# International Symposium on Combinatorial Optimization 2008 

General Information<br>\&<br>Book of Abstracts<br>16-19 March 2008<br>University of Warwick, Coventry, UK

# CO 2008 

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## Welcome to CO 2008 @ Warwick

Dear participants,
With great pleasure I welcome you to Warwick and CO2008, on behalf of the Program and Organizing Committees.

As one of a series of biennial international symposia on combinatorial optimization, CO2008 continues to attract participants all over the world. It promises an excellent array of presentations about the advances of combinatorial optimization in both theory and practice. I hope you will find CO2008 stimulating for your research and helpful for both new and existing collaborations.

As one of UK's leading universities, Warwick offers much not only to its faculty, staff and students, but also to its visitors.

I wish you all a pleasant stay at Warwick and an enjoyable and rewarding conference!

Bo Chen
The Chair of the Program Committee The Chair of the Organizing Committee

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## General Information

CO 2008 takes place in the WBS Teaching Centre and Rootes Social Building. Figure 1 shows the buildings and the shortest route between these two buildings.


Figure 1: Location of the buildings used for CO 2008

## Accommodation

Room keys are available from Rootes Building from 15:00 on the day of arrival. Your accommodation will, dependant on your conference specification, be in either standard rooms are in Rootes Residences or ensuite rooms are in Jack Martin or Arthur Vick residences (numbers 48, 29 \& 3 respectively on the plan). On your day of departure please vacate your room by 09:30. A luggage store is available in Rootes Building. Should you be arriving after 23:00 please contact reception in advance.

## Registration

Registration for CO 2008 will be in Rootes (ground floor) on Sunday evening, and in the WBS Teaching Centre all other days.

## Lecture Theatres

All presentations and all coffee breaks will take place in the WBS Teaching Centre. For the presentatations three rooms are used: M1, M2 (see Figure 2), and A2 (see Figure 3).


Figure 2: Lecture theatres M1 and M2 are located on the ground floor of WBS Teaching Centre


Figure 3: Lecture theatre A2 is located on the first floor at the far end of the corridor
The plenaries will take place in Room M1.
Parallel sessions will take place in Romm M1, M2, and A2. Every presentation is scheduled for 30 minutes, including 5 minutes for questions and discussion.

## Breakfast, Lunch and Dinner

For participants staying on campus, breakfast, lunch and dinner are served Rootes Restaurant (1st floor). Breakfast is served between 7:30-9:30, lunch 12:30-14:00, and dinner 18:30-20:00. On

Tuesday evening, dinner is served for those not participating in the conference dinner (see below). On Wednesday evening, dinner is served for those with an extended stay.
Meal tickets are provided by Warwick Conferences and are required to gain access to the building.
For participants not staying on campus, there is a restaurant in Warwick Arts Centre (EAT) for lunch.

## Social program

## Welcome reception

On Sunday evening, there will be a welcome reception with buffet style dinner, from 18:30 to 21:00. This reception will be in Rootes Social Building - Chancellor's Suite (2nd floor).

## Excursion Coventry and Conference dinner

On Tuesday afternoon, there will be an excursion to Coventry City centre with conference dinner at St. Mary's Guildhall. A ticket is required.

Time schedule:
16:30 Departure of Bus to Coventry from bus stop in front of Rootes Social Building
17:00-18:30 Guided tour Coventry Transport Museum
18:30-19:30 Guided tour "Ancient Coventry"
19:30-22:30 Conference Dinner at St. Mary's Guildhall
22:30 Departure of Bus back to University of Warwick campus

## Excursion Warwick Castle

If enough interest is expressed, we will organize transport by minibus to Warwick Castle on Wednesday afternoon. The minibus will leave at 14:00 in front of Rootes Social Building. There will be no organized transport back from the city of Warwick to the University of Warwick campus. Stagecoach bus 16 (Stratford-upon-Avon - Coventry, via Warwick, Kenilworth) can be taken to return to campus.

Please contact the CO2008 reception desk to sign up and further details.

## Facilities

There are various food and drink outlets around campus including The Bar in Rootes \& Café Bar in the Warwick Arts Centre. See http://www2.warwick.ac.uk/services/foodanddrink/ for more information.

In the mall area close to the Rootes Building there are: Natwest, HSBC and Barclays Banks with cash dispensers, a pharmacy, hairdressers, Post Office and a supermarket incorporating a newsagent . Other facilities include a cinema \& bookshop in the Arts Centre.

Sports facilities are available to delegates free of charge but you will need to take your room key in order to access these. Facilities include swimming pool, squash courts, gymnasium and outside tennis courts.

Squash and tennis courts can be booked at the Sports Centre Reception (Ext 23011).

## Internet Access

All bedrooms on campus are networked for free internet access (for computers with an Ethernet network card). Internet cables are available from Rootes Reception. Alternatively PC's with free internet and email access are available for all participants in Rootes Reception - these may be accessed 07:00-23:00 everyday. Wireless coverage is also available in the Rootes foyer.
Wireless coverage is also available in the WBS Teaching Centre. For this, you can obtain a username and password from the CO 2008 registration desk.

## Schedule of the Scientific Program

## Overview

| Monday 17 March | Room M1 | Room M2 | Room A2 |
| ---: | :---: | :---: | :---: |
| 9:30-11:00 | Network <br> Optimization 1 |  <br> Scheduling 1 | Vehicle Routing <br> Problems |
| 11:00-11:30 |  | Coffee break |  |
| 11:30-12:30 |  | Plenary Rolf Möhring (Room M1) |  |
| $12: 30-14: 00$ |  | Lunch break |  |
| 14:00-15:30 | Network <br> Optimization 2 |  <br> Heurisitics 1 | Supply Chain <br>  <br> Location Analysis |
| 16:00-17:00 | Travelling Salesman |  |  |
| Problems |  |  |  |$\quad$ Graphs \& Networks 1 | Combinatorial Theory |
| :--- |
| 17:00-18:00 |


| Tuesday 18 March | Room M1 | Room M2 | Room A2 |
| ---: | :---: | :---: | :---: |
| 9:00-11:00 | Network <br> Optimization 3 |  <br> Scheduling 2 | Graphs \& Networks 2 |
| 11:00-11:30 |  | Coffee break |  |
| 11:30-12:30 |  | Plenary Moshe Dror (Room M1) |  |
| 12:30-14:00 |  | Lunch break |  |
| 14:00-16:00 |  <br> Column Generation |  <br> Heurisitics 2 | Graphs \& Networks 3 |
| 16:00-16:30 |  | Tea break |  |
| Wednesday 19 | Room M1 | Room M2 | Room A2 |
| March | Real-life Applications |  <br> Heurisitics 3 | Assignment Problems |
| $11: 00-11: 30$ |  | Coffee break |  |
| $11: 30-12: 30$ |  | Plenary Adam N. Letchford (Room M1) |  |
| $12: 30-14: 00$ |  | Lunch break |  |
| $14: 00$ |  | End of CO 2008 |  |

## Monday 17 March 2008

| Session M1: 9:30-11:00 |
| :--- |
| Parallel session M1-A (Room M1) |
| Session title: Network Optimization 1 <br> Session chair: Arie Koster |
| 9:30-10:00 |
| Paula Carroll: Polyhedral Investigation of the Ring Spur Assignment Problem |
| 10:00-10:30 |
| Rosa Figueiredo: A Tabu Search approach to solve the mixed integer bilevel for- <br> mulation of a network design problem |

Parallel session M1-B (Room M2)
Session title: Sequencing \& Scheduling 1
Session chair: Marc Reimann

| 9:30-10:00 | Imed Kacem: 2-approximation algorithm for the weighted completion time mini- <br> mization on a single machine with a fixed non-availability interval |
| :---: | :--- |
| 10:00-10:30 | Celia A. Glass: Scheduling on parallel machines, with perishability time windows, <br> inspired by the process of micro-biological food testing |
| 10:30-11:00 | Marta Flamini: A branch and bound algorithm for a generalized Job Shop Schedul- <br> ing problem |

## Parallel session M1-C (Room A2)

Session title: Vehicle Routing Problems
Session chair: Vladimir Deineko

| 9:30-10:00 | Ulrich Pferschy: A Balanced Vehicle Routing Problem |
| ---: | :--- |
| 10:00-10:30 | Qianxin Mu: Disruption Management in Vehicle Routing and Scheduling |
| 10:30-11:00 | Nik Pearson: Good Triangulations Yield Good Tours |

## Session M2 - Invited plenary presentation (Room M1)

Invited speaker: Rolf Möhring (Technische Universität Berlin, Germany)
Session chair: Chris N. Potts
11:30-12:30 Routing in Graphs with Applications in Traffic and Logistics

## Session M3: 14:00-15:30

## Parallel session M3-A (Room M1)

Session title: Network Optimization 2
Session chair: Sebastian Orlowski
14:00-14:30 Michael Poss: Constraints generation for solving a bilayer network design problem
14:30-15:00 Christian Raack: The cut-set polytope for two-layer network design problems
15:00-15:30 Sebastian Orlowski: Branch-and-cut techniques for solving realistic two-layer network design problems

## Parallel session M3-B (Room M2)

Session title: Algorithms \& Heuristics 1
Session chair: Thomas Erlebach

| 14:00-14:30 | Wanpracha Chaovalitwongse: Approximating Several Covering/Packing Problems |
| :--- | :--- |
| 14:30-15:00 | Benny Vaksendiser: Algorithms for Storage Allocation Based on Client Preferences |
| 15:00-15:30 | Thomas Erlebach: Approximating Geometric Coverage Problems |

## Parallel session M3-C (Room A2)

Session title: Supply Chain Optimization \& Location Analysis
Session chair: Maria A. Osorio

| 14:00-14:30 | Maria Paola Scaparra: A multi-level optimization model for improving the robust- <br> ness of capacitated service and supply systems |  |
| :--- | :--- | :--- |
| $14: 30-15: 00$ | Oğuz Solyalı: Strong formulations for the one-warehouse multi-retailer problem |  |
| $15: 00-15: 30$ | Maria A. Osorio: On the use of Surrogate Constraint Analysis to fix binary variables <br> in the CFLP |  |
|  |  | Coventry, UK |


| Session M4 1600-18:00 |  |
| :---: | :---: |
| Parallel session M4-A (Room M1) |  |
| Session title: Travelling Salesman Problems / ILOG Workshop Session chair: Edward Gimadi |  |
| 16:00-16:30 | Petrică C. Pop: A Strong Integer Programming Formulation of the Generalized Travelling Salesman Problem |
| 16:30-17:00 | Edward Gimadi: On asymptotic optimality of polynomial algorithm for multi-TSP in Euclidean space |
| 17:00-18:00 | ILOG CPLEX 11 workshop |
| Parallel session M4-B (Room M2) |  |
| Session title: Graphs \& Networks 1 Session chair: Eric McDermid |  |
| 16:00-16:30 | Péter Biró: Integral stable allocation problem on graphs |
| 16:30-17:00 | Eric McDermid: Keeping partners together: Algorithmic results for the Hospitals / Residents problem with couples |
| 17:00-18:00 | See session M4-A |
| Parallel session M4-C (Room A2) |  |
| Session title: Combinatorial Theory <br> Session chair: Oliver Jenkinson |  |
| 16:00-16:30 | Fredrik Kuivinen: Submodular functions on diamonds |
| 16:30-17:00 | Oliver Jenkinson: Balanced words and majorization |
| 17:00-18:00 | See session M4-A |

## Tuesday 18 March 2008

| Session T1: 9:00-11:00 |
| :--- |
| Parallel session T1-A (Room M1) |
| Session title: Network Optimization 3 <br> Session chair: Petrică Pop |
| 9:00-9:30 |
| Teresa Gomes: An effective algorithm for obtaining the set of all minimal cost pairs <br> of disjoint paths with dual arc costs |
| $9: 30-10: 00$ |
| Lucile Denœud: A Graph-Partitioning-Based-Heuristic for Optical Network Plan- <br> ning Problems |
| 10:00-10:30 |
| Nicolas Sonnerat: Galaxy Cutsets in Graphs |

## Parallel session T1-B (Room M2)

| Session title: Sequencing \& Scheduling 2 <br> Session chair: Bo Chen |  |
| :--- | :--- |
| 9:00-9:30 | Ivan Rykov: Asymptotically exact approach to solving RCPSP with one resource <br> type |
| 9:30-10:00 | Vitaly Strusevich: Solving Make-or-Buy Trade-off Problems by Submodular Opti- <br> mization |
| 10:00-10:30 | Roberto Rossi: Scheduling Internal Audit Activities: A Stochastic Combinatorial <br> Optimization Problem |
| 10:30-11:00 | Chris N. Potts: Online Scheduling with Known Arrival Times |

## Parallel session T1-C (Room A2)

Session title: Graphs \& Networks 2
Session chair: Vadim Lozin

| 9:00-9:30 | Bert Marchal: A local search algorithm for determining tree decompositions of <br> graphs |
| :---: | :--- |
| 9:30-10:00 | Synara Brito: Forest-clique partitions of cographs |
| 10:00-10:30 | Jakub Mareček: Zykov Revisited: Engineering an Exact Solver for Graph Colouring |
| 10:30-11:00 | Vadim Lozin: Stability Preserving Transformations of Graphs |

## Session T2 - Invited plenary presentation (Room M1)

Invited speaker: Moshe Dror (University of Arizona, USA)
Session chair: Richard Eglese
11:30-12:30 Another Look at Euclidean TSP and Packing: "This" Almost Never Happens

## Session T3: 14:00-16:00

Parallel session T3-A (Room M1)
Session title: Cutting Planes \& Column Generation
Session chair: Socorro Rangel

| 14:00-14:30 | Manuel Kutschka: Seperation of $\left\{0, \frac{1}{2}\right\}$-Chvátal-Gomory cuts in general integer pro- <br> grams |
| :--- | :--- |
| 14:30-15:00 | Konstantinos Kaparis: Separation Algorithms for 0-1 Knapsack Polytopes |
| 15:00-15:30 | Stefan Ropke: Computer aided discovery of families of valid inequalities |
| $15: 30-16: 00$ | Socorro Rangel: Special cutting patterns and reduction of saw machine set ups in <br> the cutting stock problem |

Parallel session T3-B (Room M2)
Session title: Algorithms \& Heuristics 2
Session chair: Daniele Catanzaro

| 10:30-11:00 | Tuan-Vu Tran: Global Constrained Optimization of a Safety Transformer using <br> Branch-and-Bound method |
| :--- | :--- |
| 14:30-15:00 | Dmitriy Drusvyatskiy \& Robert W.H. Fisher: Scheduling Tasks on Parallel Machines <br> with Network-Based Restrictions |
| 15:00-15:30 | Shinji Imahori: Improved best-fit heuristics for rectangular strip packing and bin <br> packing problems |
| 15:30-16:00 | Daniele Catanzaro: Estimating phylogenies under maximum likelihood: A very <br> large-scale neighborhood approach |

## Parallel session T3-C (Room A2)

Session title: Graphs \& Networks 3
Session chair: Paula Zabala

| 14:00-14:30 | Marcin Kamiński: Quadratic programming on graphs without long odd cycles |
| :--- | :--- |
| 14:30-15:00 | Géraldine Heilporn: On a Network Pricing Problem with Connected Toll Arcs |
| 15:00-15:30 | Jakub Mareček: Where is the Symmetry in Vertex Colouring? |
| 15:30-16:00 | Paula Zabala: The $(k, k-1)$-coloring problem |

## Wednesday 19 March 2008

| Session W1: 9:00-11:00 |
| :--- |
| Parallel session W1-A (Room M1) |
| Session title: Real-life Applications  <br> Session chair: André Gustavo dos Santos  <br> $900-9: 30$  |
| Janny Leung: Timetable Syncronisation for Rail Mass Transit |
| 9:30-10:00 |

## Parallel session W1-B (Room M2)

| Session title: Algorithms \& Heuristics 3 <br> Session chair: Dolores Romero Morales |  |
| :---: | :---: |
| 9:00-9:30 | Haris Aziz: Computing voting power in easy weighted voting games |
| 9:30-10:00 | Sofie Coene: Profit-based Latency Problems on the Line |
| 10:00-10:30 | Olivier Hudry: Complexity of the computation of a linear order at minimum distance from a tournament |
| 10:30-11:00 | Dolores Romero Morales: Discretizing Variables for Support Vector Machines by Means of Purity Measures |

## Parallel session W1-C (Room A2)

| Session title: Assignment Problems <br> Session chair: Frits Spieksma |
| :--- |
| 9:00-9:30 |
| Socorro Rangel: A modified Lagrangian bound for a class of many-to-many assign- <br> ment problems |
| 9:30-10:00 |
| Laura Bahiense: Ship accommodations layout problem as a k-constraints quadratic <br> assignment problem |
| 10:00-10:30 |
| Yury Glazkov: Asymptotically optimal algorithms for m-layer planar 3-dimensional <br> assignment problem |
| 10:30-11:00 |
| Frits Spieksma: Multi-Index Assignment Problems: Applications and Approximation |

## Session W2 - Invited plenary presentation (Room M1)

Invited speaker: Adam N. Letchford (Lancaster University, UK)
Session chair: Vitaly Strusevich
11:30-12:30 The Maximum Cut and Maximum Clique Problems: Linear versus Semidefinite
Programming

## Abstracts of plenary presentations

# Another Look at Euclidean TSP and Packing: "This" Almost Never Happens 

Speaker: Moshe Dror (MIS, Eller School of Management, University of Arizona)

Co-author(s): James B. Orlin (Sloan School, MIT)


#### Abstract

In the first part of this talk we will examine the relation between the length of an optimal traveling salesman tour and the sum of its nodes' marginal values (a node's marginal value is the difference between the length of an optimal TSP tour over a given node set and the length of an optimal TSP tour over the node set minus the node). To our knowledge, this problem has not been studied previously. We restrict our analysis to the metric space $l_{p}^{2}$ and consider norms $L_{1}\left(L_{\infty}\right), L_{4 / 3}, L_{2}, L_{4}$. The event in which the sum of TSP marginal values is greater than the length of the optimal tour is rare. We present a number of cases for which the sum of marginal values is never greater than that for the optimal tour. We establish a worst case ratio of 2 for any metric TSP. In addition, for 6 node TSPs we determine the worst ratio for the $L_{1}\left(L_{\infty}\right)$ norm, triangle inequality, and symmetric distance, of $10 / 9,1.2$, and 1.5 respectively, by solving the appropriate mixed integer programming problems. In the second part of this talk we examine the likelihood that two shelves of integer length $L$ can be packed with items whose individual lengths are divisors of $L$, given that the individual items sum-up to $2 L$. The main thrust of this study is computational. That is, we explicitly compute the answer for this question for all integers $L, 1 \leq L \leq 1000$. We conclude that an instance of this packing problem in which we cannot pack the two shelves is very rare. Existence of packing failures is tied to the number of divisors of $L$. That is, we prove that the number of divisors has to be at least 8 for a packing failure to exist.


Keywords: Euclidean TSP • marginal Values • packing

## References:

[1] Dror, M., Y, Lee, J.B. Orlin, and V. Polishchuk, (2006). "The TSP and the sum of its marginal values", International J. of Computational Geometry\& Applications 16(4), 333-343.
Dror, M., J.B. Orlin, and M. Zhu, (2007). "Packing shelves with items that divide the shelves' length", Working Paper, MIS, University of Arizona.

Speaker's profile: Moshe Dror studied Industrial Engineering \& Management and Mathematical Methods \& Operations Research at Columbia University. In 1983 he received a Ph.D. degree from the University of Maryland. Since 1990 he is affiliated with University of Arizona in Tucson, since 2001 as Eller Professor of Management Information Systems. In June 2005, he received the Chinese Academy of Sciences International Collaboration Award.

Moshe Dror's research interests include coooperative game theory and cost allocation in inventory and supply chain management, applied combinatorial optimization in transportation, logistics and manufacturing, as well as intelligent solution systems for operations scheduling.
Contact: Moshe Dror, MIS, Eller School of Management, Tucson, AZ 85721;
email:mdror@eller.arizona.edu

# The Maximum Cut and Maximum Clique Problems: Linear versus Semidefinite Programming 

Speaker: Adam N. Letchford (Lancaster University)


#### Abstract

The Maximum Cut Problem and the Maximum Clique Problem are well-known and fundamental combinatorial optimisation problems, with a huge number of applications. Although different in many respects, they have something in common: Linear Programming (LP) techniques have produced disappointing results, whereas Semidefinite Programming (SDP) techniques appear to be much more promising. This talk will attempt to shed light on the strength and limitations of the LP and SDP approaches.

The talk will consist of three parts. The first part will be an introduction and literature survey. The second part will be concerned with recent joint work with colleagues at the University of L'Aquila [1], which shows that a pure LP-based approach can produce surprisingly strong upper bounds for MaxClique. The third and final part will report on an ongoing joint project with two PhD students [2], which attempts to do the same for Max-Cut.


Keywords: combinatorial optimization • cutting planes • semidefinite programming

## References:

[1] M. Giandomenico, A.N. Letchford, F. Rossi, S. Smriglio: An application of the Lovász-Schrijver $M(K, K)$ operator to the stable set problem. Submitted to Math. Program., 2007.
[2] D.J. Grainger, K. Kaparis, A.N. Letchford: Strong upper bounds for max-cut using knapsack facets. In preparation.

Speaker's profile: Adam N. Letchford studied Linguistics and Psychology at Nottingham Univeristy before he changed to do a MSc and PhD in Operational Research at Lancaster University. He worked as a Research Associate between 1998 and 1999 and became a Lecturer in 2000. He is now a Professor in Optimisation. In 2006 he was awarded an Advanced Research Fellowship from the Engineering and Physical Sciences Research Council (EPSRC).

Adam Letchford's research is in combinatorial optimisation. He concentrates on exact solution methods, especially methods based on row and column generation. He has a particular interest in problems defined on graphs/networks, such as vehicle routing problems and the traveling salesman problem. Recently, he is concentrating on the so-called maximum cut problem and some related problems.
Contact: Adam N. Letchford, Department of Management Science, Lancaster University, Lancaster LA1 4YX, UK.
E-mail: A.N.Letchford@lancaster.ac.uk

# Routing in Graphs with Applications in Traffic and Logistics 

Speaker: Rolf Möhring (Technische Universität Berlin)


#### Abstract

Traffic management and routing in logistic systems are optimization problem by nature. We want to utilize the available street or logistic network in such a way that the total network "load" is minimized or the "throughput" is maximized. This lecture deals with the mathematical aspects of these optimization problems from the viewpoint of network flow theory. It leads to flow models in which-in contrast to static flows-the aspects of "time" and "congestion" play a crucial role. Moreover, acceptance reasons may limit the choice of routes. We illustrate these aspects on three different applications: traffic guidance in rush hour traffic (cooperation with DaimlerChrysler), routing automated guided vehicles in container terminals (cooperation with HHLA), and timetabling in public transport (cooperation with Deutsche Bahn and Berlin Public Transport).


Keywords: routing • flows over time • traffic guidance • periodic timetabling

## References:

[1] E. Gawrilow, E. Köhler, R. H. Möhring, and B. Stenzel, Dynamic routing of automated guided vehicles in real-time, Tech. Rep. 039-2007, TU Berlin, 2007, http://www.math.tu-berlin.de/coga/publications/techreports/2007/Report-039-2007.html
[2] O. Jahn, R. H. Möhring, A. S. Schulz, and N. E. Stier Moses, System-optimal routing of traffic flows with user constraints in networks with congestion, Operations Research, 53 (2005), pp. 600-616.
[3] C. Liebchen and R. H. Möhring, The modeling power of the periodic event scheduling problem: Railway timetables - and beyond, in Algorithmic Methods for Railway Optimization, F. Geraets, L. Kroon, A. Schöbel, D. Wagner, and C. D. Zaroliagis, eds., vol. 4359 of Lecture Notes in Computer Science, Springer, Berlin/Heidelberg, 2007, pp. 3-40.

Speaker's profile: Rolf H. Möhring studied Mathematics at RWTH Aachen, Germany. In 1975 he completed his PhD and in 1982 his Habilitation. He hold faculty positions at RWTH Aachen, Universität Hildesheim and Universität Bonn, before becoming a Professor of Applied Mathematics and Computer Science at Technische Universität Berlin in 1987. His main research interests are in Graph and Network Algorithms, Combinatorial Optimization, Project Scheduling, and their Industrial Applications.

He is a member of the editorial board of various journals in the area of combinatorial optimization and has served as President of the Gesellschaft für Mathematik, Ökonomie und Operations Research (1994-1996) and the Mathematical Programming Society (2004-2007). He has been an initiative member of several DFG Research Clusters and is Scientist in Charge of the Application Area "Traffic and Communication Networks" of the DFG Research Center Matheon "Mathematics for Key Technologies".
Contact: Rolf H. Möhring, Institut für Mathematik (MA 6-1), Technische Universtät Berlin, Straße des 17. Juni 136, 10623 Berlin, Germany. E-mail: Rolf.Moehring@TU-Berlin.DE

## Abstracts of contributed presentations

(in alphabetical order of speaker surname)

# Computing voting power in easy weighted voting games 

Speaker: Haris Aziz (Department of Computer Science, University of Warwick)<br>Co-author(s): Mike Paterson (Department of Computer Science, University of Warwick)


#### Abstract

Weighted voting games are ubiquitous mathematical models which are used in economics, political science, neuroscience, threshold logic, reliability theory and distributed systems. They model situations where agents with variable voting weight vote in favour of or against a decision. A coalition of agents is winning if and only if the sum of weights of the coalition exceeds or equals a specified quota. Since the weights of the players do not always exactly reflect how the critical a player is in the decision making, voting power indices attempts to measure the ability of a player in a weighted voting game to determine the outcome of the vote. The Banzhaf index is an important voting power index for weighted voting games. It depends on the number of coalitions in which the agent is the difference in the coalition winning or losing. The computational complexity of computing Banzhaf indices in weighted voting games is well studied. It is known that that the problems of computing the Banzhaf values of players is \#P-complete. It is even NP-hard to identify a player with zero voting power or two players with same Banzhaf indices. We show that although computing Banzhaf indices of weighted voting games is a hard problem in general, it is easy for various classes of weighted voting games. We give a comprehensive characterization of weighted voting games which can be solved in polynomial time. These classes include weighted voting games which are r-geometric or satisfy the alternative dominance condition. Among other results, we provide a polynomial $O\left(k\left(\frac{n}{k}\right)^{k}\right)$ algorithm to compute the Banzhaf indices in weighted voting games in which the number of weight values is bounded by $k$. Therefore, we show that the problem of computing Banzhaf indices of a weighted voting game for fixed parameter $k$ the number of possible values of the weights is in FPT. It is known that a greedy algorithm can compute Banzhaf indices of an 'unbalanced' weighted voting game in polynomial time. We also prove that even if the weighted voting game is 2 -unbalanced instead of unbalanced, the problem of computing Banzhaf indices becomes NP-hard.


Keywords: Banzhaf indices • algorithms and complexity • game theory • weighted voting games
Contact: Haris Aziz, Computer Science Department, University of Warwick, Coventry, UK, CV4 7AL. Fax: ++44 247657 3024, Tel: ++44 247652 2350, Emails: \{Haris.Aziz, M.S.Paterson\}@warwick.ac.uk.

## Ship accommodations layout problem as a k-constraints quadratic assignment problem

Speaker: Laura Bahiense (Federal University of Rio de Janeiro, COPPE, Production Engineering, Rio de Janeiro, Brazil)

Co-author(s): Eliane M. Loiola • Nair M. M. Abreu (Federal University of Rio de Janeiro, COPPE, Production Engineering, Rio de Janeiro, Brazil)

- Richard D. Schachter (Federal University of Rio de Janeiro, POLI, Naval and Oceanic Engineering, Rio de Janeiro, Brazil)


#### Abstract

The ship accommodations layout problem is an important problem for Naval Architecture. In Brazil, this layout must comply with safety and habitability regulations of the Brazilian administration and the international institutions that regulate ship design and construction, besides ergonomic


criteria and practical rules of architecture. A model for the physical deck arrangement of a ship is a k-constraints quadratic assignment problem (QAP-k), a special case of the quadratic assignment problem (Loiola et al., 2007). The solutions set of a QAP-k instance is a co-set of a subgroup stabilizer of symmetric group. The concept of base and strong generating set introduced by Sims (1971) provides an effective computer representation for a permutation group. This representation helps us to generate random elements. We describe a method to derive solutions for facility layouts that are to have constraints of architecture, involving aspects related to tri-dimensionality. Application of heuristic methods in computer aided ship design has not been widely utilized yet. This work investigates whether such a heuristic method, can be satisfactorily applied into ship design. The ship accommodations layout problem as a QAP-k and a method for obtaining several optimized options of accommodation arrangements are presented here, in order to offer to human experts computerized heuristic analysis techniques to carry out ship design applications.
Keywords: facility layout problem • $k$-constraints quadratic assignment problem • computational group theory - ship accommodations layout problem.

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## Integral stable allocation problem on graphs

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#### Abstract

We define the stable allocation problem on an undirected graph $G(V, E)$ as follows. For every vertex $v \in V$ let $<_{v}$ be a linear order on the edges incident with $v$. We say that vertex $v$ prefers edge $f$ to edge $e$ if $e<_{v} f$ holds. Let $b: V \longrightarrow \mathbb{R}_{+}^{V}$ be the bounds of the vertices, and $c: E \longrightarrow \mathbb{R}_{+}^{E}$ be the capacities of the edges. A function $x: E \longrightarrow \mathbb{R}^{E}$ is called an allocation if $0 \leq x(e) \leq c(e)$ for every edge $e \in E$ and $x(v):=\sum_{v \in e} x(e) \leq b(v)$ for every vertex $v \in V$. An edge $e$ is saturated in the allocation $x$ if $x(e)=c(e)$ and unsaturated otherwise; similarly, a vertex $v$ is saturated if $x(v)=b(v)$ and unsaturated otherwise. An allocation $x$ is stable if for every unsaturated edge $e$ there is a vertex $v \in e$ such that $\sum_{v \in f, e \leq f} x(f)=b(v)$. We call the integral version of the stable allocation problem (i.e. if the allocation $x$ is required to be integral on every edge) as the integral stable allocation problem. If, for an integral stable allocation problem $c(e)=1$ for every edge $e$, then this special case is called the stable $b$-matching problem (e.g. Fleiner [3]). If the graph may contain parallel edges, then this problem was referred to as the stable multiple activities problem by Cechlárová and Fleiner [2]. In case of simple graphs, this problem was called the stable fixtures problem by Irving and Scott [5]. If $b(v)=1$ for every vertex, then the stable matching problem is obtained; that is called the stable roommates problem for simple graphs. Furthermore, if the given graph is simple and bipartite, then


the stable $b$-matching problem can be called the many-to-many stable matching problem. If $b(v)=1$ for every vertex on one side, then the problem can be referred to as the many-to-one stable matching problem, the college admission problem or the hospitals/residents problem. Finally, if $b(v)=1$ for every vertex, then we obtain the stable marriage problem, introduced and solved by Gale and Shapley [4].

The stable allocation problem was introduced by Baïou and Balinski [1] for bipartite graphs. They created a so-called inductive algorithm that finds a stable allocation, given an instance of the stable allocation problem for bipartite graphs, in strongly polynomial time (i.e. its running time does not depend on the bounds and capacities, only on the number of vertices and edges). Here, we generalise their inductive algorithm to the nonbipartite integral stable allocation problem. We illustrate by an example that the generalised algorithm does not remain polynomial, but, we show that this algorithm can be modified to become weakly polynomial by using scaling technique.

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Keywords: stable matching problem • roommates problem • allocation problem
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## Forest-clique partitions of cographs

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#### Abstract

: A forest-clique partition is a partition of the vertex set of a graph into two parts $F$ and $C$ such that $F$ induces an acyclic subgraph (forest) and $C$ a clique. In this case, $C$ can be seen as a cycletransversal (a subset of vertices which intersects all the cycles of the graph). In this work we study forest-clique partitions of cographs (graphs containing no induced subgraphs isomorphic to $P_{4}$, the chordless path with four vertices). Cographs which admit forest-clique partitions are called (F,C)cographs. We present a characterization of (F,C)-cographs in terms of a finite family of forbidden


induced subgraphs, showing that a cograph $G$ is ( $\mathrm{F}, \mathrm{C}$ ) if and only if it does not contain any of the graphs shown in Figure 4 as an induced subgraph. We also give a simple linear-time algorithm for recognizing ( $\mathrm{F}, \mathrm{C}$ )-cographs.


Figure 4: Forbidden induced subgraphs for (F,C)-cographs.

Keywords: graph partitioning • transversal • cographs
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# Polyhedral Investigation of the Ring Spur Assignment Problem 

Speaker: Paula Carroll (Quinn School of Business, UCD, Dublin, Ireland)


#### Abstract

This paper is about an interesting new problem that we are seeking to solve using polyhedral combinatorics. In an earlier piece of work, presented at the British Combinatorial Conference 2007, we outlined the Ring Spur Assignment Problem (RSAP) which arises in the design of Next Generation Telecommunications Networks but could also provide solution topologies for facility location with hubbing problems. The RSAP has practical applications and has not, to our knowledge, previously been addressed in the literature. The Sonet Ring Assignment problem described by Goldschmidt et al [1] and the Ring Star Problem addressed by Labbe et al [2] are related problems. In the earlier work we proved that the RSAP is NP-Hard and reported on an integer programming formulation for the problem with valid inequalities. We also presented computational results from the exact approach on benchmark problems of sizes up to 25 nodes.


We now extend that work by conducting a polyhedral investigation of the RSAP. Our aim is to identify which inequalities are facet defining and under what conditions. We intend to use this knowledge to implement a cutting plane approach on larger problems. This is a useful approach for NP-Hard problems and has been shown to produce good results in practice. The cutting plane approach starts with a relaxed (minimal) version of the problem in question, identifies cuts that can be added to strengthen the problem formulation and re-optimises. Since Facet Defining Inequalities (FDIs) are the strongest kind of inequality, they are the best to use in a cutting plane framework. We note here the definition of the separation problem for a class of inequalities which is to identify (separate) the most violated inequality of that class for the current relaxation or decide that the current solution satisfies all inequalities of that class.

We describe well known FDIs and present the initial findings of our polyhedral analysis. We ask whether published FDIs are valid for the Ring Spur Assignment Problem. We indicate where polyno-
mial separation algorithms are known for the various classes of Facet Defining Inequalities discussed, indicate the current status of our research and present our latest computational results. This paper gives a good introduction to the RSAP and indicates the current state of knowledge about the problem.

Keywords: telecommunication networks • polyhedral theory • integer linear programming • cutting planes

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## Estimating phylogenies under maximum likelihood: A very large-scale neighborhood approach

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- Michel C. Milinkovitch (Université Libre de Bruxelles)


#### Abstract

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Background: A basic problem in evolutionary genetics is the estimation of phylogenies among DNA or protein sequences. This problem is known to be $\mathcal{N P}$-Hard under most optimality criteria used for evaluating the quality of trees. Consequently, one can reasonably search for optimal phylogenies only for datasets of small sizes such that the ever increasing number of molecular sequences accumulating in public databases increases the need for new heuristic algorithms for large phylogeny estimation.


Results and Conclusion: We introduce here very large-scale neighborhood search (VLSN) techniques for the phylogeny estimation problem. VLSN techniques belong to the class of local search algorithms, and are characterized by a neighborhood size that grows exponentially with the input data. The underlying basic concept of VLSN techniques is that a greater neighborhood improves the quality of the locally optimal solutions found. These techniques provide efficient (typically polynomial) means to search for the best local optimum inside a neighborhood of a given solution. Here, we adapt these techniques to estimate phylogenies of large datasets of nucleotide sequences under the maximum likelihood criterion. We show that the use of the VLSN techniques speeds up convergency to topological local optima, and increases the overall performances of stochastic-based search algorithms.

Keywords: network design • computational biology • phylogenetic estimation • maximum likelihood
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# Approximating Several Covering/Packing Problems 

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#### Abstract

In this paper, we consider approximability of four covering/packing type problems which have important applications in computational biology. The problems considered in this paper are the triangle packing problem, the full sibling reconstruction problem under two parsimonious assumptions, the maximum profit coverage problem and the 2 -coverage problem. We provide approximation algorithms and inapproximability results for various values of parameters of interest for these problems. Our inapproximability constant for the triangle packing problem improves slightly upon the best-known inapproximability constant that can be achieved from previous results [2]; this is done by directly transforming the inapproximability gap of Håstad for the problem of maximizing the number of satisfied equations for a set of equations over $\operatorname{GF}(2)$ [4] and is interesting in its own right. Our inapproximability results on full siblings reconstruction problems answers open questions about the computational complexities of these problems posed by Berger-Wolf et al. [1]. Our results on the maximum profit coverage problem provides almost matching upper and lower bounds on the approximation ratios for this problem posed by Hassin and Or [3].


Keywords: combinatorial optimization • set covering problems • computational biology

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# Profit-based Latency Problems on the Line 

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#### Abstract

Consider the following problem. Given are a set of $n$ clients located in some metric space and profits $p_{i}$ associated with each client $i, 1 \leq i \leq n$. In addition, a single server is given, positioned at the origin at time $t=0$. The server travels at unit speed. If the server serves client $i$ at time $t$, the revenue collected by the server equals $p_{i}-t$. The goal is to select clients and to find a route for the server serving the selected clients, such that total collected revenue is maximal. We refer to this problem as the traveling repairman problem with profits, or TRPP for short. The TRPP is a generalization of the well-known traveling repairman problem (TRP), also known as the minimum latency problem (MLT). In the TRP, no profits are given and the goal is to serve all clients with minimal total latency. We restrict ourselves to the line as a metric space. Notice that in the TRPP (i) not every client needs to be served, and (ii) the revenue collected at a client depends on the time needed to reach that client.


We show:

- how a dynamic program solves the TRPP on the line in polynomial time, thereby generalizing a classical result from Afrati et al. [1],
- how this result can be generalized to the problem with multiple identical servers (referred to as MTRPP on the line),
- that the problem with multiple non-identical servers and release dates for each client, is NPhard.

In the proof of the latter result we settle the complexity of an open problem mentioned in de Paepe et al. [2].

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Keywords: minimum latency • traveling repairman • dynamic programming • complexity
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# A Graph-Partitioning-Based-Heuristic for Optical Network Planning Problems 

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#### Abstract

For a given set of connection requests, a given network topology and a given number of wavelengths, the static Routing and Wavelength Assignment problem (RWA) consists in maximazing the number of established connections (i.e. the connections for which a lightpath - a path and a wavelenght - has been assigned) under two main constraints: (i) a wavelength can be used at most once on a given link and (ii) the same wavelength must be used on all the links of any lightpath. Multicommodity flow models have been proposed to formulate this problem as an Integer Linear Program (ILP) leading to exact algorithms [1], but since this problem is NP-hard, numerous approximate algorithms have been developed [2]. We propose a graph-partitioning-based-heuristic to solve large instances of the RWA problem, consisting in decomposing the initial instance into $k$ local instances. These instances are obtained by partitioning the set of edges of the network using a tabu search whose objective is to maximize the number of connection requests belonging to the local instances (a connection request does not belong to any local instance when all the paths that are likely to support the connection are cut by the partition). Then the local instances are solved exactly using the ILP model, and all established lightpaths are brought together in order to build a solution of the global instance. The constraints of the RWA problem are satisfied for this solution since the local instances do not share edges. Finally, we merge all the remaining traffic demands and we try to route these demands in the global graph, using the ILP model with some extra constraints describing the resources already used by the established lightpaths.


We validate the proposed method on a real network and on large random instances. The obtained results are compared with the optimal ones if possible or with those of a sequential algorithm. Encouraging results have been found.
Keywords: graph partitioning • optimization • tabu search • optical networks

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## Scheduling Tasks on Parallel Machines with Network-Based Restrictions

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#### Abstract

We consider the (NP-Complete) problem of task scheduling with restrictions, in which each job $j$ has processing time $p_{j}$ on a certain subset of a set of parallel machines, and may not be processed on the other machines; the subset of machines may differ for each job. This is one of the more notorious open problems in the theory of approximation algorithms for combinatorial scheduling. A 2 -approximation algorithm for this problem has been known for close to twenty years; no algorithm can achieve better than a $\frac{3}{2}$-approximation unless $P=N P$ (2). In our first result, we consider two special cases of this problem, in which the restrictions can be modeled as a permissibility graph. The permissibility graph is a directed graph, with the nodes in the graph representing machines from the set $M$. Jobs from the set $J$ originate on machines from $M$, and a job may only be processed on machines to which there is a directed path from the job's origin machine. When the permissibility graph is a leveled hierarchy, it models machines or workers with hierarchical capabilities. The machines in such a network are separated into numbered levels, and a job may only run on machines of the same level or higher. For this model, we give a $\frac{4}{3}$-approximation algorithm. When the permissibility graph is a tree, we give a very simple and intuitive 2 -approximation algorithm.

The task scheduling with restrictions problem has seen a variety of applications, notably as a key subroutine in algorithms for task scheduling in networks. In our second set of results, we consider a natural special case of that problem, in which the network is modeled by one latency parameter, and give both centralized control and distributed approximation algorithms for task scheduling in that model. We also demonstrate a hardness bound of $\frac{3}{2}$ for the distributed case. The centralized control algorithm is a $\frac{4}{3}$-approximation, while the distributed control algorithm is a 2 -approximation with matching hardness bound when the latency parameter is initially unknown.

The leveled-hierarchy and network scheduling algorithms are polynomial-time and feature better approximation factors than any generalized algorithms for these problems. The network scheduling algorithms also yield low approximation factors in a model that can reasonably describe real world conditions.


Keywords: combinatorial optimization • network job scheduling • approximation algorithm

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## Approximating Geometric Coverage Problems

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#### Abstract

We study the approximability of geometric versions of the unique coverage problem and the minimum membership set cover problem. In the unique coverage problem, one is given a family of sets of elements from some universe and aims to select sets so that the number of elements contained in precisely one selected set is maximized. In the budgeted variant, each element has a profit, each set has a cost, and the goal is to maximize the total profit of the uniquely covered elements, subject to the constraint that the total cost of the chosen sets does not exceed the budget. The (budgeted) unique coverage problem was proposed by Demaine et al. (SODA'06) and is mainly motivated by applications in wireless networks. It is therefore natural to consider Unique Coverage in a geometric setting. We use the well-known unit disk model for wireless networks and show that Unique Coverage remains NP-hard in this case and present a polynomial-time 18 -approximation algorithm. This algorithm can be extended to the budgeted version of the problem and also to a problem variant where covering an element multiple times still yields positive profit, but the profit of an element decreases with the number of chosen sets that cover it. We give an asymptotic FPTAS in case the disks have arbitrary size but bounded ply. For the case that the geometric objects are arbitrary fat objects, we show that these problems are as hard to approximate as in the general case.

A related problem is Minimum Membership Set Cover. Given a family of sets of elements from some universe, one has to choose sets so that every element is covered and the maximum overlap (the number of chosen sets containing the same element) is minimized. Kuhn et al. (COCOON'05) showed that this problem has an $O(\ln n)$-approximation algorithm and no $(1-\epsilon) \ln n$-approximation for any $\epsilon>0$, unless $N P \subset D T I M E\left(n^{O(\log \log n)}\right)$. Considering the geometric version of the problem, we prove that approximating the problem with ratio less than 2 is NP-hard for unit disks and unit squares. We also give a 5 -approximation algorithm for unit squares provided that the optimal objective value is bounded by a constant.


An extended abstract describing these results has appeared in [1].
Keywords: combinatorial optimization • unique coverage • minimum membership set cover

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## A Tabu Search approach to solve the mixed integer bilevel formulation of a network design problem

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#### Abstract

The fixed charge network design problem (FCNDP) consist of selecting a subset of edges from a given set, in such a way that a set of given commodities can be transported from their origins to their destinations. The problem states the minimization of the sum of fixed costs (due to selecting the edges) and variable costs (depending on the flows of commodities on those edges). Both fixed and variable costs are linear and the edges are uncapacitated [2]. A variant of the FCNDP states that the commodities are routed along the shortest path on the resulting network (we call this problem FCNDP-SPR). This characteristic adds new constraints to the more general problem.

Despite the FCNDP has been widely studied, the FCNDP-SPR has been treated almost only as part of bigger problems in specific application areas. For example, in telecommunications, shortest path routing constraints are used to model the routing of the traffic demand in the context of internet protocol network design [1]. In transportation, these constraints have been used to model the traffic on urban transportation networks, but they were not explicitly formulated. To include shortest path routing constraints in a mixed integer program is not straightforward. Difficulties arise both at the modeling as well as to design efficient solution methods. The FCNDP-SPR involves two decision makers acting non-cooperatively and in a sequential way. In this work we formulate the FCNDP-SPR as a mixed integer bilevel linear programming (MIBLP) problem. The upper level objective function minimizes the sum of fixed and variable costs associated with edges. The lower level problem defines a set of independent minimum path problems on the network resulting from the decision taken by the upper level. Let us refer to this model as (IBP). We can obtain a mixed integer programming formulation to the FCNDP-SPR replacing the lower-level linear program by its optimality conditions. Let us refer this formulation as (IP).

Although linear bilevel programming is an important area in optimization, there are only a small number of solution approaches to treat the discrete and mixed cases. One reason for this is the fact that traditional branch-and-bound algorithms for mixed-integer programming cannot be applied to MIBLP problems [3]. In this work we implemented a tabu-search metaheuristic [4] using the formulation (IBP) to solve the FCNDP-SPR problem. In order to improve the performance of this method we use information coming from formulation (IP) to guide the search and select the initial points.


Keywords: network design • shortest path routing • discrete bilevel programming • tabu search

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## A branch and bound algorithm for a generalized Job Shop Scheduling problem

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#### Abstract

We deal with a generalization of the Job Shop Scheduling problem, where each operation processing must be assisted by a human operator. This problem is related to a class of scheduling models with resource constraints, introduced by Blazewicz et al. [1] and later addressed by many authors [4, 2]. Similarly, in [5], a resource constrained scheduling problem is investigated in a flow shop setting.

We are given $m$ machines, $p$ human operators and $n$ jobs, where each job $j$ is a chain of operations, where each operation has to be processed by a specific machine. The constraints of our problem are the following: at any time (i) each machine can process at most one operation; (ii) and each job can be processed at most by one machine; (iii) no preemption is allowed; (iv) each machine can process an operation only in presence of an operator; $(v)$ each operator can attend at most one operation. Hence, a feasible solution to our problem is a Job Shop Scheduling problem feasible solution, with the additional constraint that at most $p$ machines work simultaneously. The objective is the minimization of the makespan. We characterize the complexity of the problem by proving the NP-hardness of some minimal cases, namely, when $m=3$ and $p=2$, and when $n=3$ and $p=2$. As a byproduct of this contribution we may strengthen a former result by Du et al. [3] by proving that $P 2 \mid$ chains $\mid C_{\max }$ is NP -hard when the number of chains is three.


We propose heuristic procedures for some special cases and an enumeration scheme for the general case, based on a generalization of the well known disjunctive graph model used for standard scheduling problems. The disjunctive graph associated with our problem contains two types of disjunctive arcs: the classical ones and the operators ones. The set $D$ consists of classical disjunctive arcs connecting pairs of operations that have to be processed by the same machine; and the set $E$ includes disjunctive arcs connecting pairs of operations that are processed by different machines but might be attended by the same operator. In the disjunctive graph a feasible solution is obtained by selecting exactly one arc for each disjunctive pair in $D$ and at most one arc for each pair in $E$ such that $(i)$ no cycles can be detected (ii) the cardinality of the maximum antichain is at most $p$.
Keywords: scheduling • disjunctive graph • NP-completeness • resource constraints

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# Solution of the Train Platforming Problem 

Speaker: Laura Galli (DEIS, University of Bologna)

Co-author(s): Alberto Caprara • Paolo Toth (DEIS, University of Bologna)


#### Abstract

In this paper we study a general formulation of the train platforming problem, which contains as special cases all the versions previously considered in the literature as well as a case study from the Italian Infrastructure manager that we addressed. The objective of train platforming, which is the routing problem that generally follows any timetabling phase, is to find an assignment of trains to platforms in a railway station. The practical relevance of the problem inspired the definition of a few different versions, which are relatively easy for small contexts, i.e., stations with very few platforms and alternative paths to route the trains, but become extremely difficult when applied to complex railway station topologies such as those associated with the main European stations, leading to instances with hundreds of trains and tens of platforms. Moreover, most versions are not concerned with the station topology and ignore the routing phase, whereas the main European stations frequently have complex topologies and the routing issue can be quite a complicated task.


A main station typically has several external lines (also called corridors, generally with two tracks) connecting it to other main stations; these lines are called directions in our context. Moreover, there are several points at which a train may stop within the station to download/upload passengers and/or goods; these points are called platforms in our context, and can be of different type and length, some being dead-end and some being through-platforms. The connection between directions and platforms is achieved by internal lines, called paths in our context, which define a route within the station linking a given direction to a given platform. The problem aims at defining for each train the platform where it will stop and the corresponding arrival and departure paths, while ensuring that all the constraints are satisfied and minimizing the deviation from some specified "desired" arrival/departure times and stopping platforms for each train.

In particular, motivated by our case study, we consider a general quadratic objective function, and propose a new way to linearize it by using a small number of new variables along with a set of constraints that can be separated efficiently by solving an appropriate linear program. The resulting integer linear programming formulation has a continuous relaxation that leads to strong bounds on the optimal value. For the instances in our case study, we show that a simple diving heuristic based
on this relaxation produces solutions that are much better than those produced by a simple heuristic currently in use, and that often turn out to be (nearly-)optimal.
Keywords: combinatorial optimization • quadratic objective function • railway optimization $\cdot$ train platforming • train routing • exact method • branch-and-cut-and-price

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## An effective algorithm for obtaining the set of all minimal cost pairs of disjoint paths with dual arc costs

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#### Abstract

In today's telecommunications networks it is necessary, for reliability reasons, to use protection schemes involving the calculation of two (or more) disjoint paths for each node-to-node connection, especially when large amounts of traffic have to be routed in the network. This concern is particularly relevant in optical networks, namely WDM (Wavelength Division Multiplexing) networks due to the very high rates supported by lightpaths, and in the Internet using MPLS (Multiprotocol Label Switching). In this context the problem of obtaining optimal (arc or node) disjoint paths, for increasing network reliability while minimising bandwidth consumption, is extremely important.


The problem of finding $k$ disjoint paths from $s$ to (two distinct nodes), in a network with $k$ different costs on every arc such that the total cost of the paths is minimised is NP-complete even for $k=2$, when the relationship between the $k$ arc costs (in the same arc) is arbitrary. When $k=2$ these networks are usually designated as dual arc cost networks.

In this paper we propose an exact algorithm for finding the whole set of arc-disjoint path pairs, with minimal cost in a network with dual arc costs. The addressed problem can be formalised as follows. Let $G=(V, E)$ be a directed network with node set $V=\left\{v_{1}, v_{2}, \ldots, v_{n}\right\}$ and and arc set $E=$ $\left\{e_{1}, e_{2}, \ldots, e_{m}\right\}$ (were $n$ and $m$ designate the number of nodes and arcs in $G$, respectively), where two different non-negative cost functions (or metrics) in the arcs, are defined:

$$
\begin{align*}
\eta^{(j)}: & E \rightarrow \mathbb{N}_{0} \quad(j=1,2)  \tag{1}\\
& \eta^{(j)}\left(\left(v_{a}, v_{b}\right)\right)=c_{v_{a} v_{b}}^{(j)} \quad\left(v_{a}, v_{b}\right) \in E \tag{2}
\end{align*}
$$

The cost $C^{(j)}$ of a (loopless) path $p$ in $G$ with respect to metric $\eta^{(j)}$, is:

$$
\begin{equation*}
C^{(j)}(p)=\sum_{\left(v_{a}, v_{b}\right) \in p} c_{v_{a} v_{b}}^{(j)} \quad(j=1,2) \tag{3}
\end{equation*}
$$

Let path $p, p=\left\langle v_{1}, e_{1}, v_{2}, \ldots, v_{i-1}, e_{i-1}, v_{i}\right\rangle$, be given as an alternate sequence of nodes and arcs from $G$, such that the tail of $e_{k}$ is $v_{k}$ and the head of $e_{k}$ is $v_{k+1}$, for $k=1,2, \ldots, i-1$ (all the $v_{i}$ in $p$ are different). Let the set of nodes in $p$ be $V^{*}(p)$ and the set of arcs in $p$ be $E^{*}(p)$. Two paths $p=\left\langle v_{1}, e_{1}, v_{2}, \ldots, v_{i-1}, e_{i-1}, v_{i}\right\rangle$ and $q$ are arc-disjoint if $E^{*}(p) \cap E^{*}(q)=\emptyset$. Two paths $p$ and $q$ are disjoint if $V^{*}(p) \cap V^{*}(q)=\emptyset$, and are internally disjoint if $\left\{v_{2}, \ldots, v_{i-1}\right\} \cap V^{*}(q)=\emptyset$. We will say that two paths are node disjoint if they are internally disjoint.
The addressed problem is to find the whole set of pairs $(p, q)$ of arc disjoint paths which minimise the total cost of the pair, defined by:

$$
\begin{equation*}
C[(p, q)]=C^{(1)}(p)+C^{(2)}(q) \tag{4}
\end{equation*}
$$

where $p$ and $q$ have the same source and sink node.
An exact algorithm for solving this NP-complete problem will be proposed, based on a condition which guarantees that the optimal path pair cost has been obtained. This optimality condition is based on the calculation of increasingly tightened upper and lower bounds on the optimal cost. A formal proof of the correctness of the algorithm is described. Extensive experimentation is presented to show the effectiveness of the algorithm.

It will also be explained how the proposed approach can also be used for obtaining the minimal cost disjoint path pair with constraints on the maximum number of arcs allowed per path, a problem of interest in various applications, namely in telecommunication networks.
Keywords: telecommunication networks • paths with minimal cost sum • dual arc costs
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# Scheduling on parallel machines, with perishability time windows, inspired by the process of micro-biological food testing. 

Speaker: Celia A. Glass (Cass Business School, City University, London, UK)<br>Co-author(s): Konstantin Chakhlevitch (Cass Business School, City University, London, UK)


#### Abstract

This paper explores the combinatorial problems associated with co-ordinating two key processes in a food testing micro-biology laboratory. Most of the microbiology tests are carried out on glass dishes, in an agar compound. The type of agar employed is very specific to the nature of the test. Each food sample requires a whole suite of tests to be performed successfully in order to be certified fit for consumption. Thus the sequence of samples determines parallel demand for agars. The co-ordinating of this demand with production of agars is the topic of interest. The agars are produced in batches on parallel identical machines. The main complication, over and above the parallel demands for different agars, is the limited lifetime over which the agars are effective. This complication takes the problem beyond any studied in the literature. We explore the abstracted agar production problem deriving feasibility conditions on its solution, and construct a Dynamic Programming solution. In addition, a second related combinatorial problem is examined in this talk. It is that related of sequencing samples to facilitate agar production which is related to DNA sequencing,


Keywords: combinatorial optimization • scheduling • parallel machines; time windows
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## Asymptotically optimal algorithms for m-layer planar 3-dimensional assignment problem.

Speaker: Yury Glazkov (Sobolev Institute of Mathematics SB RAS)
Co-author(s): Edward Gimadi (Sobolev Institute of Mathematics SB RAS)


#### Abstract

The $m$-layer planar 3-dimensional assignment problem is for a given 3-dimensional matrix $c=\left(c_{i j k}\right)$ of size $m \times n \times n$ to find $m$ permutations $\varphi_{i}$, minimizing the sum $\sum_{i=1}^{m} \sum_{j=1}^{n} c_{i j \varphi_{i}(j)}$ under the constraint that for all $i \neq i^{\prime}$ and all $j=1,2, \ldots, n$ holds $\varphi_{i}(j) \neq \varphi_{i^{\prime}}(j)$. We consider two variations of the problem: the one mentioned above and the other with an additional constraint that all permutations $\varphi_{i}$ are needed to be cyclical, i.e. could be decomposed into one cycle (e.g. for $\varphi_{1}(1)=1, \varphi_{1}(2)=3$, $\varphi_{1}(3)=2, \varphi_{2}(1)=2, \varphi_{2}(2)=3, \varphi_{2}(3)=1$ permutation $\varphi_{2}$ is cyclical while permutation $\varphi_{1}$ is not). We consider a special type of random instances: $c_{i j k}$ are independent random variables distributed (not necessarily identically) on $\left[a_{n}, b_{n}\right], b_{n}>a_{n}>0$. For each of the problems we give an approximation algorithm which layer by layer for $i=1,2, \ldots, m$ constructs a permutation $\varphi_{i}$. The construction is made in two stages, in either a part of the permutation is constructed. The algorithm uses a greedy heuristic in the first stage to construct the part of $\varphi_{i}$ containing most of elements and a special procedure (different for the two variations of the problem) in the second stage to complete $\varphi_{i}$ to feasibility (i.e. in such a way that it satisfies the constraints). Considering probabilistic behavior of the algorithm we estimate the first parts of the permutations using Petrov's theorem and their second parts using worst-case bound thus proving asymptotical optimality for a range of distributions.


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07-07-00222) and INTAS (project 04-77-7173).
Keywords: multidimensional assignment problem • random instances • approximation algorithm • asymptotically optimal algorithm
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# A Branch-and-Price Approach to the Bandwidth Allocation Problem in Wireless Networks 

Speaker: Cristiana Gomes (INRIA/i3s(CNRS-UNSA))
Co-author(s): Gurvan Huiban • Hervé Rivano (INRIA/i3s(CNRS-UNSA))


#### Abstract

We deal with a bandwidth allocation problem called Round Weighting Problem (RWP) [1], that jointly considers the multi-commodity flow problem and the weighted fractional coloring problem. We have to find sets of compatible communication links in a network, called rounds, such that the routers bandwidth requirements are achieved.

A network topology is a communication graph $G$ where the nodes are the routers and the edges are the links. Interferences between links are given as a conflict graph $G_{c}$. We consider the case where data are sent to the gateways, therefore the flow subproblem is a single-commodity problem. This is the core optimization of wireless mesh networks provisioning [2]. The RWP input corresponds to $G$, $G_{c}$, and the network bandwidth proportion between each router and a given set of gateways. Each edge $e$ of $G$ receives a positive value $b(e)$ that represents the flow problem solution. Simultaneously, we find a set of rounds with weights $w\left(R_{i}\right)$ achieving the routers bandwidth ( $\left.\sum_{i} w\left(R_{i}\right) \geq b(e): e \in R_{i}\right)$, such that the total weight $\left(\sum_{i} w\left(R_{i}\right)\right)$ is minimized.

We present a mixed integer linear programming model and we use a branch-and-price [3] algorithm to solve the problem. A column generation approach is used to avoid dealing with the exponential set of rounds and the branch-and-bound algorithm to turn integer the flow (network packets). We run experiments on networks from the literature, with different number of gateways. Experimental results as well as theoretical insights let us conjecture that the round-up property holds for RWP considering uniform-traffic.


Keywords: bandwidth allocation problem • branch-and-price • round-up property

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## On asymptotic optimality of polynomial algorithm for multi-TSP in Euclidean space

Speaker: Edward Gimadi (Sobolev Institute of Mathematics SB RAS)


#### Abstract

A Travelling Salesman Problem (TSP) is a problem of finding an optimal Hamiltonian circuit in a given graph. TSP is called Euclidean if the vertices in the given graph correspond to points in the Euclidean space $\mathbf{R}^{k}$, and the weight of an edge is determined as the length of a corresponding interval in the space.


Multi-TSP ( $m-T S P$ ) is a problem of finding $m>1$ edge-disjoint Hamiltonian circuits of extremal total weight in a given graph. This problem is known also as a Peripatetic Salesman Problem ( $m$-PSP) formulated at first by Krarup (1975). It was shown that the existence of two edge disjoint Hamiltonian cycles involves NP-hardness of 2-PSP (De Kort, 1991).

In the report the maximum m-TSP in Euclidean space $\mathbf{R}^{k}$ is considered.
Let $S^{*}(X)$ denote an optimal solution to the problem. Assume having an algorithm $A$. We denote the solution given by the algorithm on an input set $X$ as $A(X)$. Let us define performance ratio
 $\Delta_{A} \rightarrow 1$ as $n \rightarrow \infty$.

In the case $m=1$ an asymptotically optimal algorithm solving this problem in $O\left(n^{3}\right)$ running time was constructed in $[1,2]$. In this report the condition of asymptotic optimality for the maximum multiple TSP in Euclidean space is presented.
Statement. The maximum m-TSP in Euclidean space $\mathbf{R}^{k}$ can be solved asymptotically optimal in $O\left(n^{3}\right)$ - time if the number $m=m(n)$ of edge-disjoint cycles in graph such that

$$
m(n)=o\left(n^{(k-1) /(k+1)}\right)
$$

Statement. Let $m=m(n)$ be the number of edge-disjoint cycles in graph such that

$$
m(n)=o\left(n^{(k-1) /(k+1)}\right)
$$

Then the maximum m-TSP in Euclidean space $\mathbf{R}^{\mathbf{k}}$ can be solved asymptotically optimal in $O\left(n^{3}\right)$ running time.

Keywords: combinatorial optimization • Euclidean multi-TSP • asymptotically optimal algorithm

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## On a Network Pricing Problem with Connected Toll Arcs

Speaker: Géraldine Heilporn (Université Libre de Bruxelles, Belgique. Université de Montréal, Canada.)

Co-author(s): Martine Labbé (Université Libre de Bruxelles, Belgique.)

- Patrice Marcotte (Université de Montréal, Canada.)
- Gilles Savard (école Polytechnique de Montréal, Canada.)


#### Abstract

: Consider the tarification problem of maximizing the revenue generated by tolls set on a subset of arcs of a transportation network, where origin-destination flows are assigned to shortest paths with respect to the sum of tolls and initial costs. The Network Pricing Problem with Connected Toll Arcs deals with structured networks in which all toll arcs must be connected and define a path. As those special structures can represent features specific to a real highway topology, we define a highway as the set of all connected toll arcs in a network. A user, who travels from an origin to a destination in this network, can either take the shortest toll free path from its origin to its destination, or follow the highway, using shortest paths to and from the highway. We assume that users are not allowed to reenter the highway, which implies that paths are uniquely determined by their respective entry and exit nodes on the highway. This assumption will be satisfied in most real highway systems.

Linear mixed integer models for this problem are presented in literature. It has also been proved that the Network Pricing Problem with Connected Toll Arcs with a single origin destination pair or a single toll arc is polynomially solvable. We show that the Network Pricing Problem with Connected Toll Arcs is NP-hard, using a reduction from 3-SAT. Further, we consider the same problem involving a complete toll subgraph, which allows for scale economies. We prove that this General Network Pricing Problem with Connected Toll Arcs is NPhard, using a reduction from 3-SAT. We also design some new valid inequalities for the problem. Then we restrict our attention to single origin-destination pair problems. Given the new family of valid inequalities, a complete description of the convex hull of solutions for the General Network Pricing Problem with Connected Toll Arcs can be highlighted.

Finally, we point out the links between the General Network Pricing Problem with Connected Toll Arcs and a more standard pricing problem in economics.


Keywords: price setting • networks • mixed integer programming • combinatorial optimization

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# Complexity of the computation of a linear order at minimum distance from a tournament 

Speaker: Olivier Hudry (Telecom ParisTech - LTCI - UMR 5141 CNRS)


#### Abstract

When applied for instance to an election, the result of a paired-comparison method may be represented by a tournament $T$ (i.e. a complete asymmetric directed graph) when there is no tie: there is a directed edge from a candidate $x$ to a candidate $y$ if a majority of voters prefers $x$ to $y$. This tournament may be not transitive: a majority may prefer $x$ to $y$, another majority may prefer $y$ to $z$, and a third majority may prefer $z$ to $x$. This is the well-known "Voting Paradox". If $T$ is transitive, then it is a linear order, and this order can be used to rank the candidates. Otherwise, to decide who is the winner of the election is not always an easy task. Several methods, called tournament solutions, have been designed to solve this problem. P. Slater's solution [5] consists in computing a linear order at minimum distance from $T$, i.e. a linear order which minimizes the number of disagreements with respect to $T$ (see [3] for a recent survey on this problem). From a graph theoretic point of view, this problem consists in looking for a linear order $O$ which minimizes the number of directed edges of which the orientations are not the same in $T$ and in $O$. This number is called the Slater index $i(T)$ of $T$; a linear order at minimum distance from $T$ is called a Slater order of $T$; a vertex which is preferred to any other vertex in a Slater order is called a Slater winner of $T$. In this communication, we deal with the computation of the Slater index of a given tournament $T$, of a Slater winner of $T$, of a Slater order of $T$ and of the checking that a given vertex or a given linear order are respectively a Slater winner or a Slater order of $T$. Based on the recent works by N. Alon [1], P. Charbit et alii [2], and V. Conitzer [4] on the minimum feedback arc set problem applied to tournaments, we show that these problems are NP-hard.


Keywords: complexity • graphs • tournaments • linear order • Slater problem • feedback arc set problem

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# Improved best-fit heuristics for rectangular strip packing and bin packing problems 

Speaker: Shinji Imahori (University of Tokyo)

Co-author(s): Mutsunori Yagiura (Nagoya University)


#### Abstract

: Cutting and packing problems are representative combinatorial optimization problems with many applications in various areas such as steel, glass and garment industry and VLSI design. Depending on applications, different types of cutting and packing problems need to be solved, and this paper addresses the problem of placing rectangles into a strip (a rectangular object with a fixed width and a variable height) or rectangular bins (a number of identical rectangular objects with fixed width and height). We allow non-guillotine placements (i.e., placements are not restricted to those obtainable by guillotine cuts only), and consider two cases for the rotation of rectangles: (1) Each rectangle has a fixed orientation, and (2) rotations of 90 degrees are allowed. The rectangle packing problem is known to be NP-hard, and hence heuristics and metaheuristics are important in designing practical algorithms. One of the most significant heuristic algorithms called the best-fit heuristic was proposed by Burke et al. [1], which is a greedy algorithm that examines an available space as low as possible in the rectangular object and places the rectangle that best fits the space. In our previous research, we have proposed an efficient implementation of this algorithm for the rectangular strip packing problem that requires linear space and $\mathrm{O}(n \log n)$ time, where $n$ is the number of rectangles.

In this paper, we first consider how to apply the best-fit heuristic to the rectangular bin packing problem. By using similar techniques to the case of the rectangular strip packing problem, we propose an efficient implementation that requires linear space and $\mathrm{O}(n \log n)$ time. We show practical usefulness of our implementation via computational experiments.

A drawback of using the best-fit heuristic is that some regions in the placement may have poor quality. We propose a new idea to remove rectangles placed at poor quality regions and replace them more tightly by using a modified best-fit heuristic. We evaluate the execution time and the solution quality of the proposed algorithm via computational experiments.


Keywords: cutting and packing • rectangle packing problem $\cdot$ heuristic method

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## Balanced words and majorization

Speaker: Oliver Jenkinson (Queen Mary, University of London)

Abstract: We shall be concerned with words $w=w_{1} w_{2} \ldots w_{m} \in\{0,1\}^{+}:=\bigcup_{q \geq 1}\{0,1\}^{q}$, and their base-2 expansions $E_{2}(w)=\sum_{k=1}^{m} w_{k} 2^{m-k}$. Write $|w|=m$, the length of $w$, and $|w|_{1}=\operatorname{card}(\{1 \leq i \leq$ $\left.m: w_{i}=1\right\}$ ), its 1 -length. Define the cyclic shift $\sigma:\{0,1\}^{m} \rightarrow\{0,1\}^{m}$ by $\sigma\left(w_{1} \ldots w_{m}\right)=w_{2} \ldots w_{m} w_{1}$. A cyclic subword of $w$ is any length- $q$ prefix of some $\sigma^{i-1}(w), 1 \leq i, q \leq m$. A word is balanced if $\left||z|_{1}-\left|z^{\prime}\right|_{1}\right| \leq 1$ for any cyclic sub-words $z, z^{\prime}$ with $|z|=\left|z^{\prime}\right|$.
To any word $w=w_{1} \ldots w_{m}$ we associate its orbit $\mathcal{O}(w)=\left(\mathcal{O}_{1}(w), \ldots, \mathcal{O}_{m}(w)\right)$, the vector consisting of the iterated cyclic shifts $w, \sigma(w), \ldots, \sigma^{m-1}(w)$ arranged in lexicographic order $\mathcal{O}_{1}(w) \leq$ $\ldots \leq \mathcal{O}_{m}(w)$. For $1 \leq i \leq m$, define $\mathcal{I}_{i}(w):=E_{2}\left(\mathcal{O}_{i}(w)\right), \mathcal{J}_{i}(w):=\mathcal{I}_{i}(w) /\left(2^{m}-1\right)$, and $\mathcal{S}_{i}(w):=$ $\sum_{k=1}^{i} \mathcal{I}_{k}(w)$. Define the arithmetic mean $A M(w):=\frac{1}{m} \sum_{i=1}^{m} \mathcal{J}_{i}(w)$, the geometric mean

$$
G M(w):=\left(\prod_{i=1}^{m} \mathcal{J}_{i}(w)\right)^{1 / m}=\frac{\left(\prod_{i=1}^{m} \mathcal{I}_{i}(w)\right)^{1 / m}}{2^{m}-1},
$$

and the standard deviation

$$
S D(w)=\sqrt{\frac{1}{m} \sum_{i=1}^{m}\left(\mathcal{J}_{i}(w)-A M(w)\right)^{2}} .
$$

We shall give several new characterizations of balanced words, each one related to the notion of majorization (see [1, 2]), for example:
For all orbits of a given arithmetic mean, the geometric mean is maximized, and the standard deviation is minimized, precisely when the orbit is balanced.
Suppose $w \in\{0,1\}^{Q}$ has arithmetic mean $p / q$, where $1 \leq p<q$ are coprime integers. If $b$ is balanced, with arithmetic mean $p / q$, and $b^{Q / q}$ denotes its $(Q / q)$-fold concatenation, then $\mathcal{S}_{i}\left(b^{Q / q}\right) \geq \mathcal{S}_{i}(w)$ for all $1 \leq i \leq Q$.
Keywords: optimization • majorization • balanced word • Sturmian word • binary expansion

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## 2-approximation algorithm for the weighted completion time minimization on a single machine with a fixed non-availability interval

Speaker: Imed Kacem (CNRS/ICD, UTT, France)


#### Abstract

This paper focuses on scheduling a set of jobs on a single machine on which a maintenance task has to be performed under the non-resumable scenario. The objective is to minimize the total weighted completion time. The machine is unavailable during a fixed interval. Recently, Kacem and Chu [1] studied the problem and showed that WSPT rule has a tight worst-case performance ratio of 3 under some conditions. Kellerer and Strusevich [2] proposed a 4-approximation by converting the resumable solution of Wang et al. into a feasible solution for the non-resumable scenario. Despite the fact that the approach developed in this paper is also an extension of Wang et al's method [3], it is easy to see that such an approach is usually more effective than the conversion by Kellerer and Strusevich. Note that Kellerer and Strusevich proposed also an FPTAS for this problem with $O\left(n^{4} / \epsilon^{2}\right)$ time complexity, which leads to an algorithm of $O\left(n^{4}\right)$ if we convert it into a polynomial 2-approximation algorithm. For these reasons, this paper is a successful attempt to develop a polynomial 2-approximation algorithm for the studied problem.


## Main result

The problem is to schedule a set $J$ of $n$ jobs $\{1,2, \ldots, n\}$ on a single machine, with the aim of minimizing the total weighted completion time. Every job $i$ has a processing time $p_{i}$ and a weight $w_{i}$. The machine is unavailable between $T_{1}$ and $T_{2}$ and it can process at most one job at a time. Without loss of generality, we consider that all data are integer and that jobs are indexed according to the WSPT rule (i.e., $p_{1} / w_{1} \leq p_{2} / w_{2} \leq \ldots \leq p_{n} / w_{n}$ ). Let $C_{i}$ denote the completion time of job $i$. Due to the dominance of the WSPT order, an optimal schedule is composed of two sequences of jobs scheduled in nondecreasing order of their indexes. If all the jobs can be inserted before $T_{1}$, the problem studied $(\mathcal{P})$ has obviously a trivial optimal solution obtained by the WSPT rule (Smith rule). We therefore consider only the problems in which all the jobs cannot be scheduled before $T_{1}$. Let $\varphi^{*}(Q)$ denote the minimal weighted sum of the completion times for problem $Q$ and let $\varphi_{S}(Q)$ be the weighted sum of the completion times of schedule $S$ for problem $Q$.
The proposed heuristic is based on the following algorithm, which extends the one proposed by Wang et al. for the resumable version of the problem:
(i). Let $l=0$ and $\mathcal{G}_{l}=\emptyset$.
(ii). Let $\pi(i, l)$ be the $i^{\text {th }}$ job in $J-\mathcal{G}_{l}$ according to the WSPT order. Construct a schedule $\sigma_{l}=<\pi(1, l), \pi(2, l), \ldots, \pi(g(l), l), \mathcal{G}_{l}, \pi(g(l)+1, l), \ldots, \pi\left(n-\left|\mathcal{G}_{l}\right|, l\right)>$ such that $\sum_{i \in \mathcal{G}_{l}} p_{i}+$ $\sum_{1 \leq i \leq g(l)} p_{\pi(i, l)} \leq T_{1}$ and $\sum_{i \in \mathcal{G}_{l}} p_{i}+\sum_{1 \leq i \leq g(l)+1} p_{\pi(i, l)}>T_{1}$ where jobs in $\mathcal{G}_{l}$ are sequenced according to the WSPT order.
(iii). If $\sum_{i \in \mathcal{G}_{l}} p_{i}+p_{\pi(g(l)+1, l)} \leq T 1$, then: $\mathcal{G}_{l+1}=\{\pi(g(l)+1, l)\} \cup \mathcal{G}_{l} ; l=l+1$; go to step (ii). Otherwise, go to step (iv).
(iv). $\varphi_{H}(\mathcal{P})=\min _{0 \leq h \leq l}\left\{\varphi_{\sigma_{h}}(\mathcal{P})\right\}$.

Theorem 1 Heuristic $H$ is a 2-approximation algorithm for problem $\mathcal{P}$. It can be implemented in $O\left(n^{2}\right)$ time and its worst-case performance ratio is tight.

The proof of this theorem will be presented at the conference.
Keywords: approximation algorithm • heuristic method • scheduling

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## Quadratic programming on graphs without long odd cycles

Speaker: Marcin Kamiński (Department of Computer Science, Université Libre de Bruxelles, Belgium)


#### Abstract

: In the 1980s George L. Nemhauser et al. initiated the study of computational complexity of combinatorial problems on graphs without long odd cycles. In two papers they designed algorithms for the (weighted) maximum-cut ([2]) and (weighted) MAximum-Independent-Set ([3]) problems which run in polynomial time if the input graph has no long odd cycles. More precisely, the input graph should belong to the class of graphs without odd cycles longer than $2 K+1$. We denote such class by $\mathcal{G}(K)$. In this paper we study the problem of quadratic binary programming on graphs (QBPG) and show that it can be solved efficiently in $\mathcal{G}(K)$ for every fixed $K$. QBPG contains maximum-cut and maximum-independent-set as special cases. In this sense our work generalizes [2] and [3]. Another problem expressible as QBPG is signed-GRAPH-BALANCING; for definition and details see [1]. Let $G=(V, E)$ be a weighted, undirected, loopless graph without multiple edges. The vertex set $V$ of $G$ consists of vertices $v_{1}, \ldots, v_{n}$. Each vertex is assigned its weight, a real number which we denote by $a_{i}$. Each edge (with endpoints $v_{i}, v_{j}$ ) also has its weight $w_{i j}$, a nonnegative real number. For each subset $S \subseteq V$ of vertices, we define its value as the sum of weights of vertices in $S$ minus the weights of edges whose both endpoints are in $S$. QBPG consists in finding a subset of vertices with maximum value.


The QBPG problem can be written up as a quadratic binary program:

$$
\begin{array}{ll}
\max & \sum_{v_{i} \in V} a_{i} x_{i}-\sum_{\left(v_{i}, v_{j}\right) \in E} w_{i j} x_{i} x_{j}  \tag{}\\
\text { s.t. } & x_{i} \in\{0,1\}
\end{array}
$$

Lemma 1 The QBPG problem can be solved in polynomial time in the class of bipartite graphs.

The proof of this lemma uses the technique of linearization to obtain a binary linear program equivalent to $\left(^{*}\right)$. Then we show that the constrain matrix of the linear program is totally unimodular and therefore ( ${ }^{*}$ ) can be solved in polynomial time.
To obtain our main result, we generalize the recursive method developed in [2] and [3]. Its main ingredient is the following lemma.

Lemma 2 If the QBPG problem can be solved in polynomial time in $\mathcal{G}(K)$, it also admits a polynomialtime solution in $\mathcal{G}(K+1)$.

Given the input graph $G \in \mathcal{G}(K+1)$, we create a polynomial number of graphs $G_{1}, \ldots, G_{\text {poly(n) }} \in$ $\mathcal{G}(K)$ such that the optimal solution to (*) for $G$ can be built up from optimal solutions for $G_{1}, \ldots, G_{\text {poly }(n)}$.
Applying the lemma recursively, we create a search tree whose leaves are bipartite graphs. By Lemma 2 the search tree has at most a polynomial number of leaves. That combined with the statement of Lemma 1 gives the following theorem.

Theorem 3 The QBPG problem can be solved in polynomial time in $\mathcal{G}(K)$ for any $K \geq 0$.

Our result is mainly of theoretical interest. However, it is important to identify structural properties of graphs which make hard combinatorial problems easier. The absence of long odd cycles seems to work in favor. Other problems solvable in polynomial time in $\mathcal{G}(K)$ are vertex- and edgecolorability ([4]). Certainly it is interesting to pursue the study initiated by [2,3] and find more problems which admit polynomial-time algorithms in the class of graphs without long odd cycles. Our contribution is a step in this direction.

Keywords: quadratic programming • long odd cycles • polynomial-time algorithm

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# Separation Algorithms for 0-1 Knapsack Polytopes 

Speaker: Konstantinos Kaparis (University of Southampton)

Co-author(s): Adam N. Letchford (Lancaster University)


#### Abstract

Valid inequalities for 0-1 knapsack polytopes often prove useful when tackling hard 0-1 Linear Programming problems [2]. Such polytopes have been widely studied in the past and many classes of valid inequalities are known such as the lifted cover inequalities (LCIs) of Balas [1] and the weight inequalities (WIs) of Weismantel [3]. The key element though for the successful application of such inequalities is the implementation of effective separation algorithms for detecting them. In this talk we will briefly review the literature on 0-1 knapsack polytopes and separation algorithms. Then we will present some new and effective schemes for separating LCls and extended cover inequalities (ECls), while we will reach the conclusion that the separation problem for ECls can be solved exactly in $O(n b)$ time (where $n$ is the number of items, $b$ is the knapsack capacity). We will also present two new exact separation routines for WIs with running times $O\left(n b a_{\max }\right)$ and $O\left(\left(n+a_{\max }\right) b\right)$ respectively (where $a_{\max }$ is the largest item weight). Finally, a new exact general separation algorithm for the 0-1 knapsack polytope will be presented, which is faster than existing methods. Computational results will be given to illustrate the relative performance of new and existing separation algorithms, and to guide us to our concluding remarks.


Keywords: combinatorial optimization • integer linear programming • branch-and-cut • knapsack problems • separation algorithms

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## Submodular functions on diamonds

Speaker: Fredrik Kuivinen (Department of Computer and Information Science, Linköpings Universitet)
Abstract: Let $V$ be a finite set. A set function $f: 2^{V} \rightarrow \mathbb{R}$ is said to be submodular if for all $A, B \subseteq V$ we have $f(A \cup B)+f(A \cap B) \leq f(A)+f(B)$. Submodular set functions have been studied extensively and examples of such functions include the cut function of graphs and the rank function of matroids. Given an oracle which computes $f$, it is known that the minimum of $f$ can be found in time polynomial in $|V|$. This was first established by Grötschel et al.[Combinatorica, 1 (1989), pp. 169-197]. We are interested in a generalised notion of submodular functions. The main motivation for us to do this is because the generalised concept seems to play an important role in the complexity of the maximum constraint satisfaction problem (see, Cohen et al. [Discrete Appl. Math., 149 (2005), pp. 53-72]).

Recall that a lattice is a partially ordered set in which each pair of elements have a least upper bound (join, $\sqcup$ ) and a greatest lower bound (meet, П). Given a lattice $L$ and a positive integer $n$ we can construct the product lattice $L^{n}$. We use $\mathbf{0}_{L^{n}}$ and $\mathbf{1}_{L^{n}}$ to denote the bottom and top element of $L^{n}$, respectively. We say that a function $h: L^{n} \rightarrow \mathbb{R}$ is submodular if $h(\boldsymbol{a} \sqcap \boldsymbol{b})+h(\boldsymbol{a} \sqcup \boldsymbol{b}) \leq h(\boldsymbol{a})+h(\boldsymbol{b})$ for all $\boldsymbol{a}, \boldsymbol{b} \in L^{n}$. Note that the subsets of $V$ can be seen as a lattice with union as join and intersection as meet (this lattice is a product of the two element lattice). A lattice $L$ is a diamond if the elements of the lattice form a disjoint union of $\left\{0_{L}, 1_{L}\right\}$ and $A$, for some finite set $A$ such that $|A| \geq 3$. Here $0_{L}$ is the bottom element of $L$, and $1_{L}$ is the top element of $L$, and all elements in $A$ are incomparable to each other. We want to emphasise that diamonds have a different structure compared to the lattice defined by union and intersection. In particular, diamonds are not distributive, that is they do not satisfy $x \sqcap(y \sqcup z)=(x \sqcap y) \sqcup(x \sqcap z)$ for all $x, y, z \in L$. Let $f$ be a submodular set function $f: 2^{V} \rightarrow \mathbb{R}$ such that $f(\emptyset)=0$. Let $\mathbb{R}^{V}$ be the set of all functions mapping elements of $V$ into $\mathbb{R}$. For $\boldsymbol{x} \in \mathbb{R}^{V}$ and $A \subseteq V$ we define $\boldsymbol{x}(A)=\sum_{a \in A} \boldsymbol{x}(a)$. Furthermore, if $\boldsymbol{x}(v) \leq 0$ for all $v \in V$ then $\boldsymbol{x} \leq 0$. Now, let $P(f)=\left\{\boldsymbol{x} \in \mathbb{R}^{V} \mid \forall X \subseteq V, \boldsymbol{x}(X) \leq f(X)\right\}$ and $B(f)=\left\{\boldsymbol{x} \in \mathbb{R}^{V} \mid \boldsymbol{x} \in P(f), \boldsymbol{x}(V)=f(V)\right\}$. Edmonds [Combinatorial Structures and Their Applications (1970), pp. 11-26] proved the following min-max relation: $\min _{X \subseteq V} f(X)=\max \{\boldsymbol{x}(V) \mid \boldsymbol{x} \in P(f), \boldsymbol{x} \leq 0\}=\max \left\{\boldsymbol{x}^{-}(V) \mid \boldsymbol{x} \in B(f)\right\}$.
Our result is a similar characterisation of the minimisers of a submodular function $h: L^{n} \rightarrow \mathbb{R}$ where $L$ is a diamond and $h\left(\mathbf{0}_{L^{n}}\right)=0$. Let $[n]$ denote the set $\{1, \ldots, n\}$ and let $\mathcal{V}(A)$ be the set of functions mapping $[n] \times A$ into $\mathbb{R}$. For $\boldsymbol{x} \in \mathcal{V}(A)$ we define $\boldsymbol{x}^{-} \in \mathcal{V}(A)$ as $\boldsymbol{x}^{-}(i, x)=\min \{0, \boldsymbol{x}(i, x)\}$ for all $i \in[n]$ and $x \in A$. If $x(i, x) \leq 0$ for all $i \in[n]$ and $x \in A$ we write $x \leq 0$. For $x \in \mathcal{V}(A)$ and $\boldsymbol{y} \in L^{n}$ we define $\boldsymbol{x}(\boldsymbol{y})=\sum_{i=1}^{n} g(\boldsymbol{x}, \boldsymbol{y}(i), i)$. The function $g: \mathcal{V}(A) \times L \times[n] \rightarrow \mathbb{R}$ is defined as $g\left(\boldsymbol{z}, 0_{L}, i\right)=0, g(\boldsymbol{z}, x, i)=\boldsymbol{z}(i, x)$ if $x \in A$, and $g\left(\boldsymbol{z}, 1_{L}, i\right)=\min _{a \in A} \boldsymbol{z}(i, a)+\max _{a \in A} \boldsymbol{z}(i, a)$ otherwise. We define $P_{D}(h)$ and $B_{D}(h)$ as follows, $P_{D}(h)=\left\{\boldsymbol{x} \in \mathcal{V}(A) \mid \forall \boldsymbol{y} \in L^{n}, \boldsymbol{x}(\boldsymbol{y}) \leq h(\boldsymbol{y})\right\}$ and $B_{D}(h)=\left\{\boldsymbol{x} \in \mathcal{V}(A) \mid \boldsymbol{x} \in P_{D}(h), \boldsymbol{x}\left(\mathbf{1}_{L^{n}}\right)=h\left(\mathbf{1}_{L^{n}}\right)\right\}$. Our main result is that $\min _{\boldsymbol{x} \in L^{n}} h(\boldsymbol{x})=$ $\max \left\{\boldsymbol{z}\left(\mathbf{1}_{L^{n}}\right) \mid \boldsymbol{z} \in P_{D}(h), \boldsymbol{z} \leq 0\right\}$. We also prove that these quantities are equal to $\max \left\{\boldsymbol{x}^{-}\left(\mathbf{1}_{L^{n}}\right) \mid \boldsymbol{x} \in B_{D}(h)\right\}$.

Keywords: submodular functions • combinatorial optimization • min-max theorem
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## Seperation of $\left\{0, \frac{1}{2}\right\}$-Chvátal-Gomory cuts in general integer programs

Speaker: Manuel Kutschka (BT Networks Research Centre)
Co-author(s): Arie M.C.A. Koster (Warwick Business School)

- Adrian Zymolka (Axioma (UK) Ltd.)

Abstract: Each pure integer linear program (ILP) can be written in its standard minimization form $\min \left\{c^{T} x: A x \leq b, x \geq 0, x \in \mathbb{Z}^{n}\right\}$ with integer matrix $A \in \mathbb{Z}^{m \times n}$, an integer right hand side $b \in \mathbb{Z}^{m}$, and arbitrary objective values $c \in \mathbb{R}^{n}$ (here $m$ is the number of rows and $n$ the number of columns of A).

Given a system $A x \leq b$, a Chvátal-Gomory (CG) cut is defined by

$$
\begin{equation*}
\left\lfloor u^{T} A\right\rfloor x \leq\left\lfloor u^{T} b\right\rfloor \tag{5}
\end{equation*}
$$

with $u \geq 0$. It is easy to show that undominated CG cuts have $u \in[0,1)^{m}$. By the integrality of $x$, (5) is valid. Chvátal-Gomory cuts are among the most well-known classes of cutting planes for general ILPs. In case the constraint multipliers are either 0 or $\frac{1}{2}$, such cuts are known as $\left\{0, \frac{1}{2}\right\}$-cuts. It has been proven by Caprara and Fischetti (1996) that separation of $\left\{0, \frac{1}{2}\right\}$-cuts is $\mathcal{N} \mathcal{P}$-hard.

This talk reports on our study to separate general $\left\{0, \frac{1}{2}\right\}$-cuts effectively, despite its $\mathcal{N} \mathcal{P}$-completeness. We present a number of preprocessing rules, ranging from obvious observations to a sophisticated procedure based on Gaussian elimination to eliminate rows and columns, that reduce the size of the separation problem considerably. After preprocessing, violated $\left\{0, \frac{1}{2}\right\}$-cuts can often be indicated directly as single rows of the reduced problem. Our computational experiments show that this is a very vital idea generating many violated $\left\{0, \frac{1}{2}\right\}$-cuts with small effort.
Independently from the preprocessing, an ILP is formulated to find the most violated $\left\{0, \frac{1}{2}\right\}$-cut. This auxiliary ILP can be solved either for the original separation problem or the reduced one. In a computational study we show that the exact separation can typically be sped up by a factor of at least 10 if preprocessing is performed first.

The effect of the separation of $\left\{0, \frac{1}{2}\right\}$-cuts on the performance of state-of-the-art ILP solvers is documented in a further computational study. It shows that by exact separation the number of branch\&cut nodes is reduced by $20 \%$ on average at the cost of increased overall computation times due to the auxiliary ILP that has to be solved. Moreover, it is unclear whether the most violated $\left\{0, \frac{1}{2}\right\}$-cut is also the one that strengthens the formulation the most. Therefore, we additionally propose a heuristic routine to find violated $\left\{0, \frac{1}{2}\right\}$-cuts that are likely to strengthen the formulation. Computational experiments show that $\left\{0, \frac{1}{2}\right\}$-cuts can reduce the overall computation times by $20 \%$ for moderately sized instances, compared with default settings. This success has resulted in the introduction of $\left\{0, \frac{1}{2}\right\}$-cuts in the latest release of ILOG CPLEX, the leading commercial vendor of ILP software.

Keywords: integer linear programming • Chvátal-Gomory cuts • separation algorithms

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Acknowledgement: Most us the research has been done while all authors were employed at Zuse Institute Berlin (ZIB). Most of the results have been published in [1].
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## Timetable Syncronisation for Rail Mass Transit

Speaker: Janny Leung (The Chinese University of Hong Kong)

Co-author(s): Rachel Wong (The Chinese University of Hong Kong)


#### Abstract

: Nothing to do but to look for the next transfer train is the passenger's plight when taking public transit in many places. To be able to design timetables with good co-ordination between train-lines so that passengers could enjoy "immediate" transfer is a service goal of the Mass Transit Railway Corporation (MTRC), which runs six railway lines with 13 interchange stations in Hong Kong. Whilst important, this problem has not received widespread research attention.

In this paper, we propose a mixed integer programming (MIP) optimization model for this timetable synchronization problem. The objective is to minimize the sum of all waiting times of all passengers


at interchange stations in a railway system. By adjusting the trains' run-times and station dwell-times during their trips, and their dispatch times, turnaround times and headways at the terminals, we can construct high-quality timetables that optimize the objective of minimizing passenger waiting times. A novelty in our formulation is the use of binary variables to determine the relative sequencing of trains on different lines with passenger transfers, which enables the correct representation of the waiting times for transfers to the "next available" train at interchange stations. Furthermore, in our model, we not only adjust run times and dispatch times of trains but also dwell times, turnaround times and headway of trains, which are not studied in other papers. Numerical results will be reported, which indicate that our approach improves the synchronization of the current schedule significantly.

With trains departing every few minutes from each terminal, there are a large number of trips to consider, and hence the MIP formulation for the timetable synchronization contains thousands of binary variables and tens of thousands of continuous variables and constraints. We also investigated an optimization-based heuristic for this problem, where we heuristically "fix" the values of "most" of the binary values (based on the solution to the LP-relaxation), which determine the relative sequencing of the trains on different lines. We then solve the resulting MIP formulation, which is much smaller than the original MIP. By iteratively and heuristically searching for the subsets of integer variables to fix, we can get good-quality solutions within a reasonably short time.

In our preliminary study, we consider the train schedule in the MTR system in Hong Kong for both rush-hour and non-rush-hour periods. Using our model formulation, we constructed a schedule that reduces the waiting time for transferring passengers significantly compared to the current schedule. We also explore the trade-offs among different operational parameters and flexibility and their impact on overall passenger waiting-times.
Keywords: timetables • mass transit • scheduling
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## Stability Preserving Transformations of Graphs

Speaker: Vadim Lozin (Mathematics Institute, University of Warwick)


#### Abstract

A subset of pairwise nonadjacent vertices in a graph $G$ is called stable (or independent). The cardinality of a maximum size stable set in $G$ is called the stability number of $G$ and is denoted $\alpha(G)$. Finding a maximum stable set in a graph is known to be an NP-hard problem. A useful tool to simplify the problem or solve it efficiently for graphs in special classes is based on transforming a given graph $G$ into a new graph $G^{\prime}$ in such a way that the difference $\alpha(G)-\alpha\left(G^{\prime}\right)$ is easy to compute. A trivial example is the deletion of an isolated vertex, which reduces the stability number by exactly one. A more sophisticated example comes from the matching theory and is known as the cycle shrinking (see e.g. [2]). This reduction is a key tool to solve the maximum matching problem, which is equivalent to the problem of finding a maximum stable set in the class of line graphs.

The literature provides many more examples of graph transformations that can be useful for the maximum stable set and related problems, such as minimum vertex cover or maximum clique. Originally, these transformations have been described in different terms and have been obtained by different means. We survey available results on this topic and propose a unified approach for the description and development of graph transformations for the maximum stable set problem. To this end, we employ the notion of a transformation plan, introduced in [1], and elaborate it in a non-trivial way. Our approach allows to develop transformation plans systematically and leads to generalization of several previously known transformations. We illustrate the approach by a number of new examples


and discuss possible applications of the obtained results.
Keywords: stability number • graph transformation

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# A local search algorithm for determining tree decompositions of graphs 

Speaker: Bert Marchal (Maastricht University, The Netherlands)

Co-author(s): Stan van Hoesel (Maastricht University, The Netherlands)

- Arie M.C.A. Koster (Warwick Business School)


#### Abstract

In this paper we present a local search algorithm for upper bounding the treewidth of a graph. The algorithm makes use of a new neighborhood structure with which tree decompositions or triangulations of a graph are determined.

It is well known that the treewidth of a graph $G$ is equal to the minimum over all its triangulations of the maximum clique size minus one. Our upper bound heuristic is a local search method, the neighborhood structure of which will be defined in terms of the configuration of the fill-in edges in a triangulation of $G$. Minimal triangulations of $G$ have the property that all fill-in edges form the unique chord of at least one 4-cycle. Chordality of such a triangulation of $G$ will therefore be lost when one of these fill-in edges is deleted from the triangulation. We prove that the chordality can be restored by adding the opposite chords of the 4-cycles for which the deleted fill-in edge was the unique chord. This twofold action of deleting one fill-in edge and restoring chordality by adding a set of new fill-in edges is what we will refer to as a (single) flip.


Theorem 4 Deletion of a fill-in edge in a chordal graph and addition of all opposite chords of the 4 -cycles for which the deleted fill-in edge was the unique chord, will create a chordal graph.

In addition to the single flip method, we introduce the multi flip variant. Instead of deleting one fill-in edge from the triangulation, this method tries to remove a complete set of fill-in edges simultaneously. These sets will be called multisets. We show that if a multiset satisfies some additional conditions, chordality can be restored afterwards by adding new fill-in edges that are opposite chords of the 4 -cycles for which the removed fill-in edges were unique chords.

Theorem 5 Deletion of a multiset in a chordal graph and addition of all opposite chords of the 4cycles for which its fill-in edges were unique chords, will generate a chordal graph under the condition that

- none of the edges in the multiset is part of a 4-cycle for which some other edge from the multiset forms the unique chord.
- for no pair of edges from the multiset, their end vertices induce a $K_{4}$ in the chordal graph.

Both neighborhood structures are quite large in the sense that many (sets of) fill-in edges can be selected. Moreover, it is not straightforward to compute the width of a new triangulation. Furthermore, the new triangulation need not be minimal in the sense that single fill-in edges can possibly be deleted without violating chordality. Therefore, our algorithms use a restricted set of admissible edges for both the single and multi flip neighborhoods. In the multi flip method, the size of the neighborhood structure can be further reduced by putting a limit on the size of the multisets. Already the restricted local search algorithms compare favorably to other approximation algorithms in tightness of the solutions generated. The multi flip neighborhood performs similar to the single flip neighborhood in terms of solutions generated, but needs considerably less time. Finally, in almost all cases we can improve on the results of a single run of the algorithms by performing a number of runs using random triangulations as a starting solution.

Keywords: treewidth • local search • tree decomposition
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## Where is the Symmetry in Vertex Colouring?

Speaker: Jakub Mareček (The University of Nottingham)

Co-author(s): Edmund K. Burke • Andrew J. Parkes (The University of Nottingham)


#### Abstract

Search methods for combinatorial optimisation problems suffer from symmetry within the model; they often fail to recognise that large portions of the search space are equivalent, and needlessly repeat search them separately. To mitigate this, there has been a considerable level of interest in the development of symmetry-breaking methods for general search frameworks. These general symmetry-breaking methods have, however, so far not been shown to be competitive with graph colouring solvers implementing problem-specific symmetry-breaking [1]. In graph colouring problems, a number of trivial symmetry-breaking methods are known. Preprocessing, which removes vertices of full degree, low-enough degree and any vertex, whose neighbourhood is a subset of the neighbourhood of another non-adjacent vertex, can been as a form of symmetry exploitation. The "value" symmetry between the colours can be broken either by fixing colours in a single clique and imposing constraints on colours of the remaining vertices, or by using methods that do not explicitly assign colours. For symmetries from automorphisms in some graphs naturally arising within timetabling, symmetry-exploitation based on "reversible clique partitions" has been proposed [2]. (A clique partition $Q$ of a graph $G=(V, E)$ is reversible, if for all $\left\{q_{u}, q_{v}\right\}$ in the graph induced by $Q$, for all $u \in q_{u}$, and for all $v \in q_{v}$, there is an edge $\{u, v\} \in E$.) The question as to how much of the symmetry these methods exploit once they are applied, appears to be open, though. In this contribution, we study this question empirically. Implications for the utility of general symmetry-breaking and symmetry-exploitation for graph colouring are discussed.


Keywords: graph colouring • symmetry

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# Zykov Revisited: Engineering an Exact Solver for Graph Colouring 

Speaker: Jakub Mareček (The University of Nottingham)

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#### Abstract

Graph colouring is a well-known hard-to-approximate problem. Linear programming relaxations of formulations with polynomial numbers of variables are known to be weak [1] and known cuts help only to a certain extent [2]. Pricing routines for formulations with exponential numbers of variables tend to be rather involved and the early successes [3] have been found hard to replicate. It thus seems natural to re-investigate the performance of solvers not relying on computationally-expensive linear programming relaxations. In this contribution, we focus on a natural branch-and-bound algorithm for graph colouring, based on the following theorem of Zykov [4]: for any graph $G$, and any two non-adjacent vertices, the chromatic number of $G$ is the minimum of the chromatic numbers of the two graphs produced from $G$ by either merging the two vertices or by adding an edge between them. Branches of a search tree for $k$-colouring implied by this theorem are naturally pruned when the process produces a clique of size $k+1$; conversely, if a graph on $k$ or less vertices arises, then it can be used to produce a $k$-colouring. The associated "contraction algorithms" have been studied previously, but have generally not performed competitively. We, however, believe that such algorithms can be greatly improved. Previous implementations [5] were using a single "focus clique" in branching rules, but recent advances in enumeration algorithms allow one to generate a pool of maximal cliques at a guaranteed rate and use them to prune nodes, where a $k+1$ or larger clique appears as an induced subgraph, as well as in improved branching rules. Similarly, developments in tree search methodology, such as random restarts, have greatly improved the performance of SAT solvers and arguably apply also to graph colouring. Implementation of such features is currently in progress and will be reported upon.


Keywords: graph colouring • branch-and-bound

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## Keeping partners together: Algorithmic results for the Hospitals / Residents problem with couples

Speaker: Eric McDermid (University of Glasgow)

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#### Abstract

An instance $I$ of the classical Hospitals/Residents problem (HR) [2] involves two sets, namely a set $R$ of residents and a set $H$ of hospitals. Each resident in $R$ seeks to be assigned to a hospital, whilst each hospital $h \in H$ has a positive integral capacity indicating the maximum number of residents who could be assigned to $h$. Each resident ranks a subset of $H$ in strict order of preference, and each hospital ranks, again in strict order, those residents who have ranked it. A solution of $I$ is a matching (i.e., an assignment of mutually acceptable (resident, hospital) pairs such that no resident is assigned more than one hospital, and no hospital is assigned more residents than its capacity) that is stable [2, 3]. Informally, the stability of $M$ ensures that no resident and hospital would prefer to be assigned to one another than remain with their allocations in $M$. It is known that every instance of HR admits a stable matching, and that such a matching can be found in linear time using the Gale-Shapley algorithm.


HR is a many-one extension of the classical Stable Marriage problem, [2] so-called because of its widespread application to centralised automated matching schemes that allocate graduating medical students (residents) to hospital posts. In particular the National Resident Matching Program (NRMP) in the USA [6], the Canadian Resident Matching Service [7] and the Scottish Foundation Allocation Scheme (SFAS) [4, 8] all essentially incorporate extensions of the Gale-Shapley algorithm.

In the above practical applications, the existence of couples who wish to be located at hospitals close to one another gives rise to an important variant of HR called the Hospitals / Residents problem with Couples (HRC). An instance of HRC involves both single residents and couples (pairs of residents). Each couple $(r, s)$ submits a preference list over pairs of hospitals $(h, k)$, representing the assignment of $r$ to $h$ and of $s$ to $k$. Ronn [5] formulated an appropriate notion of stability for HRC and showed that an instance of HRC need not admit a stable matching. Indeed, he showed that the problem of deciding whether an HRC instance admits a stable matching is NP-complete.

In this paper we consider a natural variant of HRC in which the members of a couple agree on the hospitals to which they wish to be assigned. That is, $h=k$ for each entry in a given couple's preference list. We therefore replace each couple $(r, s)$ by a single entity $c$ having a preference list over a subset of $H$. Thus each single resident occupies one place at a given hospital, whilst each couple occupies two places. We refer to this version of HRC as the Hospitals / Residents problem with Sizes (HRS), since effectively each single resident has size 1, whilst each couple has size 2. We formulate an appropriate notion of stability in the HRS context and show that, again, a stable matching
need not exist, and the problem of deciding whether such a matching does exist is NP-complete, thus strengthening Ronn's earlier result. Our result holds even if each preference list is of length at most 3 and each hospital capacity is at most 2 . However by contrast we show HRS is solvable in polynomial time if each hospital $h$ 's preference list is of length 2, where $h$ can have arbitrary capacity. A version of HRS, called the Unsplittable Stable Marriage problem, was studied previously by Dean et al. [1]; their version differs from ours in that they permit a hospital $h$ 's capacity to be exceeded by the assignment of a couple to $h$.

We also propose an alternative model of HRS in which the members of a couple still wish to become allocated to the same hospital; however stability in this case is defined with respect to classical (GaleShapley) stability. For this version we show that, by contrast, the problem of finding a stable matching, or reporting that none exists, is solvable in polynomial time.

Keywords: stable matching • hospitals residents problem with sizes • NP-completeness • polynomialtime algorithm

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[8] http://www.nes.scot.nhs.uk/sfas (Scottish Foundation Allocation Scheme website).

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## Disruption Management in Vehicle Routing and Scheduling

Speaker: Qianxin Mu (Lancaster University)

Co-author(s): Richard Eglese (Lancaster University)


#### Abstract

Many procedures have been developed to produce optimal or near optimal solutions to vehicle routing and scheduling problems where demands and travel times are known and fixed. However, no matter how good a plan is, various disruptions may take place at the execution stage, which can make the plan no longer an optimal or even a feasible solution. Disruptions during the execution of a VRP plan may be caused by vehicle breakdowns, traffic accidents blocking one or more links, delayed departures from the depot or any service point, new orders or cancelled orders, etc. When a disruption occurs, routes should be quickly revised to minimize the negative effect it may cause to the delivery company and their customers. The disrupted VRP problem is different from the classic vehicle routing problem because in the disrupted problem, the vehicles do not all start at the depot, but their initial location depends on when the disruption occurs. The new problem may be modelled as a multi-objective optimization problem because when the disruption occurs, apart from the normal delivery costs it may also be important to minimize the measures relating to service levels such as delays in deliveries to customers that depart from the original plan. We propose a formulation of the disrupted vehicle routing problem which involves a vehicle breakdown. A heuristic algorithm based on tabu search is developed to solve the disrupted VRP. A set of test problems has been generated based on standard VRP benchmark problems and computational results from experiments using the heuristic algorithm are presented. As a quick response with a revised plan is crucial following a disruption, the heuristic needs to produce new solutions quickly. The results from the computational experiments show what advantage can be gained in the performance of the heuristic algorithm by making use of the information obtained from solving the original problem.


Keywords: heuristic method • vehicle routing and scheduling
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# Branch-and-cut techniques for solving realistic two-layer network design problems 

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#### Abstract

We study a two multi-layer telecommunication network design problem arising, e.g., in the planning of SDH/WDM networks. We consider two connected network layers. The physical network layer consists of a set of nodes connected by undirected links, corresponding to optical fibers, for instance. The logical network layer has the same set of nodes, but the links of the logical layer correspond to paths (e.g., lightpaths) in the physical layer. Modular capacities with different capacities (bit-rates) can be installed on logical links, and demands with a low bit-rate have to be routed through the network. Furthermore, electrical cross-connects (EXCs) with enough switching capacity have to be installed at both end-nodes of each logical link in order to perform the grooming of low-rate traffic into the high lightpath capacities. The goal of the planning problem is to determine a physical and logical topology, to install node and link capacities on both layers, and to route traffic demands on the logical links such that the demands survive any physical node or link failure and total installation cost is minimized. Mathematically, this corresponds to a capacitated network design and routing problem with modular node and link capacities on the logical layer coupled with a capacitated fixed-charge network design problem on the physical layer.


In this talk we show how to model and solve this planning problem using mixed-integer programming
techniques, which give us not only feasible solutions but also dual bounds to assess the quality of solutions. It turns out that compared to using a black-box MIP solver, the computation times can be significantly reduced by adding additional user-defined plugins to a branch-and-cut algorithm. More specifically, we use problem-specific preprocessing techniques, cutting planes based on either of the two layers, and MIP-based primal heuristics within a branch-and-cut approach.
In the talk, we will briefly explain the practical background of the planning problem and present a mixed-integer programming model that has been used to solve the problem. Afterwards, we will briefly explain our preprocessing steps, the network design specific cutting planes, and the MIPbased heuristics, and describe how we use them within the branch-and-cut algorithm. Eventually, we present a computational study on several realistic instances provided by Nokia Siemens Networks to investigate the effect of the different ingredients. According to preliminary results, the additional plugins significantly reduce the computation times and optimality gaps compared to a black-box MIP solver.
Keywords: telecommunication networks • network design • multi-layer network design • preprocessing • branch-and-cut • heuristic method • cutting planes
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# On the use of Surrogate Constraint Analysis to fix binary variables in the CFLP 

Speaker: Maria A. Osorio (Department of Chemical Engineering, Universidad Autonoma de Puebla)

Co-author(s): Nohemi Machorro • Abraham Sanchez (School of Computer Sciences, Universidad Autonoma de Puebla)


#### Abstract

We use dual surrogate constraint analysis to find the best surrogate and to pair it with the objective function, in the simplest case of capacitated facility location problems (CFLP), where there are facility locations which produce a single commodity for customers each with a fixed demand. If a particular source is operating (or facility is built), it has a fixed cost and a production capacity associated with it. There is also a positive cost for shipping a unit from a source to a customer The question is where to locate the sources so that capacities are not exceeded and demands are met, all at a minimal total cost. The surrogate constraint is obtained by weighting the original problem constraints by their associated dual values in the LP relaxation. A known solution is used to convert the objective function in a constraint that forces the solution to be less or equal to it. The surrogate constraint is paired with the objective function to obtain a combined constraint where negative variables are replaced by complemented variables and the resulting constraint used to fix binary variables in the model. This paper applies the experience obtained by Osorio et al (2002), (2003) using surrogate constraint analysis to fix binary variables in MKP, to CFLP. We tested our approach with the generator of test problems presented by Hooker et al (1999) and used by Osorio et al (1999) to prove the logic cuts efficiency. We also tested problems instances generated by (IFORCF, 2006) following the proposal of Cornuejols, Sridharan and Thizy (1991). In total we report results for 24 problem instances with different problem sizes and ratio $r$ of total capacity to total demand. The test problems range in size from small instances with 25 customers and 10 potential depot sites to larger instances with 500 customers and 50 potential depot sites and a ratio of total capacity to total demand equal to 1.5 and 3 , as reported in the database used. The first integer solution obtained by CPLEX was used as the best integer solution. The performance of this methodology is not affected by the size of the problem or other characteristics as the ratio capacity or the generator used to obtain the instances tested, but it is influenced by the quality of the known integer solution. This methodology only needs one LP problem to be solved every time the fixing procedure is applied and can be used


as many times as new integer solutions acting as upper bounds, can be found. This situation allows the method to be used in combination with a branch and bound tree or with other methodologies that can take advantage of the knowledge generated by the procedure. To know in advance the real value of some of the binary variables in the problem and to be able to reduce its size, can be extremely useful in problems that cannot be solved to optimality with other methodologies. For very hard instances, it can be used in combination with heuristic procedures that generate integer solutions that can be used as part of this fixing procedure. With the variables fixed and reducing the size of the problem, the heuristic can be used again to generate a better solution as many times as the procedure can fix more binary variables. The results obtained seem to be a promising way to reduce the complexity of the capacitated facility location problem. The approach can be applied every time a branch and bound or branch and cut methods get a better integer solution or it can be used as a preprocessing algorithm (as we have done), based on assuming a bound on an optimal objective value. Our computational experiments show that this preprocessing approach creates an enhanced method that allows problems to be reduced by fixing values in their binary variables.
Keywords: capacitated facility location problem • surrogate constraints • duality • constraint pairing

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## Good Triangulations Yield Good Tours

Speaker: Nik Pearson (University of Southampton)

Co-author(s): Adam N. Letchford (Lancaster University)


#### Abstract

We present a simple heuristic for the Euclidean travelling salesman problem (TSP): select a subset of the edges which induces a planar graph and solve either the TSP (or its graphical relaxation) on that graph. The talk will outline several motivations for considering this heuristic, along with extensive computational results in which the Delaunay and Greedy triangulations are the planar graphs of interest. Our experiments show that the resulting tours are on average within $0.1 \%$ of optimality.


Keywords: Euclidean TSP • triangulations • planar graphs
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## A Balanced Vehicle Routing Problem

Speaker: Ulrich Pferschy (Department of Statistics and Operations Research, University of Graz, Austria)

Co-author(s): Gerald Hubmann (Department of Statistics and Operations Research, University of Graz, Austria)


#### Abstract

We consider a logistics problem which can be seen as an intersection of a vehicle routing problem (VRP) with a graph partitioning problem. Given a set of cities with demand values our objective is to determine a partitioning of the cities into clusters and to compute a TSP solution for the cities in every cluster such that the sum of these tours is minimal. This problem is also related to the multiple TSP but does not require a common starting point. As an extension of the classical VRP condition we have both upper and lower bounds on the total demand values in every cluster. These do not only reflect capacity constraints but mainly aim for a balanced distribution of demand values between the clusters. Similar balancing conditions are imposed on the length of the tours and on the total number of cities in every cluster. As an additional and rather unusual feature we are allowed to omit a certain fraction of cities from the solution. The total demand of the eliminated cities is bounded from above and cities with a demand exceeding a lower bound can not be omitted from the solution. This allowed omission of cities is handled by a preprocessing routine which computes for each city the maximum over the minimal distances to all other cities and eliminates cities in decreasing order of these values.

Our problem arises in the planning of tours through cities in Austria where empty cigarette packets are collected in a study of the Austrian tobacco industry to produce evidence on the amount of consumed cigarettes for which no tax was payed. These are either contraband goods or legal imports, both of which are monitored by the heavily regulated industry. We developed a heuristic algorithm which selects new starting points of clusters by a criterion based of the farthest distances from previously determined starting points. Based on these starting points each remaining city is added to one of the tours by an insertion heuristic which already takes into account the balancing and capacity constraints. The resulting tours are improved by 2-Opt and 3-Opt moves and the Lin-Kernighan improvement method.

Computational experiments on real-world instances and on instances generated from the TSP-Library illustrate the achieved balance of the tours and the quality of the tour heuristic. A comparison to the previously performed tours shows that impressive savings are attained by our solution. These can be contributed both to the lack of planning and to dishonest reporting of earlier route lengths.


Keywords: vehicle routing • graph partitioning • multiple TSP

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## On The Generalized Minimum Spanning Tree Problem

Speaker: Petrică C. Pop (Department of Mathematics and Computer Science
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Co-author(s): Corina Pop Sitar • Ioana Zelina • Ioana Taşcu (Faculty of Sciences, North University of Baia Mare, Romania)


#### Abstract

Classical combinatorial optimization problems can be generalized in a natural way by considering a related problem relative to a given partition of the nodes of the graph into node sets. In the literature one finds generalized problems such as the generalized minimum spanning tree problem, the generalized travelling salesman problem, the generalized Steiner tree problem, the generalized (subset) assignment problem, etc. These generalized problems typically belong to the class of NPcomplete problems, are harder than the classical ones and nowadays are intensively studied due to the interesting properties and applications in the real world.


We are concerned with the the Generalized Minimum Spanning Tree Problem, denoted by GMST, which is a variant of the classical Minimum Spanning Tree problem, in which the nodes of an undirected graph are partitioned into clusters and we want to find a minimum-cost tree spanning a subset of nodes which includes exactly one node from every cluster. It is known that the GMST problem is NP-hard, even when it is defined on trees.

The model fits various problems of determining the location of regional service centers (e.g. public facilities, branches, distribution centers) which should be connected by building links (e.g. highways, communication links). For example, when a company tries to establish marketing centers, one for each market segment, and construct a communication network which interconnects the established centers, the company faces a GMST problem. For another example, when designing metropolitan area networks and regional area networks, we are to interconnect a number of local area networks. For this model, we must select a node in each local network as a hub (or a gateway) and connect the hub nodes via transmission links such as optical fibers. Then, such a network design problem reduces to a GMST problem.
We describe an exact exponential time algorithm for the problem based on dynamic programming, as well we present new mixed integer programming models of the problem, mainly containing a polynomial number of constraints.

In order to give compact formulations of the GMST problem one possibility is to introduce auxiliary flow variables beyond the natural binary edge and node variables. We consider three such flow formulations: a single commodity model, a multicommodity model and a bidirectional flow model.

Moreover, we establish relationships between the polytopes corresponding to their linear relaxations and present a new solution procedure for solving the GMST problem based on a new model of the problem. Computational results are reported.

Keywords: minimum spanning tree • generalized minimum spanning tree • integer linear programming • dynamic programming

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# A Strong Integer Programming Formulation of the Generalized Travelling Salesman Problem 

Speaker: Petrică C. Pop (Department of Mathematics and Computer Science Faculty of Sciences, North University of Baia Mare, Romania)


#### Abstract

Classical combinatorial optimization problems can be generalized in a natural way by considering a related problem relative to a given partition of the nodes of the graph into node sets. In the literature one finds generalized problems such as the generalized minimum spanning tree problem, the generalized travelling salesman problem, the generalized Steiner tree problem, the generalized (subset) assignment problem, etc. These generalized problems typically belong to the class of NPcomplete problems, are harder than the classical ones and nowadays are intensively studied due to the interesting properties and applications in the real world.


We are concerned with the generalized version of the travelling salesman problem (TSP) called the generalized travelling salesman problem (GTSP). Given an undirected graph whose nodes are partitioned into a number of subsets (clusters), the GTSP is then to find a minimum-cost Hamiltonian tour which includes exactly one node from each cluster. Therefore, the TSP is a special case of the GTSP where each cluster consists of exactly one node. The GTSP has several applications to location, telecommunication problems, railway optimization, etc. The GTSP is NP-hard, as it reduces to travelling salesman problem when each cluster consists of exactly one node.
The aim of this paper is to describe a new integer programming formulation of the GTSP.
Let $G=(V, E)$ be an $n$-node weighted undirected graph whose edges are associated with nonnegative costs. We will assume w.l.o.g. that $G$ is complete (if there is no edge between two nodes, we can add it with an infinite cost). Let $V_{1}, V_{2}, \ldots, V_{m}$ be a partition of $V$ into $m$ subsets called clusters (i.e., $V=V_{1} \cup V_{2} \cup \ldots \cup V_{m}$ and $V_{l} \cap V_{k}=\emptyset$ for all $l, k \in\{1, \ldots m\}$ with $i \neq j$ ). We denote the cost of an edge $e=(i, j)$ by $c_{i j}$. Let $e=(i, j)$ be an edge with $i \in V_{k}$ and $j \in V_{l}$. If $l \neq k$, then $e$ is called an inter-cluster edge; otherwise, $e$ is called an intra-cluster edge.
The new model of the GTSP arises from distinguishing between global variables, i.e. variables modelling the inter-cluster (global) connections, and local ones, i.e. expressing wether an edge is selected between two clusters linked in the global graph $G^{\prime}$, obtained from $G$ after replacing all nodes of a cluster $V_{i}$ with a supernode representing $V_{i}$.
The computational performance of the model solved with a commercial code on test problems is also presented.
Keywords: travelling salesman problem • generalized TSP • integer linear programming

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## Constraints generation for solving a bilayer network design problem

Speaker: Michael Poss (Université Libre de Bruxelles)

Co-author(s): Bernard Fortz • Martine Labbé (Université Libre de Bruxelles)


#### Abstract

: Network design problems are central to a large number of topics, such as telecommunications, transportations or power systems. The idea is to establish a network of links (optic fibers, roads, electric lines, ...) in order to carry some commodities (people, data, electricity, ...) between network's nodes. A cost is associated with the installation of those links so that the objective is to build the cheapest network which is able to satisfy given demands. If we must install link with modular capacity, the associated decision variables are integer. This makes the problem NP-hard.

Many successful resolutions [1] of such problems are based on a Benders' decomposition approach. Some of the constraints are taken out of the model leading to a decomposition into a master problem and a subproblem. The master problem is the original one, without the considered constraints. The subproblem aims to find constraints that are needed in the master to get a feasible solution for the original problem. Those algorithms cycle between the following steps, until a feasible solution is found:


- Solve the master problem (containing integer variables) for the current set of constraints.
- Use the subproblem to test whether the master problem's solution is feasible. If not, use subproblem solution to add some constraint(s) to the master.

The drawback of this approach is that it might require many master resolutions, which are MIP. Therefore, it seems natural to see how this separation can be included into the master's resolution. This allows us to solve only one MIP instead of several ones. Then the question is when to add those feasibility cuts in the branch-and-cut.

In this work, we used such a constraints generation procedure on a kind of problem arising in telecommunications: multilayer network design [2]. First, all routing variables are projected out of the problem, replaced by the so called "metric-inequalities". Then we add some of those metric inequalities during the branch-and-cut process. Preliminary results on bilayer networks appear to be efficient regarding to existing methods.
Keywords: Benders decomposition • network design • branch-and-cut

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# Online Scheduling with Known Arrival Times 

Speaker: Chris N. Potts (School of Mathematics, University of Southampton)

Co-author(s): Nicholas G. Hall (Fisher College of Business, The Ohio State University) - Marc E. Posner (Department of Industrial, Welding and Systems Engineering, The Ohio State University)


#### Abstract

We consider an online scheduling environment where decisions are made without knowledge of the data of jobs that may arrive later. However, additional jobs can only arrive at known future times. This environment interpolates between the classical offline and online scheduling environments, and approaches the classical online environment when there are many equally spaced potential job arrival times. The objective is to minimize the sum of weighted completion times, a widely used measure of work-in-process inventory cost and customer service. For a nonpreemptive single machine environment, we show that a lower bound on the competitive ratio of any online algorithm is the solution of a mathematical program. This lower bound is between $(1+\sqrt{5}) / 2$ and 2 , with the exact value depending on the potential job arrival times. We also provide a best possible online scheduling algorithm, and show that its competitive ratio matches this lower bound. We analyze two practically motivated special cases where the potential job arrival times have a special structure. When there are many equally spaced potential job arrival times, the competitive ratio of our online algorithm approaches the best possible competitive ratio of 2 for the classical online problem.


Keywords: scheduling • online algorithm • total weighted completion time • competitive analysis
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## The cut-set polytope for two-layer network design problems

Speaker: Christian Raack (Zuse Institute Berlin)

Co-author(s): Arie M.C.A. Koster (University of Warwick)


#### Abstract

We consider a problem arising in the context of planning a two-layer telecommunication network. Every link in the (logical) upper layer is represented by a path in the (physical) lower layer. Given a set of user demands, a two-layer multi-commodity routing has to be found. Necessary capacity can be provided by installing bandwidth modules on the upper layer links. Every such module


stands for a so-called light-path in the lower layer. A lower layer edge provides a limited number of channels that is consumed by traversing light-paths.
For the performance of mixed-integer programming based approaches knowledge about the facial structure of the underlying polyhedra is crucial. The most important strong valid inequality in the context of capacitated single-layer network design problems is the cut inequality. It expresses the fact that the total capacity on the links of a network cut has to bound the demand across the cut from above.

In two-layer networks logical and physical cuts are conceptually different notions. A logical cut link uses at least one physical cut edge. In addition, there can be logical links not in the logical cut but consuming physical cut channel capacity, see Figure 5.


Figure 5: A physical cut with six physical edges used by two logical links with physical cutlength 2 and 3 . Logical links with even physical cut-length cannot transport demand across the cut (they are not in the logical cut). But they consume cut channel capacity.

In this work we study cut inequalities incorporating capacity limitations of both layers simultaneously. In particular, we study the two-layer network design polyhedron restricted to a cut in the (physical) network, referred to as being the 2-layer cut-set polytope. We present facets that extend the notion of cut inequalities to multiple layers.

Keywords: telecommunication networks • network design • multi-layer network design • cut-set polyhedra • cut-inequalities • cutting planes • combinatorial optimization

## A modified Lagrangian bound for a class of many-to-many assignment problems

Speaker: Socorro Rangel (UNESP - Brazil)

Co-author(s): Igor Litvinchev (UANL, Mexico)


#### Abstract

Lagrangian relaxation in continuous and integer optimization problems is frequently used to calculate bounds for the optimal objective. It is shown how these bounds can possibly be improved by estimating the complementarity term arising in the Lagrangian function. We propose a modified Lagrangian bound and apply it to a class of many-to-many assignment problems. Results of a numerical study comparing both Lagrangian bounds, classical and modified, indicates that for all problem instances tested the Lagrangian modified bound is stronger than the classical one, and for some clusters of data - significantly stronger. However, in the cases at which the integer gap is small, that is the linear relaxation solution is closer to the integer solution, there is little scope for bound improvement. We also discuss the connections between the proposed approach and the Lagrangian Decomposition.


Keywords: Lagrangian bounds • integer linear programming • many-to-many assignment problems

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# Special cutting patterns and reduction of saw machine set ups in the cutting stock problem 

Speaker: Socorro Rangel (UNESP - Brazil)

Co-author(s): Altamir Fiqueiredo - Gabriela Mosquera


#### Abstract

In this paper we describe a heuristic procedure to generate productive cutting patterns, that is cutting patterns that have a low operational and waste cost. We present a computational study using data from the two-dimensional cutting stock problem that arises in a medium size furniture company in Brazil. The results obtained show that the heuristic procedure proposed is useful to generate cutting patterns that produces similar or better results when compared to the cutting patterns used by the Industry. However, the number of different cutting patterns generated is higher than the number of industry cutting patterns used to satisfy the demand. A high number of cutting patterns have a high operational cost given the necessary saw machine setups. A compromise between the number of different cutting patterns and the total number of objects cut to attend the demand needs to be found. An integer programming model and an heuristic procedure are also presented to solve the two-dimensional cutting stock considering both the minimization of waste and machine set-ups.


Keywords: cutting and packing • two-dimensional cutting stock • saw cycle $\cdot \mathrm{n}$-groups cutting patterns
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# Discretizing Variables for Support Vector Machines by Means of Purity Measures 

Speaker: Dolores Romero Morales (Saïd Business School, University of Oxford)
Co-author(s): Jingbo Wang (Saïd Business School, University of Oxford)


#### Abstract

In the two-class classification problem, we are given a set of objects $I$, belonging to one of the classes in $\mathcal{C}=\{-1,1\}$. Each object $u \in I$ is described by the vector $\left(x^{u}, c^{u}\right)$. The first component $x^{u}$ is called the predictor vector and takes values in a set $X \subset \mathbb{R}^{p}$. The second component $c^{u}$, with values in the set of classes $\mathcal{C}=\{-1,1\}$, is called the class-membership of object $u$. Using this information from $I$, the aim of the two-class classification problem is to derive a classifier which predicts, as good as possible, the class-membership of objects outside I. Support Vector Machines have achieved state-of-the-art performance in many real life applications, such as text categorization and microarray gene expression data, and are based on building a separating hyperplane maximizing the margin, i.e., the distance of the closest object to the hyperplane.


In this talk we build an SVM-based classifier where the set of continuous variables are replaced by discrete ones derived through using purity measures. It has already been pointed out in the literature that discretizing continuous variables increases the intuitiveness of classifier, one of the points of criticism regarding SVM. Moreover, discretization schemes can also be used in categorical variables with a large number of values, to simplify their representation by "binning" them. We use some
databases from the UCI Machine Learning Repository to illustrate the quality of our approach. Our procedure improves, on average, the discretization approaches available in the literature as well as the performance of SVM in the original set of variables, while each continuous variable is replaced, on average, by 1.5 discrete ones.

Keywords: classification • support vector machines • margin maximization • discretization
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# Computer aided discovery of families of valid inequalities 

Speaker: Stefan Ropke (Department of Transport, Technical University of Denmark (DTU))

Co-author(s): Jean-François Cordeau (Canada Research Chair in Logistics and Transportation, HEC Montréal)

- Gilbert Laporte (Canada Research Chair in Distribution Management, HEC Montréal)


#### Abstract

The branch-and-cut (BAC) method is a popular and successful approach for solving hard combinatorial optimization problems to optimality. In this talk we focus on combinatorial optimization problems like the traveling salesman problem (TSP) that can be modeled as pure integer linear problems (ILP). That is, we are interested in minimizing a linear function over a set $X=\left\{A x \leq b, x \in \mathbb{Z}_{+}^{n}\right\}$. As we cannot minimize over $X$ directly a BAC method uses $X^{\prime}=\left\{A x \leq b, x \in \mathbb{R}_{+}^{n}\right\}$ instead. When the optimization over $X^{\prime}$ results in a solution $x^{*} \notin X$ we look for an inequality $\pi x \leq \pi_{0}$ such that $\pi x^{*}>\pi_{0}$ and $\pi x \leq \pi_{0}$ for all $x \in X$. Such an inequality can be added to $X^{\prime}$ to provide a tighter


 approximation of the convex hull of $X$.When one designs a BAC algorithm for a specific combinatorial optimization problem (e.g. the TSP) one usually identifies a number of families of valid inequalities a priori and implements separation methods for these families in the BAC system. In this talk we describe a computer program named CADVI that assists the user in discovering new families of valid inequalities for any ILP. The program currently offers two tools: Using the first tool, the user provides an instance of a problem and a linear programming (LP) solution $x^{*} \in X^{\prime}$. CADVI determines if $x^{*} \in X$. If this is not the case then it proposes one or more valid inequalities that are violated by $x^{*}$. The program attempts to generate valid inequalities that are easy to understand for the user. When using the second tool the user provides an instance of a problem and a valid inequality. CADVI strengthens (if possible) this valid inequality by increasing the coefficients on the left hand side of the inequality while keeping the right hand side fixed (a kind of lifting procedure).

One way of using the first tool is to generate a number of instances and obtain LP solutions $x^{*} \in$ $X^{\prime}$ such that no inequality from the families of known valid inequalities for the problem is violated by $x^{*}$. These instances and solutions are passed to CADVI and valid inequalities are obtained. The user analyzes these inequalities, looking for patterns and repeated structures, and eventually derives a new family of valid inequalities that contains a subset of the inequalities proposed by CADVI. The second tool is useful when attempting to improve an existing family of inequalities for which a separation procedure already exists.

Apart from the input already described, CADVI needs an exact algorithm that can optimize any linear function over the domain in question; this could be an off-the-shelf integer programming solver like CPLEX or a more sophisticated, specialized algorithm. CADVI uses a technique similar to what was proposed by [1] for finding violated valid inequalities. An LP problem with $n+1$ variables is solved. The first $n$ variables correspond to coefficients on the left hand side of the inequality we are searching for, while the last variable corresponds to the right hand side. The objective maximizes the
violation of the inequality. Each row in the LP corresponds to a solution to the original problem and expresses that the inequality we are searching for cannot be violated by the corresponding solution. Rows are generated dynamically. CADVI diverts from the approach proposed by [1] by adding further constraints to ensure that a simple inequality is generated (e.g. containing only integer coefficients). This unfortunately implies that an ILP and not an LP must be solved.
We will present new families of inequalities and strengthened versions of existing families, all found using CADVI. We will consider inequalities for the traveling salesman problem with pickup and delivery and the capacitated vehicle routing problem. Our experience from using the program indicate that it is very useful and we believe that CADVI (or similar programs) have the potential for accelerating the research on special purpose BAC methods and polyhedral studies in the coming years.

Keywords: branch-and-cut • valid inequalities • integer linear programming

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## Scheduling Internal Audit Activities: A Stochastic Combinatorial Optimization Problem

Speaker: Roberto Rossi (Centre for Telecommunication Value-Chain Research, UCC, Ireland)
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- Semra Karacaer (Hacettepe University, Department of Management, Ankara, Turkey)
- Brahim Hnich (Izmir University of Economics, Faculty of Computer Sciences, Izmir, Turkey)
- Steven D. Prestwich (Cork Constraint Computation Centre, University College Cork, Cork, Ireland)


#### Abstract

The determination of audit schedules is an integral part of every audit plan. While planning audit activities, internal audit departments have to find a balance between audit costs and the losses accrued in the absence of auditing. The cost of auditing is directly proportional to the number of audit activities conducted; hence it increases as the frequency of auditing increases. Absence of auditing, on the other hand, is also costly for firms. Costs of fraud, error and waste increase if effective control is not ensured with internal audits. Determining the best audit schedule to minimize the expected total cost presents a nontrivial stochastic combinatorial optimization problem. In this paper, we propose a Constraint Programming (CP) model which balances audit cost and losses for firms having multiple auditable units. The model minimizes the total audit costs and losses under the constraint that the probability that accrued losses will not exceed a maximum allowable loss is higher than a threshold value. The model provides the optimal timing of audit activities for each auditable unit while keeping the losses at certain level. Solutions to this combinatorial optimization problem are obtained by a specialized constraint programming algorithm. Details of the model together with numerical examples and discussions on the computational issues are presented in the article.


Keywords: combinatorial optimization • stochastic processes • scheduling • constraint programming

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# Asymptotically exact approach to solving RCPSP with one resource type 

Speaker: Ivan Rykov (Sobolev institute of mathematics)


#### Abstract

We consider one particular NP-hard subproblem of the resource-constrained project scheduling problem (RCPSP), with one renewable resource. Input parameters are independent identically distributed random variables. We compare this subproblem with the strip packing problem and use the idea of an asymptotically exact algroithm by Gimadi, Zalyubovskii and Sharygin, generalizing it to the case with special precedence relations graph, i.e. to the multiproject case.


Keywords: project scheduling • random instances • asymptotically optimal algorithm • multiproject

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## Airline Crew Scheduling: A hybrid approach using metaheuristics to improve an exact column generation algorithm

Speaker: André Gustavo dos Santos (Universidade Federal de Viçosa)

Co-author(s): Luis Felipe Hussin Bento (Universidade Federal de Viçosa)

- Geraldo Robson Mateus (Universidade Federal de Minas Gerais)


#### Abstract

In airline companies, a notable amount of money is spent with crew managing. These costs can reach around $20 \%$ of total airline costs, what makes the crew management optimization a relevant matter for studies. The Crew Scheduling problem comprises the attainment of a set of duties covering all jobs and minimizing the final cost. A duty is a sequence of flights that follows some restrictions, e.g., total flight time, total work time, minimum time between flights and total landings. For small problem instances it is possible to obtain optimal solutions using an approach similar to the Set Partition problem. This approach becomes impracticable when used on bigger instances, as there is the need of having all the possible duties stored in the main memory. The number of possible columns can reach the mark of millions. The number of duties present in the optimal solution is always a small part of the whole set, representing a good scenario for using Column Generation. New columns can be obtained solving a Shortest Path Problem with Resource Constraints.


This work presents a hybrid model using the GRASP metaheuristic to obtain an initial basis for the problem and combines the mentioned heuristic with an exact ILP model to generate new columns. Duty cost is obtained using the duty idle time and the total distance flown. A crew member is idle when he/she is not flying or having his/her compulsory rest between flights. The time left to reach the total time allowed in a duty is also added to the idle time. ILP models for the master and the slave problem were proposed to cover all the mentioned constraints. The slave problem is to generate the new columns to the master problem, which tries to obtain an optimal solution. The master problem model is rather simple, but the slave model is more complex, reaching more than 8000 variables in a simple 4 -airport instance, becoming very time consuming. Using GRASP we were able to quickly obtain an initial basis to the problem, used as an upper bound. Having some good columns in the master problem before the column generation begins would reduce the number of duties generated by the ILP slave problem. It worked well in small instances, as the upper bound obtained was often the optimal solution. The number of slave problem calls was truly reduced but, for bigger instances, the reminiscing calls were still slow, overcoming the gain offered by the initial basis generation. Thus, GRASP was used again in the slave problem. New columns are, then, generated quickly by the metaheuristic. When none is found, the slower ILP model is called to assure optimality. Even so, the time results were not satisfying, as the heuristic could only generate good columns for around $25 \%$ of the slave problem calls and the time to solve the complete problem could reach many hours in the 4 -airport instance. Despite that, a good upper bound was obtained with notable effectiveness. Analyzing the number of columns in the final solution, it is noticeably smaller than the amount of possible duties, allowing greater instances to be allocated in memory. The upper bound obtained was also considerably good, around $5 \%$ greater than the optimal solution. The problem now resides in optimizing the ILP model and the improvement of the heuristic, aiming a faster column generation.

Keywords: column generation • GRASP • metaheuristics • crew scheduling
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# A multi-level optimization model for improving the robustness of capacitated service and supply systems 

Speaker: Maria Paola Scaparra (University of Kent)

Co-author(s): Richard Church (University of California, Santa Barbara)


#### Abstract

: A crucial issue in today's distribution, supply and emergency response systems is to guarantee continuity and efficiency in service provision in the face of a variety of potential disruptions. Planning against possible disruptive acts of nature or sabotage is an enormous financial and logistical challenge, especially if one considers the scale and complexity of today's logistic systems. Since it is generally impractical to secure all assets, it is important to devise systematic approaches for identifying critical elements and optimize the protection of key system components. In this work, we consider a service-supply system where the facilities have finite capacities and develop an optimization model for identifying the most cost-effective way of protecting the facilities against worst-case scenario disruptions. More specifically, we consider a system with $p$ capacitated facilities serving a set of $n$ customers. We assume that malicious attacks can disrupt (interdict) up to $r$ facilities and that interdicted facilities become completely inoperable. After interdiction, the remaining facilities operate in the most efficient manner, where efficiency is measured in terms of supply costs and lost-sale costs. The objective is to identify the optimal allocation of a limited amount of protective


resources among the $p$ facilities so as to minimize the total costs in the event of a worst-case loss of $r$ facilities. The model assumes that exactly $q$ facilities can be hardened (where $r+q \leq p$ ) and that protected facilities cannot be interdicted.
We formulate the problems as a tri-level mixed-integer program: the top level problem involves the system planner's, or defender's, decisions about which facilities to secure; the intermediate level problem models the worst-case scenario loss of $r$ unprotected facilities; the bottom level problem reflects the fact that the system users try to operate within the system in an optimal way after interdiction. We show that the resulting defender-attacker-user tri-level problem can be reduced to a bi-level non-linear model. The bi-level model can subsequently be linearized through a suitable variable replacement and solved to optimality by an implicit enumeration approach, similar to the one proposed in [1] for the uncapacitated version of the problem. Computational results are presented for a benchmark problem with 150 customers, 30 facilities, and $p$ and $q$ ranging between 1 and 5 .
Keywords: combinatorial optimization • location • multi-level programming • exact method

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## Large-scale Call Center Agents Scheduling

Speaker: Mara Servilio (Dipartimento di Informatica, Università di L'Aquila)
Co-author(s): Fabrizio Rossi • Stefano Smriglio (Dipartimento di Informatica, Università di L'Aquila)


#### Abstract

The workforce management in large scale call centers is typically decomposed into two separate phases: staffing and staff scheduling. Given an accurate forecast of the traffic load, the staffing phase aims to determine the number of agents required in each elementary period (typically thirty minutes long) so as to meet a target Level of Service. Then, the staff scheduling phase translates the staffing levels into rosters assigned to each agent (see, [1, 2, 3]).

The work shift of an agent is made of a set of daily duties. Two different staff scheduling problems arise depending on whether this set must be decided or it is known in advance. In this study we investigate a staff scheduling problem where the set of duties is given and one has to compute an assignment of employees to duties.

The study was motivated by an industrial project funded by a major Italian telephone company having nine geographically distributed call center with about 2200 agents. Each call center deals with different market areas and the activity sectors as well as the daily working hours are fixed. The agents of each call center are grouped in service teams (STs) coordinated by a team leader and may have different contract typologies. A job rotation matrix provides the order in which the STs alternate on the different activities. Typical contract constraints characterize the problem, e.g., a fixed number of day off in a week, a minimum rest period between two consecutive working days and a general fairness in the workload balancing among the employees over the planning horizon, etc.. The Company has a


fixed number of employees (in general not sufficient to satisfy the staffing levels) and its major interest is to minimize the deviation from the desired staffing levels at minimum cost.
The problem can be formulated as a mixed-integer programming model but a direct solution to even the continuous relaxation of the model for large-scale instances is inconceivable. Therefore, we resort to a tabu search based on an exponential neighborhood searchable in polynomial time by a modified shortest path algorithm. We then discuss the development and the implementation of the algorithm in the software tool that manages the company workforce showing the advantages for the firm both at operational and tactical level.

Keywords: workforce management • staff scheduling • heuristic method • shortest path algorithm

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## Strong formulations for the one-warehouse multi-retailer problem

Speaker: Oğuz Solyalı (Department of Computer Engineering, Middle East Technical University, Northern Cyprus Campus)

Co-author(s): Haldun Süral (Department of Industrial Engineering, Middle East Technical University, Turkey)

- Meltem Denizel (Faculty of Management, Sabancı University, Turkey)


#### Abstract

We consider a two-echelon supply chain system in which a warehouse orders from its supplier and replenishes multi-retailers over a finite time horizon with direct shipping policy. We assume a periodic review system in that all replenishments occur in discrete periods. The system operates in a vendor-managed inventory (VMI) setting such that the warehouse has the complete knowledge of time-varying external demand occurring at the retailers and manages the inventories of the retailers while ensuring no stock-out both at the retailers and at itself. The problem is to determine the optimal replenishment policy for the warehouse and retailers such that the total of inventory carrying, fixed and variable (order) shipment costs for both warehouse and retailers is minimized. We consider several mixed integer programming (MIP) formulations of the problem, two of which are proposed in the literature. In addition to the zero stock case, we explicitly consider nonzero initial inventory at the warehouse although the most studies on multi-level lot-sizing in the literature tacitly assume no stocks in the beginning of horizon. Actually, inclusion of initial inventories increases the complexity of the problem unlike single level lot sizing problems because the initial inventory acts as an additional capacitated source of supply at the warehouse. We test the performance of the MIP formulations on a set of randomly generated instances. Computational results indicate that our strong MIP formulations are quite satisfactory to close the gap between the integer and the continuous solution values.


Keywords: one-warehouse multi-retailer problem • integer linear programming • lot sizing • strong formulations

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## Galaxy Cutsets in Graphs

Speaker: Nicolas Sonnerat (Department of Mathematics and Statistics, McGill University)

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#### Abstract

Graph connectivity is a fundamental concept in network design and has been extremely well studied from a complexity viewpoint. The motivation for desiring highly connected networks is to provide resilience against network failures at vertices and/or edges. These failures may arise due to equipment malfunctions or due to malicious attacks on the network. Here we generalise the form in which network failures may occur: Instead of just affecting isolated vertices/edges, we allow them to spread through the network. This models e.g. virus-like malicious attacks, where a virus is transmitted from a vertex to some or all of its neighbours, or subversion in a spy network, where subverting some agents compromises the reliability of the spies with whom the subverted agents had been communicating.

Our model is as follows. We are given a network $G=(V, E)$ that is susceptible to attacks taking the following form: An attack occurs at a subset of $k$ vertices and begins to spread through the network.


Any vertex within distance $r$ of one of the initially attacked vertices can be infected. Thus an attack corresponds to a collection of subsets of vertices, each of which is spanned by a subtree of depth at most $r$. The question we focus on is whether a given network remains connected after such an attack.

The main results of this paper are the following. If $r=1$, an attack corresponds to a subset of vertices which is the union of at most $k$ stars. We call such a set a galaxy of order $k$. We show that it is NPhard to determine whether a given network contains a cutset which is a galaxy of order $k$, if $k$ is part of the input. To do this, we consider a reduction from the Vertex Cover problem. (If $k$ is fixed, there is a straightforward polynomial time algorithm to determine whether a given network contains a cutset which is a galaxy of order $k$.)

In stark contrast to the situation where multiple attacks occur, testing whether a graph can be disconnected by a single attack (i.e. $k=1$ ) can be done very efficiently. Such an attack corresponds to a set of vertices spanned by a tree of depth at most $r$. We present an $O(r n m)$ algorithm that determines if a given network contains such a set as a cutset, improving on a basic $O\left(n^{3} m\right)$ algorithm.
In addition to being useful for designing highly connected networks that are resilient to attacks, the notions of galaxy cutsets and cutsets spanned by trees of depth $r$ are natural extensions of the usual vertex- and edge-connectivity of graphs and of star-cutsets, and therefore interesting from a purely graph theoretic point of view as well.

Keywords: graph connectivity • star-cutsets • complexity
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## Multi-Index Assignment Problems: Applications and Approximation

Speaker: Frits Spieksma (KULeuven)


#### Abstract

In this presentation we give an overview of approximation algorithms for, and applications of, multi index assignment problems (MIAPs). MIAPs and relatives of it have a relatively long history both in applications as well as in theoretical results, starting at least in the 1950's (see e.g. Motzkin [2]). In particular, we focus here on the complexity and the approximability of special cases of MIAPs. A prominent example of a MIAP is the so-called axial three index assignment problem, which has many applications in different areas of production planning, target tracking, and other fields.


Let us shortly describe this problem. Given are $3 n$-sets $A_{1}, A_{2}$, and $A_{3}$. For each triple in $A_{1} \times A_{2} \times A_{3}$ a number is known (either a profit $w_{i j k}$ or a cost $c_{i j k}$ ). The problem is now to find $n$ triples such that each element in $A_{1} \cup A_{2} \cup A_{3}$ is in exactly one triple. So in the axial 3IAP one is asked to output $n$ triples that maximize total profit or minimize total cost. (This contrasts with another 3-dimensional assignment problem, the planar problem, which asks for $n^{2}$ triples such that each pair of elements occurs exactly once).

We overview the special cases that arise when the cost coefficients $c_{i j k}$ satisfy:

$$
\begin{aligned}
& c_{i j k}=d_{i j}+d_{i k}+d_{j k} \text { for all } i, j, k \text { (for nonnegative distances } d \text { ), } \\
& c_{i j k}=\min \left\{d_{i j}+d_{i k}, d_{i j}++d_{j k}, d_{i k}+d_{j k}\right\} \text { for all } i, j, k \text { (for nonnegative distances } d \text { ), } \\
& c_{i j k}=a_{i} \cdot b_{j} \cdot e_{k} \text { for all } i, j, k \text { (for positive numbers } a, b, e \text { ), and } \\
& \left.c_{i j k}=e_{k} /\left(a_{i}+b_{j}\right) \text { for all } i, j, k \text { (for positive numbers } a, b, e\right) .
\end{aligned}
$$

We focus in particular on the last cost-structure mentioned above which was recently introduced by Isler et al. [1].

Keywords: combinatorial optimization • multi-index assignment • approximation algorithm

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# Solving Make-or-Buy Trade-off Problems by Submodular Optimization 

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#### Abstract

In production management, if a manager realizes that the existing production capabilities are insufficient to fulfill all orders internally, the orders can be partly subcontracted.

Formally, the make-or-buy trade-off problem is as follows. The orders of the set $N=\{1,2, \ldots, n\}$ are simultaneously available at time zero at the production facility. For each order $j \in N$, its processing requirement is given by $\bar{p}_{j}$. Order $j$ can be processed internally in full or it can be partially processed internally for $p_{j}$ time units and partially by a subcontractor for $h_{j}$ time units, i.e., $\bar{p}_{j}=p_{j}+h_{j}$. Additionally, due to technological and/or managerial constraints there is a given mandatory limit $\underline{p}_{j}$ on the amount of internal manufacturing, i.e., $\underline{p}_{j} \leq p_{j} \leq \bar{p}_{j}$. Producing an order $j \in N$ incurs the following two costs: the work-in-process cost $f_{j}\left(C_{j}\right)$, where $C_{j}$ is the completion time of order $j$ at the main production facility, and the subcontracting cost $\alpha_{j} h_{j}$, where the coefficients $\alpha_{j}$ are non-negative. Function $f_{j}\left(C_{j}\right)$ measures the work-in-process cost related to completing order $j$ at time $C_{j}$. Each $f_{j}$ is a non-decreasing piecewise linear function consisting of $m_{j}$ linear pieces. We denote the total number of pieces of all functions $f_{j}$ by $L$. The overall quality of a schedule is measured in terms of the maximum work-in-process cost $F=$ $\max \left\{f_{j}\left(C_{j}\right) \mid j \in N\right\}$ and the total subcontracting cost $K=\sum_{j \in N} \alpha_{j} h_{j}$. We reduce the bicriteria problem of simultaneous minimization of the functions $F$ and $K$ to a series of parametric linear programs defined over the intersection of a submodular polyhedron with a box. We demonstrate that the feasible region is represented by a so-called base polyhedron and the corresponding problem can be solved by the greedy algorithm that runs in $O\left(L^{2}\left(n^{2}+\log L\right)\right)$ time, two orders of magnitude faster than those previously known. For each of the associated single criterion problems (to minimize one of the function, provided that the value of the other function does not


exceed a given limit) we develop algorithms that deliver the optimum faster than it can be deduced from a solution to the bicriteria problem.

Keywords: submodular optimization • single machine scheduling • controllable processing times
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# Global Constrained Optimization of a Safety Transformer using Branch-and-Bound method 

Speaker: Tuan-Vu Tran (L2EP - Ecole Centrale de Lille)


#### Abstract

The design of electrical machines is often presented in term of problem with continuous parameters. However, these problems are in the second part of the design process and often limited to the fine tuning of some parameters corresponding to the structure selected in the first part (working structure). There is a lack of decision tools for the choice of the structure and materials (embodiment design). In this stage, the parameters are mainly discrete and non-classable. Moreover, the production in very small series practiced by some small and medium companies must be necessarily supported by standards. Then, the problem is to choose among a great but limited number of solutions rather than to optimize finely some dimensions.

In this paper, the global optimization algorithm based on Branch-and-Bound (BB) method [1] is detailed and applied to design optimally a safety isolating transformer with categorical and integer design variables [2]. Other methods: commercial software package (Pro@Design) [3], Exhaustive Enumeration (EE) and Genetic Algorithm (GA) are also compared in terms of optimal solution reached and computational time to highlight the efficiency of this global optimization algorithm.

The combinatorial optimization problem contains 7 discrete design variables and 7 non-linear inequality constraints (figure 1). There are three parameters a, b, c for the shape of the lamination, one for the frame d , two for the section of conductors $\mathrm{S} 1, \mathrm{~S} 2$, and one for the number of primary turns n 1 . There are 62 possible combinations for the lamination and the frame, and 62 types of conductor. The number of primary turns $n 1$ is integer but only 1000 values are allowed, leading to $246,078,000$ possible combinations. The objective function is to minimize the mass Mtot of iron and copper materials. The multi-physical phenomena within transformer (electric, magnetic, thermal) are modelled by an analytical model.


The simplest combinatorial optimization technique (EE) performed by MATLAB Distributed Computing Toolbox [4] in order to reduce the CPU time, ensures the global solution but the computational time is huge ( 23 days with 8 PCs). A stochastic method as GA is tested with a specific coding [5, 6]. The BB algorithm is detailed. It obtains the global optimum compared with EE method. Numerical results comparing BB with Pro@Design, EE and GA are presented in Table 1. They show high efficiency of the BB algorithm, reflected by a global solution obtained and a significantly overall computational effort.

Keywords: combinatorial constrained optimization • global optimization • branch-and-bound • genetic algorithm - safety transformer


Figure 6: Combinatorial constrained optimization problem of a safety isolating transformer

| Methods | $\mathrm{M}_{\text {tot }}$ | Time | \{a; b; c; d\} | ( | , | $\left\{n_{1}\right\}$ | $T_{c o}$ | $T_{i r}$ | $\eta$ | $I_{4} / I_{1}$ | $\Delta V_{2} / V_{2}$ | $f_{i}$ | $f_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | kg | s | mm | $\mathrm{mm}^{2}$ | $\mathrm{mm}^{2}$ | - | ${ }^{\circ} \mathrm{C}$ | ${ }^{\circ} \mathrm{C}$ | \% | \% | \% | \% | \% |
| Pro@Design | 2.840 | 38 | \{18; 54; 18; 33.5\} | 0.3318 | 2.835 | 722 | 103.60 | 94.49 | 88.54 | 4.90 | 8.20 | 49.29 | 47.56 |
| EE | 2.594 | $1.8 .10^{6}$ | $\{18 ; 54 ; 18 ; 33.5\}$ | 0.2827 | 2.270 | 610 | 109.20 | 99.58 | 87.60 | 9.98 | 8.20 | 35.50 | 32.20 |
| GA | 2.633 | $3.10^{3}$ | $\{18 ; 54 ; 18 ; 33.5\}$ | 0.2376 | 2.835 | 614 | 106.25 | 97.13 | 88.04 | 9.58 | 7.86 | 30.20 | 37.98 |
| sBB | 2.614 | 816 | \{18; 54; 18; 33.5\} | 0.2642 | 2.545 | 611 | 106.48 | 97.34 | 88.00 | 9.90 | 7.90 | 35.60 | 36.00 |
| gBB | 2.594 | 1.7.10 ${ }^{3}$ | \{18; 54; 18; 33.5\} | 0.2827 | 2.270 | 610 | 109.20 | 99.58 | 87.60 | 9.98 | 8.20 | 35.50 | 32.20 |

Table 1: Optimization results of Pro@Design, EE, GA and BB methods

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## Algorithms for Storage Allocation Based on Client Preferences

Speaker: Benny Vaksendiser (The Interdisciplinary Center)

Co-author(s): Tami Tamir (The Interdisciplinary Center)


#### Abstract

We consider a variant of the knapsack problem arising in storage management of Video on Demand (VoD) systems. Formally, a VoD system services $n$ clients, that are interested in watching movies from a collection of $M$ movies. The system has limited resources: it consists of disks, each having a limited storage capacity, $C$, and a limited bandwidth (load capacity), $L$. Each transmission requires a dedicated stream of one bandwidth (load) unit. This implies that each of the disks can store movies of total size $C$ and can transmit broadcasts to at most $L$ clients simultaneously.

The problem is therefore reduced to a class-constrained packing problem, in which the items to be packed (streams) are drawn from $M$ classes (movies) and have the same unit size. The bins (disks) have a limited capacity, $L$, and can pack items from at most $C$ classes. This storage management problem motivated the study of class-constrained packing in recent years. However, in all previous work it is assumed that each client specifies a single movie he wishes to watch and the goal is to allocate storage to movies and transmissions to clients in a way that maximizes the number of clients whose request is granted. In this work we define and study a more generalized setting: For each client $j$ and movie $i$, let $b_{i, j} \geq 0$ denote the payment that client $j$ is willing to pay for watching movie $i$. That is, each client provides his complete preference over the whole collection of movies. The objective is to allocate storage to movies and transmissions to clients in a way that maximizes the system's profit given by $\sum_{j=1}^{n}\left\{b_{i, j} \mid\right.$ movie i is transmitted to client $\left.j\right\}$.

For a VoD system with a single disk we present a hardness proof and a ( $1-1 / e$ )-approximation algorithm. We then extend the approximation algorithm for systems with variable file sizes and/or storage costs, and for the $k$-round problem, in which there are $k$ synchronized broadcasting rounds.


For multiple disks we first present a $(C-1)(e-1) / C e$-approximation algorithm. Next, we propose algorithms for solving the problem in two stages. In the first stage an allocation of movies to the disks is determined. In the second stage, given the storage allocation, the bandwidth allocation problem is to decide which of the clients will be serviced by which disk. We present two heuristics for the first task, and an optimal algorithm for the second task.

In order to better evaluate the performance of our algorithms we simulated a VoD system, and compared their performances. In our simulated system, as in the real-world, client preferences and payment vectors are power-law distributed.
Our results can be applied to other packing and subset selection problems (e.g. auctions) in which clients provide preferences over the elements.
Keywords: storage allocation • approximation algorithm • packing • subset selection
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## The $(k, k-1)$-coloring problem

Speaker: Paula Zabala (Depto. de Computación, FCEyN, UBA, Argentina) Co-author(s): Isabel Méndez-Díaz (Depto. de Computación, FCEyN, UBA, Argentina) Abstract:

In many real-life situations, resources need to be shared among users with conflicting requirements and graph coloring and its generalizations are useful tools in modelling a wide variety of these problems [2]. Consider a set $V$ of $n$ projects (papers) and a set $R$ of referees. Each project $v$ has to be evaluated by $k(v)$ referees chosen from $R$. Associated with each project $v$ is a set $r(v) \subseteq R$ of possible referees. Given a pair of projects $u, v$ having participants (authors) in common, it is reasonable to restrict the referees assignment to the ones not having referees in common. However, the number
of available referees may not be enough and it could became necessary to relax this constraint and allow to share at most $c_{u v}$ referees. For organizational or economical reasons, we want to manage the minimum number of referees for the evaluation process.
The problem can be modeled by an arbitrary conflict graph $G=(V, E)$, in which the vertices represent the projects, and an edge between two vertices $u, v$ means that the set of referees assigned to the endpoints must have intersection size less than or equal to $c_{u v}$. More formally, let an undirected graph $G=(V, E), R$ a set of colors, $n c o l=|R|, k(v)$ the demanded number of colors of node $v, r(v) \subseteq R$ the set of feasible colors for $v$ and $c_{u v}$ the maximum number of colors that can be shared by adjacent vertices $u, v$. A $(k, c, r)$-coloring of $G$ is a mapping $f: V \rightarrow 2^{R}$ such that $f(v) \subseteq r(v),|f(v)|=k(v)$ and $|f(u) \cap f(v)| \leq c_{u v}$ The graph $(k, c, r)$-coloring problem is to find a ( $k, c, r$ )-coloring using as few colors as possible.

In case that we impose $c_{u v}=0 \forall u v \in E$ and $r(v)=R, \forall v \in V$, this problem can be formalized as the well-known multicoloring problem [3, 4] and $c_{u v}=0 \forall u v \in E$ and $k(v)=1, \forall v \in V$ corresponds to list coloring problem [1]. We focus on the particular case that $k(v)=k, r(v)=\{1, \ldots, n k\} \forall v \in V$ and $c_{u v}=k-1 \forall u v \in E$ and we call it the $(k, k-1)$-coloring problem.

In this work, we analize the complexity and present an integer programming formulation for $(k, k-$ $1)$-coloring problem. We perform a polyhedral study of the polytope associated with the proposed model. Computational experience is also presented for a Branch-and-Cut algoritm based on our theoretical polyhedral results. We also take into account many others factors like preprocessing, search and branching strategies, lower and upper bounds and streghthening of the LP-relaxation. We evaluate the performance of our algorithm on randomly generated instances. We conclude that the proposed algorithm is very effective. Experimental tests show that the use of the valid inequalities proposed here has shown particularly useful in reducing of nodes in the Branch-and-Bound tree as well as CPU time.

Keywords: combinatorial optimization • graph multi-colouring • branch-and-cut

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