Activism, Costly Participation, and Polarization

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Abstract

I develop a model of activism and polarization in the context of electoral competition. Two candidates simultaneously announce policy platforms and seek the support of ideologically inclined party activists. Activists compete to influence electoral outcomes by expending costly support for their respective candidates. I show that the presence of activists always moderates the platform choice of candidates, compared to the case of no activism. Further, I establish that the relationship between ideological polarization of activists and political polarization depends critically on the activists’ willingness to engage in the campaign. Specifically, when the willingness to engage exceeds a threshold, increased polarization among activists leads to lower polarization of candidate platforms – meaning candidates compromise rather than diverge. I precisely characterize the conditions under which the presence of activism, and increasing partisanship among activists are both welfare-improving for voters. Finally, I identify a novel crowding out effect of big money on the demand for activism. My analysis suggests public funding of elections as an important institutional reform that would create a demand for activism, mitigating the pernicious effects of high polarization.

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1 Introduction

“People at the top might devote time and resources to supporting a political party strongly opposed to redistribution. People at the bottom would have an opposite response.” - McCarty, Poole and Rosenthal, Polarized America

Median voter convergence with office-motivated candidates, as propagated by Downs [18] and Black [13], has been widely considered a benchmark for the analysis of political competition. However, a cursory look at electoral campaigns in the US, especially from the 1960’s onwards, suggests a widening polarization between the elected representatives of the two major parties.\(^1\) This could be interpreted as a manifestation of increasingly polarized preferences among the electorate. Another interpretation is that participation of more ideologically partisan groups of activists may give rise to such platform polarization. In this paper, I look at the question of polarization through the lens of political activism. The role of a partisan political activist include persuading voters, donating money, helping with door-to-door canvassing and leafleting, attending national party convention, and mobilizing voters through grass-roots campaigning.\(^2\) Activists on either sides of the political spectrum, through their active participation in campaigns, influence voters’ opinion and therefore affect the electoral prospects of candidates. When political activists engage with the electorate and try to influence outcomes, candidates behave strategically and choose platforms in order to attract greater participation from these activists. In this way, the presence of activism affects the very nature of competition in democratic polities.

In fact, political activism has played an important role in several instances. For example, Obama’s 2008 and 2012 campaigns were propelled by grass-roots activists and mobilization heretofore unseen in US presidential elections. Further, tea-party activists similarly played a crucial role in the Republicans winning back the House of Representatives in the 2010 mid-term elections. More recently, Bernie Sanders’ campaign against the Democratic party’s front

\(^1\)For example, McCarty, Poole and Rosenthal [36], henceforth MPR, record this increasing polarization by looking at the roll-call votes in both the US Senate and House of representatives. They find a growing divide between Democrats and Republicans across issues, and a decrease in moderates in both chambers of the US congress.

\(^2\)See Norris [43] for a more detailed analysis of the nature of political activism.
runner Hillary Clinton witnessed grass-roots participation from a wide array of activists.\(^3\) Political activism, therefore, is an important channel of electoral participation in democratic countries with competitive party systems.

Activist influence can either mobilize support (get out the vote campaign) in favor of a candidate, or persuade voters by directly affecting their preferences. Traditionally, the role of activists as mobilizing agents has been widely studied.\(^4\) However, activists spend a lot of their efforts on convincing \textit{undecided voters} of their candidate’s platform and influencing their final voting decision. The National Election Survey (NES) data from the US between 1952-2000 shows evidence for this kind of a persuasive role. Specifically, the survey data finds that a significant proportion (around 30-40\%) of the electorate in the US indulged in persuasive activism – engaging with potential voters about the candidates’ policy – over these years.\(^5\) More recently, Madestam et al. find evidence from the tea party activism of 2010 for a similar persuasive role.\(^6\)

Aldrich [1] [2], in his seminal work, provides a rational-choice explanation for participation in activism. Specifically, Aldrich predicted divergence in preferences of activists belonging to the two parties. His work also laid down the strategic nature by which activists trade-off benefits and costs associated with participation. One implication of Aldrich’s work is that activists may decide on how much to contribute (costly effort) depending on the platforms announced by the candidates. Candidates, anticipating this, may find it beneficial to cater to their preferences. This way, party activists may themselves become a potential source of

\(^3\)To give examples from outside the US context, the 2015 state elections in New Delhi saw a fledgling party AAP win 95\% of the seats by promising a platform of ending governmental corruption and nepotism. The campaign witnessed mass mobilization by grass roots activists and volunteers who were able to influence voters’ preferences towards supporting the party. In the 1990’s, a similar activism led campaign by right-wing groups led to the formation of the first successful non-Congress national government in India, almost 50 years after its independence.

\(^4\)For example, empirical work on the role of party activists in voter mobilization has been studied by Gerber and Green [22] and, McClurg and Holbrook [33].

\(^5\)See Norris [42], Chapter 6, Table 6.4 for a more detailed analysis of the persuasive effect of campaign activism.

\(^6\)Madestam et al [34] investigate whether political protests (or activism) alter voter preferences and impact political outcomes in the context of Tea-party activism in US during the 2010 midterm elections. Their main finding is that political activism had significant multiplier effects in terms of affecting the number of votes secured by Republican candidates, and also resulted in more conservative stances by policymakers in congress. They conclude ”...these results are consistent with larger political protests creating a stronger political movement that is able to more effectively persuade the populace about its policy agenda come election time, which ultimately affects both incumbent behavior and election outcomes”.
polarization. Given the importance and relevance of activism for political competition, it is surprising that there is relatively very little work in terms of the theoretical literature on political activism. Though Aldrich’s work provides a framework for studying the motivations for becoming an activist, questions of how this partisanship influences political platforms, what affects the extent of participation by partisan activists, and the welfare effects of activism remain unanswered, and very pertinent, especially in the current context of the polarization debate surrounding US politics.

In this paper, I bridge the gap by developing a simple model of activism and political participation that captures these intuitive trade-offs. Specifically, I present a unified model of Downsian competition that captures the role of activists, the voting decisions of voters, and candidates’ platform selection. My model incorporates three key features – i) candidates care about ideology and benefits of office (they are “responsible”, in the spirit of Calvert [14] and Wittman [54]); ii) activists are ideological price-takers (as modeled by Aldrich, they take candidate platforms as given); iii) activists persuade voters but face participation costs.

I model the political process as follows: candidates simultaneously announce platforms, party activists expend effort to influence voters given the set of platforms, and (median) voter decides whom to vote for; in that order. Candidates, when announcing platforms, and activists, when deciding on levels of participation, are unaware of the median voter’s preferred policy, which is drawn from a uniform distribution. Activist participation plays a role of direct influence, in the sense that their effort affects median voter’s utility, either by shifting their preferences towards a candidate or by providing them with a direct utility benefit from activism. Two important trade-offs emerge in this setup. Candidates trade-off their ideology to elicit greater participation from activists; and, activists trade-off benefits from participation and the costs of doing so. Together, these twin trade-offs provides for a novel set of results.

First, I find that political polarization decreases in the presence of party activists, com-

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7This kind of role for activists could be equated to the influence seeking behavior in organizations, as modeled by Milgrom [39], and Milgrom and Roberts [38]. Activists may potentially provide some informative benefits (e.g., valence characteristics of a candidate or information regarding certain policies in the manifesto) that the voters deem as useful in making an informed choice.

8Throughout this paper, I will refer to candidates’ platform polarization as political polarization, and
pared to settings with no activists. The result is driven by the fact that when activists are price-takers, they punish both their own candidate (by reducing effort) and the other candidate (increasing effort) for polarizing. As a result, competition between activists decreases equilibrium political polarization, irrespective of the activists’ ideological preferences. This suggests that contrary to the case of organized lobbies and interest groups (see Grossman and Helpman [26] [24]), the presence of activists moderates platforms and brings about political compromise. An important implication of this result is that activism disciplines the platform choice candidates in electoral democracies. They do so by not only restricting the extent to which their candidate polarizes, but also how much the other side polarizes. That is, when a candidate takes a more extreme platform, the activists supporting the other side’s candidate increase their participation to counter balance this extreme policy platform. This, in equilibrium, enforces greater discipline among candidates and reduces the overall level of polarization.

Second, I find that increased partisanship between activists does not necessarily increase platform polarization of candidates. Specifically, when activists’ elasticity of engagement in the political process exceeds a critical threshold, increased partisanship leads to greater convergence in candidate platforms, resulting in lesser political polarization. The nature of this relationship is determined by the activists’ elasticity of engagement. I precisely characterize the threshold elasticity above which activist polarization leads to candidate convergence. The reason for this result is twofold. As the activists become more extreme, the participation goes up. Activists trade off the marginal effect of this higher participation and the marginal costs of doing so. When the elasticity is above a threshold, the marginal increase in costs is balanced by a steeper increase in the marginal benefits. Candidates understand these trade-offs. Specifically, when the participation elasticity is above the threshold, polarizing hurts the activists, thereby eliciting greater punishment from them – the own activist’s polarization as partisanship.

9This kind of punishment for polarization is the opposite of what Virag [52] finds. In their model, voters belonging to a party are better informed only about their party’s platform. This pushes candidates to extremize in order to elicit participation from the more extreme voters. In my model with polarized activists, this form of extremizing by candidates is punished by the other side’s activist, leading to lesser polarization of candidate platforms.
participation decreases, and opposing activist participation increases. Further, these effects are stronger when the elasticity is above the critical threshold. This decreases the benefit for either candidate to polarize, resulting in lesser political polarization in equilibrium. On the other hand, when the elasticity of engagement goes below the threshold, the candidate’s marginal benefit from moving closer to his preferred platform is greater than the perceived punishment costs. This implies that greater partisanship also results in increased political polarization.

An important implication of this result is that a widening partisan gap is neither necessary nor sufficient for causing increased polarization of platforms. In fact, my analysis suggests that a combination of partisanship and decreased willingness to engage is what drives political polarization. The fact that activists are ideologically risk-averse and prefer moderation implies that as long as they have a greater sense of engagement in the campaign process, more moderate platforms would prevail as an equilibrium of electoral competition.

I derive a rich set of comparative statics results with respect to the exogenous parameters of the model. Specifically, I find that an increase in the importance of activism relative to pure ideological preferences of voters, or a decrease in the uncertainty regarding median voter’s ideal point both lead to more moderation in equilibrium. The reason for this is twofold. Any increase in the importance of activism relative to pure ideology, or reduced uncertainty about the median voter increases the participation of activists in the elections. This, combined with the fact that party activists have a moderating influence on platforms, implies that candidates would compromise by more in equilibrium.

Next, I investigate the effect of activist polarization on voters’ welfare. There are two important set of results. The first pertains to the introduction of activists and its effect on welfare. The second is the relationship between voter welfare and the level of activist polarization. From a purely normative angle, my analysis suggests that the presence of activists may or may not improve overall voter welfare. That is, as long as the level of divergence in the absence of activism exceeds a threshold (the ex-ante welfare maximizing level of polarization), the introduction of activism can be beneficial. However, it could be that the presence of activists hurts voters. This happens when the
is already too moderate in terms of the choice offered by candidates, introducing activists only makes the choice more moderate, to the point that it hurts voters’ welfare.

On a similar vein, as activists themselves polarize, the platforms of candidates could either diverge or converge. Depending on the extent of resulting polarization, this may improve or decrease voter welfare. I precisely characterize the conditions under which increasing (decreasing) activist polarization decreases (increases) the welfare of voters. The intuition is the following. When activists’ elasticity of engagement is high enough, increased partisanship between activists decreases polarization of platforms by candidates. However, when the levels of polarization is already below the socially optimum level, then as candidates moderate their platforms even further, the overall welfare of voters decreases. My finding suggests a delicate balance between the level of polarization and voter welfare. In particular, when the existing levels of polarization is low, any increase in the polarization of activists, as long as their willingness to engage is within a threshold, could end up providing more choice to the voters and thereby improve welfare. This result suggests that democratic societies with greater (lesser) barriers to political participation could actually benefit from increased partisanship among the political activists, as long as the existing choices provided by candidates are highly similar (dissimilar).

In Section 5, I consider two extensions. First, I investigate the role of activism in a noisy campaign, in the sense of Austen-Smith [5]. Activism, instead of influencing voters’ utility, instead plays an informative role. The median voter observes an imperfect (noisy) signal of the actual platform, and greater activism reduces the variance of this noise, rendering platforms more informative. The elasticity of engagement in this case is dependent on the efficiency of activism (in reducing the noise of platforms) and the participation-cost aversion of activists. The results in this modified setup are similar to the original game. Specifically, the noisy campaign game yields an unique equilibrium and the comparative statics results do not change.

I then study the role of soft money in the electoral competition game. Apart from seeking the support of party activists, candidates are also endowed with campaign money (e.g. big donors, organized PACs or super PACs), which they use to influence voter preferences.
In this sense, money and activist participation both perform similar roles and supplement each other. That is, money and activist effort are substitutable goods. In this setting, my analysis suggests a novel crowding-out effect of soft money on activism – a greater pool of soft money reduces participation of activists, resulting in more polarized platforms. My analysis suggests that introducing public funding of elections could be useful as a potential policy intervention. Public funding restricts the resources available to a candidate, limiting the crowding-out effect of big money. Capping the extent of campaign finance and restricting the influence of big money would help shorten the length of the campaign cycle, and, increase the demand for grass-roots political activism. This way, the political process restores the dependence of candidates on activists and engenders greater consensus in the polity, resulting in reduced platform polarization.

The rest of the paper is organized as follows. In Section 2, I briefly discuss related literature. Section 3 presents the benchmark model and characterizes the equilibrium of the electoral game. Section 4 discusses the main results from comparative statics analysis. Section 5 details the welfare results and Section 6 contains two extensions to the model. A brief discussion and concluding remarks follow in Section 7. All proofs are confined to the appendix.

2 Related Literature

My paper, at its core, is a model of electoral competition which induces platform separation in equilibrium. Several models have explored the idea of platform divergence\textsuperscript{10}. This paper looks into electoral activism as a possible channel for divergence, and more importantly, the main goal of my work is to theoretically investigate the impact of partisanship on polarization.

The work of Aldrich [1] is a natural starting point for my analysis. Aldrich derives two important results with respect to the existence of ”party cleavages”: firstly, within a party, the distribution of activists is cohesive in ideology; and secondly, across party lines,

\textsuperscript{10}See Bernhardt et al. [9], Osborne and Slivinski [44], Besley and Coate [12], Gul and Pesendorfer [27], Ashworth and Mesquita [4], Aragones and Palfrey [3]
Ideologies are distinctly polarized in terms of their distributions. In this paper, I use the modeling assumption about the existence of partisan cleavages to analyze the extent of participation and its impact on political competition. I specifically concentrate my analysis on the relationship between activist polarization, as envisaged by Aldrich, and political polarization in the sense of candidate platforms.

My model is related to the work by Bernhardt, Duggan and Squintani [10] (henceforth BDS). They consider the case for responsible parties in the presence of uncertainty around the median voter’s ideal policy. They present an important normative result – a small level of polarization actually improves voter welfare. I introduce political participation (through activists) to this setup and consider the impact of activist polarization and participation on candidate platforms.

The role of activism in my work is similar to the models of influence as studied by Milgrom [39], Milgrom and Roberts [38], and Meyer et al. [37]. While these papers study the role of influence seeking in organizations, I apply analogous ideas to a model of political competition. In my work, activists engage with voters in order to influence their choice of candidate, and further, this engagement in influence seeking is costly for the activists. This brings about a novel trade off between the activists’ preferences and the extent of participation (influence seeking).

This paper is also related to the work on political participation of voters. On the theoretical side, this strand of literature could be broadly categorized into two classes of turnout models\textsuperscript{11} – turnout driven by costly voting (Riker and Ordeshook [48], Palfrey and Rosenthal [45] [46], Morton [40], and Feddersen and Sandroni [21] [20]) and turnout driven by candidates or activists (Shachar and Nalebuff [51], Herrera and Martinelli [30], Herrera et al. [29], Feddersen and Gul [19]).

In first set of models, the act of voting is a costly decision, and voters makes a decision to participate based on this. In the second set of models, the act of voting is not costly, but getting the vote out is. Candidates therefore expend costly effort (spending) in order

\textsuperscript{11}See Herrera, Morelli and Nummari [31] and Herrera, Morelli and Palfrey [32] for work that relate turnout to institutional arrangements.
to ensure voter participation. The model close to my setup is the one by Herrera and Martinelli [30] and Herrera et al. [29]. In the former, citizens decide whether to participate by becoming an activist (influencer), who can then mobilize support among the rest; in the latter, candidates announce platforms and spend costly effort to increase turnout of voters. My model of activism differs from this in two ways. Firstly, my paper is not a model of turnout but one of costly influence. Secondly, the effort is borne not by candidates but by activists who belong to their party. In my model, the act of voting is costless (passive participation), but activism is costly. Party activists undertake costly campaign effort just so their preferred candidate wins. This shifts the costs from the voter to the activist and fundamentally changes the nature of trade-offs to one between activists’ influence and the costs associated with it.

My model is also closely related to the work on direct informative role\textsuperscript{12} of campaign spending, notably, by Austen-Smith [5]. In Austen-Smith [5], candidates simultaneously announce policy, and elicit contributions from two firms. The electoral game analyzed by Austen-Smith differs from mine in two aspects. They consider an informative role whereas I focus on the persuasive role of activism\textsuperscript{13} (in that they affect median voter’s ideological preferences). This apart, the contribution decision of firms in Austen-Smith are not constrained in that there is no party affiliation and donors can choose to contribute to either candidate. The motivations for activists in my work is to associate with a single party (left or right) and strictly support only their party’s candidate.

Finally, it is important to differentiate activism from interest groups or lobbies. The literature on campaign contributions and influence seeking by interest groups or lobbies is vast and has been extensively studied. Baron ([8],[7]), Grossman and Helpman ([23], [25],[24]), Bernheim and Whinston [11], Austen-Smith [6] investigate various aspects of influence seeking by interest groups. The distinctions between these models and mine is twofold. Firstly, activists are quite different in their objectives, in the sense that their support is ideologically partisan and is not driven by any form of non-partisan or economic considerations. Further,

\\textsuperscript{12}Coate [15] presents an alternate model of informative campaign spending.
\textsuperscript{13}We extend our model to include noisy campaigns, and show that the fundamental predictions on equilibrium polarization and activist participation holds. See Section 6.
there is a lack of commitment by activists towards a candidate. This implies that the relationship is temporary and short-term, and activists do not directly influence the platforms of candidates. Interest groups and organized lobbies, on the other hand, can commit credibly to contracts that specify a certain kind of quid pro quo relationship. In this limited sense, their relationship with candidates extends beyond elections and is not short term. The focus of my work, alternatively, is purely on electoral campaigns and the indirect role of activists in influencing political platforms of candidates.

3 Model

Two candidates, who care about ideology and benefits of office, contest elections on an unidimensional policy space \([-1, 1]\]. Candidate \(L\) has an ideal point \(p^C_L = -\alpha\) and Candidate \(R\) has an ideal point \(p^C_R = \alpha\), where \(\alpha \in (0, 1)\). The candidates simultaneously announce policy \(X_i\) (where \(i \in \{L, R\}\)), and the winning candidate enjoys benefits from office, \(b > 0\). The winner implements the ex-ante chosen policy. The (symmetric) candidate utilities are given by,

\[
U^C_i = \begin{cases} 
-(X_i - p^C_i)^2 + b & \text{if } i \text{ wins} \\
-(X_{-i} - p^C_i)^2 & \text{otherwise}
\end{cases}
\]

Candidates, after announcing platforms, seek the support of party activists belonging to each party. The activists’ payoff consists of two components. First, the two activists \(A_L\) and \(A_R\) have an ideological preference that is different from that of their respective candidate, and given by \(p^A_L = -\beta\) and \(p^A_R = \beta\) respectively. Second, activists contribute to the electoral process by making a costly effort/participation decision. This is captured by a convex cost function, \(M(c_i)\), such that \(M' > 0, M'' > 0, M(0) \geq 0, M'(0) > 0\). Let \(\gamma_m(c_i) = c_i \frac{M''}{M'}\) be defined as the elasticity of marginal cost of participation for activists.\(^{14}\)

Note that party polarization is defined by the ideological distance between activists\(^ {15}\), \(2\beta\). An increase in \(\beta\) could be interpreted as a reflection of more partisan parties (or party

\(^{14}\)This provides a measure analogous to risk aversion, except that measures the elasticity of the marginal costs involved with participation.

\(^{15}\)Aldrich refers to this partisan identification as ”party cleavages”.}
activists). For example, greater $\beta$ could be thought of as more extreme views (on the right and left) on tax policy, gay rights, regulations, or minimum wages, and so on. Given $\beta$, the mobilization $c_i$ is very loosely defined to capture any form of contribution by activists. This could be either direct small donations to candidates or indirect ones, namely, door-to-door canvassing, attending campaign events (national conventions, among others), talking to potential supporters in local districts, and so on. Broadly, any measure of time, effort, or money spent on endorsing and campaigning for the candidate could be accounted for by the variable $c_i$. The utility for an activist is therefore given by,

$$U^A_i = \begin{cases} 
-(X_L - p^A_i)^2 - M(c_i) & \text{if } L \text{ wins} \\
-(X_R - p^A_i)^2 - M(c_i) & \text{if } R \text{ wins}
\end{cases}$$

Finally, there is a continuum of voters $v \in [-1, 1]$ distributed uniformly. There is uncertainty around the median voter’s bliss point. I model this by assuming that after candidates announce platforms and activists decide on contributions, the voters experience a shock $\mu$ distributed uniformly\(^{16}\) on $[-\sigma, \sigma]$ that shifts their ideal points. That is, the bliss point of a voter is given by, $\theta_v = v + \mu$, where $v$ is the ex-ante ideal point. Therefore, the candidates, while announcing platforms, and activists, when they choose their levels of participation, do not observe the true bliss point of the median voter. This helps us to focus attention on the (ex-ante) median voter’s decision problem.

Further, the effort spent by the party activist affects the median voter’s utility directly by providing a positive utility according to an ”influence function” $P(c_i)$, where $P(.)$ is twice continuously differentiable in $c_i$ such that $P' > 0, P'' \leq 0, P''' \geq 0, P(0) = 0, P'(0) > 0$. This influence function could be interpreted in one of two ways. It could be thought off as a preference shock that is induced by activists on voters, similar to the effect identified by Madestam et al.[34]. Alternatively, the influence activity may also be interpreted as a direct utility benefit that voters derive from activism.\(^{17}\) Either way, what is important is

\(^{16}\)The results hold for any generic symmetric distribution $G(.)$ around the ex-ante median. However, the uniform distribution provides us with precise characterizations without the expositional burdens.

\(^{17}\)This may be a form of informative benefit or persuasion utility that is similar to the role of influence seeking (see Milgrom [38]) or advertising (see Dixit and Norman [17]) in the organizational literature.
that activism plays a role of influence, rather than one of aiding turn-out. This remains a crucial departure from existing models of electoral competition.

In a similar vein as before, it is useful to define $\gamma_p(c_i) = -c_i \frac{P''}{P'}$ as the elasticity of marginal influence. $\gamma_p(c_i)$ describes the curvature of the influence function and measures the effectiveness of activism in influencing median voter’s utility. For sake of exposition, I will assume that the elasticities are independent of the activist contribution $c_i$.

**Assumption 1:** $\gamma_p(c_i) = \gamma_p, \gamma_m(c_i) = \gamma_m$

The utility of the median voter is given by:

$$U_m = \begin{cases} -(X_L - \mu)^2 + \eta \cdot P(c_L) & \text{if } L \text{ wins} \\ -(X_R - \mu)^2 + \eta \cdot P(c_R) & \text{if } R \text{ wins} \end{cases}$$

This is a reduced form utility function\(^\text{19}\), to capture the fact that increased participation by activists persuades the voter by shifting preferences towards their favored candidate. Contributions reflect the extent of mobilization by activists during the election cycle. The salience of activism is captured by the $\eta > 0$ parameter. A greater $\eta$ implies that activist participation is weighed more significantly by the median voter, thereby increasing their relevance in the campaign process. Therefore, $\eta$ represents the “relative demand for activism” from candidates during electoral competition\(^\text{20}\).

When $\eta > 0$, candidates care about activists because of their influence on median voter’s preferences. This influence, $P(.)$, could be interpreted along the lines of Madestam et al.\(^\text{34}\). That is, the effort of activists could be seen as providing a kind of direct payoff to the median voter, and the function $P(.)$ captures this direct benefit from campaign activism. Given this nature of dependence on activists, candidates announce platforms taking into account the possibility of greater participation and the improved chances of winning from their contributions. The timing of the game is summarized as follows:

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\(^{18}\)For a broad class of power functions of the form $f(c) = c^\rho$, the elasticities are constant. To see this, notice that $c \cdot f'' = (\rho - 1)$.

\(^{19}\)Using such an utility form for the voter provides a tractable equilibrium solution to the electoral framework. In the extension presented in Section 4.2, we change this assumption to include noisy campaigns.

\(^{20}\)When $\eta = 0$, the median voter is unaffected by activism, and the game resembles a variant of the BDS paper in which candidates with mixed motivations compete for an electoral office, in the presence of uncertainty about median voter’s ideal preference.
1. Candidates $L$ and $R$ simultaneously announce policy platforms $X_L, X_R$

2. Activists observe platforms, and simultaneously choose contributions $c_L$ and $c_R$

3. Nature draws the median voter’s bliss point $\mu$ from an uniform distribution $[-\sigma, \sigma]$

4. The median voter observes policy platforms of candidates, contribution of activists, and decides the winner

Observe that this sequence of play highlights the "price taking" behavior of party activists, as envisaged by Aldrich. We could think of candidates moving first as the equivalent of party conventions, in which the party nominees (or the winners of the respective primaries) announce their platforms in front of party donors and activists, who then take this as given and decide on contributions for the campaign\footnote{Notice that elections involve many other facets which have been ignored in this set-up. Instead, by isolating and concentrating on the channel of electoral activism, I intend to draw critical insights on the effects of activism on political polarization.}. All the exogenous parameters ($\alpha, \beta, \eta, \sigma, b$) and the functional forms of $P(.)$ and $M(.)$ are common knowledge. The equilibrium concept is sub-game perfect Nash equilibrium (SPNE) in symmetric pure strategies.

### 3.1 Median Voter Subgame

The (median) voter chooses the party which gives a higher payoff, ie, the voter prefers candidate $L$ over candidate $R$ iff,

$$-(X_L - \mu)^2 + \eta P(c_L) \geq -(X_R - \mu)^2 + \eta P(c_R)$$

Therefore the cutoff $\mu$, below which the median voter will vote for party $L$ is,

$$\hat{\mu}(X_R, X_L, c_R, c_L) = \frac{(X_R + X_L)}{2} + \frac{\eta P(c_L) - P(c_R)}{2 (X_R - X_L)}$$

Let $\lambda(X_R, X_L, c_R, c_L)$ denote the probability with which candidate $L$ wins when $X_L \neq X_R$. Given the distribution of $\mu$, the probability of candidate $L$ winning is, therefore,
\[ \lambda(X_R, X_L, c_R, c_L) = \frac{1}{2} + \frac{(X_R + X_L)}{4\sigma} + \frac{\eta}{4\sigma} \frac{P(c_L) - P(c_R)}{(X_R - X_L)} \] (3.1)

Notice that the win-probability of candidate \( L \) is increasing in the contributions from activist \( A_L \), and decreasing in the contribution of activists \( A_R \). Fixing one of the side’s contribution constant (say \( c_L \)), and increasing the other \( (c_R) \) reduces candidate \( L \)’s winnability, and vice-versa (\( \frac{\partial \lambda}{\partial c_R} < 0, \frac{\partial \lambda}{\partial c_L} > 0 \)). This provides an incentive for activists to mobilize during the campaign. However, activists also face convex effort costs, and the participation decision essentially balances these two opposing forces.

### 3.2 Activist Contribution Subgame

Consider the contribution decision of the activist. The activist \( A_{L(R)} \) evaluates the winnability of candidate \( L(R) \), which in turn is dependent, among other variables, on the difference in contributions of the two activists. Before exploring the contribution decision of party activists, it is useful to establish what happens when candidates converge on the same platform in the first stage.

**Lemma 1.** When platforms are not differentiated \( (X_L = X_R) \), the activist subgame has an unique equilibrium in which neither activist participates in the campaign, \( c_L = c_R = 0 \).

**Proof.** See Appendix A.1

This result follows from the costly participation decision of activists. When both party candidates converge towards the same platform, the expected ideological loss is constant for the activists. Since activists in the model share purely ideological motives, any positive effort level cannot be optimal given the costs involved, since \( M(c_i) > 0 \) for \( c_i > 0 \). In other words, in the absence of any platform polarization, party activists do not stand to gain ideologically (in expected terms) from participation. On the other hand, when platforms are polarized, it introduces incentives for participation.
Lemma 2. When $X_L \neq X_R \neq 0$, each activist chooses a level of contribution given by,

\[
\frac{M'(c_L)}{P'(c_L)} = \frac{\eta}{4\sigma} [2\beta + (X_L + X_R)]
\]

and

\[
\frac{M'(c_R)}{P'(c_R)} = \frac{\eta}{4\sigma} [2\beta - (X_L + X_R)]
\]

Proof. See Appendix A.2

The contribution function represents the extent of mobilization by activists, and is dependent on activists’ ideology, the platforms announced by the candidates, the demand parameter for activism, and degree of uncertainty in the median voter’s ideology. Observe that the contributions are an increasing function of $2\beta$, the polarization in party activists’ ideologies. When activists are more polarized, it becomes more salient for them to support their candidate, since not doing so would result in the opposite party winning and implementing a more extreme policy. Since activists are purely ideologically motivated and risk averse, greater polarization between the two parties, ceteris paribus, implies that the marginal benefits from contribution is higher. This increased marginal benefit implies contributions by activists must increase in order to offset the marginal benefit. However, this increase in contribution is dampened by the presence of participation costs ($M'' > 0$). The optimal contributions are then determined precisely by the expressions above, balancing both the marginal benefit and marginal costs of increased participation.

The other important point to note is the relationship between contributions of activists and candidates’ announced platforms, $(X_L, X_R)$. There are two effects at play - i) preference for moderation effect, and ii) counter-mobilization effect. Specifically, when one of the candidate becomes more extreme, the party activist supporting the candidate reduces contributions to the campaign ($\frac{\partial c_R}{\partial X_R} < 0$ and $\frac{\partial c_L}{\partial X_L} > 0$). The preference for moderation effect implies that ideologically risk averse party activists tend to have a propensity for moderation. As a result, more extreme platforms are disliked by activists, and a willingness to compromise by a candidate (moving closer to the other candidates’ platform, say) increases
participation from the party activist supporting that candidate.

In addition to this direct effect, there is an additional indirect effect. Specifically, when one candidate becomes more extreme, the activist supporting the other candidate contributes even more (\(\frac{\partial c}{\partial X_L} < 0\) and \(\frac{\partial c}{\partial X_R} > 0\)). This stems from the fact that when a candidate polarizes, the expected ideological loss is higher for the other party’s activist, and hence, the marginal benefit of contribution is greater. Activists, therefore, not only care about whether their candidate moves closer to the centre, but equally care about whether the other candidate becomes more extreme. In this sense, the preferences of party activists are such that it favors greater moderation and compromise from candidates during the campaign process.

### 3.3 Symmetric platforms and supply of activism

Since we restrict attention to symmetric candidate platforms, it is useful to first analyze the form and structure of the supply function of activism under this formulation. When platforms are symmetric, meaning \(X_R = -X_L\), the contributions by activists are independent of the announced platforms \((X_L, X_R)\), since \(X_R + X_L = 0\). This implies that equilibrium supply of activism is purely a function of the exogenous parameters of the model – \((\beta, \eta, \sigma)\) – and is independent of the extent of polarization in platforms. This property is due to the fact that there is no strategic interaction between ideology and the persuasion function \(P(.).\) This additive separability in the median voter’s preferences implies that as long as the two platforms are symmetric on either side of the political spectrum, the optimal contributions of the party activists are unaffected by the extent of platform polarization.

**Lemma 3.** When candidate platforms are symmetric, ie \(X_R = -X_L\), supply of activism is given by \(c^*_L(\beta, \eta, \sigma) = c^*_R(\beta, \eta, \sigma) = c^*(\beta, \eta, \sigma)\) that solves \(M'(c) = \frac{\eta^2}{2\sigma}P'(c)\). Moreover, the following holds: \(\frac{\partial c^*}{\partial \beta}, \frac{\partial c^*}{\partial \eta} > 0\), \(\frac{\partial c^*}{\partial \sigma} < 0\)

**Proof.** See Appendix A.3

Notice that the equilibrium supply of activism has a simple structure. The characterization equates marginal costs and marginal benefits of contribution, resulting in an unique
equilibrium of the activist subgame. Since platforms are symmetric, the contributions of both activists are the same in equilibrium. Further, the marginal benefit is weighted by the term \( \frac{\eta \beta}{\sigma} \). Whenever the demand parameter of activism (\( \eta \)) is higher, the supply of activism is also greater. Similarly, when there is lesser uncertainty (or variance) regarding the median voter’s ideal point, participation increases. These two results stem from the fact that an increase in the demand for activism or reduced variance in median voter’s ideal point shifts the marginal benefit curve up thereby increasing the contributions in equilibrium. On the other hand, as explained earlier, as party activists become more divergent and the partisan gap increases, there is greater participation among the activists. Therefore, any increase in \( \eta \) and \( \beta \), or a decrease in \( \sigma \), leads to greater participation in equilibrium.\(^{22}\)

### 3.4 Candidate platforms

Candidates anticipate contributions and the winning probability as a function of their chosen platforms. A (subgame perfect) Nash Equilibrium strategy for a candidate is a policy platform that maximizes their payoff, given the other candidate’s platform choice and the subsequent play of the game. I restrict attention to symmetric candidate platforms. Before characterizing equilibrium platforms with party activists, it would be useful to consider the case when there is no demand for activists, meaning \( \eta = 0 \). This describes a political environment devoid of activists, and the equilibrium is determined by candidates with mixed motivations and median voter uncertainty. The following proposition characterizes the equilibrium platforms in the absence of activism.

**Proposition 1.** The electoral game without activism has a symmetric equilibrium \((-\bar{x}, \bar{x})\) such that, if \( \alpha > \frac{b}{4\sigma} \) then \( \bar{x} = \frac{4\sigma \alpha - b}{4(\alpha + \sigma)} \), and if \( \alpha \leq \frac{b}{4\sigma} \) then \( \bar{x} = 0 \).

*Proof. See Appendix A.4* \(\square\)

\(^{22}\) Although participation in equilibrium is positive when platforms are polarized, it is nevertheless wasteful, in the sense that both the activists’ contributions are equal and therefore do not have any relative impact on the winnability of the candidate. However, the reason why they are positive is precisely because if one activist were to reduce the level of contributions, it decreases their candidate’s winnability. The other activist, as a consequence, has a greater incentive to contribute, since the marginal benefits of contributing exceeds the marginal costs of doing so. This interdependence between winnability and activism prompts either party activists to contribute a positive level in the campaign, even though in equilibrium the two contributions cancel off each other resulting in zero net effect of activism.
An important point to note in the above proposition is that \( \bar{x} < \alpha \). This implies that responsible candidates never choose their ideal policy and always moderate in equilibrium. Given this, introducing ideologically risk averse party activists further changes the incentives for candidates. On top of targeting the median voter’s ideal policy, candidate’s also have to cater to the preferences of activists. As I had shown earlier, party activists prefer moderation in platforms. This indicates that candidates may further trade-off ideology in order to elicit greater participation from activists by moderating platforms in equilibrium, irrespective of the extent of partisan gap between party activists.

**Proposition 2.** The electoral game with activism has an unique symmetric pure strategy equilibrium in candidate platforms \((−x^*, x^*)\) that solves,

\[
4(\alpha + \sigma)x^2 - [4\alpha(\sigma - \frac{1}{2}D(c^*(\eta, \beta, \sigma), \eta, \beta))] - bx + \frac{b}{2}D(c^*(\eta, \beta, \sigma), \eta, \beta) = 0
\]

where \( D(c^*(\eta, \beta, \sigma), \eta, \beta) = \frac{\eta \beta c^*P'(c^*)}{\gamma_m(c^*) + \gamma_p(c^*)} \)

such that if \( \alpha > \frac{b}{4(\sigma - \frac{1}{2}D(\cdot))} \) then \( x^* > 0 \); if \( \alpha \leq \frac{b}{4(\sigma - \frac{1}{2}D(\cdot))} \), then \( x^* = 0 \). Furthermore, \( \bar{x} \geq x^* \).

**Proof.** See Appendix A.5

The equilibrium with activists is more moderate than in the absence of them. This is an interesting finding since it illustrates an important role for political activism. Political activists in representative democracies help constrain extreme platforms and build consensus. In the absence of activists, candidates with mixed motivations would tend to move away from each other and closer to their preferred platform, causing greater polarization. In polarized societies like the present day US, activism is indeed good. The inability of activists, unlike lobbies and organized interest groups, to commit to (implicit) policy contingent contracts with the candidates implies that candidate platforms are more moderate in the presence of activism. Political compromise is brought about by their mere presence, irrespective of the extent of partisanship between the activists.

Given this interesting finding, it is pertinent to study the relationship between polarization among activists and platform polarization of candidates. Doing so requires a way
to describe the incentives for engaging in activism. During campaigns, party activists engage with potential supporters and the wider public in order to mobilize support for their candidate. This means that the willingness to engage of activists plays a crucial role in determining what platforms are chosen in the first place. When activists have a greater willingness to engage, their participation affects the campaign in a more significant way, and candidates would then have incentives to moderate.

I define willingness to engage as $WTE = \frac{1}{\gamma_m + \gamma_p}$. That is, willingness to engage is simply the inverse of the sum of elasticities of marginal cost of participation and marginal influence. This gives us an intuitive way to think about the participation decision of activists in the electoral process. For example, a lower elasticity of the marginal cost function raises the willingness to engage for activists, just as a lower elasticity of marginal influence does.

### 4 Partisan gap and candidate polarization

In my analysis, party activists are driven by an ideological inclination. The mere presence of such activists reduces the polarization of candidate platforms. An important question that arises is - "How does polarization change as activists themselves becoming more partisan?"

Take, e.g., the Pew Research Center’s study\(^\text{23}\) in 2014 that documents this partisanship, and I quote - "Today, 92% of Republicans are to the right of the median Democrat, and 94% of Democrats are to the left of the median Republican.". Further, it adds, "But on every measure of engagement, political participation is strongly related to ideology and partisan antipathy; those who hold consistently liberal or conservative views, and who hold strongly negative views of the other political party, are far more likely to participate in the political process than the rest of the nation. This results in a consistent “U-shaped” pattern, with higher levels of engagement on the right and left of the ideological spectrum, and lower levels in the center."

When activists become more extreme (partisan), given the risk-averseness of their ide-
logical preferences, more moderation and compromise provides activists with lesser expected ideological loss. However, moving closer to ex-ante median implies the candidates suffer greater ideological losses. Therefore, while activists prefer moderation, the candidates, on the other hand, prefer greater divergence. This partisanship between activists, combined with the candidates’ dependence on them, could then increase the levels of political polarization, to the detriment of voters’ welfare. Whether the candidate platforms diverge further or converge depends crucially on the activists’ willingness to engage. The next result precisely outlines this relationship.

**Proposition 3.** The following holds:

(i) As party activists become more extreme, equilibrium platforms are less polarized if \( \gamma_p < \frac{1 - \gamma_m}{2} \)

(ii) As activists become more extreme, equilibrium platforms are more polarized if \( \gamma_p > \frac{1 - \gamma_m}{2} \)

(iii) The equilibrium polarization is independent of the ideological preferences of party activists if \( \gamma_p = \frac{1 - \gamma_m}{2} \)

**Proof.** See Appendix A.6

The above proposition states that there is a non-monotonic relationship between activist polarization and political polarization of candidates, and the nature of this relationship is captured by the willingness to engage of activists. Specifically, as the partisan gap between parties widen, it need not result in greater political polarization in equilibrium as long as the willingness to engage of party activists is sufficiently high (specifically, \( WTE > \frac{1}{1-\gamma_p} \)). This stems from the fact that as activists become more extreme, they also suffer a greater ideological loss when the other party’s candidate wins the election. As a result, irrespective of candidate platforms, when activists become more polarized, the equilibrium contribution goes up (\( \frac{\partial c^*}{\partial \beta} > 0 \)), since the stakes are higher for the activists.

Candidates, on the other hand, understand these trade-offs.\(^{24}\) By moderating their plat-  

\(^{24}\)This kind of political compromise was witnessed in the recent US primary battle between Hillary Clinton and Bernie Sanders. Clinton, during the course of the battle, adopted platforms far more progressive than her perceived ideological moorings (as a centrist candidate). Platforms that resulted in compromise were, e.g., college tuition and $15 minimum wage. A similar compromise was witnessed in the 2015 New Delhi elections in which both the main competing parties decided to adopt a variant of the anti-corruption bill, which remained the main campaign issue.
forms, they can extract greater participation from their activist. The downside is that they move further away from their own ideal platform $\alpha$. This tension between preference for polarization on one hand and activism driven moderation on the other implies that equilibrium platforms could either diverge further or converge towards each other. When activists have a high willingness to engage, as one of the candidate polarizes, it pushes the two effects – counter-mobilization and preference-for-moderation – in favor of the other, less polarized, candidate. These two effects are more stronger when the willingness to engage is greater. This implies that for the polarizing candidate, the marginal loss is greater than the marginal benefit of doing so.

The opposite is true when the willingness to engage is low. In this case, candidates find it optimal to polarize since doing so does not change the participation by enough in order to compensate for the increased gains of moving closer to their preferred platform ($-\alpha, \alpha$). That is, both the counter-mobilization and preference-for-moderation effects are weaker than the previous case. Candidates understand this trade-off while announcing their platforms. They recognize that a lower $WTE$ means that activists do not react to polarization as strongly, and this provides them incentives to move closer to their ideal points.

Finally, when the willingness to engage is such that it satisfies the third condition, candidate polarization is unaffected by increasing partisanship among the activists. These results are of fundamental importance in understanding the intricate relationship between party polarization and candidates’ platform polarization. What this proposition implies is that a widening partisan gap between parties is neither necessary nor sufficient for increased polarization in campaigns. What is important is the interaction between partisan gap and the willingness to engage of activists in the electoral process, meaning, a combination of $(\beta, WTE)$ is critical in determining whether polarization increases.

For example, consider the issue of rising income inequality. This may push political activists towards supporting policies that are more progressive on one side and more conservative on the other. However, this does not necessarily guarantee that candidates representing their interests would further polarize their platforms to reflect this partisanship. In fact, as my analysis suggests, a compromise could be reached in equilibrium if both parties’ willing-
ness to engage in the political process is high enough. In this case, a high $WTE$ implies that either the curvature of the cost function is low, or activism is more effective in influencing voter preferences, or both. Then, any increase in the $WTE$ implies that party activists react more severely to polarization by either candidate, and this precludes them from doing so, even though parties themselves have a widened partisan gap on issues.

This result helps refocus attention on the role of party structure in electoral campaigns and democratic polities. If party activists show a greater willingness to engage with, and persuade, ordinary voters, then even if their own policy preferences diverge, the fact that there are imminent risks associated with electoral competition would imply that candidates may adopt more moderate stances reflecting the risky nature of campaigns. Parties and activists in democratic states could then provide a natural barrier against polarization as long as they remain actively engaged in the political process.

4.1 An Example

Consider a simple example in which I assume specific functional forms to the influence function and the participation costs. Firstly, for simplicity, let $\eta = \sigma = 1$. This is useful in order to understand the theoretical underpinnings of the model more clearly. Further, let participation costs be a power function of the form $M(c_i) = c_i^\rho \ (\rho > 1)$ and let the influence function be linear, $P(c_i) = c_i$. This makes the median voter’s utility quasi-linear in activist participation, and $\eta$ the marginal utility of activist influence.

In this simple parameterization, the equilibrium participation then becomes a solution to the following equation,

$$\rho c^\rho - 1 = \beta$$

$$\Rightarrow c^* = \left(\frac{\beta}{\rho}\right)^{\frac{1}{\rho-1}}$$

The two aversion parameters are then $\gamma_m = (\rho - 1)$ and $\gamma_p = 0$. In the absence of activism, $c^* = 0$ and $\bar{x} = \max\{0, \frac{4\alpha-b}{4(\alpha+1)}\}$. Further, as the demand for activism increases, the candidate
platforms decrease and move closer to zero (if $\bar{x} > 0$). Now, the willingness to engage for the activists is given simply by $WTE = \frac{1}{\gamma_m} = \frac{1}{(\rho - 1)}$. This means that any lowering of $\rho$, the elasticity of the marginal cost function, increases the $WTE$ parameter. In fact the main condition for the result then simplifies to,

$$\frac{\partial x^*}{\partial \beta} < 0 \text{ if } WTE > 1 \text{ or } \rho < 2 \quad (4.1)$$

Therefore, $\frac{\partial x^*}{\partial \beta} < 0$ when the marginal cost function is concave (less steep). To gain the intuition behind this, it is important to understand the relationship between marginal costs and benefits of activism. Take the case of activist $A_R$. When $\rho < 2$, the marginal impact of ideology on participation ($\frac{\partial c^*}{\partial \beta} = \frac{1}{\beta(\rho - 1)} \left( \frac{\beta}{\rho} \right)^{\frac{1}{\rho - 1}}$) is increasing but convex $-\frac{\partial^2 c^*}{\partial \beta^2} = \frac{2 - \rho}{\rho - 1} \frac{1}{\beta} \frac{\partial c^*}{\partial \beta}$ is positive only when $\rho < 2$. This has two effects. Firstly, this directly translates into increasing and concave marginal costs of ideological polarization for the activist. Secondly, a convex change in marginal impact of ideology on participation has to be matched by a corresponding increase in marginal benefit, in terms of reduced expected ideological loss. Candidate $R$ would have to choose platforms such that this trade-off of activist $R$ is satisfied.

Since activists are ideologically risk averse, platforms that reduce variance (more convergent platforms) provide them with marginal benefits that equate marginal costs of increased polarization. As activists continue to become more polarized, and $\rho < 2$, the marginal costs to the activist is concave and therefore, the contributions are increasing in a convex manner. To compensate for this increase, candidate $R$ chooses a platform closer to ex-ante median ($\frac{\partial x^*}{\partial \beta} < 0$). Simultaneously, candidate $L$ does the same, and the overall platform polarization reduces, leading to greater convergence.

### 4.2 Other results

**Proposition 4.** When $x^* > 0$, candidate platforms become more extreme if i) candidates’ ideological polarization $\alpha$ increases; ii) benefits of office $b$ decreases; iii) the demand for activism $\eta$ decreases; iv) variance in median voter’s ideological preference $\sigma$ increases.

**Proof.** See Appendix A.7
The relation between platform divergence and \( \eta \) is along expected lines. The rationale is the following. As the demand for activism decreases, it implies that voters weight candidate ideology more heavily compared to activist engagement. This implies activists end up decreasing their participation in the political process. Besides, the increased weight on candidate platforms means that the candidates rely less (in marginal terms) on activist participation. This decreased dependency on activists, therefore, translates into more divergent platforms in equilibrium.

As \( \eta \) goes to zero, notice that the equilibrium platform is the same as the case with no activism. That is, as \( \eta \to 0, x^* \to \bar{x} \). This can be gleaned by substituting \( \eta = 0 \) into the equilibrium equation in Proposition 2. Therefore, as the demand for activism increases in the electoral process, the platforms decrease from \( \bar{x} \) and move towards \( x^* = 0 \) (perfect Downsian convergence), meaning there is lesser polarization as the demand for activism increases.

Lastly, platform divergence increases when there is greater uncertainty regarding voter’s preferences. This makes intuitive sense in that, ceteris paribus, candidates in the model are trying to locate the median voter’s bliss point. Remember that greater uncertainty reduces activist participation because the possibility of more extreme median platforms reduces the marginal benefits for the activists. As this uncertainty or the variance increases, candidates adjust their platform in a way so as to account for this reduced participation from party activists and move more closer to their ideal policy.

5 Welfare implications of activism

In this section, I study the welfare effects of activism and polarization on the voters. Two questions need answering. First, are activists welfare-improving for the voters, compared to polities without their presence? Second, does increased polarization among activists adversely affect the welfare of voters? Though I motivated this work on the basis that polarization hurts democratic societies, normative results on voter welfare have shown that this need not be the case for low levels of divergence in platforms (see BDS). This fundamental result is the starting point of my analysis.
Consider the ex-ante welfare of voters first under no activism. Let $W_v(-x, x)$ be the welfare of any voter $v$ under the symmetric equilibrium platforms of the two candidates. Then,

$$W_v(-x, x) = -\frac{1}{2\sigma} \left[ \int_{-\sigma}^{0} (x + v + \mu)^2 d\mu - \int_{0}^{\sigma} (x - v - \mu)^2 d\mu \right]$$

The sum of all voters’ welfare is given by,

$$W_{tot}(-x, x) = -\frac{1}{4\sigma} \int_{-1}^{1} \left[ \int_{-\sigma}^{0} (x + v + \mu)^2 d\mu + \int_{0}^{\sigma} (x - v - \mu)^2 d\mu \right] dv$$

**Lemma 4.** If $\alpha > \frac{b}{4(\sigma - \frac{1}{2}D(c))}$, then welfare of voters is such that, i) $W_{tot}(-x, x) > W_{tot}(0, 0)$; ii) it is maximized at $x_{vo} = \frac{\sigma}{2}$.

When there is uncertainty about the median voter’s bliss point, voters are better off when there is a small degree of divergence in platforms, compared to Downsian convergence on the ex-ante median. Normatively speaking, this implies that voters prefer some level of polarization in platforms. When there are activists engaging voters, they generate a positive welfare effect on voters. However, I am interested in the polarization induced by the presence of activists. Since in any symmetric equilibrium, the two activists’ contributions are the same, they cancel each other out in the welfare function. Besides, activist participation is independent of the symmetric platform choice of candidates, $(-x^*, x^*)$. Therefore, the first best levels of polarization remain unchanged in the presence of activism.

Given this, it is imperative to characterize the conditions under which the introduction of activists is actually welfare improving. From the previous analysis, we know that compared to the case of no activism, the presence of activists moderates the platforms. That is, the equilibrium polarization with activism is,

$$x^* = \frac{[4\alpha(\sigma - \frac{1}{2}D(c, \eta, \beta)) - b]}{8(\alpha + \sigma)} + \frac{\sqrt{[4\alpha(\sigma - \frac{1}{2}D(c, \eta, \beta)) - b]^2 - 8b(\alpha + \sigma)D(c, \eta, \beta)}}{8(\alpha + \sigma)}$$
In this case, the welfare increase is possible only if two conditions hold: i) $\bar{x} > \frac{\sigma}{2}$; and ii) the equilibrium with activists $(-x^*, x^*)$ is not low enough that $W_{\text{tot}}(-\bar{x}, \bar{x}) > W_{\text{tot}}(-x^*, x^*)$. The first condition ensures that in the absence of activism, the level of polarization is above the social optimal (welfare is decreasing in polarization), and the second implies that the presence of activists must not moderate the platforms to below a threshold (given by $\sigma - \bar{x}$).

The following proposition precisely characterizes these conditions.

**Proposition 5.** When $b < 2\sigma(\alpha - \sigma)$ and $x^* > \frac{4\sigma^2 + b}{4(\alpha + \sigma)}$, activism improves welfare of voters. When either $b > 2\sigma(\alpha - \sigma)$ or, $b < 2\sigma(\alpha - \sigma)$ and $x^* < \frac{4\sigma^2 + b}{4(\alpha + \sigma)}$, the presence of activists hurts the welfare of voters.

**Proof.** See Appendix A.8

The presence of activists need not always improve welfare. The reason for this is that by their preference for compromise, activists may constrain the choices of candidates to the point where they are too similar. One way by which this compromise happens is when one of the candidates adopts some features of the platform offered by the other, therefore compromising on their ideological preferences. By doing so, the candidates may end up providing very little choice to the electorate, harming their welfare.

**Lemma 5.** Given an equilibrium level of polarization $x^* \in (0, \frac{\sigma}{2}]$, any increase in the partisan gap between activists reduces total welfare when activists’ $WTE > \frac{1}{1 - \gamma_p}$ and improves welfare when $WTE < \frac{1}{1 - \gamma_p}$; and vice versa for $x > \frac{\sigma}{2}$.

**Proof.** See Appendix A.9

The welfare effects of activist polarization is ambiguous. That is, the impact of activist divergence on overall welfare depends on the extent of prevailing polarization in the political process. As we observed earlier, when the $WTE$ of activists is high (condition i) of Proposition 3), the equilibrium level of polarization decreases as activists become more diverged. Now, when the level of polarization is in the interval $[0, \frac{\sigma}{2}]$ (welfare is increasing in polarization in this interval), a high $WTE$ implies that activist divergence results in greater moderation, and this reduces overall welfare. However, when the level of polarization with
activists, $x^*$ is higher than the first best, then increased divergence of activists improves voter welfare when the WTE is high.

The implication of this result is that increased partisanship calls for either lowering or increasing the WTE of activists, depending on the existing levels of polarization. The lowering of willingness to engage could be seen as introducing barriers to participation, or increasing the noise such that activism is rendered ineffective. On the other hand, increasing the willingness to engage could be interpreted as a way in which the marginal costs are decreased for activism, and/or increasing the effectiveness of activists. Therefore, by varying the barriers to participation, the campaign process with activism can indeed lead to welfare improving outcomes.

Lemma 6. Consider a social planner trying to implement the socially optimal level of polarization. 1. As long as $b < 2\sigma(\alpha - \sigma)$, there exists a $\eta^*$ such that the first best levels of polarization can be achieved. 2. As long as $D(.) < \frac{\sigma}{\alpha}(\alpha - \sigma)$, there exists a $b^*$ such that the social planner can implement the first best levels of polarization.

Proof. See Appendix A.10

To show the first part follows, notice that when $\eta = 0$, the level of polarization is given by proposition 1, $\bar{x}$. Given this, when the level of office of benefits is low enough, then the level of polarization is above the social optimum $\frac{\sigma}{2}$. But, since the level of polarization is increasing in $\eta$, the planner can increase the relative demand for activism until the level of polarization drops to the social optimum. The second part asserts that as long as the level of polarization at the limit $b \rightarrow 0$ is above the first best levels, increasing the benefits of office can moderate the platforms to achieve the first best.
6 Extensions

6.1 Activism in Noisy campaigns

Suppose the policy platform of candidates are observed with noise by median voter, and activists' role is to inform the median voter of the precise position. If $X_i$ is the true position of the candidate, the policy observed by the median voter is $\tilde{X}_i = X_i + \eta_i$, where $\eta_i$ is a random variable (noise term) with expectation zero and variance $\sigma^2_i$. Further, contribution from activists reduces the variance of the noise term. If $c_i$ is the contribution from the activist, then $\sigma^2_i = a(c_i)$. The following assumptions are made on the functional form of $a(.)$:\footnote{We additionally assume that the noise reduction mechanism $a(\cdot)$ is the same for both candidates.}: $a'(\cdot) < 0$, $a''(\cdot) > 0$, $a'''(\cdot) < 0$ and $a(0) > 0$. First two conditions ensure that as activists contribute more, the variance of noise function is decreasing, and convex. The subsequent condition implies that the concavity of marginal variance, and the last condition states that, in the absence of activism, there is a positive level of noise in platforms, meaning voters imperfectly observe platform of candidates.

This formulation naturally implies that greater activist participation is beneficial for candidates since it reduces the variance of the platforms, and since the voter is risk-averse, less variance is preferred. Activists or volunteers, then, have an important role in conveying - through door-to-door canvassing or phone calls - the true policy stance of their candidate.

Before presenting the results, it is important to glean the role of noise reduction function. Remember, activism is now not a persuasive tool, but restricted to only reducing the variance of the noisy platform. I introduce the parameter $\gamma_n(c)$ to define the efficiency of activism in reducing the noisiness of platforms. That is, Efficiency of Marginal Noise Reduction, $\gamma_n(c) = -c \cdot \frac{a''}{a'}$. As in the baseline model, the willingness to engage is defined by $WTE_N(c) = \frac{1}{\gamma_m(c) + \gamma_n(c)}$. For the sake of simpler exposition, let $WTE_N$ be independent of $c$ ($\gamma_m(c) = \gamma_m$ and $\gamma_n(c) = \gamma_n$).

Proposition 6. In a noisy electoral campaign with activism, there exists an unique symmetric equilibrium in candidate platforms. Furthermore,
i. $\frac{\partial x}{\partial \beta} < 0 \text{ if } \gamma_n < \frac{1-\gamma_m}{2}$;  
ii. $\frac{\partial x}{\partial \beta} > 0 \text{ if } \gamma_n > \frac{1-\gamma_m}{2}$

Proof. See Appendix B.1

Proposition 8 shows that the main equilibrium and comparative statics result holds. The details of the same are confined to Appendix B.

6.2 The role of soft money

Though the reliance of grassroots activists is an important avenue of campaigning, the role of soft money in the form of PACs or super PACs also have been playing an increasingly important role. McCarty, Poole and Rosenthal [36], e.g., find that large contributions and contributors on average were more extreme\textsuperscript{26} (on either side of the political spectrum), and moreover, they tended to favor extreme candidates. In some way, this type of soft money contributions act as a substitutable good to activism. Therefore, when candidates have access to big money, their reliance on the party organization and its activists goes down. This in turn may provoke candidates to polarize away from each other, and towards more extreme ideologies.

To see this mechanism, I will modify the model to consider the role of big money\textsuperscript{27}. Suppose $S$ is the the available soft money contributions for either candidate. Then, the candidates have two goods that are employed for influence in elections – activist participation $c_i$ and soft money $S$. Moreover, I modify the influence function to include soft money parameter. That is, $P(S, c_i)$ is the total influence generated by campaigning, such that $P_2(.) > 0$, $P_{22}(.) < 0$, $P_1(.) > 0$, $P_{11}(.) < 0$, and $P_{12}(.) < 0$. The concavity assumption holds as before, and the last assumption states that $S$ and $c_i$ are strategic substitutes. This makes intuitive sense. Candidates need money to spend on advertisements, hiring campaign staff, on personalized communication to voters, and so on. Therefore, money supplements the traditional grassroots campaign of activists.

\textsuperscript{26}Please refer to chapter 5 of MPR.

\textsuperscript{27}For the sake of exposition, I will abstract away from strategic interaction of big donors. I will exogenously assume that candidates have a share of soft money contributions, and then use this to study its impact on polarization and activism.
What this suggests is that the presence of big soft money may crowd out the role of activism, thereby decreasing the level of participation. This crowding out of activists may then provide candidates to polarize for much the same reasons as discussed earlier. Any decrease in the marginal effect of activism reduces the candidates’ incentive to compromise. This leads to greater polarization in equilibrium.

**Lemma 7.** *In the presence of soft money in campaigns, the equilibrium polarization and participation are such that, i) \( \frac{dc^*}{dS} < 0 \) and ii) \( \frac{dx^*}{dS} > 0 \).*

*Proof.* See Appendix B.2

Lemma 3 shows how soft money has a crowding out effect on activism and pushes platforms to more extremes as a result. The fact that this kind of soft money reaches more extreme candidates in the first place would only exacerbate its effect on political polarization (\( \frac{dx^*}{dS}, \frac{dx^*}{da} > 0 \)). As a result, this suggests a rationale for curbing this kind of big money spending by individual contributors or organized groups. My analysis suggests curbing the use of soft money and increasing the dependence on activists. This could help counterbalance the current trend of excessive role of big money in the political process and potentially reduce platform polarization.

7 Discussion and Concluding remarks

I have analyzed a model of political competition that addresses the question of whether, and how, the participation of activists in the electoral process affects political polarization. I find that, interestingly, when activists are price-takers in Aldrichian sense, candidate platforms are always more moderate than the case without activists. That is, the political process with activists yields a greater compromise between the two candidates. The inability of activists to make policy contingent (implicit) contributions and commit to *quid pro quo contracts*\(^{29}\)

\(^{28}\)The so called ”527 group” spending, e.g., places no upper bounds on how much and who to contribute. Some prominent ones include Club for Growth, MoveOn.org, New Democrat Network, among others.

\(^{29}\)For more on organized interest groups and lobbies that are able to commit to such policy contingent contracts, see Grossman and Helpman [24], and Morton and Myerson [41].
implies that candidates take into account the preferences of activists when announcing policy platforms, and this leads to moderation in platforms.

Moreover, I show that when activists become more polarized, and their willingness to engage is above a threshold, the equilibrium platforms of candidates tend to converge, or in other words, political polarization decreases further. The main finding of my analysis provides a clear testable threshold on the willingness to engage of activists. Below this threshold, any increase in divergence between activists must reflect on the candidate platforms by polarizing them. The implication of this result is that activist polarization, on its own, is insufficient to explain political polarization. The combination of activist partisanship and the willingness to engage in the political process together determine the extent of political polarization. The other testable result of my analysis is that polarization reduces as the demand for activism increases. That is, there is a negative correlation between polarization and the salience of activism in campaigns.

The willingness to engage can be reconciled by looking at both the supply and demand side factors that affect activism. On the supply side, one possible interpretation of the willingness to engage parameter is that it captures the costs incurred by activists during the electoral process. Specifically, modern day campaigns, especially so in the context of US, happen over a period of time. Electoral cycles are a continuous and arduous process, taking up a lot of time and resources on the part of candidates and activists, starting with the announcement of platforms up until election day. In some sense, the elasticity parameter reflects the marginal costs involved, and captures the length of the electoral campaign cycle. If this is indeed the case, then local (municipal bodies or city councils) elections where stakes are lower and length of the campaign is substantially shorter, may have lower levels of polarization compared to state or federal elections, controlling for activist ideologies.

Some important welfare implications emerge from my analysis. Specifically, I establish that when political polarization is very high, increased divergence among activists can actually be welfare improving for voters as long as the willingness to engage for the activists is high. That is, highly polarized polities may benefit from activism in campaigns, irrespective of the ideological inclinations of the activists. Even when activists become more diverged,
they could end up moderating platforms further, and increase voter welfare. However, on
the downside, democratic societies with high levels of participation and less polarization may
indeed provide lesser choice (decreased polarization) to the voters making them worse off in
welfare terms.

My welfare analysis also raises an important point that is related to the issue of party
strength and polarization in democracies. Specifically, several studies (e.g., Norris [42] [43],
Dalton and Wattenberg [16]) have pointed out the phenomenon of decreasing political en-
gagement and weakening party structures in advanced economies. My work suggests that
this may be due to lack of incentives to associate with political outfits (high marginal costs,
say), or alternatively, a result of reduced demand for them (crowding out effect of soft
money). The empirical evidence suggests that the willingness to engage of the wider elec-
torate in activism has decreased over time in industrialized nations.\(^{30}\) In the case of US, it
may be that the decline of traditional participation is closely linked to the demand for such
activism.\(^{31}\) For example, modern communication and messaging techniques employed by
candidates preclude the need for a more grass-roots campaign. The candidates rely instead
on big campaign expenditure to communicate directly to the electorate. Activists, who as-
sociate with parties and act as intermediaries, get crowded out in the political process. The
polarization in American politics, therefore, can be pinned down to such a mix of supply-side
and demand-side considerations of activism.

Finally, my theoretical findings suggests a possible link between publicly funded elections
and political polarization. Public funding of campaigns could potentially have two effects.
First, they reduce the crowding out effect of big money on activism. This would increase the
reliance of candidates on activists and tilt the balance of demand towards smaller volunteers
and a grass-roots campaign. Second, public funding\(^{32}\) may automatically reduce the length

\(^{30}\) Refer to Dalton and Wattenberg [16], Schmitt and Holmberg [49], and Mair and Van Biezen [35] for a
comprehensive account of the decline in partisanship in Western Democracies.

\(^{31}\) See Seyd and Whiteley [53] [50] for evidence concerning the reduced demand for activists in Britain's
political process.

\(^{32}\) Some other prominent advantages of public funding is that is reduces candidate dependence on per-
nicious interest group considerations, allowing for more electoral competition, and possibly decreasing
incumbency advantage. In U.S, the states of Maine and Arizona have had publicly funded state legis-
slature elections. Look at the report by Government Accountability Office (GAO) for more on this:
of the campaign cycle, thereby encouraging moderate supporters (those with high marginal costs of participation) to effectively contribute. This creates greater inclusiveness in the campaign by encouraging participation of more moderate political activists. These two effects, according to the predictions of my model, would decrease the levels of polarization and improve welfare of voters. Therefore, curbing independent expenditures (e.g., restrictions on PACs and super-PACs spending) by introducing stricter laws, akin to ones previously articulated by Prat [47] and Gul and Pesendorfer [28], and simultaneously introducing publicly funded campaigns would act as a useful institutional reform to encourage wider participation and bringing down political polarization.
A Proofs - Benchmark model

All the proofs are carried out for candidate $R$ and activist $A_R$, and are symmetric for candidate $L$ and activist $A_L$.

A.1 Proof of Lemma 1

When platforms are not differentiated, irrespective of who wins the election, the ideological loss faced by the activist is constant and independent of $c_R$. As a result, there is no incentive to contribute since $M'(0) > 0$. Therefore, the only equilibrium of the activist contribution subgame is one where $c_R = c_L = 0$.

A.2 Proof of Lemma 2

Consider activist $A_R$ and their contribution decision. Given candidate R’s win probability, the expected utility of activist $A_R$ is,

$$EU_{A_R}^R(c_R; \beta) = K - \lambda(X_L - \beta)^2 - (1 - \lambda)(X_R - \beta)^2 - M(c_R)$$

Taking the first order condition, we get,

$$M'(c_R) - \frac{\eta}{4\sigma} P'(c_R)[2\beta - (X_L + X_R)] = 0$$

Rearranging, we get the necessary result.

$$\frac{M'(c_R)}{P'(c_R)} = \frac{\eta}{4\sigma}[2\beta - (X_L + X_R)] \quad (A.1)$$

An analogous argument holds for the activist $A_L$.

$$\frac{M'(c_L)}{P'(c_L)} = \frac{\eta}{4\sigma}[2\beta + (X_L + X_R)] \quad (A.2)$$

A.3 Proof of Lemma 3

Under symmetric platforms, $X_L + X_R = 0$ and from equation A.1, it must be that the optimal contribution by Activists $A_R$ and $A_L$ are equal. That is, $c_R = c_L = c^*$. Further, this $c^*$ solves,
\[ M'(c) = \frac{\eta \beta}{2\sigma} P'(c) \]  

(A.3)

For the second part of the lemma, we apply implicit function theorem to equation A.3.

Let \( \Psi = \eta \beta \frac{\sigma}{2\sigma} P'(c) - M'(c) \)

\[
\frac{dc^*}{d\beta} = -\frac{\eta}{2\sigma} \frac{\sigma}{2\sigma} P''(c) - M''(c)
\]

Since \( P''(.) \leq 0 \) and \( M''(c) > 0 \), it follows that \( \frac{dc^*}{d\beta} > 0 \).

Similarly, \( \frac{dc^*}{d\eta} = -\frac{\beta}{2\sigma} \frac{\sigma}{2\sigma} P''(c) - M''(c) > 0 \) and \( \frac{dc^*}{d\sigma} = \frac{\eta \beta}{2\sigma} \frac{\sigma}{2\sigma} P''(c) - M''(c) < 0 \) for the same arguments.

### A.4 Proof of Proposition 1

When \( \eta = 0 \), the win probability of candidate \( R \) is just \( 1 - \frac{\lambda}{2} = \frac{1}{4} (X_R + X_L) \). Each candidate chooses a platform to maximize their payoffs, taking as given the platform chosen by the other candidate.

The SPNE is such a pair of platform choices that maximizes the expected utility of both the candidates in the first stage. Let us consider candidate R.

\[
EU_C(X_L, X_R) = -\lambda (X_L - \alpha)^2 - (1 - \lambda)(X_R - \alpha)^2 + (1 - \lambda) b
\]

Supposing that candidate L chooses \( X_L = -x \). Taking the FOC and evaluating the expression at \((-x,x)\),

\[
\frac{dEUC_{R}(-x,X_R)}{dX_R} \bigg|_{(-x,x)} = X_R' \left[ 4\alpha x + b \right] - (\alpha - x) = 0 \text{ where } X_R' = \frac{1}{4\sigma}
\]

Solving this gives us \( \bar{x} = \frac{4\sigma \alpha - b}{4(\alpha + \sigma)} \). If \( \alpha > \frac{b}{4\sigma} \), then \( \bar{x} > 0 \) follows from the expression.

### A.5 Proof of Proposition 2

In the case of \( \eta > 0 \), the win-probability is affected by activism.

That is, \( X_R' = \frac{1}{4\sigma} + \frac{\eta}{4\sigma} \left[ \frac{(X_R - X_L)[P'(c_L) - P'(c_R)]}{(X_R + X_L)^2} \right] \).

Applying implicit function theorem to equations A.1 and A.2, we can compute \( \frac{dc_R}{dX_R} \) and \( \frac{dc_L}{dX_R} \) respectively.

\[
\frac{dc_R}{dX_R} = \frac{\eta}{4\sigma} \frac{P'(c_R)}{(2\beta - (X_L + X_R))P''(c_R) - M''(c_R)}
\]

\[
\frac{dc_L}{dX_R} = -\frac{\eta}{4\sigma} \frac{P'(c_L)}{(2\beta + (X_L + X_R))P''(c_L) - M''(c_L)}
\]
At \((-x, x)\), it is easy to check that \(\frac{dc_R}{dX_R} \big|_{(-x,x)} = -\frac{dc_L}{dX_R} \big|_{(-x,x)}\) and,

\[
\frac{dc_R}{dX_R} \big|_{(-x,x)} = \frac{\eta}{4\sigma} \frac{\eta^3 P''(c) - M''(c)}{P'(c)}
\]

Therefore, the FOC evaluated at \((-x, x)\) gives us,

\[
\lambda'_{X_R} = \frac{1}{4\sigma} - \frac{\eta^2}{16\sigma^2 x} \frac{(P'(c))^2}{\frac{\eta}{2\sigma} P''(c) - M''(c)}
\]

\[
\lambda'_{X_R} = \frac{1}{4\sigma} \left[ 1 + \frac{\eta^2}{4\sigma^2} \frac{(P'(c))^2}{\frac{\eta}{2\sigma} P''(c) - M''(c)} \right]
\]

But we know that \(c\) solves \(M'(c) = \frac{\eta^3}{2\sigma} P'(c)\). Simplifying the above expression and using the fact that \(\gamma_m = c \frac{M''}{M'}\) and \(\gamma_p = -c \frac{P''}{P'}\),

\[
\Rightarrow \lambda'_{X_R} = \frac{1}{4\sigma} \left[ 1 + \frac{\eta^2}{2\sigma^2} \frac{c P'(c)}{\gamma_m + \gamma_p} \right] > 0
\]

Rewriting the above as \(\lambda'_{X_R} = \frac{1}{4\sigma} \left[ 1 + \frac{1}{2} D(c, \eta, \beta) \right]\) and using this in the FOC evaluated at \((-x, x)\),

\[
\lambda'_{X_R}[4\alpha x + b] - (\alpha - x) = 0
\]

This yields us the required condition,

\[
4(\alpha + \sigma)x^2 - [4\alpha(\sigma - \frac{1}{2} D(c, \eta, \beta))] - b)|x + \frac{b}{2} D(c, \eta, \beta) = 0 \quad (A.4)
\]

where \(D(c, \eta, \beta) = \frac{\eta}{\beta} \frac{c P'(c)}{\gamma_m + \gamma_p}\)

\(x = 0\) if \(4\alpha(\sigma - \frac{1}{2} D(c, \eta, \beta)) - b < 0\)

This implies that \(\alpha \leq \frac{b}{4(\sigma - \frac{1}{2} D(.))}\) for \(x^* = 0\).

Further, notice that \(x\) is decreasing in \(D(.)\). As a result, when \(\eta = 0\) and \(D(.) = 0\) the resulting equilibrium \(\bar{x} > x_{D(.)>0}\). This proves the proposition.

### A.6 Proof of Proposition 3

From the equilibrium equation A.4, let \(\phi = 4(\alpha + \sigma)x^2 - [4\alpha(\sigma - \frac{1}{2} D(c, \eta, \beta))] - b)|x + \frac{b}{2} D(c, \eta, \beta)\). Then, the following holds: \(\frac{d\phi}{dx} > 0\) at the equilibrium \((-x, x)\). To see this,

\[
\frac{d\phi}{dx} = 8(\alpha + \sigma)x - [4\alpha(\sigma - \frac{1}{2} D(c, \eta, \beta))] - b
\]
That is, \( \frac{d\phi}{dx} > 0 \) iff \( x > \frac{[4\alpha(\sigma - \frac{1}{2} D(c, \eta, \beta)) - b]}{8(\alpha + \sigma)} \). But,

\[
x = \frac{1}{8(\alpha + \sigma)} \left[ [4\alpha(\sigma - \frac{1}{2} D(c, \eta, \beta)) - b] + \sqrt{[4\alpha(\sigma - \frac{1}{2} D(c, \eta, \beta)) - b]^2 - 8b(\alpha + \sigma)D(c, \eta, \beta)} \right]
\]

\[\Rightarrow x = \frac{[4\alpha(\sigma - \frac{1}{2} D(c, \eta, \beta)) - b]}{8(\alpha + \sigma)} + \sqrt{[4\alpha(\sigma - \frac{1}{2} D(c, \eta, \beta)) - b]^2 - 8b(\alpha + \sigma)D(c, \eta, \beta)} \]

Therefore it holds that \( \frac{d\phi}{dx} > 0 \). Given this,

\[
\frac{dx}{d\beta} = -\frac{\frac{d\phi}{dx}}{\frac{d\phi}{db}}
\]

This implies that \( \frac{dx}{d\beta} > 0 \) if \( \frac{d\phi}{db} < 0 \).

\[
\frac{d\phi}{d\beta} = \frac{d\phi}{d\beta} \frac{d\phi}{d\beta} = \left( 4\alpha x + b \right) \frac{d\phi}{d\beta} \frac{d\phi}{d\beta}
\]

\[
\frac{d\phi}{d\beta} = \frac{d\phi}{d\beta} \frac{d\phi}{d\beta} + \frac{d\phi}{d\beta}
\]

where,

\[
\frac{d\phi}{d\beta} = -\frac{\eta}{\beta} \frac{P'(c)}{P''(c)} + \frac{1}{\beta} \frac{c}{(\gamma_m + \gamma_p)}
\]

\[\Rightarrow \frac{d\phi}{d\beta} = \frac{P'(c)}{\eta P''(c)} \]

\[
\frac{d\phi}{d\beta} = -\frac{\eta}{\beta^2} \frac{c P'(c)}{(\gamma_m + \gamma_p)} = -\frac{1}{\beta} D(.)
\]

\[
\frac{d\phi}{d\beta} = \frac{\eta}{\beta} \left[ \frac{c P''(c) + P'(c)}{(\gamma_m + \gamma_p)} \right]
\]

\[
\frac{d\phi}{d\beta} = D(.) \left[ -\frac{1}{\eta P''(c)} + \frac{1}{\beta} \frac{c P''(c) + P'(c)}{(\gamma_m + \gamma_p)} \right]
\]

Therefore, \( \frac{d\phi}{d\beta} > 0 \iff \frac{\eta P'(c)}{\beta} > \frac{d\phi}{d\beta} \).

\[\Rightarrow \frac{(c P''(c) + P'(c))}{(\gamma_m + \gamma_p)} > P' \]

Simplifying the above equation implies that \( \frac{d\phi}{d\beta} > 0 \) when \( \frac{1 - \gamma_p}{\gamma_m + \gamma_p} > 1 \).

However, \( \frac{d\phi}{d\beta} > 0 \Rightarrow \frac{d\phi}{d\beta} > 0 \Rightarrow \frac{dx}{d\beta} < 0 \). The above condition simplifies to,

\[\Rightarrow \gamma_p < \frac{1 - \gamma_m}{2} \]

A similar argument follows for the other two cases. This completes the proof.

### A.7 Proof of Proposition 4

The comparative statics result with respect to \( \alpha \) and \( b \) follows from Bernhardt, Duggan and Squintani [10]. I will therefore concentrate on the parameters \( \eta \) and \( \sigma \). As before, the sign of \( \frac{dD(.)}{d\eta} \) determines the sign of \( \frac{dx}{d\eta} \).
\[
\frac{dD(.)}{d\eta} = \frac{\partial D(.)}{\partial c} \frac{\partial c}{\partial \eta} + \frac{\partial D(.)}{\partial \eta}
\]

Since \(\frac{\partial c}{\partial \eta}, \frac{\partial D(.)}{\partial \eta} > 0\), and \(\frac{\partial D(.)}{\partial c} > 0\), it must be that \(\frac{dD(.)}{d\eta} > 0\) which further implies that \(\frac{dx}{d\eta} < 0\).

\[
\frac{dD(.)}{d\sigma} = \frac{\partial D(.)}{\partial c} \frac{\partial c}{\partial \sigma} + \frac{\partial D(.)}{\partial \sigma} = \frac{\partial D(.)}{\partial c} \frac{\partial c}{\partial \sigma}
\]

Since \(\frac{\partial c}{\partial \sigma} < 0\), this implies that \(\frac{dD(.)}{d\sigma} < 0\) and \(\frac{dx}{d\sigma} > 0\). This completes the proof.

### A.8 Proof of Proposition 5

Notice that the total welfare of voters is symmetric around \(\frac{\sigma}{2}\) such that \(W_{tot}(0, 0) = W_{tot}(-\sigma, \sigma)\).

This means that the welfare function is increasing in the interval \([0, \frac{\sigma}{2}]\) and decreasing in the interval \((\frac{\sigma}{2}, \sigma]\). This, combined with the fact that activism reduces platform divergence, implies that for the presence of activism to be welfare improving, a necessary condition is that the candidate platforms in the absence of activism must be above the social optimum.

This is guaranteed when \(\bar{x} = \frac{\alpha (\sigma - b)}{4(\alpha + \sigma)} > \frac{\sigma}{2}\). Simplifying this gives us the required condition.

However, this condition is not sufficient for welfare improvement. The reason is that activism could potentially moderate the platforms to an extent that it reduces the overall welfare. For this to not happen, it must be that the platforms under activism must be above a threshold.

This threshold is easily calculated using the symmetry property of the welfare functions. Specifically, as long as \(x^* > (\sigma - \frac{4\sigma b}{4(\alpha + \sigma)})\), the presence of activists improves welfare compared to the no activists case. This gives us the required conditions. It is easy to observe that when either or both of these fail, the introduction of activists reduces welfare of voters unambiguously. This completes the proof.

### A.9 Proof of Lemma 5

When the levels of polarization induced by activism is in the interval \((0, \frac{\sigma}{2})\), the welfare function is increasing in \(x\). This implies that as \(\beta\) increases, and \(WTE\) is above the threshold \(\frac{1}{(1-\gamma_p)}\), the candidate platforms in equilibrium decrease (or converge towards each other).

This reduces the welfare of voters as a result. The opposite holds when \(WTE < \frac{1}{(1-\gamma_p)}\), as in this case the polarization of activists induces more divergence in candidate platforms,
leading to improved welfare. Similar arguments hold when the levels of polarization is above \( \frac{\sigma}{2} \).

A.10 Proof for Lemma 6

To prove the first part, notice that as \( \eta \to 0 \), the equilibrium polarization of the electoral game converges to \( \bar{x} \). When \( b < 2\sigma(\alpha - \sigma) \), the equilibrium platform is \( \bar{x} > \frac{\sigma}{2} \). But, since \( \frac{dx}{d\eta} < 0 \), it is always possible to find a \( \eta^* > 0 \) such that the equilibrium polarization falls to the social optimal \( \frac{\sigma}{2} \).

In a similar way, as \( b \to 0 \), the equilibrium polarization is such that \( \lim_{b \to 0} x = \frac{a(\sigma - \frac{1}{2}D(c,\eta,\beta))}{(\alpha + \sigma)} \). And, as long as this is above the social optimal levels, the social planner can always increase the benefits of office and achieve the social optimum levels of polarization (since \( \frac{dx}{db} < 0 \)).

This completes the proof.

B Proofs - Extensions

B.1 Activism and Noisy Campaigns

Suppose, for sake of exposition, \( \sigma = 1 \). The solution to the electoral model is solved backwards, as previously. As before, we present the results for candidate (and activist) \( R \).

The median voter observes the platforms with a noise: \( \tilde{X}_L = X_L + \eta_L \), \( \tilde{X}_R = X_R + \eta_R \).

Voter prefers candidate \( L \) if,

\[
E[(-\tilde{X}_L - \mu)^2] \geq E[(-\tilde{X}_R - \mu)^2] \\
\iff E[(X_L - \mu)^2 + \eta_L^2 + 2\eta_L(X_L - \mu)] \leq E[(X_R - \mu)^2 + \eta_R^2 + 2\eta_R(X_R - \mu)] \\
\iff E(\eta_L^2) - E(\eta_R^2) \leq (X_R - \mu)^2 - (X_L - \mu)^2 \\
\iff a(c_L) - a(c_R) \leq (X_R - \mu)^2 - (X_L - \mu)^2 \\
\mu \leq \frac{(X_R + X_L)}{2} + \frac{a(c_R) - a(c_L)}{2(X_R - X_L)}
\]

The win-probability for Candidate \( L \) is

\[
\lambda = \frac{1}{2} + \frac{(X_R + X_L)}{4} + \frac{a(c_R) - a(c_L)}{4(X_R - X_L)}
\]

As in the original model, the the equilibrium condition for participation is

\[
\frac{M'(c_R)}{a'(c_R)} = \frac{[X_R + X_L - 2\beta]}{4}
\] (B.1)
By symmetry, the equivalent condition for activist $L$ is,

\[
\frac{M'(c_L)}{a'(c_L)} = -\frac{[X_R + X_L + 2\beta]}{4}
\] (B.2)

At the symmetric equilibrium $(-x, x)$, equations B.1 and B.2 reduce to \( \frac{M'(c)}{a(c)} = -\frac{\beta}{2} \). Let \( D(c(\beta), \beta) = -\frac{1}{\beta} \cdot \frac{c a'(c)}{\gamma_m + \gamma_n} \approx D \). Then, the equation that solves for the symmetric equilibrium is given by the following:

\[
4(\alpha + 1)x^2 - (4\alpha - b - 2\alpha D)x + \frac{b}{2}D = 0 \quad (B.3)
\]

\[
\frac{M'(c)}{a'(c)} = -\frac{\beta}{2} \quad (B.4)
\]

Together, the above two equations determine the symmetric equilibrium platform of candidates, and mobilization by activists. The dependence of equilibrium platform $x$ on $\alpha$ and $b$ are along the lines of the main model. To derive comparative statics with respect to $\beta$, let \( \Phi = 4(\alpha + 1)x^2 - (4\alpha - b - 2\alpha D)x + \frac{b}{2}D \).

\[
\frac{d\Phi}{d\beta} = \left[ \frac{\partial \Phi}{\partial D} \frac{\partial D}{\partial \beta} + \frac{\partial \Phi}{\partial c} \frac{\partial c}{\partial \beta} \right]
\]

The rest of the proof follows from arguments similar to one in Proposition 3. The sign of \( \frac{\partial \Phi}{\partial \beta} \) depends on the expression \((-2 + \frac{1+\gamma_m}{\gamma_m+\gamma_n})\), which is greater than zero when \( \gamma_n < \frac{1-\gamma_m}{2} \). When this is satisfied, \( \frac{\partial \Phi}{\partial \beta} > 0 \) and further, \( \frac{\partial x}{\partial \beta} < 0 \). When the sign of this inequality is reversed, that is \( \gamma_n > \frac{1-\gamma_m}{2} \), then \( \frac{\partial x}{\partial \beta} > 0 \). This concludes the analysis.

**B.2 The role of big money in campaigns**

The participation in equilibrium is modified to

\[ M'(c) = \frac{\eta \beta}{2\sigma} P'_2(S, c) \]

Applying implicit function theorem yields, \( \frac{dc}{d\beta} = -\frac{\frac{\eta \beta}{2\sigma} P''_2(S, c)}{\frac{\eta \beta}{2\sigma} P'_2(S, c) - M'(c)} < 0 \).
Similarly, the equilibrium is just the modified equation where,

\[ D(c, \eta, \beta, S) = \frac{\eta \cdot cP'_2(S, c)}{\beta \cdot \gamma_m(c) + \gamma_p(c)} \]

The sign of \( \frac{dD}{dS} \) determines the sign of \( \frac{dx}{dS} \), as before.

\[ \frac{dD}{dS} = \frac{\partial D(\cdot, \cdot)}{\partial c} \frac{\partial c}{\partial S} + \frac{\partial D(\cdot, \cdot)}{\partial S} \]

Since \( \frac{\partial D(\cdot, \cdot)}{\partial S} < 0 \) and \( \frac{\partial c}{\partial S} < 0 \), and from our earlier assumption on WTE, it is true that \( \frac{dD}{dS} < 0 \). This further implies that \( \frac{dx}{dS} > 0 \).

References


