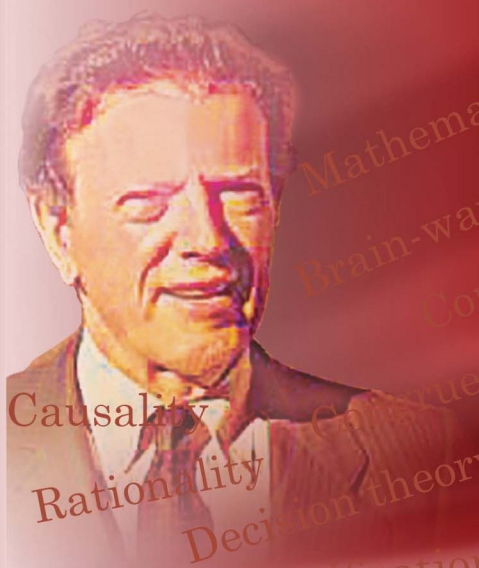
Tributes 28

Conceptual Clarifications

Tributes to Patrick Suppes (1922–2014)



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Tributes

Volume 28

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Tributes to Patrick Suppes (1922-2014)

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Conceptual Clarifications, Tributes to Patrick Suppes (1922 - 2014)
A Preface

JEAN-YVES BÉZIAU, DÉCIO KRAUSE AND JONAS ARENHART

This is a collection of papers dedicated to the memory of Patrick Colonel Suppes (1922-2014) by people who have been closely connected with him and his work. It was first thought of as a kind of natural follow up of a special issue of *Synthese* (Volume 154, Issue 3, February 2007) edited by the first two editors of this book commemorating the 80th birthday of Pat Suppes. The title of this issue was *New Trends in the Foundations of Science*. The subtitle of the present volume reflects — in some measure — the situation in which it must be presented to the public, given Suppes' passing away during its production.

So, with this laudatory intention in mind, the whole volume was first thought of as a Tribute to Suppes due to the occasion of his 90th anniversary. We shall not speculate on the reasons why people think that anniversaries should be commemorated with a volume — when they are commemorated at all — from ten to ten years (why don't we find a homage to someone's 87th birthday?). The first plan, anyway, was that the volume should be a well deserved homage to Suppes, commemorating his long life of productive interaction, influence, and direct collaboration with a great variety of researchers. This did not mean that the homage took it as a fact that Suppes' work was finished, or that Suppes was only an influent philosopher of the past that had nothing else to say: he was still active and developing influential new ideas in many fields. In fact, one of the papers in this collection is co-authored by Suppes himself. So, the idea was clearly not merely to praise someone, but to continue an ongoing debate on some of the issues discussed in the work of Suppes himself.

The work of Suppes touches many different areas, ranging from meteorology to physics, through logic, mathematics, psychology, neuroscience, education, painting, but he was first of all and/or above all a philosopher, always questioning, but not in vain. Part of Suppes' research in the foundations of science culminates in his book *Representation and invariance of scientific structures* (CSLI, Stanford, 2002). This

book is a synthesis showing clearly the relations between all the topics he has investigated. There are not many philosophers who can be proud of having written influential math textbooks, contributed decisively to the philosophical foundations of science, helped to develop research in real labs, and much more. Suppes is such a singular figure. Not only did he think about some of those important issues, but also helped to bring some of them about, as in the particular case of computer-assisted learning; perhaps, that is the aspect of his work for which he will be reminded by most of the public out of philosophical circles (again, not many philosophers can be proud of that too).

Since the range of interest of Suppes was very broad, so is the variety of topics dealt with in this volume. In fact, the work of a researcher is certainly not limited to his own writings, but has to be appreciated also through the work of the people he has been working with and influenced. From this point of view the work of Suppes is very impressive, and the present book contributes to show that. This feature —wide range of interest and great competence to deal with such interests properly — appears very clearly in this collection; on the one hand the volume clearly bears the sign of Suppes' influence and wide ranging interests as a scholar: most authors have had direct contact and have felt the need to discuss Suppes ideas; the themes are varied and their main thread is. . ., well, their relation, direct or indirect, with the work of Suppes. On the other hand, with such a long life span and such a great variety of interests, the volume should certainly contain a broad spectrum of themes that may seem at first to lack a unifying thread. In fact, as readers may notice, it is not possible to contemplate — not even in a whole volume — the whole spectrum of areas of interest featuring in the works of Suppes. This volume illustrates this: hardly two papers deal with the same subject. Suppes was such an apt thinker as to see unity in many disparate areas.

Most of the authors of the papers collected here had, at first, the expectation that their contribution would be indeed a further step in their fruitful interaction with Suppes; most of the papers develop themes that were among Suppes' areas of direct research, in the present or in some moment of his life. Anyway, the celebratory tone of the volume was not meant to obscure the fact that another round of stimulating intelectual dialog with Suppes himself was expected. It is not everyday that we meet someone with such a great expertise in so many areas of intelectual investigation, so, the opportunity should not be missed by anyone.

However, as we have mentioned, during the preparation of the volume Suppes passed away. This happening made the present volume

into a posthumous homage celebrating his work and influence. The fact that it is now a posthumous homage does not change that Suppes' contribution is a lively one; the debate should go on following the steps of Suppes' fruitful contributions. We are proud of having a paper by Suppes himself and his collaborators in our volume. Not only was he a philosopher deserving to be praised, but was also a philosopher in the scientific sense of the word. For those investigating the nature of science there is no stoping point, science always presents another challenge and the philosopher must be always on guard. Suppes did that in an exemplary fashion, contributing to both science and philosophy.

The editors would like the take the opportunity to thank all the authors that contributed to this volume. The quality of the papers here gathered certainly are a fair tribute to the greatness of the philosopher Suppes was. We would also like to thank the authors for their immense patience it took for the production of the volume. As we know, academic pressures and other contingencies sometimes provide for unwanted and unexpected delays. However, at last, it is ready and out for the public judgement. May the volume meet its public and help to enlighten the themes here touched on, themes that were dear to Suppes, and it will have reached its aim.

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Reconditioning the Conditional

DAVID MILLER

ABSTRACT. Many authors have hoped to understand the indicative conditional construction in everyday language by means of what are usually called conditional probabilities. Other authors have hoped to make sense of conditional probabilities in terms of the absolute probabilities of conditional statements. Although all such hopes were disappointed by the triviality theorems of [15], there have been copious subsequent attempts both to rescue CCCP (*the conditional construal of conditional probability*) and to extend and to intensify the arguments against it. In this paper it will be shown that triviality is avoidable if the probability function is replaced by an alternative generalization of the deducibility relation, the measure of *deductive dependence* of [19]. It will be suggested further that this alternative way of orchestrating conditionals is nicely in harmony with the test proposed in [29], and also with the idea that it is not the truth value of a conditional statement that is of primary concern but its assertability or acceptability.

0 A Critical Memorial to Patrick Suppes

Twenty years ago Karl Popper and I marked Patrick Suppes's 70th birthday with a technical paper [28] that was quite in sympathy with his view of probability as 'perhaps the single most important concept in the philosophy of science' ([35], p. 14). The present tribute, however, though written in gratitude and appreciation, respectfully breaks step. In open disagreement with Suppes's thesis that '[t]he theory of rationality is intrinsically probabilistic in character' ([36], p. 10), I shall sketch, and illustrate the fertility of, a fundamentally non-probabilistic way in which deductive dereliction can be accommodated in the theory of rationality. In short, I shall take exception, not to Suppes's *probabilistic metaphysics*, his view, with which I largely agree, that '[t]he

 The main idea of this paper (that some indicative conditionals are better understood in terms of deductive dependence than in terms of probability) was mentioned during my presentation 'On Deductive Dependence' at the meeting UNCERTAINTY: REASONING ABOUT PROBABILITY AND VAGUENESS held at the Academy of Sciences of the Czech Republic in September 2006. The details were worked out during a visit to the University of Sassari in the spring of 2013. Warm thanks are due to my Sardinian audience, and also to Alan Hájek and Richard Bradley, who commented on an earlier version of the paper.

fundamental laws of natural phenomena are essentially probabilistic rather than deterministic in character' (*ibidem*), but to his *probabilistic epistemology*. Rejection of a probabilistic approach to rationality is of course to be expected of an adherent of deductivism ([22]). I hold, indeed, that the speculative character of our knowledge can be neither palliated nor controlled by the introduction of probabilities, although its worth may be augmented by sustained criticism. In this paper, however, the thesis to be advanced is a less radical one: that the proposed relaxation of deductive austerity better ministers to the purposes of traditional justificationist epistemology than does an approach that uses probabilities in its management. Whether rationality in any way involves justification will not be examined here.

1 Degrees of Deducibility

Since the time of [5], if not earlier, it has been appreciated that, when $\mathfrak{p}$ is a probability measure, the identity $\mathfrak{p}(\mathfrak{c} \mid \mathfrak{a}) = 1$ is a necessary, but generally insufficient, condition for the deducibility in classical logic of the conclusion $\mathfrak{c}$ from the assumption(s) $\mathfrak{a}$. What has been less often recognized is that there are other legitimate ways in which *degrees of deducibility* may be measured. In particular, since $\mathfrak{c}$ is deducible from $\mathfrak{a}$ if and only if $\mathfrak{a}'$ is deducible from $\mathfrak{c}'$ (here the prime stands for negation), the identity $\mathfrak{p}(\mathfrak{a}' \mid \mathfrak{c}') = 1$, which is not equivalent to $\mathfrak{p}(\mathfrak{c} \mid \mathfrak{a}) = 1$, also gives a necessary condition for the deducibility of $\mathfrak{c}$ from $\mathfrak{a}$. There are a number of other interesting possibilities, which I shall elaborate on elsewhere, but they are not the concern of this paper.

A few historical remarks about the function $\mathfrak{q}(\mathfrak{c} \mid \mathfrak{a}) = \mathfrak{p}(\mathfrak{a}' \mid \mathfrak{c}')$ are offered in § 8 below. Following [19], § 1, we shall call $\mathfrak{q}(\mathfrak{c} \mid \mathfrak{a})$ the *(degree of) deductive dependence* of the statement $\mathfrak{c}$ on the statement $\mathfrak{a}$, where $\mathfrak{c}$ is typically the conclusion of an inference from the assumption(s) or premise(s) $\mathfrak{a}$. Although, as just noted, $\mathfrak{q}(\mathfrak{c} \mid \mathfrak{a})$, like $\mathfrak{p}(\mathfrak{c} \mid \mathfrak{a})$, equals 1 when $\mathfrak{c}$ is deducible from $\mathfrak{a}$, the two functions take the value 0 in different circumstances. Whereas $\mathfrak{p}(\mathfrak{c} \mid \mathfrak{a}) = 0$ when $\mathfrak{c}'$ is deducible from $\mathfrak{a}$, that is, when $\mathfrak{a}$ and $\mathfrak{c}$ are mutual *contraries*, $\mathfrak{q}(\mathfrak{c} \mid \mathfrak{a}) = 0$ when $\mathfrak{c}$ is deducible from $\mathfrak{a}'$, that is, when $\mathfrak{a}$ and $\mathfrak{c}$ are mutual *subcontraries*. In other words, $\mathfrak{q}(\mathfrak{c} \mid \mathfrak{a})$ assumes the value 1 when $\mathfrak{c}$ is deductively wholly dependent on $\mathfrak{a}$, in the sense of being deducible from $\mathfrak{a}$, and the value 0 when $\mathfrak{c}$ is deductively wholly independent of $\mathfrak{a}$, in the sense of having only tautological consequences in common with $\mathfrak{a}$. (This relation of deductive independence is closely related to *maximal independence*, as defined by [32].) The interpretation of the function $\mathfrak{q}$ as a measure of deductive dependence is encouraged by the fact that, if the familiar function $1 - \mathfrak{p}(\mathfrak{b}) = \mathfrak{p}(\mathfrak{b}')$ is adopted as a measure of the (informative) *content* $\mathfrak{ct}(\mathfrak{b})$ of the statement $\mathfrak{b}$, and if $\mathfrak{ct}(\mathfrak{c}) \neq 0$, then $\mathfrak{q}(\mathfrak{c} \mid \mathfrak{a})$ is equal to $\mathfrak{ct}(\mathfrak{c} \vee \mathfrak{a}) / \mathfrak{ct}(\mathfrak{c})$, the 'proportion' of the content of $\mathfrak{c}$ that resides within the content of $\mathfrak{a}$ ([13], p. 110; [19], *ibidem*; [17], Chapter 10.4*c*).

Although the deductive dependence function $\mathfrak{q}$ has been defined above in terms of the probability function $\mathfrak{p}$, this is not supposed to attribute to $\mathfrak{p}$ any conceptual priority. A more correct treatment would begin with an abstract measure $\mathfrak{m}$, and define each of $\mathfrak{p}$ and $\mathfrak{q}$ from $\mathfrak{m}$. But we forgo such niceties here.

2 Formalities

The function $\mathfrak{p}$ is required to satisfy the axiom system of [24], appendix $*$v, which is based on the operations of negation $'$ and conjunction (inconspicuously represented by concatenation). A dual axiomatic system for the function $\mathfrak{q}$, based on the operations $'$ and $\vee$, is presented in [19], § 2. In these systems the terms $\mathfrak{p}(\mathbf{c} \mid \mathbf{a})$ and $\mathfrak{q}(\mathbf{c} \mid \mathbf{a})$ are well defined for every $\mathbf{a}, \mathbf{c}$, including the contradiction $\perp$ and the tautology $\top$. Indeed, $\mathfrak{p}(c \mid \perp) = 1 = \mathfrak{q}(\top \mid a)$ for every a and c. The usual addition or complementation law therefore fails in general, since $\mathfrak{p}(c \mid \perp) + \mathfrak{p}(c' \mid \perp) = 2$. But it holds when the second argument of $\mathfrak{p}$ is not the contradiction $\perp$. Other theorems of the systems will be cited, without much proof, when they are needed. In interpreting Popper's system it is safe to restrict attention to functions $\mathfrak{p}$ for which $\forall \mathbf{b}\, \mathfrak{p}(\mathbf{c} \mid \mathbf{b}) \geq \mathfrak{p}(\mathbf{a} \mid \mathbf{b})$ if and only if $\mathbf{c}$ is deducible from $\mathbf{a}$. (Since $\mathbf{c}$ is deducible from $\mathbf{a}$ if and only if $\mathbf{a}'$ is deducible from $\mathbf{c}'$, the deducibility of $\mathbf{c}$ from $\mathbf{a}$ can evidently be characterized also by $\forall \mathbf{b}\, \mathfrak{q}(\mathbf{b} \mid \mathbf{c}) \leq \mathfrak{q}(\mathbf{b} \mid \mathbf{a})$.) It follows that $\mathbf{a}$ and $\mathbf{c}$ are interdeducible if and only if they are *probabilistically indistinguishable*: that is, $\forall \mathbf{b}\, \mathfrak{p}(\mathbf{c} \mid \mathbf{b}) = \mathfrak{p}(\mathbf{a} \mid \mathbf{b})$. It should be recorded also that, although $\mathfrak{p}(\mathbf{c} \mid \mathbf{a}) = 1$ is in general insufficient for $\mathbf{c}$ to be deducible from $\mathbf{a}$, the formula $\forall \mathbf{b}\, \mathfrak{p}(\mathbf{c} \mid \mathbf{ab}) = 1$ (whose equivalence to the formula $\forall \mathbf{b}\, \mathfrak{p}(\mathbf{c} \mid \mathbf{b}) \geq \mathfrak{p}(\mathbf{a} \mid \mathbf{b})$ is easily demonstrated within Popper's system[1]) is both necessary and sufficient for deducibility, as is the formula $\forall \mathbf{b}\, \mathfrak{p}(\mathbf{a}' \mid \mathbf{c}'\mathbf{b}) = 1$. In other words, $\mathbf{c}$ is deducible from $\mathbf{a}$ if and only if $\forall \mathbf{b}\, \mathfrak{q}(\mathbf{b} \to \mathbf{c} \mid \mathbf{a}) = 1$, where the arrow $\to$ represents the material conditional.

3 Conditionals

The appearance here of the material conditional $\mathbf{b} \to \mathbf{c}$ in the first argument of $\mathfrak{q}$ may quicken the hope that the substitution of the function $\mathfrak{q}$ for the probability function $\mathfrak{p}$ can in some way shed light on the *problem of indicative conditionals*, one of the most tenaciously unsolved problems of modern philosophical logic, and especially on the hypothesis of the *conditional construal of conditional probability* (facetiously dubbed CCCP by [11]). It is the objective of this paper substantially to consummate this hope. But it should be said at once that the matter is not entirely straightforward. Pretty well the simplest form of the CCCP hypothesis worth attending to may be written as the universal identity $\forall \mathbf{a} \forall \mathbf{c} \forall \mathbf{b}\, \mathfrak{p}(\mathbf{a} \rightsquigarrow \mathbf{c} \mid \mathbf{b}) = \mathfrak{p}(\mathbf{c} \mid \mathbf{ab})$, according to which the absolute probability of the indicative conditional *if* $\mathbf{a}$ *then* $\mathbf{c}$ in ordinary language, here shortened to $\mathbf{a} \rightsquigarrow \mathbf{c}$, is equal to the conditional probability of $\mathbf{c}$ given $\mathbf{a}$, not only under the measure $\mathfrak{p}$ but under any measure obtained from $\mathfrak{p}$ by *conditionalization* on the statement $\mathbf{b}$. We shall see below that this form of the CCCP hypothesis can hold only for the material conditional $\to$, and that when it does hold, the function $\mathfrak{p}$ is necessarily two-valued, and no more than a distribution of truth values ([14]). But the identity $\forall \mathbf{a} \forall \mathbf{c} \forall \mathbf{b}\, \mathfrak{q}(\mathbf{a} \to \mathbf{c} \mid \mathbf{b}) = \mathfrak{q}(\mathbf{c} \mid \mathbf{ab})$, its analogue in

[1] If $\mathfrak{p}(\mathbf{c} \mid \mathbf{b}) \geq \mathfrak{p}(\mathbf{a} \mid \mathbf{b})$ for every $\mathbf{b}$, then $\mathfrak{p}(\mathbf{c} \mid \mathbf{ab}) \geq \mathfrak{p}(\mathbf{a} \mid \mathbf{ab})$. The latter term equals 1, which is the upper bound of the function $\mathfrak{p}$. It follows that $\mathfrak{p}(\mathbf{c} \mid \mathbf{ab}) = 1$. For the converse we may note that, if $\mathfrak{p}(\mathbf{c} \mid \mathbf{ab}) = 1$ for every $\mathbf{b}$, then, by the monotony law for the first argument of $\mathfrak{p}$ and the general multiplication law, $\mathfrak{p}(\mathbf{c} \mid \mathbf{b}) \geq \mathfrak{p}(\mathbf{ca} \mid \mathbf{b}) = \mathfrak{p}(\mathbf{c} \mid \mathbf{ab})\mathfrak{p}(\mathbf{a} \mid \mathbf{b}) = \mathfrak{p}(\mathbf{a} \mid \mathbf{b})$ for every $\mathbf{b}$.

terms of deductive dependence, may be shown to be equivalent to the CCCP hypothesis, and so to force q to be two-valued too.[2] Moving from p to q in this way does little to avoid triviality.

This result notwithstanding, it is the material conditional $a \to c$ that will be rehabilitated, in § 6 below, in terms of the deductive dependence function q.

A great deal has been written on various versions of the CCCP hypothesis and, in particular, on the crucial results of [15] that show that, in the usual Kolmogorov axiomatizations of probability, the hypothesis is condemned in one way or another to triviality. In § 4 below it will be shown that, within Popper's axiom system, the triviality of the CCCP hypothesis follows from a result in [25] that is closely related to the theorems of [27]. I shall not discuss directly the implosion of the CCCP hypothesis in Kolmogorov's systems. Nor shall I attempt to summarize the many extensions to Lewis's results and the many responses that have been made to them. For a useful (if dated) discussion, the reader may consult [11], and other papers in the same volume [9], including [31]; and for surveys of the principal philosophical and technical problems posed by conditionals, [8], [4], and the works cited therein. Mention should be made also of [21], which deepens and corrects the theory of tri-events propounded in [6].

4 Triviality of the CCCP hypothesis

In order visibly not to prejudge the question of whether the connective $\rightsquigarrow$ introduced above is or is not worthy of the title of an indicative conditional, in this section we shall state the CCCP hypothesis in the ostensibly weaker form

$$\text{CCCP}_0 \qquad \forall a \forall c \exists y \forall b \; p(y \mid b) = p(c \mid ab).$$

We shall show that within Popper's axiomatic system this universal hypothesis implies that for each a, c, the object y can only be the material conditional $a \to c$ and, furthermore, that the values of the function p can only be 0 and 1.

We assume that b is not the contradiction $\perp$. Using a version of the addition law, then the multiplication law, and finally CCCP_0 twice, we may then derive

$$\begin{aligned} p(ya' \mid b) &= p(y \mid b) - p(ya \mid b) \\ &= p(y \mid b) - p(y \mid ab)p(a \mid b) \\ &= p(c \mid ab) - p(c \mid a(ab))p(a \mid b) \\ &= p(c \mid ab)(1 - p(a \mid b)). \end{aligned}$$

[2] By the definition of q, the identities $q(a \to c \mid b) = q(c \mid ab)$ and $p(b' \mid ac') = p(a' \vee b' \mid c')$ are equivalent. The hypothesis in question therefore holds if and only if $\forall a \forall c \forall b \; p(b' \mid ac') = p(a' \vee b' \mid c')$. By simultaneously replacing in this expression a by b, b by c', and c by a', suppressing the double negations that materialize, and massaging the quantifiers, we obtain $\forall a \forall c \forall b \; p(c \mid ba) = p(b' \vee c \mid a)$. By interchanging a and b, and writing $a \to c$ for $a' \vee c$, we reach $\forall a \forall c \forall b \; p(c \mid ab) = p(a \to c \mid b)$, and finally the CCCP hypothesis for $\to$, as announced.

Using the multiplication law, CCCP$_0$, and the law $\mathfrak{p}(c \mid \bot) = 1$ we may derive

$$\begin{aligned}\mathfrak{p}(ya' \mid b) &= \mathfrak{p}(y \mid a'b)\mathfrak{p}(a' \mid b)\\ &= \mathfrak{p}(c \mid a(a'b))\mathfrak{p}(a' \mid b)\\ &= 1 - \mathfrak{p}(a \mid b),\end{aligned}$$

by a second use of the addition law (which is valid here since b is not $\bot$). It follows that if $b \not\equiv \bot$ then $\mathfrak{p}(c \mid ab)(1 - \mathfrak{p}(a \mid b)) = 1 - \mathfrak{p}(a \mid b)$ for all a, c, and hence that $(1 - \mathfrak{p}(c \mid ab))(1 - \mathfrak{p}(a \mid b)) = 0$ for all a, c. Now formula (22) in Addendum 3 of [25] states without proof (and in different notation) that $(1 - \mathfrak{p}(c \mid a))(1 - \mathfrak{p}(a))$ is equal to the value of the arithmetical difference between the probability $\mathfrak{p}(a \to c)$ and the probability $\mathfrak{p}(c \mid a)$. It may be shown more generally that $\mathfrak{p}(a \to c \mid b) - \mathfrak{p}(c \mid ab) = (1 - \mathfrak{p}(c \mid ab))(1 - \mathfrak{p}(a \mid b))$ when $ab \not\equiv \bot$,[3] which implies that $\mathfrak{p}(a \to c \mid b) - \mathfrak{p}(c \mid ab) = 0$ when $ab \not\equiv \bot$. But $ab \equiv \bot$ implies the deducibility of $a \to c$ from b, and hence that $\mathfrak{p}(a \to c \mid b) = 1 = \mathfrak{p}(c \mid ab)$. We conclude that $\mathfrak{p}(a \to c \mid b) - \mathfrak{p}(c \mid ab) = 0$ for every a, b, c.

It follows from CCCP$_0$ above that for all a, c, there exists a statement y such that $\mathfrak{p}(a \to c \mid b) - \mathfrak{p}(y \mid b) = 0$ holds for all b. What this means is that the statement y is probabilistically indistinguishable from the material conditional $a \to c$, in the sense of § 2 above, and thus interdeducible with it. The equation $\mathfrak{p}(y \mid b) = \mathfrak{p}(c \mid ab)$ can hold for every b if and only if y is the statement $a \to c$.

To show that the function $\mathfrak{p}(c \mid a)$ can take only the values 0 and 1, we may set aside the case of inconsistent a (since $\mathfrak{p}(c \mid \bot)$ always equals 1). We have proved above that if $b \not\equiv \bot$ then $(1 - \mathfrak{p}(c \mid ab))(1 - \mathfrak{p}(a \mid b)) = 0$, from which it follows that if $\mathfrak{p}(a \mid b) \neq 1$ then $\mathfrak{p}(c \mid ab) = 1$ for every c. In particular, $\mathfrak{p}(a' \mid ab) = 1$. But $\mathfrak{p}(a'a \mid b) = 0$ if $b \not\equiv \bot$, and so by the multiplication law, $\mathfrak{p}(a' \mid ab)\mathfrak{p}(a \mid b) = 0$. It may be concluded that if $\mathfrak{p}(a \mid b) \neq 1$ then $\mathfrak{p}(a \mid b) = 0$.

What is so damaging about these results is not that the only conditional conforming to the CCCP hypothesis is the familiar material conditional, for several authors have held that indicative conditionals are, in their semantics, material conditionals, but that all probabilities have to be either 0 or 1. There is nothing but disappointment for the hope that since 'the abstract calculus [of probability] is a relatively well defined and well established mathematical theory ... [and i]n contrast, there is little agreement about the logic of conditional sentences ... [p]robability theory could be a source of insight into [their] formal structure' ([34], p. 64). Indeed, the recourse to probability is otiose, since a two-valued probability function is no more than an assignment of truth values: we may define b to be *true* if $\mathfrak{p}(b \mid \top) = 1$, and *false* if $\mathfrak{p}(b \mid \top) = 0$. Matters are actually worse than this, for all true statements turn out to be probabilistically indistinguishable from $\top$, and all false statements probabilistically

[3]The right-hand side of the equation, $(1 - \mathfrak{p}(c \mid ab))(1 - \mathfrak{p}(a \mid b))$, can be expanded, and by the multiplication law shown equal to $1 - \mathfrak{p}(c \mid ab) - \mathfrak{p}(a \mid b) + \mathfrak{p}(ac \mid b)$. By two applications of the addition law, this can be shown equal to $1 - \mathfrak{p}(c \mid ab) - \mathfrak{p}(ac' \mid b) = \mathfrak{p}(a \to c \mid b) - \mathfrak{p}(c \mid ab)$.

indistinguishable from $\bot$. This belies the assumption of § 2 that probabilistic indistinguishability ought to coincide with interdeducibility.[4]

The first proof that, in Popper's system, CCCP_0 implies the two-valuedness of $\mathfrak{p}$ was given by [14]. The present proof dates from about 1992. The Basic Triviality Result of [20], pp. 301f., which is derivable in Kolmogorov's less general (finite) system, is related but less general.

5 Updating and Relativization

One of the factors that has made the CCCP hypothesis attractive is surely the multiple uses of the word *conditional* and its cognates. As [11] put it, the hypothesis 'sounds right' (p. 80). What is not always realized, however, is that, aside from the word *conditional* in logic, here endorsed, there are two distinct uses of the words in probability theory. There is the process of (Bayesian) *conditionalization*, the generally agreed way in which a probability distribution is updated on the receipt of new information or new knowledge. There is also the result of applying the probability functor $\mathfrak{p}$ not to a single argument (in the present paper, a statement) but to two arguments, or to one statement relative to another, yielding a binary measure $\mathfrak{p}(\mathfrak{c} \mid \mathfrak{a})$ that is standardly called *conditional probability*. These processes of *updating* and *relativization*, as they will hereafter be called, happen to have the same mathematical effect: the result of updating the singulary measure $\mathfrak{p}$ with the information $\mathfrak{b}$ is the same as relativizing it to $\mathfrak{b}$. It follows that updating $\mathfrak{p}(\mathfrak{c})$ with $\mathfrak{b}$, and then relativizing it to $\mathfrak{a}$, is the same as relativizing $\mathfrak{p}(\mathfrak{c})$ to $\mathfrak{a}$, and then updating it with $\mathfrak{b}$. Since conjunction in the second argument of $\mathfrak{p}$ is commutative, the outcomes $\mathfrak{p}(\mathfrak{c} \mid \mathfrak{b}\mathfrak{a})$ and $\mathfrak{p}(\mathfrak{c} \mid \mathfrak{a}\mathfrak{b})$ are identical. Although relativization and updating are therefore formally dead ringers for each other, they deserve to be understood as distinct undertakings. In particular, if $\mathfrak{p}(\mathfrak{c} \mid \mathfrak{a}) = r$ is a declaration of relative probability there is no presumption that the statement $\mathfrak{a}$ is known to be true, or even supposed to be true ([37], § 2), any more than this is the case in the metalogical declaration $\mathfrak{a} \vdash \mathfrak{c}$. (But the interpretation of $\mathfrak{a}$ as a statement of evidence, and of $\mathfrak{c}$ as a hypothesis, is not excluded.) This is not idle pedantry. With the function $\mathfrak{q}$, the distinction between updating and relativization emerges as a distinction with a difference.

The axiomatic system of [24] that we adopted in § 2 above is a system of relative probability $\mathfrak{p}(\mathfrak{c} \mid \mathfrak{a})$. It is easy to check that if the function $\mathfrak{p}$ satisfies the axioms, and if $\mathfrak{b} \not\equiv \bot$, then $\mathfrak{p}_{\mathfrak{b}}(\mathfrak{a} \mid \mathfrak{c}) = \mathfrak{p}(\mathfrak{a} \mid \mathfrak{c}\mathfrak{b})$ also satisfies them. (The function $\mathfrak{p}_{\bot}$ is identically equal to 1, and violates the axiom that requires the function $\mathfrak{p}$ to have at least two distinct values.) The subscript notation embodied in $\mathfrak{p}_{\mathfrak{b}}$ will be used whenever we wish to refer to the updating of a function with the information $\mathfrak{b}$. Since $\mathfrak{p}_{\mathfrak{b}}(\mathfrak{c} \mid \mathfrak{a})$ equals $\mathfrak{p}(\mathfrak{c} \mid \mathfrak{a}\mathfrak{b})$ for every $\mathfrak{a}$,

[4] The two-valuedness of $\mathfrak{p}$ settles the truth table for negation. The other tables need also the addition and monotony laws. For example, by the general addition law, $\mathfrak{p}(\mathfrak{a} \to \mathfrak{c} \mid \top) = 0$ if and only if $\mathfrak{p}(\mathfrak{a} \mid \top) = 1 - \mathfrak{p}(\mathfrak{a}\mathfrak{c} \mid \top)$. By monotony and two-valuedness, this holds if and only if $\mathfrak{p}(\mathfrak{a} \mid \top) = 1$ and $\mathfrak{p}(\mathfrak{a}\mathfrak{c} \mid \top) = 0$. In short, $\mathfrak{a} \to \mathfrak{c}$ is false if and only if $\mathfrak{a}$ is true and $\mathfrak{c}$ is false. The CCCP hypothesis implies that in addition $\mathfrak{a} \to \mathfrak{c}$ is false if and only if $\mathfrak{p}(\mathfrak{c} \mid \mathfrak{a}) = 0$. But if $\mathfrak{c}$ is true, $\mathfrak{a} \to \mathfrak{c}$ is true for every $\mathfrak{a}$, and accordingly $\mathfrak{p}(\mathfrak{c} \mid \mathfrak{a}) = 1 = \mathfrak{p}(\top \mid \mathfrak{a})$ for every $\mathfrak{a}$.

and hence $\mathfrak{p}_b(b \mid a) = \mathfrak{p}(b \mid ab) = 1 = \mathfrak{p}(\top \mid ab) = \mathfrak{p}_b(\top \mid a)$, updating with b amounts to a decision to treat b as probabilistically indistinguishable from $\top$.

Since $q(c \mid a) = \mathfrak{p}(a' \mid c')$, the updated function q_b is defined by $q_b(c \mid a) = \mathfrak{p}_b(a' \mid c') = \mathfrak{p}(a' \mid c'b) = q((c'b)' \mid a)$, which equals $q(b \to c \mid a)$. In general, this term differs from $q(c \mid ab)$. Updating with b is not the same as relativizing to b. The distinction is especially transparent when the second argument of the function q is the tautology $\top$. For except when $a \equiv \bot$, the value of $\mathfrak{p}(\bot \mid a)$ is 0 for every probability measure; and therefore $q(c \mid \top) = 0$ except when $c \equiv \top$. (The function q, unlike the function $\mathfrak{p}$, has an almost flat prior distribution.) Updating $\mathfrak{p}$ to $\mathfrak{p}_b$ does not change matters: $q_b(c \mid \top)$ still equals 0 (unless $c \equiv \top$). But relativization of $q(c)$ to b yields $q(c \mid b)$, which may well not be 0.

6 The Reconditioned Conditional

Armed with these considerations we are at last in a position to understand how and why the replacement in the CCCP hypothesis of the probability measure $\mathfrak{p}$ by the deductive dependence measure q makes such a dramatic difference. The first formula displayed below is CCCP_0, exactly as it was displayed in § 4. The formula CCCP_1 is a notational variant, obtained from CCCP_0 by writing $\mathfrak{p}_b(c \mid a)$ for $\mathfrak{p}(c \mid ab)$. The formula CCCP_2 is obtained from CCCP_0 by first commuting the terms in the conjunction ab, then interchanging the letters a and b throughout, and finally writing $\mathfrak{p}_b(c \mid a)$ for $\mathfrak{p}(c \mid ab)$, as before. It is because updating and relativization are formally equivalent manoeuvres that each of CCCP_1 and CCCP_2 is equivalent to CCCP_0, though they look different.

CCCP_0 $\quad \forall a \forall c \exists y \forall b\ \mathfrak{p}(y \mid b) = \mathfrak{p}(c \mid ab)$

CCCP_1 $\quad \forall a \forall c \exists y \forall b\ \mathfrak{p}(y \mid b) = \mathfrak{p}_b(c \mid a)$

CCCP_2 $\quad \forall b \forall c \exists y \forall a\ \mathfrak{p}(y \mid a) = \mathfrak{p}_b(c \mid a)$.

We now replace $\mathfrak{p}$ by q in both CCCP_1 and CCCP_2, to produce the formulas

CCCQ_1 $\quad \forall a \forall c \exists y \forall b\ q(y \mid b) = q_b(c \mid a)$

CCCQ_2 $\quad \forall b \forall c \exists y \forall a\ q(y \mid a) = q_b(c \mid a)$.

These formulas are far from equivalent to each other: one is refutable, the other is demonstrable. CCCQ_1 is refuted by identifying b with $\top$. This shows that, for each a and c, $q(c \mid a) = q_\top(c \mid a)$ can take only the value 1 or the value 0; the value 1 if y (which may depend on a and c) is equivalent to $\top$, and the value 0 if it is not. In contrast, CCCQ_2 is demonstrable, since y may be the conditional $b \to c$. As was shown near the end of § 5 above, $\forall b \forall c \forall a\ q(b \to c \mid a) = q_b(c \mid a)$.

7 Discussion

In the interests of amity and brevity, I shall limit my discussion of these results to three items. One concerns their relation to the well-known Ramsey test. A

second concerns the tenability of the thesis that, at least with regard to conditionals, measures of deductive dependence offer an attractive alternative to measures of probability. The third matter, dealt with first, and in only a couple of sentences, is whether the unassailability of CCCQ_2 vindicates the identification of all indicative conditionals, at a semantic level, with material conditionals. This remains an open question. But I am not able here to provide solace to those who, having resolved to learn about indicative conditionals by studying their synergy with probabilities, are dismayed by what has been learnt.

Ramsey's test Much work on the connection between conditionals and probability has been guided by the words of Ramsey in 1929 ([29], p. 247): 'If two people are arguing "If p, will q?" and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge, and arguing on that basis about q; ... We can say that they are fixing their degrees of belief in q given p. If p turns out false, these degrees of belief are rendered *void*.' In [33], p. 101, this description becomes a piece of advice: 'your deliberation ... should consist of a simple thought experiment: add the antecedent (hypothetically) to your stock of knowledge (or beliefs), and then consider whether or not the consequent is true. Your belief about the conditional should be the same as your hypothetical belief, under this condition, about the consequent.' [11], p. 80, add that the agent's system of beliefs may need to be revised (but as little as possible) if it is to accommodate the antecedent consistently, a qualification that imports new problems. What lies behind the advice, if I understand it, is the idea that evaluating the probability of the consequent of a conditional, relative to its antecedent, is a way in which the agent might 'consider whether or not the consequent is true'.

I suggest that the explicit identity that we may extract from CCCQ_2, namely $\mathfrak{q}(\mathfrak{b} \to \mathfrak{c} \mid \mathfrak{a}) = \mathfrak{q}_{\mathfrak{b}}(\mathfrak{c} \mid \mathfrak{a})$, heeds this advice as well as does any identity derivable from the CCCP hypothesis. To be sure, there is a difference. In the case of an identity of the form $\mathfrak{p}(\mathfrak{a} \rightsquigarrow \mathfrak{c} \mid \mathfrak{b}) = \mathfrak{p}(\mathfrak{c} \mid \mathfrak{a}\mathfrak{b})$, it is likely that what Stalnaker (and others) had in mind was that the antecedent of the conditional $\mathfrak{a} \rightsquigarrow \mathfrak{c}$ be 'added to your stock of knowledge (or beliefs)' by further relativizing $\mathfrak{p}(\mathfrak{c} \mid \mathfrak{b})$ to $\mathfrak{a}$. I do not know that this strategy has ever been described (equivalently) as one of updating of $\mathfrak{p}(\mathfrak{c} \mid \mathfrak{b})$ with $\mathfrak{a}$. But in the identity $\mathfrak{q}(\mathfrak{b} \to \mathfrak{c} \mid \mathfrak{a}) = \mathfrak{q}_{\mathfrak{b}}(\mathfrak{c} \mid \mathfrak{a})$, the antecedent of the conditional $\mathfrak{b} \to \mathfrak{c}$ is unambiguously used to update the function $\mathfrak{q}$. This is how $\mathfrak{b}$ is to be 'added to your stock of knowledge (or beliefs)'.

Stated quite literally, what is here being proposed is this: in order to assess the deductive dependence of the material conditional $\mathfrak{b} \to \mathfrak{c}$ on the statement $\mathfrak{a}$, the agent should (provisionally and hypothetically) update the function $\mathfrak{q}$ to $\mathfrak{q}_{\mathfrak{b}}$ and then, using this updated function, assess the deductive dependence of $\mathfrak{c}$ on $\mathfrak{a}$. This procedure cannot properly be described as 'evaluating the dependence of the consequent of a conditional on its antecedent'. But if $\mathfrak{a}$ is supposed to state truthfully some information about the world, it is surely one way in which the agent might 'consider whether or not the consequent is true'.

Assertability and Acceptability of Conditionals It has been suggested by several writers, especially [1], that conditionals cannot be true or false, and

that $\mathfrak{p}(c \mid a)$ measures not the probability of the truth of $a \rightsquigarrow c$, but its *assertability*; that is to to say, the appropriateness of its utterance. Others, including Adams himself in a later phase ([2]), have favoured the term *acceptability*, that is to say, the reasonableness of the belief in $a \rightsquigarrow c$. [10], § 2, has ventured the neologism *assentability*. Although this has to my ears a subjectivist ring that is absent from *acceptability* and, to a lesser extent, *assertability*, for our present purposes the differences between these ideas are less important than what they have in common, which is an origin in the justificationist doctrine that an agent is entitled fully to assert or to accept or to assent to a statement only if he knows it to be true. The word *probably*, and similar expressions such as *in my opinion* and *I think*, are often used to qualify statements that are not fully asserted. The less probable that c is, given a, the less the agent is entitled to assert it, or the more tentatively he asserts it. In this vein, [16], Chapter 1, called probability 'a guarded guide'.

Those of us who dismiss as not quite serious the goal of justified truth never worry that we are not entitled to assert a statement. We think that we are entitled to say what we like, whatever the epistemological authorities may enjoin. But we may worry whether a proposition asserted is true, and if we suspect that it is not, we may qualify our assertion by such expressions as *about* or *or so* or *roughly* or *more or less*. Since the quantity $\mathfrak{q}(c \mid a)$, the deductive dependence of a non-tautological statement c on a statement a, is a straightforward measure of how well (the content of) c is approximated by (the content of) a, ranging from 0, when a contains none of c, to 1 when it contains it all, it does appear that $\mathfrak{q}(c \mid a)$ may serve also as a measure of the assertability or the acceptability of the statement c in the presence of a. If our aim is truth, then the higher $\mathfrak{q}(c \mid a)$ is, the more successful is the statement (or hypothesis c), given the statement (or evidence) a. More generally, the assertability or acceptability of the conditional $b \to c$ may be measured by $\mathfrak{q}(b \to c \mid a)$, that is, by $\mathfrak{q}_b(c \mid a)$. It is vigorously denied here that the 'highly entrenched tenet of probabilistic semantics ... [that] the assertability of conditionals goes by conditional probability' ([3], p. 584) exhausts the senses in which a conditional statement may be assertable or acceptable, but not completely so.

8 Conclusion

The goal of this paper has been to elucidate one of the gains that can be made in epistemology by replacing probability measures (understood as degrees of belief) by measures of deductive dependence (understood as degrees of approximation). On this theme, much more needs to be said than can be said here. In the first place, it must be recognized that variants of the function $\mathfrak{q}$ of deductive dependence have been introduced before, in rather different contexts. [12], Part IV, for example, interpreted $\mathfrak{q}(a \mid c)$ as a measure of the *systematic power* of the hypothesis c to organize the evidence a. [30], appendix, espied in the divergence between the functions $\mathfrak{p}$ and $\mathfrak{q}$ a potential solution to Hempel's paradoxes of confirmation. [13], § IV, interpreted $\mathfrak{q}(a \mid c)$ as a measure of the *information transmitted* by the evidence a about the hypothesis c, and used it to answer Ayer's question of why those who assay hypotheses by their relative probabilities ever search for new evidence. The function $\mathfrak{q}$ has simi-

larities also with the idea of probabilistic validity advanced in [2], and especially with the use of p-values in modern classical (non-Bayesian) statistics. All these connections will have to be explored in due course. Interested readers may glean from [18] meanwhile a glimpse of the versatility of the function $\mathfrak{q}$, and of the role that it may perform in a saner philosophy of knowledge than is fashionable at present.

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