The Impact of Credit Rating Agencies on Capital Markets *

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Abstract

In this paper, I argue that the source for potential inefficiencies arising from CRAs might be more pathological than the literature recognizes; even in the absence of conflicts of interest or other distortions resulting from players’ behavior, a CRA might have an adverse effect on critical economics variables. I develop a model of investment financing which, similarly to capital markets, is characterized by information asymmetry and lack of commitment. In the benchmark setting, the CRA is capable of perfect monitoring and reveals its private information truthfully and without cost. I explore the impact of such an “ideal” CRA on the interest rate and the probabilities of project financing and default. I find that introducing such a CRA may lead to under-financing of projects with a positive net present value (NPV) that would otherwise be financed; a higher expected interest rate; and a higher expected probability of default. These findings relate to the feedback effect, which is inherent in capital markets, and its asymmetric impact on firms of different quality. I evaluate the policy of restricting CRAs to provide hard evidence with their ratings, and suggest that it might have an unfavorable effect on the probabilities of project financing and default.

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1 Introduction

The 2007-2008 financial crisis raised unprecedented scrutiny on credit rating agencies (CRAs). CRAs have been accused of failing to predict the financial crisis (Mason and Rosner, 2007), and of following investors’ opinions rather than leading them (Richard and Steward, 2003). However, the main criticism regarding CRAs relates to the conflicts of interest, which might arise in an issuer-pays regime (Mathis, McAndrews, and Rochet, 2009; Bouvard and Levy, 2012).

In this paper, I shift the attention from the CRA’s behavior to the framework in which CRAs operate. I explore the impact of a CRA in an environment which, similarly to capital markets, is characterized by information asymmetries and lack of commitment. I explore a model where i) there is no conflict of interest, ii) the CRA is truthful and issues ratings for free, and iii) these ratings are accurate. I argue that even in this environment, introducing a CRA might lead to i) under-financing of projects with positive NPV that would otherwise be financed, and ii) higher expected probability of default.

CRAs play a significant role in capital markets by affecting major economic variables, such as interest rate and probability of default. The main mechanism through which credit ratings affect such variables is by providing information which shapes investors’ beliefs, and hence investment decisions. CRA, however, differs from other information intermediaries in a critical direction: unlike meteorologists, who predict the weather without affecting it, CRAs produce ratings that do affect the quality of the asset they rate. Kühner (2001) points out that a credit rating can take the form of a self-fulfilling prophecy. Similarly, Manso (2013), captures the self-fulfilling nature of ratings, by showing that a rating deterioration may trigger a “death spiral”. This self-fulfilling characteristic is also highlighted in Holden, Natvik, and Vigier (2014). The feedback effect, which is inherent in capital markets, lies at the heart of my model.

I develop a model of project’s financing, where the entrepreneur is privately informed about his cost of effort, and cannot commit to exerting effort in the implementation of the project. In particular, the entrepreneur’s cost is drawn from a “good” or a “bad” distribution, where the good distribution dominates the bad distribution in the first order stochastic dominance sense. Thus, an entrepreneur whose cost is drawn from a good distribution is of low expected cost (LEC), and an entrepreneur whose cost is drawn from a bad distribution is of high expected cost (HEC). I focus on where the planner would finance both types. The information asymmetry about the entrepreneur’s cost is partly resolved by a CRA. The combination of lack of commitment and information asymmetry allows me to endogenize the probability of default and the interest rate. In this environment, a feedback effect arises: the interest rate is determined by the expected probability of default, which in turn, determines the realized probability of default.

I argue that introducing a truthful CRA which resolves, at zero cost, part of the information asymmetry, may not result in a better outcome from a social perspective. This is because the
interaction information asymmetry and lack of commitment results in a trade-off when a CRA is introduced into the market. On the one hand, better information alleviates the information asymmetry problem; on the other hand, it exacerbates the adverse effect of non-commitment. The main findings suggest that the impact of introducing a CRA depends on the features of the environment. In particular, the introduction of a CRA: (i) leads to a higher expected probability of default when the entrepreneur is efficient enough to raise funds independently of the presence of a CRA (mild information asymmetry), (ii) leads to under-financing when financing of an HEC entrepreneur is feasible only if he is pooled with an LEC entrepreneur (moderate information asymmetry), and (iii) alleviates under-financing when financing of an LEC entrepreneur is feasible only if he can be differentiated from an HEC entrepreneur (severe information asymmetry). Besides, I characterize the level of accuracy of the CRA’s ratings which leads to the best allocation of resources, and show that some degree of inaccuracy might be optimal. Also, I consider the case where the CRA charges a profit-maximizing fee. I find that this fee adversely affects the probability of default, but it does not influence the project’s financing opportunities.

The intuition behind the findings presented in the previous paragraph relates to the feedback effect in capital markets. In order to capture the feedback effect, some insights of the model are required. First, I assume that the project succeeds as long as effort is exerted. Thus, a critical role in our model is played by threshold $\hat{c}$, which denotes the maximum value of effort cost for which an entrepreneur exerts effort in the implementation of his project. This threshold is negatively related to the interest rate the entrepreneur is expected to pay. Suppose now that an investor considers financing an entrepreneur. First, he forms his beliefs about the probability of default, which in this setup coincides with the probability that the entrepreneur’s cost is above $\hat{c}$. Based on his beliefs, the investor demands an interest rate which allows him to break even. This interest rate affects the threshold $\hat{c}$, which in turn, affects investors beliefs about the probability of default, and so on (feedback effect). We show that the increase in the interest rate and the corresponding probability of default, due to the feedback effect, is decreasing in the entrepreneur’s ex-ante efficiency. Thus, for mild information asymmetry, when introducing a CRA, the negative effect on an HEC entrepreneur dominates the positive effect on an LEC entrepreneur. Along these lines, Kliger and Sarig (2000) use a natural experiment to show that credit ratings affect the cost of capital, and Kisgen (2006) shows that a firm’s structural decision is directly affected by credit ratings.

The intuition underlying the finding that a CRA may lead to under-financing of positive NPV projects is similar. Consider a case where an HEC entrepreneur has a bad rating and investors are not willing to finance him. Suppose now there is no CRA and an HEC entrepreneur is mixed with an LEC entrepreneur. As far as an HEC entrepreneur is concerned, investors are now more optimistic about the entrepreneur’s creditworthiness and require a lower interest rate. This lower interest rate decreases the probability of default of an HEC entrepreneur,
which decreases the interest rate even more. This feedback effect can thus turn the financing of HEC entrepreneur to a credit-worthy investment.

Finally, I address the impact of restricting a CRA to provide hard evidence with its ratings. Notice that the hard-evidence assumption effectively restricts the rating policy to truthful disclosure, which subsequently diminishes the role of the CRA. Relaxing this assumption gives rise to a model similar to the Bayesian Persuasion setting developed by Kamenica and Gentzkow (2009). In this environment, I explore the optimal rating rule when the CRA can commit in advance to this rule. I find that the optimal rating rule is either a truthful revelation of the entrepreneur’s type or a babbling equilibrium, where no information is revealed. Hence, both “rating-inflation” and “rating-deflation” can be part of the optimal rating policy. In addition, I show that restricting a CRA to provide hard evidence can only worsen financing opportunities. In contrast, “rating-inflation” and “rating-deflation” can improve the allocation of resources.

I argue that this paper has implications for the information disclosure policy implemented by a government. Note that the introduction of a CRA in capital markets can also be interpreted as a revelation of a signal about the creditworthiness of an agent or institution. For instance, a key question after the recent crisis is whether the results of stress tests for banks should be revealed (Goldstein and Leitner 2013). The answer that my analysis suggests is that concealing these results could improve the allocation of resources, unless the market breaks down in the absence of additional information. Moreover, the implications of this paper can be applied to the sovereign debt in euro zone countries. Finally, my work suggests that future research on the regulation of CRAs should take into account feedback effect, which is inherent in capital markets.

This paper is organized as follows: Section 2 reviews the related literature. Section 3 introduces the model. Section 4 explores the benchmark case. Section 5 presents the regime with and without a CRA. Section 6 explores the impact of introducing a CRA into the market. Section 7 relaxes the assumption of hard evidence and explores the optimal rating rule when the CRA can pre-commit to it. In Section 8, I explore an environment where a CRA can choose a profit-maximizing fee. Section 9 discusses and concludes.

2 Literature Review

This paper relates primarily to two strands of the literature: the literature dealing with the effect of CRAs on real economic variables, and the literature that recognizes the adverse effect of better information.

Much of the literature on CRAs focuses on the quality of the reported ratings, rather than the impact of CRAs. There are three main approaches in this strand. The first approach explores whether CRAs have incentive to inflate their ratings. The second approach examines the way CRAs choose to disclose their private information. The third approach highlights the
role of rating shopping by issuers on the quality of ratings.

The most popular way of addressing whether a CRA has incentive to inflate its rating, is by testing the validity of the the reputation-concerns argument. According to this, inflating the ratings would harm a CRA’s reputation, and ultimately force it out of the market; hence, such conflict of interests does not exist. Mathis et al. (2009) develop a model with rating-contingent fees, and demonstrate that the reputation-concerns argument only works when a significant part of a CRA’s income comes from sources other than rating complex projects. Bouvard and Levy (2012) also test the reputation concerns argument and show that reputation for transparency is not always desirable because it demotivates low-quality firms to ask for a rating.

The second approach links the quality of ratings with information disclosure. Lizzeri (1999) shows that a monopolist CRA only reveals the minimum level of information, but if there are multiple CRAs, information is disclosed fully. In contrast, Faure-Grimaud, Peyrache, and Quesada (2007) show that competition reduces information revelation.

The third approach attributes rating inflation to behavioral biases of investors and rating shopping by issuers. Bolton, Freixas, and Shapiro (2012) develop a model with the interaction of sophisticated and naive investors, and the results show that a duopoly might be less efficient than a monopoly, because the entrepreneur has the opportunity to shop for a good rating to exploit naive investors. Skreta and Veldkamp (2009) obtain similar findings, where the direct implication of rating shopping is the systemic bias in disclosed ratings, even if each CRA produces unbiased ratings. Opp, Opp, and Harris (2013) show that rating inflation can emerge if the face value - not the informational value - of the rating that matters.

My central approach deviates from this literature; In the benchmark setting, I focus on a seemingly best case scenario, where the CRA always reports its private information truthfully, and hard evidence supports each rating. In this environment, I explore the impact of CRAs on capital markets. Save for Boot, Milbourn, and Schmeits (2006), Kuhner (2001), Manso (2013) and to some extent Mathis et al. (2009), the literature has overlooked this effect. Boot et al. (2006) propose that credit ratings can serve as a coordination mechanism in situations where multiple equilibria exist. Mathis et al. (2009) show that the behavior of CRAs can lead to reputation cycles, with implications for credit spreads. Kuhner (2001) shows that when CRAs care about reputation, they are more likely to reveal their private information if their ratings cannot become self-fulfilling ex-post.

Manso (2013) deals with the feedback effect of credit ratings. He describes an environment where a single CRA repeatedly interacts with a firm that holds performance-sensitive debt, and whose payout flows are linked to its rating. This framework enables him to incorporate the feedback effect of credit ratings in a dynamic credit-rating model. He finds that, when forming its rating policy, the CRA should focus not only on ratings accuracy, but also on the effect of the ratings on the borrower’s probability of survival.

This paper differs from Manso (2013) in three critical dimensions: (i) the nature of the
feedback effect, (ii) the information available to the CRA and (iii) the main focus. First, this paper explores the feedback loop between the project’s quality and the investors’ decisions rather than the feedback loop between project’s quality and credit rating. The second departure from Manso (2013) is that the CRA has an information advantage over potential investors. In Manso (2013) the cash flow process, which is the only parameter which determines firm’s creditworthiness, is observed by all market participants. In contrast, in this paper, the CRA obtains a private signal about the firm’s creditworthiness. That characteristic enables me to provide micro-foundations for the impact of CRAs on the cost of capital. Another departure from Manso (2013) concerns the determination of capital cost. In Manso (2013) the capital cost depends on ratings exogenously, whereas in this paper, the capital cost is endogenously determined, capturing all the parameters of the model. Finally, regarding the main focus of the paper, Manso (2013) is interested in exploring the effect of the rating policy, i.e., the function which maps the cash flow into a rating, in the economy. This work focuses instead on the impact of providing information via a CRA on capital markets.

Another study which explores the feedback effect of ratings is Holden et al. (2014). The authors follow a global games approach where agents choose whether to roll over or withdraw their loan. They show that the rating agency exacerbates default risk when it is high, and alleviates default risk when it is low. As opposed to this paper, my work focuses on the capital markets, with its main emphasis being on financing opportunities.

In addition, my work pertains to the literature on the adverse welfare consequence of information disclosure. In his seminal work, Hirshleifer (1971) argues that more information leads to welfare reduction because it destroys hedging opportunities. More recently, Amador and Weill (2010) show that the effect of releasing partial information about a monetary or productivity shock is two-fold: on the one hand, providing more information benefits the economy; but on the other hand, it forces households to value the newly released public information more and their private information less. As a result, the situation leads to reduction of the endogenous informational content of prices. Kondor (2013) shows that when the correlation between the private information of different groups is low, the release of public information increases disagreement among short-horizon traders about the expected selling price. Kurlat and Veldkamp (2012) argue that disclosure of information reduces an asset’s risk and hence its return. As a result, high-risk, high-return investments disappear and investor welfare falls.

The mechanism through which better information can be harmful differs from the existing literature. I suggest that the inefficient allocation of resources is a consequence of the coexistence of asymmetric information and lack of commitment. In this environment, resolving part of the information asymmetry amplifies the distortion arises from lack of commitment.

This paper also relates to the literature on Bayesian Persuasion. In Section 7 I show that if the CRA is not obliged to provide supporting evidence for its ratings, and can commit to a rating policy, then the emerging setup is similar to Kamenica and Gentzkow (2009).
3 Model

Environment: I consider a setting with three risk neutral players: an cashless entrepreneur, a CRA, and a representative investor. The entrepreneur seeks capital to finance an investment project. The project is non-divisible, and its implementation requires an investment equal to $1. The output of the project depends on whether the entrepreneur exerts effort in its implementation; if the entrepreneur exerts effort, the project succeeds with probability one. Otherwise, the project fails with probability one. The project returns $R$, in case of success, and zero otherwise. The outcome of the project is observed by all parties, and $R$ is common knowledge. Exerting effort is costly, unobserved by investors, and the entrepreneur cannot commit to it.

Entrepreneur’s types: The entrepreneur can be of two types, $i \in \{H, L\}$, where the type refers to the entrepreneur’s cost of exerting effort, denoted by $c$. In particular, the cost of type $i$ is drawn from a distribution where $f_i(c)$ and $F_i(c)$ denote the probability and density function, respectively. Throughout this paper, I assume the distribution which corresponds to type $i = L$ dominates the distribution which corresponds to type $i = H$ in the first order stochastic dominance sense i.e., for each $c' \in [0, \bar{c}]$, $F_L(c') \leq F_H(c')$. Thereafter, I refer to type $i = L$ as low-expected-cost (LEC) entrepreneur, and type $i = H$ as high-expected-cost (LEC) entrepreneur. Thus, an LEC entrepreneur can be interpreted as an efficient (in expectation) entrepreneur, whereas an HEC entrepreneur as an inefficient (in expectation) entrepreneur. I restrict the analysis to the case where both types have positive ex-ante net present value, i.e., $R > 1 + \mathbb{E}[c_i]$ for each $i \in \{H, L\}$. Hence, if the aim of the planner is to maximize total surplus, both types would be financed.

Information sets: The entrepreneur has private information about his type. In contrast, investors hold prior beliefs about the entrepreneurs type. In particular, investors expect the the entrepreneur to be of low expected cost (LEC) with probability $\lambda$, and of high expected cost (HEC) with probability $1 - \lambda$. These beliefs are common knowledge. Besides, I assume that the entrepreneur learns his realized cost, $c$, after carrying out the investment, and only then he takes the effort decision. The rationale of this assumption is that the cost of exerting effort in implementing a project depends not only on the entrepreneur’s type, but also on the

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1Raising funds in return for a security is the reduced form of a setup where the entrepreneur sells a security at a price $P$ in order to finance a project of size 1. This is because a direct consequence of a pooling equilibrium in the contracting stage is that there is a loss for the efficient entrepreneur (LEC). Thus, the entrepreneur does not have the incentive to raise more than the capital required for the investment, i.e. $P = 1$.

2The main findings are robust if we instead assume that exerting effort increases the probability of success.

3This is without loss of generality as long as the return in case of failure, $R_F$ is lower than $R$.

4The main findings are robust to the case where $R$ is drawn from a distribution which is known to the entrepreneur and investors.
project itself. Thus, the cost of exerting effort is not known until the agent starts implementing the project.

Unlike potential investors, the CRA can monitor the type of the entrepreneur. The intuition behind this assumption is the following. First, it captures the empirical observation that CRAs have better information than investors about the creditworthiness of an entrepreneur/firm. This is because they have access to critical economic indexes, and more experience in monitoring. Second, this assumption reflects the idea that regardless of the experience that CRAs may have, they cannot have better information about the creditworthiness of a firm, than what the firm itself has.

It is worth highlighting that this model is qualitatively equivalent to a setting where: i) the role of the entrepreneur is played by a manager of a firm, ii) the cost of effort coincides with the cost of allocating production resources to the implementation of the project, and iii) the manager’s objective is to maximize the firm’s profitability.

**Actions of each player:** The entrepreneur faces three sets of actions. First, he decides whether to ask for a rating. Second, he decides about the structure of the security he issues. Finally, he decides whether to exert effort in the implementation of the project.

Investors’ only action is to choose whether to finance an entrepreneur or not. We assume that capital markets are competitive, and that the interest rate investors demand is normalized to zero.

In order to disentangle the impact of a CRA, we allow for different sets of actions. In the benchmark setting we restrict the CRA to provide truthful rating at zero rating fee. In Section 7, the CRA chooses its rating policy, which maps the entrepreneur’s type to a rating. In Section 8, the CRA chooses the rating fee.

**Timing:** The timing of the events is as follows:

1. Nature determines the type of the entrepreneur.
2. The entrepreneur chooses whether to ask for a rating.
3. Conditional on the entrepreneur asking for a rating, the CRA issues a rating.
4. The entrepreneur observes his rating (if any), and chooses the security design.

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5 We show in the Appendix that the model is qualitatively similar to a setting where the entrepreneur observes a noisy signal of the realized cost, rather than the realized cost itself.

6 The reason the CRA only knows the type of the entrepreneur/firm, i.e., the distribution from which the actual cost is derived and not the realized cost, relates to two particular characteristics of the rating industry: the monitoring method and the clustering of ratings. Regarding the monitoring method, CRAs usually base their ratings on a mix of indexes. Hence, two firms with similar index values are likely to receive a similar rating, even though the indexes do not reflect other parameters that are vital for a firm’s profitability. Regarding the clustering of ratings, one of the characteristics of the industry is that CRAs issue ratings by following a specific rating scale, and refrain from giving predictions about the exact economic outcome.
5. Investors observe the entrepreneur’s rating (if any) and security, and then decide whether to invest.

6. Conditional on financing, the entrepreneur observes his realized cost, and chooses whether to exert effort in the project’s implementation.

7. The output is realized, and the security is executed.

**Equilibrium Concept:** The equilibrium concept is *Perfect Bayesian Equilibrium*, where the CRA, the entrepreneur, and investors, choose their corresponding actions in order to maximize expected profits/utility. Finally, on-equilibrium beliefs are consistent.

## 4 Benchmark Case

In the benchmark case, I explore an environment where the CRA: i) can monitor the type of the entrepreneur perfectly, ii) reveals its private information truthfully, and iii) does not charge a rating fee. These assumptions suppressing any conflict of interests, and allow me to isolate the impact of better information, provided by a CRA, on capital markets. In Section 2.7, I relax the hard evidence assumption, whereas in Section 2.8, I relax the zero-fee assumption.

### 4.1 Entrepreneur’s Problem

First, the entrepreneur decides whether to ask for a rating. In doing so, he takes into consideration: i) the expected rating, ii) what would be the security that he would issue with and without a rating, and iii) when he would exert effort.

A consequence of the lack of a rating fee is that an LEC entrepreneur can costlessly differentiate himself, which allows him to promise a lower return to investors. An implication of this remark is that an HEC entrepreneur is indifferent between asking and not asking for a rating. This finding relates to the signaling component of the decision of asking for a rating. Note that if an HEC entrepreneur asks for a rating, his type is revealed by the CRA with certainty. Also, if he does not ask for a rating, the market will be able to infer his type, since anticipates that an LEC entrepreneur would always find it optimal to ask for a rating. Following the previous reasoning, we can assume without loss of generality that both types ask for a rating.

Second, the entrepreneur chooses the security design which maximizes his expected utility. It is shown in the Appendix that the optimal security promises a payment \((1+r)\), if the project succeeds, and zero otherwise. This simple form of the optimal security is a consequence of non-verifiability of effort cost.

Third, conditional on raising capital and starting implementing the project, the entrepreneur observes his actual cost of exerting effort \(c\), and decides whether to exert effort. Recall that the project is successful only when the entrepreneur exerts effort. Thus, the entrepreneur’s utility, depending on whether he exerts effort, is given by:
\[ U(\text{effort}) = R - (1 + r) - c \]
\[ U(\text{not effort}) = 0 \]

Hence, the entrepreneur exerts effort as long as:

\[ c \leq R - (1 + r) \equiv \hat{c} \quad (1) \]

Threshold \( \hat{c} \) is a critical variable of the model. The intuition behind this threshold is straightforward: the entrepreneur exerts effort as long as the benefit from doing so, \( R - (1 + r) \), exceeds the cost, \( c \).

I define as “default” the event where the entrepreneur fails to pay back his loan. In this setting, a default occurs when the project returns zero, which coincides with the case where no effort is exerted. This leads to the following definition of the probability of default.

**Definition 1**

The probability of default is defined as the probability of financing an entrepreneur whose realized cost of effort exceeds \( \hat{c} \):

\[ \Pr(\text{default}) \equiv \Pr(c > \hat{c}(r)) \quad (2) \]

Similarly, the probability of success is defined as the probability of financing an entrepreneur whose realized cost of effort is below or equal to \( \hat{c} \):

\[ \Pr(\text{success}) \equiv \Pr(c \leq \hat{c}(r)) \quad (3) \]

Because \( \hat{c} \) is negatively related to the payment \( (1 + r) \), the probability of default is positively related to \( (1 + r) \). To avoid the trivial case where an entrepreneur can finance his project through risk-free debt, I restrict the probability of default to strictly positive values. This is true if there exists at least one value of \( c \) such that the entrepreneur does not exert effort even if the interest rate is zero, i.e., \( \hat{c} > R - 1 \).

### 4.2 The Investors’ Problem

Investors form their beliefs about the probability of success, and subsequently require an interest rate that satisfies their participation constraint:

\[ \mathbb{E}[\Pr(\text{success})|\Omega] \times (1 + r) + \mathbb{E}[\Pr(\text{default})|\Omega] \times 0 \geq 1 \quad (4) \]

\(^7\)What matters for the main findings to go through, is that the effort threshold, which determines the probability of default, is negatively related to the payment \( (1 + r) \). Alternatively, this could be achieved by assuming that the project succeeds with certainty when the entrepreneur exerts costly effort and succeeds with probability \( q \) if there is no effort. In such a setup the entrepreneur chooses to exert effort if and only if \( c \leq (R - (1 + r))(1 - q) \) which is associated to a probability of success equal to \( \Pr(c \leq \hat{c}) + q\Pr(c > \hat{c}) = q + (1 - q)\mathbb{E}[\Pr(c \leq \hat{c})|\Omega] \).
where the LHS of (4) is the investors’ expected benefit, whereas the RHS of (4) is the capital they lend to the entrepreneur. More specifically, \(1+r\) stands for the payment in case of success, whereas \(E[Pr(success)|\Omega]\) stands for investors’ beliefs about the probability of success, given their information set \(\Omega\). Similarly, \(E[Pr(default)|\Omega]\) stands for investors’ beliefs about the probability of default. Following Definition 1, the investors’s participation constraint becomes:

\[
E[Prob(c \leq \hat{c})|\Omega] \times (1 + r) \geq 1
\] (5)

Thus, investors are willing to finance an entrepreneur as long as \(E[Prob(c \leq \hat{c})|\Omega] \geq (1 + r)^{-1}\). If the previous condition does not hold, the market collapses.

### 4.3 Equilibrium Condition

Recall that the entrepreneur exerts effort as long as:

\[
c \leq R - (1 + r) \equiv \hat{c}(r)
\] (6)

Note that investors’ participation constraint is binding, as a consequence of the assumption that capital markets are perfectly competitive. Thus, the equilibrium interest rate is the minimum \(r\) which solves:

\[
E[Prob(c \leq \hat{c}(r))|\Omega] = (1 + r)^{-1}
\] (7)

The inherent feedback effect can be seen in the previous fixed-point equation; the interest rate \(r\) which solves (7) depends on the threshold \(\hat{c}(r)\), which in turn, depends on the interest rate \(r\) via (6).

It is worth highlighting that it is the investors’ beliefs about the probability of default - not the probability of default itself - that determine the interest rate.

For notational convenience, I denote \(E[Prob(c \leq \hat{c}(r))|\Omega]\) as \(\tilde{s}(\hat{c}(r))\). Thus, \(\tilde{s}(\hat{c}(r))\) captures investors’ beliefs about the probability of success, given their information set \(\Omega\). Note the there might be more than one combinations of \(r\) and \(\hat{c}(r)\) which solve (7). The combination, however, which maximizes entrepreneur’s expected utility is the one which corresponds to the lowest interest rate.

The previous analysis highlights the importance of investors’ beliefs about the default probability on the determination of the equilibrium interest rate. The introduction of a CRA affects investors’ beliefs as follows: when investors observe a bad rating, they anticipate that the entrepreneur is of HEC type. Consequently, they downgrade their beliefs about the probability of success, compared to the case where there is no CRA. Similarly, when investors observe a good rating, they anticipate that the entrepreneur is of LEC type. Subsequently, they upgrade their beliefs about the probability of success, compared to the case with no CRA. As a result, in-
investors require a higher (lower) interest rate to finance an HEC (LEC) entrepreneur, compared to the regime without a CRA.

5 Regime with and without a CRA

In this section, I characterize the equilibrium in two different regimes: (i) a regime without a CRA, and (ii) a regime with a CRA. Recall that the only distributional assumption is First Order Stochastic Dominance, i.e., for each \( c' \in [0, \bar{c}] \), \( F_L(c') \leq F_H(c') \).

5.1 Regime without a CRA

The Investors’ Problem: Investors’ beliefs about the probability of success are given by:

\[
\tilde{s}_{II}(\hat{c}(r)) = \Pr(c \leq \hat{c}(r)|i = L) \mathbb{E}[\Pr(i = L)|\Omega] + \Pr(c \leq \hat{c}(r)|i = H) \mathbb{E}[\Pr(i = H)|\Omega]
\] (8)

Given that investors have no additional information, their beliefs about the entrepreneur’s type coincide with their prior beliefs. Note that \( \Pr(c \leq \hat{c}(r)|i = L) = F_L(\hat{c}(r)) \) and \( \Pr(c \leq \hat{c}(r)|i = H) = F_H(\hat{c}(r)) \), where \( F_L(.) \) is the c.d.f of the cost of an LEC entrepreneur, and \( F_H(.) \) is the c.d.f of the cost of an HEC entrepreneur. Thus,

\[
\tilde{s}_{II}(\hat{c}(r)) = \lambda F_L(\hat{c}(r)) + (1 - \lambda) F_H(\hat{c}(r))
\] (9)

Market survival and equilibrium interest rate: I show in the Appendix that the market survives as long as there exists an interest rate, \( r \), such as \( \tilde{s}_{II}(\hat{c}(r)) \geq (1 + r)^{-1} \). Also, the equilibrium interest rate, which is denoted by \( r^{*}_{II} \), is minimum interest rate which solves:

\[
(1 + r)^{-1} = \lambda F_L(\hat{c}(r)) + (1 - \lambda) F_H(\hat{c}(r))
\] (10)

5.2 Regime with a CRA

In this section, I introduce a CRA that has perfect monitoring, and its ratings are, by assumption, truthful. The CRA affects the equilibrium interest rate by shaping investors’ beliefs via changing their information set. As a result, the equilibrium interest rates are conditional on the rating.

The Entrepreneur’s Problem: We argue in section 4 that an LEC entrepreneur always asks for a rating, whereas an HEC entrepreneur is indifferent between asking and not asking for a rating since, in both cases, his type is disclosed with certainty. Without loss of generality, we assume that when an LEC entrepreneur is indifferent, he asks for a rating.
Recall that the threshold $\hat{c}$ depends on the interest rate, which depends on investor’s beliefs about the probability of success, which in turn, rely on the rating. Thus, as long as the interest rate differs depending on the rating, the resulting threshold also varies. Thus, there are two critical values: $\hat{c}(r_{GR})$, if the rating is good, and $\hat{c}(r_{BR})$, if the rating is bad.

**The Investors Problem:** Conditional on the rating, investors form their beliefs about the probability of success. $\tilde{s}(\hat{c}(r_{GR}))$ denotes investors’ beliefs when the rating is good, and $\tilde{s}(\hat{c}(r_{BR}))$ when the rating is bad, where:

\[
\tilde{s}(\hat{c}(r_{GR})) = Pr(c \leq \hat{c}(r_{GR})|i = L)Pr(i = L|GR) + Pr(c \leq \hat{c}(r_{GR})|i = H)Pr(i = H|GR)
\]

\[
\tilde{s}(\hat{c}(r_{BR})) = Pr(c \leq \hat{c}(r_{BR})|i = L)Pr(i = L|BR) + Pr(c \leq \hat{c}(r_{BR})|i = H)Pr(i = H|BR)
\]

Note that $Pr(i = L|GR) = 1$ and $Pr(i = H|BR) = 1$, as the CRA has perfect monitoring and reports its private information truthfully. Hence, I can re-write investors’ beliefs as follows:

\[
\tilde{s}(\hat{c}(r_{GR})) = Pr(c \leq \hat{c}(r_{GR})|i = L) = F_{L}(\hat{c}(r_{GR}))
\]

\[
\tilde{s}(\hat{c}(r_{BR})) = Pr(c \leq \hat{c}(r_{BR})|i = H) = F_{H}(\hat{c}(r_{BR}))
\]

**Market survival and equilibrium interest rate:** Conditional on a good rating, the market survives as long as there exists an interest rate, $r$, such as $\tilde{s}(\hat{c}(r_{GR})) \geq (1 + r)^{-1}$. Similarly, conditional on a bad rating, the market survives as long as there exists an interest rate, $r$, such as $\tilde{s}(\hat{c}(r_{BR})) \geq (1 + r)^{-1}$. Also, the equilibrium interest rate which corresponds to a good rating, denoted by $r^*_{GR}$, is minimum interest rate which solves:

\[
(1 + r) = (F_{L}(\hat{c}(r)))^{-1}
\]

Similarly, the equilibrium interest rate which corresponds to a bad rating, denoted by $r^*_{BR}$, is the minimum interest rate which solves:

\[
(1 + r) = (F_{H}(\hat{c}(r)))^{-1}
\]
6 Impact of introducing a CRA

6.1 Comparison

In Section 5, I characterize the necessary and sufficient conditions for a market to survive. Besides, I characterized the equilibrium conditions for the regimes with and without a CRA. This section compares those regimes regarding three critical market variables: the probability of project financing, the expected probability of default, and the expected interest rate.

6.1.1 Impact on Project Financing

A fundamental aspect of capital markets is financing opportunities. The following Proposition explores the impact of introducing a CRA on the probability of raising capital. Recall that a social planner, whose objective is to maximize net surplus, would finance both types of entrepreneurs. This is because both types correspond to positive NPV projects.

**Proposition 1** (Probability of Financing)

(i) If there is \( r \) such as \( F_L(\hat{c}(r)) \geq (1 + r)^{-1} \) and \( \lambda F_L(\hat{c}(r)) + (1 - \lambda) F_H(\hat{c}(r)) < (1 + r)^{-1} \), introducing a CRA alleviates under-financing.

(ii) If there is \( r \) such as \( F_H(\hat{c}(r)) < (1 + r)^{-1} \) and \( \lambda F_L(\hat{c}(r)) + (1 - \lambda) F_H(\hat{c}(r)) \geq (1 + r)^{-1} \), introducing a CRA leads to under-financing.

*Proof.* See Appendix.

The intuition behind Proposition 1 is captured in Figure 1 and 2. These figures illustrate investors’ beliefs about the probability of success, as a function of \( r \), in three cases: i) when the entrepreneur holds a good rating (dash-dotted line), ii) when the entrepreneur holds a bad rating (dense-dotted line), and iii) when there is no rating/CRA (loose-dotted line). The solid curve depicts the inverse of the payment \( 1 + r \). Recall that \( \hat{c} = R - (1 - r) \). An implication of the First Order Stochastic Dominance assumption is that the graphical illustration of \( F_L(\hat{c}(r)) \) lies above the graphical illustration of \( F_H(\hat{c}(r)) \). Recall that for the market to survive, the should be at least one value of \( r \), such as the solid curve is above the line which corresponds to investors’s beliefs.

Part one and two of Proposition 1 is depicted in Figure 1 and 2 respectively. Figure 1 illustrates the case where asymmetric information is severe, and an LEC entrepreneur can raise capital only if he can differentiate himself. Hence, introducing a CRA alleviates under-financing. Figure 2 illustrates the case where information asymmetry is moderate, and an HEC

---

8We present the case where the entrepreneur is of an LEC or an HEC type with equal probability, and the cost of effort is uniformly distributed.
entrepreneur can raise capital only if he is pooled with an LEC entrepreneur. Thus, introducing a CRA prevents pooling, and leads to under-financing of an HEC entrepreneur, even though finance him is socially optimal.

The intuition behind the last observation relates to the fact that an entrepreneur cannot commit to exerting effort. Also, we argue later in this section, that the distortion due to lack of commitment is greater, the lower ex-ante efficiency is. Hence, pooling an HEC with an LEC entrepreneur alleviates the resulting distortion. This is evident in the case where an HEC
entrepreneur cannot be funded in isolation, though he can be funded by being pooled with an LEC entrepreneur. The reason is that pooling leads investors to upgrade their beliefs about the probability of success, which in turn, reduces the interest rate they require. If the lower interest rate alters the choice of exerting effort from unprofitable to profitable, then investors anticipate this, and they are willing to finance the entrepreneur.

6.1.2 Impact on Probability of Default

Once the equilibrium interest rate is derived, I can compute the expected interest rate and the expected probability of default. The expected interest rate (probability of default) consists of the interest rate (probability of default) of each type of entrepreneur, $i$, weighted by the probability that the entrepreneur is of type $i$. Recall that, as ratings are perfectly accurate, a good rating is associated with an LEC entrepreneur and a bad rating is associated with an HEC entrepreneur. I present the effect of introducing a CRA on these variables for both an LEC and an HEC entrepreneur; however, my main goal is the effect at the expected/market level.

**Proposition 2** (Probability of Default)

*If information asymmetry is mild, i.e., there exists $r$ such as $F_H(\hat{c}(r)) \geq (1 + r)^{-1}$ (the market survives independently of whether a CRA exists), then the relation among the equilibrium probabilities of default is:*

$$1 - F_L(\hat{c}(r_{GR}^*)) < 1 - F_L(\hat{c}(r_{II}^*)) < 1 - F_H(\hat{c}(r_{II}^*)) < 1 - F_H(\hat{c}(r_{BR}^*)) \quad (13)$$

*In addition, the expected probability of default in absence of a CRA is lower than the expected probability of default when a CRA exists:*

$$\lambda(1 - F_L(\hat{c}(r_{II}^*))) + (1 - \lambda)(1 - F_H(\hat{c}(r_{II}^*))) <$$

$$\lambda(1 - F_L(\hat{c}(r_{GR}^*))) + (1 - \lambda)(1 - F_H(\hat{c}(r_{BR}^*))) \quad (14)$$

**Proof.** See Appendix.

Figure 3 illustrates Proposition 2 and 3. The distance of point $G$ ($B$) from the vertical axis captures the equilibrium interest rate which corresponds to a good rating, and the distance of $G$ ($B$) from the horizontal axis captures the probability of success of an LEC (HEC) entrepreneur. Also, the distance of point $A$ from the vertical axis captures the expected equilibrium interest rate, and the distance of $A$ from the horizontal axis captures the probability of success, in the regime with a CRA. Similarly, point $N$ indicates the expected equilibrium interest rate and the expected probability of success when there is no CRA. The first part of the proposition is a
consequence of the FOSD assumption. The second part relates to the convexity of $(1 + r)^{-1}$: in Figure 3, point $N$ is always above point $A$\footnote{In $G$: $(1 + r_{GR})^{-1} = F_L(\hat{c}(r_{GR}))$, in $B$: $(1 + r_{BR})^{-1} = F_H(\hat{c}(r_{BR}))$ and in $N$: $(1 + r_{GR})^{-1} = 0.5F_L(\hat{c}(r_{GR}))+ 0.5F_H(\hat{c}(r_{BR}))$.} Thus, the expected probability of success (default) is higher in the regime without a CRA.

6.1.3 Impact on Interest Rate  

**Proposition 3** (Equilibrium Interest Rates)  
If information asymmetry is mild, i.e., there exists $r$ such as $F_H(\hat{c}(r)) \geq (1 + r)^{-1}$ (the market survives independently of whether a CRA exists), then the relation among the equilibrium interest rates is given by:

\[ (1 + r_{GR}^*) < (1 + r_{II}^*) < (1 + r_{BR}^*) \]  

(15)

In addition, the expected equilibrium interest rate in the absence of a CRA is always lower than the expected equilibrium interest rate when a CRA exists:

\[ (1 + r_{II}^*) < \lambda(1 + r_{GR}^*) + (1 - \lambda)(1 + r_{BR}^*) \]  

(16)

**Proof.** See Appendix. \(\square\)

Similarly to proposition 2, part one follows directly from the FOSD assumption, whereas part two relates to the convexity of $(1 + r)^{-1}$. 

Figure 3: Interest rate and default probability with and without CRA.
Here I am trying to shed light on the intuition behind Proposition 2 and 3. The main message of Proposition 2 and 3 is that introducing a CRA has a negative effect on an HEC, and a positive effect on an LEC entrepreneur. I show that the former always offsets the latter in the case where asymmetric information is not severe enough to lead to a market breakdown. These findings arise from the feedback effect in capital markets and its asymmetric impact on entrepreneurs of different quality. In order to understand this asymmetry, it is crucial first to understand how the feedback effect works in practice.

After investors observe the rating and form their beliefs about the expected cost of effort and the probability of default, they demand an interest rate which allows them to break even. The raised funds adjusted by the interest rate (capital cost) need to be paid back by the entrepreneur. For an entrepreneur who was indifferent between exerting and not effort, adding the capital cost makes exerting effort unprofitable, and thus the effort threshold, \( \hat{c} \), drops. The lower effort threshold coincides with a higher probability of default, which results in investors demanding a higher interest rate. This feedback loop continues until the interest rate converges to its equilibrium value.

Note that every loop leads to continuous updates in beliefs about the probability of default, and the demanded interest rate. The magnitude, however, of the increase in the probability of default and the interest rate after each loop, diminishes. This is a consequence of the always decreasing expected cost, conditional on exerting effort.

The previous reasoning implies that the change in the interest rate and the probability of default, due to this feedback mechanism, is smaller for more efficient entrepreneurs, i.e., the equilibrium interest rate is a convex function of the expected value of the cost of effort. A direct consequence of this remark is that the negative effect on an HEC entrepreneur dominates the positive effect on an LEC entrepreneur.

6.1.4 Optimal Level of Rating Precision

The main message of Propositions 1-3 is that better monitoring does not necessarily correspond to better allocation of resources. For example, consider an environment where CRA receives a signal about the entrepreneur’s type, and the social planner can affect the precision of this signal. This could be achieved through, for instance, regulating the evidence that an entrepreneur should provide to the CRA during the evaluation process. A question which arises naturally is what would be the precision level that maximizes financing opportunities. To explore this, I allow for the following modifications: I assume that the CRA receives a binary signal, \( \sigma = \{GS, BS\} \), which reveals the true state with probability \( \alpha \), i.e.,

\[
Prob(i = L|GS) = Prob(i = H|BS) = \alpha
\]
where $\alpha \in (0.5, 1]$ is common knowledge. To avoid any unnecessary complications, I assume, without loss of generality, that the CRA reveals his private signal truthfully, i.e., the CRA gives a good rating, if the signal is good, and a bad rating, if the signal is bad. The level of optimal precision is given in Proposition 4.

**Proposition 4:** (Optimal Level of Precision)
The level of precision $\alpha$ that maximizes an entrepreneur’s financing opportunities does not necessarily coincide with 1 (perfectly precise signal). If an HEC entrepreneur’s financing is feasible only if there is some pooling with an LEC entrepreneur, the financing opportunities are maximized for a value of $\alpha$ which is weakly smaller than one. The optimal level of precision, $\alpha^*$, solves:

$$F_L(\hat{c}(\tilde{r}_{BR}))(1 - \alpha^*) + F_H(\hat{c}(\tilde{r}_{BR}))\alpha^* = (1 + \tilde{r}_{BR})^{-1} \quad (17)$$

*Proof.* See Appendix.

The following Corollary summarizes the impact of introducing a CRA. Figure 4 and 5 illustrate findings of Corollary 1, for the case where the cost is uniformly distributed.

**Corollary 1** (Impact of better Information)

The introduction of a CRA:

(i) leads to a higher expected probability of default and interest rate, when both an LEC and an HEC entrepreneur can be financed independently of whether a CRA exists. (Area A)

(ii) leads to under-financing, when an HEC entrepreneur’s financing is feasible only if he is pooled with an LEC entrepreneur. (Area B)

(iii) alleviates under-financing, when the LEC entrepreneur’s financing is feasible only if he can be differentiated by an HEC entrepreneur. (Area C)

The intuition behind Corollary 1, which summarizes the main findings, relates to the co-existence of asymmetric information and a lack of commitment to exerting effort. In this environment, resolving part of the information asymmetry amplifies the impact of non-commitment. Thus, there is an inherent trade-off in introducing a CRA; on one hand, the CRA alleviates the information asymmetry problem, but on the other hand, exacerbates the adverse effect of non-commitment. It can be shown that, if the entrepreneur can commit to exerting effort, then better information always improves the allocation of resources, an outcome that may not be true if the entrepreneur is unable to commit. The net effect of a CRA on the allocation of resources depends on the relative extent of each problem. For instance, if the information

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10 For $\alpha = 1$ the signal is perfectly informative.
asymmetry is severe (such the market collapses in the absence of a CRA), introducing a CRA improves the allocation of resources. Antithetically, if the information asymmetry is mild, the positive effect of the CRA in resolving part of the asymmetry is dominated by the negative effect due to lack of commitment. This is captured in Figure 4 and 5 where, as the probability of the entrepreneur being LEC, \( \lambda \), increases, area B spreads over area C.

\[
\begin{align*}
\text{x-axis: } \bar{c}, \text{ y-axis: } \gamma
\end{align*}
\]

Figure 4: Equilibrium existence regions

\[
\begin{align*}
\text{x-axis: } \bar{c}, \text{ y-axis: } \gamma
\end{align*}
\]

Figure 5: Equilibrium existence regions
7 Optimal Rating Policy

In the previous analysis, I assumed that the ratings should be accompanied by hard evidence. This assumption restricts the rating policy to truthful disclosure. Even though this assumption is useful in capturing the impact of better information, it diminishes the role of a CRA. In this section, I relax the hard-evidence assumption, and explore the CRA’s optimal rating policy. The rationale behind relaxing the assumption of hard evidence is that, although it is true that CRAs provide supporting evidence with the ratings, there are cases where this evidence does not reveal perfectly the type of the firm.

The CRA’s objective when designing the rating rule is to maximize expected profits,

\[ \mathbb{E} [\Pi | \Omega_{CRA}] = \mathbb{E} [(P - d)D | \Omega_{CRA}] \]

where \( P \), \( d \) and \( D \) stand for the rating fee, the cost of acquiring a signal, and the demand for ratings, respectively.

In order to disentangle the rating policy from the decision of acquiring a private signal and the rating fee determination, I assume that the CRA receives a perfect signal about the entrepreneur’s type at cost \( d = 0 \), and that it is a price taker. In this environment, the objective of maximizing expected profit coincides with the objective of maximizing the expected demand for ratings:

\[ \mathbb{E} [D | \Omega] = \mathbb{E} [\lambda I_L + (1 - \lambda) I_H | \Omega] \]

where \( I_L (I_H) \) equals one if an LEC (HEC) type ask for a rating, and zero otherwise. Note that the entrepreneur asks for a rating if he expects that having a rating will enable him to raise capital at a lower cost. Thus, the objective of maximizing demand pins down to designing a rating rule, such that the entrepreneur asks for a rating independently of his type.

This setup is similar to the persuasion setting developed in Kamenica and Gentzkow (2009), where a sender (CRA) observes a private signal (type of entrepreneur), and then sends a message (rating) to a receiver (investor). Subsequently, the receiver takes an action (decides whether to finance and the interest rate in case of financing), which affects the payoff of both the sender and receiver.

The persuasion game is relevant when the message is pivotal. In this setting, the message is pivotal if it can affect financing opportunities: an HEC entrepreneur cannot raise capital if his type is known.

To avoid any unnecessary complications, I assume that the signal can be of two values, good and bad (\( \tilde{\sigma} = \{GS, BS\} \)) and that it perfectly reveals the type of the entrepreneur, i.e. \( Pr(i = L|GS) = 1 \) and \( Pr(i = H|BS) = 1 \). The rating policy is a mapping from a signal realization to a rating, where the rating can take two values, good and bad (\( \tilde{R} = \{GR, BR\} \)). Thus, the rating policy consists two parameters, \( \alpha_G \) and \( \alpha_B \), where \( \alpha_G = Prob(GR|GS) \) and
\( \alpha_B = \text{Prob}(BR|BS) \) [11]

As long as both ratings are issued in equilibrium, the market survives if there exists an \( r \), such that the following conditions hold:

\[
Pr(i = L|GR)F_L(\hat{c}(r)) + Pr(i = H|GR)F_H(\hat{c}(r)) \geq (1 + r)^{-1} \tag{18}
\]
\[
Pr(i = L|BR)F_L(\hat{c}(r)) + Pr(i = H|BR)F_H(\hat{c}(r)) \geq (1 + r)^{-1} \tag{19}
\]

where condition (18) refers to the case where a good rating is issued, whereas condition (19) refers to the case where a bad rating is issued. By implementing the Bayes rule, I can re-write conditions (18) and (19) as follows:

\[
\frac{\alpha_G \lambda}{\alpha_G \lambda + (1 - \alpha_B)(1 - \lambda)} F_L(\hat{c}(r)) + \frac{(1 - \alpha_B)(1 - \lambda)}{\alpha_G \lambda + (1 - \alpha_B)(1 - \lambda)} F_H(\hat{c}(r)) \geq (1 + r)^{-1}
\]
\[
\frac{(1 - \alpha_G) \lambda}{(1 - \alpha_G) \lambda + \alpha_B(1 - \lambda)} F_L(\hat{c}(r)) + \frac{\alpha_B(1 - \lambda)}{(1 - \alpha_G) \lambda + \alpha_B(1 - \lambda)} F_H(\hat{c}(r)) \geq (1 + r)^{-1}
\]

Recall that by the FOSD assumption, \( F_L(\hat{c}(r)) > F_H(\hat{c}(r)) \) for any \( r \). This property implies that the market survives as long as the probability that the investors attribute to the entrepreneur being of LEC type is sufficiently large. I define as \( \hat{\lambda} \) the minimum probability that investors should attribute to the entrepreneur being of LEC type, such as financing takes place. \( \hat{\lambda} \) solves:

\[
\hat{\lambda} F_L(\hat{c}(r)) + (1 - \hat{\lambda}) F_H(\hat{c}(r)) = (1 + r)^{-1}
\]

Thus, a CRA can achieve project financing for both types \( (I_{LEC} = I_{HEC} = 1) \) as long as \( \alpha_G \) and \( \alpha_B \) are chosen in a way that investors’ beliefs about the probability that the entrepreneur is of LEC type exceed \( \hat{\lambda} \), independently of the rating. This translates into the CRA choosing a combination of \( \alpha_G \) and \( \alpha_B \) such that:

\[
\min \left\{ \frac{\alpha_G \lambda}{\alpha_G \lambda + (1 - \alpha_B)(1 - \lambda)} \cdot \frac{(1 - \alpha_B)(1 - \lambda)}{(1 - \alpha_G) \lambda + \alpha_B(1 - \lambda)} \right\} \geq \hat{\lambda}
\]

An question which arises naturally is to what extent the CRA can affect investor’s beliefs, and more specifically, what is the maximum \( \hat{\lambda} \), denoted as \( \hat{\lambda}^{\text{max}} \), that could be achieved by a CRA when it can pre-commit to a rating policy. \( \hat{\lambda}^{\text{max}} \) solves the following problem:

Maximize \( \min \{ Pr(i = L|GR), Pr(i = H|BR) \} \)

subject to \( 0 \leq \alpha_G \leq 1 \), \( 0 \leq \alpha_B \leq 1 \) and \( \alpha_G \geq \alpha_B \).

Note that the constraint \( \alpha_G \geq \alpha_B \) relates the signaling component of the decision to ask for

[11] Truthful disclosure is implemented when \( \alpha_G = \alpha_B = 1 \).
a rating, and in particular to off-equilibrium beliefs. This condition guarantees that an LEC has stronger incentive than an HEC entrepreneur to ask for a rating. Otherwise, there could be an equilibrium when neither an LEC nor an HEC entrepreneur asks for a rating. The solution to the maximization problem satisfies the following equation:

\[ \alpha^*_G = 1 - \alpha^*_B \]  

(20)

Note that the equilibrium that maximizes financing opportunities is a babbling equilibrium. In a babbling equilibrium the updated beliefs equal the prior beliefs in a regime without CRA, i.e.,

\[ \hat{\lambda}_{max} \equiv Pr(LEC|\alpha^*_G, \alpha^*_B, GR) = Pr(LEC|\alpha^*_G, \alpha^*_B, BR) = \lambda \]

**Proposition 5:** Optimal Rating Policy

The optimal rating policy when CRA’s objective is to maximize its profit is:

i) \( \alpha^*_G = \alpha^*_B = 0.5 \) (babbling equilibrium), if there is a value of \( r \) such that \( \lambda F_L(\hat{c}(r)) + (1 - \lambda)F_H(\hat{c}(r)) \geq (1 + r)^{-1} \),

ii) \( \alpha^*_G = \alpha^*_B = 1 \) (truthful disclosure), otherwise.

**Rating Inflation, Rating Deflation and their Impact**

A strictly positive value of \( \alpha^*_B \) implies that the CRA inflates its rating, i.e., an HEC entrepreneur receives a rating which corresponds to an entrepreneur of a higher level of creditworthiness. In addition, a value of \( \alpha^*_G \) smaller than 1 implies that the CRA deflates its rating, i.e., an LEC entrepreneur receives a rating which corresponds to an entrepreneur of a lower level of creditworthiness. The intuition is straightforward: by giving bad rating to an LEC type, the CRA prevents a significant downgrade in investors’ beliefs when the observe a bad rating. Hence, the entrepreneur can raise capital even after a bad rating, thus, he has incentive to ask for a rating.

Also, regarding the allocation of resources, we show that once we allow for the feedback effect inherent in capital markets, rating inflation and deflation might lead to financing of an entrepreneur with positive NPV, that would not be financed if the rating were truthful. The message of this finding is that, if information asymmetry is not severe, restricting CRAs to provide hard evidence with their ratings might have a negative effect on the probabilities of project financing and default.

**Corollary 2:** Impact of Rating Inflation/Deflation

When the CRA can pre-commit to a rating rule, rating inflation/deflation can be part of the equilibrium even if the fee is not rating-contingent. Besides, rating inflation/deflation might lead to financing of positive NPV projects that would not otherwise be financed.
8 Profit-Maximizing Fee

In this section I allow the CRA to choose a profit-maximizing fee. A direct consequence of the positive rating fee is that it modifies the implementation threshold, $\hat{c}$. The comparison with the case where the rating fee is restricted to zero allows me to isolate the distortion that a rating fee incurs.

**The CRA’s Problem:** The CRA anticipates that only an LEC entrepreneur has incentive to ask for a rating; thus, it chooses the rating fee $P$, such that an LEC entrepreneur is indifferent between having and not having a rating. The functional form of the profit-maximizing fee, denoted as $P^{\text{max}}$ and specified below, is related to the entrepreneur’s outside option, which is determined by whether financing is feasible without a rating. If having a rating is not necessary for raising capital, the CRA will charge a fee that makes an LEC entrepreneur indifferent between asking for a rating and differentiating himself, or not asking for a rating and pooled with an HEC entrepreneur. In contrast, when the absence of a good rating leads to no financing, the entrepreneur’s outside option is zero. Hence the profit-maximizing CRA extracts all the surplus, i.e., it charges the maximum fee for which an equilibrium exists. The following conditions characterize the profit maximizing rating fee:\[12\]

\[
P^{\text{max}} \equiv \begin{cases} 
P^{\text{max}} = \frac{(r_{II} - \hat{r}_R)}{(1 + \hat{r}_R)} & \text{If rating not necessary for financing} \\
P^{\text{max}} : F(\hat{c}|P = \bar{P}^{\text{max}}) = 0.5 & \text{If rating necessary for financing} 
\end{cases}
\]  

(21)

**The Entrepreneur’s Problem:** The entrepreneur’s problem is more complicated than before: now, before facing the problem of the security design or whether to exert effort, the entrepreneur has to choose whether to ask for a rating. An HEC entrepreneur has no incentive to ask for a rating because by doing so, he would reveal his type.

I first explore the incentives of an LEC entrepreneur in the case where the market does not collapse in the absence of a good rating. The decision of an LEC entrepreneur to ask for a rating depends on which of the following two forces dominates. On one hand, asking for a rating differentiates him from an HEC entrepreneur, and allows him to promise a lower interest rate, $\hat{r}_R$ instead of $r_{II}$. On the other hand, asking for a rating implies that he needs to borrow a higher amount to cover - apart from the investment cost- the rating fee, $P^{\text{max}}$. Thus, an LEC entrepreneur faces a trade-off between repaying a smaller loan (1, instead of $1 + P^{\text{max}}$) with higher interest rate, or a larger loan with a lower interest rate.\[13\] Thus, in an environment where a rating is not necessary for financing, the entrepreneur chooses the action which leads

\[^{12}\bar{P}^{\text{max}} = \arg \max P \text{ s.t.}(1 + \hat{r}_R)^{-1} = F(\hat{c}|P = \bar{P}^{\text{max}})\]

\[^{13}\text{Here I implicitly assume that there is a lag between the time that the CRA gives the rating and the time CRA is paid. Due to rational expectations of the CRA, such setup is feasible if the entrepreneur can commit to paying the fee after the loan is taken.}\]
to the highest expected utility, where:

\[
U(\text{without rating}) = \max\{R - (1 + r_{II}) - c, 0\}
\]

(22)

\[
U(\text{with rating}) = \max\{R - (1 + \hat{r}_R)(1 + P^{max}) - c, 0\}
\]

(23)

where the max function reflects the fact that the entrepreneur always has the choice not to exert effort. Thus, an LEC entrepreneur asks for a rating as long as:

\[
(1 + \hat{r}_R)(1 + P^{max}) \leq (1 + r_{II})
\]

(24)

and exerts effort as long as:

\[
c \leq \hat{c} \equiv \max\{R - (1 + r_{II}), R - (1 + \hat{r}_R)(1 + P^{max})\}
\]

(25)

I now proceed to the case where the market collapses in the absence of a good rating. In this case, the CRA will charge a fee which extracts all the surplus of the entrepreneur. Following the previous analysis, and the CRA’s problem, I re-write the implementation threshold of an LEC entrepreneur, \(\hat{c}_L\), as:

\[
\hat{c}_L \equiv \begin{cases} 
\hat{c}_L = R - 1 + r_{II}^* \quad \text{If rating & rating not necessary} \\
\hat{c}_L = R - (1 + \hat{r}_R)(1 + P^{max}) \quad \text{If rating & rating necessary}
\end{cases}
\]

(26)

The related threshold for an HEC entrepreneur is:

\[
\hat{c}_H \equiv R - (1 + r_{NR})
\]

(27)

The Investors’ Problem: Investors know whether project financing is feasible without the rating, and update their beliefs after observing whether the entrepreneur holds a rating. Because the CRA reports its private information truthfully, and charges a fee that an LEC entrepreneur buys a rating in equilibrium: \(\text{Prob}(i = L|GR) = 1\) and \(\text{Prob}(i = H|NR) = 1\). Hence, investors’ beliefs are as follows:

\[
\tilde{s}(.) = \begin{cases} 
\tilde{s}(r_R) = F_L(\hat{c}(r_R^*)) \quad \text{If GR & rating not necessary for financing} \\
\tilde{s}(r_R) = F_L(\hat{c}(\tilde{r}_R^*)) \quad \text{If GR & rating necessary for financing} \\
\tilde{s}(r_{NR}) = F_H(\hat{c}(r_{NR})) \quad \text{If No Rating}
\end{cases}
\]

(28)

 Investors would be willing to finance an entrepreneur who holds a rating as long as \(\tilde{s}(r_R) \geq (1 + r_R)^{-1}\), if rating is necessary, and as long as \(\tilde{s}(r_R) \geq (1 + r_R)^{-1}\), if rating is not necessary for raising capital. Similarly, investors would be willing to finance an entrepreneur with no rating as long as \(\tilde{s}_{NR} \geq (1 + r_{NR})^{-1}\).
8.1 Equilibrium Interest Rates

The combined problems of the CRA, the entrepreneur, and investors, determine the equilibrium interest rates $r^*_R$ and $r^*_{NR}$, for an entrepreneur with or without rating respectively.

$$(1 + r^*_R) ≡ \begin{cases} 
(1 + r^*_R) = F_L(\hat{c}(r^*_R))^{-1} \quad \text{If rating & rating not necessary} \\
(1 + r^*_R) = F_L(\hat{c}(r^*_R))^{-1} = 2 \quad \text{If rating & rating necessary}
\end{cases}$$

(29)

Note that the interest rate $r^*_R$ of an LEC entrepreneur is affected by the probability of the entrepreneur being LEC. A higher $\lambda$, or a lower cost of an HEC entrepreneur, reduces the interest rate $r^*_R$, improves the outside option of an LEC entrepreneur, and results in a lower rating fee and interest rate. Moreover, a higher the fee increases the probability of default.

**Proposition 6:** (Effect of Profit Maximizer CRA)

The profit-maximizing fee is negatively related to the probability $\lambda$ and the cost of effort. Also, allowing the CRA to charge the profit-maximizing fee increases the interest rate and the probability of default of an LEC entrepreneur, but it does not affect the probability of project financing.

*Proof.* See Appendix.

9 Concluding Remarks and Future Research

In this paper, I evaluate the impact of a CRA, in a setup of project financing, which is characterized by the coexistence of information asymmetry and lack of commitment. I show that even in an ideal environment, where a CRA has access to perfect monitoring and reveals its rating truthfully, introducing a CRA might lead to a higher probability of default and hurt financing opportunities of positive-NPV projects. Moreover, I evaluate the policy of requiring CRAs to provide hard evidence with their ratings, and argue that it might have an adverse effect on project’s financing opportunities. Finally, I show that rating inflation or deflation might lead to better allocation of resources.

My findings have implications for the optimal information rating policy of a government or a central bank. For instance, a key question after the recent crisis is whether the results of stress tests for banks should be publicized. My analysis suggests that concealing these results might improve the allocation of resources. This is a consequence of the finding that the amplification in the probability of default of bad banks might dominate the beneficial effect on good banks. **Goldstein and Leitner (2013)** arrive at a similar conclusion by adopting a different setting.

14Disclosure of some information may be necessary to prevent a market breakdown, but disclosing too much destroys risk-sharing opportunities.

25
Corollary 1 could also relate to the recent debate on the borrowing interests rates that countries in the eurozone face. Interest rates differ across countries due to differences in credit risk; this has resulted in some peripheral countries borrowing with high spreads, which kept rising over time and eventually led to some countries being close to default. This paper suggests that a policy that requires countries with low credit risk to guarantee for countries with high credit risks, could improve the allocation of resources and decrease the expected probability of default.

My analysis suggests that future research on CRAs’ behavior should account for the inherent in capital markets feedback effect. Note that this feedback effect implies a self-fulfilling effect of ratings: keeping the efficiency of an entrepreneur fixed, the probability of default which corresponds to a bad rating exceeds the one which corresponds to a good rating. The literature has overlooked this self-fulfilling effect of ratings -a concept that could be applied in future work in the context of testing CRAs’ arguments on reputation concerns.

The mechanism explained in the previous paragraph opens the door to policy considerations, and it raises concerns regarding CRAs’ regulation. A message from this paper is that regulation of CRAs has yet to consider their ability to affect crucial variables, such as the probability of default or project financing opportunities.
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Appendix A: Optimal Security

A security, \( w \), can be contingent on the realized value of \( c \), and the final output of the project \((R \text{ or } 0)\). Recall that the cost of effort is not verifiable. An implication of lack of verifiability is that the optimal security pins down to two relevant-in-equilibrium payments. The intuition is the following. First, note that the entrepreneur’s limited liability implies that the payment in the case where the output is zero, is also zero. We now consider the case where the project succeeds. Suppose that the payment which corresponds to the case where the project succeeds is contingent on \( c \). Since \( c \) is non-verifiable, the entrepreneur would always find it optimal to report the value of cost which corresponds to the minimum payment, denoted by \( w_{\text{min}} \). This implies that, in equilibrium, the only relevant payments would be 0 if the project’s return is 0, and \( w_{\text{min}} \) if the return is \( R \). In order to be consistent with the benchmark model, we denote \( w_{\text{min}} \) as \((1 + r)\). The previous analysis allows me to assume, without loss of generality, that the security is contingent on the project’s outcome. Hence, the security is associated with the following return for potential investors:

\[
\text{Investor’s return} = \begin{cases} 
(1 + r) & \text{if project returns } R \\
0 & \text{otherwise}
\end{cases}
\]  

(31)

In addition, following the intuition Nachman and Noel (1994), asymmetric information regarding the entrepreneur’s type implies the HEC entrepreneur mimics the LEC entrepreneur, by offering the same security.

Here I deal with the uniqueness of the optimal security. Since both HEC and LEC entrepreneurs offer the same security, the expected revenue of potential investors is \( 1 + r \) times the probability of financing an entrepreneur who exerts effort, which coincides with the probability that \( c < R - (1 + r) \). This last point implies that the entrepreneur chooses the value of \( r \) by taking into consideration the probability of default, such that the investors’ zero-profit condition is satisfied. Note that depending on the distribution of \( c \), there could be multiple combinations of interest rate-default probabilities that satisfy the zero-profit condition. However, the only combination/equilibrium which survives is the one that corresponds to the lowest value of \( r \), denoted as \( r^* \). This is because that equilibrium dominates all other equilibria from the entrepreneur’s perspective.

The question that emerges is whether such securities are observed in capital markets. Offering such securities is a method of financing similar to debt financing; the entrepreneur issues a bond that returns \( \min\{(1+r), V\} \), where \( V \) is the firm’s value and \( r \) the interest rate. Since the entrepreneur has no wealth other than what the project returns, the value of the firm will be either \( R \) if effort is exerted, or 0 otherwise. Note also that if \((1+r) \geq V\) the entrepreneur would choose not to exert effort because this would lead to negative profit. Hence, the return can only be \((1+r)\) if the project is implemented \((V = R)\), or 0 if the project is not implemented
(V = 0).

**Appendix B: Equilibrium Existence Theorem**

In order to find the conditions for the existence of an equilibrium I use the Bolzano Theorem: for two real numbers $r_\alpha$ and $r_\beta$, $r_\alpha < r_\beta$, if $G(r)$ is (i) a continuous function and (ii) $G(r_\alpha)$, $G(r_\beta)$ are of opposite signs, then there exists an $r^* \in [r_\alpha, r_\beta]$ such that $G(r^*) = 0$.

To find the equilibrium conditions I set $E[Prob(c \leq \hat{c})|\Omega] \equiv \Phi(\hat{c}(r)|\Omega)$, where $\hat{c} = R - (1+r)$, and I create a new function, $G(r) = (1 + r)^{-1} - \Phi(\hat{c}(r)|\Omega)$. Note that $G(r)$ satisfies condition (i) as $\Phi(\hat{c}(r)|\Omega)$ and $(1 + r)^{-1}$ are continuous functions. To show that condition (ii) is satisfied, I set $r_\alpha = 0$ where the function $(1 + r)^{-1}$ reaches its maximum value, 1. Note that $G(0)$ is non-negative as $\Phi(\hat{c}(r)|\Omega, r = 0) < 1$. Hence, for an equilibrium to exist it is sufficient to show that there is at least one value of $r$, denoted as $\tilde{r}$, such that $G(\tilde{r}) < 0$. Figure B.1 illustrates the case where at least one equilibrium exists, and Figure B.2 the case where there is no equilibrium (market breakdown).

![Figure 6: Case where market collapses.](image)

![Figure 7: Case where market survives.](image)
Appendix C: Proof of Main Propositions

C.1 Proof of Proposition 1

Part one: The case where the introduction of a CRA alleviates under-financing of a project with positive probability refers to a state of the world where an LEC entrepreneur is financed only if he can differentiate himself from an HEC entrepreneur. This is true when the following conditions are satisfied:

\[ \lambda F_L(\hat{c}(r')) + (1 - \lambda)F_H(\hat{c}(r')) < (1 + r)^{-1} \]
\[ F_L(r') \geq (1 + r)^{-1} \]

Thus the proof of part one, pins down to showing that (32) and (33) are not mutually exclusive. By combining (32) and (33), we obtain:

\[ F_L(\hat{c}(r')) > (1 + r')^{-1} > \lambda F_L(\hat{c}(r')) + (1 - \lambda)F_H(\hat{c}(r')) \]

Since, \( F_L(r') \geq F_H(r') \) (due to the FOSD assumption), the is always an interest rate, denoted by \( r' \) which satisfies (34).

Part two: The case where the introduction of a CRA leads to under-financing of a project with positive probability refers to a state of the world where an HEC entrepreneur is financed only if he is pooled with an LEC entrepreneur. This case emerges when the following conditions are satisfied:

\[ \lambda F_L(\hat{c}(r'')) + (1 - \lambda)F_H(\hat{c}(r'')) \geq (1 + r'')^{-1} \]
\[ F_L(\hat{c}(r'')) < (1 + r'')^{-1} \]

Thus, the proof of part two pins down to showing that (35) and (36) are not mutually exclusive. By combining (35) and (36), we obtain:

\[ \lambda F_L(\hat{c}(r'')) + (1 - \lambda)F_H(\hat{c}(r'')) > (1 + r'')^{-1} > F_L(\hat{c}(r'')) \]

Since, \( F_L(r'') \geq F_H(r'') \) (due to the FOSD assumption), the is always an interest rate, denoted by \( r'' \) which satisfies (37).
C.2 Proof of Proposition 2

Part one: I present the case where the entrepreneur is of LEC type. Similar intuition applies for the case where the entrepreneur is of HEC type.

Recall that when the type of the entrepreneur is known, the probability of default is $1 - F_L(\hat{c}(r))$, if the entrepreneur is of LEC type, and $1 - F_H(\hat{c}(r))$, if the entrepreneur is of HEC type. Also $\hat{c} = R - (1 + r)$. Recall that an LEC entrepreneur promises $1 + r^*_G$, when a CRA exists, and $1 + r^*_I$, otherwise, where $r^*_I > r^*_G$. Recall also that $F_i(\hat{c}(r))$ is a non-decreasing function of $\hat{c}$. As a result, $\hat{c}_G > \hat{c}_I$. Following the previous remarks, $F_L(\hat{c}(r_G)) > F_L(\hat{c}(r_I))$, which implies that the probability of default is higher in the regime without a CRA.

![Figure 8: $r$ and $\hat{s}$ with and without CRA](image)

Part two: Suppose that we consider the ex-ante probability of success in a regime with a CRA. If the entrepreneur is of LEC type, he receives a good rating, and the probability of success equals $G$, whereas the interest rate equals $r^*_G$. Similarly, if the entrepreneur is of HEC type, he receives a bad rating, and the probability of success equals to $B$, whereas the interest rate equals to $r^*_B$. Thus, in the regime with a CRA, the expected probability of effort equals $\lambda G + (1 - \lambda)B$, which is the linear combination of $G$ and $B$. The blue line depicts the expected probability of success, which equals $C$ when $\lambda = 0.5$.

Suppose now the regime without a CRA. The red line depicts the investor’s beliefs about the probability of success as a function of $r$, for the case where $\lambda = 0.5$. Note that for any value of $\lambda$, there is a non-empty set $[r^*_G, \bar{r}]$ where the red line lies above the blue line. This is because for $r^*_G$, the red line captures the linear combination of $G$ and $D$, whereas the blue line captures the linear combination of $G$ with $B$, where $B$ is always lower than $D$. Note also that
for \( r^*_C \), the green curve is above the red line, due to its continuity. In order to show the expected probability of effort when there is no CRA exceeds the expected probability of effort when there is a CRA, it is sufficient to show that the green curve crosses the red line for \( r^*_II \) smaller than \( \bar{r} \). This is because at the crossing point, the value of \((1 + r^*_II)^{-1}\) equals the expected probability of success when there is no CRA. We prove that this is the case by the method of contradiction. The negative slope of the red line implies that the only case that the green curve crosses first the blue line and then the red line, i.e., crossing the red line for \( r > \bar{r} \), is if the green curve is concave. The green curve (depicting \((1 + r)^{-1}\)), however, is convex for each value of \( r \). Thus, the green curve crosses the red line for \( r^*_II \) smaller than \( \bar{r} \). Hence, the expected probability of success in a regime without a CRA exceeds the corresponding probability in a regime with a CRA, i.e.,

\[
\lambda F_L(\hat{c}(r^*_II)) + (1 - \lambda) F_H(\hat{c}(r^*_II)) \geq \lambda F_L(\hat{c}(r^*_GR)) + (1 - \lambda) F_H(\hat{c}(r^*_BR)) \quad (38)
\]

which implies part two of the Proposition 2:

\[
\lambda[1 - F_L(\hat{c}(r^*_GR))] + (1 - \lambda)[1 - F_H(\hat{c}(r^*_BR))] \geq \lambda[1 - F_L(\hat{c}(r^*_II))] + (1 - \lambda)[1 - F_H(\hat{c}(r^*_II))] \quad (39)
\]

**C.3 Proof of Proposition 3**

Recall that the entrepreneur exerts effort as long as \( c < R - (1 + r_k) \), and that the equilibrium condition for each regime is given by:

\[
(1 + r_k^*)^{-1} = \begin{cases} 
F_L(\hat{c}(r^*_GR)) & \text{if } k = GR \\
F_H(\hat{c}(r^*_BR)) & \text{if } k = BR \\
\lambda F_L(\hat{c}(r^*_II)) + (1 - \lambda) F_H(\hat{c}(r^*_II)) & \text{if } k = II \text{ (no CRA)}
\end{cases} \quad (40)
\]

**Part one:** Part one is an implication of FOSD. The proof is rather intuitive and is based on the fact that, for a given interest rate \( r \), the probability of default is weakly higher for an HEC project. As \( F(\hat{c}(r)) \) is an non-increasing function of \( \hat{c} \) and \( \hat{c} \) is a decreasing function of \( r \), the curve of \( F_L(\hat{c}(r)) \) crosses the curve of \((1 + r)^{-1}\) before the curve of \( F_H(\hat{c}(r)) \) does, which implies that \((1 + r^*_GR) < (1 + r^*_BR)\). Similarly, the curve \( \lambda F_L(\hat{c}(r)) + (1 - \lambda) F_H(\hat{c}(r)) \) crosses the curve of \((1 + r)^{-1}\) on the right of the curve of \( F_L(\hat{c}(r)) \) and on the left of the curve of \( F_H(\hat{c}(r)) \) does, which implies that \( r^*_GR < r^*_II < r^*_BR \).

**Part two:** In order to show part two I use Proposition 2. From part two of Proposition 2, I obtain:

\[
\lambda F_L(\hat{c}(r^*_II)) + (1 - \lambda) F_H(\hat{c}(r^*_II)) \geq \lambda F_L(\hat{c}(r^*_GR)) + (1 - \lambda) F_H(\hat{c}(r^*_BR)) \quad (41)
\]
After I substitute the equilibrium conditions into equation (41), I obtain:

\[(1 + r^*_II)^{-1} \geq \lambda (1 + r^*_GR)^{-1} + (1 - \lambda)(1 + r^*_BR)^{-1}\]  

which implies that:

\[(1 + r^*_II) \leq \frac{(1 + r^*_GR)(1 + r^*_BR)}{\lambda(1 + r^*_BR) + (1 - \lambda)(1 + r^*_GR)}\]  

(43)

The aim of this proof is to show that:

\[\lambda(1 + r^*_GR) + (1 - \lambda)(1 + r^*_BR) \geq (1 + r^*_II)\]  

(44)

In order to show that (44) is satisfied, it is sufficient to show that:

\[\lambda(1 + r^*_GR) + (1 - \lambda)(1 + r^*_BR) \geq \frac{(1 + r^*_GR)(1 + r^*_BR)}{\lambda(1 + r^*_BR) + (1 - \lambda)(1 + r^*_GR)}\]  

(45)

Hence, the goal is to show that (45) holds. Relation (45) simplifies to:

\[2(\lambda^2 - \lambda)(1 + r^*_GR)(1 + r^*_BR) + (1 - \lambda)\lambda(1 + r^*_GR)^2 + (1 - \lambda)\lambda(1 + r^*_BR)^2 \geq 0\]

which simplifies further to:

\[(1 - \lambda)\lambda[(1 + r^*_GR)^2 + (1 + r^*_BR)^2 - 2(1 + r^*_GR)(1 + r^*_BR)] \geq 0\]

which is satisfied as long as:

\[(1 - \lambda)\lambda[(1 + r^*_GR) - (1 + r^*_BR)]^2 \geq 0\]  

(46)

where (46) holds, given that \(\lambda \in (0, 1)\). Thus, (44) is satisfied.

C.4 Proof of Proposition 4

Following the assumption that the CRA reveals its private signal truthfully and without cost, and given that the CRA’s precision level is captured by \(\alpha\), the investors’ beliefs about the probability of success are formed as follows:

\[
\tilde{s} \equiv \begin{cases} 
\tilde{s}(\tilde{r}_{GS}) = F_L(\hat{c}(\tilde{r}_{GS}))\alpha + F_H(\hat{c}(\tilde{r}_{GS}))(1 - \alpha) & \text{if Good Signal (GS)} \\
\tilde{s}(\tilde{r}_{BS}) = F_L(\hat{c}(\tilde{r}_{BS}))(1 - \alpha) + F_H(\hat{c}(\tilde{r}_{BS}))\alpha & \text{if Bad Signal (BS)}
\end{cases}
\]  

(47)

Given the level of precision \(\alpha\), the equilibrium condition for an entrepreneur with a good or a
bad signal, respectively, are:

\[
(1 + \hat{r}^*) = \begin{cases} 
(1 + \hat{r}_{GS}^*) = [F_L(\hat{c}(\hat{r}_{GS}))\alpha + F_H(\hat{c}(\hat{r}_{GS}))(1 - \alpha)]^{-1} & \text{if GS} \\
(1 + \hat{r}_{BS}^*) = [F_L(\hat{c}(\hat{r}_{BS})) (1 - \alpha) + F_H(\hat{c}(\hat{r}_{BS})))\alpha]^{-1} & \text{if BS}
\end{cases}
\]  

(48)

Note that since \(\alpha > 0.5\), if the market survives after a bad signal, this must be also true after a good signal. The proof of proposition 4 pins down to showing that for a given \(\alpha = \hat{\alpha} < 1\), there is at least an interest rate, denoted as \(\hat{r}\), where the following two conditions are not mutually exclusive.

\[
(1 + \hat{r}_{BR|\hat{\alpha}})^{-1} \leq [F_L(\hat{c}(\hat{r}_{BS}))(1 - \hat{\alpha}) + F_H(\hat{c}(\hat{r}_{BS}))\hat{\alpha}] \\
(1 + \hat{r}_{BR|\alpha=1})^{-1} > F_H(\hat{r}_{BR|\alpha=1})
\]

(49) \hspace{1cm} (50)

where (49) implies that for \(\alpha = \hat{\alpha}\) an HEC entrepreneur raises funds (the market survives), and the (50) implies that for \(\alpha = 1\) (benchmark case) an HEC entrepreneur cannot raise funds (the market breaks down). Note that (49) and (50) are not mutually exclusive because \(F_H(\hat{c}(\hat{r})) < F_L(\hat{c}(\hat{r}))\), for any \(\hat{r}\).

C.5 Proof of Proposition 6

Note that the fee is strictly positive as long as there is at least one value of \(r\) such that:

\[
F_L(\hat{c}(r)|P = 0) \geq (1 + r)^{-1}
\]

(51)

Note also that a positive fee shifts the \(F_L(r)\) curve downwards, thus, (51) is the sufficient condition for an equilibrium to exist. The fee is zero if for any \(r\) but \(r^*\), \(F_L(\hat{c}(r)|P = 0) < (1 + r)^{-1}\) and for \(r = r^*\), \(F_L(\hat{c}(r^*)|P = 0) \geq (1 + r^*)^{-1}\). Observe that these two conditions reflect the minimum value of \(F_L(\cdot)\) for which the market does not collapse, which is unaffected by the profit-maximizing assumption due to the non-positive fee. Hence, allowing a CRA to charge a fee does not affect financing opportunities.

The proof of proposition 6 relies on the remarks that \(\hat{c}_L\) is an decreasing function of \(P^{max}\), and \(F_L(\hat{c}(r))\) is a non-decreasing function of \(\hat{c}_L\). Hence, \(F_L(\hat{c}(r)|P = P^{max}) \leq F_L(\hat{c}(r)|P = 0)\), and the equilibrium interest rate given by \((1 + r^*)^{-1} = F_L(\hat{c}(r^*))\), will be higher if the CRA charges the profit maximizing fee.
Appendix D: Case where the entrepreneur observes a signal about the realized cost

The aim of this section is to show that the benchmark setting is qualitatively similar to a setup where the entrepreneur observes an informative signal about the cost, rather than its realized value. For mathematical convenience, and without loss of generality, I assume that this signal is unbiased:

$$\sigma_c = c + \epsilon$$

where $$\epsilon \sim N(0, \sigma^2)$$.

This interpretation implies that if the realized signal is $$\sigma_c'$$, then Bayesian updating results in

$$E[c|\sigma_c = \sigma_c'] = \sigma_c'$$.

I first deal with the implications on the entrepreneur’s problem. Recall that exerting effort is profitable as long as

$$c \leq R - (1 + r).$$

Since the entrepreneur observes $$\sigma_c$$ and not $$c$$, his critical condition for exerting effort will depend on $$\sigma_c$$ (rather than $$c$$). Thus, the entrepreneur exerts effort as long as

$$E[c|\sigma_c] \leq R - (1 + r),$$

which is true if $$\sigma_c \leq R - (1 + r)$$. Hence the effort threshold $$c^* \equiv R - (1 + r)$$ switches to $$\sigma_c^* \equiv R - (1 + r)$$.

I now deal with the implications on the investors’ problem. Recall that investors’ beliefs about the probability of an entrepreneur exerting effort determine $$(1 + r)$$. Following the analysis of the previous paragraph, the investors’ beliefs about that probability equals

$$E[Prob(c \leq \hat{c})|\Omega]$$

instead of

$$E[Prob(c \leq \hat{c})|\Omega],$$

where $$\Omega$$ is investor’s information set. Observe that even though $$\hat{c} = \sigma_c$$, in general $$E[Prob(\sigma_c \leq \hat{\sigma}_c)|\Omega]$$ differs from $$E[Prob(c \leq \hat{c})|\Omega]$$. In order to show that the two models are identical, it is sufficient to show that there exists a unique $$\hat{\sigma}_c$$ such that

$$E[Prob(\sigma_c \leq \hat{\sigma}_c)|\Omega] = E[Prob(c \leq \hat{c})|\Omega].$$

If this is the case, then replacing $$\hat{c}$$ with $$\hat{\sigma}_c$$ generates the same model. Hence, the whole exercise pins down to showing that there exists a $$\hat{\sigma}_c$$ which satisfies the aforementioned property exists.

The proof of that is straightforward. Since the cumulative distribution function of $$\sigma_c$$ and $$c$$ is a continuous and strictly increasing function, then for each $$(1 + r)$$ there is a unique $$\hat{c}$$ associated to the value $$E[Prob(c \leq \hat{c})|\Omega]$$. In addition, for each value of $$E[Prob(s \leq \hat{s})|\Omega]$$, there is a unique $$\hat{\sigma}_c$$ associated with it. Hence, there exists a unique $$\hat{\sigma}_c$$ for which $$E[Prob(\sigma_c \leq \hat{\sigma}_c)|\Omega] = E[Prob(c \leq \hat{c})|\Omega].$$

Appendix E: Uniform Distribution Example

In this section, I assume that the cost of effort is uniformly distributed with

$$c_L \sim U[\alpha \bar{c}, \beta \bar{c}]$$

and

$$c_H \sim U[\gamma \bar{c}, \delta \bar{c}].$$

The analysis is meaningful only if the probability of default is strictly positive. The probability of default is zero if effort is exerted for any feasible value of $$c$$. Thus, the probability of default is always positive as long as $$\bar{c} > R - 1$$. Additionally, I keep the assumption that the implied expected rate of return of both types is non-negative, i.e.,
\[ \frac{R - 1 - E[c_i]}{1 + E[c_i]} \geq 0. \] In order to satisfy these two conditions and simplify calculations, I set \( \alpha = 0 \) and \( \beta = \delta = 1. \) The parameter \( \gamma \) captures the efficiency level of the HEC type compare to the efficiency level of the LEC type; as \( \gamma \) approaches 1, the efficiency level of the HEC type approaches the efficiency level of the LEC type.

**E.1 Regime without a CRA**

After I incorporate the entrepreneur’s problem, investors’ beliefs are given by:

\[ \tilde{s}_{II} \equiv \lambda \tilde{s}_L + (1 - \lambda) \tilde{s}_H = \lambda \frac{R - (1 + r_{II})}{\bar{c}} + (1 - \lambda) \frac{R - (1 + r_{II}) - \gamma \bar{c}}{\bar{c}(1 - \gamma)} \]  

Substituting \( \tilde{s}_{II} \) into the investors’ zero profit condition, the equilibrium interest rate, \( \bar{r}^s_{II} \), is given by:

\[ 1 + \bar{r}^s_{II} = \frac{[(1 - \lambda \gamma)(R) - \gamma \bar{c}(1 - \lambda)] - \sqrt{[(1 - \lambda \gamma)(R) - \gamma \bar{c}(1 - \lambda)]^2 - 4(1 - \lambda \gamma)(1 - \gamma) \bar{c}^2}}{2(1 - \lambda \gamma)} \]

It can be shown that the probability \( \lambda \) and the return \( (R) \) of the project is positively related to the probability of market survival. In contrast, the cost of effort is negatively related to the probability of market survival. For this set of distributions, the equilibrium condition is quadratic in \( r \). In the case where the equation has two distinct roots, the smallest root is the one which maximizes entrepreneur’s profits.

**E.2 Regime with a non-profit maximizer CRA**

The investor’s beliefs depend on the rating as follows:

\[ \tilde{s} = \begin{cases} \\ \tilde{s}_{GR} = \frac{R - (1 + r_{GR})}{\bar{c}} & \text{if Good Rating} \\ \tilde{s}_{BR} = \frac{R - (1 + r_{BR}) - \gamma \bar{c}}{\bar{c}(1 - \gamma)} & \text{if Bad Rating} \end{cases} \]

The equilibrium interest rates are given by:

\[ 1 + r^* = \begin{cases} \\ 1 + r^*_GR = \frac{R - \sqrt{R^2 - 4\bar{c}}}{2} & \text{if Good Rating} \\ 1 + r^*_BR = \frac{(R - \gamma \bar{c}) - \sqrt{(R - \gamma \bar{c})^2 - 4\bar{c}(1 - \gamma)}}{2} & \text{if Bad Rating} \end{cases} \]

The market existence condition when an entrepreneur holds a bad rating is \( (R - \gamma \bar{c})^2 \geq 4\bar{c}(1 - \gamma) \). Similarly, when an entrepreneur holds a good rating, the market existence condition is given by \( R^2 \geq 4\bar{c} \). When these conditions are satisfied, the introduction of CRA leads to inferior results, as it does not improve financing opportunities and it increases the expected default probability.

An interesting case is when an HEC entrepreneur is not efficient enough to prevent market breakdown, but he can raise capital if he is pooled with an LEC entrepreneur. This is because
the emerging interest rate is smaller, and the condition which prevents market breakdown is less restrictive \( \left( \frac{(1-\lambda\gamma)R-\gamma\bar{c}(1-\lambda)}{(1-\lambda\gamma)} \right)^2 \geq 4\bar{c}(1-\gamma) > (R - \gamma\bar{c})^2 \). Without a CRA both types are financed. In contrast, in the regime with a CRA, this pooling is prevented, and only an LEC entrepreneur is financed. Hence, the introduction of a CRA leads to under-financing of an entrepreneur with positive expected net return.

Lastly, when an LEC entrepreneur can be financed only if they can be distinguished from an HEC entrepreneur \( \left( \frac{(1-\lambda\gamma)R-\gamma\bar{c}(1-\lambda)}{(1-\lambda\gamma)(1-\gamma))} \right)^2 < 4\bar{c} \leq R^2 \), then without a CRA, the market collapses and neither an LEC nor an HEC entrepreneur is financed. Thus, the introduction of a CRA prevents the market of an LEC entrepreneur from collapsing, alleviating the problem of under-financing.

### E.3 Regime with a profit-maximizer CRA

Investors observe the rating and whether a rating is necessary for financing \( \left( \frac{(1-\lambda\gamma)R-\gamma\bar{c}(1-\lambda)}{(1-\lambda\gamma)(1-\gamma))} \right)^2 \geq 4(1-\lambda\gamma)(1-\gamma)\bar{c} \), and they form their beliefs about the probability of default as follows:

\[
\tilde{s} \equiv \begin{cases} 
\tilde{s}_{GR} = \frac{R-(1+\bar{r}_R)(1+P_{max})}{\bar{c}} & \text{If GR & rating not necessary} \\
\tilde{s}_{GR} = \frac{R-(1+\bar{r}_R)(1+\bar{P}_{max})}{\bar{c}} & \text{If GR & rating necessary} \\
\tilde{s}_{NR} = \frac{R-(1+\bar{r}_NR)}{\bar{c}} & \text{If No Rating}
\end{cases}
\]  

The combination of the CRA’s, the entrepreneur’s and the investors’ problem, determine the equilibrium interest rates \( r^*_R \) and \( r^*_{NR} \) for an entrepreneur with or without rating respectively.

\[
(1 + r^*_R) \equiv \begin{cases} 
(1 + r^*_R) = \frac{\bar{c}}{R-(1+\bar{r}_R)} & \text{if rating not necessary} \\
(1 + r^*_R) = 2 & \text{if rating necessary}
\end{cases}
\]  

\[
(1 + r^*_{NR}) = \frac{(R - \gamma\bar{c}) - \sqrt{(R - \gamma\bar{c})^2 - 4\bar{c}(1-\gamma)}}{2}
\]  

As Proposition 6 indicates, the market existence conditions are unaffected by the profit maximizing assumption.