequence, aggregate home profits exhibit a hump-shaped dynamics too. In the presence of external positions, wealth effects inherit this pattern, and the responses of foreign consumption, hours and output are hump-shaped accordingly.

Figure 7: Impulse responses to a labour market cost-push shock with and without external positions (blue and red)

6.3 Monetary policy and welfare

If the volatility of cost-push shocks in the labour market is as small as it is in Figure 7, that is $\text{std}(e_{\psi}) = 0.01$, the cost of ignoring output fluctuations is small and no clear monetary policy tradeoff emerges: the control of price and wage inflation remains the sole goal of the monetary authority.

If cost-push shocks are volatile enough, instead, significant departures of the natural level
of output from its efficient counterpart occur, and the stabilisation of economic activity becomes an important additional goal of monetary policy. Table 4 shows what happens to the calibration of the Taylor rules as the size of the external positions varies, under the assumption that \( \text{std}(e_\psi) = \text{std}(e_x) = 0.10 \).

The pattern that emerges is not dissimilar to that observed in Table 2. Welfare is monotonically increasing in both \( \gamma_\pi \) and \( \gamma_w \), so I constrain both coefficients from exceeding 4. The welfare-maximising \( \gamma_y \) stays constant as external positions range between 0% and 50%, and then declines.

Table 4: External positions and Taylor rule parameters in the presence of shocks to \( \psi \)

<table>
<thead>
<tr>
<th>External positions</th>
<th>Optimal ((\gamma_R; \gamma_\pi; \gamma_w; \gamma_y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>(0, 4, 4, 3)</td>
</tr>
<tr>
<td>25%</td>
<td>(0, 4, 4, 3)</td>
</tr>
<tr>
<td>50%</td>
<td>(0, 4, 4, 3)</td>
</tr>
<tr>
<td>75%</td>
<td>(0, 4, 4, 2.5)</td>
</tr>
<tr>
<td>100%</td>
<td>(0, 4, 4, 2)</td>
</tr>
</tbody>
</table>

Whether external positions mitigate or exacerbate the overall volatility of consumption in this model depends on the direction of the wealth effects set into motion by exogenous shocks. This, in turn, depends on the comovement of output and profits, which varies across different types of macroeconomic disturbances.

On one hand, output and profits comove positively after (i) technology shocks and (ii) labour market cost-push shocks. External positions exert a stabilising role on consumption in these cases, because they determine countercyclical wealth effects: resources are transferred to the foreign country when home output and profits are high, and vice-versa.

On the other hand, output and profits comove negatively after (iii) monetary shocks and (iv) goods market cost-push shocks. External positions exert a destabilising role on consumption now, because they determine procyclical wealth effects: resources are transferred to the foreign country when domestic output is low, because this is associated with high domestic profits.

The output stabilisation objective tends to take lower priority when the volatility of output is small relative to that of consumption; strict inflation targeting tends to emerge as the most desirable policy in this case. This is what happens in economies dominated by shocks (iii) and (iv). By the same logic, output stabilisation becomes an important objective of monetary policy in economies dominated by shocks (i) and (ii), where flexible inflation targeting tends to be the welfare-maximising policy.

To confirm this intuition, let us increase the relative importance of technology and labour
cost-push shocks by setting

\[ \text{std}(e_a) = \text{std}(e_\psi) = 0.10, \quad \text{std}(e_m) = \text{std}(e_\varepsilon) = 0.01. \]

Table 5 shows that a greater emphasis on output stabilisation does indeed emerge as external positions get larger, because output becomes more variable than consumption.

### Table 5: External positions and Taylor rule parameters with larger shocks to \( a \) and \( \psi \)

<table>
<thead>
<tr>
<th>External positions</th>
<th>Optimal ( (\gamma_R, \gamma_\pi, \gamma_w, \gamma_y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>( (0, 4, 4, 1) )</td>
</tr>
<tr>
<td>25%</td>
<td>( (0, 4, 4, 1) )</td>
</tr>
<tr>
<td>50%</td>
<td>( (0, 4, 4, 1) )</td>
</tr>
<tr>
<td>75%</td>
<td>( (0, 4, 4, 1.5) )</td>
</tr>
<tr>
<td>100%</td>
<td>( (0, 4, 4, 1.5) )</td>
</tr>
</tbody>
</table>

Comparing Tables 4 and 5, we see that in order to draw conclusions about the most appropriate monetary policy mix, one must take a stance on the relative importance of the different sources of business cycles. This issue is still the object of an ongoing debate.

Smets and Wouters (2007) investigated the sources of business cycle fluctuations in the United States between 1966 and 2004 in the context of a loglinearised DSGE model with both price and wage stickiness. Their Bayesian methodology pointed to the following posterior mean estimates for the volatilities of the shocks:

\[ \text{std}(e_a) = 0.45, \quad \text{std}(e_m) = 0.24, \quad \text{std}(e_\varepsilon) = 0.14, \quad \text{std}(e_\psi) = 0.24. \]

If the exogenous processes of the present model were parameterised according to these estimates, the presence of larger external positions would appear to strengthen the case for output stabilisation. The reason for this is intuitive: technology shocks are the overwhelmingly dominant source of output and consumption fluctuations under this calibration\(^9\), so countercyclical wealth effects prevail. As the volatility of consumption is mitigated and that of output is exacerbated by the presence of external positions, the case for flexible inflation targeting is strengthened.

Two notes of caution are in order at this point. First, the Smets and Wouters posterior estimates imply implausibly large volatilities of macroeconomic variables in the context of the present model. More reasonable volatilities would emerge if we calibrated the shocks using

\(^9\)Domestic technology shocks represent as much as 98% of the variance of output and almost 30% of the variance of consumption in the home country (with the remaining 70% being almost entirely explained by foreign technology shocks).
the Smets and Wouters priors:

\[
\text{std} (e_a) = \text{std} (e_m) = \text{std} (e_\varepsilon) = \text{std} (e_\psi) = 0.10.
\]

In this case, the welfare-maximising monetary policy mix would appear nearly invariant to external positions, due to the flatness of the welfare surface along the \( \gamma_y \) dimension.

Second, maximum likelihood estimates of a standard closed economy New Keynesian model by Ireland (2004) point to monetary disturbances and cost-push shocks in the goods markets as the main drivers of macroeconomic fluctuations, as opposed to technology. For the reasons explained above, procyclical wealth effects would prevail in this case, and larger external positions would require that central banks adopt weaker output reaction coefficients.

7 Concluding remarks

The rise of international financial integration documented by Lane and Milesi-Ferretti (2001, 2006, 2007) has directed attention to the role of external positions in the international transmission of economic shocks. The purpose of the present work is to explore their impact on the conduct of monetary policy, in the context of an explicitly optimisation-based framework.

This paper shows that if external positions are introduced into an otherwise standard two-country New Keynesian model in the form of fixed claims on foreign profits, important international wealth effects materialise following technology, monetary and cost-push shocks. These effects reallocate consumption across countries, and cause macroeconomic adjustment to differ from what would be observed in an economy without financial integration.

The key mechanism at work is as follows. As the comovement of aggregate output and profits varies across distinct kinds of disturbances, so does the direction of wealth effects—because these are tied to dividend payments. The procyclicality or countercyclicality of these transfers, in turn, determines whether consumption is stabilised or destabilised relative to output in the presence of external positions. The implication for monetary policy is that central banks should place more emphasis on output stabilisation in the presence of shocks that cause a positive comovement of output and profits (such as shocks to technology or to the wage markup), and less emphasis on such a goal in the presence of shocks that cause a negative comovement of output and profits (such as shocks to interest rates or to the price markup).

Since the choice of the monetary policy mix depends on the relative importance of different macroeconomic disturbances in the presence of external positions, the model stresses the importance of a correct identification of the sources of business cycles for the conduct of monetary policy in a financially integrated world.
References


Appendices

A - Aggregate production function

Imposing domestic labour market clearing and making use of the production function (3) and the demand schedule (13) we get

\[ n_t = \int_0^1 n_t(i) \, di \]

\[ = \int_0^1 \left[ \frac{y_{h,t}(i)}{a_t} \right]^{\frac{1}{1-\alpha}} \, di \]

\[ = \left( \frac{y_{h,t}}{a_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left( \frac{p_{h,t}(i)}{p_{h,t}} \right)^{-\frac{\alpha}{1-\alpha}} \, di. \]

Define the last integral as in (17) and solve for output to get the aggregate production function (16).

B - Relative marginal cost

Firm \( i \)'s real marginal cost and the economy’s average real marginal cost are respectively

\[ mc_{h,t}(i) = \frac{w_t}{p_{h,t}} \frac{1}{mpn_t(i)}, \quad mc_{h,t} = \frac{w_t}{p_{h,t}} \frac{1}{mpn_t}. \]

Working out the individual and average marginal products of labour from equations (3) and (16), we find

\[ \frac{mc_{h,t}(i)}{mc_{h,t}} = \frac{mpn_t}{mpn_t(i)} \]

\[ = \left( \frac{1}{d_{h,t}} \right)^{1-\alpha} \left( \frac{n_t(i)}{n_t} \right)^{\alpha} \]

\[ = \frac{1}{d_{h,t}} \left( \frac{y_{h,t}(i)}{y_{h,t}} \right)^{\frac{\alpha}{1-\alpha}} \]

\[ = \frac{1}{d_{h,t}} \left( \frac{p_{h,t}(i)}{p_{h,t}} \right)^{-\frac{\alpha}{1-\alpha}}. \]

Notice that the real marginal cost becomes constant across firms under constant returns to scale (\( \alpha = 0 \)):

\[ mc_{h,t}(i) = \frac{mc_{h,t}}{d_{h,t}}. \]
C - Aggregate profits

Under the production function specification (3), the total cost of production of a given variety \( i \) in the home country is

\[
\Psi (y_{h,t} (i)) = \frac{w_t}{p_{h,t}} n_t (i)
\]

\[
= \frac{w_t}{p_{h,t}} \left( \frac{y_{h,t} (i)}{a_t} \right)^{\frac{1}{1-\alpha}}
\]

\[
\Psi (p_{h,t} (i)) = \frac{w_t}{p_{h,t}} \left( \frac{y_{h,t} (i)}{a_t} \right)^{\frac{1}{1-\alpha}} \left( \frac{p_{h,t} (i)}{p_{h,t}} \right)^{-\frac{\varepsilon_t}{1-\alpha}}.
\]

The total aggregate profits of home firms are decreasing in the dispersion of prices:

\[
\Pi_{h,t} \equiv \int_0^1 \Pi_{h,t} (i) \, di
\]

\[
= \int_0^1 y_{h,t} (i) \frac{p_{h,t} (i)}{p_{h,t}} - \Psi (y_{h,t} (i)) \, di
\]

\[
= y_{h,t} \int_0^1 \left( \frac{p_{h,t} (i)}{p_{h,t}} \right)^{1-\varepsilon_t} \, di - \frac{w_t}{p_{h,t}} \int_0^1 n_t (i) \, di
\]

\[
= y_{h,t} - \frac{w_t}{p_{h,t}} \left( \frac{y_{h,t}}{a_t} \right)^{\frac{1}{1-\alpha}} d_{h,t}.
\]

Likewise, the total cost of production in the foreign country using equation (4) is

\[
\Psi (p_{f,t} (i)) = \frac{w^*_t}{p_{f,t}} \left( \frac{y_{f,t}}{a^*_t} \right)^{\frac{1}{1-\alpha}} \left( \frac{p_{f,t} (i)}{p_{f,t}} \right)^{-\frac{\varepsilon_t}{1-\alpha}}
\]

and the total aggregate profits of foreign firms are

\[
\Pi_{f,t} \equiv \int_0^1 \Pi_{f,t} (i) \, di
\]

\[
= y_{f,t} - \frac{w^*_t}{p_{f,t}} \left( \frac{y_{f,t}}{a^*_t} \right)^{\frac{1}{1-\alpha}} d_{f,t}.
\]

D - Optimal price setting

By direct substitution of the constraints in the price setter’s objective function, the problem becomes

\[
\max_{p_{h,t}} \mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_{p/\beta})^\tau \left( \frac{\lambda_{t+\tau}}{\lambda_t} \right) \left\{ y_{h,t+\tau} \left( \frac{\bar{p}_{h,t}}{p_{h,t+\tau}} \right)^{1-\varepsilon_t} - \Psi (y_{h,t+\tau} (i)) \right\}.
\]
The FOCs are
\[
\begin{align*}
E_t \sum_{\tau=0}^{\infty} (\theta_p^t)^\tau \left( \frac{\lambda_{t+\tau}}{\lambda_t} \right) & \left\{ (1 - \varepsilon_t) \frac{y_{h,t+\tau}}{p_{h,t+\tau}} \left( \frac{\bar{p}_{h,t}}{p_{h,t+\tau}} \right)^{-\varepsilon_t} \right\} \\
= E_t \sum_{\tau=0}^{\infty} (\theta_p^t)^\tau \left( \frac{\lambda_{t+\tau}}{\lambda_t} \right) & \left\{ m_{ch,t+\tau} \left( i \right) \left( -\varepsilon_t \right) \frac{y_{h,t+\tau}}{p_{h,t+\tau}} \left( \frac{\bar{p}_{h,t}}{p_{h,t+\tau}} \right)^{-\varepsilon_t-1} \right\}.
\end{align*}
\]

We can rewrite these conditions in terms of the economy-wide marginal cost using (18):
\[
\begin{align*}
E_t \sum_{\tau=0}^{\infty} (\theta_p^t)^\tau \left( \frac{\lambda_{t+\tau}}{\lambda_t} \right) & \left\{ \frac{mc_{h,t+\tau}}{d_{h,t+\tau}} \left( \frac{\varepsilon_t}{\varepsilon_t - 1} \right) \frac{y_{h,t+\tau}}{p_{h,t+\tau}} \left( \frac{\bar{p}_{h,t}}{p_{h,t+\tau}} \right)^{\frac{\alpha - 1 - \varepsilon_t}{1 - \alpha}} \right\}.
\end{align*}
\]

If we define auxiliary variables
\[
\begin{align*}
g^2_{h,t} & \equiv E_t \sum_{\tau=0}^{\infty} (\theta_p^t)^\tau \left( \frac{\lambda_{t+\tau}}{\lambda_t} \right) \left\{ \frac{y_{h,t+\tau}}{p_{h,t+\tau}} \left( \frac{\bar{p}_{h,t}}{p_{h,t+\tau}} \right)^{-\varepsilon_t} \right\}, \\
g^1_{h,t} & \equiv E_t \sum_{\tau=0}^{\infty} (\theta_p^t)^\tau \left( \frac{\lambda_{t+\tau}}{\lambda_t} \right) \left\{ \frac{mc_{h,t+\tau}}{d_{h,t+\tau}} \left( \frac{\varepsilon_t}{\varepsilon_t - 1} \right) \frac{y_{h,t+\tau}}{p_{h,t+\tau}} \left( \frac{\bar{p}_{h,t}}{p_{h,t+\tau}} \right)^{\frac{\alpha - 1 - \varepsilon_t}{1 - \alpha}} \right\},
\end{align*}
\]
we can rewrite the FOCs compactly as
\[
g^2_{h,t} = \left( \frac{\varepsilon_t}{\varepsilon_t - 1} \right) g^1_{h,t}.
\]

Additional manipulation of these auxiliary variables yields the recursive equations (21) and (22). Similar conditions hold for the pricing of foreign goods.

**E - Equilibrium conditions in real terms**

Since nominal variables are not uniquely determined in the present model, we must rewrite the entire system of equilibrium conditions in terms of real variables and relative prices.

Keeping the definitions of optimal relative prices stated in (19), we define *real wages* as
\[
w_t \equiv \frac{w_t^*}{p_t}, \quad w_t^* \equiv \frac{w_t^*}{p_t^*},
\]
and we rewrite prices in terms of *PPI-to-CPI ratios*:
\[
\mathcal{P}_{h,t} \equiv \frac{p_{h,t}}{p_t}, \quad \mathcal{P}_{f,t} \equiv \frac{p_{f,t}}{p_t}, \quad \mathcal{P}^*_h \equiv \frac{p_{h,t}}{p_t^*}, \quad \mathcal{P}^*_f \equiv \frac{p_{f,t}}{p_t^*}.
\]
Then, we rewrite these as functions of \( s_t \) only:

\[
\mathcal{P}_{h,t} = \left[ (1 - \zeta) + (\zeta) (s_t)^{1-\eta} \right]^{\frac{1}{\eta-1}},
\]

\[
\mathcal{P}_{f,t} = \left[ (1 - \zeta) \left( \frac{1}{s_t} \right)^{1-\eta} + (\zeta) \right]^{\frac{1}{\eta-1}},
\]

\[
\mathcal{P}_{h,t}^* = \left[ (1 - \zeta) (s_t)^{1-\eta} + (\zeta) \right]^{\frac{1}{\eta-1}},
\]

\[
\mathcal{P}_{f,t}^* = \left[ (1 - \zeta) + (\zeta) \left( \frac{1}{s_t} \right)^{1-\eta} \right]^{\frac{1}{\eta-1}}.
\]

The definitions of the terms of trade and the CPIs are embedded in these four price ratios and need not enter the equilibrium conditions again. The remaining conditions are as follows:

\[
c_{h,t} = (1 - \zeta) (\mathcal{P}_{h,t})^{-\eta} c_t,
\]

\[
c_{f,t} = (\zeta) (\mathcal{P}_{f,t})^{-\eta} c_t,
\]

\[
c_{f,t}^* = (1 - \zeta) \left( \mathcal{P}_{f,t}^* \right)^{-\eta} c_t^*,
\]

\[
c_{h,t}^* = (\zeta) \left( \mathcal{P}_{h,t}^* \right)^{-\eta} c_t^*.
\]

\[
y_{h,t} = c_{h,t} + c_{h,t}^*,
\]

\[
y_{f,t} = c_{f,t} + c_{f,t}^*,
\]

\[
u_c(c_t, n_t) = \lambda_t,
\]

\[
u_n(c_t, n_t) = \lambda_t,
\]

\[
\frac{w_t}{\lambda_t} = \lambda_t,
\]

\[
q_t = \beta \mathcal{E}_t \left( \frac{\lambda_{t+1} + \frac{p_t}{\lambda_t} p_t}{p_t} \right),
\]

\[
c_t + \frac{q_t}{p_t} b_{t+1} = \frac{b_t}{p_t} + w_t n_t + s_h \mathcal{P}_{h,t} \Pi_{h,t} + s_f \mathcal{P}_{f,t} \Pi_{f,t} + \frac{t_t}{p_t},
\]

\[
u_c(c_t^*, n_t^*) = \lambda_t^*,
\]

\[
u_n(c_t^*, n_t^*) = \lambda_t^*,
\]

\[
\frac{w_t^*}{\lambda_t^*} = \lambda_t^*,
\]

\[
q_t^* = \beta \mathcal{E}_t \left( \frac{\lambda_{t+1}^* + \frac{p_t^*}{\lambda_t^*} p_t^*}{p_t^*} \right),
\]

\[
c_t^* + \frac{q_t^*}{p_t^*} b_{t+1} = \frac{b_t^*}{p_t^*} + w_t^* n_t^* + (1 - s_h) \mathcal{P}_{h,t}^* \Pi_{h,t} + (1 - s_f) \mathcal{P}_{f,t}^* \Pi_{f,t} + \frac{t_t^*}{p_t^*},
\]
\[ y_{h,t} = a_t \left( \frac{n_t}{d_{h,t}} \right)^{1-\alpha}, \]
\[ y_{f,t} = a_t^* \left( \frac{n_t^*}{d_{f,t}} \right)^{1-\alpha}, \]
\[ m_{c,h,t} = \frac{w_t}{\mathcal{P}_{h,t} (1 - \alpha) a_t} (d_{h,t})^{-\alpha} (n_t)^\alpha, \]
\[ m_{c,f,t} = \frac{w_t^*}{\mathcal{P}_{f,t}^* (1 - \alpha) a_t^*} (d_{f,t})^{-\alpha} (n_t^*)^\alpha, \]
\[ d_{h,t} = \theta_p (\pi_{h,t})^{\frac{\epsilon_t}{1+\alpha}} d_{h,t-1} + (1 - \theta_p) (\bar{p}_{h,t})^{-\frac{\epsilon_t}{1+\alpha}}, \]
\[ d_{f,t} = \theta_p (\pi_{f,t})^{\frac{\epsilon_t^*}{1+\alpha}} d_{f,t-1} + (1 - \theta_p) (\bar{p}_{f,t})^{-\frac{\epsilon_t^*}{1+\alpha}}, \]
\[ \bar{p}_{h,t} = \left[ 1 - \theta_p (\pi_{h,t})^{\epsilon_t - 1} \right] \frac{1}{1 - \theta_p}, \]
\[ \bar{p}_{f,t} = \left[ 1 - \theta_p (\pi_{f,t})^{\epsilon_t^* - 1} \right] \frac{1}{1 - \theta_p}, \]
\[ \pi_{h,t} = \frac{\mathcal{P}_{h,t}}{\mathcal{P}_{h,t-1}} \pi_t, \]
\[ \pi_{f,t} = \frac{\mathcal{P}_{f,t}}{\mathcal{P}_{f,t-1}} \pi_t^*, \]
\[ g_{h,t}^1 = \mathcal{M}_{p,t} g_{h,t}^1, \]
\[ \mathcal{M}_{p,t} = \frac{\epsilon_t}{\epsilon_t - 1}, \]
\[ g_{h,t}^1 = y_{h,t} \frac{m_{c,h,t}}{d_{h,t}} (\bar{p}_{h,t})^{\frac{\alpha-1-\epsilon_t}{1+\alpha}} + \theta_p \beta \mathbb{E}_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \left( \frac{\bar{p}_{h,t}}{\bar{p}_{h,t+1}} \right)^{\frac{\alpha-1-\epsilon_t}{1+\alpha}} (\pi_{h,t+1})^{\frac{\epsilon_t}{1+\alpha}} g_{h,t+1}^1, \]
\[ g_{h,t}^2 = y_{h,t} (\bar{p}_{h,t})^{-\epsilon_t} + \theta_p \beta \mathbb{E}_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \left( \frac{\bar{p}_{h,t}}{\bar{p}_{h,t+1}} \right)^{-\epsilon_t} (\pi_{h,t+1})^{\frac{\epsilon_t}{1+\alpha}} g_{h,t+1}^2, \]
\[ g_{f,t}^1 = \mathcal{M}_{p,t}^* g_{f,t}^1, \]
\[ \mathcal{M}_{p,t}^* = \frac{\epsilon_t^*}{\epsilon_t^* - 1}, \]
\[ g_{f,t}^1 = y_{f,t} \frac{m_{c,f,t}}{d_{f,t}} (\bar{p}_{f,t})^{\frac{\alpha-1-\epsilon_t^*}{1+\alpha}} + \theta_p \beta \mathbb{E}_t \left( \frac{\lambda_{t+1}^*}{\lambda_t^*} \right) \left( \frac{\bar{p}_{f,t}}{\bar{p}_{f,t+1}} \right)^{\frac{\alpha-1-\epsilon_t^*}{1+\alpha}} (\pi_{f,t+1})^{\frac{\epsilon_t^*}{1+\alpha}} g_{f,t+1}^1, \]
\[ g_{f,t}^2 = y_{f,t} (\tilde{p}_{f,t})^{-\varepsilon_t^*} + \theta_p \beta \mathbb{E}_t \left( \frac{\lambda_{t+1}^f}{\lambda_t^f} \right)^{-\varepsilon_t^*} (\pi_{f,t+1})^{\varepsilon_{t+1}^* - 1} g_{f,t+1}^2, \]

\[ R_t = \frac{1}{q_t}, \]

\[ R_t^* = \frac{1}{q_t^*}, \]

\[ \frac{R_t}{\pi/\beta} = \left( \frac{R_{t-1}}{\pi/\beta} \right)^{\gamma_R} \left[ \left( \frac{\pi_{h,t}}{\pi_{h}} \right)^{\gamma_R} \left( \frac{y_{h,t}}{y_{h,t-1}} \right)^{\gamma_R} \right]^{1-\gamma_R} m_t, \]

\[ \frac{R_t^*}{\pi^*/\beta} = \left( \frac{R_{t-1}^*}{\pi^*/\beta} \right)^{\gamma_R} \left[ \left( \frac{\pi_{f,t}}{\pi_{f}} \right)^{\gamma_R} \left( \frac{y_{f,t}}{y_{f,t-1}} \right)^{\gamma_R} \right]^{1-\gamma_R} m_t^*, \]

\[ \log m_t = \rho_m \log m_{t-1} + \nu_m \log m_{t-1}^* + e_{m,t}, \]

\[ \log m_t^* = \rho_m \log m_{t-1}^* + \nu_m \log m_{t-1}^* + e_{m,t}^*, \]

\[ \log a_t = \rho_a \log a_{t-1} + \nu_a \log a_{t-1}^* + e_{a,t}, \]

\[ \log a_t^* = \rho_a \log a_{t-1}^* + \nu_a \log a_{t-1}^* + e_{a,t}^*, \]

\[ \log \varepsilon_t = \rho_{\varepsilon} \log \varepsilon_{t-1} + \nu_{\varepsilon} \log \varepsilon_{t-1}^* + e_{\varepsilon,t}, \]

\[ \log \varepsilon_t^* = \rho_{\varepsilon} \log \varepsilon_{t-1}^* + \nu_{\varepsilon} \log \varepsilon_{t-1}^* + e_{\varepsilon,t}^*, \]

\[ t_t = q_t b_{t+1} - b_t, \]

\[ t_t^* = q_t^* b_{t+1}^* - b_t^*, \]

\[ b_t = 0, \]

\[ b_t^* = 0, \]

\[ \Pi_{h,t} = y_{h,t} - \frac{w_t}{\partial_{h,t}} \left( \frac{y_{h,t}}{a_t} \right)^{\frac{1}{1-\alpha}} d_{h,t}, \]

\[ \Pi_{f,t} = y_{f,t} - \frac{w_t^*}{\partial_{f,t}^*} \left( \frac{y_{f,t}}{a_t^*} \right)^{\frac{1}{1-\alpha}} d_{f,t}, \]

\[ V_{h,t} = u(e_t, n_t) + \beta \mathbb{E}_t V_{h,t+1}, \]

\[ V_{f,t} = u(e_t^*, n_t^*) + \beta \mathbb{E}_t V_{f,t+1}. \]

**F - Optimal wage setting**

Since nominal variables are not uniquely defined, the optimal wage setting problem must be rewritten in terms of real variables.
To begin with, let us write the labour demand schedule (24), the aggregate wage index and the household’s budget constraint in real terms:

\[ n_t(j) = \left( \frac{w_t(j)}{w_t} \right)^{-\psi_t} n_t, \]

\[ w_t = \left( \int_0^1 w_t(j)^{1-\psi_t} \, dj \right)^{\frac{1}{1-\psi_t}}, \]

\[ c_{t+\tau} + \frac{q_{t+\tau} b_{t+\tau+1}}{p_{t+\tau}} = \frac{b_{t+\tau}}{p_{t+\tau}} + \int_0^1 \bar{w}_t(j) \frac{n_{t+\tau}(j)}{\bar{w}_{t+\tau}} \, dj \]

\[ + s_h \frac{p_{h,t+\tau}}{p_{t+\tau}} \Pi_{h,t+\tau} + s_f \frac{p_{f,t+\tau}}{p_{t+\tau}} \Pi_{f,t+\tau} + \frac{t_{t+\tau}}{p_{t+\tau}}. \]  

(28)

Since workers supply any quantity of labour that satisfies demand at the wage charged, the hours worked at time \( t+\tau \) by a worker who has been unable to reset his wage since time \( t \) are as follows:

\[ n_{t+\tau}(j) = \left( \frac{\bar{w}_t(j)}{\bar{w}_{t+\tau}} \right)^{-\psi_t} n_{t+\tau} \]

\[ = \left( \frac{\bar{w}_t(j) \prod_{s=1}^{\tau} \frac{1}{\pi_{s+s}}}{\bar{w}_{t+\tau}} \right)^{-\psi_t} n_{t+\tau}. \]  

(29)

By direct substitution of (29) into (28) we obtain the Lagrangian of the wage setting problem:

\[ \mathcal{L} = \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta \theta_w)^\tau u(c_{t+\tau}, n_{t+\tau}) \]

\[ + (\beta \theta_w)^\tau \lambda_{t+\tau} \left( \frac{b_{t+\tau}}{p_{t+\tau}} + \left( \prod_{s=1}^{\tau} \frac{1}{\pi_{t+s}} \right)^{1-\psi_t} \int_0^1 \bar{w}_t(j)^{1-\psi_t} n_{t+\tau}(\bar{w}_{t+\tau})^{\psi_t} \, dj \right) \]

\[ + s_h \mathcal{P}_{h,t+\tau} \Pi_{h,t+\tau} + s_f \mathcal{P}_{f,t+\tau} \Pi_{f,t+\tau} + \frac{t_{t+\tau}}{p_{t+\tau}} - c_{t+\tau} - \frac{q_{t+\tau} b_{t+\tau+1}}{p_{t+\tau}}. \]

Assuming full consumption risk sharing across workers, the cost of supplying work is identical across labour types. Since the labour demand schedule is the same across labour types, we can focus on a symmetric equilibrium where all resetters choose the same wage \( \bar{w}_t \).

The first-order conditions are

\[ \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta \theta_w)^\tau u_n(c_{t+\tau}, n_{t+\tau}) n_{t+\tau} (-\psi_t) (\bar{w}_{t+\tau})^{\psi_t} \left( \bar{w}_t \right)^{-\psi_t-1} \left( \prod_{s=1}^{\tau} \frac{1}{\pi_{t+s}} \right)^{-\psi_t} \]

\[ + \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta \theta_w)^\tau \lambda_{t+\tau} \left( \prod_{s=1}^{\tau} \frac{1}{\pi_{t+s}} \right)^{1-\psi_t} n_{t+\tau} (\bar{w}_{t+\tau})^{\psi_t} (1-\psi_t) \left( \bar{w}_t \right)^{-\psi_t} = 0. \]
These can be rewritten compactly by means of auxiliary variables:

\[ f_{1h,t}^1 \equiv \bar{w}_t \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta \theta_w)^\tau \lambda_{t+\tau} \left( \prod_{s=1}^{\tau} \frac{1}{\pi_{t+s}} \right) n_{t+\tau} \left( \frac{\bar{w}_t}{w_{t+\tau}} \right)^{-\psi_t}, \]

\[ f_{1h,t}^2 \equiv \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta \theta_w)^\tau u_n (c_{t+\tau}, n_{t+\tau}) \left( \prod_{s=1}^{\tau} \frac{1}{\pi_{t+s}} \right)^{-\psi_t} n_{t+\tau} \left( \frac{\bar{w}_t}{w_{t+\tau}} \right)^{-\psi_t}, \]

\[ f_{1h,t}^1 = \left( \frac{\psi_t}{\psi_t - 1} \right) f_{1h,t}^2. \]

Additional manipulation yields equations (26) and (27). Analogous conditions hold abroad.

**G - Aggregation under sticky wages: technology, hours and output**

The aggregate supply of labour is found by integrating the hours purchased by employment agencies over labour types \( j \):

\[ n^s_t = \int_0^1 n_t (j) \, dj = \int_0^1 \left( \frac{w_t (j)}{w_t} \right)^{-\psi_t} n^d_t \, dj = d_{h,t}^w n^d_t. \]

Notice that \( n^s_t \) depends on the aggregation technology adopted by the labour packers, and it includes a first source of inefficiency: nominal wages dispersion.

The aggregate demand for labour, in turn, is found by integrating individual demands for composite labour services over intermediate goods producers \( i \):

\[ n^d_t = \int_0^1 n^d_t (i) \, di = \int_0^1 \left( \frac{y_{h,t}}{a_t} \right)^{\frac{1}{1-\alpha}} a_t \, di = \left( \frac{y_{h,t}}{a_t} \right)^{\frac{1}{1-\alpha}} d_{h,t}^p. \]

\( n^d_t \) depends on the aggregation technology adopted by the producers of final goods, and includes a second source of inefficiency: price dispersion across varieties of intermediate goods.

Imposing the equality of labour demand and supply, we obtain the exact relationship between aggregate output, employment and technology in this environment:

\[ n_t = d_{h,t}^w \left( \frac{y_{h,t}}{a_t} \right)^{\frac{1}{1-\alpha}} d_{h,t}^p. \]
Accordingly, we make use of the following relative marginal cost relationship to rewrite the optimal price setting problem of the firms in terms of aggregate variables:

\[ \frac{mc_{h,t}(i)}{mc_{h,t}} = \frac{1}{d_{w,h,t} d_{p,h,t}} \left( \frac{p_{h,t}(i)}{p_{h,t}} \right)^{-\frac{\psi_t}{\psi_2}}. \]

H - Equilibrium conditions under sticky wages

Keeping the definitions of optimal relative prices, real wages and PPI-to-CPI ratios stated in Appendix E, we get the following set of equilibrium conditions written in real terms:

\[ \mathcal{P}_{h,t} = \left[ (1 - \zeta) + (\zeta) (s_t)^{1-\eta} \right] \frac{1}{\pi_t}, \]

\[ \mathcal{P}_{f,t} = \left[ (1 - \zeta) \left( \frac{1}{s_t} \right)^{1-\eta} + (\zeta) \right] \frac{1}{\eta - \frac{1}{\psi_t}}, \]

\[ \mathcal{P}_{h,t}^* = \left[ (1 - \zeta) (s_t)^{1-\eta} + (\zeta) \right] \frac{1}{\pi_t}, \]

\[ \mathcal{P}_{f,t}^* = \left[ (1 - \zeta) + (\zeta) \left( \frac{1}{s_t} \right)^{1-\eta} \right] \frac{1}{\eta - \frac{1}{\psi_t}}, \]

\[ c_{h,t} = \frac{1 - \zeta}{\mathcal{P}_{h,t}} c_t, \]

\[ c_{f,t} = \frac{(1 - \zeta) \mathcal{P}_{f,t}}{\mathcal{P}_{f,t}} c_t, \]

\[ c_{f,t}^* = \frac{(1 - \zeta) \mathcal{P}_{f,t}^*}{\mathcal{P}_{f,t}^*} c_t^*, \]

\[ c_{h,t}^* = \frac{(1 - \zeta) \mathcal{P}_{h,t}^*}{\mathcal{P}_{h,t}^*} c_t^*, \]

\[ y_{h,t} = c_{h,t} + c_{h,t}^*, \]

\[ y_{f,t} = c_{f,t} + c_{f,t}^*, \]

\[ u_c(c_t,n_t) = \lambda_t, \]

\[ f_{h,t}^1 = \mathcal{M}_{w,t} f_{h,t}^2, \]

\[ f_{h,t}^1 = \bar{w}_t \left( \frac{\bar{w}_t}{\bar{w}_t} \right)^{\psi_t} \lambda_t n_t + \beta \theta_w \bar{E}_t (\pi_{t+1})^{\psi_t-1} \left( \frac{\bar{w}_t}{\bar{w}_{t+1}} \right)^{1-\psi_t} f_{h,t+1}^1, \]

\[ f_{h,t}^2 = -u_n(c_t,n_t) n_t \left( \frac{\bar{w}_t}{\bar{w}_t} \right)^{-\psi_t} + \beta \theta_w \bar{E}_t (\pi_{t+1})^{\psi_t} \left( \frac{\bar{w}_t}{\bar{w}_{t+1}} \right)^{-\psi_t} f_{h,t+1}^2, \]

\[ \mathcal{M}_{w,t} = \frac{\psi_t}{\psi_2 - 1}, \]
\[ q_t = \beta \mathbb{E}_t \left( \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}} \right), \]

\[ c_t + \frac{q_t}{p_t} b_{t+1} = \frac{b_t}{p_t} + w_t n_t + s_h \mathcal{P}_{h,t} \Pi_{h,t} + s_f \mathcal{P}_{f,t} \Pi_{f,t} + \frac{t_t}{p_t}, \]

\[ u_c(c_t^*, n_t^*) = \lambda_t^*, \]

\[ f_{f,t}^1 = M_{w,t} f_{f,t}^2, \]

\[ f_{f,t}^1 = \bar{w}_t^* \left( \frac{w_{t}}{w_t^*} \right)^{\psi_t^*} \lambda_t^* n_t^* + \beta \theta_w \mathbb{E}_t (\pi_{t+1}^*)^{\psi_t^* - 1} \left( \frac{\bar{w}_t^*}{w_{t+1}^*} \right)^{1 - \psi_t^*} f_{f,t+1}^1, \]

\[ f_{f,t}^2 = -u_n(c_t^*, n_t^*) n_t^* \left( \frac{\bar{w}_t^*}{w_{t+1}^*} \right)^{-\psi_t^*} + \beta \theta_w \mathbb{E}_t (\pi_{t+1}^*)^{\psi_t^*} \left( \frac{\bar{w}_t^*}{w_{t+1}^*} \right)^{-\psi_t^*} f_{f,t+1}^2, \]

\[ M_{w,t}^* = \frac{\psi_t^*}{\psi_t^* - 1}, \]

\[ q_t^* = \beta \mathbb{E}_t \left( \frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{1}{\pi_{t+1}^*} \right), \]

\[ c_t^* + \frac{q_t^*}{p_t^*} b_{t+1}^* = \frac{b_t^*}{p_t^*} + w_t^* n_t^* + (1 - s_h) \mathcal{P}_{h,t}^* \Pi_{h,t} + (1 - s_f) \mathcal{P}_{f,t}^* \Pi_{f,t} + \frac{t_t^*}{p_t^*}, \]

\[ y_{h,t} = a_t \left( \frac{n_t}{d_{w,h,t}^w d_{h,t}^w} \right)^{1 - \alpha}, \]

\[ y_{f,t} = a_t^* \left( \frac{n_t^*}{d_{w,f,t}^w d_{f,t}^w} \right)^{1 - \alpha}, \]

\[ m_{ch,t} = \frac{w_t}{\mathcal{P}_{h,t} (1 - \alpha) a_t} \left( d_{h,t}^{w,h} \right)^{1-\alpha} (n_t)^\alpha, \]

\[ m_{cf,t} = \frac{w_t^*}{\mathcal{P}_{f,t}^* (1 - \alpha) a_t^*} \left( d_{f,t}^{w,f} \right)^{1-\alpha} (n_t^*)^\alpha, \]

\[ d_{h,t}^w = \theta_w \left( \frac{w_{t-1}}{w_t} \right)^{-\psi_t} \left( \pi_t \right)^{\psi_t} d_{h,t-1}^w + (1 - \theta_w) \left( \frac{\bar{w}_t}{w_t} \right)^{1 - \psi_t}, \]

\[ d_{f,t}^w = \theta_w \left( \frac{w_{t-1}}{w_t^*} \right)^{-\psi_t^*} \left( \pi_t^* \right)^{\psi_t^*} d_{f,t-1}^w + (1 - \theta_w) \left( \frac{\bar{w}_t^*}{w_t^*} \right)^{1 - \psi_t^*}, \]

\[ \frac{\bar{w}_t}{w_t} = \left[ \frac{1 - \theta_w (\pi_{w,t} \pi_t)^{\psi_t - 1}}{1 - \theta_w} \right]^{\frac{1}{1 - \psi_t}}, \]

\[ \frac{\bar{w}_t^*}{w_t^*} = \left[ \frac{1 - \theta_w (\pi_{w,t}^* \pi_t^*)^{\psi_t^* - 1}}{1 - \theta_w} \right]^{\frac{1}{1 - \psi_t^*}}. \]
\[
d_{h,t} = \theta_p (\pi_{h,t})^{\frac{\varepsilon_t}{1-\alpha}} d_{h,t-1} + (1 - \theta_p) (\tilde{p}_{h,t})^{-\frac{\varepsilon_t}{1-\alpha}},
\]
\[
d_{f,t} = \theta_p (\pi_{f,t})^{\frac{\varepsilon_t^*}{1-\alpha}} d_{f,t-1} + (1 - \theta_p) (\tilde{p}_{f,t})^{-\frac{\varepsilon_t^*}{1-\alpha}},
\]
\[
\tilde{p}_{h,t} = \left[ \frac{1 - \theta_p (\pi_{h,t})^{\varepsilon_t-1}} {1 - \theta_p} \right]^{\frac{1}{1-\varepsilon_t}},
\]
\[
\tilde{p}_{f,t} = \left[ \frac{1 - \theta_p (\pi_{f,t})^{\varepsilon_t^*-1}} {1 - \theta_p} \right]^{\frac{1}{1-\varepsilon_t^*}},
\]
\[
\pi_{h,t} = \frac{\mathcal{P}_{h,t}} {\mathcal{P}_{h,t-1}} \pi_{t},
\]
\[
\pi_{f,t} = \frac{\mathcal{P}_{f,t}} {\mathcal{P}_{f,t-1}} \pi_{t}^*,
\]
\[
g_{h,t}^2 = M_{p,t} g_{1,h,t}^1,
\]
\[
g_{h,t}^1 = y_{h,t} \frac{m c_{h,t}} {d_{h,t} d_{h,t}^p} (\tilde{p}_{h,t})^{\frac{\alpha-1-\varepsilon_t}{1-\alpha}} + \theta_p \beta E_t \left( \frac{\lambda_{t+1}} {\lambda_t} \right) \left( \frac{\tilde{p}_{h,t}} {\tilde{p}_{h,t+1}} \right)^{\frac{\alpha-1-\varepsilon_t}{1-\alpha}} (\pi_{h,t+1})^{\frac{\varepsilon_t}{1-\alpha}} g_{h,t+1}^1,
\]
\[
g_{h,t}^2 = y_{h,t} (\tilde{p}_{h,t})^{-\varepsilon_t} + \theta_p \beta E_t \left( \frac{\lambda_{t+1}} {\lambda_t} \right) \left( \frac{\tilde{p}_{h,t}} {\tilde{p}_{h,t+1}} \right)^{-\varepsilon_t} (\pi_{h,t+1})^{\varepsilon_t-1} g_{h,t+1}^2,
\]
\[
\mathcal{M}_{p,t} = \frac{\varepsilon_t}{\varepsilon_t - 1},
\]
\[
g_{f,t}^2 = M_{f,t}^* g_{1,f,t}^1,
\]
\[
g_{f,t}^1 = y_{f,t} \frac{m c_{f,t}} {d_{f,t} d_{f,t}^p} (\tilde{p}_{f,t})^{\frac{\alpha-1-\varepsilon_t^*}{1-\alpha}} + \theta_p \beta E_t \left( \frac{\lambda_{t+1}^*} {\lambda_t^*} \right) \left( \frac{\tilde{p}_{f,t}} {\tilde{p}_{f,t+1}} \right)^{\frac{\alpha-1-\varepsilon_t^*}{1-\alpha}} (\pi_{f,t+1})^{\frac{\varepsilon_t^*}{1-\alpha}} g_{f,t+1}^1,
\]
\[
g_{f,t}^2 = y_{f,t} (\tilde{p}_{f,t})^{-\varepsilon_t^*} + \theta_p \beta E_t \left( \frac{\lambda_{t+1}^*} {\lambda_t^*} \right) \left( \frac{\tilde{p}_{f,t}} {\tilde{p}_{f,t+1}} \right)^{-\varepsilon_t^*} (\pi_{f,t+1})^{\varepsilon_t^*-1} g_{f,t+1}^2,
\]
\[
\mathcal{M}_{f,t}^* = \frac{\varepsilon_t^*}{\varepsilon_t^* - 1},
\]
\[
R_t = \frac{1}{q_t},
\]
\[
R_t^* = \frac{1}{q_t^*},
\]
\[
\frac{R_t}{\pi / \beta} = \left( \frac{R_{t-1}} {\pi / \beta} \right)^{\gamma_R} \left[ \left( \frac{\pi_{h,t}} {\pi_h} \right)^{\gamma_w} \left( \frac{\pi_{w,t}} {\pi_w} \right)^{\gamma_w} \left( \frac{y_{h,t}} {y_{h,t-1}} \right)^{\gamma_y} \right]^{1-\gamma_R} m_t,
\]
\[
\frac{R_t^*}{\pi^*/\beta} = \left( \frac{R_{t-1}^*}{\pi^*/\beta} \right)^{\gamma_R} \left[ \left( \frac{\pi^*_{f,t}}{\pi^*_f} \right)^{\gamma_f} \left( \frac{\pi^*_{w,t}}{\pi^*_w} \right)^{\gamma_w} \left( \frac{y_{f,t}}{y_{f,t-1}} \right)^{\gamma_y} \right]^{1-\gamma_R} m_t^*,
\]

\[
\pi_{w,t} = \frac{w_{t-1}}{w_t} \frac{1}{\pi_t},
\]

\[
\pi^*_{w,t} = \frac{w_{t-1}}{w_t^*} \frac{1}{\pi^*_t},
\]

\[
\log m_t = \rho_m \log m_{t-1} + \nu_m \log m^*_{t-1} + e_{m,t},
\]

\[
\log m^*_t = \rho_m \log m^*_{t-1} + \nu_m \log m_{t-1} + e^*_{m,t},
\]

\[
\log a_t = \rho_a \log a_{t-1} + \nu_a \log a^*_{t-1} + e_{a,t},
\]

\[
\log a^*_t = \rho_a \log a^*_{t-1} + \nu_a \log a_{t-1} + e^*_{a,t},
\]

\[
\log \varepsilon_t = \rho_e \log \varepsilon_{t-1} + \nu_e \log \varepsilon^*_{t-1} + e_{\varepsilon,t},
\]

\[
\log \varepsilon^*_t = \rho_e \log \varepsilon^*_{t-1} + \nu_e \log \varepsilon_{t-1} + e^*_{\varepsilon,t},
\]

\[
\log \psi_t = \rho_\psi \log \psi_{t-1} + \nu_\psi \log \psi^*_{t-1} + e_{\psi,t},
\]

\[
\log \psi^*_t = \rho_\psi \log \psi^*_{t-1} + \nu_\psi \log \psi_{t-1} + e^*_{\psi,t},
\]

\[
t_t = q_t b_{t+1} - b_t,
\]

\[
t^*_t = q_t^* b^*_{t+1} - b^*_t,
\]

\[
b_t = 0,
\]

\[
b^*_t = 0,
\]

\[
\Pi_{h,t} = y_{h,t} - \frac{w_t}{\mathcal{P}_{h,t}} \left( \frac{y_{h,t}}{a_t} \right)^{\frac{1}{\alpha}} d_{h,t} \delta_{h,t},
\]

\[
\Pi_{f,t} = y_{f,t} - \frac{w_t^*}{\mathcal{P}_{f,t}^*} \left( \frac{y_{f,t}}{a_t^*} \right)^{\frac{1}{\alpha}} d_{f,t} \delta_{f,t}.
\]