Security Design with Endogenous Implementation Choice *

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Abstract

We study an economy where an entrepreneur raises capital to finance an investment project. Our focus is on an environment where the entrepreneur shares the same characteristics as the representative entrepreneur in crowdfunding platforms: i) there is no record regarding her ability, ii) she might be associated with a negative-NPV project, and iii) she has limited liability. Asymmetric information regarding the entrepreneur’s ability between the entrepreneur and potential investors gives rise to a signaling game when the former issues securities to raise capital. We characterize the optimal security, and show that it is always optimal to reward the non-implementation of the project after financing takes place. We show that compared to a case where the entrepreneur is obliged to implement the project after raising capital, endogenizing the project implementation choice: i) prevents market breakdown, ii) leads to a more efficient allocation of resources, and iii) strengthens the incentive of an entrepreneur to invest in her productivity.

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1 Introduction

Crowdfunding (CF) is a new method for financing projects by raising capital from a large pool of investors, performed via an internet platform. The forecasts for capital raised in 2015 through CF platforms exceeds $34 billion, when the venture capital (VC) industry invests an average of $30 billion each year[1]. CF started as a method for raising capital from crowds whose contributions were driven mostly by non-monetary incentives (non-equity CF). However, since April 2012, it entered a new era: the Jump-start Our Business Startups (JOBS) Act legalized equity CF by relaxing a series of restrictions regarding the sale of securities. The aim of this paper is to shed light on the role of securities on the allocation of resources, and to show that rewarding the non-implementation of a project is always part of the optimal security.

A key characteristic of CF is the easier access to potential capital. This relates to the fundamental idea in crowdfunding: it is the crowd, rather than a venture capitalist or a bank that decides about the creditworthiness of a project. Simplifying the process of accessing potential capital has a critical impact on the type of entrepreneurs and projects that this method attracts. On the plus side, crowdfunding is more open to innovative ideas. On the downside, it might attract entrepreneurs who: i) are associated with negative-NPV projects, ii) have little experience, and as a result, no record regarding their ability, and iii) have limited liability[2].

This paper is based on two observations. First, compared to VC, investors in CF do not participate in the determination of the terms of financing. In CF platforms, entrepreneurs offer a take-or-leave security to potential investors. Thus, the determination of the optimal security when raising capital is a signaling rather than a screening problem. This signaling problem has been studied thoroughly in the security design literature, where prominent examples are Myers and Majluf (1984) and Nachman and Noe (1994). This brings us to the second observation, which regards the main characteristics of the capital seeking party. In particular, we recognize that the representative entrepreneur in CF platforms differs from a typical large company; hence, applying the main findings of the security design literature in this environment is not straightforward. Following the previous two observations, the goal of this paper to characterize the optimal security issued by an entrepreneur which shares the same characteristics as the representative entrepreneur in CF platforms, and explore its main implications on the allocation of resources.

We study a setup where a cashless entrepreneur seeks capital to finance an investment project. The entrepreneur is either of high-productivity (a good type) or of low productivity (a bad type). The entrepreneur is privately informed about her type whereas potential...
investors only hold beliefs about the entrepreneur’s type. Asymmetric information between
the entrepreneur and potential investors gives rise to a signaling game when the former issues
securities to raise capital. We differ from the security design literature in two crucial dimen-
sions. First, to capture better the crowdfunding example, we do not restrict our analysis to
entrepreneurs which are associated with projects of positive net present value (NPV). In par-
ticular, we assume that the good type corresponds to a project of positive NPV, whereas the
bad type corresponds to a project of negative NPV. Second, we relax the implicit or explicit
assumption in the security design literature that the entrepreneur is obliged to implementing
the project after raising funds; in our model, whether the project is implemented is determined
dependently. These two features not only enrich the security design problem by allowing the
security to be also contingent on the implementation choice, but also enables us to explore the
allocational impact of the optimal security.

The first set of findings refers to the characterization of the optimal security. We find
that the unique equilibrium in the contracting game is a pooling equilibrium where the bad
type offers the same security as the good type. The optimal security is characterized by two
components: a payment scheme if the project is implemented, and a fixed payment if the project
is not implemented. The payment scheme is similar to the standard debt; the investors become
the claimants of the return of the project if that fails, whereas they receive a predetermined
amount otherwise. The payment in case of non-implementation is such as the bad type’s
expected utility when not implementing the project coincides with her expected utility when
implementing the project. The intuition behind a debt-like contract is two-fold. First, offering
a security where the entrepreneur’s return is zero when the project fails -which is more likely
when the entrepreneur is of bad type- is aligned with the incentive of the good type to separate
from the bad type. This idea is similar to the intuition of Nachman and Noel (1994). Second,
a debt-like security minimizes the expected utility of the bad type when implementing the
project. Worsening the bad type’s option of implementing the project effectively minimizes the
cost of preventing the bad type from implementing her project, which in turn, minimizes the
negative externality imposed by the bad to the good type.

The second set of findings refers to the implications of the optimal security in the allocation
of resources. We show that once we endogenize the choice of project implementation, the
market survives, and a positive NPV entrepreneur implements her projects, independently
of the extent of information asymmetry. Besides, the optimal security achieves separation
in the implementation of the project; the negative-NPV type never implements her project.
These findings differ significantly from the case where the entrepreneur is obliged to project
implementation, which leads to either under-implementation or over-implementation. Finally,
we show that commitment to project implementation is never profitable for the entrepreneur.
This is because preventing a negative-NPV type from implementation mitigates its negative
externality to the positive-NPV type. These findings highlight that focusing on securities that
do not allow for the possibility of non-implementing the project has negative ramifications on welfare.

In Section 5 we develop a richer environment where the entrepreneur chooses her productivity level in equilibrium. We derive the equilibrium productivity level, and explore how it relates to the assumption that the entrepreneur is obliged to project implementation. In particular, we are interested in examining whether the finding of rewarding an entrepreneur when not implementing her project could weaken her incentive to invest in increasing her productivity, given that productivity is irrelevant when the project is not implemented. To explore this, we allow the entrepreneur to take a costly action, which is unobservable to investors and increases her productivity. We find that allowing the project implementation to be at the entrepreneur’s discretion increases the entrepreneur’s expected productivity. This follows from the finding that high productivity is rewarded more due to the better allocation of resources, which, effectively, leads to a steeper incentive pay. This analysis indicates that allowing for non-implementation of projects is aligned with the CF platform’s incentives of attracting high productivity projects.3

\[ \text{Relevant Literature} \]

Our work pertains mainly to the literature on security design under asymmetric information, initiated by Myers and Majluf (1984) and Nachman and Noe (1994), and followed by DeMarzo and Duffie (1999) and DeMarzo (2005). Technically, our setup is closer to Nachman and Noe (1994), apart from two major modifications: first, the entrepreneur might be associated with a negative-NPV project, and second, we relax the (implicit) assumption that the entrepreneur is obliged to implement the project. Allowing for the project implementation to be at the entrepreneur’s discretion differentiates our work from the papers in this strand of the literature.

Besides, this work relates to the literature which explores the question of security design in an environment which is characterized by moral hazard. Prominent examples include Innes (1990), Crémér, Khalil, and Rochet (1998), Hartman-Glaser, Piskorski, and Tchistyi (2012). We differ from this strand of the literature with respect to the form of the hidden action. In our paper, the hidden action refers to the decision of the agent to invest in her productivity level. Regarding the finding of alleviating adverse selection, our paper relates to Brennan and Kraus (1987). In our model, alleviating adverse selection is achieved by endogenizing the choice of project implementation. In contrast, Brennan and Kraus (1987) is interested in financial settings which alleviate adverse selection.

This paper is also related to the literature which highlights the optimality of CEO’s severance pay, known as “golden parachutes”. “Golden parachutes” refers to the compensation of a CEO when her executive is terminated, as a result of a merger or takeover. Similarly to our paper, the idea behind the optimality of severance pay to an informed agent is that it might

\[ ^3 \text{The revenue of platforms comes from a commission of 4-5% on the amount raised.} \]
prevent her from taking an action, which results in an inefficient allocation of resources. In our setup, the inefficient allocation comes from implementing a negative-NPV project, whereas in the CEO example, comes from taking an investment decision which differs from the profit-maximizing one. For example, Inderst and Mueller (2010) show that the rewarding an agent to quit could be optimal, as long as it is accompanied by a steep incentive pay. In a slightly different environment, Levitt and Snyder (1997) argue in favor of rewarding a CEO to reveal bad news which could, in turn, lead the principal to cancel an inefficient project. As opposed to Levitt and Snyder (1997), where the possibility of a project cancellation leads to lower effort and productivity, we find that rewarding non-implementation of the project results in higher productivity. This is a critical departure from the literature on “golden parachutes”: in our setup, the optimality of this payment not only does not fade off when the productivity is endogenous, but it is reinforced. Also, in the technical part, a major difference of our work from the literature on CEO’s compensation contracts is that, in our setup, it is the informed party who offers the contract. Hence, we explore a signaling rather than a screening problem.

A key finding of our paper is that allowing entrepreneurs to decide whether to implement their project, and to include this decision in the security, prevents market breakdown. Preventing market breakdown is also one of the main goals in Philippon and Skreta (2010), Tirole (2012) and Camargo, Kim, and Lester (2014), who focus on the role of the optimal government intervention, rather than financial securities.

This paper is organized as follows. Section 2 introduces the model. Section 3 explores the case where the entrepreneur is obliged to project implementation. Section 4 explores the case where the project implementation is at the entrepreneur’s discretion. Section 5 presents the case where the productivity of the entrepreneur is endogenously determined. Section 6 discusses and concludes.

2 The Model

Environment: We consider an environment where a cashless, risk-neutral entrepreneur seeks capital to finance a project. The project is non-divisible, and its implementation requires an investment equal to $I$. The entrepreneur raises capital by potential investors, in exchange for securities. We assume that capital market is perfectly competitive, and that investors demand a net return normalized to zero. Once the necessary capital is raised, and the project is implemented, a flow $x$ is generated. We allow for a binary cash flow, i.e., $x = \{S, F\}$, where $S > F$. Thus, $x = S$ can be interpreted as the cash flow when the project succeeds, and $x = F$ as the cash flow when the project fails. In order to be consistent with the crowdfunding example, we assume that the entrepreneur does not have any wealth other than the project’s return. Also, both the entrepreneur and investors are protected by limited liability.
Entrepreneur’s types & Information sets: The entrepreneur can be of two types, bad or good, i.e., \( t \in T = \{B, G\} \). We assume that the good (bad) type generates a success with probability \( p_G \) (\( p_B \)), where \( p_G > p_B \). A good type can be interpreted as a high-productivity entrepreneur, whereas a bad type can be interpreted as a low-productivity entrepreneur, where the productivity is determined by the probability of success of the project. The entrepreneur has private information regarding her type. In contrast, investors hold prior beliefs about the entrepreneur’s type. In particular, investors expect the entrepreneur to be of good type, with probability \( \lambda_G \), and of bad type, with the complementary probability \( \lambda_B = 1 - \lambda_G \). Both the probability of success of each type, and investors’ beliefs regarding the entrepreneur’s type are common knowledge.

It is worth highlighting a critical departure of our paper from other papers on the security design literature such as Nachman and Noe (1994). In this paper, we do not restrict the analysis to projects of positive net present value (NPV). Instead, we assume that the bad type corresponds to a negative-NPV project, whereas the good type corresponds to a positive-NPV project, i.e.,

\[
 p_G S + (1 - p_G) F > I > p_B S + (1 - p_B) F.
\]

Finally, we do not impose any restriction on the ex-ante NPV of the project:

\[
 \lambda_G [(p_G S + (1 - p_G) F] + \lambda_B] [p_B S + (1 - p_B) F] - I
\]

i.e., conditional on investor’s prior beliefs, the entrepreneur’s project can have positive, negative or zero NPV. We explore each case separately.

A possible concern behind this assumption, is why an entrepreneur would choose to implement a negative-NPV project. The answer to this relates to the combination of two characteristics, which closely reflect what is documented in the crowd-funding industry: i) the project is funded fully by a third party, and ii) the entrepreneur has limited liability. A direct consequence of these two characteristics is that although the entrepreneur’s project has negative expected net return, the entrepreneur’s expected utility is always non-negative. Note that allowing for a negative-NPV project has implications for the planner’s problem: if the aim of the planner is to maximize total welfare, only type \( t = G \) is worth financing.

Contracting game: The information asymmetry between the entrepreneur and investors, regarding the type of the former, turns the choice of the security design into a signaling game. A significant part of the literature on security design, such as Myers and Majluf (1984) and Nachman and Noe (1994) assumes, either implicitly or explicitly, that once financing takes place, the entrepreneur is obliged to project implementation.\(^4\) The goal of this paper is to show

\(^4\) In fact, Nachman and Noe (1994) allow for securities which are contingent only on the realized cash flow.
that this assumption plays a critical role when the entrepreneur shares the same characteristics as the representative entrepreneur in crowdfunding platforms. In particular, we are going to show that this assumption leads to an inefficient allocation of resources, and if information asymmetry is severe, to a market breakdown. Hence, applying the main findings of Myers and Majluf (1984) and Nachman and Noe (1994) to a crowdfunding environment might be problematic.

Endogenizing the implementation choice enriches the contracting game; the security is contingent not only on the realized cash flow, but also on the implementation choice. We denote by $g$ a security which consists of two sets of payments, $g(x)$ and $\bar{g}$. $g(x)$ denotes the payment when the project is implemented and the realize cash flow is $x$. In contrast, $\bar{g}$ denotes the payment if no implementation takes place. Thus, the security is defined by the triple: $g(S), g(F), \bar{g}$. Finally, given the price $P_g$ of the security, the assumption of limited liability imposes the following restrictions:

\begin{align*}
0 \leq g(x) &\leq P_g - I + x, \quad \text{if the project is implemented} \\
0 \leq \bar{g} &\leq P_g, \quad \text{if the project is not implemented}
\end{align*}

where $g(x) \geq 0$ and $\bar{g} \geq 0$ capture investors’ limited liability, whereas $g(x) \leq P_g - I + x$ and $\bar{g} \leq P_g$ capture the entrepreneur’s limited liability. We denote by $G$ the set of admissible securities, i.e., securities which are characterized by $g(x)$ and $\bar{g}$, and satisfy the limited liability assumption.

**Entrepreneur’s Maximization problem:** An entrepreneur of type $t \in T$ issues a security $g_t$, which consists of $g_t(x)$ and $\bar{g}_t$, in order to maximize her expected utility:

\[ V(t, g_t, P_{g_t}) = \max \{ \mathbb{E}_t[P_{g_t} - I + x - g_t(x)], P_{g_t} - \bar{g}_t \} \]

where $P_{g_t}$ represents the price of security $g_t$. Note that $\mathbb{E}_t[P_{g_t} - I + x - g_t(x)]$ is the entrepreneur’s expected utility when implementing her project, whereas $P_{g_t} - \bar{g}_t$ is her expected utility in the case where the project is not implemented. The $\max$ function indicates that the entrepreneur will choose the option which maximizes her expected return. Note that $P_{g_t}$ cannot be lower than $I$, otherwise there would be insufficient funds for the implementation of the project. This remark, combined with the limited liability assumption, imply that $V(t, g_t, P_{g_t})$ is non-negative, independently of the type of the entrepreneur.

**Timing:** The sequence of events is as follows:

1. The entrepreneur of type $t \in T$ sells a security $g_t$ at price $P_{g_t}$.
2. The entrepreneur decides whether to implement, by investing $I$. 
3. If the project is implemented, its cash flow $x$ is realized.

4. Contract is executed.

**Equilibria characterization:** A candidate for an equilibrium is a triple of functions $e^* = (g^*, \mu^*, P^*)$, where: i) $g^* : T \rightarrow G$, where $g_t^*$ is the security design chosen by the type $t$, ii) $\mu^* : G \rightarrow \Delta_T$, and $\mu_g$ is the market’s posterior beliefs given that the security $g$ is offered by the entrepreneur, and iii) $P^* : G \rightarrow R_+$. A Perfect Bayesian Equilibrium is a triple $e^* = (g^*, \mu^*, P^*)$, which satisfies the following conditions:

- **Sequential Rationality:** For each $t \in T$, $g_t^*$ maximizes $V(t, g_t, P_{g_t})$, subject to the constraints that $g \in G$ and $P_g^* \geq I$.

- **Beliefs Consistency:** When security $g$ is such that $g = g_t^*$ for some $t \in T$, $g$ is “on the equilibrium” and $\mu$ is determined by Bayes’ rule. When $g$ is such that $g \neq g_t^*$ for every $t \in T$, $g$ is “off the equilibrium”, then it is only required that $\mu_g \in \Delta_T$.

- **Competitive Rationality:** $P^*_G = E_{\mu^*g}[g]$, for all $g \in G$.

Regarding the “off-equilibrium beliefs”, we adopt the D1 refinement criterion discussed in Cho and Kreps (1987). D1 places zero weight on a type $t = t'$ deviating to an off-equilibrium design if there exists a type $t = t''$ who has strong incentive to deviate, whenever type $t = t'$ has weak incentive to deviate.

### 3 Entrepreneur obliged to project implementation

In order to evaluate the impact of endogenizing the choice of project implementation, we use the case where the entrepreneur is obliged to implement the project, as a benchmark. To this end, we start by characterizing the optimal security, under the assumption that the entrepreneur is obliged to project implementation. This environment is consistent with the setup in Nachman and Noe (1994), except for the modification that we allow for a negative-NPV project. Note that, in this case, the security $g$ is characterized only by the payment scheme $g(x)$; the payment $\bar{g}$ becomes irrelevant. To avoid any confusion, when the entrepreneur is obliged to implement the project, we denote the security as $g'$ and the corresponding payment scheme as $g(x)'$. In contrast, when the choice of project implementation is endogenous, we denote the security as $g$, and the corresponding payment scheme as $g(x)$.

**Lemma 1**

_in equilibrium, the bad type offers the same security as the good type (pooling equilibrium)._
The intuition behind Lemma 1 is straightforward. If the bad type offered a different security, she would reveal herself, given that “on the equilibrium beliefs” should be correct. In that case, the bad type would not be able to raise capital, because investors anticipate that she corresponds to a negative-NPV project. Hence, the only equilibrium in the contracting game, is an equilibrium where the bad type offers the same security as the good type. Consequently, investors posterior beliefs about the entrepreneur’s type, coincide with their prior. Also, given that the bad type mimics the good type, we can focus on the maximization problem of the good type.

Lemma 2

In equilibrium, the price of a security \( g \), \( P_g \) equals \( I \).

Lemma 2 is a consequence of a pooling equilibrium, where the bad type mimics the good type. The rationale behind Lemma 2 is that the good type, when raising capital, suffers a negative externality from the bad type. This cross-subsidization implies that the good type ends up paying a higher capital cost, than the one that would correspond to her type. As a result, it is never optimal for the entrepreneur to raise more capital than the amount that is necessary for undertaking the project. Consequently, the entrepreneur would never offer a security \( g' \), whose corresponding price exceeds the cost of financing the project, i.e. \( P_g \leq I \). Since the project is non-divisible, \( P_g = I \). Given Lemma 1 and Lemma 2, the optimal security when the entrepreneur is obliged to project implementation, solves the following maximization problem:

\[
\begin{align*}
\text{Maximize} \quad & p_G(S - g(S)) + (1 - p_G)(F - g(F)) \\
\text{s.t.} \quad & \lambda_G[p_G g(S) + (1 - p_G)g(F)] + \lambda_B[p_B g(s) + (1 - p_B)g(F)] = I \\
& 0 \leq g(S) \leq S \\
& 0 \leq g(F) \leq F
\end{align*}
\]

where (3) is the investors’ participation constraint, which is binding, given that capital markets are assumed to be perfectly competitive. In particular, the LHS of (3) captures investors’ expected return, whereas the RHS captures the amount investors pay for the security, denoted by \( P_g \), which, following the Lemma 2, equals \( I \). The last two constraints capture the limited liability of the entrepreneur and investors.

Before deriving the optimal security, we explore under what conditions the market survives. Given that the equilibrium in the contracting game is pooling, the market collapses and no financing takes place if there is no feasible security which satisfies investors’ participation
constraint:
\[
\lambda_G [(p_G g(S) + (1 - p_G) g(F))' + \lambda_B [p_B g(S) + (1 - p_B) g(F)')] \geq \frac{I}{\text{Cost}}
\]

Note that if the market collapses, the entrepreneur’s expected utility is zero. In contrast, if the market survives, the expected utility of the entrepreneur is non-negative. Hence, the market collapses if, even for the maximum feasible payments \((g(S)' = S, g(F)' = F)\), the cost of financing exceeds its expected revenue, i.e.,

\[
\lambda_G (p_G S + (1 - p_G) F) + \lambda_B [p_B S + (1 - p_B) F] < I
\]

Condition (6) holds if the prior probability that investors attribute to the entrepreneur being of good type is sufficiently small. Thus, the market collapses if:

\[
\lambda_G < \lambda_G^{\min} \equiv \frac{I - [p_B S + (1 - p_B) F]}{(p_G - p_B)(S - F)}
\]

Proposition 1 characterizes the optimal security when the entrepreneur is obliged, by assumption, to project implementation.

**Proposition 1**

If \(\lambda_G \geq \lambda_G^{\min}\), financing takes place, and the optimal security is given by:

\[
g^*(F)' = F, \quad g^*(S)' = \frac{I - F}{\lambda_G p_G + \lambda_B p_B} + F
\]

If \(\lambda_G < \lambda_G^{\min}\), no financing takes place (market collapses).

**Proof.** See Appendix C.

The intuition behind the optimal security relates to the core idea in [Nachman and Noe (1994)]. Namely, the good type tries to separate herself from the bad type, by offering a contract which promises the maximum feasible payment when the cash realization is low, because this cash flow is more likely to arise when the entrepreneur is of bad rather than good type.

Note that there are two types of distortions depending on the value of \(\lambda_G\). If \(\lambda_G < \lambda_G^{\min}\), there is under-implementation compared to the first best: the positive-NPV type does not raise the necessary capital to implement her project. In contrast, if \(\lambda_G \geq \lambda_G^{\min}\), the market survives, and there is over-implementation: the negative-NPV type implements her project.
4 Endogenous project implementation

This section explores the case where the choice to implement the project is at the entrepreneur’s discretion, and this choice is contractible. We showed that restricting the entrepreneur to implement the project leads to under-implementation, when the market collapses, and over-implementation, when the market survives. The goal of this section is to show that endogenizing the choice of project implementation: i) increases the entrepreneur’s expected utility, independently of her type, ii) improves the allocation of resources, and iii) prevents the market from collapsing. Besides, we show in Section 5 that endogenizing project’s implementation strengthens the incentive of the entrepreneur to invest in her productivity.

Recall that the entrepreneur issues a security $g$ which consists of three payments; $g(S)$, if the project is implemented and it succeeds, $g(F)$, if the project is implemented and it fails, and $\bar{g}$, if the project is not implemented. Similarly to the case where the entrepreneur is obliged to project implementation, the bad type offers the same security as the good type, i.e., Lemma 1 goes through. Besides, following Lemma 1, Lemma 2 holds as well. Although there is pooling at the contracting stage, endogenizing the choice of project implementation might lead to separation at the stage of project implementation. Four are the possible scenarios regarding the implementation of the project:

- Scenario 1: Both types implement their project. As long as beliefs are consistent, this equilibrium is effectively the same as the equilibrium when the entrepreneur is obliged to project implementation.
- Scenario 2: Neither the good nor the bad type implements her project.
- Scenario 3: Only the bad type chooses to implement the project. This scenario can not emerge in equilibrium because it violates investors’ participation constraint. Even for the maximum feasible payments, i.e., $g(F) = G$, $g(S) = S$, $\bar{g} = P_g$, investors would make zero profits if the entrepreneur is of good type, and negative profits otherwise.
- Scenario 4: Only the good type chooses to implement the project. This is the only case where separation can be achieved, in the sense that the implementation choice differs depending on the entrepreneur’s type. The following analysis shows that this scenario can emerge in equilibrium.

We postpone exploring scenario 1 and 2 for the end of this section, and we start with analyzing scenario 4. We are interested in finding the security which maximizes the expected utility of the good type, subject to the constraint that only the good type implements the project. Compared to the case where the entrepreneur is obliged to project implementation, the maximization
problem is augmented by one incentive compatibility condition for each type. The maximization problem of the good type is given by:

\[
\begin{align*}
\text{Maximize} & \quad EU(\text{impl}|t = G) \quad \text{s.t.} \\
EU(\text{impl}|t = G) & \geq EU(\text{not impl}|t = G) \quad (ICC_G) \\
EU(\text{impl}|t = B) & \leq EU(\text{not impl}|t = B) \quad (ICC_B) \\
\lambda_G [p_G g(S) + (1 - p_G) g(F)] + \lambda_B \bar{g} & \geq I \quad (PC_I) \\
0 & \leq g(S) \leq S, \quad 0 \leq g(F) \leq F, \quad 0 \leq \bar{g} \leq I \quad (LL)
\end{align*}
\]

where,

\[
\begin{align*}
EU(\text{impl}|t = G) &= p_G(S - g(S)) + (1 - p_G)(F - g(F)) \\
EU(\text{impl}|t = B) &= p_B(S - g(S)) + (1 - p_B)(F - g(F)) \\
EU(\text{not impl}|t = G) &= EU(\text{not impl}|t = G) = I - \bar{g}
\end{align*}
\]

\(ICC_G\) (\(ICC_B\)) strands for the incentive compatibility constraint of the good (bad) type, whereas, \(PC_I\) stands for the investors’ participation constraint.

We show in the Appendix that under the optimal security, \(ICC_B\) is binding, i.e. if the entrepreneur is of a bad type, her expected utility when not implementing the project equals her expected utility when implementing the project. If this is not the case, there is always a deviation to a higher level \(\bar{g}\) and a lower level of \(g(S)\), which does not violate the incentive constraints, and it is strictly preferred by the good type. Also, we show in the Appendix that \(ICC_G\) is slack. Finally, \(PC_I\) is binding, as an implication of the perfectly competitive capital market. Then, by substituting \(ICC_B\) into \(PC_I\), and solving with respect to \(g(S)\), we obtain:

\[
g^*(S) = \frac{\lambda_B(-Fp_B + F + p_BS) + \lambda_G I}{\lambda_G p_G + \lambda_B p_B} - \frac{R}{\lambda_G (1 - p_G) + \lambda_B (1 - p_B)} g^*(F) \tag{8}
\]

Condition [8] implies that the entrepreneur could increase \(g(F)\) by \(\epsilon\) and decrease \(g(S)\) by \(R \times \epsilon\) without violating investors’ participation constraint. Increasing \(g(F)\) by \(\epsilon\) decreases the entrepreneur’s expected utility by \((1 - p_G) \times \epsilon\), whereas decreasing \(g(S)\) by \(R \times \epsilon\) increases the entrepreneur’s expected utility by \(p_G \times R \times \epsilon\). It can be shown that \(p_G \times R \times \epsilon > (1 - p_G) \times \epsilon\) as long as \(p_G > p_B\). Hence, under the optimal security, \(g^*(F)\) reaches its maximum value, i.e. \(g^*(F) = F\). Substituting \(g^*(F) = F\) into [8], we derive the optimal value of \(g^*(S)\), and subsequently, by substituting \(g^*(F)\) and \(g^*(S)\) into \(ICC_B\), we derive \(\bar{g}^*\). Note also that the security of Lemma 3 is the unique security which survives the Intuitive Criterion.
Lemma 3: Optimal security in scenario 4.

\[ g_{s4}^*(F) = F \]
\[ g_{s4}^*(S) = \frac{\lambda_G(I - F(1 - p_G)) + \lambda_B p_B S}{\lambda_G p G + \lambda_B p_B S} \]
\[ \tilde{g}_{s4}^* = I - p_B (S - g_{s4}^*(S)) \]

Proof. See Appendix.

The intuition behind \( g_{s4}^*(F) = F \) is two-fold. First, similarly to the intuition in Proposition 1, offering a security which promises the maximum feasible payment when the realized cash flow is low, is aligned with the entrepreneur’s incentive to separate from the bad type. This is because a low cash flow is more likely to arise if the entrepreneur is of bad rather than good type. Second, compared to any other security which satisfies investors’ participation constraint, \( g(F) = F \) minimizes the expected utility of the bad type when implementing the project. Worsening the bad type’s option of implementing the project, allows the good entrepreneur to minimize the negative externality imposed by the bad type. Subsequently, mitigating the cross-subsidization, enables the good entrepreneur to offer a lower payment when the project is implemented and generates a success.

Here, we show that scenario 1 cannot be an equilibrium. In scenario 1, similarly to the case where the entrepreneur is obliged to project implementation, both types implement their project. Thus, \( \tilde{g} \) is never realized. Although \( \tilde{g} \) does not affect the entrepreneur’s utility directly, it plays a critical role when it comes to beliefs’ formulation. Hence, for values of \( \tilde{g} \) for which the bad entrepreneur prefers implementing the project, the maximization problem in scenario 1 is effectively the same as the maximization problem when the entrepreneur is obliged to project implementation. As a result, \( g_{s1}^*(F) = g^*(F)' \) and \( g_{s1}^*(S) = g^*(S)' \). However, the good type is better-off under scenario 4 because \( g_{s4}^*(S) \leq g^*(S)' \) and \( g_{s4}^*(F) = g^*(F)' \). Thus, scenario 1 cannot emerge in equilibrium: the good type has incentive to deviate to scenario 4, by choosing \( \tilde{g} \), such as the bad type prefers not implementing her project.

Similar intuition applies to scenario 2. In this scenario, the expected utility of investors equals \(-P_g + \tilde{g}\). Following Lemma 2 and the assumption of perfectly competitive markets, \( P_g = \tilde{g} \). In this equilibrium, assuming that such an equilibrium exists, the entrepreneur’s utility is zero. However, the good type is better-off under scenario 4 because she achieves positive expected utility. Thus, scenario 2 cannot emerge in equilibrium: the good type always has the incentive to deviate to scenario 4, by offering the security characterized in Lemma 3.

Corollary 1

Independently of her type, the entrepreneur is better-off when the project implementation is
endogenous and contractible.

Proof. See Appendix.

Hence, the optimal security, provided in Lemma 3, is the unique optimal security. Note that the improvement in the entrepreneur’s expected utility does not come at the cost of lower expected utility for investors; under both setups, the investors’ participation constraint is binding. This Pareto improvement stems from a more efficient allocation of resources, due to the finding that the negative-NPV type does not implement her project.

**Market breakdown when project implementation is endogenous.**

When the entrepreneur is allowed to choose whether to implement her project, and to offer securities contingent on this choice, the market survives if there is at least one feasible security \( g \in G \), such as the investors’ participation constraint is satisfied:

\[
\frac{\lambda_B \bar{g} + \lambda_G [p_G g(S) + (1 - p_G) g(F)]}{\text{Expected Revenue}} \geq \frac{I}{\text{Cost}}
\]

where the LHS of (9) denotes the expected revenue, whereas the RHS denotes the cost of investors. Recall that for a security to be feasible, it must satisfy limited liability, i.e., \( 0 \leq g(S) \leq S, 0 \leq g(F) \leq F, 0 \leq \bar{g} \leq I \). We now explore relation (9) for the optimal security of Lemma 3, which is, by construction, feasible. Hence, (9) becomes:

\[
\lambda_B \bar{g}^* + \lambda_G [p_G g^*_S(S) + (1 - p_G) g^*_F] \geq I
\]

By substituting the optimal values of \( \bar{g}^*_S, g^*_S(S) \) and \( g^*_F(F) \) into (10), we obtain that the LHS of (10) equals \( I \). Thus, (9) is satisfied for the security of Lemma 3. Note that if the investors’ participation constraint is not satisfied (market breakdown), the entrepreneur’s expected utility is zero. Recall also that the entrepreneur’s expected utility under the security of Lemma 3 is non-negative. Hence, we can conclude that the market never collapses, because compared to that case, the entrepreneur is better-off when offering the security of Lemma 3, which always satisfies the investor’s participation constraint.

**Corollary 2**

*If the implementation choice is endogenous and contractible, there always exists a security which satisfies investors’ participation constraint (market always survives), and the positive-NPV project is implemented.*

The reason the market always survives is that, once we endogenize the implementation choice, the positive-NPV project is always implemented, whereas the negative-NPV project is not.
The intuition behind this finding can be captured in the following example. To simplify the algebra, suppose an environment where the bad type always fails ($p_B = 0$). In this case, the best strategy for the good type is to offer a security which pays $g(F) = F$. Such a payment leaves no surplus to the bad type when implementing the project. Consequently, the bad type is willing to forgo implementation as long as $\bar{g} \leq I$. Note that the combination of $g(F) = F$ with $\bar{g} = I$ minimizes the loss of investors when financing a bad type, which effectively, minimizes the distortion imposed by the bad type to the good type. Eliminating the cross-subsidization enables the good type to offer a payment in case of success which equals the fair payment, i.e., the payment which corresponds to her type, $g(S) = \frac{(I - F(1 - p_G))}{p_G}$.

It is worth highlighting that the extent of the decrease in cross-subsidization is negatively related to the probability of success of the bad type, $p_B$. This is because there is a monotonic relation between $p_B$ and the cost of preventing the bad type from implementation. This relation is captured by the binding $ICC_B$.

The previous example captures the main idea of this paper; once the choice of project implementation is endogenous and contractible, the good entrepreneur can offer a security which, effectively, provides insurance to investors against the event of financing a bad entrepreneur. This insurance allows the entrepreneur to offer a lower payment in the case where the project is implemented and succeeds.

The combination of Lemma 3, Corollary 1 and Corollary 2 leads to Proposition 2, which characterizes the optimal security when the choice of implementing the project is endogenous and contractible.

**Proposition 2:**

*When the choice of project implementation is endogenous and contractible, project’s financing always takes place, and the unique optimal security is given by:*

$$g^*(F) = F$$

$$g^*(S) = \frac{\lambda_G(I - F(1 - p_G)) + \lambda_B p_B S}{\lambda_G p_G + \lambda_B p_B}$$

$$\bar{g}^* = I - p_B(S - g^*(S))$$

5 **Endogenizing Entrepreneur’s Productivity**

The main source of revenues of crowdfunding platforms is a commission of 4-5% on the total capital raised. Thus, one of the main objectives of crowdfunding platforms is to attract high-
productivity entrepreneurs. We showed in the previous section that endogenizing the choice of project implementation implies that a subset of entrepreneur’s types does not proceed with the implementation of the project. Hence, a question which arises naturally, is whether this characteristic could demotivate entrepreneurs to invest in their productivity, given that it does not affect the expected return in case of non-implementation. If this concern is valid, it could potentially cancel out the benefits of preventing implementation of negative-NPV projects.

In this section, we shed light on this concern by allowing the entrepreneur to invest in her productivity, before seeking capital to finance her project. The final goal of this exercise is two-fold. First, to develop a richer environment, where the entrepreneur chooses her productivity in equilibrium. Second, to explore the impact of endogenizing the choice of project implementation on the entrepreneur’s productivity.

**Environment & Technology:** For the sake of tractability, and in order to be consistent with the previous analysis, we allow for the simplest investment-in-productivity technology: the entrepreneur is of low productivity \( t = B \) unless she takes a costly action, which upgrades her to a high-productivity entrepreneur \( t = G \). This action can be interpreted as the acquisition of skills or relevant information, which would increase the probability of developing a successful project. In this setting, the probability that the entrepreneur invests in her productivity coincides with \( \lambda_G \), which is now determined endogenously.

Endogenizing \( \lambda_G \) leads to a game consisting of two stages: the investment-in-productivity stage, and the contracting/implementation stage. The second stage is effectively the same as in the case where the probability \( \lambda_G \) is exogenous. The first stage refers to the entrepreneur’s decision to take a costly action, which determines her productivity.

**Information Sets:** In order to maintain the setting of asymmetric information in the contracting stage, we assume that investors cannot observe whether the agent has invested in her productivity level. For instance, suppose that the entrepreneur’s project is a new application for a smartphone. In this case, the entrepreneur can acquire costly information regarding similar applications, which will enable her to design her application better, and increase its probability of success. We assume that the cost of acquiring skills, \( c \), is not verifiable, and as opposed to the entrepreneur who observes \( c \), investors only hold beliefs about it. In particular, investors anticipate that \( c \) is drawn from an interval \([c, \bar{c}]\), according to a continuous probability density function \( \phi(c) \), with \( \Phi(c) \) standing for the corresponding cumulative distribution function. The rationale behind this assumption is that the entrepreneur, mainly because of her expertise, knows the cost of increasing the project’s productivity, whereas investors have an imperfect

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6 This is also consistent with the fact that it is a reputation-based industry.
7 The results are robust to any technology where taking the costly action increases the probability of becoming a high-productivity entrepreneur.
estimate about it. Thus, the investment decision, which is denoted as \( d \), is a mapping from \( c \) to the type \( t \in T \).

**Timing:** The sequence of events is as follows:

1. The entrepreneur observes her cost, and decides whether to invest in her productivity.
2. The type \( t \in T \) of the entrepreneur is determined.
3. The entrepreneur of type \( t \in T \) sells a security \( g_t \) at price \( P_{g_t} \).
4. The entrepreneur decides whether to implement the project.
5. If the project is implemented, its cash flow \( x \) is realized.
6. Contract is executed.

Thus, compared to the benchmark model, the game is augmented by the investment-in-productivity stage.

### 5.1 Derivation of probability \( \lambda_G \).

A property of the equilibrium is that investors’ beliefs about \( \lambda_G \) are correct. Note also that the probability that the entrepreneur invests in her productivity, i.e., \( \lambda_G \), depends on the security, which in turn, depends on \( \lambda_G \) through investors’ beliefs. Hence, the optimal security and the optimal value of \( \lambda_G \), are determined jointly in equilibrium.

We solve the game backwards. We start from the contracting stage, by taking investors’ beliefs about \( \lambda_G \) as given. Then, we proceed with the investment-in-productivity stage, by taking the security \( g \) as given.

#### 5.1.1 Contracting stage

Recall that the entrepreneur’s decision to invest in her productivity is unobservable. Thus, this stage is identical to the contracting stage at Section 4, apart from a critical modification: probability \( \lambda_G \) is now replaced by the beliefs of investors about the probability that the entrepreneur has invested in her productivity, denoted by \( \tilde{\lambda}_G \). Hence, by Proposition 2, the security which maximizes the entrepreneur’s expected utility, subject to the constraint that \( \lambda_G = \tilde{\lambda}_G \), is captured in Lemma 4.

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\(^8\)Note the entrepreneur offers the security *after* investing in her productivity. However, as long as \( c \) is not verifiable, the equilibrium security is the same independently of whether the entrepreneur offers the security before or after investing in her productivity.
Lemma 4

\[ g^*(F) = F, \quad \tilde{g}^* = I - p_B(S - g^*(S, \tilde{\lambda}_G)) \]

\[ g^*(S, \tilde{\lambda}_G) = \frac{\tilde{\lambda}_G(I - F(1 - p_G)) + \tilde{\lambda}_B p_B S}{\lambda_{GPG} + \tilde{\lambda}_B p_B} \]

5.1.2 Investment-in-productivity stage

When the entrepreneur considers investing in her productivity, she anticipates that the payments in the financing stage will be given by Lemma 4. Thus the entrepreneur’s expected utility in each case is:

\[ EU(\text{invest}) = p_G(S - g^*(S, \tilde{\lambda}_G)) + (1 - p_G)(F - g^*(F)) - c \]

\[ EU(\text{not invest}) = p_B(S - g^*(S, \tilde{\lambda}_G)) + (1 - p_B)(F - g^*(F)) \]

Since \( g^*(F) = F \), the entrepreneur invest in her productivity as long as:

\[ \frac{c}{\text{cost}} \leq \frac{(p_G - p_B)(S - g^*(S, \tilde{\lambda}_G))}{\text{benefit}} \equiv \hat{c} \quad (11) \]

where \( \hat{c} \) can be interpreted as the investment threshold. Figure 1 represents the relationship between \( \tilde{\lambda}_G \) (on the horizontal axis) and the benefit of investing in productivity (dashed curve), for four different distributions of \( c \). As expected, there is a negative relation between the probability \( \tilde{\lambda}_G \), and the payment in case of success that investors are willing to accept to finance an entrepreneur. In addition, the higher the \( \tilde{\lambda}_G \), the stronger the entrepreneur’s incentive to invest in productivity. This is captured in equation (11).

5.1.3 Equilibrium existence and uniqueness of interior equilibrium

We start the analysis by focusing on equilibria for which \( \tilde{\lambda}_G^* \in (0, 1) \), thereafter, “interior equilibria”. We analyze the equilibrium conditions when the project implementation is at the entrepreneur’s discretion- similar intuition applies when the entrepreneur is obliged to implement the project. We consider monotone or threshold equilibria in which the investment strategy is monotonic in \( c \). In an interior threshold equilibrium, the following conditions need to hold:

\[ \hat{c}^* = (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G^*)) \quad (12) \]

\[ Pr(c \leq \hat{c}^*) = \tilde{\lambda}_G^* \quad (13) \]

Condition (12) provides the critical value \( \hat{c}^* \) for which the entrepreneur is indifferent between investing and not investing. Condition (13) relates to the fact that investor’s beliefs should
be correct. In particular, the probability that investors attribute to the entrepreneur being of good type coincides with the probability that the entrepreneur’s cost is below $\hat{c}$. Besides, for investors’ beliefs to be correct, the entrepreneur must prefer investing when $c \leq \hat{c}$, and not investing otherwise. An implication of a threshold equilibrium is that $\hat{c}$ depends on $g^*(S, \tilde{\lambda}_G)$, which in turn, depends on $\hat{c}$. Thus, the optimal values $\hat{c}^*$ and $g^*(S, \tilde{\lambda}_G^*)$, are determined jointly in equilibrium.

Recall that $Pr(c \leq \hat{c}^*) \equiv \Phi(\hat{c}^*)$. In order to illustrate graphically the equilibrium existence, it is more informative to use the inverse of $\Phi(\hat{c})$:

$$\Phi^{-1}(\tilde{\lambda}_G) = \hat{c}$$

where $\Phi(\hat{c})$ is invertible as a continuous and strictly increasing function. Hence, every interior equilibrium in the investment-in-productivity stage satisfies the following condition:

$$\Phi^{-1}(\tilde{\lambda}_G) = (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G))$$  \hspace{1cm} (14)

where $\Phi^{-1}(\tilde{\lambda}_G)$ is depicted by the solid curve in Figure 1. Note that $\Phi^{-1}(\tilde{\lambda}_G)$ is strictly in-
creasing in $\tilde{\lambda}_G$, as a consequence of the fact that $\Phi(\hat{c})$ is strictly increasing in $\hat{c}$. Thus, relation (14) characterizes value of $\tilde{\lambda}_G$ where the dashed and the solid curve intersect. Relation (14), however, is not a sufficient condition for an interior equilibrium to exist. For example, point A in panel A of Figure 1 satisfies (14) but it can not be an equilibrium; for cost $\hat{c}_A + \epsilon$, with $\epsilon > 0$, and as long as the beliefs are consistent, the expected benefit exceeds the cost. Hence, for cost $\hat{c}_A + \epsilon$ the entrepreneur has incentive to invest in productivity, which contradicts the definition of a threshold equilibrium: in a threshold equilibrium, the entrepreneur invests in her productivity only if $c \leq \hat{c}_A$. Thus, for an interior equilibrium to exist, the solid curve should cross the dashed curve from below. Following that, the unique interior equilibrium is given by point $E$ in panel A of Figure 1.

Note that there could be more than one combinations of $\tilde{\lambda}_G$ and $g^*(S, \tilde{\lambda}_G)$ which satisfy (14) and the solid curve crosses the dashed curve from below. If this the case, the unique interior equilibrium is the one which corresponds to the maximum $\tilde{\lambda}_G$. This is because the expected utility of the entrepreneur is decreasing in $\tilde{\lambda}_G$, due to the impact of the latter on $g^*(S, \tilde{\lambda}_G)$.

We conclude this subsection with the case where there is no interior equilibrium. If it is very costly to invest in productivity, the only equilibrium in the investment stage is for $\tilde{\lambda}_G = 0$, for which the market collapses (Panel C). In contrast, if the cost of investing is very low, the unique equilibrium is for $\tilde{\lambda}_G = 1$ (Panel D). Lemma 5 presents the sufficient conditions for interior equilibrium to exist, where the payment $g^*(S, \tilde{\lambda}_G)$ is determined in Lemma 4.

**Lemma 5:** Interior and corner equilibrium - sufficient conditions

- If there is no $c \in [\underline{c}, \overline{c}]$ which satisfies:

\[
c \leq (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G = \Phi(c)))
\]  
(15)

then, the unique equilibrium is for $\tilde{\lambda}_G = 0$, for which the market collapses.

- If $\overline{c}$ satisfies:

\[
\overline{c} > (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G = \Phi(\overline{c}))
\]  
(16)

then, the unique equilibrium is for $\tilde{\lambda}_G = 1$.

- If (16) is violated, and there exists $c \in [\underline{c}, \overline{c}]$ which satisfies (15), then the unique interior equilibrium is characterized by the maximum $\tilde{\lambda}_G \in (0, 1)$ which solves:

\[
\Phi^{-1}(\tilde{\lambda}_G) = (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G))
\]  
(17)

**Proof.** See Appendix.  

20
5.2 Optimal security when types are endogenously determined

In this subsection, we characterize the optimal security when the choice of project implementation is endogenous, and the entrepreneur has to option to invest in her productivity level before financing takes place. In order to allow for an environment of asymmetric information between the entrepreneur and potential investors, we focus our analysis on the case where the type of the entrepreneur is uncertain, i.e., there exists an interior equilibrium in investment-in-productivity stage. The combination of Lemma 4 and Lemma 5, leads to Proposition 3.

**Proposition 3**

As long an interior equilibrium exists, the optimal security is given by:

\[
g^*(F) = F \\
g^*(S, \tilde{\lambda}_G^*) = \frac{\tilde{\lambda}_G^*(I - F(1 - p_G)) + (1 - \tilde{\lambda}_G^*)p_BS}{\tilde{\lambda}_G^*p_G + (1 - \tilde{\lambda}_G^*)p_B} \\
\tilde{g}^* = I - p_B(S - g^*(S, \tilde{\lambda}_G^*))
\]

where \(\tilde{\lambda}_G^*\) is the maximum \(\tilde{\lambda}_G \in (0, 1)\) which solves:

\[
\Phi^{-1}(\tilde{\lambda}_G) = (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G)) \quad (18)
\]

For the sake of completeness we present in Proposition 4 the optimal security when the entrepreneur is obliged, by assumption, to the project implementation. Similarly to the previous case, we focus our analysis on the case where there is uncertainty about the type of the entrepreneur.

**Proposition 4**

As long as an interior equilibrium exists, the optimal security is given by:

\[
g^*(F)' = F \quad , \quad g^*(S, \tilde{\lambda}_G)^* = \frac{I - F}{\tilde{\lambda}_G^*p_G + (1 - \tilde{\lambda}_G^*)p_B} + F
\]

where \(\tilde{\lambda}_G^*\) is the the maximum \(\tilde{\lambda}_G \in (0, 1)\) which solves:

\[
\Phi^{-1}(\tilde{\lambda}_G) = (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G)) \quad (19)
\]

**Proof.** See Appendix. \qed
5.3 Impact of endogenizing project implementation on entrepreneur’s productivity

In this subsection, we explore the impact of relaxing the assumption that the entrepreneur is obliged to project implementation on probability $\tilde{\lambda}_G$. This finding is summarized in Proposition 5.

**Proposition 5:**
*Allowing for the project implementation to be endogenous and contractible, increases the probability that an entrepreneur invests in her productivity.*

**Proof.** See Appendix.

The intuition behind this finding is straightforward: being productive is rewarded more when the project implementation is endogenous, due to the steeper incentive pay. Thus, for any given value of the cost $c$, the entrepreneur’s incentive to invest in her productivity is stronger.

Figure 2 allows us to illustrate the main idea behind Proposition 5. Figure 2 similarly to Figure 1 represents the relationship between $\tilde{\lambda}_G$ (on the horizontal axis) and the benefit of investing in productivity. The dashed curve illustrates the benefit when the choice of project implementation is endogenous, whereas the dotted curve illustrates the benefit when the entrepreneur is obliged to the project implementation. Note that the dashed curve starts from the origin (excluding the origin point), highlighting the finding that the market survives as long as $\tilde{\lambda}_G > 0$. In contrast, the dotted curve starts from $\tilde{\lambda}_G^{\min}$, which is the minimum value of $\tilde{\lambda}_G$ for which the market survives. The dashed curve is always above the dotted curve, capturing the finding presented in Corollary 1, that for any given value of $\tilde{\lambda}_G$, $g^*(S,\tilde{\lambda}_G) \geq g^*(S,\tilde{\lambda}_G)$.

Panels A to D illustrate four different distributions of $c$, where in each case, point “E” denotes the equilibrium when the project implementation is endogenous, whereas point “e” denotes the equilibrium when the entrepreneur is obliged to project implementation. We denote as $\tilde{\lambda}_G^*$ the maximum value of $\tilde{\lambda}_G$ which solves (18) and as $\tilde{\lambda}_G^{*'}$ the maximum value of $\tilde{\lambda}_G$ which solves (19). Notice that only panels A and B represent environments where there is uncertainty about the type of the entrepreneur. The previous analysis gives rise to the following cases regarding the relationship between $\tilde{\lambda}_G^*$ and $\tilde{\lambda}_G^{*'}$.

- $1 > \tilde{\lambda}_G^* > \tilde{\lambda}_G^{*'} > \tilde{\lambda}_G^{\min}$: The probability that the entrepreneur invests in her productivity is higher when the project implementation is at the entrepreneur’s discretion. (panel A).
- $\tilde{\lambda}_G^* > \tilde{\lambda}_G^{*'} = 0$: The market survives only in the environment where the choice of project implementation is endogenous. (panel B)

\[ \tilde{\lambda}_G^{\min} = \frac{I - [p_B S + (1 - p_B) F]}{[(p_G S + (1 - p_G) F) - p_B S + (1 - p_B) F]}. \]
• $\tilde{\lambda}_G^* = \tilde{\lambda}_G^* = 0$: The entrepreneur never invests in her productivity, and the market collapses independently of whether the choice of project implementation is endogenous (panel C).

• $\tilde{\lambda}_G^* = \tilde{\lambda}_G^* = 1$: The entrepreneur invests in her productivity with certainty, independently of whether the choice of project implementation is endogenous (panel D).

To conclude, we show in this section that allowing the entrepreneur to choose whether to implement her project, and to offer securities contingent on this choice, leads to a higher expected productivity. This finding has implications for the crowdfunding example. In particular, Proposition 5 suggests that allowing the entrepreneur to offer securities contingent on the implementation choice, is aligned with the objective of crowdfunding platforms is to attract productive entrepreneurs/projects.

Figure 2: Equilibrium in the invest-in-productivity stage.
6 Conclusion and Further Discussion

In this paper, we develop a simple model of investment financing, where the entrepreneur shares the same characteristics as the representative entrepreneur in crowd-funding platforms. In particular, the entrepreneur has private information regarding her productivity, is protected by limited liability, and is associated with a negative-NPV project with positive probability.

The main message of this work is that, allowing the entrepreneur to offer contracts contingent on the choice of project implementation, leads to a better allocation of resources, prevents market breakdown and strengthens the entrepreneur’s incentive to invest in her productivity. Hence, this work indicates that crowd-funding platforms should promote the use of these securities.

The findings of this work can also be applied to the literature on venture capital financing. In venture capital financing, similar to our model, the entrepreneur is privately informed about the productivity of her project. A key difference between the two settings is that, in venture capital financing, it is the uninformed party (venture capitalist) who offers a security. This work suggests that, rewarding the non-implementation of a project, could prevent an entrepreneur from wasting the venture capitalist’s resources in negative NPV projects.

Besides, Section 5 suggests that, by rewarding the non-implementation of a project, the venture capitalists would be able to offer a steeper incentive pay, in case of implementation. In turn, the steeper incentive pay could motivate the entrepreneur to undertake costly, hidden actions to increase the probability of success.

Lastly, our work has implication for the literature on the compensation contracts of CEOs, and more specifically, on the literature which highlights the optimality of severance pay. An insight of our paper is that allowing for a severance pay could be optimal, since it can prevent a CEO from taking an action which results in inefficient allocation of resources.
References


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Appendix A: Proofs

A.1 Proof of Proposition 1

Conditional that the market survives, i.e., \( \lambda_G > \lambda_G^\text{min} = \frac{I - [p_B S + (1 - p_B) F]}{(p_G - p_B)(S - F)} \), the optimal security solves the following maximization problem.

Maximize \( p_G (S - g(S))' + (1 - p_G)(F - g(F))' \) s.t.

\[
\lambda_G [(p_G g(S))' + (1 - p_G)g(F)'] + \lambda_B [p_B g(s)'] + (1 - p_B)g(F)' \right) = I \tag{20}
\]

\[
0 \leq g(S)' \leq S \tag{21}
\]

\[
0 \leq g(F)' \leq F \tag{22}
\]

Optimality of \( g^*(F)' = F \)

The investors’ participation constraint can be written as:

\[
\left[ \frac{C_S}{\lambda_G p_G + \lambda_B p_B} g(S)' + \frac{C_F}{\lambda_G (1 - p_G) + \lambda_B (1 - p_B)} g(F)' \right] = I \tag{23}
\]

Suppose that the entrepreneur offers a security with corresponding payments \( g^*(F)' = F \) and \( g^*(S)' \) which solves (23) given that \( g^*(F)' = F \). Suppose now that the entrepreneur considers switching from \( g^*(F)' = F \) to \( g(F)'' = F - \epsilon \). For the investor’s participation constraint to be satisfied, the payment in case of success should be not lower than \( g(S)'' = g^*(S)' + \frac{C_F}{C_S} \times \epsilon \).

This deviation is profitable as long as its benefit, which is given by \( (1 - p_G) \times \epsilon \), exceeds its cost, which is given by \( p_G \times \frac{C_F}{C_S} \times \epsilon \). By simple algebra, we obtain that:

\[
(1 - p_G) \times \epsilon > p_G \times \frac{C_F}{C_S} \times \epsilon \implies \frac{\lambda_B (p_G - p_B)}{-\lambda_G p_G - \lambda_B p_B} > 0 \tag{24}
\]

where (24) is never satisfied because \( p_G > p_B \). Hence, under the optimal security \( g^*(F) \) reaches its maximum value, i.e. \( g^*(F) = F \). By substituting \( g^*(F)' = F \) into (20) and rearranging, we obtain:

\[
g^*(S)' = \frac{I - F}{\lambda_G p_G + \lambda_B p_B} + F
\]

Intuitive Criterion

Suppose the security which characterized in the previous analysis, \( g^* \), i.e., \( g^*(S) = \frac{I - F}{\lambda_G p_G + \lambda_B p_B} + F, g^*(F) = F \). Let us allow for an alternative security \( g' \), where its corresponding payments are denoted as \( g(S)' \) and \( g(F)' \). For security \( g^* not \) to be an equilibrium in the contracting game, it must be that there is a deviation to security \( g' \), which is preferred by a good type but
not by a bad type, i.e.,
\[ \mathbb{E} U[g'|t = G, \mu] > \mathbb{E} U[g^*|t = G, \mu = \lambda] \]
\[ \mathbb{E} U[g'|t = B, \mu] < \mathbb{E} U[g^*|t = B, \mu = \lambda] \]
which can be rearranged as:
\[ p_G(g^*(S) - g'(S)) + (1 - p_G)(g^*(F) - g'(F)) > 0 \] (25)
\[ p_B(g^*(S) - g'(S)) + (1 - p_B)(g^*(F) - g'(F)) < 0 \] (26)

Recall that \( p_G > p_B \). Thus, for (25) and (26) not to be mutually exclusive, it must be the case that \( g'(F) > g^*(F) \) and \( g'(S) < g^*(S) \). However, \( g'(F) \) cannot exceed \( g^*(F) \), since \( g^*(F) \) reaches its maximum feasible value, i.e., \( g^*(F) = F \).

### A.2 Proof of Corollary 1

Both types are better-off as long as \( g^*(S) < g^*(S)' \), i.e.
\[
\frac{I\lambda_G - F\lambda_G(1 - p_G) + \lambda_B p_B S}{\lambda_G p_G + \lambda_B p_B S} < \frac{I - F}{\lambda_G p_G + \lambda_B p_B} + F \quad \Rightarrow \\
I\lambda_G - F\lambda_G(1 - p_G) + \lambda_B p_B S < I - F\lambda_B - F\lambda_G + F\lambda_G p_G + F\lambda_B p_B \quad \Rightarrow \\
I\lambda_G + \lambda_B p_B S < I - F\lambda_B + F\lambda_B p_B \quad \Rightarrow \\
I\lambda_G + \lambda_B [p_B S + (1 - p_B)F] < I
\]
which holds since, by assumption, \([p_B S + (1 - p_B)F] < I\).

### A.3 Proof of Lemma 3 - Proposition 2

The optimal security solves the following maximization problem.

\[
\text{Maximize } p_G(S - g(S)) + (1 - p_G)(F - g(F)) \quad \text{s.t.} \\
p_G(S - g(S)) + (1 - p_G)(F - g(F)) \geq I - \bar{g} \quad (ICCG_G) \\
p_B(S - g(S)) + (1 - p_B)(F - g(F)) \leq I - \bar{g} \quad (ICCG_B) \\
\lambda_G[p_G g(S) + (1 - p_G)g(F)] + \lambda_B \bar{g} \geq I \quad (PC_1) \\
0 \leq g(S) \leq S \quad , \quad 0 \leq g(F) \leq F \quad , \quad 0 \leq \bar{g} \leq I \quad (LL)
\]

**Binding ICC_B**

First, we show that under the optimal security ICC_B binds. We do this by following the method
of contradiction. Suppose that the optimal security is given by \( g(S), g(F) \) and \( \bar{g} \), which satisfy \( PC_I, ICC_G \) and \( ICC_B \) is not binding, i.e.,

\[
p_B(S - g(S)) + (1 - p_B)(F - g(F)) < I - \bar{g}
\]

Now consider the following deviation. Suppose that we increase \( \bar{g} \) to \( \bar{g}'' = \bar{g} + \epsilon \). Then, the LHS of \( PC_I \) increases, thus, \( PC_I \) is still satisfied. Also, the RHS of \( ICC_G \) decreases, thus, \( ICC_G \) is still satisfied. This implies that, as long as \( \eta \leq \frac{\lambda_B}{\lambda_{GBP}} \), there is always a deviation where we decrease \( g(S) \) to \( g(S)' = g(S) - \eta \), which does violate \( ICC_G \) or \( PC_I \). Such deviation would be profitable, because it would increase the expected utility of the good type. Hence, the initial hypothesis that under the optimal security \( ICC_B \) is binding does not hold.

**Optimality of** \( g^*(F) = F \)

Suppose for now that \( ICC_G \) is slack (we will come back to this in the end of the proof). Given that \( ICC_B \) binds, by substituting \( ICC_B \) into \( PC_I \) and solving with respect to \( g(S) \), we obtain:

\[
g^*(S) = \frac{\lambda_B(-Fp_B + F + p_BS) + \lambda_G I}{\lambda_{GBP} + \lambda_{BPB}} - \frac{\lambda_G(1 - p_G) + \lambda_B(1 - p_B)}{\lambda_{GBP} + \lambda_{BPB}} g^*(F) \quad (27)
\]

Condition \((27)\) implies that the entrepreneur could increase \( g(F) \) by \( \epsilon \) and decrease \( g(S) \) by \( R \times \epsilon \), without violating \( PC_I \). Increasing \( g(F) \) by \( \epsilon \) decreases the entrepreneur’s expected utility by \((1 - p_G) \times \epsilon \), whereas decreasing \( g(S) \) by \( R \times \epsilon \) increases entrepreneur’s expected utility by \( p_G \times R \times \epsilon \). By simple algebra, we obtain that:

\[
p_G \times R \times \epsilon > (1 - p_G) \times \epsilon \implies \frac{(\lambda_B)(p_G - p_B)}{\lambda_{GBP} + \lambda_{BPB}} \geq 0 \quad (28)
\]

where \((28)\) is always satisfied, given that \( p_G > p_B \). Hence, under the optimal security \( g^*(F) \) reaches its maximum value, i.e. \( g^*(F) = F \). Substituting \( g^*(F) = F \) into \( PC_I \) we obtain:

\[
g^*(S) = \frac{\lambda_G(I - F(1 - p_G)) + \lambda_{BPB}S}{\lambda_{GBP} + \lambda_{BPB}}
\]

Also, by substituting \( g^*(F) = F \) into \( ICC_B \) we obtain:

\[
\bar{g}^* = I - p_B(S - g^*(S))
\]

The last part of the proof is to explore whether \( ICC_G \) is slack under the optimal security. By
substituting $g^*(F) = F$ into $ICC_G$, we obtain:

$$p_G(S - g^*(S)) \geq I - \bar{g}^*$$

Recall that under the optimal contract $ICC_B$ binds, i.e.,

$$p_B(S - g^*(S)) = I - \bar{g}^*$$

Hence, given that $p_G > p_B$, under the optimal contract $ICC_G$ is redundant by $ICC_B$.

**Intuitive Criterion**

Suppose the security which characterized in the previous analysis, $g^*$, i.e., $g^*(S) = \frac{\lambda_G(I - F(1 - p_G)) + \lambda_{BP} S}{\lambda_G + \lambda_{BP}}$, $g^*(F) = F$, and $\bar{g}^* = I - p_B(S - g^*(S))$. Let us allow for an alternative security $g'$, where its corresponding payments are denoted as $g(S)'$, $g(F)'$ and $\bar{g}'$. For the security $g^*$ not to be an equilibrium in the contracting game, it must be that there is a deviation to security $g'$ which, is preferred by a good type but not by a bad type, i.e.,

$$\mathbb{E}U[g'|t = G, \mu] > \mathbb{E}U[g^*|t = G, \mu = \lambda]$$  \hspace{1cm} (29)
$$\mathbb{E}U[g'|t = B, \mu] < \mathbb{E}U[g^*|t = B, \mu = \lambda]$$  \hspace{1cm} (30)

which can be rearranged as:

$$p_G(g^*(S) - g'(S)) + (1 - p_G)(g^*(F) - g'(F)) > 0$$  \hspace{1cm} (31)
$$p_B(g^*(S) - g'(S)) + (1 - p_B)(g^*(F) - g'(F)) < 0$$  \hspace{1cm} (32)

Recall that $p_G > p_B$. Thus, for (31) and (32) not to be mutually exclusive, it must be the case that $g'(F) > g^*(F)$ and $g'(S) < g^*(S)$. However, $g'(F)$ cannot exceed $g^*(F)$ since $g^*(F)$ reaches its maximum feasible value, i.e., $g^*(F) = F$.

**A.4 Proof of Lemma 5**

Recall that if an interior equilibrium exists, it must satisfy the following condition:

$$\Phi^{-1}(\tilde{\lambda}_G) = (p_G - p_B)(S - g(S, \tilde{\lambda}_G))$$  \hspace{1cm} (33)

where $\tilde{\lambda}_G = \Phi(\hat{c})$.

If there is no value of $c' \in [\underline{c}, \bar{c}]$ for which the expected benefit, $((p_G - p_B)(S - g(S, \Phi(c'))))$, exceeds $c'$, then, there is no interior equilibrium. In this case, the entrepreneur never invests in her productivity level, and the market collapses. This case is captured in Panel A of Figure 3.
Figure 3: Cases where there is no interior equilibrium.

Suppose now that there is \( \hat{c} \in [\underline{c}, \bar{c}] \) which satisfies (33). For \( \hat{c} \) to be an interior equilibrium, investors’ beliefs should be consistent, i.e., for cost \( c' \in [\underline{c}, \bar{c}] \) the expected benefit always exceeds \( c' \), whereas for cost \( c'' \in (\hat{c}, \bar{c}] \), the expected benefit is always below \( c'' \). This necessary condition implies that if the expected benefit \( (p_G - p_B)(S - g^*(S, \Phi(\bar{c}))) \) exceeds \( \bar{c} \), then the entrepreneur always invests in her productivity. Hence, the unique equilibrium in this case is for \( \tilde{\lambda}_G = 1 \). For instance, point \( A \) in Panel B of Figure 3 can not be an interior equilibrium, because there is deviation to \( \tilde{\lambda}_G = 1 \), where the beliefs are consistent and the entrepreneur is better-off.

A.5 Proof of Proposition 4

Lemma 6 provides the necessary and sufficient conditions for an interior equilibrium to exist, for the regime where the entrepreneur is obliged to the project implementation. The payment \( g^*(S, \tilde{\lambda}_G)' \) is determined in Lemma 7.

**Lemma 6: Interior equilibrium - sufficient conditions**

- If there is no \( c \in [\underline{c}, \bar{c}] \) which satisfies:

\[
c \leq (p_G - p_B)(S - g^*(S, \Phi(c)))'
\]

(34)

then, the unique equilibrium is for \( \tilde{\lambda}_G = 0 \), for which the market collapses.

- If \( \bar{c} \) satisfies:

\[
\bar{c} > (p_G - p_B)(S - g^*(S, \Phi(\bar{c})))'
\]

(35)

then, there is a unique equilibrium is for \( \tilde{\lambda}_G = 1 \).
If (35) is violated, and there exists \( c \in [\bar{c}, \tilde{c}] \) which satisfies (36), then, the unique interior equilibrium is characterized by the maximum \( \tilde{\lambda}_G \in (0, 1) \) which solves:

\[
\Phi^{-1}(\tilde{\lambda}_G) = (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G)')
\] (36)

Given Proposition 1, the optimal security when the entrepreneur is obliged to project implementation, given \( \tilde{\lambda}_G \), is captured in Lemma 7.

**Lemma 7**

\[
g^*(F)' = F, \quad g^*(S)' = \frac{I - F}{\lambda_{GPG} + \lambda_{BPP}} + F
\]

In this regime, the entrepreneur invest in her productivity as long as:

\[
\begin{aligned}
c \leq \left( p_G - p_B \right) (S - g^*(S, \tilde{\lambda}_G)') & \equiv \hat{c}
\end{aligned}
\] (37)

The methodology that we follow in order to characterize the equilibrium conditions is identical with the methodology we followed in Lemma 5.

The proof of Lemma 6 is identical to the proof of Lemma 4. The main difference is that the payment in case of success is defined by Lemma 7 rather than Lemma 6.

### A.6 Proof of Proposition 5

Recall that for a interior equilibrium to exist, the solid curve, which depicts \( \Phi^{-1}(\tilde{\lambda}_G) \), should cross the expected benefit curve from below. Recall also that \( \Phi^{-1}(\tilde{\lambda}_G) \) is continuous and increasing in \( \tilde{\lambda}_G \). Also, by Corollary 1, and for any given value of \( \lambda_G \), it holds:

\[
g^*(S, \tilde{\lambda}_G)' \geq g^*(S, \tilde{\lambda}_G)
\]

Hence, for any given value of \( \bar{\lambda}_G \in [\bar{\lambda}_G^{\text{min}}, 1] \), for the expected benefit of investing in productivity, it holds:

\[
(p_G - p_B)(S - g^*(S, \bar{\lambda}_G)) \leq (p_G - p_B)(S - g^*(S, \tilde{\lambda}_G))
\]

Thus, the dashed curve is always above the dotted curve. As a result, the intersection of the solid curve with the dashed curve always corresponds to a higher value of \( \tilde{\lambda}_G \) than the one which corresponds to the intersection of the solid curve with the dotted curve.

Note that for \( \tilde{\lambda}_G \in [0, \tilde{\lambda}_G^{\text{min}}) \), the market collapses if the entrepreneur is obliged to project implementation. This corresponds to \( \tilde{\lambda}_G^* = 0 \).