Why Financial Frictions and Currency Mismatches do not Affect Traditional Mundell-Fleming Results

Santiago L. E. Acosta Ormaechea
University of Warwick, UK and University of Wisconsin-Madison, USA
S.L.E.Acosta-Ormaechea@warwick.ac.uk
acostaormaec@wisc.edu

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Abstract

This paper develops a dynamic general equilibrium model derived from first principles to determine whether traditional Mundell-Fleming results hold when currency mismatches and financial frictions are explicitly introduced. For this, we extend the small-open economy model of Obstfeld and Rogoff (1995) along the lines of Céspedes, Chang and Velasco (2004); being critical to us deriving the form in which financial frictions affect the economy from primitive assumptions in a model that can be studied analytically. We consider two different unexpected and permanent policy shocks: a monetary expansion (floating exchange rate) and a currency devaluation (fixed exchange rate). We find that traditional Mundell-Fleming results hold, since both shocks have expansionary effects on the economy. These are amplified, although moderately, by the presence of credit market imperfections. The mechanics at work seem to be traditional: with preset prices the unforeseen shocks produce a trade balance surplus on impact (i.e., there is an expenditure-switching effect) and hence an accumulation of net foreign assets. It follows a short- and long-run boom in the economy implying that the non-neutrality property of nominal shocks is not satisfied. We also show that capital and net worth act as a propagation mechanism in this model.

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1 Introduction

Several authors have pointed out after observing the events in South-East Asia in 1997-8 that the presence of currency mismatches and credit market imperfections could be key elements to understand the financial crisis that affected these economies. A far from exhaustive list of researchers that first supported this argument includes Krugman (1999) and Aghion et al (2000). Implicitly in their work lies the idea that expenditure-switching effects as emphasized in the traditional Mundell-Fleming textbook model can be counter-balanced by a debt burden of those agents indebted in foreign currency. Whenever the latter effect is strong enough, the reduction in the value of the domestic *vis a vis* the foreign currency can generate a contraction in output (i.e., the balance-sheet effect). This is, as Calvo and Reinhart (1999) argued, a reinterpretation of the debt-deflation argument popularized by Fisher (1933) but now in the context of small-open economies. Krugman and Aghion et al, however, only develop highly-stylized partial-equilibrium models in which the long-run dynamics are not considered.

In this regard, it is nowadays accepted that any well-respected macroeconomic model should be derived from primitive behavioral assumptions. This, in conjunction with budget constraints and some form of optimization process, allow the researcher to study the dynamic properties of the model in full. Along these lines of reasoning Obstfeld and Rogoff (1995) (henceforth OR) have been able to obtain the typical positive expenditure switching effects emphasized by Mundell and Fleming in a well-microfounded dynamic general equilibrium model. This paper has been influential not only because traditional Keynesian results were obtained from first economic principles, but also due to the high tractability of their model[1]. It gave birth, moreover, to the so-called 'New-Open-Economy-Macroeconomics' (henceforth NOEM) generation of models within international finance[2].

Whether traditional Mundell-Fleming results[3] also hold when currency mismatches and credit market imperfections are included in a NOEM-type of model has been an active area of research. Although this is not a novel question[4], what is new is the fact that the recent literature almost entirely concentrates on financial channels where the presence of credit market imperfections is particularly emphasized. The latter is in turn rationalized by the presence of asymmetric information problems between lenders and borrowers. Broadly speaking, this literature combines three key ingredients: i. currency mismatches, ii. nominal price rigidities and iii. credit market imperfections.

Currency mismatches imply that certain agents in the economy, for some reason taken as given[5],

[1]It is noteworthy that they were able to show the main results of the paper analytically. The cost of this has been, however, the introduction of assumptions that could be thought of as 'restrictive' (for instance, they ignore capital as an input in production and they also restrict Households' preferences in a way that allows them to derive tractable results).

[2]See Lane (2001) and Sarno and Taylor (2002) for a complete survey of the NOEM literature.

[3]By traditional Mundell-Fleming results we mean a monetary expansion generating a boost in output under a floating exchange rate regime and a similar result considering a currency devaluation under a fixed exchange rate regime.

[4]For a thorough review of this older literature see Agénor and Montiel (1999).

[5]Several papers have addressed the deeper question of understanding why agents may undertake contracts in a currency other than the one legally accepted as a mean of payment. Different explanations coexist, being mostly

have access to a credit market in which loans are essentially denominated in the foreign currency. Their assets, however, are denominated in the domestic currency or highly indexed to the domestic price index. The assumption about nominal price rigidities allows monetary or exchange rate policy to have non-trivial effects. Note also that the presence of prices that do not fully adjust after a policy shock is at the heart of the existence of balance-sheet problems: the local-currency indexation of domestic assets becomes restricted in such a case. In contrast, the domestic-value of foreign currency denominated liabilities freely adjusts with the nominal exchange rate. Indebted agents may suffer then a negative net worth effect as the exchange rate depreciates, thereby generating a contractionary effect on the economy. The presence of credit market imperfections, moreover, will restrict the availability of credit and will also imply the existence of a risk premium which is inversely associated with the evolution of net worth. Therefore, there is an endogenous mechanism that amplifies shocks and affects the economy. This is in general modelled following two seminal contributions: Carlstrom and Fuerst (1997) and Bernanke et al (1999), which provide an elegant and relatively simple way of introducing financial frictions into an otherwise standard dynamic general equilibrium model.

Most of the papers developed along these lines have focused in studying quantitatively different monetary and exchange rate policies. For instance, Cook (2004) finds numerically that an expansionary monetary policy can have contractionary effects in the short-run due to the presence of the balance-sheet channel. Choi and Cook (2004), in a similar framework (where currency mismatches are localized in the banking sector) explore a slightly different question: which exchange rate regime provides better stabilization properties? They find evidence in favor of a peg. Devereux et al (2006) show that although the presence of financial frictions amplifies shocks, the ranking in terms of the stability properties of an exchange rate regime is not affected. This ranking is sensitive, however, to the degree of exchange-rate pass through. Moreover, they argue that in welfare terms there seems to be a well-established preference for a flexible regime. Gertler et al (2007) develop a similar framework to reproduce the Korean crisis in 1997-8, finding that their model seems to match fairly well the data; and in welfare terms they also find that a fixed exchange rate regime seems to imply higher losses.

Although constructing models that can be evaluated numerically through impulse-response functions and other relevant empirical measures is a useful exercise, there is a sort of ‘black box’ when someone is willing to carefully understand how the dynamic properties of the model are affected by the presence of financial frictions. A qualitative or analytical approach seems to be the appropriate way of dealing with this issue. There are fewer researchers, however, that have explored this avenue; being the only published paper that we are aware of Céspedes, Chang and Velasco (2004) (henceforth CCV). They manage to provide an analytical solution constructing a microfounded dynamic general equilibrium model but relying on a number of strong simplifications. In particular,

associated with variations of a lack of credibility problem in domestic monetary policy (see IADB 2004, specially Ch. 4, where the problems associated with financial dollarization and currency mismatches are discussed at length). A similar argument is given by Hausmann (1999) but under the more artistic name, ‘original sin problem’.

they essentially postulate the form in which credit market imperfections affect the economy. This assumption, however, seems to go against the objective of developing a model from first principles.

The present paper's intention is to fill this gap. It can be viewed as an extension of the small-open economy version of OR with the spirit of CCV. In particular, our intention is to construct a simple though realistic framework developed from first principles that can be analyzed analytically[6] to study the role of credit market imperfections in an otherwise standard NOEM model affected by one-period nominal price rigidities and currency mismatches. In this setup we are further asking whether traditional Mundell-Fleming results still hold. For expositional simplicity we now outline the structure of the model.

1.1 Overview of the model

We consider a tractable two-sectors general equilibrium model for a small-open economy. To understand how the different sectors are interrelated in this economy we show in Figure 1, below, a flow chart of the model.

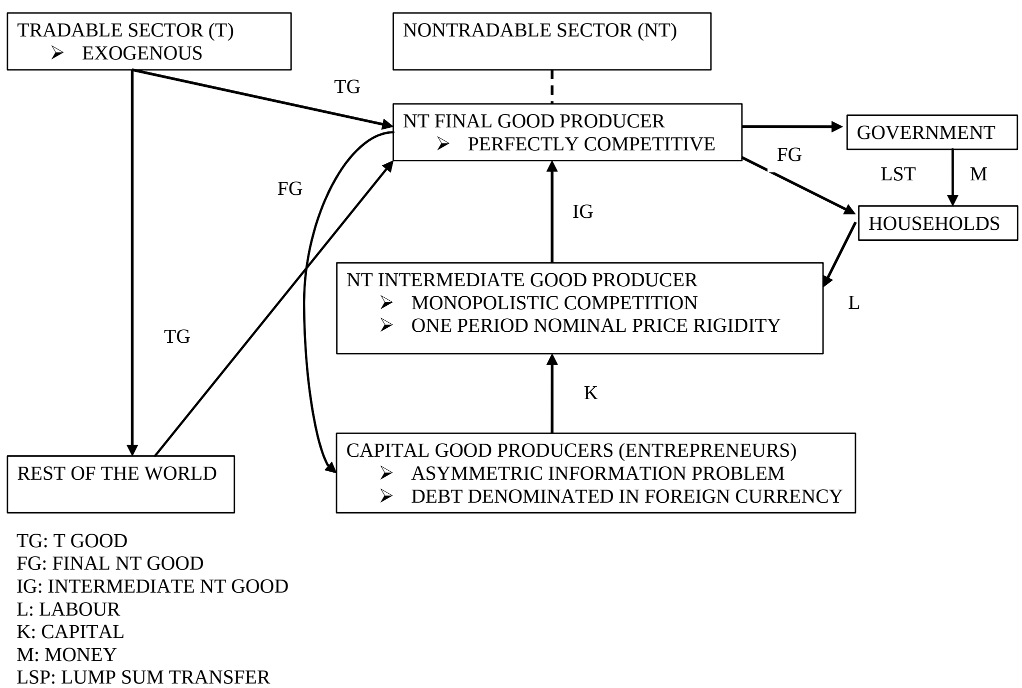

Figure 1. Flow of goods in the model

Following OR we assume that the tradable sector is exogenous. Although this seems to be a particularly strong assumption, we justify it emphasizing that we think of a small economy in which there is a natural resource (e.g., oil) which is an endowment for the economy at each point in time. It follows then that the richness of the model is associated with the behavior of the nontradable sector.

[6]It is worth pointing out that although our main concern is to solve the model analytically, the final results of the model rely on some form of numerical exercise. To be more precise, our solution could be thought of as 'quasi-analytical' since the steady state and the minimum state-space form of the model (to study its dynamic behavior) are derived analytically in full.

This is essentially composed by a final and an intermediate goods sectors. The former is assumed to be perfectly competitive, requiring a tradable and a combination of differentiated nontradable inputs to undertake production. With this specification we want to emphasize that to produce a final good a firm requires different inputs such as transport, retailing and so forth; all of which are nontradable. Note that the demand for the tradable input by final producer firms is the only source of absorption of tradable goods. It then follows that the evolution of the trade balance surplus will be directly associated with the evolution of this demand. The final nontradable good is sold to domestic agents either for consumption (to households and government) or as an input for the production of capital (to entrepreneurs).

Since we want to introduce nominal price rigidities in its simplest form, but it becomes difficult to justify it in a context of perfectly-competitive markets, we consider the existence of a continuum of intermediate firms producing differentiated inputs (i.e., there is monopolistic competition in this sector) that face one-period nominal price rigidities. We justify this assumption considering Burstein et al (2005)'s argument where, analyzing 5 recent episodes of large devaluations[7], they show that the main source of changes in the real exchange rate has been the slow adjustment in nontradable prices. Note that the combination of nominal rigidities and monopolistic competition rationalizes that output can be demand-determined in the short-run.

This is a model which gathers together different elements that are not often observed in the NOEM literature. Since our aim is to understand thoroughly the implications of each of these non-conventional features, we decided to separate the sector that faces monopolistic competition and price rigidities from the one exposed to credit market imperfections. Without loss of generality, and following Carlstrom and Fuerst (1997)[8], we assume that the latter only affects the production of capital. This production is undertaken by entrepreneurs, who require external funding to invest. We also assume that entrepreneurs can only obtain debt denominated in foreign currency. A key element of this approach is the fact that the production function is linear but with a productivity factor which is stochastic and idiosyncratic. In order to simplify things we further assume that this random variable follows a uniform distribution[9]. We then show that financial frictions will imply that the entrepreneur has to pay a premium over a risk-free interest rate when contracting the loan. This premium is in turn negatively associated with the evolution of net worth, as can be expected intuitively. Translated into macroeconomics' jargon, we are essentially departing from OR by introducing capital and net worth as two state variables of the model that will propagate any initial perturbation that affects the economy.

We introduce Households and Government in a way that essentially resembles OR. We depart from them, however, assuming that preferences over real money balances take the logarithmic instead

[7] They consider the cases of Argentina (2002), Brazil (1999), Korea (1997), Mexico (1994) and Thailand (1997).

[8] An important difference is, however, the fact that our modelling device considers nominal instead of real variables.

[9] In this regard, we are departing from most of the literature that considers financial frictions, where the log-normal distribution seems to be the most popular choice (e.g., Carlstrom and Fuerst (1997), Bernanke et al (1999) and Devereux et al (2006) to name just a few). Our hope is that taking the simpler uniform distribution does not alter substantially the results and helps us in providing a better understanding of the properties of the model.

of the constant risk-aversion form. This assumption, a helpful simplification in view of the other complexities of the model, will imply that the domestic nominal interest rate is unaffected by the policy shock. Thereby the nominal exchange rate does not show any overshooting effect as it does in the case of Dornbusch (1976). Finally, this is a model with Ricardian agents in which the main role of the Government is to introduce money into the economy. Therefore, we only consider private-sector bonds. With this general background at hand we can now proceed to explain the main findings of the paper.

1.2 Main results and structure of the paper

The main message of our paper is: although the presence of financial frictions seems to amplify the initial shock, though moderately, the results are essentially the same as those predicted by a simple Mundell and Fleming textbook model. Therefore, a monetary expansion under a floating exchange rate or a currency devaluation under a fixed exchange rate both have positive effects on output[10].

Recall that we are assuming that prices are preset only for a period. It is then necessary to divide the analysis of the model's solution into two differentiated phases. There is a short-run solution which lasts only the initial period which coincides with the presence of the unexpected and permanent shock. There is also a long-run solution which holds afterward, in which prices are fully flexible and no other shock takes place in the economy. Observe also that capital and net worth are the only state variables of the model. Their values, therefore, are given at each point in time. From their respective laws of motion there is a connection between the short and the long-run model's solutions. Short-run investment is therefore constrained: today's investment should guarantee that tomorrow's capital stock is effectively supplied[11]. Considering these elements in conjunction with the intertemporal national budget constraint of the economy as a whole (henceforth INBC), it is then possible to obtain the complete time-path of the endogenous variables.

Consider an unexpected and permanent monetary expansion (floating regime.) Since consumption moves *pari passu* with real money balances under our specification of preferences, consumption increases and through it there is a boost in the final nontradable sector. There is then a pressure toward a trade balance deficit. We should keep in mind two key elements of the model at this stage, however: i. any plausible solution must be consistent with the resource constraint of the economy as a whole and ii. the stock of capital is given on impact.

To make feasible these endogenous changes with the INBC it is required to reduce today's

[10]Note that the shocks that we are considering in this paper and the effects on the current account dynamics give a sense of complementarity to those evaluated by CCV (a rise in the foreign interest rate and a negative shock to the terms of trade). This is due to the differences in our modelling device. Real money balances are explicitly introduced here. This allows us to explore the effects of the canonical exercise popularized by Dornbusch (1976): an unexpected and permanent monetary expansion. We also allow households to accumulate bonds, and so our model provides interesting current account dynamics not explored by them. Our source of nominal rigidity is also different: Instead of assuming preset wages we consider that prices of intermediate nontradable goods are preset for a period. This will modify the adjustment process of different endogenous variables after the policy shocks.

[11]This is an important aspect of the solution of the model. As we will show in the paper, moreover, this constraint will imply that the optimal contracting problem between the entrepreneur and the lender does not hold in the short-run.

absorption. The exchange rate then rises toward its new steady state level and remains there forever. Notice also that in the short-run any excess demand of intermediate goods is adjusted through quantities. But since capital does not adjust within the period, there is a limit on intermediate goods' production. It follows that the nominal exchange rate increases less than the money supply. There is more capital demanded on impact, fact that is reflected in a higher short-run real rental price of capital. This boosts short-run investment thereby raising (future) capital stock. The economy, therefore, should generate a trade balance surplus that provides the tradable resources that will be absorbed in the future (a higher supply of capital will imply a higher production of the final nontradable good).

This trade balance surplus can also be seen as the result of an expenditure-switching effect: the final nontradable sector substitutes tradable *vis a vis* nontradable inputs as the relative price of the latter decreases. There is also a short-run positive wealth effect (i.e., an accumulation of net foreign assets), thereby implying that the long-run non-neutrality of money is not satisfied. This result seems to be a novelty since in the small-open economy version of OR current account effects are absent. We also show that when capital is eliminated there are no current account effects and the long-run neutrality property is recovered. In this sense, we argue that capital can be thought of as an element that provides an additional source of rigidity in the economy.

Regarding the role played by credit market imperfections, recall that the supply of capital in the period immediately after the shock is higher. Entrepreneurs then face a boom, which implies that their net worth also raises[12]. The real amount of debt contracted in the short-run must be at a level that guarantees next-period supply of capital (in the case considered here debt slightly rises). The essence of these results holds regardless the currency of denomination of entrepreneurs' debt. Finally, notice that when the exchange rate regime is a peg and the policy shock is a currency devaluation all the previous results are preserved; being the main difference that the real effects are slightly amplified.

The rest of the paper is organized as follows. Section 2 explains in the detail the model. Section 3 defines equilibrium. Section 4 and 5 solve for the steady state and for a log-linear approximation of the model, respectively. Sections 6 and 7 discuss its dynamic properties under a floating (monetary expansion) and a fixed (currency devaluation) exchange rate regimes, respectively. Finally, in Section 8 we present some concluding remarks.

[12]In this setup net worth is a fraction of entrepreneurs expected profits. Whenever the latter increases, for instance as a consequence of a boom in the production of capital, the former also rises.

2 The model

2.1 Firms

2.1.1 Tradable sector

There is a single homogeneous tradable good whose supply is constant and exogenously given each period t and is denoted by $Y_{T,t} = \overline{Y}_T$. This output, in turn, becomes each period households' endowment.

2.1.2 Nontradable sector

The nontradable sector is composed of a continuum of intermediate firms that produce differentiated inputs and a perfectly competitive producer of the nontradable final good. There are a large number of firms indexed by i in the intermediate sector, where each one specializes in producing a particular input. The intermediate output of firm i at period t is produced by combining capital and labor services with a Cobb-Douglas production function as follows,

$$Z_{i,t} = A_t K_{i,t}^{\alpha} L_{i,t}^{1-\alpha},\ i \in [0,1],\ 0 < \alpha \leq 1, \tag{1}$$

where $Z_{i,t}$ indicates the production of input i, A_t is a technology parameter assumed to be common to all firms, $K_{i,t}$ is the stock of capital rented to entrepreneurs at the beginning of period t, $L_{i,t}$ indicates labor services obtained from households and α is the share of capital in the nontradable intermediate input (which is assumed to be the same for all firms).

The producer of the final nontradable good combines the inputs provided by intermediate firms and a tradable input with a Cobb-Douglas-type production function. This output is afterward sold to domestic agents for consumption or to entrepreneurs for using it as an input in the production of the capital good. The production function of a representative firm is defined as follows,

$$Y_t = \{[\int_0^1 (Z_{i,t})^{\frac{\theta-1}{\theta}} di]^{\frac{\theta}{\theta-1}}\}^{\gamma}\{X_{T,t}\}^{1-\gamma},\ \theta > 1,\ 0 < \gamma < 1, \tag{2}$$

where Y_t is the final nontradable good, θ is the elasticity of substitution between different nontradable inputs (or elasticity of demand), γ is the share of nontradable components in the final nontradable good and $X_{T,t}$ is the tradable input that is used in producing the final good. Each intermediate firm in the nontradable sector, therefore, faces the following downward sloping demand curve[13],

$$Z_{i,t} = Y_t \gamma^{\theta} [\frac{(1-\gamma)}{P_{T,t}}]^{\frac{(1-\gamma)(\theta-1)}{\gamma}} (P_{i,t})^{-\theta} (P_t)^{\frac{\theta-1+\gamma}{\gamma}}, \tag{3}$$

[13]The final good producer solves the following cost minimization problem:
$\min \int_0^1 Z_{i,t} P_{i,t} di + P_{T,t} X_{T,t}$ s.t. $Y_t = \{[\int_0^1 (Z_{i,t})^{\frac{\theta-1}{\theta}} di]^{\frac{\theta}{\theta-1}}\}^{\gamma}\{X_{T,t}\}^{1-\gamma}$, giving the inverse demand function stated in Eq. 3.

where $P_{T,t}$, $P_{i,t}$ and P_t are the prices of the tradable good, the intermediate good and the final good, respectively. It is worth noting that the marginal cost of the final producer firm is defined as $MC_t = \gamma^{-\gamma}(1-\gamma)^{(\gamma-1)}(P_{T,t})^{1-\gamma}[\int_0^1 P_{i,t}^{1-\theta}di]^{\frac{\gamma}{1-\theta}}$[14].

Let us define $P_{N,t} = [\int_0^1 P_{i,t}^{1-\theta}di]^{\frac{1}{1-\theta}} = [P_t\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}(P_{T,t})^{\gamma-1}]^{\frac{1}{\gamma}}$. We can therefore rewrite the demand curve that each intermediate firm faces as,

$$Z_{i,t} = Y_t(1-\gamma)^{\frac{\gamma-1}{\gamma}}(\frac{P_t}{P_{T,t}})^{\frac{\gamma-1}{\gamma}}(\frac{P_{i,t}}{P_{N,t}})^{-\theta}. \tag{4}$$

Observe that Eq. 4 is a conditional demand correspondence obtained as a result of a cost minimization problem taken Y_t as given. Moreover, it is worth noticing that since the production function is homogeneous of degree one the scale of production becomes indeterminate (and therefore the demand for factors). In this context the appropriate way of deriving the conditional demand correspondence is by solving a cost minimization problem as we have already done here.

It will be considered that the law of one price (LOOP) holds for tradable goods at all t, implying that,

$$P_{T,t} = S_t,$$

where S_t denotes the nominal exchange rate measured as the domestic price of foreign exchange. Note that the foreign price of the tradable good was normalized to one.

2.1.3 Demand for factors by intermediate firms

Turning to intermediate firms, it is worth observing that their decision problem can be analyzed in two stages. As in the case of final producers the production function of intermediate firms is also homogeneous of degree one. The conditional demand correspondences are therefore obtained as the result of a cost minimization problem (first stage). Since intermediate firms produce differentiated products they are able to choose the price level that maximizes their profits (second stage).

The cost minimization problem (taking the output level $Z_{i,t}$ as given) is then defined as:

$$\min_{\{K_{i,t},L_{i,t}\}} R_t^k K_{i,t} + W_t L_{i,t} \text{ s.t. } Z_{i,t} = A_t K_{i,t}^{\alpha} L_{i,t}^{1-\alpha}, \tag{5}$$

where R_t^k indicates the nominal rental price of capital and W_t denotes the nominal wage. It is worth highlighting that $K_{i,t}$ is a homogeneous capital good demanded by intermediate firms and supplied by a large number of entrepreneurs. $L_{i,t}$, on the other hand, is a homogeneous type of labor demanded by intermediate firms and supplied by a large number of households. Since both inputs are homogeneous, supplied by a large number of agents and demanded by a large number of firms, at the individual level each firm takes the nominal rental price of capital R_t^k and the nominal wage W_t as given.

The first order conditions associated with this problem are,

[14]Note that in equilibrium the marginal cost of the final producer firm will be equal to the price of the final good, P_t.

$$K_{i,t}^{*} = (\frac{1-\alpha}{\alpha})^{\alpha-1}\frac{Z_{i,t}}{A_t}(\frac{W_t}{R_t^k})^{1-\alpha} \tag{6}$$

and

$$L_{i,t}^{*} = (\frac{1-\alpha}{\alpha})^{\alpha}\frac{Z_{i,t}}{A_t}(\frac{W_t}{R_t^k})^{-\alpha}. \tag{7}$$

Note that the cost function evaluated at $K_{i,t}^{*}$ and $L_{i,t}^{*}$ takes the form,

$$C_{i,t}(Z_{i,t}, R_t^k, W_t) = C_{i,t}^{*} = \alpha^{-\alpha}(1-\alpha)^{\alpha-1}A_t^{-1}Z_{i,t}W_t^{1-\alpha}(R_t^k)^{\alpha}. \tag{8}$$

2.1.4 Profit maximization problem of intermediate firms

In the second stage intermediate firms determine the price level $P_{i,t}$ and output $Z_{i,t}$ that maximize profits subject to the cost function obtained in Eq. 8 and the inverse demand function stated in Eq. 4,

$$\max_{\{P_{i,t}\}} \pi_{i,t} = P_{i,t}Z_{i,t} - C_{i,t}^{*} \text{ s.t. } Z_{i,t} = Y_t(1-\gamma)^{\frac{\gamma-1}{\gamma}}(\frac{P_t}{P_{T,t}})^{\frac{\gamma-1}{\gamma}}(\frac{P_{i,t}}{P_{N,t}})^{-\theta}.$$

The solution of this problem gives the following price setting equation,

$$P_{i,t} = \frac{\theta}{\theta-1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1}A_t^{-1}W_t^{1-\alpha}(R_t^k)^{\alpha}, \tag{9}$$

where $\frac{\theta}{\theta-1}$ is a markup over marginal costs[15].

This equation defines how intermediate firms optimally set the price level of their output $P_{i,t}$. It is worth highlighting that firms decide the price level that will prevail at period t at the end of period $t-1$. To be more precise, we can think of Eq. 9 as implicitly given by the following expression,

$$P_{i,t} = E_{i,t-1}\{\frac{\theta}{\theta-1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1}A_t^{-1}W_t^{1-\alpha}(R^k)^{\alpha}\},$$

where $E_{i,t-1}$ indicates the expectation hold by agent i at the end of period $t-1$ given the information available at that time. This model assumes perfect foresight. Therefore, the above expression will be identical to Eq. 9 for all periods but $t=0$, when an unexpected shock hits the economy. During that period, the price $P_{i,0}$ differs from what firm i would have optimally chosen had it known the shock in advance. It is in this context that we can consider the price level of the intermediate firm i as 'given' at period $t=0$.

[15] Note that in the perfectly competitive case, when $\theta \to \infty$, the price of the intermediate firm is equal to the marginal cost.

2.2 Households

The representative household obtains utility from consumption of the final good C_t, real money balances $\frac{M_t}{P_t}$[16] and leisure (given by the disutility associated with working in the production of the nontradable input $-\frac{\kappa}{2}(L_t)^2$). Therefore, lifetime utility of the representative agent takes the form,

$$U_t = \sum_{t=0}^{\infty} \beta^t [\log C_t + \chi \log(\frac{M_t}{P_t}) - \frac{\kappa}{2}(L_t)^2]. \tag{10}$$

The budget constraint that the household faces when maximizing utility, expressed in nominal terms, is defined by,

$$P_t C_t + M_t + S_t D_{t+1} = P_{T,t}\overline{Y}_T + W_t L_t + \pi_t + S_t R_t^* D_t + M_{t-1} + P_t T_t. \tag{11}$$

Household's sources of funding are given by the endowment of the tradable good $P_{T,t}\overline{Y}_T$, wage earnings for working in the nontradable intermediate sector $W_t L_t$, dividends from owning intermediate firms π_t, nominal gross return from previous-period foreign currency denominated deposits $S_t R_t^* D_t$[17], holdings of previous period nominal money balances M_{t-1} and lump-sum government transfers $P_t T_t$[18]. These resources are used to purchase consumption goods $P_t C_t$, to accumulate nominal money balances M_t or to acquire new interest-bearing deposits $S_t D_{t+1}$.

The first order conditions associated with this problem are obtained by maximizing Eq. 10 with respect to D_{t+1}, M_t, and L_t subject to the budget constraint stated in Eq. 11. We therefore have,

$$C_{t+1} = \beta R_{t+1}^* \frac{S_{t+1}}{S_t} \frac{P_t}{P_{t+1}} C_t \tag{12}$$

$$\frac{M_t}{P_t} = \chi C_t \frac{R_{t+1}}{R_{t+1} - 1} \tag{13}$$

$$\frac{1}{\kappa} \frac{1}{C_t} \frac{W_t}{P_t} = L_t. \tag{14}$$

Eq. 12 is a Euler equation indicating that the marginal rate of substitution of consumption in two subsequent periods must be equal to the real interest rate. We assume that the UIP holds in this model thus yielding $R_{t+1} = R_{t+1}^* \frac{S_{t+1}}{S_t}$, where R_{t+1} and R_{t+1}^* indicate the gross nominal risk-free domestic and foreign interest rates, respectively[19]. Observe that the demand for real balances stated in Eq. 13 is positively associated with consumption and the weight in the utility function of having

[16] The fact that households obtain utility from real balances is common in the Money-in-the-Utility function literature. It can be thought of as money generating utility owing to the services that it provides in facilitating transactions (see Walsh 2003).

[17] Since one of the main objectives of the model is highlighting problems associated with currency mismatches, it is assumed that households only hold deposits denominated in the foreign currency.

[18] In facilitating the analysis it is assumed that government's transfers are only made in the final nontradable good.

[19] Although we directly assumed that the UIP condition holds, we could have also derived it by allowing households to accumulate a bond denominated in the local currency. Arbitrage between the domestic and the foreign bond will then imply that $R_{t+1} = R_{t+1}^* \frac{S_{t+1}}{S_t}$ must hold.

an extra-unit of real balances; and negatively related to the gross nominal interest rate. This is a standard result in models with Money-in-the-Utility function and infinitely-lived agents. Finally, the labor supply equation shown in Eq. 14 increases in the real wage, while decreases in consumption and in the weight that the household gives to the disutility of working.

2.3 Government

For simplicity we assume that Ricardian equivalence holds, and therefore we will abstract from government debt. It is further assumed that government spending affects only the final nontradable good. In this simple setting, the only source of funding for the government's current spending and the lump-sum transfer that is made toward households is therefore real seigniorage. Observe that the interpretation of T_t is twofold: whenever it takes a positive value it refers to a lump-sum transfer from the government to households, while if it takes a negative value it implies a lump-sum tax paid from households to the government. The government's budget constraint is expressed as,

$$G_t + T_t = \frac{(M_t - M_{t-1})}{P_t}, \tag{15}$$

where G_t indicates government's expenditure on the final good and $\frac{(M_t - M_{t-1})}{P_t}$ is the real seigniorage that the government is obtaining for issuing money between t and $t-1$. In facilitating the analysis it will be assumed that $G_t \equiv 0$ and therefore any revenue due to seigniorage is immediately rebated to households in a lump-sum way.

2.4 Entrepreneurs

Entrepreneurs will play a central role in the model. They will produce the capital good that is afterward rented to firms. In producing capital, however, they must obtain external funding, which is denominated in foreign currency and subject to frictions, as described below. The present section provides a detailed analysis of the entrepreneurs' behavior and their interactions with the credit market.

2.4.1 Partial equilibrium contracting problem

The analysis of the debt contracting problem under asymmetric information developed in this section closely follows Carlstrom and Fuerst (1997). It is assumed the existence of a continuum of entrepreneurs indexed by j in the interval $[0, 1]$ producing a homogeneous capital good. Each entrepreneur has the following stochastic linear technology,

$$K_{j,t+1} = \omega_{j,t} i_{j,t}, \tag{16}$$

where $K_{j,t+1}$ indicates the capital good produced by entrepreneur j at period t, that will be incorporated in the production process of intermediate firms in period $t + 1$; $i_{j,t}$ denotes the input utilized by entrepreneur j to produce the capital good, which is part of the final good produced in the economy; $\omega_{j,t}$ is a *iid* random variable with a common distribution across j, where the cumulative and density functions have positive supports and are denoted by $\Phi(\cdot)$ and $\phi(\cdot)$, respectively. To simplify the analysis it is assumed that $E(\omega) = 1$.

When the entrepreneur decides how much to invest at period t, he or she faces the following budget constraint,

$$S_t B^*_{j,t+1} = P_t(i_{j,t} - n_{j,t}), \tag{17}$$

where $S_t B^*_{j,t+1}$ indicates the domestic value of the foreign currency denominated debt contracted at period t to be repaid at period $t+1$ and $n_{j,t}$ is the net worth of entrepreneur j at the beginning of period t. This constraint simply indicates that the entrepreneur can purchase inputs beyond his or her net worth only by contracting foreign currency denominated debt.

Following Townsend (1979) and Gale and Hellwig (1985) among others, the model assumes a costly state verification problem. In this context the optimal contract between the borrower and the lender will take the form of a standard non-contingent debt contract. To simplify the model it will be assumed that there is enough anonymity in the credit market, so as to avoid issues related to how past records of interactions between entrepreneurs and lenders may affect the characteristics of the financial contract.

The contract specifies a fixed payment to the lender in all states where the project generates a nominal gross return above the fixed nominal value of the debt repayment. In contrast, when this condition is not satisfied, the entrepreneur defaults on the debt and the lender recoups as much as he or she can from the project, after paying a fixed monitoring cost.

The random variable $\omega_{j,t}$, which can be thought of as a productivity parameter, is neither observed by the entrepreneur nor by the lender ex-ante. For the entrepreneur, however, it is costless to observe the ex-post value of $\omega_{j,t}$. The lender, in contrast, must incur in a monitoring cost to observe the true value of $\omega_{j,t}$.

The monitoring cost is given by the payment of $\mu i_{j,t}$ units of the final capital good, where $0 \leq \mu \leq 1$[20]. The payment to observe $\omega_{j,t}$, however, is only made in case the entrepreneur defaults on the debt. It is clear now where the costly state verification problem arises in the model: in order to observe the true realization of $\omega_{j,t}$, the lender must incur in a deterministic pecuniary cost.

It is worth highlighting that for simplicity we will assume that capital fully depreciates within the period and that there are no adjustment costs of any form in the law of motion of capital [21]. From these assumptions it follows that the relative price of capital in terms of the final nontradable good

[20] The assumption regarding the form of the monitoring cost implies that there is a fixed cost $\mu i_{j,t}$, known ex-ante by the lender, for observing the true realization of the project. Note that this cost depends on the scale of the investment $i_{j,t}$, but is independent of the ex-post realization of $\omega_{j,t}$. A slightly different approach is taken in Bernanke et al (1999), where the monitoring cost is a fraction of the ex-post realization of the project. It is worth observing, however, that the main results of the model remain the same, independently of the form in which monitoring costs are defined.

[21] Although these assumptions can be seen as particularly strong, it is noteworthy that are critical to obtain a model which can be handled analytically. Departing from these assumptions complicates the analysis and, in our view, is not critical for a neat understanding of the role played by credit market imperfections in a dynamic general equilibrium model. Note, moreover, that CCV also introduce these simplifications in their paper.

is equal to 1. Similarly, it can be easily seen that the (gross) nominal rate of return of producing the capital good will be equal to R^k_{t+1}, its rental price.

Let $\overline{\omega}_{j,t}$ denote the minimum value of $\omega_{j,t}$ at which default does not occur and let $R^{nd}_{j,t+1}$ indicate the non-default gross nominal interest rate charged on entrepreneur j when contracting the debt at period t. $R^{nd}_{j,t+1}$ and $\overline{\omega}_{j,t}$ therefore satisfy,

$$R^k_{t+1}\overline{\omega}_{j,t}i_{j,t} = R^{nd}_{j,t+1}S_t B^*_{j,t+1} = R^{nd}_{j,t+1}P_t(i_{j,t} - n_{j,t}). \tag{18}$$

Eq. 18 indicates that entrepreneur j, with the associated value for the productivity parameter given by $\overline{\omega}_{j,t}$, produces $\overline{\omega}_{j,t}i_{j,t}$ units of the capital good that are afterward rented to firms at the nominal rental price R^k_{t+1}. The term $R^k_{t+1}\overline{\omega}_{j,t}i_{j,t}$, therefore, represents the minimum nominal gross return of the produced capital required to repay the principal and interests on the debt, $R^{nd}_{j,t+1}S_t B^*_{j,t+1}$.

Note that Eq. 18 can be rewritten as follows,

$$R^{nd}_{j,t+1} = \frac{R^k_{t+1}\overline{\omega}_{j,t}}{P_t(1 - \frac{n_{j,t}}{i_{j,t}})}. \tag{19}$$

Eq. 19 gives a simple relation between $R^{nd}_{j,t+1}$ and $\overline{\omega}_{j,t}$. It is worth highlighting that R^k_{t+1} is a market price, and as such it will be determined by the equilibrium conditions between aggregate supply and aggregate demand for capital. The general price index, P_t, is also a market price determined by equilibrium conditions in the market for goods. Therefore, from the entrepreneur's viewpoint, these variables are taken as given.

Also note that taking the net worth of entrepreneur j as given, the contractual problem between the lender and the entrepreneur is fully specified once we solve either for $R^{nd}_{j,t+1}$ and $i_{j,t}$, or $\overline{\omega}_{j,t}$ and $i_{j,t}$ (see Eq. 18). Since the contract in terms $\overline{\omega}_{j,t}$ and $i_{j,t}$ is slightly easier to study, we analyze the optimal contractual problem only in terms of these two variables.

2.4.2 Expected profits

In determining the optimal contract it is assumed that both the entrepreneur and the lender are risk neutral. Recalling that capital fully depreciates within the period, the net expected profit of the entrepreneur in nominal terms can be expressed as follows,

$$R^k_{t+1}\int_{\overline{\omega}_{j,t}}^{\infty} i_{j,t}\omega\phi(\omega)d\omega - [1 - \Phi(\overline{\omega}_{j,t})]R^{nd}_{j,t+1}P_t(i_{j,t} - n_{j,t}),$$

where the first term indicates the expected gross income for producing the capital good whenever $\omega_{j,t} > \overline{\omega}_{j,t}$, while the second term shows the expected cost of the debt repayment in case the entrepreneur repays the debt as established in the contract (i.e., whenever $\omega_{j,t} > \overline{\omega}_{j,t}$). The term $[1 - \Phi(\overline{\omega}_{j,t})]$ thus indicates the probability that the entrepreneur repays the debt. Observe that in case of default, or whenever $\omega_{j,t} < \overline{\omega}_{j,t}$, the entrepreneur receives nothing, and any remaining value of the project is completely seized by the lender.

Using Eq. 18 it is possible to rewrite the above expression as follows,

$$R^k_{t+1} i_{j,t} f(\overline{\omega}_{j,t}) = R^k_{t+1} i_{j,t} \{ \int_{\overline{\omega}_{j,t}}^{\infty} \omega\phi(\omega)d\omega - [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t} \}$$

where $f(\overline{\omega}_{j,t}) = \{ \int_{\overline{\omega}_{j,t}}^{\infty} \omega\phi(\omega)d\omega - [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t} \}$ indicates the expected share of the investment that the entrepreneur keeps when undertaking a successful project.

Following a similar way of reasoning, the net expected profit of the lender can be expressed as follows,

$$R^k_{t+1} \int_0^{\overline{\omega}_{j,t}} i_{j,t}\omega\phi(\omega)d\omega - R^k_{t+1}\mu i_{j,t}\Phi(\overline{\omega}_{j,t}) + [1 - \Phi(\overline{\omega}_{j,t})]R^{nd}_{j,t+1} P_t(i_{j,t} - n_{j,t}).$$

In this case $R^k_{t+1} \int_0^{\overline{\omega}_{j,t}} i_{j,t}\omega\phi(\omega)d\omega$ indicates the expected gross income generated by the project that is seized by the lender whenever $\omega_{j,t} < \overline{\omega}_{j,t}$ and $R^k_{t+1}\mu i_{j,t}\Phi(\overline{\omega}_{j,t})$ denotes the expected payment of monitoring costs[22]. Note that $\Phi(\overline{\omega}_{j,t})$ indicates the probability that entrepreneur j defaults on the debt. In the case in which $\omega_{j,t} > \overline{\omega}_{j,t}$, on the other hand, the entrepreneur repays the loan as established in the contract, and thus the lender expects to receive $[1 - \Phi(\overline{\omega}_{j,t})]R^{nd}_{j,t+1} P_t(i_{j,t} - n_{j,t})$.

Using Eq. 18 it is possible to define the expected profit of the lender as,

$$R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t}) = R^k_{t+1} i_{j,t} \{ \int_0^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - \mu\Phi(\overline{\omega}_{j,t}) + [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t} \},$$

where $g(\overline{\omega}_{j,t}) = \int_0^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - \mu\Phi(\overline{\omega}_{j,t}) + [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}$ indicates the expected share of the investment that the lender keeps from the project.

Considering the definitions of $f(\overline{\omega}_{j,t})$ and $g(\overline{\omega}_{j,t})$ it is possible to show that $f(\overline{\omega}_{j,t}) + g(\overline{\omega}_{j,t}) = 1 - \mu\Phi(\overline{\omega}_{j,t})$[23]. This fact implies that a fraction $\mu\Phi(\overline{\omega}_{j,t})$ of the total investment made by entrepreneur j is expected to be lost owing to the presence of monitoring costs.

[22] Recall that when the entrepreneur defaults on the debt, the lender must pay $\mu i_{j,t}$ units of the capital good, which must therefore be priced at the rental price of capital R^k_{t+1}.

[23] To obtain this result, notice that $f(\overline{\omega}_{j,t}) + g(\overline{\omega}_{j,t})$ can be written as $\int_0^{\infty} \omega\phi(\omega)d\omega - \mu\Phi(\overline{\omega}_{j,t})$. Recalling that $E(\omega) = \int_0^{\infty} \omega\phi(\omega)d\omega = 1$ gives $f(\overline{\omega}_{j,t}) + g(\overline{\omega}_{j,t}) = 1 - \mu\Phi(\overline{\omega}_{j,t})$.

2.4.3 A note on the behavior of $f(\overline{\omega}_{j,t})$ and $g(\overline{\omega}_{j,t})$

Let us consider again the fraction of the investment that the entrepreneur and the lender keep from the project $f(\overline{\omega}_{j,t})$ and $g(\overline{\omega}_{j,t})$, respectively. In Appendix B it is shown that $f'(\overline{\omega}_{j,t}) = -[1 - \Phi(\overline{\omega}_{j,t})]$ and that $f''(\overline{\omega}_{j,t}) = \phi(\overline{\omega}_{j,t})$, implying that $f(\overline{\omega}_{j,t})$ is a convex function of $\overline{\omega}_{j,t}$ (notice that monitoring costs, μ, do not affect $f(\overline{\omega}_{j,t})$) . In particular, note that $f'(\overline{\omega}_{j,t})$ will always be negative, unless $\overline{\omega}_{j,t}$ takes the highest value for which ω is defined, in which case $f'(\overline{\omega}_{j,t}) = 0$. Therefore, for a given level of investment $i_{j,t}$, entrepreneur's expected profits, $R^k_{t+1} i_{j,t} f(\overline{\omega}_{j,t})$, do not increase in $\overline{\omega}_{j,t}$.

Regarding $g(\overline{\omega}_{j,t})$, in this Appendix it is also shown that $g'(\overline{\omega}_{j,t}) = -\mu\phi(\overline{\omega}_{j,t}) + [1 - \Phi(\overline{\omega}_{j,t})]$ and that $g''(\overline{\omega}_{j,t}) = -[\mu \frac{\partial \phi(\overline{\omega}_{j,t})}{\partial \overline{\omega}_{j,t}} + \phi(\overline{\omega}_{j,t})]$. Note that, without monitoring costs (i.e., whenever $\mu = 0$), $g(\overline{\omega}_{j,t})$ is concave in $\overline{\omega}_{j,t}$ and $g'(\overline{\omega}_{j,t}) \geq 0$[24]. When monitoring costs are introduced in the model, there is an additional effect on $g(\overline{\omega}_{j,t})$. It can be shown that $g''(\overline{\omega}_{j,t}) < 0$ but, for sufficiently high values of $\overline{\omega}_{j,t}$, $g'(\overline{\omega}_{j,t}) < 0$ (i.e., whenever $\mu\phi(\overline{\omega}_{j,t}) > [1 - \Phi(\overline{\omega}_{j,t})]$). Therefore, with monitoring costs $g(\overline{\omega}_{j,t})$ becomes a hump shaped concave function, with a maximum at the value of $\overline{\omega}_{j,t}$ for which $\mu\phi(\overline{\omega}_{j,t}) = [1 - \Phi(\overline{\omega}_{j,t})]$, call it $\overline{\omega}^*_{j,t}$.

Observe that for a given $i_{j,t}$, the behavior of the lender's expected profits, $R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t})$, depends on the the behavior of $g(\overline{\omega}_{j,t})$. In particular, whenever $\overline{\omega}_{j,t} < \overline{\omega}^*_{j,t}$ a small rise in $\overline{\omega}_{j,t}$ must increase lender's expected profits. To gain intuition on this result, note that a small rise in $\overline{\omega}_{j,t}$ has three effects on lender's expected profits: i. Increases the expected gross revenue of what the lender would recoup when the entrepreneur defaults on the debt, ii. Increases the expected monitoring costs and iii. Reduces the expected nominal value of the debt repayment. Therefore, it must be true that the first effect overcomes the second and third effects when $g'(\overline{\omega}_{j,t}) > 0$, so as to have that the lender's expected profits increase in $\overline{\omega}_{j,t}$ when $\overline{\omega}_{j,t} < \overline{\omega}^*_{j,t}$.

2.4.4 Determining the optimal contract

The optimal debt contract will be determined by a pair of values of $i_{j,t}$ and $\overline{\omega}_{j,t}$ that maximizes the entrepreneur's expected profits, subject to the lender receiving at least the opportunity cost of the loan. Note that to ensure the optimality of the contract entrepreneurs expected profits must be maximized. Otherwise, there might be another lender that provides a different contract under better conditions.

In what follows, it is assumed that the entrepreneur's participation constraint, given by $R^k_{t+1} i_{j,t} f(\overline{\omega}_{j,t}) > R_{t+1} P_t n_{j,t}$, holds. This condition indicates that the gross nominal rate of return that the entrepreneur expects to obtain for undertaking the project, $\frac{R^k_{t+1} i_{j,t} f(\overline{\omega}_{j,t})}{P_t n_{j,t}}$, must be greater than the gross nominal interest, R_{t+1}.

The lender's participation constraint, in turn, is given by $R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t}) \geq R_{t+1} P_t (i_{j,t} - n_{j,t})$, indicating that the lender's expected gross nominal rate of return, $\frac{R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t})}{P_t (i_{j,t} - n_{j,t})}$, must be at least the opportunity cost of the loan, R_{t+1}. Assuming that there are a large number of lenders in

[24] $g'(\overline{\omega}_{j,t})$ will always be positive unless $\overline{\omega}_{j,t}$ takes the highest value for which ω is defined, in which case $g'(\overline{\omega}_{j,t}) = 0$.

this economy, we can invoke arbitrage conditions so as to guarantee that the lender's participation constraint binds.

The optimization problem that defines the optimal contract, therefore, can be stated as follows,

$$\max_{\{i_{j,t},\overline{\omega}_{j,t}\}} R^k_{t+1} i_{j,t} f(\overline{\omega}_{j,t}) \text{ s.t. } R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t}) = R_{t+1} P_t (i_{j,t} - n_{j,t}).$$

From the first order conditions it is possible to obtain,[25]

$$R^k_{t+1}\{g(\overline{\omega}_{j,t}) - \frac{f(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})} g'(\overline{\omega}_{j,t})\} = R_{t+1} P_t, \tag{20}$$

and

$$i_{j,t} = \frac{n_{j,t}}{1 - \frac{R^k_{t+1}}{R_{t+1}P_t} g(\overline{\omega}_{j,t})}. \tag{21}$$

It is worth observing that Eqs. 19, 20 and 21 constitute a system of three equations in three unknowns ($\overline{\omega}_{j,t}$, $R^{nd}_{j,t+1}$ and $i_{j,t}$), since $n_{j,t}$, P_t, R^k_{t+1}, and R_{t+1} are taken as given. In solving this system, notice that Eq. 20 gives an implicit function of the form,[26]

$$\overline{\omega}_{j,t} = F(\frac{R^k_{t+1}}{R_{t+1}P_t}) = \overline{\omega}_t, \text{ where } \frac{\partial \overline{\omega}_t}{\partial R^k_{t+1}} = \frac{F'(\cdot)}{R_{t+1}P_t} > 0. \tag{22}$$

Eq. 22 implies that, in equilibrium, the value of $\overline{\omega}_{j,t}$ is the same for all entrepreneurs (and thus it is denoted by $\overline{\omega}_t$). Using Eqs. 21 and 22 it is possible to rewrite the demand function for the input $i_{j,t}$ as,

$$i_{j,t} = \frac{n_{j,t}}{1 - \frac{R^k_{t+1}}{R_{t+1}P_t} g(\overline{\omega}_t)}. \tag{23}$$

This expression indicates that the demand function for the input $i_{j,t}$ linearly depends on the net worth of entrepreneur j, fact that facilitates aggregation. This result is a direct consequence of the linearity of the production function of capital, the monitoring technology and the entrepreneur budget constraint. Taking as given the net worth of entrepreneur j, moreover, Eq. 23 gives a positive relation between the rental price of capital, R^k_{t+1}, and the investment demand, $i_{j,t}$. Formally, differentiating this equation with respect to R^k_{t+1} it is possible to obtain,

$$\frac{\partial i_{j,t}}{\partial R^k_{t+1}} = \frac{1}{R_{t+1}P_t} \frac{i^2_{j,t}}{n_{j,t}} [g(\overline{\omega}_t) + R^k_{t+1} g'(\overline{\omega}_t) \frac{\partial \overline{\omega}_t}{\partial R^k_{t+1}}] > 0^{27}, {}^{28}. \tag{24}$$

[25]See Appendix A for details.

[26]See Appendix B for details.

[27]A sufficient condition for having $\frac{\partial i_{j,t}}{\partial R^k_{t+1}} > 0$ is that $g'(\overline{\omega}_t) > 0$. In Appendix C it is shown that this condition must hold in order to satisfy the second order conditions of the entrepreneur's maximization problem when ω is uniformly distributed in the interval $[0, 2]$. Moreover, the fact that in equilibrium $g'(\overline{\omega}_t) > 0$ can also be determined by analyzing the maximand and the constraint of the entrepreneur's optimization problem. To see this: let $g'(\overline{\omega}_t) < 0$. From the constraint, this fact implies that $\frac{\partial \overline{\omega}_t}{\partial i_t} < 0$. Using this result, we can see from the maximand that the entrepreneur can always increase expected profits by increasing investment (since $f'(\overline{\omega}_t) \leq 0$), fact that must not be true in equilibrium (i.e., entrepreneur's profits must be bounded).

[28]It is worth emphasizing the close link between the entrepreneur's optimal contracting problem and the modern

Also notice that Eq. 20 can now be rewritten as,

$$R_{t+1}^{k} = \{g(\overline{\omega}_t) - \frac{f(\overline{\omega}_t)}{f'(\overline{\omega}_t)}g'(\overline{\omega}_t)\}^{-1}R_{t+1}P_t. \tag{25}$$

Combining Eqs. 19, 22 and 23 we can compute the solution for $R_{j,t+1}^{nd}$,

$$R_{j,t+1}^{nd} = \frac{R_{t+1}^{k}\overline{\omega}_{j,t}}{P_t(1-\frac{n_{j,t}}{i_{j,t}})} = R_{t+1}\overline{\omega}_t g(\overline{\omega}_t)^{-1} = R_{t+1}^{nd}. \tag{26}$$

This equation indicates that, in equilibrium, the non-default interest rate will be the same for all entrepreneurs since it does not depend on any variable of entrepreneur j.

3 Equilibrium

3.1 Aggregate net worth and aggregate investment

A key variable of the model is given by the entrepreneur's net worth. For simplicity and taking the modelling device considered in Bernanke et al (1999), it will be assumed that there is a constant fraction of entrepreneurs υ dying each period, where death implies leaving the economy and consuming the accumulated net worth up to that point. The size of the population remains constant since for each entrepreneur that dies there is a newcomer entering the economy. As Carlstrom and Fuerst (2001, p. 7) point out, it can be thought of that those entrepreneurs who do not survive are 'informed' at the beginning of the period and thus they consume their end-of-period profits just an instant before dying.

Recall that $R_{t+1}^{k}f(\overline{\omega}_{j,t})i_{j,t}$ denotes the expected net profits of entrepreneur j at period t. Using the fact that in equilibrium $\overline{\omega}_{j,t} = \overline{\omega}_t$ and summing over j, we can define the net expected profits of the entrepreneurial sector as $R_{t+1}^{k}f(\overline{\omega}_t)i_t$, where i_t denotes aggregate investment (defined below). It then follows that:

$$P_{t+1}n_{t+1} = (1-\upsilon)R_{t+1}^{k}f(\overline{\omega}_t)i_t. \tag{27}$$

Recalling that $f(\overline{\omega}_t) = 1 - \mu\Phi(\overline{\omega}_t) - g(\overline{\omega}_t)$, nominal aggregate net worth at the beginning of period $t+1$ can be defined as,

$$P_{t+1}n_{t+1} = (1-\upsilon)R_{t+1}^{k}(1-\mu\Phi(\overline{\omega}_t) - g(\overline{\omega}_t))i_t.$$

Notice that the lender's constraint in the entrepreneur's maximization problem can be written in the aggregate as $R_{t+1}^{k}i_t g(\overline{\omega}_t) = R_{t+1}P_t(i_t - n_t)$. We can then obtain,

$$P_{t+1}n_{t+1} = (1-\upsilon)\{R_{t+1}^{k}(1-\mu\Phi(\overline{\omega}_t))i_t - R_{t+1}S_t B_{t+1}^{*}\}. \tag{28}$$

literature on credit rationing. In particular, the fact that in equilibrium it must be true that $g'(\overline{\omega}_t) > 0$, suggests that this model does not show 'equilibrium credit rationing' in the sense of Stiglitz and Weiss (1981). Therefore, on the upward sloping part of $g(\overline{\omega}_t)$, the lender may provide any extra funding required by the entrepreneur at a higher interest rate R_{t+1}^{nd}, since lender's expected profits increase in that region. In contrast, whenever $g'(\overline{\omega}_t) \leq 0$, any further increase in the interest rate, which is associated with a higher probability of default of the entrepreneur, reduces lender's expected profits thus giving place to a situation where credit rationing holds.

To gain intuition on the mechanism by which balance-sheet effects may affect the economy, it is useful to write the expression for net worth (in real terms) at the beginning of period $t+1$ noticing that $(1-\mu\Phi(\overline{\omega}_t))i_t = K_{t+1}$[29] :

$$n_{t+1} = (1-\upsilon)\{\alpha\frac{P_{N,t+1}}{P_{t+1}}Z_{t+1} - \frac{S_{t+1}}{P_{t+1}}B^*_{t+1}\},$$

where we have made use of Eq. 6 and the UIP condition. *Ceteris paribus,* an unexpected real exchange rate depreciation measured by a rise in S_{t+1}/P_{t+1} negatively affects n_{t+1}. There might be then a negative effect on the production of capital available at the beginning of period $t+2$. Note, however, that this is a partial equilibrium exercise. In general equilibrium, however, B^*_{t+1} can be affected by the evolution of i_t (notice that n_t will be predetermined), and similarly either $P_{N,t}/P_t$ and/or Z_t might be modified by any unexpected shock. In such a case, therefore, the effect on n_{t+1} can be different.

Aggregate consumption at period $t+1$, C^e_{t+1}, is hence given by,

$$P_{t+1}C^e_{t+1} = \upsilon\{R^k_{t+1}(1-\mu\Phi(\overline{\omega}_t))i_t - R_{t+1}S_tB^*_{t+1}\}. \tag{29}$$

Considering Eqs. 28 and 29 lagged one period, the budget constraint of the entrepreneurial sector at period t (*i.e.*, $S_tB^*_{t+1} = P_t(i_t - n_t)$) yields,

$$P_ti_t + P_tC^e_t + R_tS_{t-1}B^*_t = R^k_t(1-\mu\Phi(\overline{\omega}_{t-1}))i_{t-1} + S_tB^*_{t+1}. \tag{30}$$

Each period t, entrepreneurs invest P_ti_t to produce the capital good, consume $P_tC^e_t$ of the final produced good and repay capital and interests of the debt contracted at period $t-1$, $R_tS_{t-1}B^*_t$[30]. These expenditures are financed with the aggregate income obtained for renting the produced capital good to firms, $R^k_t(1-\mu\Phi(\overline{\omega}_{t-1}))i_{t-1}$, and by issuing new debt $S_tB^*_{t+1}$.

Aggregate investment can be obtained by summing over j the demand function for the input $i_{j,t}$ stated in Eq. 23, thus yielding,

$$i_t = (1-\frac{R^k_{t+1}}{R_{t+1}P_t}g(\overline{\omega}_t))^{-1}n_t = (1-\frac{f'(\overline{\omega}_t)g(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)})n_t, \tag{31}$$

where the last term in this expression is derived from Eq. 25. Eq. 31 shows that i_t linearly depends on n_t, the aggregate net worth available at the beginning of period t. It also indicates that, in equilibrium, aggregate investment at period t is determined by the aggregate net worth in the same period scaled by the factor $(1-\frac{f'(\overline{\omega}_t)g(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)})$, which can be thought of as a measure of the leverage ratio of the entrepreneurial sector as a whole.

Introducing Eqs. 25 and 31 into Eq. 28 it is possible to obtain,

[29]The fact that $(1-\mu\Phi(\overline{\omega}_t))i_t$ is equal to the supply of capital at period $t+1$ is discussed in detail in the next subsection.

[30]It is worth noting that, although each individual entrepreneur has to repay $R^{nd}_{j,t}S_{t-1}B^*_{j,t}$ to lenders (whenever the debt is repaid as established in the contract), at the aggregate level lenders receive the opportunity cost of the loan, $R_tS_{t-1}B^*_t$.

$$P_{t+1}n_{t+1} = (1-\upsilon)R_{t+1}\{-\frac{f'(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)}(1-\mu\Phi(\overline{\omega}_t))P_t n_t - S_t B^*_{t+1}\}. \tag{32}$$

Similarly, entrepreneurs' consumption can be expressed as,

$$P_{t+1}C^e_{t+1} = \upsilon R_{t+1}\{-\frac{f'(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)}(1-\mu\Phi(\overline{\omega}_t))P_t n_t - S_t B^*_{t+1}\}. \tag{33}$$

Eq. 32 defines the evolution of aggregate net worth. It indicates that entrepreneurs obtain in the aggregate the gross nominal return $-\frac{f'(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)}(1-\mu\Phi(\overline{\omega}_t))R_{t+1}$ for investing their aggregate net worth $P_t n_t$ to produce the capital good. They utilize this return to repay the amount $R_{t+1}S_t B^*_{t+1}$ for the debt contracted at period t. The difference between these two flows multiplied by $(1-\upsilon)$, the fraction of entrepreneurs' net profits not consumed, defines aggregate net worth at the beginning of period $t+1$.

Finally, note that from the budget constraint of the entrepreneurial sector and Eq. 31 it is possible to obtain the aggregate demand for credit:

$$S_t B^*_{t+1} = -P_t n_t \frac{f'(\overline{\omega}_t)g(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)}. \tag{34}$$

3.2 Aggregate supply of capital

From the previous discussion we know that a fraction $\mu\Phi(\overline{\omega}_t)$ of the total investment made by entrepreneur j at period t is expected to be lost owing to the presence of monitoring costs. The expected aggregate supply of capital is hence defined as,

$$K^s_{t+1} = i_t(1-\mu\Phi(\overline{\omega}_t)) = (1-\frac{f'(\overline{\omega}_t)g(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)})(1-\mu\Phi(\overline{\omega}_t))n_t, \tag{35}$$

where the last equality is obtained from Eq. 31. The existence of asymmetric information problems between lenders and entrepreneurs implies that the aggregate supply of capital, K^s_{t+1}, is a fraction $(1-\mu\Phi(\overline{\omega}_t))$ of what would be supplied under perfect information (i.e., whenever $\mu = 0$). Notice that, as R^k_{t+1} increases K^s_{t+1} is affected by two effects: i. Aggregate investment increases, positively affecting K^s_{t+1} and ii. Expected monitoring costs rises, negatively affecting K^s_{t+1}. The first effect, however, overcomes the second effect. This fact is formally assessed in the following remark:

Remark 1 *The model with monitoring costs provides an upward sloping supply curve of capital in the (R^k_{t+1}, K_{t+1}) space.*

Proof. Recalling that $f(\overline{\omega}_t) + g(\overline{\omega}_t) = 1 - \mu\Phi(\overline{\omega}_t)$, Eq. 35 can be rewritten as: $K^s_{t+1} = (1 - \frac{f'(\overline{\omega}_t)g(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)})(f(\overline{\omega}_t) + g(\overline{\omega}_t))n_t$. Since in equilibrium $g'(\overline{\omega}_t) > 0$, we know that i_t is an increasing function of R^k_{t+1}. We also know that $\frac{\partial\overline{\omega}_t}{\partial R^k_{t+1}} > 0$. To demonstrate that $\frac{\partial K^s_{t+1}}{\partial R^k_{t+1}} > 0$, it thus suffices to show that $\frac{\partial}{\partial\overline{\omega}_t}(f(\overline{\omega}_t) - \frac{f'(\overline{\omega}_t)g(\overline{\omega}_t)}{g'(\overline{\omega}_t)}) > 0$. Notice that $\frac{\partial}{\partial\overline{\omega}_t}(f(\overline{\omega}_t) - \frac{f'(\overline{\omega}_t)g(\overline{\omega}_t)}{g'(\overline{\omega}_t)}) = \frac{g(\overline{\omega}_t)}{g'(\overline{\omega}_t)}\{-f''(\overline{\omega}_t) + \frac{f'(\overline{\omega}_t)g''(\overline{\omega}_t)}{g'(\overline{\omega}_t)}\}$; expression that must be positive to satisfy the second order condition of the entrepreneur's maximization problem (see Appendix C). ■

3.3 Equilibrium conditions

To define the equilibrium of the model it is still necessary to specify: i. Money market equilibrium, ii. Goods market equilibrium, iii. Capital good market equilibrium, iv. Labor market equilibrium, v. Intertemporal balance of trade equilibrium and vi. Domestic credit market equilibrium.

3.3.1 Money market equilibrium

Money market equilibrium is given by Eq. 13 under the assumption that aggregate supply equals aggregate demand for real money balances.

3.3.2 Goods market equilibrium

To determine the equilibrium conditions in the goods market it is worth recalling that there are two sectors in this economy: one tradable and one nontradable. Since the only source of absorption of tradables is given by the demand for tradable inputs by the final producer firm we can denote the (nominal) trade balance at period t as,

$$P_{T,t}(\overline{Y}_T - X_{T,t}) = TB_t. \tag{36}$$

In a symmetric equilibrium we have that $P_{i,t} = P_{N,t}$ and that $Z_{i,t} = Z_t$ for all i. Therefore, the production function of the nontradable intermediate firm becomes $Z_t = A_t K_t^{\alpha} L_t^{1-\alpha}$. Owing to the existence of imperfect competition in this sector, it must be true that the aggregate income of intermediate firms equates the payment of the two factors of production plus any remaining profits or: $P_{N,t} Z_t = R_t^k K_t + W_t L_t + \pi_t$[31].

Regarding the final producer firm, in equilibrium the marginal cost equals the price level thus yielding:

$$P_t = \gamma^{-\gamma}(1-\gamma)^{(\gamma-1)} {P_{T,t}}^{1-\gamma} P_{N,t}^{\gamma}, \tag{37}$$

where $P_{N,t}$ is given by,

$$P_{N,t} = \frac{\theta}{\theta - 1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1} A_t^{-1} W_t^{1-\alpha} (R_t^k)^{\alpha}. \tag{38}$$

Note that in equilibrium the production function of the final producer is $Y_t = Z_t^{\gamma} X_{T,t}^{1-\gamma}$. Cost minimization hence implies $P_t Y_t = P_{N,t} Z_t + P_{T,t} X_{T,t}$, and thus the demand functions for the tradable and the nontradable inputs are given by:

$$P_{T,t} X_{T,t} = (1-\gamma) P_t Y_t, \tag{39}$$

and

[31] The profits of intermediate firms are given by $\pi_t = \frac{1}{\theta} P_{N,t} Z_t$. There is an inverse relation between π_t and θ, where the former converges to zero as the demand for intermediate inputs becomes perfectly elastic (i.e., $\theta \to \infty$).

$$P_{N,t}Z_t = \gamma P_t Y_t. \tag{40}$$

Finally, the clearing market condition for the final nontradable good implies,

$$Y_t = C_t + C_t^e + i_t. \tag{41}$$

3.3.3 Capital good market equilibrium

From Eq. 6, the aggregate demand for the capital good in period $t+1$ takes the form,

$$K_{t+1}^d = (\frac{1-\alpha}{\alpha})^{\alpha-1}\frac{Z_{t+1}}{A_{t+1}}(\frac{W_{t+1}}{R_{t+1}^k})^{1-\alpha}. \tag{42}$$

In equilibrium R_t^k will adjust so as to equate aggregate supply and aggregate demand for the capital good: $K_t^s = K_t^d = K_t$. From Eqs. 35, 42 and 38 we thus have that,

$$R_t^k = \alpha\frac{\theta-1}{\theta}\frac{P_{N,t}Z_t}{K_t}, \tag{43}$$

where K_t is given by Eq. 35.

3.3.4 Labor market equilibrium

From Eq. 7, in a symmetric equilibrium the aggregate demand for labor is given by: $L_t^d = (\frac{1-\alpha}{\alpha})^{\alpha}\frac{Z_t}{A_t}(\frac{W_t}{R_t^k})^{-\alpha}$. Using Eq. 38 and the equilibrium condition $L_t^s = L_t^d = L_t$ yields,

$$W_t = (1-\alpha)\frac{\theta-1}{\theta}\frac{P_{N,t}Z_t}{L_t}, \tag{44}$$

where L_t is given by Eq. 14.

3.3.5 Intertemporal balance of trade equilibrium

By adding the budget constraints of households, government and entrepreneurs we can obtain the budget constraint of the economy as a whole (i.e., the balance of payment):

$$\begin{aligned} & P_tC_t + P_tC_t^e + P_ti_t + S_tR_t^*B_t^* + S_tD_{t+1} \\ = \quad & P_{T,t}\overline{Y}_T + S_tB_{t+1}^* + R_t^k(1-\Phi(\overline{\omega}_{t-1}))i_{t-1} + W_tL_t + \pi_t + S_tR_t^*D_t. \end{aligned}$$

Using the fact that $R_t^k(1-\mu\Phi(\overline{\omega}_{t-1}))i_{t-1} = R_t^kK_t$; $P_{N,t}Z_t = R_t^kK_t + W_tL_t + \pi_t$; $P_tY_t = P_{N,t}Z_t + P_{T,t}X_{T,t}$, the definition of TB_t and the clearing market condition of the nontradable sector gives,

$$S_t(D_{t+1} - B_{t+1}^*) = TB_t + S_tR_t^*(D_t - B_t^*).$$

Let $F_t = D_t - B_t^*$ and $F_{t+1} = D_{t+1} - B_{t+1}^*$ denote the net foreign assets accumulated by the economy as a whole at periods t and $t+1$, respectively, in foreign currency. The balance of payments of the economy can thus be written as,

$$S_t F_{t+1} = TB_t + S_t R_t^* F_t. \tag{45}$$

Observe that Eq. 45 implicitly defines an equilibrium condition between the domestic economy and the rest of the world, in which any excess supply (demand) of tradable goods is automatically satisfied at the price $P_{T,t}$ while any excess supply (demand) of foreign assets is cleared at the foreign risk-free interest rate R_t^*. We can iterate forward Eq. 45 obtaining:

$$-S_0 R_0^* F_0 = \sum_{t=0}^{\infty} TB_t(1.S_0(S_1R_1^*)^{-1}...S_{t-1}(S_tR_t^*)^{-1}) + \lim_{T\to\infty} S_0(-F_{T+1})(1.(R_1^*)^{-1}...(R_T^*)^{-1}).$$

In order to guarantee that the economy is solvent from an intertemporal perspective we impose the 'no-Ponzi-game-condition': $\lim_{T\to\infty} S_0(-F_{T+1})(1.(R_1^*)^{-1}...(R_T^*)^{-1}) = 0$, thus obtaining the intertemporal national budget constraint,

$$-S_0 R_0^* F_0 = \sum_{t=0}^{\infty} TB_t(1.S_0(S_1R_1^*)^{-1}...S_{t-1}(S_tR_t^*)^{-1}). \tag{46}$$

3.3.6 Domestic credit market equilibrium

By Walras' law, equilibrium in the domestic credit market is guaranteed whenever the remaining markets of the economy are in equilibrium. In this context, the condition becomes an identity which is given by:

$$S_t B_{t+1}^* \equiv S_t(D_{t+1} - F_{t+1}),$$

where the aggregate demand for credit (i.e., $S_t B_{t+1}^*$) is defined in Eq. 34; while the aggregate supply is given by $D_{t+1} - F_{t+1}$. The total credit available in the economy is thus provided by domestic households in the form of deposits D_{t+1}, and by foreigners in the form of net foreign liabilities, $-F_{t+1}$.

4 Zero-inflation symmetric steady state

In this section the solution of the model in a zero-inflation symmetric steady state is described[32]. For brevity we leave the derivation of the main equations discussed here to Appendix E. Nevertheless, since $\overline{\omega}_t$ is a non-conventional variable present in this model it is worth going with some detail through its derivation. Let us start with the steady state version of Eq. 32:

[32] It could have been also possible to choose a different steady state to undertake the analysis; for instance one in which the money supply growth rate and therefore the inflation rate are both constant. For simplicity, and since our main results do not depend on the chosen steady state we have decided to consider the analytically simpler zero-inflation case.

$$n[f(\overline{\omega})g'(\overline{\omega}) + (1-\upsilon)Rf'(\overline{\omega})(f(\overline{\omega}) + g(\overline{\omega}))] = -b(1-\upsilon)Rf(\overline{\omega})g'(\overline{\omega}), \tag{47}$$

where $b \equiv \frac{SB^*}{P}$. Observe that this equation involves four endogenous variables: $\overline{\omega}$, n, R and b. However, from the Euler equation derived in the households' maximization problem, we have that R is pinned-down and given by $R = \beta^{-1}$. It follows then that Eq. 47 only contains three endogenous variables: $\overline{\omega}$, n and b. It is possible to obtain another expression in the same variables considering the aggregate demand for credit defined in Eq. 34, thus yielding:

$$b = -\frac{f'(\overline{\omega})g(\overline{\omega})}{f(\overline{\omega})g'(\overline{\omega})}n. \tag{48}$$

These two equations pin-down $\overline{\omega}$. To see this, notice that the combination of Eqs. 47 and 48 gives,

$$-\frac{g'(\overline{\omega})}{f'(\overline{\omega})} = (1-\upsilon)\beta^{-1}. \tag{49}$$

In this equation there is only one endogenous variable $\overline{\omega}$, whose solution can now be obtained (call it $\overline{\omega}^*$). With the value of $\overline{\omega}^*$ at hand, it will later be possible to solve for n and b. It can also be shown that entrepreneurial consumption takes the form: $C^e = \frac{\upsilon}{1-\upsilon}n$, indicating that a share $\frac{\upsilon}{1-\upsilon}$ of entrepreneur's aggregate net worth is devoted to consumption in the steady state.

Since ω is a random variable, solving the model analytically requires assuming a distribution function for it. To simplify matters, we assume that ω is uniformly distributed in the interval $[0, 2]$. Therefore, Eq. 49 yields (see Appendix D for details):

$$\overline{\omega} = 2 + \frac{\mu\beta}{1-(\beta+\upsilon)} \equiv \overline{\omega}^*. \tag{50}$$

Note that for $\overline{\omega}^*$ to be within the interval $[0, 2]$ it is further required that $\beta + \upsilon > 1$ and that $-2 \leq \frac{\mu\beta}{1-(\beta+\upsilon)} \leq 0$. From here onwards we will assume that these two steady state restrictions are satisfied. It is worth observing also that the parameters affecting $\overline{\omega}^*$ are only μ, β and υ. It follows then that the steady state solution of $\overline{\omega}$ is not affected by those parameters related to production functions or preferences other than β. Observe that having derived $\overline{\omega}^*$ we can obtain the associated steady state values $f(\overline{\omega}^*)$ and $g(\overline{\omega}^*)$. With the expression of $g(\overline{\omega}^*)$ it is then easy to see that the restriction $g'(\overline{\omega}^*) > 0$ requires $\upsilon < 1$, condition that is always satisfied since $\upsilon \in (0, 1)$[33].

4.1 Analysis of the remaining steady state variables of the model

In this subsection we briefly discuss the solutions of some endogenous variables of the model not directly related to the presence of credit market imperfections. With a few exceptions, the model does not provide simple closed-form solutions unless we set the steady state value of (real) net

[33]This can be easily seen by noting that $g'(\overline{\omega}^*) = \frac{\mu}{2}\frac{1-\upsilon}{\beta+\upsilon-1}$ when ω is uniformly distributed in the interval $[0, 2]$ (see Appendix D).

foreign assets to zero: $\digamma = \frac{SF}{P} = 0$[34]. From Appendix E it is then possible to show that in this case $X_T = \overline{Y}_T$ and thus the steady state real exchange rate takes the form $s = Y(1-\gamma)\overline{Y}_T^{-1}$. The solutions for output, consumption, net worth and labor are given by:

$$Y = \{1 - \alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*) + \beta g(\overline{\omega}^*)}{f(\overline{\omega}^*) + g(\overline{\omega}^*)}\}^{\frac{(1-\alpha)\gamma}{2(\alpha\gamma-1)}}\{A^{-1}\overline{Y}_T^{\frac{\gamma-1}{\gamma}}(\frac{r^{k*}}{\alpha})^{\alpha}(\gamma\frac{\theta-1}{\theta})^{-\frac{1+\alpha}{2}}(\frac{\kappa}{1-\alpha})^{\frac{1-\alpha}{2}}\}^{\frac{\gamma}{\alpha\gamma-1}} \tag{51}$$

$$C = Y\{1 - \alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*) + \beta g(\overline{\omega}^*)}{f(\overline{\omega}^*) + g(\overline{\omega}^*)}\}, \tag{52}$$

$$n = Y\alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)(1-\upsilon)}{f(\overline{\omega}^*) + g(\overline{\omega}^*)}, \tag{53}$$

and

$$L = (\gamma\frac{\theta-1}{\theta}\frac{1}{\kappa}\frac{1}{1 - \alpha\beta\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}})^{1/2}. \tag{54}$$

In the general case in which $\digamma \neq 0$, it can be shown that taking this variable as 'given'[35] a higher $\digamma$ raises Y, and through it there is a positive impact on C and n but L remains unaffected. More generally, it can be shown that the solution of L is independent of $\digamma$, result that is directly associated with the type of preferences assumed in the model[36].

Another interesting exercise to consider is an increase in the exogenous variable $\overline{Y}_T$. From Eq. 51 we can see that Y must be at a higher level after the change in $\overline{Y}_T$ and thereby C and n (note that L is also unaffected in this case). It is possible to also see that s decreases and that the real wage w rises. At the steady state level, we can thus observe some sort of 'Dutch-disease' phenomenon, where an exogenous expansion in the tradable sector is spread into the rest of the economy. Hence, there is an expansion of nontradable output and consumption, but there is also a more appreciated real exchange rate and a higher real wage in the long-run. In fact, the rise in $\overline{Y}_T$ can be viewed as a positive wealth shock affecting the economy, with implications that are somehow intuitive.

We close the steady state analysis discussing the solution of a number of variables relative to K. Notice that these ratios are unaffected by steady state net foreign assets. For a better understanding of the implications of financial frictions we also consider the case in which $\mu = 0$.

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[34] A clear exception is $\overline{\omega}$, whose solution was already derived.

[35] To be more precise, $\digamma$ is an endogenous variable determined in general equilibrium where domestic choices are mutually consistent with the restriction imposed by the national intertemporal budget constraint.

[36] The implications of this property will become apparent when studying the dynamic behavior of the model.

Table 1. Selected steady state solutions relative to K

Variable	Credit markets imperfections case: $\mu \in (0,1]$	RBC case: $\mu = 0$
$\frac{Y}{K}$	$r^{k*}\frac{1}{\alpha\gamma}\frac{\theta}{\theta-1}$	$\frac{1}{\beta}\frac{1}{\alpha\gamma}\frac{\theta}{\theta-1}$
$\frac{i}{K}$	$\frac{1}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}$	1
$\frac{b}{K}$	$r^{k*}\frac{\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}$	1
$\frac{n}{K}$	$r^{k*}\frac{(1-\upsilon)f(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}$	0

Without credit market imperfections it can be easily seen that $f(\overline{\omega}^*) = 0$ and that $g(\overline{\omega}^*) = 1$[37]. In this situation we arrive to a fairly standard RBC model, similar to that developed, for instance, in McCallum (1989)[38],[39]. Since we have assumed that capital fully depreciates within the period, when $\mu = 0$ investment i and the capital stock K coincide, as expected: $i/K = 1$. With credit market imperfections we have a different result: $i/K > 1$. The fact that $\mu > 0$ implies that there is a fraction of investment which is lost due to monitoring.

Notice that, although the steady state level of output is higher in the RBC case, the ratio Y/K is lower compared to that when $\mu > 0$. This fact is a direct consequence of the relatively higher increase in K compared to that of Y, which is obtained when monitoring costs are eliminated.

An interesting feature of the model is the fact that as $\mu \to 0$, $n \to 0$. Observe that since the real amount of debt b and the stock of capital K coincide we also have that $b/K = 1$ and hence $n/K = 0$ as shown in Table 1. In this situation we also have that $C^e \to 0$, implying that entrepreneurs' consumption must also be zero. We emphasize this property of the model in the following remark:

Remark 2 *Without credit market imperfections entrepreneurs disappear as a differentiated class from households. In this case there is no need to accumulate net worth. The supply of capital in the economy is hence directly provided by households in the form of savings for future consumption; and thus can be interpreted as a one to one linear transformation from savings:* $b \equiv K = i = Y - C$.

5 Linearized model

To study the dynamics of the model it is useful to take advantage of the dichotomy which exists between the monetary and the real side of the economy, due to the assumed households' logarithmic preferences. A similar approach for solving their respective models is taken, for instance, in Benassy (1995) and in Fender and Rankin (2003).

[37]See Appendix D for details.

[38]For the case in which McCallum assumes full capital depreciation, a Cobb-Douglas production function and logarithmic preferences.

[39]It should be noted, however, that other relevant differences between our model and McCallum's framework remain. He considers a closed-economy model, with flexible prices and without money. However, the assumption previously made that $f \equiv 0$ and that the nominal price rigidity only lasts one period imply that the steady state solutions of the two models do not essentially differ for that reasons. Both solutions are different, however, since McCallum assumes perfectly competitive sectors. Since we are introducing monopolistic competition there are slight differences in the steady state solutions.

Since this model is highly non-linear, however, we will analyze its dynamic properties undertaking a linear approximation of it about a reference steady state in which inflation is zero. With a few exceptions, a lower-case variable denotes a percentage deviation of the original variable with respect to that reference steady state. For instance, for any variable X_t we define $x_t \equiv \frac{X_t - X^{RSS}}{X^{RSS}}$ $\left(\approx \log \frac{X_t}{X^{RSS}}\right)$, where X^{RSS} is the value of X_t in the reference steady state[40]. We now start analyzing the linear approximation of those variables associated with the monetary side of the model.

$$x_t \equiv m_t - p_t - c_t \tag{55}$$

$$h_t \equiv m_t - m_{t+1} \tag{56}$$

$$x_{t+1} = \beta^{-1} x_t - h_t \tag{57}$$

$$x_t = -\frac{\beta}{1-\beta} r_{t+1} \tag{58}$$

$$s_{t+1} - s_t = r_{t+1}. \tag{59}$$

Defining the demand for real money balances per unit of consumption as X_t ($\equiv \frac{M_t}{P_t C_t}$) and the inverse of the (gross) growth rate of money supply between $t+1$ and t as H_t ($\equiv \frac{M_t}{M_{t+1}}$), it is easy to see that Eqs. 55 and 56 are log-linear versions of these two equations. Solving for R_{t+1} Eq. 12 (introducing the UIP condition) and replacing the obtained result into the demand for money equation derived in Eq. 13 gives, after linearizing, Eq. 57. The log-linearization of the money demand equation yields Eq. 58. The UIP condition does not require approximation and, assuming that the risk-free foreign interest rate R^*_{t+1} is constant over time, applying logarithms to it and differentiating yields Eq. 59.

The solution of the model becomes easier by firstly solving Eq. 57, which is a first order linear difference equation in the forward-looking variable x_t. Since $\beta < 1$, this difference equation is unstable in its forward dynamics. Assuming that h_t is constant over time (i.e., $h_t = h \ \forall \ t$), saddle point stability requires that x_t immediately jumps to the steady state value $\frac{h}{\beta^{-1}-1}$.

Since the economic policy exercise in which we are interested is a permanent and unanticipated change in the log-deviation of the money supply at time t, we further have that $h = 0$ (i.e., $m_t = m_{t+1} \equiv \overline{m}$)[41]. In this case, therefore, x_t is unaffected by the monetary shock ($x_t = 0$).

Notice that Eq. 55 implies that $c_t = \overline{m} - p_t$, and thus consumption and real money balances move together over time. From Eq. 58 we also have that $r_{t+1} = 0$ and hence Eq. 59 gives $s_t = s_{t+1} \equiv \overline{s}$.

[40] A convenient way to obtain log-deviations is proposed in Uhlig (1999). For any variable X_t we can define: $X_t = X^{RSS} e^{x_t}$. A first order Taylor approximation about the point $x_t = 0$ gives, after rearranging, $x_t \equiv \frac{X_t - X^{RSS}}{X^{RSS}}$.

[41] Observe that $\overline{m}$ can be thought of as the percentage deviation of the new steady state level of the money supply ($=M^{RSS\prime}$) with respect to its pre-shock steady state level ($=M^{RSS}$): $\overline{m} \equiv \frac{M^{RSS\prime} - M^{RSS}}{M^{RSS}}$.

This is an important implication of the model since it embeds the fact that the nominal exchange rate immediately jumps to its new steady state value after the shock. This model does not show, therefore, non-trivial (nominal) exchange rate dynamics as in the case of the well-known overshooting model of Dornbusch (1976)[42].

It is worth highlighting that this dichotomy between the monetary and the real side of the model is not complete since $\overline{s}$ is an endogenous variable and we still have to solve for it. To do that, it will be necessary to consider the real side of the model, the direction in which we are now moving.

To facilitate the exposition, below is presented a list with the key variables of the model, where we have made use of the results previously obtained for the monetary sector.

$$c_{t+1} = p_t - p_{t+1} + c_t \tag{60}$$

$$c_t = \overline{m} - p_t \tag{61}$$

$$z_t = a_t + \alpha k_t + (1-\alpha) l_t \tag{62}$$

$$y_t = \gamma z_t + (1-\gamma) x_{T,t} \tag{63}$$

$$y_{T,t} = 0 \tag{64}$$

$$x_{T,t} = y_t - (\overline{s} - p_t) \tag{65}$$

$$z_t = y_t - (p_{N,t} - p_t) \tag{66}$$

$$p_t = (1-\gamma)\overline{s} + \gamma p_{N,t} \tag{67}$$

$$p_{N,t} = -a_t + (1-\alpha) w_t + \alpha r_t^k \tag{68}$$

$$\widehat{i}_t = \sigma_1 \widehat{n}_t + \sigma_2 b_{t+1} \tag{69}$$

$$\widehat{\omega}_t = \sigma_6 \widehat{n}_t - \sigma_6 b_{t+1} \tag{70}$$

[42] The fact that the time-path of the domestic interest rate r_{t+1} is unaffected by the monetary shock is the reason behind the no-overshooting result. It can be shown that with a more general specification of preferences such as $U_t = \sum_{t=0}^{\infty} \beta^t [\log C_t + \frac{\chi}{1-\epsilon} \log(\frac{M_t}{P_t})^{1-\epsilon} - \frac{\kappa}{2}(L_t)^2]$, where the consumption-elasticity of money demand ϵ is different from 1, it is possible to recover the overshooting effect. In this case r_{t+1} is affected by the monetary shock.

$$r^k_{t+1} - p_t = \sigma_1 b_{t+1} - \sigma_1 \widehat{n}_t + \sigma_3 \widehat{\omega}_t \tag{71}$$

$$r^{nd}_{t+1} = (1+\sigma_3)\widehat{\omega}_t \tag{72}$$

$$\widehat{n}_{t+1} = c^e_{t+1} = b_{t+1} - (p_{t+1} - p_t) + \sigma_4 \widehat{\omega}_t \tag{73}$$

$$k_{t+1} = -\sigma_5 \widehat{\omega}_t + \sigma_1 \widehat{n}_t + \sigma_2 b_{t+1} \tag{74}$$

$$r^k_t - p_{N,t} = z_t - k_t \tag{75}$$

$$w_t - p_{N,t} = z_t - l_t \tag{76}$$

$$l_t = w_t - p_t - c_t \tag{77}$$

$$y_t = \sigma_7 c_t + \sigma_8 c^e_t + \sigma_9 \widehat{i}_t \tag{78}$$

$$-x_{T,t} = \tau_t \tag{79}$$

$$-\beta^{-1} F_0 = \sum_{t=0}^{\infty} \beta^t \tau_t \tag{80}$$

where:

$\sigma_1 \equiv (1 + \frac{\beta g(\overline{\omega}^*)}{(1-\upsilon) f(\overline{\omega}^*)})^{-1}; \sigma_{1/\mu=0} \equiv 0$
$\sigma_2 \equiv (1 + \frac{(1-\upsilon) f(\overline{\omega}^*)}{\beta g(\overline{\omega}^*)})^{-1};\ \sigma_{2/\mu=0} \equiv 1$
$\sigma_3 \equiv \frac{1}{4}\frac{(\overline{\omega}^*)^2}{g(\overline{\omega}^*)} - 1;\ \sigma_{3/\mu=0} \equiv 0$
$\sigma_4 \equiv \frac{1}{4}(\overline{\omega}^*)^2 (g(\overline{\omega}^*)^{-1} + f(\overline{\omega}^*)^{-1}) - f(\overline{\omega}^*)^{-1};\ \sigma_{4/\mu=0} \to -\infty$
$\sigma_5 \equiv (f(\overline{\omega}^*) + g(\overline{\omega}^*))^{-1} - 1;\ \sigma_{5/\mu=0} \equiv 0$
$\sigma_6 \equiv \frac{(\overline{\omega}^*-2)(\mu+\overline{\omega}^*-2)}{2\overline{\omega}^{*2}+\mu\overline{\omega}^*-4\overline{\omega}^*+(2\overline{\omega}^{*2}+2\mu\overline{\omega}^*-4\overline{\omega}^*)\frac{(1-\upsilon)}{\beta}\frac{f(\overline{\omega}^*)}{g(\overline{\omega}^*)}};\ \sigma_{6/\mu=0} \equiv 0$
$\sigma_7 \equiv \frac{C}{Y} = 1 - \alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)};\ \sigma_{7/\mu=0} \equiv 1 - \alpha\gamma\beta\frac{\theta-1}{\theta}$
$\sigma_8 \equiv \frac{C^e}{Y} = \alpha\upsilon\gamma\frac{\theta-1}{\theta}(1 + \frac{g(\overline{\omega}^*)}{f(\overline{\omega}^*)})^{-1};\ \sigma_{8/\mu=0} \equiv 0$
$\sigma_9 \equiv \frac{i}{Y} = \alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)(1-\upsilon)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)};\ \sigma_{9/\mu=0} \equiv \alpha\gamma\beta\frac{\theta-1}{\theta}.$

Eq. 60 is derived from the Euler equation for consumption, where we have considered the fact that $r_{t+1} = 0$. Eq. 55 is rewritten in Eq. 61 for the case in which $x_t = 0$. The supply side of the model is described in Eqs. 62-64. The log-linear production function of intermediate firms is defined in Eq. 62, while that of the final producer firm is given by Eq. 63. The log-linear expression for the tradable output, which is equal to zero owing to the assumption that its supply is exogenous and constant over time, is given by Eq. 64.

Eqs. 65 and 66 are log-linear versions of the input demand functions of the final producer firm stated in Eqs. 39 and 40, respectively. We have used the fact that the LOOP holds and thus $p_{T,t} = \overline{s}$. The linearized price index of the economy is represented by Eq. 67; while that of nontradable goods is given by Eq. 68.

The presence of credit market imperfections is essentially reflected in Eqs. 69-74. The linear approximation of aggregate investment (i.e., Eq. 31), is given by Eq. 69[43], where we have used the following definition $b_{t+1} \equiv b^*_{t+1} + \overline{s} - p_t$. Similarly, Eq. 70 is the linear approximation of the aggregate demand for credit defined in Eq. 34. Eqs. 71-74 are linear approximations of Eqs. 25, 26, 32, 33 and 35, respectively, considering Eq. 34. It is worth observing that since entrepreneurs' net worth and consumption are constant fractions of expected profits ($(1-\upsilon)$ and υ, respectively), the log-deviation of these variables is the same. Eqs. 75-77 are log-linear versions of Eqs. 43, 44 and 14, respectively. Eqs. 78 and 79 are linear approximations of the clearing market condition for the nontradable sector and the definition of the trade balance surplus incorporating the result that $y_{T,t} = 0$. Regarding this linearization, it is worth highlighting that to facilitate studying the analytics of the model we have approximated it about a reference steady state in which the trade balance surplus is zero. It follows then that in this steady state net foreign assets are also zero. We thus define τ_t ($\equiv \frac{TB_t}{(S\overline{Y}_T)^{RSS}}$) as the absolute deviation of the trade balance surplus at period t deflated by the value of tradable output in the steady state of reference. The linear approximation of the intertemporal national budget constraint gives Eq. 80. In this expression we define $\digamma_0\{\equiv S_0F_0/(S\overline{Y}_T)^{RSS}\}$ as the absolute deviation of (inherited) net foreign assets deflated by the value of tradable output at that reference steady state. Notice that F_0 is given by the previous history of the model. The coefficients σ_i, $i = 1, ..., 9$, are complicated functions of the structural parameters of the model. For future comparisons we also include the values for the case without credit market imperfections (i.e., $\mu = 0$), which we denote as $\sigma_{i/\mu=0}$, $i = 1, ..., 9$. Notice that $\sigma_{4/\mu=0} \to -\infty$, implying that net worth is not well defined in such a case. This is a direct consequence of the fact that the steady state level of net worth is zero when $\mu = 0$.

6 Dynamics

Since we are studying a monetary shock, to simplify the analysis it will be assumed that $a_t = 0 \; \forall \; t$[44]. Let $\overline{e} \equiv \overline{s} - \overline{m}$ denote the difference between the log-deviation of the exchange rate and the money supply, respectively. In Appendix F it is shown that Eqs. 60-78 can be reduced to the following

[43]In obtaining Eq. 69 we also considered the aggregate demand for credit: $\frac{S_t B^*_{t+1}}{P_t} = -n_t \frac{f'(\overline{\omega}_t) g(\overline{\omega}_t)}{f(\overline{\omega}_t) g'(\overline{\omega}_t)}$.

[44]The model can be extended to also consider technology shocks. It will then be required to relax the assumption that $a_t = 0 \; \forall t$ and to define a law of motion for this variable. We leave this interesting exercise for future research.

minimum state-space form:

$$\begin{bmatrix} k_{t+1} \\ \widehat{n}_{t+1} \\ c_{t+1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} k_t \\ \widehat{n}_t \\ c_t \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \overline{e}, \tag{81}$$

where the coefficients $a_{i,j}$ and b_j, $i, j = 1, 2, 3$, are complicated expressions of the structural parameters of the model, also defined in this appendix. Notice that this representation takes the value of the endogenous variable $\overline{s}$ as given. This variable is not affecting the matrix of coefficients that pre-multiplies the vector of endogenous variables, however, and thus will not affect the speed of convergence toward the new steady state after the shock. But we then have to solve for $\overline{s}$. The strategy for doing this is as follows. We first obtain the time-paths of k_t, $\widehat{n}_t$ and c_t. Then, using Eq. 79 and the intertemporal national budget constraint which is defined in Eq. 80, we should obtain one equation in one unknown (i.e., $\overline{s}$); from which the nominal exchange rate is determined. There will be a unique value of the exchange rate that guarantees the solvency of the economy from an intertemporal perspective.

Observe that Eq. 81 defines a non-homogeneous first-order system of linear difference equations, in which the capital stock k_t and net worth n_t are predetermined or state variables at each period t, while c_t is a non-predetermined or 'jump' variable. At $t = 0$, in particular, notice that $k_0 (\equiv \log K_0/K^{RSS})$ and $\widehat{n}_0$ ($\equiv \log N_0/N^{RSS}$) are given by the previous history of the model. If we assume that before the shock hits the economy the system was located at an initial steady state equal to the reference steady state, it then follows that $k_0 = \widehat{n}_0 = 0$. We will assume that this is the case here. To study the dynamic behavior of the model it is necessary to first evaluate the roots of the above system of linear difference equations. Notice that satisfying the saddle point stability property will require to have two roots lying inside and one root lying outside the unit circle in absolute value. Since the coefficients $a_{i,j}$ and b_j, $i, j = 1, 2, 3$ are highly nonlinear in the structural parameters of the model, however, to proceed studying its dynamics it becomes necessary to undertake some sort of numerical exercise.

6.1 Parameterization

In the following table we define the baseline parameter values considered in the paper:

Table 2. Baseline parameter values

μ	υ	α	β	θ	γ	χ
0.12	0.12	0.33	0.99	10	0.925	0.342

We firstly discuss those parameters associated with the presence of credit market imperfections. Regarding μ we follow Carlstrom and Fuerst (1997)'s notion that monitoring costs are equivalent to bankruptcy costs. In case of default, hence, the lender will have to incur in direct costs (e.g., legal costs) but also indirect costs to seize the entrepreneur's project. The latter are associated with those costs in which the lender must incur for having entrepreneur's assets idle while these are being liquidated. We then set $\mu = 0.12$. This value is the same as in Bernanke et al (1999) but is below of that considered in Carlstrom and Fuerst ($\mu = 0.25$). Turning to υ, notice that its value is restricted by the following steady state relations: $\beta + \upsilon > 1$ and $-2 \leq \frac{\mu\beta}{1-(\beta+\upsilon)} \leq 0$, under the assumption that ω_t is uniformly distributed in the interval $[0, 2]$. Since the share of expected profits consumed by entrepreneurs is likely to be relatively low we define $\upsilon = 0.12$. This value implies that, in steady state, entrepreneurs consume 12% of their net expected profits.

The rest of the parameters are taken, essentially, from Chari et al (2002). From their paper we consider then: $\alpha = 0.33$, $\beta = 0.99$, and $\theta = 10$. Taking US annual data for the period 1948-2006 we take a simple average of the ratio exports / GDP, in order to obtain a rough figure of $1 - \gamma$ (the share of tradable inputs in the final nontradable good). This calculation gives $\gamma \simeq 0.925$. Finally, to define the value of χ, the weight in the utility function of real money balances, we have taken an indirect approach. Chari et al estimate the following regression for the US economy (period 1960:1-1995:4): $\log \frac{M}{P} = -\eta \log \frac{\omega}{1-\omega} + \log c - \eta \log \frac{R-1}{R}$. They obtain the values: $\eta = 0.39$ and $\omega = 0.94$. Considering our money-demand specification (see Eq. 13) we can indirectly obtain the following relation: $\chi = (\frac{\omega}{1-\omega})^{-\eta} \simeq 0.342$, which is the value at which we set χ.

6.2 Analysis under flexible prices

The saddle point stability property is satisfied since, under the parameterization specified above, we have two roots inside, λ_1 and λ_2, and one root outside, λ_3, the unit circle (see Table 3).

There is an extensive literature that discusses how to solve dynamic models under the Rational Expectations hypothesis. In this regard, a widely used approach is to follow the method of undetermined coefficients[45]. In our view, however, sometimes it is difficult to understand in depth the economic and mathematical conditions that this method requires to obtain finite or bounded solutions.

An economically more transparent approach and, perhaps, mathematically more elegant is that developed by Blanchard and Kahn (1980). Essentially this method consists in solving the system of linear difference equations as a boundary-value problem; where a set of initial and terminal conditions are imposed so as to obtain the unique convergent saddle-path. This solution is, of course, equivalent to the one that can be obtained following the method of undetermined coefficients.

It is possible to show that we can express the stable solution (conditional on $\overline{s}$) by solving mathematically the system of difference equations and setting to zero the constant of integration associated with the unstable root[46],[47]. Being explicit, its solution takes the form,

[45] See Uhlig (1999) for a thorough exposition.

[46] This approach is extensively used in Sargent (1987).

[47] To be more precise, in the present case we are imposing the following set of conditions: i) initial conditions on the predetermined or state variables k_t and $\widehat{n}_t$, indicating that their initial values k_0 and $\widehat{n}_0$ are taken as given and ii) a final condition on the 'jump' or non-predetermined variable c_t, $\lim_{T\to\infty} \lambda_3^{-T} c_T = 0$, implying that consumption must grow at a rate lower that λ_3.

$$
\begin{bmatrix} k_t \\ \widehat{n}_t \\ c_t \end{bmatrix} = \kappa_1 \begin{bmatrix} v_{11} \\ v_{21} \\ v_{31} \end{bmatrix} (\lambda_1)^t + \kappa_2 \begin{bmatrix} v_{12} \\ v_{22} \\ v_{32} \end{bmatrix} (\lambda_2)^t + \begin{bmatrix} ss_1 \\ ss_2 \\ ss_3 \end{bmatrix} \overline{e}, \tag{82}
$$

where letting $i = 1,2,3$ and $j = 1,2$ we have that: $v_{i,j}$ are components of the associated eigenvectors of each eigenvalue λ_j; κ_j are constants of integration to be determined using the fact that k_0 and $\widehat{n}_0$ are given values and ss_i are steady state coefficients. Observe then that the solutions of the constants of integration will be of the form: $\kappa_i = \eta_{\kappa j,e}\overline{e}$. Considering the baseline specification presented previously, the solution of the system is completed with the components of the right hand side of Eq. 82 taking the following values:

Table 3. Solution under baseline parameter values

λ_1	λ_2	λ_3	v_{11}	v_{21}	v_{31}	v_{12}	v_{22}	v_{32}	ss_1	ss_2	ss_3	$\eta_{\kappa 1,e}$	$\eta_{\kappa 2,e}$
0.31	0.94	3.75	-0.92	-0.28	-0.28	-0.03	-1.00	-0.01	-0.11	-0.11	-0.11	-0.12	-0.08

For simplicity, it becomes easier to analyze the solution of the model in terms of the recursive equilibrium law of motion of the three endogenous variables conditional on $\overline{s}$:

$$
\begin{aligned}
c_t &= \eta_{c,k}k_t + \eta_{c,n}\widehat{n}_t + \eta_{c,e}\overline{e}, \\
k_{t+1} &= \eta_{k,k}k_t + \eta_{k,n}\widehat{n}_t + \eta_{k,e}\overline{e}, \\
\widehat{n}_{t+1} &= \eta_{n,k}k_t + \eta_{n,n}\widehat{n}_t + \eta_{n,e}\overline{e},
\end{aligned}
$$

where, as it is conventional in dynamic macroeconomic models, the coefficients $\eta_{p,q}$ are the elasticities of each variable p with respect to variable q. Observe that $\overline{e}(\equiv \overline{s} - \overline{m})$ acts as an impulse affecting the dynamics of the endogenous variables only if there is a discrepancy between $\overline{s}$ and $\overline{m}$. Hence, in the case in which $\overline{e} = 0$ there is no perturbation that moves the system away from the initial steady state.

We are now ready to start solving for $\overline{s}$. For doing this we require the intertemporal national budget constraint. But we first need the solution for $x_{T,t}$, which can be expressed as $x_{T,t} = \eta_{x,k}k_t + \eta_{x,n}n_t + \eta_{x,e}\overline{e}$. Using this solution in combination with Eq. 82, we are then able to solve for $\overline{s}$. For concreteness, we present in the table below the elasticities $\eta_{p,q}$ for a set of endogenous variables considering different values of μ but leaving the rest of the parameters as in the benchmark specification. In particular, we address here three cases: i. $\mu = 0.01$, which essentially reflects a situation in which credit market imperfections are negligible, ii. $\mu = 0.12$, which gives the baseline value of μ and finally iii. $\mu = 0.2$, case in which financial frictions are at its highest level while at the same time the steady state restrictions on μ, β and υ are satisfied.

Table 4. Equilibrium law of motion under flexible prices

	$\mu = 0.01$			$\mu = 0.12$			$\mu = 0.2$		
	k_t	$\widehat{n}_t$	$\overline{e}$	k_t	$\widehat{n}_t$	$\overline{e}$	k_t	$\widehat{n}_t$	$\overline{e}$
c_t	0.305	0.000	-0.075	0.307	-0.005	-0.075	0.310	-0.014	-0.076
k_{t+1}	0.305	1e-04	-0.075	0.299	0.020	-0.074	0.287	0.060	-0.071
$\widehat{n}_{t+1}$	-0.194	0.941	-0.027	-0.196	0.946	-0.027	-0.198	0.955	-0.026
τ_t	0.000	0.000	1.000	0.002	-0.008	0.999	0.007	-0.022	0.998
y_t	0.305	0.000	-0.075	0.304	0.003	-0.075	0.303	0.009	-0.074
$\widehat{i}_t$	0.305	0.000	-0.075	-0.309	-0.014	-0.076	0.316	-0.035	-0.078
z_t	0.330	0.000	0.000	0.329	0.003	2e-04	0.328	0.008	6e-04
l_t	0.000	0.000	0.000	-0.001	0.004	3e-04	-0.003	0.011	8e-04
b_{t+1}	0.306	-0.002	-0.075	0.432	-0.416	-0.106	1.654	-4.419	-0.406

There are a number of interesting properties of the model worth emphasizing[48]. Notice that the degree of credit market imperfections has a very mild effect on the elasticities of the variables of the table with respect to k_t and $\overline{e}$ with the exception of l_t and b_{t+1}. The elasticities with respect to $\widehat{n}_t$ show slightly more variation, although their absolute values are relatively low. It follows then that varying μ may not add significant differences in terms of the model's dynamics. This observation seems to be an important characteristic of the model, which is also appearing at later stages of the analysis. In contrast, the elasticities associated with l_t and b_{t+1} become more relevant as μ increases. A similar pattern is also observed in those variables directly affected by the entrepreneur's contracting problem (see Appendix G).

Focusing on the elasticities with respect to $\overline{e}$, it is worth noting that are negative in all cases but τ_t, z_t and l_t (although for the latter two variables are almost insignificant). It follows that a negative value of $\overline{e}$ (i.e., $\overline{s}$ rising less than $\overline{m}$) will be expansionary, but it will affect negatively the trade balance surplus.

Finally, it is important to notice that the elasticity of c_t with respect to $\widehat{n}_t$ is negative. This is a direct consequence of the fact that a higher $\widehat{n}_t$ also implies a higher entrepreneurial consumption, c_t^e, which *ceteris paribus* reduces c_t. A similar negative relation is also observed between b_{t+1} and $\widehat{n}_t$: higher net worth reduces the need of external funding thereby decreasing b_{t+1}. This effect becomes more important as μ rises. There is as well a negative relation between $\widehat{i}_t$ and $\widehat{n}_t$, which can be understood observing Eq. 69. A rise in net worth positively affects $\widehat{i}_t$ but also reduces b_{t+1} which in turn reduces $\widehat{i}_t$ (under the specification considered here we have that $\sigma_1 > 0$ and $\sigma_2 > 0$). When the latter effect is strong enough the overall result might be a reduction of investment, as the table shows.

We are now ready to solve for $\overline{s}$. Observe that assuming an initial steady state which is equal to the reference steady state will also imply that $\digamma_0 = 0$[49]. Eq. 80 then reduces to: $0 = \sum_{t=0}^{\infty} \beta^t \tau_t$.

[48] We leave for Appendix G a similar table considering other variables of the model.

[49] Recall that $\digamma_0 \equiv S_0 F_0 / (S\overline{Y}_T)^{RSS}$ is defined as the absolute deviation of net foreign assets with respect to a reference steady state in which the trade balance surplus is zero. It follows that net foreign assets must also be zero in this steady state. If initially there is a zero trade balance surplus, it must be the case then that $F_0 = 0$ which in turn implies that $\digamma_0 = 0$.

Using the recursive equilibrium solution for $x_{T,t}$ in combination with Eq. 82 and the intertemporal national budget constraint finally yields:

$$\begin{aligned} 0 \;=\; & [(\eta_{x,k}v_{11}+\eta_{x,n}v_{21})\frac{1-\beta}{1-\beta\lambda_1}\eta_{\kappa 1,e}+(\eta_{x,k}v_{12}+\eta_{x,n}v_{22})\frac{1-\beta}{1-\beta\lambda_2}\eta_{\kappa 2,e}+ \\ & \eta_{x,k}SS_1+\eta_{x,n}SS_2+\eta_{x,e}]\overline{e}. \end{aligned}$$

Notice that whenever the term in curly brackets in the expression above is different from 0, satisfying this equation requires that $\overline{e}=0$[50]. It follows then that $\overline{s}=\overline{m}$, and therefore the real variables of the model are unaffected by the monetary shock. As mentioned previously, this is a direct consequence of the fact that in the recursive equilibrium law of motion of the model the propagation mechanism is given by $\overline{e}$. Whence, the neutrality of money is satisfied when prices are fully flexible, as one would predict.

6.3 Analysis including nominal price rigidities

The case in which intermediate nontradable firms do not adjust prices at period $t=0$ adds interesting elements into the analysis, as we will now proceed to discuss. In order to solve for $\overline{s}$ we will also consider the intertemporal national budget constraint. Before doing this, however, we have to find the expression that defines the short-run trade balance surplus, the direction in which we are now moving.

In what follows the subindex 0 indicates a short-run value of any given variable (i.e., while prices of intermediate nontradable goods remain unchanged). By assumption, therefore, $p_{N,0}=0$. Notice that nontradable output will be demand-determined in the short-run (as explained below) and that Eqs. 68, 75 and 76 do not hold on impact[51]. From the general price index of the economy it then follows that $p_0=(1-\gamma)\overline{s}$. It is now easy to derive the impact level of consumption, which is defined as: $c_0=\overline{m}-(1-\gamma)\overline{s}$. It can also be seen that the demand for the tradable input takes the form $x_{T,0}=y_0-\gamma\overline{s}$, expression from which we can later derive the short-run trade balance surplus. For doing this we first ought to find y_0. First observe that the assumption $\widehat{n}_0=0$ also implies that $c_0^e=0$. The clearing market condition can then be written as $y_0=\sigma_7 c_0+\sigma_9\widehat{i}_0$. We are now interested in obtaining an expression for $\widehat{i}_0$ as a function of only $\overline{s}$ and $\overline{m}$. We proceed as follows.

Since the non-tradable output is demand-determined in the short-run, the level of $\widehat{i}_0$ is now constrained. It should be noticed, in particular, that $\widehat{i}_0$ must satisfy the clearing market condition of the final nontradable good. It has to also guarantee that k_1 is effectively supplied when prices are fully flexible at $t=1$. Owing to these reasons, the optimal contracting problem of entrepreneurs do not hold in the short-run. Therefore, entrepreneurs do not have the possibility of choosing $\widehat{i}_0$ to

[50] Under the benchmark specification the term in curly brackets is always different from zero. Moreover, this result seems to be robust to variations of the baseline parameter values.

[51] To be more precise, since wages and the rental price of capital freely adjust in the model in the short and long-run Eqs. 75 and 76 are used to derive the short-run values of w_0-p_0 and $r_0^k-p_0$ once the solutions for z_0, p_0 and l_0 are obtained.

maximize their expected profits. In other words, at period $t = 0$ the supply of capital should satisfy the following set of conditions:

$$k_1 = -\sigma_5 \widehat{\omega}_0 + \widehat{i}_0, \tag{83}$$

$$\widehat{n}_1 = r_1^k - p_1 + \widehat{\omega}_0(\sigma_4 - \sigma_3) + \widehat{i}_0, \tag{84}$$

and

$$\widehat{i}_0 = \sigma_2 b_1. \tag{85}$$

Eqs. 83 and 84 are log-linear versions of Eqs. 35 and 28 when $t = 0$, describing the laws of motion of capital and net worth, respectively. Eq. 85 is simply the log-linear version of the aggregate budget constraint of entrepreneurs: $S_t B_{t+1}^* = P_t(i_t - n_t)$, where $b_1 \equiv b_1^* + \overline{s} - p_0$. It is worth highlighting that k_1, $\widehat{n}_1$, and $r_1^k - p_1$ are flexible price solutions, which can be easily obtained considering their respective equilibrium law of motion as discussed in the previous subsection. Putting it another way, these variables are solved according to the saddle-path solution of the model. Recalling that k_0 and $\widehat{n}_0$ are given and equal to 0, it follows then that Eqs. 83 and 84 provide the solutions of the two unknowns $\widehat{\omega}_0$ and $\widehat{i}_0$ as functions of $\overline{s}$ and $\overline{m}$. Call these solutions $\widehat{\omega}_0 = \widehat{\omega}_0(\overline{s}, \overline{m})$ and $\widehat{i}_0 = \widehat{i}_0(\overline{s}, \overline{m})$. Once $\widehat{i}_0$ is obtained, moreover, entrepreneurs' debt b_1 is simply given by: $b_1 = \widehat{i}_0(\overline{s}, \overline{m})/\sigma_2$. To be more precise, it can be shown that $\widehat{i}_0$ takes the form:

$$\widehat{i}_0 = \{\eta_{k,e}\varphi_1 + [\eta_{n,e} - (\eta_{rk,k}\eta_{k,e} + \eta_{rk,n}\eta_{n,e} + \eta_{rk,e})]\varphi_2\}\overline{e},$$

where $\varphi_1 \equiv \frac{\sigma_4 - \sigma_3}{\sigma_5 + \sigma_4 - \sigma_3}$ and $\varphi_2 \equiv \frac{\sigma_5}{\sigma_5 + \sigma_4 - \sigma_3}$. Notice that we have made use of the following equilibrium law of motion: $r_t^k - p_t = \eta_{rk,k} k_t + \eta_{rk,n} \widehat{n}_t + \eta_{rk,e} \overline{e}$ that holds $\forall t > 0$. The short-run trade balance surplus can now be expressed as a function of only $\overline{s}$ and $\overline{m}$[52]. Denote the latter as: $\tau_0 \equiv \tau_0(\overline{s}, \overline{m})$. Recalling that $\forall t > 0$ prices are fully flexible, from the previous subsection we already have the result that that $\tau_t \equiv \tau_t(\overline{s}, \overline{m})$ $\forall t > 0$. The trade balance surplus is hence divided into two different phases that describe its whole time-path. There is a short-run effect that lasts only one period, while prices of intermediate nontradable goods are unchanged. There is also a second phase which is characterized by price flexibility and holds $\forall t = 1...\infty$.

We are now ready to solve for $\overline{s}$ considering the intertemporal national budget constraint stated in Eq. 80. Since $F_0 = 0$ it must be the case that $\tau_0(\overline{s}, \overline{m}) + \sum_{t=1}^{\infty} \beta^t \tau_t(\overline{s}, \overline{m}) = 0$, equation that will provide a unique value of $\overline{s}$. In the table below, we present the solution of $\overline{s}$ as well as the short-run values of a set of endogenous variables of the model modifying those parameters associated with the entrepreneurs' contracting problem: μ and υ. We again modify certain (specified) parameters while leaving the rest as in the baseline specification. For simplicity we set $\overline{m} = 1$[53].

[52]It can be shown that: $\tau_0 = \overline{s}\{(1-\gamma)\sigma_7 + \gamma - \sigma_9[\eta_{k,e}\varphi_1 + [\eta_{n,e} - (\eta_{rk,k}\eta_{k,e} + \eta_{rk,n}\eta_{n,e} + \eta_{rk,e})]\varphi_2]\}$
$-\overline{m}\{\sigma_7 - \sigma_9[\eta_{k,e}\varphi_1 + [\eta_{n,e} - (\eta_{rk,k}\eta_{k,e} + \eta_{rk,n}\eta_{n,e} + \eta_{rk,e})]\varphi_2]\}$.

[53]We leave the short-run values for a different sub-set of endogenous variables for Appendix G.

Table 5. Short-run solutions of the model

	$\mu = 0.01$	$\mu = 0.12$	$\mu = 0.2$	$\upsilon = 0.07$	$\upsilon = 0.15$
$\overline{s}$	0.998	0.998	0.998	0.998	0.998
$\overline{e}$	-0.003	-0.003	-0.003	-0.003	-0.003
τ_0	0.249	0.250	0.251	0.252	0.250
y_0	0.674	0.673	0.672	0.671	0.673
c_0	0.925	0.925	0.925	0.925	0.925
$\widehat{i}_0$	1.9e-04	1.9e-04	2e-04	1.9e-04	1.9e-04
z_0	0.748	0.748	0.746	0.746	0.748
l_0	1.117	1.116	1.114	1.113	1.116
b_1	1.9e-04	2.7e-04	0.001	0.01	2.3e-04

From this table we can observe that $\overline{s} < \overline{m}$ (i.e., $\overline{e} < 0$). In this model it then follows that the exchange rate is less volatile than the money supply. It is also clear that the unexpected and permanent monetary shock has expansionary effects on output (both intermediate and final) as well as on households' consumption. Observe also that since $k_0 = 0$, production in the intermediate sector can only increase if labor rises, which is the case here. Notice that the monetary shock has generated a positive wealth effect on impact. The economy accumulates net-foreign assets (i.e., there is a short-run trade balance surplus). Thereby this model shows the path-dependence property that often arises in dynamic models with infinitely-lived agents[54]. Notice, moreover, that investment $\widehat{i}_0$ also rises on impact. Its magnitude, however, is almost negligible. This is in turn associated with the low value of k_1, as shown below. It follows then that as $\widehat{i}_0$ rises only slightly entrepreneurs do not face the necessity of increasing indebtedness by a large amount, and thereby neither b_1 nor $\widehat{\omega}_0$ are very sensitive to the policy shock. Other variables related to the presence of credit market imperfections also show a similar pattern (see Appendix G). Note that variations in either μ or υ have mild effects on the real variables of the model. It is worth noticing, moreover, that higher values of μ seem to be associated with a lower supply of nontradable goods.

For a better understanding of the long-run effects of the model we now discuss the associated impulse-response functions for the case in which $\overline{m} = 1$ under the baseline parameter specification.

Table 6. Impulse-response functions after a monetary shock

t	τ_t	k_t	$\widehat{n}_t$	y_t	c_t	$\widehat{i}_t$	z_t	l_t	b_{t+1}	$\widehat{\omega}_t$
-1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.2500	0.0000	0.0000	0.6728	0.9252	1.9e-04	0.7476	1.1159	2.7e-04	1.1e-04
1	-0.0025	1.8e-04	6.7e-0.5	0.0002	0.0002	2.5e-04	0.0001	0.0000	3.2e-04	1.0e-04
2	-0.0025	2.4e-04	9.5e-05	0.0002	0.0003	2.6e-04	0.0001	0.0000	3.3e-04	0.0000
50	-0.0025	2.7e-04	2.6e-04	0.0003	0.0003	2.7e-04	0.0001	0.0000	2.7e-04	0.0000
100	-0.0025	2.7e-04	2.7e-04	0.0003	0.0003	2.7e-04	0.0001	0.0000	2.7e-04	0.0000

Note that the accumulation of net foreign assets on impact allows the country to run a permanent trade balance deficit. To be more precise, in the new steady state the following relation must hold:

[54]For instance, the path-dependence property is obtained in the two-countries version model of Obstfeld and Rogoff (1995), although the small-open economy version discussed therein does not produce it.

$F\frac{1-\beta}{\beta} = -tb$, where F denotes (real) net foreign assets and tb defines the trade balance surplus also in real terms. Since the real (gross) interest rate in this steady state is equal to β^{-1}, the country is financing the steady state trade balance deficit with the return on the net foreign assets accumulated in the short-run.

Net worth and capital will propagate the initial shock. It is noteworthy, however, the fact that k_t and $\widehat{n}_t$ take values close to zero, and hence the effect on the remaining endogenous variables is relatively mild, as previously mentioned. Regarding credit market imperfections, note that $\widehat{\omega}_t$ goes up since there is a small increase in indebtedness. Finally, observe that labor is unaffected by the positive wealth effect. This is an important property of the model, implying that labor is not affected when prices are fully flexible as we already noticed at the steady state level [55].

It is also interesting to understand the effect on the relative prices of the economy. At an intuitive level we should expect that the positive wealth effect raises real wages and appreciates the real exchange rate. These effects can be formally confirmed in the table below (under the baseline specification):

Table 7. Impulse-response functions after a monetary shock (cont.)

t	$w_t - p_t$	$\overline{s} - p_t$	$r_t^k - p_t$	r_{t+1}^{nd}
-1	0.0000	0.0000	0.0000	0.0000
0	2.0411	0.9227	0.6728	3.5e-05
1	0.0002	-0.0023	5.9e-05	3.3e-05
2	0.0003	-0.0022	2.0e-05	3.1e-05
50	0.0003	-0.0022	0.0000	0.0000
100	0.0003	-0.0022	0.0000	0.0000

It follows that although the nominal exchange rate immediately jumps to its new steady state value, the real exchange rate shows an overshooting effect. On impact there is a real depreciation since intermediate good prices are pre-set and therefore the general price index adjusts only partially. However, the increase in the domestic price level at period $t = 1$ (when prices of nontradable goods freely adjust) generates a strong real exchange rate appreciation. In the new steady state the real exchange rate is lower (i.e., more appreciated). Also notice that the real rental price rises on impact, and it decreases toward a new steady state where its value is essentially zero. As the supply of capital rises in the transition toward the new steady state there is a negative pressure on $r_t^k - p_t$. An important transmission mechanism that has been emphasized in the literature on balance-sheet effects is the evolution of the risk premium. We can define it as $r_{t+1}^{nd} - r_{t+1}$ (or simply r_{t+1}^{nd}, since r_{t+1} is unaffected by the shock). We can notice by inspecting Table 7 that the monetary expansion moderately raises the risk premium (both in nominal and real terms), result which is consistent with the higher degree of entrepreneurs' leverage.

[55] This result is sometimes referred to as a case in which the income and substitution effects originated in the increase of the money supply exactly cancel each other. For a detailed discussion of this property, although in a different model and considering a different shock, see King, Plosser and Rebelo (2002).

Since the result behind the long-run non-neutrality of money is associated with the impact effect of the monetary shock on the trade balance surplus, a sensitivity analysis of the model can be undertaken by focusing only on this variable (varying a particular parameter at each time).

Table 8. Sensitivity analysis of the trade balance surplus

t	$\mu = 0.01$	$\mu = 0.2$	$\upsilon = 0.07$	$\upsilon = 0.15$	$\alpha = 0.01$	$\alpha = 0.5$	$\gamma = 0.01$	$\mu = \alpha = 0.01$
-1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	0.2491	0.2512	0.2516	0.2496	0.0076	0.3786	0.0000	0.0075
1	-0.0025	-0.0025	-0.0025	-0.0025	-1e-04	-0.0038	0.0000	-1e-04
2	-0.0025	-0.0025	-0.0025	-0.0025	-1e-04	-0.0038	0.0000	-1e-04
50	-0.0025	-0.0025	-0.0025	-0.0025	-1e-04	-0.0038	0.0000	-1e-04
100	-0.0025	-0.0025	-0.0025	-0.0025	-1e-04	-0.0038	0.0000	-1e-04

Notice, again, that the case in which $\mu \to 0$ gives an otherwise conventional RBC-type model for a small-open economy. Comparing the second and third columns in Table 8 we see that a higher degree of credit market imperfections amplifies the trade balance surplus in the short-run, though moderately. Modifying the fraction of expected profits that entrepreneurs consume each period neither has substantive effects on τ_t. A higher level of entrepreneurs' consumption, however, seems to be associated with a reduction of the trade balance surplus as one would expect. Results are dramatically changed, in contrast, when α or γ are modified. Note that γ can be thought of as a measure of openness. As $\gamma \to 0$ the economy tends to be completely open, thereby isolating it from what is happening in the nontradable sector. In particular, note that the evolution of capital and net worth, as well as the one-period nominal price rigidity assumption, will become irrelevant in this case. This is why we do not observe current account effects. Notice also that by setting $\alpha \to 0$ we are eliminating capital from the model (and indirectly the relevance of credit market imperfections). In such a case the trade balance surplus also becomes negligible. This effect is even more attenuated when $\mu \to 0$. We now summarize the discussion in the following remark:

Remark 3 *The introduction of capital in the model in combination with credit market imperfections and one-period nominal price rigidities bring the result that money is not neutral in the long-run after considering an unexpected and permanent rise in the money supply. The presence of capital hence incorporates an additional source of 'rigidity', since at each point in time the capital stock is given. In this scenario the monetary shock generates a positive wealth effect via a trade balance surplus, which is the reason behind the departure from the long-run non-neutrality of money. To recover equilibrium the nominal exchange rate does not have to depreciate as it would have to do otherwise.*

We close this section emphasizing that the in the small-open-economy version of OR, in which labor is the only input of production and credit market imperfections are absent, the long-run neutrality of money is satisfied. From our previous discussion it follows that the case $\alpha = \mu = 0$ (which roughly collapses to the OR version), recovers the long-run neutrality of money even under one-period nominal price rigidities since there are no current account effects (i.e., $\tau_t = 0 \ \forall \ t = 0, .., \infty$).

7 A currency devaluation

In the previous sections we have analyzed how a permanent and unanticipated monetary expansion affects the economy under a pure floating regime. In this section we explore the implications of a different economic policy shock: an unexpected and permanent devaluation under a fixed exchange rate regime. With some specific differences to be analyzed, most of the previous procedure for solving the model when considering a monetary expansion remains valid. A key difference to notice is, however, the fact that the nominal exchange rate now becomes the policy variable (i.e., it is exogenous) and is fixed by the economic authority at a certain particular level. In contrast, the money supply is endogenous and adjusts after the policy shock.

We assume that at time $t = 0$ there is an unexpected and permanent nominal exchange rate devaluation. The log-deviation of the nominal exchange rate, therefore, changes from 0 to some arbitrary level denoted $\overline{s}$[56]. As we did before, we can start solving the model considering only the monetary side of the economy. Since the devaluation is permanent it follows from Eq. 59 that $r_{t+1} = 0$, result that in turn implies that $x_t = 0$ (see Eq. 58). Consumption is thus given by:

$$c_t = m_t - p_t. \tag{86}$$

To solve for this economic policy exercise, we now have to replace Eq. 61 with Eq. 86 in the system given by Eqs. 60-78. Letting in this case $\overline{e}_t \equiv \overline{s} - m_t$ (instead of $\overline{e}$), we can proceed to solve the model exactly as before. Notice that in order to have a non-trivial role for the exchange rate policy, we are still assuming that prices of intermediate nontradable goods do not adjust in the short-run (i.e., $p_{N,0} = 0$). The final solution for m_t must then be obtained considering also the short-run equilibrium of the model. In particular, note that there will be a unique level of m_t which satisfies a mutually consistent internal and external equilibrium for a given value of $\overline{s}$.

Turning to the numerical results, for comparability with our previous policy exercise we set $\overline{s} = 1$. It can be shown that the currency devaluation produces effects which go exactly in the same direction as in the case of the monetary expansion. Nevertheless, under the baseline parameter values the devaluation has slightly larger short- and long-run effects[57]. This is a direct consequence of the fact that the trade balance surplus is always larger on impact under a devaluation, thereby implying a larger positive wealth effect at period $t = 0$ in this case.

[56]Observe that $\overline{s} \equiv \frac{S^{RSS'} - S^{RSS}}{S^{RSS}}$, being $S^{RSS'}$ and S^{RSS} the steady state level of the nominal exchange rate after and before the shock, respectively. Since we have assumed that before the shock takes place the economy is located at the reference steady state, it then follows that initially $\overline{s} = 0$.

[57]To preserve space we decided not to repeat results essentially similar to those already discussed under a monetary shock.

8 Concluding remarks

We have developed here a fully microfounded dynamic general equilibrium model which can be thought of as an extension of the small-open economy model of Obstfeld and Rogoff (1995) along the lines of Céspedes, Chang and Velasco (2004). We essentially extended OR incorporating currency mismatches, investment and the presence of credit market imperfections in the production of capital. We somehow complemented Céspedes and colleagues since we have addressed two unexpected and permanent policy shocks not considered by them: a monetary expansion and a currency devaluation. The main difference with their framework is, nevertheless, the fact that we derived the form in which financial frictions affect the economy instead of postulate it.

Although we had derived analytically the steady state and the minimum state space form of the model, we have had to rely on a numerical calibration exercise to study its dynamic properties. This fact is a consequence of the high degree of non-linearities in the coefficients of the endogenous variables of the model after log-linearizing it. Nonetheless, our approach allowed us to study in detail the properties of the dynamic system and to trace back thoroughly the transmission mechanisms of the shocks.

We showed that the presence of currency mismatches and credit market imperfections do not seem to invalidate traditional Mundell-Fleming results. On the contrary, these results seem to be amplified, although moderately, by introducing an additional state variable into the model (i.e., net worth), compared to what is obtained when capital is the unique propagation mechanism.

To explain, whenever prices do not fully adjust in the short-run, the presence of capital (and at a lesser extent financial frictions) allows the economy to accumulate net foreign assets (on impact) after the policy shock. This current account surplus is essentially driven by a typical expenditure-switching effect. Hence, there is a positive wealth effect which implies that nominal shocks are not neutral in the long-run. We further explain that the neutrality property is recovered by eliminating capital (indirectly reducing the relevance of financial frictions); avoiding therefore a source of 'real rigidity', since the stock of capital is given at each point in time. In that case the model roughly collapses to that of OR (where current account effects are absent). The currency of denomination of entrepreneurs' debt does not seem to have any critical effect on the results obtained here.

We leave for further research two interesting extensions. We have assumed that the risk-free foreign interest rate is constant over time. Relaxing this assumption will allow us to explore exogenous changes in this variable as Céspedes and colleagues do. To facilitate the analysis of the model's dynamics we have eliminated the possibility of having technology socks. Relaxing this assumption will allow us to explore different propagation mechanisms (in this case real), getting closer to the RBC-type literature and to those shocks emphasized in Aghion et al (2000).

Appendix A

The maximization problem can be written in terms of the following Lagrangean,

$$\max_{\{i_{j,t},\overline{\omega}_{j,t},\lambda\}} L = R^k_{t+1} i_{j,t} f(\overline{\omega}_{j,t}) + \lambda[R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t}) - R_{t+1} P_t (i_{j,t} - n_{j,t})], \tag{AA1}$$

The associated first order conditions are:

$$\frac{\partial L}{\partial i_{j,t}} = R^k_{t+1} f(\overline{\omega}_{j,t}) + \lambda[R^k_{t+1} g(\overline{\omega}_{j,t}) - R_{t+1} P_t] = 0, \tag{AA2}$$

$$\frac{\partial L}{\partial \overline{\omega}_{j,t}} = R^k_{t+1} i_{j,t} f'(\overline{\omega}_{j,t}) + \lambda[R^k_{t+1} i_{j,t} g'(\overline{\omega}_{j,t})] = 0, \tag{AA3}$$

and

$$\frac{\partial L}{\partial \lambda} = R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t}) - R_{t+1} P_t (i_{j,t} - n_{j,t}) = 0. \tag{AA4}$$

Note that Eq. AA3 implies $\lambda = -\frac{f'(\overline{\omega}_{j,t})}{g'(\overline{\omega}_{j,t})}$. Replacing this expression in Eq. AA2 and rearranging gives Eq. 20. Solving Eq. AA4 for $i_{j,t}$ gives Eq. 21.

Appendix B

From the main text we have: $f(\overline{\omega}_{j,t}) = \int_{\overline{\omega}_{j,t}}^{\infty} \omega\phi(\omega)d\omega - [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}$. Observe that $f(\overline{\omega}_{j,t})$ can be written as $f(\overline{\omega}_{j,t}) = \int_0^{\infty} \omega\phi(\omega)d\omega - \int_0^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}$. Recalling that $E(\omega) = \int_0^{\infty} \omega\phi(\omega)d\omega = 1$, we can obtain,

$$f(\overline{\omega}_{j,t}) = 1 - \int_0^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}.$$

Taking derivatives with respect to $\overline{\omega}_{j,t}$ gives,

$$f'(\overline{\omega}_{j,t}) = -[1 - \Phi(\overline{\omega}_{j,t})], \tag{AB1}$$

and

$$f''(\overline{\omega}_{j,t}) = \phi(\overline{\omega}_{j,t}), \tag{AB2}$$

implying that $f(\overline{\omega}_{j,t})$ is a convex function of $\overline{\omega}_{j,t}$.

Similarly, from the main text we have,

$$g(\overline{\omega}_{j,t}) = \int_0^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - \mu\Phi(\overline{\omega}_{j,t}) + [1-\Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}.$$

Taking derivatives with respect to $\overline{\omega}_{j,t}$ gives,

$$g'(\overline{\omega}_{j,t}) = -\mu\phi(\overline{\omega}_{j,t}) + [1-\Phi(\overline{\omega}_{j,t})], \tag{AB3}$$

and

$$g''(\overline{\omega}_{j,t}) = -[\mu\frac{\partial\phi(\overline{\omega}_{j,t})}{\partial\overline{\omega}_{j,t}} + \phi(\overline{\omega}_{j,t})]. \tag{AB4}$$

Let us now consider the first order condition stated in Eq. 20. After rearranging terms, this equation takes the form,

$$\frac{R^k_{t+1}}{R_{t+1}P_t} = \{g(\overline{\omega}_{j,t}) - \frac{f(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})}g'(\overline{\omega}_{j,t})\}^{-1}.$$

Let us define $G(\overline{\omega}_{j,t}) = \{g(\overline{\omega}_{j,t}) - \frac{f(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})}g'(\overline{\omega}_{j,t})\}^{-1}$. Therefore,

$$\frac{\partial G(\overline{\omega}_{j,t})}{\partial\overline{\omega}_{j,t}} = \{g(\overline{\omega}_{j,t}) - \frac{f(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})}g'(\overline{\omega}_{j,t})\}^{-2}\frac{f(\overline{\omega}_{j,t})g'(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})^2}\{\frac{g''(\overline{\omega}_{j,t})f'(\overline{\omega}_{j,t})}{g'(\overline{\omega}_{j,t})} - f''(\overline{\omega}_{j,t})\} > 0,$$

considering the second order conditions of the entrepreneur's maximization problem (see Appendix C) and the fact that in equilibrium $g'(\overline{\omega}_{j,t}) > 0$. This implies that $\frac{\partial\overline{\omega}_{j,t}}{\partial R^k_{t+1}} > 0$, as explained in the main text.

Appendix C

To obtain the second order conditions we also need the following partial derivatives of the Lagrangean analyzed in Appendix A:

$$\frac{\partial^2 L}{\partial i^2_{j,t}} = 0,$$

$$\frac{\partial^2 L}{\partial i_{j,t}\partial\overline{\omega}_{j,t}} = \frac{\partial^2 L}{\partial\overline{\omega}_{j,t}\partial i_{j,t}} = R^k_{t+1}[f'(\overline{\omega}_{j,t}) + \lambda g'(\overline{\omega}_{j,t})] = 0^{58},$$

$$\frac{\partial^2 L}{\partial\overline{\omega}^2_{j,t}} = R^k_{t+1}i_{j,t}[f''(\overline{\omega}_{j,t}) + \lambda g''(\overline{\omega}_{j,t})]$$

$$\frac{\partial}{\partial i_{j,t}}(\frac{\partial L}{\partial\lambda}) = R^k_{t+1}g(\overline{\omega}_{j,t}) - R_{t+1}P_t,$$

and

[58] To obtain this result, it is worth recalling that $\lambda = -\frac{f'(\overline{\omega}_{j,t})}{g'(\overline{\omega}_{j,t})}$.

$$\frac{\partial}{\partial \overline{\omega}_{j,t}}(\frac{\partial L}{\partial \lambda}) = R^k_{t+1} i_{j,t} g'(\overline{\omega}_{j,t}).$$

We can now form the bordered Hessian,

$$H = \begin{bmatrix} 0 & R^k_{t+1} g(\overline{\omega}_{j,t}) - R_{t+1} P_t & R^k_{t+1} i_{j,t} g'(\overline{\omega}_{j,t}) \\ R^k_{t+1} g(\overline{\omega}_{j,t}) - R_{t+1} P_t & 0 & 0 \\ R^k_{t+1} i_{j,t} g'(\overline{\omega}_{j,t}) & 0 & R^k_{t+1} i_{j,t} [f''(\overline{\omega}_{j,t}) + \lambda g''(\overline{\omega}_{j,t})] \end{bmatrix}.$$

To satisfy the associated second order condition for a maximum, we need the determinant of the matrix H to be greater or equal to zero (see, for instance, Simon and Blume 1994, p. 461). This determinant takes the form,

$$|H| = -R^k_{t+1} i_{j,t} [R^k_{t+1} g(\overline{\omega}_{j,t}) - R_{t+1} P_t]^2 [f''(\overline{\omega}_{j,t}) + \lambda g''(\overline{\omega}_{j,t})].$$

Since $[R^k_{t+1} g(\overline{\omega}_{j,t}) - R_{t+1} P_t]^2 \geq 0$, to satisfy the second order condition it is needed that $[f''(\overline{\omega}_{j,t}) - \frac{f'(\overline{\omega}_{j,t})}{g'(\overline{\omega}_{j,t})} g''(\overline{\omega}_{j,t})] \leq 0$. Whenever ω is uniformly distributed in the interval $[0, 2]$, the second order condition can be written as $1 \leq \frac{(1-\frac{1}{2}\overline{\omega})}{1-\frac{1}{2}(\overline{\omega}+\mu)}$ (see Appendix D). For this inequality to be satisfied, we further require that $g'(\overline{\omega}) = 1 - \frac{1}{2}(\overline{\omega} + \mu) > 0$ (or that $\mu < 2 - \overline{\omega}$). Moreover, providing that $\mu > 0$ the second order condition is satisfied as an strict inequality.

Appendix D

In this appendix we consider the particular case in which ω is uniformly distributed in the interval $[0, 2]$; therefore the mean of ω is 1, as stated in the main text. The following results immediately follows: $\phi(\overline{\omega}) = \frac{1}{2}$, $\frac{\partial \phi(\overline{\omega})}{\partial \overline{\omega}} = 0$, $\Phi(\overline{\omega}) = \frac{1}{2}\overline{\omega}$ and $1-\ \Phi(\overline{\omega}) = 1 - \frac{1}{2}\overline{\omega}$. Using the results obtained in Appendix B, we can easily obtain:

$$g(\overline{\omega}) = -\frac{1}{4}\overline{\omega}^2 + \overline{\omega}(1 - \frac{\mu}{2}),$$

$$g'(\overline{\omega}) = 1 - \frac{1}{2}(\mu + \overline{\omega}),$$

$$g''(\overline{\omega}) = -\frac{1}{2},$$

$$f(\overline{\omega}) = \frac{1}{4}\overline{\omega}^2 - \overline{\omega} + 1,$$

$$f'(\overline{\omega}) = -(1 - \frac{1}{2}\overline{\omega}),$$

and

$$f''(\overline{\omega}) = \frac{1}{2}.$$

Appendix E

In this appendix we explain how to derive the steady state solutions of the main endogenous variables of the model. The analysis becomes simpler by defining all variables in real terms as follows: $r^k \equiv \frac{R^k}{P}$, $R^{nd} \equiv r^{nd}$, $p_N \equiv \frac{P_N}{P}$, $s = p_T \equiv \frac{S}{P}$, $w \equiv \frac{W}{P}$, $m \equiv \frac{M}{P}$, $b \equiv \frac{SB^*}{P}$, $tb \equiv \frac{TB}{P}$, and $F \equiv \frac{SF}{P}$. In facilitating the exposition we list below the key endogenous variables of the model in a zero-inflation steady state:

$$C = m\frac{(1-\beta)}{\chi} \tag{AE1}$$

$$r^{nd} = \beta^{-1}\overline{\omega}g(\overline{\omega})^{-1} \tag{AE2}$$

$$r^k = \beta^{-1}(g(\overline{\omega}) - \frac{f(\overline{\omega})}{f'(\overline{\omega})}g'(\overline{\omega}))^{-1} \tag{AE3}$$

$$r^k = \alpha p_N \frac{\theta - 1}{\theta}\frac{Z}{K} \tag{AE4}$$

$$K = i(1 - \mu\Phi(\overline{\omega}_t)) \tag{AE5}$$

$$w = (1-\alpha)p_N \frac{\theta - 1}{\theta}\frac{Z}{L} \tag{AE6}$$

$$L = \frac{1}{\kappa}\frac{1}{C}w \tag{AE7}$$

$$Z = AK^{\alpha}L^{1-\alpha} \tag{AE8}$$

$$Y = Z^{\gamma}X_T^{1-\gamma} \tag{AE9}$$

$$\gamma Y = p_N Z \tag{AE10}$$

$$(1-\gamma)Y = sX_T \tag{AE11}$$

$$1 = \gamma^{-\gamma}(1-\gamma)^{(\gamma-1)}s^{1-\gamma}p_N^{\gamma} \tag{AE12}$$

$$p_N = \frac{\theta}{\theta - 1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1}A^{-1}w^{1-\alpha}(r^k)^{\alpha} \tag{AE13}$$

$$C^e = \frac{\upsilon}{1-\upsilon}n \tag{AE14}$$

$$i = (1 - \frac{f'(\overline{\omega})g(\overline{\omega})}{f(\overline{\omega})g'(\overline{\omega})})n \tag{AE15}$$

$$s(\overline{Y}_T - X_T) = tb \tag{AE16}$$

$$Y = C + C^e + i \tag{AE17}$$

$$b = -\frac{f'(\overline{\omega})g(\overline{\omega})}{f(\overline{\omega})g'(\overline{\omega})}n \tag{AE18}$$

$$-\frac{g'(\overline{\omega})}{f'(\overline{\omega})} = (1-\upsilon)\beta^{-1} \tag{AE19}$$

$$-\digamma\beta^{-1} = (1-\beta)^{-1}tb. \tag{AE20}$$

Recalling that when $\digamma = 0$ we have that $s = Y(1-\gamma)\overline{Y}_T^{-1}$, from Eq. AE12 it is possible to obtain,

$$Y = \overline{Y}_T(\frac{p_N}{\gamma})^{\frac{\gamma}{\gamma-1}}.$$

This equation, however, does not pin-down Y since p_N is an endogenous variable. To solve for p_N note that the equilibrium condition in the labor market, described by Eqs. AE6 and AE7, in conjunction with Eq. AE10 bring the relation,

$$w = \{\gamma(1-\alpha)\frac{\theta-1}{\theta}\kappa Y C\}^{\frac{1}{2}}. \tag{AE21}$$

Hence, the price index for the nontradable good given in Eq. AE13 can be expressed as,

$$p_N = A^{-1}(\frac{r^{k*}}{\alpha})^{\alpha}(\frac{\theta}{\theta-1})^{\frac{1+\alpha}{2}}\{\frac{\gamma\kappa}{(1-\alpha)}YC\}^{\frac{1-\alpha}{2}}.$$

Combining the above equation with the expression for Y obtained previously and solving for C gives,

$$C = Y^{\frac{\gamma(1+\alpha)-2}{(1-\alpha)\gamma}}\{A^{-1}(\frac{r^{k*}}{\alpha})^{\alpha}(\frac{1}{\gamma}\frac{\theta}{\theta-1})^{\frac{1+\alpha}{2}}(\frac{\kappa}{1-\alpha})^{\frac{1-\alpha}{2}}\overline{Y}_T^{\frac{\gamma-1}{\gamma}}\}^{\frac{2}{\alpha-1}}.$$

This equation gives a relation between two endogenous variables: C and Y. The clearing market condition for the nontradable good, Eq. AE17, in combination with Eqs. AE14, AE15 and AE19 also give,

$$n=(1-\upsilon)(1+\beta\frac{g(\overline{\omega}^{*})}{f(\overline{\omega}^{*})})^{-1}\{Y-Y^{\frac{\gamma(1+\alpha)-2}{(1-\alpha)\gamma}}\{A^{-1}(\frac{r^{k*}}{\alpha})^{\alpha}(\frac{1}{\gamma}\frac{\theta}{\theta-1})^{\frac{1+\alpha}{2}}(\frac{\kappa}{1-\alpha})^{\frac{1-\alpha}{2}}\overline{Y}_{T}^{\frac{\gamma-1}{\gamma}}\}^{\frac{2}{\alpha-1}}\}, \quad \text{(AE22)}$$

after substituting for C. Observe that Eq. AE22 relates two endogenous variables: n and Y.

We can obtain a second expression in these two variables as follows. The production function of intermediate firms, $Z=AK^{\alpha}L^{1-\alpha}$, and Eq. AE10 give,

$$Y=A\gamma^{-1}K^{\alpha}L^{1-\alpha}p_{N}.$$

Notice that $L^{1-\alpha}p_{N}=A^{-1}(\frac{\theta}{\theta-1}\frac{r^{k*}}{\alpha})^{\alpha}(\gamma Y)^{1-\alpha}$ (from Eqs. AE7, AE13 and AE21). Also observe that $r^{k*}K=(1-\upsilon)^{-1}(1+\frac{g(\overline{\omega}^{*})}{f(\overline{\omega}^{*})})n$ (from Eqs. AE3, AE5, AE15 and AE19).

Therefore, it is possible to obtain,

$$n=Y\alpha\gamma\frac{\theta-1}{\theta}(1-\upsilon)(1+\frac{g(\overline{\omega}^{*})}{f(\overline{\omega}^{*})})^{-1}. \quad \text{(AE23)}$$

Substituting Eq. AE23 into Eq. AE22 and rearranging gives the solution for Y,

$$Y=\{1-\alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^{*})+\beta g(\overline{\omega}^{*})}{f(\overline{\omega}^{*})+g(\overline{\omega}^{*})}\}^{\frac{(1-\alpha)\gamma}{2(\alpha\gamma-1)}}\{A^{-1}(\frac{r^{k*}}{\alpha})^{\alpha}(\frac{1}{\gamma}\frac{\theta}{\theta-1})^{\frac{1+\alpha}{2}}(\frac{\kappa}{1-\alpha})^{\frac{1-\alpha}{2}}\overline{Y}_{T}^{\frac{\gamma-1}{\gamma}}\}^{\frac{\gamma}{\alpha\gamma-1}}.$$

Having solved for Y we can now obtain the steady state solutions of all the remaining endogenous variables of the model.

Appendix F

This appendix explains how to derive the minimum state-space system described in Eq. 81. Without loss of generality it is assumed that $a_t=0\ \forall\ t$. Let $\overline{e}\equiv\overline{s}-\overline{m}$ denote the difference between the steady state level of the exchange rate and the money supply, respectively. To express the system in terms of the endogenous variables k_{t+1}, $\widehat{n}_{t+1}$ and c_{t+1}, as a first order linear system of difference equations, it is necessary to firstly obtain the following intermediate results: (i) $p_{t+1}-p_t$ as a function of k_{t+1}, c_{t+1}, c_t, $\widehat{n}_t$ and $\overline{e}$; (ii) c_{t+1} as a function of k_{t+1}, c_t, $\widehat{n}_t$ and $\overline{e}$; (iii) $\widehat{n}_{t+1}$ as a function of k_{t+1}, c_{t+1}, c_t, $\widehat{n}_t$ and $\overline{e}$; (iv) k_{t+1} as a function of k_t, $\widehat{n}_t$, c_t, $\overline{e}$ and (v) c_{t+1} and $\widehat{n}_{t+1}$ as a function of k_t, $\widehat{n}_t$, c_t, $\overline{e}$. In what follows, we explain in detail how to obtain each of these results.

(i) From Eqs. 61, 62, 63 and 65 it is possible to write $x_{T,t}$ as:

$$x_{T,t}=\alpha k_t+(1-\alpha)l_t-\gamma^{-1}(\overline{e}+c_t). \quad \text{(AF1)}$$

Substituting Eqs. 62, 63 and AF1 into Eq. 66 gives:

$$p_{N,t+1}=p_{t+1}+\frac{\gamma-1}{\gamma}\overline{e}+\frac{\gamma-1}{\gamma}c_{t+1.} \quad \text{(AF2)}$$

Notice that using Eqs. 62, 76 and 77 it is possible to obtain the following expression for w_{t+1},

$$w_{t+1} = \frac{\alpha}{1+\alpha}(\overline{m} + k_{t+1}) + \frac{1}{1+\alpha} p_{N,t+1}. \tag{AF3}$$

Eqs. 68, 71 and AF3, in turn, bring:

$$p_{N,t+1} - p_t = \frac{1-\alpha}{2}(c_t + k_{t+1}) + \frac{1+\alpha}{2}(\sigma_1 b_{t+1} - \sigma_1 \widehat{n}_t + \sigma_3 \widehat{\omega}_t). \tag{AF4}$$

Substituting Eq. AF2 into Eq. AF4 yields,

$$p_{t+1} - p_t = \frac{1-\gamma}{\gamma} c_{t+1} + \frac{1-\gamma}{\gamma}\overline{e} + \frac{1-\alpha}{2}(c_t + k_{t+1}) + \frac{1+\alpha}{2}(\sigma_1 b_{t+1} - \sigma_1 \widehat{n}_t + \sigma_3 \widehat{\omega}_t). \tag{AF5}$$

Notice that from Eqs. 70 and 74 b_{t+1} and $\widehat{\omega}_t$ can be expressed as:

$$\widehat{\omega}_t = \frac{\sigma_6(\sigma_1 + \sigma_2)}{\sigma_2 + \sigma_5\sigma_6}\widehat{n}_t - \frac{\sigma_6}{\sigma_2 + \sigma_5\sigma_6} k_{t+1}, \tag{AF6}$$

$$b_{t+1} = \frac{\sigma_5\sigma_6 - \sigma_1}{\sigma_2 + \sigma_5\sigma_6}\widehat{n}_t + \frac{1}{\sigma_2 + \sigma_5\sigma_6} k_{t+1}. \tag{AF7}$$

Therefore, $p_{t+1} - p_t$ is given by,

$$\begin{aligned} p_{t+1} - p_t \quad &= \quad \frac{1-\gamma}{\gamma} c_{t+1} + \frac{1}{2}\{1 - \alpha + (1+\alpha)\frac{(\sigma_1 - \sigma_3\sigma_6)}{\sigma_2 + \sigma_5\sigma_6}\} k_{t+1} + \frac{1-\alpha}{2} c_t \\ &\quad - \frac{1+\alpha}{2}\frac{(\sigma_1+\sigma_2)(\sigma_1 - \sigma_3\sigma_6)}{\sigma_2 + \sigma_5\sigma_6}\widehat{n}_t + \frac{1-\gamma}{\gamma}\overline{e}. \end{aligned} \tag{AF8}$$

(ii) From the linearized Euler equation for consumption given in Eq. 60 and using Eq. AF8 it is possible to obtain:

$$\begin{aligned} c_{t+1} \quad &= \quad -\frac{1}{2}\gamma\{1 - \alpha + (1+\alpha)\frac{(\sigma_1 - \sigma_3\sigma_6)}{\sigma_2 + \sigma_5\sigma_6}\} k_{t+1} + \frac{(1+\alpha)}{2}\gamma c_t \\ &\quad + \frac{1}{2}\gamma(1+\alpha)\frac{(\sigma_1+\sigma_2)(\sigma_1 - \sigma_3\sigma_6)}{\sigma_2 + \sigma_5\sigma_6}\widehat{n}_t + (\gamma - 1)\overline{e}. \end{aligned} \tag{AF9}$$

(iii) Substituting Eqs. AF5, AF6 and AF7 into Eq. 73 yields,

$$\begin{aligned} \widehat{n}_{t+1} \quad &= \quad \{\frac{1 - \sigma_6\sigma_4}{\sigma_2 + \sigma_5\sigma_6} - \frac{1}{2}(1 - \alpha + \frac{1+\alpha}{\sigma_2 + \sigma_5\sigma_6}(\sigma_1 - \sigma_3\sigma_6))\} k_{t+1} + \frac{\gamma - 1}{\gamma} c_{t+1} \\ &\quad + \frac{\{(\sigma_1+\sigma_2)(\sigma_6\sigma_4 + \frac{1+\alpha}{2}(\sigma_1 - \sigma_3\sigma_6)) + \sigma_5\sigma_6 - \sigma_1\}}{\sigma_2 + \sigma_5\sigma_6} n_t - \frac{1-\alpha}{2} c_t + \frac{\gamma - 1}{\gamma}\overline{e}. \end{aligned} \tag{AF10}$$

(iv) Observe that Eqs. 77, AF2 and AF3 give:

$$l_t = -\frac{1}{1+\alpha}\frac{1}{\gamma} c_t + \frac{\alpha}{1+\alpha} k_t + \frac{1}{1+\alpha}\frac{\gamma - 1}{\gamma}\overline{e}.$$

Substituting this expression into Eq. 63 and using Eqs. 62 and AF1:

$$y_t = \frac{2\alpha}{1+\alpha}k_t + \frac{1}{\gamma}((\gamma-1) - \frac{1-\alpha}{1+\alpha})c_t + \frac{2}{1+\alpha}\frac{\gamma-1}{\gamma}\overline{e}.$$

Finally, substituting the above equation in the clearing market condition stated in Eq. 78 and using Eqs. 69, 73 and AF7 it is possible to obtain:

$$k_{t+1} = 2\frac{\alpha}{1+\alpha}\Delta_1 k_t - \Delta_1\Delta_2\widehat{n}_t + \Delta_1\Delta_3 c_t + \frac{2}{1+\alpha}\frac{\gamma-1}{\gamma}\Delta_1\overline{e}, \tag{AF11}$$

where:

$\Delta_1 \equiv \frac{\sigma_2+\sigma_5\sigma_6}{\sigma_9\sigma_2}$

$\Delta_2 \equiv \sigma_8 + \frac{\sigma_9\sigma_5\sigma_6(\sigma_1+\sigma_2)}{\sigma_2+\sigma_5\sigma_6}$

$\Delta_3 \equiv \frac{\gamma-1}{\gamma} - \frac{1}{\gamma}\frac{1-\alpha}{1+\alpha} - \sigma_7$

(v) Replacing Eq. AF11 into Eq. AF9 gives, after some manipulation:

$$c_{t+1} = -\frac{\alpha\gamma}{1+\alpha}\Delta_4 k_t + \frac{\gamma}{2}\{(1+\alpha)\Delta_6 + \Delta_2\Delta_4\}\widehat{n}_t + \frac{\gamma}{2}\{1+\alpha-\Delta_3\Delta_4\}c_t + (\gamma-1)(1-\frac{\Delta_4}{1+\alpha})\overline{e}. \tag{AF12}$$

Introducing Eqs. AF11 and AF12 into Eq. AF10 yields,

$$\begin{aligned}\widehat{n}_{t+1} \quad = \quad & \frac{\alpha}{1+\alpha}\{2\Delta_5\Delta_1 - \gamma\Delta_4\}k_t + \{\gamma\frac{(1+\alpha)}{2}\Delta_6 + 1 - \Delta_2\Delta_5\Delta_1 + \frac{\gamma}{2}\Delta_2\Delta_4 - (\sigma_1+\sigma_2)\Delta_5\}\widehat{n}_t \\ & \{\Delta_3\Delta_5\Delta_1 - \frac{1}{2}(1-\gamma)(1+\alpha) - \frac{\gamma}{2}\Delta_3\Delta_4 - \frac{1-\alpha}{2}\}c_t + \frac{\gamma-1}{\gamma}\{\frac{2}{1+\alpha}\Delta_5\Delta_1 + \gamma - \frac{\gamma}{1+\alpha}\Delta_4\}\overline{e},\end{aligned} \tag{AF13}$$

where

$\Delta_4 \equiv \frac{(1-\alpha)(\sigma_2+\sigma_5\sigma_6)}{\sigma_9\sigma_2} + \frac{(1+\alpha)(\sigma_1-\sigma_3\sigma_6)}{\sigma_9\sigma_2}$

$\Delta_5 \equiv \frac{1-\sigma_4\sigma_6}{\sigma_2+\sigma_5\sigma_6}$

$\Delta_6 \equiv \frac{(\sigma_1+\sigma_2)(\sigma_1-\sigma_3\sigma_6)}{\sigma_2+\sigma_5\sigma_6}.$

Collecting Eqs.AF11, AF12 and AF13 in matrix form we have:

$$\begin{bmatrix} k_{t+1} \\ \widehat{n}_{t+1} \\ c_{t+1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} k_t \\ \widehat{n}_t \\ c_t \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \overline{e},$$

where:

$a_{11} \equiv 2\frac{\alpha}{1+\alpha}\Delta_1$

$a_{12} \equiv -\Delta_1\Delta_2$

$a_{13} \equiv \Delta_1\Delta_3$

$a_{21} \equiv \frac{2\alpha}{1+\alpha}\Delta_5\Delta_1 - \frac{\gamma\alpha}{1+\alpha}\Delta_4$

$a_{22} \equiv \gamma\frac{(1+\alpha)}{2}\Delta_6 + 1 - \Delta_2\Delta_5\Delta_1 + \frac{\gamma}{2}\Delta_2\Delta_4 - (\sigma_1+\sigma_2)\Delta_5$

$a_{23} \equiv \Delta_3\Delta_5\Delta_1 - \frac{1}{2}(1-\gamma)(1+\alpha) - \frac{\gamma}{2}\Delta_3\Delta_4 - \frac{1-\alpha}{2}$

$a_{31} \equiv -\frac{\alpha\gamma}{1+\alpha}\Delta_4$

$a_{32} \equiv \frac{\gamma}{2}\{(1+\alpha)\Delta_6 + \Delta_2\Delta_4\}$

$a_{33} \equiv \frac{\gamma}{2}\{1+\alpha-\Delta_3\Delta_4\}$
$b_1 = \frac{2}{1+\alpha}\frac{\gamma-1}{\gamma}\Delta_1$
$b_2 = \frac{\gamma-1}{\gamma}\{\frac{2}{1+\alpha}\Delta_5\Delta_1+\gamma-\frac{\gamma}{1+\alpha}\Delta_4\}$
$b_3 = (\gamma-1)(1-\frac{\Delta_4}{1+\alpha})$.

Appendix G

In this appendix we present Tables AG1 and AG2 which extend Tables 4 and 5 of the main text, respectively.

Table AG1

	$\mu = 0.01$			$\mu = 0.12$			$\mu = 0.2$		
	k_t	n_t	$\overline{e}$	k_t	n_t	$\overline{e}$	k_t	n_t	$\overline{e}$
$\widehat{\omega}_t$	0.0068	-0.022	-0.002	0.174	-0.569	-0.043	1.405	-4.603	-0.345
$w_t - p_t$	0.305	0.000	-0.075	0.306	-9e-04	-0.075	0.306	-0.003	-0.075
$\overline{s} - p_t$	0.305	0.000	0.925	0.307	-0.005	0.925	0.310	-0.014	0.924
$r_t^k - p_t$	-0.645	0.000	-0.075	-0.696	0.003	-0.075	-0.697	0.009	-0.074
r_{t+1}^{nd}	0.006	-0.021	-0.002	0.056	-0.184	-0.014	0.083	-0.271	-0.020

Table AG2

	$\mu = 0.01$	$\mu = 0.12$	$\mu = 0.2$	$\upsilon = 0.07$	$\upsilon = 0.15$
$\widehat{\omega}_0$	0.000	1.1e-04	8.7e-04	0.010	6.5e-05
$w_0 - p_0$	2.042	2.041	2.039	2.039	2.042
$\overline{s} - p_0$	0.923	0.923	0.923	0.923	0.923
$r_0^k - p_0$	0.674	0.673	0.672	0.671	0.673
r_1^{nd}	0.000	3.5e-05	5.1e-05	5.1e-05	2.9e-05

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