Logic III: Incompleteness and Undecidability
Term 2, 2015-2016

- **Module tutor**
  - Walter Dean (W.H.Dean@warwick.ac.uk)
  - Put "PH340" in the subject line of your email and sign your *full name*.
- **Website**

http://go.warwick.ac.uk/ph340 (See "Course Materials" at the bottom.)
Readings, announcements, and problem sets will be posted at this address.

- **Logistics:**
  - Lecture 1: Tuesday 11:00-13:00 in F1.11
  - Lecture 2: Wednesday 12:00-13:00 in S0.18
  - Seminar: Wednesday 13:00-14:00 S2.04/5 (Science Concourse)
- **Description:**

The focus of this module are the Incompleteness Theorems of Kurt Gödel, first obtained in 1931. Gödel's theorems are often regarded as among the most important results in mathematical logic both because of the light they shed on the notions of truth and provability in mathematics and also because of the techniques involved in their proofs. The first theorem roughly states that any consistent formal mathematical theory capable of expressing basic facts about arithmetic is incomplete in the sense that there are true arithmetical statements which it cannot prove. Again roughly, the second theorem states that any such theory is incapable of proving its own consistency.

Although these results apply to a wide variety of axiomatic theories, they are commonly formulated with respect to a particular system known as first-order Peano Arithmetic [PA]. Part of the module will accordingly be spent studying this and related arithmetical theories. We will first study the technique known as *arithmetization* whereby it is shown that syntactic notions like well-formedness and provability can be expressed using a purely arithmetical language. After proving Gödel's theorems themselves, we will then study the phenomena of self-reference more generally and obtain several other limitative results due to Church, Rosser, Tarski, and Löb. Time permitting, we will then cover additional material about the model theory of arithmetic, reflection principles, and the use of modal logic to reason about provability as a sentential operator.

- **Related module:**

PH345 Philosophy of Computation (http://go.warwick.ac.uk/ph345) will cover related material about computability theory, models of computation, and Church's Thesis.

- **Recources:**

Our primary text will be a version of the *Open Logic Text* (part of the Open Logic Project. Some topics will be supplemented with material from

> *Computability and logic*[CL], 5th ed. by George Boolos, John Burgess, and Richard Jeffrey, Cambridge, 2007.

The relevant portions of these are available on the "Readings" section of the module website.

- **Assessment**

Assessment for this module will consist of two components:

1) Assessed problem sets [15%]. There will be three problem sets posted on the module website. These are to be submitted at the beginning of seminar (= Monday 13:00) in weeks 5 and 8 and to either to my pigeonhole in the department or by email in week 11.[1] Solutions will be posted on the module website.
2) Two hour exam [85%]. This will be problem-based and administered during the summer exam period.

- **Approximate schedule and readings** (-) = required, (+) = recommended
  - **Week 1**: Introduction, arithmetical languages and theories
    (-) Gödel's Incompleteness Theorems [SEP]
    (-) Open Logic 1.1, 1.2, 2.3
    (+) Kurt Gödel, Hilbert's program [SEP]
  - **Week 2**: Model theory review, models of arithmetical theories
    (-) Open Logic 4.8-4.10, chapter 5
    (+) Boolos chapter 25
  - **Week 3**: Primitive recursion, recursive functions and relations
    (-) Open Logic chapter 6
    (+) Boolos chapters 6 and 7
    (+) Recursive functions [SEP]
  - **Week 4**: The arithemitization of syntax
    (-) Open Logic chapter 8
    (+) Boolos chapter 15
  - **Week 5**: Representability in Q
    (-) Open Logic chapter 9
    (+) Boolos chapter 16
    ($\star$) **Problem set 1 due**
  - **Week 6**: Reading week (NO LECTURE OR SEMINAR)
  - **Week 7**: The fixed-point lemma and the First Incompleteness Theorem
    (-) Open Logic chapter 10
  - **Week 8**: The theorems of Church, Rosser, and Tarski
    (-) Open Logic chapter 10
    (-) Boolos chapter 17
    ($\star$) **Problem set 2 due**
  - **Week 9**: The derivability conditions and the Second Incompleteness Theorem
    (-) Open Logic chapter 10
    (-) Boolos chapter 18
  - **Week 10**: Reflection principles and provability logic
    (-) Provability Logic [SEP]
    (-) Boolos chapter 27
  - **Week 11** End of term.
    ($\star$) **Problem set 3 due**

[1] Extensions can only be granted by the Senior Tutor for Philosophy upon submissions of evidence of extenuating circumstances.