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Small Firm Debt
Rescheduling
Versus Insolvency
The Bank's Decision Problem

by

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SMALL FIRM DEBT
RESCHEDULING
VERSUS INSOLVENCY:
THE BANK’S DECISION PROBLEM

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ABSTRACT

A model of loan rescheduling as an alternative to liquidation is developed which emphasises the decision problem of the firm's lender (the Bank). The model explains a set of stylised empirical facts about bank-small firm relationships including the occurrence of loan rescheduling as an alternative to bankruptcy and liquidation.

Firms are divided into two types, defaulters and non-defaulters, defined by the firm's operating profitability relative to debt servicing obligations. The bank lends money to the firm of initially unknown type over two periods with contracted payments in each period. The contract is such that should the firm default in the first period the bank may either reschedule the loan or call in the Receiver leading to liquidation of the firm. The conditions under which the creditor will choose to reschedule rather than precipitate bankruptcy are represented by an optimal reservation price rule which simultaneously identifies firm types to the creditor on the basis of debt servicing performance and separates client firms into the two decision categories, reschedulings and liquidations. Liquidation now only results if the firm's default is 'drastic'; rescheduling occurs otherwise. Rescheduling always results in a reduction in debt servicing obligations. A key influence on the rescheduling decision is the magnitude of asset values and bankruptcy costs relative to liquidation values and profits the firm is expected to generate if rescheduling is chosen as the solution to the default problem.

We find that firms with higher asset values and lower bankruptcy costs get larger loans since the expected return on lending to such customers is higher. These firms however have a smaller chance of
their loans being rescheduled in the event of default since they are relatively more valuable dead than alive. However higher expected earnings may both increase lending to firms and enhance the likelihood of rescheduling in the event of default. The latter arises because higher expected earnings reduce bank writeoffs in the second period.
SMALL FIRM DEBT RESCHEDULING VERSUS LIQUIDATION:  
THE BANK'S DECISION PROBLEM

A brief perusal of the statistics on firm liquidations reveals the following stylised facts:

-The vast majority of businesses in the economy are 'small' and so therefore are the majority of business liquidations.

-Firm failure is a frequent occurrence in economic life: between 1985 and 1989 there were on average 12,125 business failures per annum in the UK alone (Central Statistical Office of the UK, 1990). Such failures seem to be crucial to the efficient allocation of resources in the economy.

-Although hard empirical data on rescheduling of bank loans to (particularly) small firms is difficult to come by banks are often quoted as saying that they prefer other things equal not to have to call in the Receiver under loan default. Negotiations thus often take place after the firm has technically breached the conditions of the loan in the form of attempts at debt rescheduling. The latter if successful involves a reduction in the debt servicing obligations (capital repayments and/or interest charges - henceforth DSO) originally imposed in the loan agreement.

-More than one third of (UK) liquidations are Compulsory i.e. enforced by the firm's creditors through the Court (Central Statistical Office of the UK, 1990). For small firms the (UK) figure is around 16%. However of the Voluntary liquidations some two-thirds are Creditors' Voluntary, i.e. brought about by action of the creditors without need for court intervention (Storey et al,
1987, p.45). The evidence thus suggests that the majority of liquidations are in fact decided by the firm's creditors rather than its owners.

-There is substantial cross-sectional variation in financial characteristics of the firms involved in liquidations. This fact has not gone unnoticed amongst academics and a financial industry has grown up in the last two decades attempting to predict on the basis of accounting ratios which firms are more likely to be bankruptcy-prone. (See e.g. Altman(1983) for references, Zavgren(1986) for a recent survey, and Palepu(1986) or Cressy(1991) for a critique).

Despite the obvious importance of bankruptcy in the economy it is rare to find any microeconomic predictive-theoretical treatment of firm liquidation in the literature before the last decade. An early exception is Van Horne (1976) who examines liquidation as an optimal decision of the firm's creditors under loan default. We believe this approach to be consonant with the evidence. However Van Horne's paper is aimed mainly at the provision of a theoretical algorithm for financial decision-making by businesses rather than at producing a predictive theory of liquidations. Later economic theories of firm closure however (e.g. Jovanovic, 1982) assume that liquidation is invariably an optimising decision of the firm's owners. From the empirical evidence cited it is clear that such theories cannot account for a very substantial proportion of observed bankruptcy cases or for the phenomenon of rescheduling.

In the more recent economic literature the theory of 'involuntary' liquidation has been developed further. Wadhwani (1986) presents a model of bankruptcy based on expected profit maximisation by firms but allowing for unanticipated inflation that upvalues a firm's
debt servicing requirements. Firms go bankrupt because they do not foresee random increases in debt servicing costs through inflation. Whilst the theory has been shown to have empirical support it is essentially macroeconomic in structure and cannot by its nature explain observed cross-sectional variations in bankruptcy rates.

Much of the recent finance literature on bankruptcy (see White (1989) for a good survey and critique) is preoccupied with a game-theoretic analysis of the bankruptcy decision with the principal players the firm's equity-holders and creditors acting in coalition. Whilst this kind of analysis is relevant to large firm behaviour it has little or no relevance to small firms where the bankruptcy decision is dominated by the power of the firm's bankers. The vast majority of such liquidations will have been prompted by bank decisions to recover outstanding debts with little or no reference to equity-holders in the firms.

Three deficiencies in the literature have now been identified which the present paper will attempt to remedy.

Firstly, we have shown that a theory of involuntary liquidation is needed that explains cross-sectional variations in bankruptcy rates. To this purpose we propose a theory based on firm/creditor-specific characteristics. Since financial problems for the firm can occur for a variety of reasons maximum generality is retained for our theory by allowing that any cause of financial decline must finally impinge on cash-flow profits.

Secondly, a theory is required that generates a nonzero probability of debt-rescheduling as a response to default. This theory must demonstrate that it is optimal for the bank to reschedule debt
under certain circumstances and that such rescheduling involves a reduction in the size of the DSO. To achieve this we propose a theory of firm types under which the creditor gains information on type as a result of a firm's debt-servicing performance. This information update allows the bank to 'rethink' its debt servicing requirements from the firm for the second period.

Thirdly, it is desirable to know which factors influence the probability of rescheduling versus liquidation. Such results are conspicuously absent from most of the literature on bankruptcy. The theory proposed in the present paper identifies influences on the probability of liquidation versus debt rescheduling by deriving a set comparative statics results.

A summary of the model follows.

Firms are divided into two types, Defaulters and Non-defaulters, defined by the firm's operating profitability relative to debt servicing obligations. The bank lends money to the firm of initially unknown type over two periods with contracted payments in each period. The contract is such that should the firm default in the first period the bank may either reschedule the loan or call in the Receiver leading to liquidation of the firm. The conditions under which the creditor will choose to reschedule rather than precipitate bankruptcy are represented by an optimal reservation price rule which simultaneously identifies firm types to the creditor on the basis of debt-servicing performance and separates client firms into the two decision categories, reschedulings and liquidations. Liquidation now only results if the firm's default is 'drastic'; rescheduling occurs otherwise. Rescheduling always results in a reduction in debt-servicing obligations. A key
influence on the rescheduling decision is the magnitude of asset values and bankruptcy costs relative to liquidation values and profits expected from the firm if rescheduling is chosen as a solution to the default problem.

We find that firms with higher asset values and lower bankruptcy costs get larger loans since the expected return on lending to such customers is higher. These firms however have a smaller chance of their loans being rescheduled in the event of default since they are relatively more valuable dead than alive. However higher expected earnings may both increase lending and enhance the likelihood of rescheduling in the event of default. The latter arises because higher expected earnings reduce bank writeoffs to the rescheduling option in the second period.

The remainder of the paper is arranged as follows. In Section I we describe and present the theoretical model. In Section II the comparative statics of the model are derived. A concluding Section provides a summary and overview and indicates directions for future research.

1. The Model
A firm and its creditor, a large wealth-maximising bank, exist for two periods. The firm borrows from the bank to finance its operations. A loan contract formulated at the beginning of the first period stipulates a fixed payment \( k_1 \) (interest plus principal) at the end of both periods from the firm. We shall refer to this payment as the firm's Debt Servicing Obligation (henceforth DSO) or more loosely as its loan amount.
At the end of the first period the firm's cash-flow operating profits are \( \pi_1 \). Profits viewed from the beginning of period 1 are random. The bank knows only their distribution at the time it makes its decision to lend. The bank as a risk neutral wealth-maximiser therefore decides the loan amount and interest rate at the beginning of period 1 to maximise expected wealth, expectation being taken over the first-period distribution of firm profits. These satisfy the cdf \( \Phi(\pi_1) \), with \( \Phi'(\pi_1) > 0 \) all \( \pi_1 \geq 0 \).

The firm distributes operating profits in excess of debt servicing payments to shareholders and thus has no reserves. Its share capital (A) is realisable as cash only in the event of liquidation. Positive bankruptcy costs (B) are assumed to be incurred to liquidate the firm. Although these are formally paid by the firm since they must be taken from the firm's carcass-value they reduce the amount available to service the bank's loan.

The foregoing assumptions imply that if the firm's cash-flow profits are insufficient to service its debt it will automatically default. Therefore at the end of the first period the firm either pays the scheduled amount \( k_1 \) or some amount less than \( k_1 \) depending on the level of its profits. We assume \( k_1 \) is paid for \( \pi_1 \geq k_1 \) and \( \pi_2 \) is paid if profits are less than \( k_1 \).

This payment provides the bank with perfect information on the firm's type in the following sense. The firm's profits in period two, \( \pi_2 \), are given by

\[
\pi_2 = \begin{cases} 
\pi_{2D} & \text{with probability 1 if } \pi_1 < k_1 \\
\pi_{2N} \text{ w.p.1} & \text{if } \pi_1 \geq k_1 
\end{cases}
\] (1)
where $0 < \pi_{2D} < \pi_{2N}$. Rescheduling consists of a decision made at the beginning of period 2 (thus before $\pi_2$ is realised) to alter the DSO from its original value $k_1^*$ in the light of new information provided by the realisation of $\pi_1$. The optimal period 2 rescheduling, $k_2^*$, is thus as follows. Thus if the firm's first period profits are less than the DSO the bank knows at the beginning of period 2 that the firm is a Defaulter and will default next period also. If on the other hand profits cover the DSO then the bank knows the firm is a Non-defaulter and will repay next period also.

Consider now the optimal DSO for the second period. The bank now has information on realised first-period profits $\pi_1$:

i) If $\pi_1 \geq k_1$ then the bank will set $k_2^* = k_1$ since the old contract is still in force and it cannot enforce revision. The bank's period two return is then the minimum of the firm's second period profits and the optimally chosen DSO, i.e. $\min\{ k_2^*, \pi_{2N} \} = \min\{ k_1, \pi_{2N} \}$.

ii) If $\pi_1 < k_1$ then the firm technically owes $k_1 - \pi_1$ from period one but rescheduling in the light of new information allows the bank to choose $k_2$ to maximise period two wealth. However it will not set $k_2 > \pi_{2D}$ since at the beginning of period one the firm's type has been identified as Defaulter. Thus setting $k_2 > \pi_{2D}$ invites a certain loss of $k_2 - \pi_{2D}$ and choosing $k_2$ subject to $k_2 \leq \pi_{2D}$ it therefore sets $k_2^* = \pi_{2D}$. The bank's period two return is then $\min\{ k_2^*, \pi_{2D} \} = \pi_{2D}$.

Summarising, the optimal DSO set at the beginning of period 2 satisfies
\[ k^*_2 = \begin{cases} \pi_{2D}, & \pi_1 < k_1 \\ \min\{k_2, \pi_{2N}\}, & \pi_1 \geq k_1 \end{cases} \] (2)

If the firm pays less than \( k_1 \) at the end of period 1 it is technically in breach of contract. Then and only then can the bank decide whether to bring in the Receiver. If it does so it gets the minimum of the firm's scrap or liquidation value \( K(\pi_1) = \pi_1 + A \) and the discounted value of the outstanding DSO (assumed to be \( L(k_1) = (1+\beta)k_1 \) where \( \beta \) is the bank's discount factor) both quantities being net of the bankruptcy costs \( B \). We assume that this liquidation value \( K(\pi_1) \) is known to the bank and has two components, current profits \( \pi_1 \) and share capital \( A \). Thus the bank calculates its gross return to bankruptcy as the minimum of \( K(\pi_1) \) and \( L(k_1) \). We shall assume in what follows that \( A \) is cashable only by liquidation of the firm.

Assume that the firm distributes all profits net of interest and tax in the period in which they are earned. Assume also that there are costs to the bank to making a loan of size \( k_1 \), though not marginal costs. Costs of making a loan of size \( k_1 \) are given by \( C > 0 \). These costs will primarily be of an administrative nature involving associated set-up and monitoring costs of \( k_1 \) but independent of its size.

The bank's objective function viewed from the beginning of period 1 is written
\[ W(k_i) = \beta \left[ \int_0^{k_i} \max\{ l(\pi_i) - B, R(\pi_i) \} \, d\pi_i \right] + L(k_i) \int_{k_i}^{\infty} d\pi_i \right] - C \]

\[ \text{where} \]

\[ l(\pi_i) = \min\{ L(k_i), K(\pi_i) \} = \text{bank's gross liquidation return} \]

\[ R(\pi_i) = \pi_i + \beta \pi_{2D} = \text{bank's return to rescheduling} \]

\[ L(k_i) = (1 + \beta)k_i = \text{bank's loan return under 'adequate collateral'} \]

\[ K(\pi_i) = \pi_i + A = \text{gross carcass value of firm} \]

The first term in 3 indicates what is received by the bank should the firm default: the expected maximum of the firm's liquidation value net of bankruptcy costs \( l(\pi_i) - B \) and its continuation value under rescheduling \( R(\pi_i) \). The former consists of the minimum of the firm's gross liquidation value \( K(\pi_i) \) and its outstanding debt obligation \( L(k_i) \) minus bankruptcy costs \( B \). The second term indicates what is received by the bank should the firm honour its debt obligations and consists of the value of the contracted payment schedule \( L(k_i) \). The relationship between these four functions is depicted in Figure 1.
Collateral Cutoff

The expression for the bank's return in the event of liquidation as the minimum of the firms liquidation value and the initial value of the loan contained in 3 implies the existence of a cutoff profitability $\hat{\pi}$ such that at or above this the bank gets the full value of its investment returned whilst below it loses some of its original (contracted) wealth (See Figure 1). This value is defined by

$$K(\pi) = L(\pi)$$  \hspace{1cm} (4)

Solving for $\pi$ we get the collateral cutoff $\hat{\pi}$:

$$\hat{\pi} = (1+\beta)k - A$$  \hspace{1cm} (5)

For realism we assume $\hat{\pi} > 0$ since otherwise (Figure 1) the bank is guaranteed to get its loan repaid in full regardless of realised profits $\pi_0$. Thus for $\pi$ less than $\hat{\pi}$ the bank loses some of the value of either interest or principal on its debt under default.

Rescheduling Cutoff

If the return to rescheduling were always less than that to liquidation ($R(\pi)$ were to intersect $L-B$ to the right of $k^*$ in Figure 1) then rescheduling would never be chosen. Likewise if the return to liquidation were always less than that to rescheduling ($R(\pi)$ were to intersect $K(\pi)-B$ at $\pi < 0$) then liquidation would never be chosen. Thus realistically we need the line $R(\pi)$ to cut the line $L-B$ above the collateral cutoff $\hat{\pi}$ but below the optimal
DSO $k_1^*$. Given the shape of the two functions this implies that there is a unique value of $\pi_1$, say $\pi_1^*$, defined by

$$\pi_1^* = (1 + \beta)k_1 - \beta \pi_{2D} - B \quad (6)$$

Whilst no decision is possible for the bank with respect to the collateral cutoff a decision rule is implicit in the formula for the rescheduling cutoff. This can be seen intuitively from Figure 1 and is formulated as

$$\begin{align*}
\text{IF } \pi_1 < \pi_1^* & \quad \text{LIQUIDATE}; \\
\text{IF } k_1 > \pi_1 > \pi_1^* & \quad \text{RESCHEDULE}; \\
\text{IF } \pi_1 > k_1 & \quad \text{DO NOTHING}\end{align*} \quad (7)$$

Optimisation

So far we have merely defined the conditions for the existence of rescheduling as an alternative to liquidation. Of course the bank's objective at the beginning of period 1 is to choose $k_1^*$ optimally and thereby to calculate an optimal cutoff $\pi_1^*$ to maximise its expected wealth over the two periods of the loan given the fact that in the second period it may possibly renegotiate the terms of the loan contract having then acquired better information on the borrower's type. This value will in turn be a function of the optimal rescheduled DSO in the event of default, $k_2^*$.

Using 5 and 6 the bank's value function 3 can now be written
\[
W(k_1) = \beta \left[ \int_{\pi_1^*}^{\hat{\pi}_1} \{K(\pi_1) - B_1\} d\pi_1 + \int_{\pi_1^*}^{k_1} \{L(k_1) - B_1\} d\pi_1 + \int_{\pi_0}^{\infty} R(\pi_1) d\pi_1 \right] - C (8)
\]

By rearrangement using the definitions of the collateral and rescheduling cutoffs equation 8 can be written in the simpler form

\[
W(k_1) = \beta \left[ \int_{\pi_1^*}^{\hat{\pi}_1} K(\pi_1) d\pi_1 + \int_{\pi_1^*}^{k_1} R(\pi_1) d\pi_1 - B\pi(\pi_1^*) \right. \\
+ \left. L(k_1)[\rho(\hat{\pi}_1) - \rho(\pi_1^*) + 1 - \rho(k_1)] \right] \\
(8')
\]

The first term in square brackets on the right is the expected (gross) return to liquidation if collateral is inadequate, the second term is the expected return to rescheduling, and the third term is the expected bankruptcy costs in liquidation. The last term
the expected return to the bank given the loan is repaid in full.
(The term $\Phi(\pi^*) - \Phi(\hat{\pi})$ represents the probability of marginal
default, and $1 - \Phi(k_1)$ represents the probability of no default.
Since minor default implies the loan will be repaid in full as with
no default, the sum of these terms gives the probability of full
repayment.)

At the beginning of period 1 the bank wishes to maximise this
expression with respect to $k_1$. The first order condition for an
interior maximum is (see Appendix for derivation):

$$W'(k_1) = \beta \left[ (1+\beta) [1 - \Phi(k_1) + \Phi(\pi^*) - \Phi(\hat{\pi})] + \beta \phi(k_1) \{ \pi - k_1 \} \right] = 0$$

\[ (9) \]

In words this expression states that the initial Debt Servicing
Obligation (DSO) will be chosen so that the last £1 lent equates
the expected return to full repayment $(1+\beta)[.]$ to the expected
loss from marginal debt rescheduling $\beta \phi(.) (k_{2D}^* - k_1)$. (Recall that
$k_{2D}^* = \pi_{2D}$ is the rescheduled DSO.) Notice that the former is the
discounted value over both periods whilst the latter is the
discounted value only over the second period. This is explained by
the fact that the loss from rescheduling occurs after first period
profits have been realised.

Equation 9 is of course the key to the comparative statics that
follow. Note that the maximisation problem has a solution only if
the the second term is negative. Thus we can state our first
result:

Proposition 1 (Rescheduled DSO value): The optimally rescheduled DSO $k^*_2 = \pi^*_2$ is less than the amount originally scheduled $k^*_1$, the amount by which it is less being determined by equation 9. Thus rescheduling involves either a reduction in the interest rate charged or capital repayments required.

Proof: This follows from 9 and the fact that $k^*_1 > \pi^*_1 > \pi^*_2$ and $\phi(k^*_1) > 0$.

Interpretation: The expected marginal benefit from lending under full repayment is clearly positive since an extra £ lent returns both capital and interest in full with positive probability. The expected marginal cost to lending under non-drastic default will likewise be positive. To see this (Figure 2) assume that the bank maintains the DSO of period 1 rather than reducing its value and sets $k^*_1 = k^*_2$. Then the first term in equation 9 would be zero and the second positive. Hence the bank would expand lending leading to $k^*_1 > k^*_2$ making the optimal rescheduled DSO less than the initial amount. (The same logic of course applies a fortiori if the bank starts with $k^*_1 < k^*_2$.) Thus we conclude that it is optimal for the bank to reduce the DSO below its initial value under marginal or non-drastic default. Equation 9 tells us that the optimal rescheduled DSO involves an expected reduction in payments of precisely $k^*_1 - k^*_2 = k^*_1 - \pi^*_2$.

Proposition 2 (Decision to reschedule): For $\pi^*_1 \in (\pi^*_1, k^*_1)$ (non-drastic default) it is optimal for the bank to reschedule the loan rather than call in the Receiver.
Proof: From Figure 1 for \( \pi \in (\pi^*_1, k^*_1) \) we have \( R(\pi_1) > l(\pi_1) - B \), i.e. the return to rescheduling in this region exceeds the return to liquidation. Hence after first-period profits are realised and default is revealed to be non-drastic the bank's profit-maximising choice is to reschedule. □

Proposition 3 (Liquidation Decision): Liquidation is optimal for \( \pi_1 \in (0, \pi^*_1) \) (drastic default).

Proof: From Figure 1 for \( \pi_1 \in (0, \pi^*_1) \) we have \( R(\pi_1) < l(\pi_1) - B \), i.e. the return to liquidation exceeds the return to rescheduling under drastic default and will be chosen by the bank. □

II. Comparative Statics

The comparative statics of the model are done with respect to parameters of the cdf of profits (\( \alpha \)), liquidation values (\( A \)), the bank's discount factor (\( \beta \)), second-period profits (\( \pi_{2D} \)) and the costs of bankruptcy (\( B \)).

Definition: A density \( \phi(\pi_1) \) is said to be nice if it satisfies the condition that \( \phi'(\pi_1) \gg 0 \) as \( \pi_1 \gg \mu_1 \) where \( \mu_1 \) is the mode of the distribution and \( \phi \) is everywhere positive. (We shall loosely refer to the mode as the average of the distribution in what follows). Note that this characterisation is roughly that of a differentiable unimodal distribution and covers many of the distributions familiar to economics and finance (e.g., Lognormal, Normal, Pareto etc.)

Assumption: In what follows the profitability distribution is assumed to be nice.
Proposition 4 (Loan size, liquidation values and bankruptcy costs):
A ceteris paribus increase in liquidation values (decrease in bankruptcy costs) increases the optimal DSO.

Proof: Differentiate 9 with respect to A and B respectively and use the Second Order condition on \( k^* \).

Interpretation: As liquidation values increase (bankruptcy costs decrease) expected profits from an extra £1 lent increase and so larger DSOs (higher period 1 loan schedules) will be made. In terms of the marginal benefit-marginal cost schedules of equation 9 and Figure 2 a lower \( B \) will leave the marginal rescheduling costs unaffected but increase the chances of full repayment of the marginal £1 lent by lowering the rescheduling cutoff \( \pi^*_1 \) (Figure 1). Hence the marginal benefit curve shifts to the right implying it is optimal to lend more. A higher \( A \) likewise shifts the marginal benefit curve to the right since the chances of adequate collateral \((1 - \Phi(\hat{\pi}_1))\) have increased (\( \hat{\pi}_1 \) increases in Figure 1) as the return to liquidation increases.

Considering now the comparative static effects on probabilities, the probability of bankruptcy is given by

\[
\Pr\{\text{Bankruptcy}\} = \Pr\{\pi_1 < k_1, \pi_1 < \pi^*_1 \} \\
= \Phi(\pi^*_1) \text{ since } \pi^*_1 < k_1 \text{ by assumption.} \tag{10}
\]

The probability of rescheduling is likewise
\[ \Pr(\text{Rescheduling}) = \Pr(\pi_1 > \pi_1^*, \pi_1 < k_1^*) \]
\[ = \Phi(k_1^*) - \Phi(\pi_1^*) \quad (11) \]

We shall henceforth abbreviate these two probabilities to \( \Pr(B) \) and \( \Pr(R) \).

Proposition 5 (Loan size and bankruptcy): A ceteris paribus increase(decrease) in DSO (loan size/interest rate) always increases(decreases) the probability of bankruptcy and for larger DSOs reduces(increases) the probability of rescheduling.

Proof: Differentiate 14 and 15 with respect to \( k_1 \) to get respectively

\[ \frac{\partial \Pr(B)}{\partial k_1} = \phi(\pi_1^*)(1 + \beta) > 0 \quad (12) \]

and

\[ \frac{\partial \Pr(R)}{\partial k_1} = \phi(k_1^*) - \phi(\pi_1^*)(1 + \beta) < 0. \quad (13) \]

The former is positive by definition of \( \phi \) and the latter is negative for \( \phi' \leq 0 \), i.e. for 'large' DSOs.

Interpretation: From 6 we know that \( \frac{\partial \pi_1^*}{\partial k_1} = 1 + \beta > 0 \) so that the rise in \( k_1 \) is accompanied by a rise in \( \pi_1^* \). Thus it becomes more likely that \( \pi_1 \) falls below \( \pi_1^* \) and the probability of bankruptcy unambiguously increases. This is the familiar effect of higher
bankruptcy risk from higher gearing.

The effect of a unit increase in $k_1^*$ on the probability of rescheduling depends critically on the slope of the density at the optimum (Figure 3). With a uniform density on $[0,1] \phi = 1$ and equation 13 becomes

$$\frac{\partial \text{Pr}(R)}{\partial k_1^*} = 1 - (1 + \beta) = -\beta < 0.$$ 

Thus the probability of rescheduling unambiguously decreases with an increase in $k_1^*$. However for typical economic densities the pdf of profits is not constant. Thus it is possible that the probability of rescheduling may increase if the effect of an increase in $k_1^*$ on the cdf at $k_1^*$ outweighs the effect of the increase in the cdf at $\pi_1^*$ (see equation 15.) In Figure 2 the area $X$ is greater than the area $Y (\phi(\pi_1^*) > \phi(k_1^*))$ so that this negative relationship is preserved. However to the left of $\mu_1 Y$ may be greater than $X$ and the conclusion will be reversed.

[Figure 3 here]

Proposition 6 (Probability of bankruptcy/rescheduling, liquidation values and bankruptcy costs): An increase in liquidation values (decrease in bankruptcy costs) increases the probability of bankruptcy for all DSOs and decreases the probability of rescheduling for large DSOs.

Proof: Immediate from Props. 4 and 5 ■

Interpretation: The effect of either an increase in liquidation values or a decrease in bankruptcy costs is to unambiguously
increase the return to liquidation under default. The optimal DSO increases by Prop. 4 and by Prop. 5 this will increase the probability of bankruptcy. By Prop. 5 a higher DSO decreases the probability of rescheduling for larger DSOs.

Proposition 7 (Period 2 default profitability): An increase(decrease) in the period 2 default profitability ($\pi_{2D}$) increases(decreases) the rescheduling probability for firms of all sizes and reduces(increases) the optimal DSO size and bankruptcy probability for large DSOs.

Proof: The effect on the loan size and bankruptcy/rescheduling probabilities is given by differentiating 13, 14 and 15 with respect to $\pi_{2D}$:

$$\frac{\partial k^{*}_1}{\partial \pi_{2D}} = \beta^2 [\phi - \phi^*(1 + \beta)]/(W'')$$

$$\frac{\partial \Pr(B)}{\partial \pi_{2D}} = \phi^*[1 + \beta)\frac{\partial k^{*}_1}{\partial \pi_{2D}} - \beta]$$

$$\frac{\partial Pr(R)}{\partial \pi_{2D}} = \phi \frac{\partial k^{*}_1}{\partial \pi_{2D}} - \frac{\partial Pr(B)}{\partial \pi_{2D}}$$

$$= \beta^2 [\phi - \phi^*(1 + \beta)]^2(W''')^{-1} + \phi^* \beta$$

The first of these expressions is clearly negative for $\phi' \leq 0$ and the last is positive for all $\phi'$.

Interpretation: The effect of an increase in $\pi_{1D}$ is to increase the return to rescheduling by reducing the amount written off by the
bank (namely, $\pi_{1D}^* - k^*_0$). The direct effect on lending is thus positive. The indirect effect on lending via the probability of full repayment is negative since a higher $\pi_{1D}$ will raise the rescheduling cutoff. Thus the impact on DSO size is ambiguous. However the effect on the probability of rescheduling turns out to be unambiguously positive.

Proposition 8 (Variance-preserving mean shift in profitability): For nice profitability distributions, a variance-preserving increase (decrease) in profitability ($\alpha$) will increase (decrease) the optimal DSO (loan size/interest rate) for small firms. This in turn will increase (decrease) the probability of bankruptcy for small firms and have an ambiguous effect on the probability of rescheduling.

Proof: See Appendix A

Interpretation: As the distribution shifts to the right the probability of returns below the original mean $\mu_1$ decrease in probability and those above the mean increase. This will from 13 increase the chances of full repayment and therefore expand lending. The sign of the effect is identical for all firms and will ceteris paribus increase the chances of bankruptcy as the debt-equity ratio rises. However the slope of the density determines the impact of the change on expected rescheduling losses from the marginal defaulter. This effect however is in the opposite direction: below-average profitability firms have lower slopes than before, i.e. they become less costly under marginal default. Thus for small firms an increase in the mean of the distribution decreases the expected costs from rescheduling at the old DSO. The
bank finding the smaller firm now more profitable expands lending to it. This in turn increases the firm's debt-equity ratio and increases the chances of bankruptcy.

Proposition 9 (Bank's discount factor): The effect of a change in the bank's discount factor on loan size and the probability of rescheduling and bankruptcy is ambiguous.

Proof: See Appendix ■

Interpretation: The direct effect of an increase in $\beta$ is to increase the return to full repayment but also to increase the costs associated with rescheduling. The former occurs because a higher $\beta$ signals a decrease in the opportunity cost of funds to the bank. The latter occurs because the opportunity cost of monies lost through the need for rescheduling also increases. Thus the effect is to shift the marginal benefit curve to the right (thus increasing $k^*_1$) but to shift the marginal cost curve to the left (thus decreasing $k^*_1$). The net effect on $k^*_1$ is thus ambiguous. Hence the effect on $Pr(B)$ and $Pr(R)$ is ambiguous also.

AA summary table of derivatives is provided below.
Table 1: Comparative Statics Derivatives

<table>
<thead>
<tr>
<th>Notes</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial k}{\partial A} )</td>
<td>( \frac{\partial k}{\partial B} )</td>
</tr>
<tr>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>No condition</td>
<td>No condition</td>
</tr>
<tr>
<td>( \phi' &lt; 0 )</td>
<td>( \phi' &lt; 0 )</td>
</tr>
</tbody>
</table>
III. Summary and Conclusions

We have shown that it is possible to derive a predictive theory of small firm bankruptcy in which liquidation, whilst ultimately driven by firm profitability, is modelled as the optimising decision of a large creditor. The model was been shown to yield results that accord with intuition and practice. In particular the theory 'reproduced' the familiar empirical fact that loan rescheduling takes the form of a reduction in the debt payments of the defaulter. However this result was derived as a consequence of an optimising decision by a large creditor. Non-drastic defaulters were optimally rescheduled and drastic defaulters liquidated.

The theory emphasised the importance of bankruptcy costs and liquidation values on the bank's decision. We found that firms with higher asset values and lower bankruptcy costs got larger initial loans (DSOs) since the expected return on lending to such customers was higher. These firms however had a smaller chance of their loans being rescheduled in the event of default since they were relatively more valuable dead than alive. However higher expected earnings from such firms enhanced the likelihood of rescheduling in the event of default. This arose because higher expected earnings reduced bank writeoffs from the rescheduling option in the second period.

A theoretical development intended to be the subject of a future paper is to investigate the possibility that the loan size may influence the parameters of the distribution of profitability. In the present paper we assumed these two things to be independent. However if capital is productive we should expect the profitability distribution to change with the loan policy of banks.
Finally testing of the theory on a suitable database will throw light on the quantitative importance of the various parameters in the model. Recent work by Reid (Reid, 1992) attempts to assess the empirical importance of rescheduling amongst small businesses and the Warwick Startup Tracking Excercise (see Cressy (1992) for details) has some data on rescheduling of overdraft facilities by NatWest bank of the UK. Neither of these databases appears to have explicit data on bankruptcy costs, though writeoff information is available in the Warwick study.
Proof of Equation 9

Differentiating equation 7 with respect to \( k_1 \) bearing in mind the dependence of the limits of integration on \( k_1 \) we get

\[
W' = \beta \left[ (1 + \beta)^2 k_1 \hat{\phi} + (k_1 - \beta \pi_{1D}) \phi - ((1 + \beta)k_1 - \beta) \phi^*(1 + \beta) \right. \\
- \beta \phi^*(1 + \beta) + (1 + \beta)k_1 \{ \phi^*(1 + \beta) - \hat{\phi}(1 + \beta) - \phi \} \\
+ \left[ \hat{\phi}^* - \hat{\phi} + 1 - \hat{\phi} \right] (1 + \beta)
\]

Cancelling terms and rearranging we get equation 9 \( \blacksquare \)

Proof of Proposition 7

Define \( \bar{\Phi}(\cdot) \) by

\[
\bar{\Phi}(\pi \mid \alpha) = \bar{\Phi}(\pi - \alpha).
\]

Thus an increase in \( \alpha \) represents a variance-preserving mean shift in \( \bar{\Phi} \). Evaluated at \( \alpha=0 \) we have the effect of an infinitesimal rightward shift in the CDF:

\[
\frac{\partial \bar{\Phi}}{\partial \alpha} \bigg|_{\alpha=0} = -\phi(\pi) < 0
\]

In other words the probability of \( \pi \) falling above any fixed value now increases. Differentiating the First-order condition 9 with respect to \( \alpha \) we get

\[
\frac{\partial W' \ast}{\partial \alpha} = \beta \left[ (1 + \beta)[\phi(\pi) + \phi(k_1) - \phi(\pi^*)] + \beta(k_1 - \pi_{2D}) \phi(k_1) \right]
\]
From Prop. 1 the term \((k^*_1 - \pi^*_2)\) is positive. We have shown that \(\pi^*_1 < k^*_1\) so that provided \(\phi\) is increasing the term in small square braces is positive. Thus, the derivative above is positive for \(\phi' \geq 0\) and using the Second Order Condition so is \(\partial k^*_1 / \partial \alpha\). Therefore the optimal DSO increases as the profitability distribution shifts to the left. Using Prop. 5, we have

\[
\partial \Pr(B) > 0 \text{ for } \phi'(\pi^*_1) \leq 0
\]

and

\[
\partial \Pr(R) \leq 0
\]

Proof of Proposition 8

Differentiating 9 with respect to \(\beta\) and using equation 9 itself we get

\[
\partial w' / \partial \beta = \beta[(1 + \beta)\phi(\pi^*_1) - \phi(k_1)](\pi^*_1 - \pi^*_2) - (1 + \beta)\phi(\hat{\pi}_1)
\]

\[
+ \Phi(\pi^*_1) - \Phi(\hat{\pi}_1) + 1 - \Phi(k^*_1)
\]

which is ambiguous due to the existence of the term \(-(1 + \beta)\phi(\hat{\pi}_1)\)
References


Gale D and Hellwig M "Incentive-Compatible Debt Contracts: The One-Period Problem", Review of Economic Studies, LIII, 1985, 647-663


Maddala, G S  Limited Dependent and Qualitative Variables in Econometrics, Cambridge, 1983


Prais, SJ The Evolution of the Giant Firm in Britain, Cambridge University Press, 1976


Footnotes

1 Ganguly(1985) quotes figures of 88-99% of total manufacturing businesses in the 'small' category (defined as single-plant establishments with less than 200 employees). In services the percentages of small businesses are of course higher.

2 Given the negative association of size and failure probability (See e.g. Prais, 1973).

3 See White(1989) for a contrary view based on the US experience.

4 A more general model in which the firm's type is not revealed with probability one is the subject of a future paper.

5 $A_0$ can be interpreted alternatively as the trade sale value of the firm achieved by the Receiver if the firm does not go into liquidation and $B_0$ as the costs of Receivership. Then the model predicts which defaulting firms will be rescheduled versus which will be either liquidated or sold off. In both cases $A_0$ is paid to the bank net of $B_0$. It appears that very little is known empirically about the true extent of trade sales as an alternative to liquidation.

6 The implication of this is that whilst fixed costs may be relevant to the bank's profitability they do not necessarily influence the size of the loan.
We have assumed in the event of bankruptcy proceedings that the bank refuses payment of $π_0 < k_0$. This amount is then added to the firm's reserves. The analysis is identical if we assume $π_0$ is accepted as part-payment and reserves go down by the same amount.

The marginal defaulter is a firm whose profits in the first period are just insufficient to pay the DSO $k_1^*$. Thus the (undiscounted) loss from such a defaulter is $π_{2D} - k_1^*$. $k_2^*$ is assumed to have been determined optimally in the second stage of the game. It is now treated as a fixed number for the analysis of the first period decision on $k_1$. 


FIGURE 1: Bank's returns to do-nothing, reschedule and liquidate as a function of firm's realised first-period profits.
FIGURE 2: Determination of Optimal Initial DSO

Notes: 
mbf = marginal benefit from full repayment  
mor = marginal cost of rescheduling
Figure 3a: Effect of DSO Size on Rescheduling
Figure 3b: Effect of DSO Size on Rescheduling