Equilibrium Yield Curves: The Role of Durable Goods

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Abstract

In the data, durable consumption growth is very persistent and strongly predictable by nominal yields and inflation, which suggests that durable consumption is an important source of long-run and inflation risks in the economy. We set up and estimate an equilibrium two-good nominal economy which features nonseparable utility over durable and nondurable consumption, persistent variations in real expected growth and inflation, fluctuations in relative preference for durables, and recursive utility of investors. Our model can explain unconditional moments and conditional movements in the nominal term structure in the data; as in the data, equilibrium nominal yields negatively predict future durable consumption. Model-implied equilibrium real yields are upward sloping which we verify for a range of numeraire choices. Empirically, most of the inflation premium in the model comes from a durable risk channel.

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1 Introduction

Traditional consumption-based asset-pricing models start from the assumption of a single perishable consumption good. While a single consumption good and time-additive CRRA utility setup, in its simplicity, fails to explain key stylized facts of financial market data, Bansal and Yaron (2004) show that long-run predictability of future consumption is potentially an important driver of asset prices and risk premia. In this paper, we argue that durable goods provide an additional source of the long-run risks which plays a key role to understand the movements in the term structure. First, we find that nominal interest rates forecast negatively and significantly future durable consumption growth several years out; this is consistent with an evidence of high persistence and predictability of durables by the aggregate equity prices shown in Yang (2010). Second, we document a significant negative relationship between durables and inflation in the data: while positive expected inflation shock leads to a decline in future consumption growth (see e.g., Piazzesi and Schneider (2006), Bansal and Shaliastovich (2010)), this effect appears to be stronger for durables than non-durables. This suggests that durables play an important role to generate the inflation premium in nominal yields. To explain and quantify the role of durable risks for the nominal terms structure, we set up and estimate an equilibrium, two-good long-run risks type economy. The model is consistent with above empirical findings; it further generates an upward sloping real term structure, and in all, it underscores the role of durable risks as one of the key risk factors for the term structure.

Specifically, we derive a long-run risk model based in a two-good nominal economy. Preferences are described by the general recursive preferences of Epstein and Zin (1989), Kreps and Porteus (1978) and Weil (1989) over future consumption of the two goods. The per-period utility over the respective goods is specified through a CES-aggregator over the non-durable consumption and service flow from durables, as in Gomes, Kogan, and Yogo (2009) and Yogo (2006). The novel element in our preference specification is that we allow for a unit service flow of durables to be time-varying. This can be interpreted as a preference shock, capturing the fluctuations of consumer tastes in durables relative to non-durables over time. An alternative interpretation is based on technological change where innovation and variation in the nature and quality of durable supply translate to fluctuations in the service flow per unit of durables. The main motivation for extending the preference structure in this way comes from the empirical evidence for the role of the cointegration residual of user cost on ratio of durables to non-durables in predicting macroeconomic and financial variables in the data. Siegel (2006) concludes that such forecasting properties of the cointegration residuals are inconsistent with a frictionless Euler equation; indeed in a frictionless economy without preference shocks, such a residual captures just measurement errors in the data, and cannot forecast future variables. In our model, however, the cointegration residual is directly linked to the preference shocks to durables, so that an increase in the preference weight to durables predicts an increase in their future consumption. While we do not model the primitive channels for this effect directly in the model (e.g., through a production sector, as in Gomes et al. (2009)), we find them economically plausible and statistically important in the data.

The key ingredients of our model are non-separability of durable and nondurable goods in preferences, persistent fluctuations in expected growth rates, and recursive utility. With these features, we show that investors are concerned about the risks in non-durable consumption and the expenditure on durable consumption, valued at its user cost, relative to non-durable one. This is similar to the characterization of the discount factor in Piazzesi, Schneider, and Tuzel (2007) in the expected utility case and with fully-depreciating goods. We solve the equilibrium real and nominal term structure in terms of the fundamental consumption news and state variables in the economy. In particular, we exogenously specify a joint dynamics of non-durable consumption growth, durable consumption growth, inflation rate and change in preference shock, which feature a low-frequency variation in expected growth components, as in Bansal and Yaron (2004). Based on this specification, we show that the signs and magnitudes of risk prices and exposures critically depend on the dynamic interactions of expected non-durable and durable consumption shocks and expected inflation, importance of durable goods in the economy, inter-temporal elasticity of substitution and elasticity of substitution between the two goods, and the risk aversion coefficient of the representative agent.

One of the key novel implications of our model is that while the real bonds respond positively to expected non-durable consumption news, their sensitivity to expected durable consumption is negative if elasticity of substitution between the two goods is less than the inter-temporal elasticity of substitution. Thus, the real bonds hedge expected non-durable growth risks, however, they are risky with respect to fluctuations in expected durable consumption. If the durable channel is strong enough, the overall risk premium on real bonds can be positive, which will lead to an upward-sloping real term structure. The positive slope of the real term structure cannot be obtained in a standard restriction of the model to one non-durable consumption good.

The correlation between shocks to expected inflation and expected real growth is a key determinant of the inflation premium in the model. In the raw data, the real durable and non-durable growth rates are both negatively correlated with changes in inflation. Such negative correlation gives rise to a positive inflation risk premium for nominal bonds, as nominal bonds are risky and load negatively on expected consumption states. Empirically, this inflation risk premium is further amplified through the durable channel due to the increased persistence of the durable goods.

We formulate a fairly general statistical model of the growth rates of the two goods (durable and non-durable) and inflation. Our model is fitted to these three variables, as well as a fourth variable we define through a linear combination of the two goods and user cost of durables, which in our theoretical model corresponds to a preference shock. Using these four variables, we fit a latent variable model using Kalman filtering to the data. For the sake of trying to explain historical term structure innovations, it is crucial that we capture the interaction between the shocks to expected growth in the two consumption goods, inflation and preferences. In doing so we specify a first order unrestricted VAR for the state-variables. Our full model has thirty parameters and we estimate this model using likelihood based inference through Bayesian MCMC simulation. The results show that we capture interesting and novel interactions between the expected growth rates of the two consumption goods and the expected inflation growth. While we find that the expected non-durable consumption nearly unaffected by shocks to expected inflation, expected durable consumption responds negatively to expected inflation shocks.

In fitting our model to term structure data, we take as given the estimates from the macro economic described above. Since our econometric model does not identify estimates of the preference parameters, we investigate several preference parameter constellations. In our baseline model we consider a general Epstein-Zin preference structure where the intertemporal elasticity of substitution (IES), is set equal to two. We show that when setting the IES parameter ψ equal to our benchmark value of two and setting risk aversion γ to 17, our model matches a number of important features of the term structure data. In particular, the model matches both the average nominal yield in our sample (6.1%) as well as the slope of the nominal term structure (60 BP) exactly. Both the nominal and real yield curves are upward sloping in our model. We show that this feature is to to the inclusion of the durable good in our model: if we constrain our preference structure to be such that the representative agent only derives utility from the non-durable good, both nominal and real yield curves become negatively sloped. The relatively higher yields earned on long dated bonds in our model is due to the negative interaction between the durable goods and inflation that we estimate in our VAR model for the unobserved states. Also, the ability of our model to fit the historical upward sloping yield curve patterns that one observe in the United States depends crucially on our models' separation of risk aversion from intertemporal elasticity of substitution in the Epstein-Zin utility framework. In particular, imposing the parametric constraints associated with CRRA utility leads uniformly to near disastrous model performance in a number of dimensions including the average rates, and yield curve slope. Indeed, the yield curve is only upward sloping in our model when we keep the durable channel active and use the full Epstein-Zin preference structure.

We investigate our model's ability to explain historical yield curve patterns. In particular, we compare our model to historically observed short rates using our Kalman filtered state-variables to compute the model implied trajectories of level and slope. Overall, our full model produces an interest rate and term spread which are very similar that those observed int the data. Various parametric restrictions on our full model produce large discrepancies between the model implied and observed trajectories of level and slope. In particular, the model without the durable good in the preference specification produces an almost identical MSE for the level, but fails to capture the dynamics of the slope of the yield curve. All CRRA based models produce large MSE for both level and slope.

Another important prediction that distinguishes our full model with recursive preferences and durable consumption from other model specification is a negative loading of yield on expected durable consumption growth. Indeed, in the data, in the regressions of a short-term interest rates on extracted state variables the loading on expected durable growth is negative and significant. We find that the full model can match the signs of the coefficients obtained from regression, and all the model-implied coefficients are close to their counterparts in the data. In particular, the model-implied bond loading on expected durables is negative, as in the data. On the contrary, all other constrained model specification imply counterfactual loadings on this state variable.

In the last step, we study the model implications for the equilibrium real yields, under different assumptions for the underlying numeraire. We find that real yield curve is always upward sloping, but its slopes and level under varies under different choices of the numeraire. When the real bond is assumed to deliver one unit of non-durable good, the real yield curve is relatively flat. However, when we assume that the real bond delivers one unit of durable good measured at the purchase price, the real yield curve becomes steeper, because now durable good takes effect through two channels: the durable consumption growth affects the stochastic discount factor and the durable purchase price inflation determines the payoff value of real bonds. Last, if the real bond is assumed to deliver one unit of consumption bundle, the real yield curve becomes the steepest. In this case, durable good influences real yield through three channels: the durable consumption growth affects the SDF, and the preference shock and durable user cost inflation determines the payoff of real bonds on maturity.

Our paper is related to the recent literature which explores the asset-pricing implications of recursive utility with several goods. In a context of general equilibrium, long-run risks type models, Yang (2010) specifies a model where non-durable consumption is a random walk and durable consumption has a persistent, long run risk component. Yang calibrates his model to unconditional moments of of equity and bond markets and his model produces an upward sloping average real yield curve. Fillat (2010) and Ready (2010) use a similar framework to address the importance of housing consumption risks and oil consumption risks, respectively. Pakos (2007) highlights the implications of a high intra-temporal complementarity between non-durables and durables for the asset prices and risk premia, and shows that with a preference for early resolution of uncertainty, the durable good channel go a long way to explain the equity premium, the risk-free rate puzzle and size and value puzzles. Colacito and Croce (2011) study the implications of a two-good economy in the international context. Yogo (2006) uses the stochastic discount factor implied by the recursive preference structure to study the cross-section of asset returns, while Lustig and Verdelhan (2007) explores it in the cross-section of currency returns. These papers do not focus on the implications of durable risks for the term structure of interest rates.

In a CRRA expected utility framework, Piazzesi, Schneider, and Tuzel (2007) build a two-good model to analyze the implications of the composition risk in housing consumption for the asset prices in the economy. Pakos (2007) discusses the role of perfect complementarity between durables and non-durables for a range of asset-pricing puzzles, while Gomes et al. (2009) addresses the implications of durability of goods for the cross-section of asset returns in a production setting. Some of the earlier prominent literature on multiple consumption goods includes Eichenbaum and Hansen (1990), Dunn and Singleton (1986), Ogaki and Reinhart (1998). In particular, Dunn and Singleton (1986), based on term-structure data, find evidence against a specification of expected non-separable utility over durables and non-durables. Our specification features a long-run risks economy with recursive utility and fluctuations in expected growth components, which gives rise to additional risk premia components due to long-run risks and enables to better capture the risk and return in financial markets. Notably, the restrictions of expected utility in our setting makes the market prices of expected growth risks to be equal to zero, so the resulting real and nominal term structures are flat.

There is a large literature on structural, consumption-based term structure models. In the long-run risks one-good economy, papers which analyze the implications of long-run consumption risks include Bansal and Yaron (2004), Piazzesi and Schneider (2006), Bansal and Shaliastovich (2010), Eraker (2006), Doh (2010), Hasseltoft (2010). Wachter (2006) addresses the term structure implications in the habits model, while Bekaert, Hodrick, and Marshall (2001) use a peso-problem argument to address the violations of the expectations hypothesis and explain the term structure of interest rates in United States, United Kingdom and Germany. To the best of our knowledge, the term-structure link to durable risks has not been entertained in a fully-specified general equilibrium context.

The paper is organized as follows. The next Section presents the preference model setup, and the equilibrium solution to the model. In Section 3 we present and solve a benchmark model of the economy to highlight the qualitative role of the durable risk channel. Section 4 focuses on empirical estimation results and model implications for the term structure. Section 5 concludes the paper. Model derivations are given in the Appendix.

2 Model Setup

2.1 Preferences

The life-time utility of the agent is described by an Epstein-Zin-Weil recursive function,

$$U_t = \left[(1-\beta)u_t^{1-\frac{1}{\psi}} + \beta\left(E_t U_{t+1}^{1-\gamma}\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}}, \tag{1}$$

where u_t is a period utility function, ψ is the elasticity of inter-temporal substitution, and γ is the relative risk aversion coefficient. For ease of notations, we define $\theta = (1-\gamma)/(1-1/\psi)$.

In our economy, the agent derives utility from non-durable consumption C_t and service flow from durable goods Y_t. With a CES-type aggregator, we can write down

$$u(C,Y) = \left[(1-\alpha)C^{1-\frac{1}{\epsilon}} + \alpha Y^{1-\frac{1}{\epsilon}}\right]^{\frac{1}{1-\frac{1}{\epsilon}}}. \tag{2}$$

Parameter α determines the utility weight to durable consumption versus the non-durable one, while ϵ captures the elasticity of substitution between the two goods.

Let S_t denote the quantity of durable goods in the economy. The total stock of durables this period is equal to the amount last period net of depreciation at the rate δ and plus the new purchases in the current period E_t :

$$S_t = (1-\delta)S_{t-1} + E_t. \tag{3}$$

The typical assumption in the literature is that the service flow Y_t is linearly proportional to its stock S_t with a constant proportionality coefficient, which, after re-normalization, allows to write down $Y_t = S_t$ (see e.g. Ogaki and Reinhart (1998); Yogo

2006; Piazzesi et al. (2007); Gomes, Kogan and Yogo, 2009; Yang 2010). However, such a specification is shown to have difficulties capturing the joint behavior of durable and non-durable consumption and the relative price in the data. For example, Siegel (2010) examines cointegration properties of the three time-series series and concludes that they are inconsistent with the aggregate Euler equation. To help explain the data, Pakos (2005) argues for a non-homothetic utility function over durable and non-durable consumption, while Fillat (2010) introduces time-variation in the preferences of the agents to capture their dynamics in the data.

In our paper, we keep the preference specification standard, and instead introduce time-variation in a linear relationship between durable stock and service flow from durables, e.g. we specify

$$Y_t = A_t S_t, \tag{4}$$

where A_t captures the fluctuations in the relationship between durable stock and flow. The parameter A_t can be interpreted as a preference shock, capturing the time-variation in consumer tastes to durables relative to non-durables. An alternative interpretation is the one based on technological change and innovation affecting the per-durable-unit utility of the agent. Due to technological progress and increase in variety of durable goods, one unit of durables has a different effect on the utility of the agent now relative to the past, and such fluctuations in the service flow of durables are captured in parameter A_t. We provide empirical evidence for such fluctuations and their importance for the asset prices in the empirical part of the paper.

2.2 Equilibrium Solution

The solution to the model is similar to Yogo (2006), and all the details are provided in the Appendix.

Each period, the agent consumes C_t of non-durable goods, makes purchases E_t of durable goods at price P_t^d, and invests h_{it} shares into asset i whose price is P_{it} and dividend is D_{it} $(i = 1, 2, \ldots N)$. Hence, we can write down the budget constraint of the agent in the usual form,

$$\sum_{i=1}^{N}(P_{it} + D_{it})h_{i,t-1} = C_t + P_t^d E_t + \sum_{i=1}^{N} P_{it}h_{it}. \tag{5}$$

Let W_t denote the beginning-of-period total wealth of the agent, which includes financial wealth of investing into financial assets as well as the value of the stock of durable goods:

$$W_t = \sum_{i=1}^{N}(P_{it} + D_{it})h_{i,t-1} + (1-\delta)P_t^d S_{t-1}. \tag{6}$$

The purchase price of durable goods is denoted by P_t^d. However, it is different from a user cost of durable goods, as durable goods last more than one period. Define the user cost of durable goods as the ratio of marginal utilities of durable goods to non-durable ones:

$$Q_t = \frac{u_{st}}{u_{ct}} = \frac{\alpha}{1-\alpha} A_t \left(\frac{Y_t}{C_t}\right)^{-\frac{1}{\epsilon}}. \tag{7}$$

In equilibrium, the user cost of durable goods is equal to its rental price, and is determined through the following condition:

$$Q_t = P_t^d - (1-\delta)E_t M_{t+1} P_{t+1}^d. \tag{8}$$

Let us define $R_{c,t+1}$ to be the return on consumption asset which delivers non-durable consumption and durable consumption valued at its user cost, $C_t + Q_t S_t$. That is, $R_{c,t+1}$ satisfies

$$R_{c,t+1} = \frac{W_{t+1}}{W_t - C_t - Q_t S_t}. \tag{9}$$

Then, in the Appendix we show that we can price any asset in the economy, including the consumption asset, using the usual Euler equation:

$$E_t M_{t+1} R_{i,t+1} = 1. \tag{10}$$

The stochastic discount factor satisfies

$$M_{t+1} = \beta^{\theta} \left(\frac{V_{t+1}}{V_t} \right)^{\theta\left(\frac{1}{\epsilon} - \frac{1}{\psi}\right)} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} R_{c,t+1}^{\theta-1}, \tag{11}$$

where V_t denotes period utility per non-durable consumption: $V_t = u_t/C_t$, that is

$$V_t = \left[(1-\alpha) + \alpha (A_t S_t / C_t)^{1-\frac{1}{\epsilon}} \right]^{\frac{1}{1-\frac{1}{\epsilon}}}. \tag{12}$$

In a one-good economy with only non-durable consumption ($\alpha = 0$), we obtain a standard equation for the discount factor featuring non-durable consumption risks and the risks in the consumption asset. The novel component of the discount factor in the economy with durables is the composition risk in the term V_t. Indeed, define z_t the log of relative expenditure on durables, valued at its user cost, over the non-durables:

$$z_t = \log \frac{Q_t S_t}{C_t}. \tag{13}$$

Then, using the equilibrium condition for the user cost in Equation (7), we obtain that

$$\log V_t = \frac{1}{1-\frac{1}{\epsilon}} \log(1-\alpha) + \frac{1}{1-\frac{1}{\epsilon}} \log\left[1 + e^{z_t}\right]. \tag{14}$$

Hence, the fluctuations in the composition risk in V_t are directly related to the relative share of durables in total consumption.

Further, the effective dividend on the consumption asset is also directly related to non-durable consumption and the relative share z_t. Indeed,

$$\log(C_t + Q_t S_t) = c_t + \log(1 + e^{z_t}). \tag{15}$$

From here it follows that the key variables which determine the real discount factor and ultimately, the real asset prices and the risk premia, are those capture the conditional dynamics of the relative share z_t and the non-durable consumption growth Δc_t; this observation is similar to one made in Piazzesi et al. (2007) in the context of expected utility. In the next section, we specify the exogenous dynamics of the macroeconomic inputs, and solve for the equilibrium asset prices using the Euler condition in Equation (10).

3 Model Specification

3.1 Economy Specification

Denote g_t the vector of non-durable consumption growth, non-durable inflation rate, growth rate of stock of durables, and change in preference shock: $g_t = \begin{bmatrix} \Delta c_t & \Delta \pi_t & \Delta s_t & \Delta a_t \end{bmatrix}'$. Following the long-run risks model, we model their dynamics by incorporating a time-varying component x_t in the drift:

$$g_{t+1} = \mu_g + F x_t + \Sigma_g \eta_{t+1}, \tag{16}$$

where η_{t+1} represents the vector of macroeconomic shocks, and x_t captures the persistent variation in expected growth opportunities in the economy, specified in the following way:

$$x_{t+1} = \Pi x_t + \Sigma_x \eta_{t+1}. \tag{17}$$

In particular, x_t includes expected growth components of durable and non-durable consumption, inflation rate and preference shock. For parsimony, we assume that short-run news and news into expected growth are pair-wise, i.e. $\Sigma_g \Sigma_x' = 0$.

As we discussed in the previous section, what matters for the asset prices are the dynamics of non-durable consumption and relative expenditure share. The growth rate of the latter can be written in terms of the underlying states of the economy:

$$\begin{aligned}\Delta z_{t+1} &= \left(1 - \frac{1}{\epsilon}\right)(\Delta a_{t+1} + \Delta s_{t+1} - \Delta c_{t+1}) \\ &= \left(1 - \frac{1}{\epsilon}\right)(i_a + i_s - i_c)' g_{t+1},\end{aligned} \tag{18}$$

where i_a, i_c and i_s are the vectors which pick out preference shock, non-durable and durable consumption growth from the vector z_t.

3.2 Discount Factor

To obtain closed-form analytical solutions to the asset prices, we approximate the dynamics of $log(1 + e^{z_t})$ in the following way[1]

$$\Delta log(1 + e^{z_t}) \approx \chi \Delta z_t. \tag{19}$$

The parameter $\chi \in (0, 1)$ is an approximating constant equal to the average expenditure on durables in the economy, $\chi = \frac{\bar{Q}\bar{S}}{\bar{Q}\bar{S}+\bar{C}}$. Hence, this parameter captures the importance of durable goods in the economy. The dynamics of V_t term and total consumption can the be stated in the following way:

$$\begin{aligned}\Delta \log V_t &= \frac{1}{1-\frac{1}{\epsilon}} \Delta \log\left[1 + e^{z_t}\right] \approx \frac{1}{1-\frac{1}{\epsilon}} \chi \Delta z_{t+1}, \\ \Delta \log(C_t + Q_t S_t) &= \Delta c_t + \Delta \log(1 + e^{z_t}) \approx \Delta c_{t+1} + \chi \Delta z_{t+1}.\end{aligned} \tag{20}$$

Hence, for $\chi = 0$ (e.g., when preference weight to durables $\alpha = 0$), the specification reduces to a one-good economy, while high χ indicates relative importance of the durable channel.

[1]Our linearization approach shuts-off the impact of relative share fluctuations for the second moments of approximating variables; closed-form solutions which avoid linearization are discussed in Cochrane, Longstaff, and Santa-Clara (2008).

To solve the model, conjecture that the equilibrium price-consumption ratio is a function of the economic states x_t :

$$pc_t = A_0 + A_x' x_t. \tag{21}$$

Using the Euler equation for the consumption asset, we obtain that the price-consumption loadings satisfy:

$$A_x = \left(1 - \frac{1}{\psi}\right) \left(I - \kappa_1 \Pi'\right)^{-1} F' \left((1-\chi) i_c + \chi (i_s + i_a)\right), \tag{22}$$

where κ_1 is the log-linearization coefficient whose value is provided in the Appendix. The intuition for the signs of the responses of asset valuations to economic state naturally extends that in one-good long-run risks model (see Bansal and Yaron (2004)). Investors are concerned about future long-run expectations of non-durable consumption c_t and effective durable consumption $a_t s_t$, where the relative weight to the two is determined by the average expenditure parameter χ. When inter-temporal elasticity of substitution ψ is above one, substitution effect dominates wealth effect. So, the states which increase expected non-durable or effective durable consumption positively affect asset valuations.

The real stochastic discount factor, un units of non-durable numeraire, can be written in terms of the fundamental states and shocks in the economy in the following way:

$$m_{t+1} = m_0 + m_x' x_t - (\lambda_g' \Sigma_g + \lambda_x' \Sigma_x) \eta_{t+1}, \tag{23}$$

where m_x captures the loadings of the discount factor on expected growth components and preference variable, and λ_g and λ_x encode risk compensation for short-run and long-run economic shocks. To gain further intuition on the sources and compensation for risks in the economy, we can decompose discount factor loadings and market prices of risks to the components related to expected non-durable and durable consumption and preference state variable. Specifically,

$$m_x = -\left(\frac{1}{\psi}(1-\chi) + \frac{1}{\epsilon}\chi\right) F' i_c + \chi \left(\frac{1}{\epsilon} - \frac{1}{\psi}\right) F'(i_s + i_a). \tag{24}$$

The three components in the brackets capture the loadings of the discount factor to expected non-durable consumption, expected durable consumption, and preference state variable. Notably, when $\chi = 0$ the specification reduces to a one-good non-durable model, and the loading captures the expected non-durable consumption divided by the IES. With durable goods, both the inter-temporal elasticity of substitution and elasticity of substitution between the two goods play a key role in determining the response of the discount factor to the underlying economic states. In a two-good economy, similar to a one-good one, the loading to expected non-durable consumption is negative. When $\epsilon < \psi$, the loading to expected durable consumption and expected preference shock is positive.

In a similar way, we can decompose the market prices of risks in the economy:

$$\begin{aligned} \lambda_z &= \left(\gamma(1-\chi) + \frac{1}{\epsilon}\chi\right) i_c + \left(\gamma - \frac{1}{\epsilon}\right)\chi(i_s + i_a), \\ \lambda_x &= (1-\theta)\kappa_1 A_x. \end{aligned} \tag{25}$$

In a one-good non-durable economy, $\chi = 0$, and price of short-run consumption news is γ, while the price of expected growth news is given by $(1-\theta)\kappa_1 A_x$. With durables, the price of non-durable consumption risks changes to a weighted average between the risk aversion and the inverse of elasticity of substitution, where the weight is determined by the importance of durables in the economy. The prices of short-run durable risk and preference shocks depend on the relative magnitude of the risk-aversion coefficient and the inverse of elasticity of substitution, and are expected to be positive when intra-temporal elasticity ϵ is small enough.

It is important to note that with CRRA expected utility, $\gamma = 1/\psi$, and market-prices of expected durable and non-durable consumption risks are equal to zero. Then, investors are concerned only with short-run innovations in consumption and preference shocks.

3.3 Equilibrium Bond Yields

We first start with the equilibrium solution for the real yields. Notably, our economy is solved under the assumption that non-durable consumption is the numeraire. Denote

$p_{t,n}^{nd}$ the price of a risk-free bond which delivers one unit of non-durable consumption n periods in the future. Using Euler condition in Equation (10), we can show that the equilibrium price is linear in the states of the economy:

$$p_{t,n}^{nd} = -B_{0,n}^{nd} - B_{x,n}^{nd\prime} x_t, \tag{26}$$

where the recursive solutions for the bond loadings are provided in the Appendix. In particular, the solution to a one-period bond yield satisfies,

$$r_t^{nd} = B_{0,n}^{nd} - m_x' x_t. \tag{27}$$

Given our discussion in the previous section, the one-period claim to unit nondurables positively responds to expected consumption news, and negatively to news to expected durables if $\epsilon < \psi$.

Denote $rx_{t+m,n}^{nd}$ the excess log return on buying an n month bond at time t and selling it at time $t+m$ as an $n-m$ period bond as

$$rx_{t+m,n}^{nd} = -py_{t,n}^{nd} + p_{t+m,n-m}^{nd} + py_{t,m}^{nd}. \tag{28}$$

The expected excess return for 1-period strategies is given by the covariance of the discount factor with the excess bond return, up to Jensen's term,

$$\begin{aligned} E_t rx_{n,t+1}^{nd} + \frac{1}{2} Var_t rx_{n,t+1}^{nd} &= -Cov_t(m_{t+1}, rx_{t+1,n-1}^{nd}) \\ &= -B_{x,n-1}^{nd\prime} \Sigma_x \Sigma_x' \lambda_x. \end{aligned} \tag{29}$$

The overall risk premium depends on market prices of long-run risks, bond loadings to those risks, as well as the variance-covariance matrix of expected growth shocks. When the real bond premium is positive, this leads to a positive slope of the term structure.

In the approach above the real bonds deliver one unit of nondurable consumption. Unlike a one-good economy, in multiple good economy this is not the only way to think about the real risk-free asset. More generally, we can define a real bond as delivering a basket of goods in the future. Define P_t^* the price of the basked in units of nondurables. Then, the price of the bond which delivers this basked satisfies the following Euler equation:

$$P^*_{t,n} = E_t M_{t+1} \frac{P^*_t}{P^*_{t+1}} P^*_{t+1,n-1}. \tag{30}$$

For instance, taking $P^*_t = 1$ we are back to the definition of real bond delivering unit of nondurables. If $P^*_t = Q_t$ or $P_t = P^d_t$, this represents an Euler condition for a claim to a unit of durable consumption in the future, where the price deflator uses rental cost or purchase price of durables, respectively. We can further take P^*_t to be the ideal price index corresponding to investors preferences. For example, using the rental values, the ideal price index can be defined

$$P^*_t = \left((1-\alpha)^\epsilon + \alpha\epsilon \left(\frac{Q_t}{A_t} \right)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}, \tag{31}$$

and similar using the purchase price of durables. In a particular case of of a claim on durable good and rental price as a deflator, we can solve for the bond price analytically using similar approach as for a claim on durables. For other definitions, we use log-linearizations to compute the prices in a closed form.

Another particular case of the Euler condition above obtains when we use nominal price of non-durables in P^*_t. This allows us to compute equilibrium prices of nominal claims. Indeed, the nominal discount factor takes into account the dynamics of the inflation process, and can be solved in the following way:

$$\begin{aligned} m^\$_{t+1} &= m_{t+1} - \pi_{t+1} \\ &= m^\$_0 + m^{\$\prime}_x x_t + m_z z_t - (\lambda^{\$\prime}_g \Sigma_g + \lambda'_x \Sigma_x)\eta_{t+1}. \end{aligned} \tag{32}$$

The nominal discount factor parameters are provided in the Appendix. For nominal bonds, the exposure of expected non-durable and durable consumption to expected inflation risks changes the exposure of nominal bonds to expected growth risks. This, the inflation premium can come through both the non-durable and durable channel: if both expected non-durables and durables respond negatively to expected inflation, then non-durable and durable based inflation premia are all positive.

Notably, with expected CRRA utility, market prices of expected durable and non-durable risks are equal to zero. Hence, in this benchmark model, the real and nominal premia are equal to zero, which will lead to a flat real and nominal term structure. We examine the quantitative importance of the durable risk channel in the empirical part of the paper.

4 Empirical Results

4.1 Data

The Bureau of Economic Analysis (BEA) reports quarterly data on nominal non-durable good and service expenditures, nominal durable good expenditures, and non-durable and durable good price levels. Following the literature convention, we assume that consumers derive utility from the real service flow of consumption goods. The non-durable goods and services are consumed at the time of purchase, so the consumption of non-durable goods and services is equal to their expenditure. Durable goods, once purchased, are consumed over multiple periods. We assume that the service flow of the durable goods is proportional to the stock level, consistent with the convention in the literature (i.e., Yogo (2006)). Since the BEA only reports the year-end durable good stock levels, we back out the quarterly durable good stock level using the depreciation and expenditure in the same way as Yogo (2006). Aggregate nominal service flows are then deflated by the appropriate price levels and divided by the the total population to yield the real service flow data per capita. The service flows of consumption goods and the inflation rate exhibit strong seasonality in the data. Since the seasonality has no theoretical economic implications for equilibrium bond yields we pre-filter the seasonality from the macro-economic data.

[Table 1 about here.]

[Figure 1 about here.]

Table 1 presents basic descriptive statistics for our sample data. The data are seasonally adjusted real quarterly logarithmic growth rates. As seen, the non-durable consumption growth rate is about half that of durables. Durable growth is slightly more volatile than non-durable, and importantly, it has significantly more persistence with almost double the first order autocorrelation of non-durable (63% vs. 38%). As shown in Figure 1, durable growth autocorrelation coefficient remains positive and large up to ten lags, while that of non-durable consumption falls to zero at about 5 lags.

Consistent with high persistence of durable growth in the data and the evidence in Yogo (2006) and Yang (2010), durables are significantly predictable by the asset prices and macroeconomic variables in the data. In Table 2 we show that nominal interest rate predicts very significantly future durable growth, and the R^2 reach above 20% at 2 and 3 year horizons. The predictability evidence is much stronger for durable consumption than for non-durable one. Similarly, inflation rate strongly and negatively predicts future durables, as shown in Table 3, with the R^2 of 35% at 3 year horizons. This effect is again stronger for durables than nondurables.

[Table 2 about here.]

[Table 3 about here.]

4.2 Cointegration Specification and Preference Shock

Durable goods are consumed over multiple periods so the user cost of durable goods is not equal to the purchase price. The formal definition of user cost is given in equation 8. It is usually infeasible to obtain the exact sequence of user cost because the Stochastic Discount Factor (SDF) itself depends on the user cost indirectly. Following Ogaki and Reinhart (1998), we approximate the user cost by the conditional expectation of durable good price next period and the one period risk free rate [2]:

$$Q_t \approx P_t^d - \frac{(1-\delta)E_t(P_{t+1}^d)}{E_t(R_{f,t+1})}$$

[2] Refining the approximation using the extracted state variables is work-in-progress.

Numerous studies in the literature have used the following cointegration relationship between the user cost of durables and the ratio of durable stock to non-durable consumption:

$$const - (s_t - c_t) - (\epsilon q_t - \epsilon p_t^c) = res_t. \tag{33}$$

where s_t is the log level durable service flow, c_t is the log level non-durable expenditure, q_t is the durable good user cost and p_t^c is the non-durable good purchase price. ϵ is the elasticity of substitution between two goods. Under the assumption of cointegration, this relation can be used to estimate the elasticity of substitution between two goods ϵ (see e.g. Ogake and Reinhart (1998) and the references therein, and also Yogo (2006), Pakos (2007), and Piazzesi et al. (2007)). The OLS regression yields a super-consistent estimate of ϵ when $s_t - c_t$ is robust to various frictions in the specification. In our estimation, the elasticity of substitution ϵ is about 0.83, consistent with Yogo's (2006) estimate of 0.8.

In the frictionless model without preference shock the cointegration equation (33) should hold with identity, and any remaining residual can at best be interpreted as a measurement error in the observed macroeconomic variables. With this interpretation, the residual should not have predictive power to forecast future macroeconomic or financial variables. In the data, however, we find that the cointegration residual strongly predicts future multi-period durable growth and inflation, and also that the residual is strongly related to the nominal yields. Siegel (2010) concludes that these dynamic characteristics of the cointegration residual are inconsistent with a frictionless Euler equation. In our model we interpret this residual as being directly linked to the preference shocks to durables. Equation (7) implies

$$res_t = (1 - \epsilon)a_t. \tag{34}$$

Since in the model the residual res_t captures the fluctuations in durable preference shocks, in principle, it can predict future durable consumption: a contemporaneous durables preference shock will increase the amount of durables consumed in the future. Empirically, we find that the residual res_t strongly predicts durable consumption growth and inflation in the future; for example, it positively and significantly predicts future durable consumption with R^2 of $35 - 43\%$ at two to five year horizons (as shown in Table 3). Further, as shown in Figure 5, shocks to expected preference shock significantly raise future durables, while their impact on nondurables is negative after 2 years. This evidence of predictability is a key motivating factor in including the preference shock in the theoretical model. While we do not specify primitive economic channels for this effect (e.g., production of durables), we believe this specification is economically plausible and can successfully capture the salient features of the macroeconomic variables in the data.

4.3 Macroeconomic Model Estimation

We estimate our full general model using Bayesian Markov-Chain-Monte-Carlo under uninformative (uniform) priors. The likelihood function were computed using standard Kalman filtering. The advantage of Kalman filtering is that we will recover estimates of the unobserved latent state-variables, X_i for $i = \{c, s, \pi, a\}$. We will use these filtered state-variables to compute our model implied yield curve. Importantly, we construct the estimates of the filtered state-variables and the model parameters without revealing financial markets data to the estimator. Thus, the financial market implications are effectively computed "out-of-sample", as no financial market data are used to influence the model based financial market implications.

[Table 4 about here.]

[Figure 2 about here.]

[Figure 3 about here.]

[Figure 4 about here.]

[Figure 5 about here.]

Table 4 gives the parameter estimates of our model. The table again reveals that expected durable consumption shocks have a more persistent impact than do non-durable (0.86 vs 0.77). Inflation is very persistent with a quarterly autocorrelation of

0.89. Inflation shocks have a small negative impact on non-durable (real) consumption, and a relatively larger impact on non-durable consumption (-0.0637). Overall, the model matches very well the autocorrelation patterns in the data, as shown in Figure 1. To provide further evidence on dynamic long-run interactions between the variables, we plot impulse response functions of the four macroeconomic inputs to expected growth shocks in Figures 3 - 5. First, a positive, one-standard deviation shock to non-durable expected growth increases future non-durable consumption growth, and generally decreases durable consumption growth. It also predicts an increase in future inflation. Shocks to durable consumption is similarly associated with increases in future durable consumption, but lower non-durable consumption.

The north-west quadrant of figure 3 shows that shocks in durable and non-durable consumption have opposite effects on expected inflation: shocks to durable growth are associated with lower future expected inflation while shocks to non-durable growth are associated with increased expected inflation. The difference in the inflation impacts of shocks to the two goods is important for the term structure because long maturity bonds are inflation sensitive. The difference in impacts of these shocks on future inflation can be traced back to the estimates of $\Pi(2,1) = 0.4243$ and $\Pi(2,3) = -0.1053$.

Figure 4 shows the response of inflation shocks on the real variables. As can be seen, inflation has a much larger impact on durable consumption than non-durable consumption. This negative long run impact of inflation shocks on real durable consumption implies that long dated real bond yields carry a positive inflation risk premium due to a durable risk channel.

4.4 Model-Implied Yields

In the previous subsection, we estimate all the parameters that determine the dynamics of the macro-economic variables using these macro variables only. We also obtain the whole time series of latent variables without using any information contained in the historical yield curve data.This step does not allow us to identify the preference parameters δ, ψ and γ. In this subsection, we take a second step to calibrate the preference parameters to match historical yield curves in our benchmark model. We set subjective discount factor $\delta = 0.9975$, IES $\psi = 2$ and risk aversion parameter

$\gamma = 17$.

[Table 5 about here.]

In Table 5 we document the term structure of the level, volatility and first order autocorrelation for nominal yields implied by our models. Our benchmark model (reported in the row denoted by "Full") produces a upward sloping nominal yield curve and matches the data nearly perfectly on level. The volatility of yield predicted by our benchmark model slightly fall short of that in data, and this is common in many macroeconomic term structure models. As in the data, the autocorrelation of yields implied by the model increases as the maturity grows.

[Table 6 about here.]

[Figure 6 about here.]

[Figure 7 about here.]

In addition to the unconditional moments, an important criteria for the success of any asset pricing model is its ability to explain a conditional time-series evidence. We summarize the model fit in Table 6, we report the Mean-Squared Errors (MSE) as a measure of overall fit of the whole time series. Our benchmark model implications (reported in the column denoted by "Full") match the average one year nominal yield and five year nominal spread perfectly. The MSE are very low for our benchmark model, implying a very good fit for the whole time series. Figures 6 and 7 clearly plot the short rate and term spread observed in the data and implied by our model. Recall that the model implied yields are computed using time-series parameters and filtered state-variables which are estimated using the macro data only, so the model prediction is "out-of-sample". Generally, the model-implied yields track the empirical yields in the data quite well. Some of the noticeable deviations of the model predictions to the data include the mid eighties, where from about 83-87 interest rates peaked significantly above what is predicted by our model. This period spells the aftermath of the 79-82 monetary policy experiment. The second period of particular interest is the most recent from about 2001-2006. During this period the trajectory of interest rates was below our equilibrium prediction, which again might have to do with particularities of the interest rate policy of this period which are outside our model.

[Table 7 about here.]

Our equilibrium model predicts that the one period risk free bond yield loads positively on non-durable consumption growth and negatively on durable consumption growth. To verify this in the data, we regress the observed three month nominal yield on the filtered state variables X_i for $i = \{c, s, \pi, a\}$ and compare the regression coefficients with the loading implied by our model. The R^2 of the regression is about 65% and the signs of coefficients on non-durable and durable match our model predictions. The data actually implies an even stronger negative loading on durable consumption growth and an even stronger positive loading on non-durable consumption growth. The magnitude of our model prediction stands within the 90% confidence interval of the empirical estimates.

[Table 8 about here.]

Though our empirical evaluation of the benchmark model mainly focuses on nominal yield and spread, the model gives rich implication on real yield as well. It is well known that in one-good economy the real yield curve is downward sloping when investors prefer early resolution of risk and dislike persistent shocks. In two-good economy, however, our model predicts that the real yield may become upward sloping if the durable good channel is important enough. As we have discussed in the model solution section, when consumption bundle includes both non-durable and durable goods, the numeraire used in the model is also important in determining the real yield. We document the model implied real yield term structure in Table 9.

[Table 9 about here.]

In general, real yield curve remains upward sloping under different numeraires, but the slope and level of the curves are different. When the real bond delivers one unit of non-durable good, the real yield is relatively flatter because the durable good only affects the real yield through the SDF channel and the payoff of the real bond is not affected on maturity. However, when the real bond delivers one unit of durable good measured at the purchase price, the real yield curve becomes steeper, because now durable good takes effect through two channels. The durable consumption growth affects the SDF and the durable purchase price inflation determines the payoff value of real bonds. Taking the ideal price index as numeraire generates the steepest real yield curve. In this case, durable good influences real yield through three channels: the durable consumption growth affects SDF, and the preference shock and durable user cost inflation determines the payoff of real bonds on maturity.

[Table 10 about here.]

Once we have both nominal and real yields, we are able to investigate the term structure of inflation premium in the two-good economy. We show in Table 3 that inflation predicts negatively future consumption growth in data, and its impact on durable consumption growth is far more significant than that on non-durable. If inflation brings bad news and the news is particularly bad on durable good side, then we expect our model to generate positive inflation premium. The unconditional inflation premium is defined as the unconditional nominal yield minus unconditional expected inflation minus unconditional real yield for the same maturity. We report the model implied term structure of unconditional inflation premium (assume non-durable good as numeraire) in Table 10. The inflation premium grows over maturities and reaches 66 bps for a five year nominal bond.

4.5 Role of Model Ingredients for Equilibrium Yields

In theoretical Section 3.3 we established several key results for the novel and important role of durable risks for the term structure when investors have EZ preference. First, we showed that expected durable growth risks can lead to a positive risk premium on real bonds. This result is in a sharp contrast to standard one-good economies with only non-durable consumption, as in these economies real bonds hedge expected consumption risks and thus always require negative risk premium, as discussed in Bansal and Shaliastovich (2010) and Bansal and Yaron (2004), among others. Second, in a two-good economy setup bond prices react to state variables which drive the expected durable growth, in addition to the state variables which capture the dynamics of non-durable consumption. In particular, this implies that the inflation premium now can come through the durable channel, due to a negative correlation of expected durable growth and expected inflation.

To clearly evaluate the importance of durable good channel and EZ preference, in this section, we evaluate three other models (EZ preference with only non-durable good, power utility with both durable good and non-durable good, and power utility with only non-durable good), and compare their implications with those in our benchmark model (EZ preference with both non-durable and durable good). The first model, i.e. EZ preference with only non-durable good, is constructed by setting the utility weight on durable to zero in our benchmark model ($\alpha = 0$ and thus $\chi = 0$). The second and the third model replace EZ preference in benchmark model by power utility; the second model retains both durable and non-durable channels while the third model further removes the durable channel by setting the utility weight on durable to zero. For comparison purpose, we evaluate the three models with the same preference parameters (i.e. subjective discount factor $\delta = 0.9975$, IES $\psi = 2$ and risk aversion parameter $\gamma = 17$). Meanwhile, we also test the two models with power utility with a relatively lower risk aversion parameter $\gamma = 2$ and higher $\delta = 0.9998$ to better match risk free rate level.

We first start with the implications for the real yields. As shown in Table 8, only the benchmark model generates a upward sloping real yield curve, and all other models produce downward-sloping real yields. The importance of durable channel is evident by comparing the real yields reported in row 1 (denoted by "Full") and row 2 (denoted by "No-dur"). As expected, when we shut down the durable good channel, the hedging effect of real bond dominates and the real yield curve becomes downward sloping even with EZ preference. This is consistent with findings in literature. Meanwhile, the importance of EZ preference is illustrated by comparing the real yields reported in row 1 and row 3 (denoted by "PW1"). As we discussed in theoretical section, if we replace EZ preference by CRRA expected utility (e.g. power utility), the market prices of expected durable and non-durable consumption risks become zero and investors are concerned only with short-run innovations in consumption and preference shocks. In this case, the persistence of durable consumption growth plays no role in generating risk premium and the real yield curve becomes downward sloping. It is also clear that the models with power utility reproduce the well-known interest rate puzzle when the risk aversion is high (i.e. $\gamma = 17$). In the rows denoted by "PW1" and "NPW1", we report that the model implied risk free rates are above 29% for all maturities. We tune down the risk aversion parameter (reduce γ to 2) in row "PW2" and "NPW2" to produce a more reasonable real yield level, but the whole curve is still downward sloping.

Now let's turn to evaluate the implications of nominal yields. The term structure of mean, volatility and first order autocorrelation of nominal yields are reported in Table 5 for all models discussed in this paper. Only the benchmark model can reproduce the term structure of nominal yields well. The five year nominal spreads are negative for all models except the benchmark model, showing the importance of durable good and EZ preference in generating inflation risk premium. Though the EZ preference with only non-durable good predicts a negative spread with our current preference parameters ($\gamma = 17$ and $\delta = 0.9975$), we realize that it may be able to produce a upward sloping yield curve with a very high risk aversion parameter and a subjective discount factor larger than 1 (e.g. Piazzesi and Schneider (2006) use $\gamma = 59$ and $\delta = 1.005$). Table 6 reports the overall fit of the time series of one yield nominal rate and five year spread for different models. The benchmark model yields the lowest MSE and fits the data best. Different models also predicts different loadings of one period bond yield on filtered state variables. As we illustrate in Table 7, models with only non-durable good place zero loadings on the state variables that represent durable consumption growth and preference shock (i.e. X_s and X_a). Power utility models with both non-durable and durable consumption predict a positive loading on the state variable that represents the durable consumption growth. All these model predictions conflict with what we find in data, and the magnitude of the predicted loading on durable consumption lies out of the 95% confidence interval of the empirical estimate.

Conclusion

In the data, durable consumption is very persistent and highly and negatively predictable by nominal yields and inflation, and these effects are much more pronounced for durable consumption then the nondurable one. Motivated by these findings, we set up a two-good, long-run risks type nominal economy which features nonseparable utility over consumption of durable and nondurable goods, fluctuations in relative preference for durables, inflation, and recursive utility function with preference for early resolution of uncertainty. We show that the model is consistent with the above empirical facts. Further, the model can successfully, and effectively out-of-sample, explain unconditional moments and the conditional movements in the term structure. Model-implied equilibrium real yields are upward sloping, which cannot be obtained in a one-good economy, for a range of numeraire choices. Empirically, we find that most of the inflation premium in the model comes from a durable risk channel. Overall, our findings suggests that long-run durable risks and preference for early resolution of uncertainty play a key role to explain the bond prices in the data.

In the current model specification, all the risk premia are constant, so future bond returns are not predictable. We leave extensions of the model to generate time-variation in risk premia, such as through time-varying aggregate volatility as in Bansal and Yaron (2004), Bansal and Shaliastovich (2010) and Hasseltoft (2010), for future research.

A Equilibrium Model Solution

Denote W_t the total wealth of the agent, which includes financial wealth from investing h_{it} shares of asset i with price P_{it} and dividend stream D_{it} ($i = 1, 2, \dots N$), as well as the wealth from holding S_t units of durable goods with purchase price P_t^d. Then, we can write down the budget constraint of the agent in the following way:

$$W_t = \sum_{i=1}^{N} (P_{it} + D_{it}) h_{i,t-1} + (1-\delta) P_t^d S_{t-1} = C_t + P_t^d S_t + \sum_{i=1}^{N} P_{it} h_{it}. \tag{A.1}$$

Denote $\omega_{it} = P_{it} h_{it} / (W_t - C_t)$ the fraction of wealth invested in asset i. Similar, define stock of durable goods relative to wealth $\omega_{N+1,t} = P_t^d S_t / (W_t - C_t)$, and denote durable good return $R_{N+1,t+1} = (1-\delta) P_{t+1}^d / P_t$. Then, we can rewrite the budget constrain in Equation (A.1) in the following way:

$$W_{t+1} = R_{p,t+1} (W_t - C_t), \tag{A.2}$$

where $R_{p,t+1}$ is the total portfolio return:

$$R_{p,t+1} = \sum_{i=1}^{N+1} \omega_{i,t} R_{i,t+1}, \tag{A.3}$$

and portfolio weights ω_{it} sum up to one:

$$\sum_{i=1}^{N+1} \omega_{it} = 1. \tag{A.4}$$

We solve for the optimal non-durable consumption level C and portfolio weights ω which maximize the life-time utility of the agent in Equation (1) subject to the budget and portfolio constraints (A.2)-(A.4).

To simplify exposition, denote

$$\zeta_t = E_t \left(\phi_{t+1} R_{p,t+1} \right)^{1-\gamma}. \tag{A.5}$$

Due to homotheticity of the problem, the life-time utility is proportional to wealth, so we denote $U_t = \phi_t W_t$. Then, the optimization problem of the agent can be stated in the following way:

$$U_{t+1} = \phi_t W_t = \max_{C,\omega} \left[(1-\beta) u_t^{1-\frac{1}{\psi}} + \beta (W_t - C_t)^{1-\frac{1}{\psi}} \zeta_t^{\frac{1}{\theta}} \right]^{\frac{1}{1-\frac{1}{\psi}}}. \tag{A.6}$$

The first-order condition with respect to non-durable consumption choice is given by, after some algebra,

$$(1-\beta)(1-\alpha)V_t^{\frac{1}{\epsilon}-\frac{1}{\psi}}C_t^{-\frac{1}{\psi}}f_t = \beta(W_t - C_t)^{-\frac{1}{\psi}}\zeta_t^{\frac{1}{\theta}}, \tag{A.7}$$

where V_t is specified in Equation (14), and the component f_t is defined to be,

$$f_t = 1 - \frac{\alpha}{1-\alpha}\left(\frac{Y_t}{C_t}\right)^{-\frac{1}{\epsilon}}\frac{A_t\omega_{N+1,t}}{P_t^d}. \tag{A.8}$$

From Equation (A.7) and definition of utility per wealth ratio in Equation (A.6), it follows that

$$\phi_t = \left((1-\beta)(1-\alpha)V_t^{\frac{1}{\epsilon}-\frac{1}{\psi}}\left(\frac{C_t}{W_t}\right)^{-\frac{1}{\psi}}\right)^{\frac{1}{1-\frac{1}{\psi}}}. \tag{A.9}$$

Let us simplify the first-order condition in Equation (A.7). Using the definition of f_t in Equation (A.8) and definition of user cost of durables in Equation (7), it follows that

$$f_t = 1 - \frac{Q_t S_t}{W_t - C_t}. \tag{A.10}$$

Further, note that the the consumption return $R_{c,t+1}$ defined in Equation (9) is related to the total portfolio return through an adjustment

$$R_{c,t+1} = R_{p,t+1}/f_t. \tag{A.11}$$

Now let us rewrite the first-order condition in Equation (A.7) using the definition of ζ_t, the solution for utility per wealth ratio in (A.1), and the definition of consumption asset above. First, from the definition of ζ_t,

$$\begin{aligned} \zeta_t^{\frac{1}{\theta}} &= \left(E_t\left(\phi_{t+1}R_{p,t+1}\right)^{1-\gamma}\right)^{\frac{1}{\theta}} \\ &= (1-\beta)(1-\alpha)f_t\left(W_t - C_t\right)^{\frac{1}{\psi}}\left(E_t V_{t+1}^{\theta\left(\frac{1}{\epsilon}-\frac{1}{\psi}\right)}C_{t+1}^{-\frac{\theta}{\psi}}R_{c,t+1}^{\theta}\right)^{\frac{1}{\theta}}. \end{aligned} \tag{A.12}$$

Use it with the Equation (A.7) to obtain the Euler condition for the consumption asset,

$$E_t M_{t+1} R_{c,t+1} = 1, \tag{A.13}$$

where the stochastic discount factor M_{t+1} satisfies Equation (11).

Now let us take first-order conditions with respect to asset holdings and relative weight in the durable asset. These two conditions satisfy, respectively:

$$
\begin{aligned}
\varrho_t &= \beta\left(1-\frac{1}{\psi}\right)(W_t - C_t)^{1-\frac{1}{\psi}}\,\zeta_t^{\frac{1}{\theta}-1} E_t \phi_{t+1}^{1-\gamma} R_{p,t+1}^{-\gamma} R_{i,t+1}, \\
\varrho_t &= \alpha(1-\beta)\left(1-\frac{1}{\psi}\right) V_t^{\frac{1}{\epsilon}-\frac{1}{\psi}} C_t^{\frac{1}{\epsilon}-\frac{1}{\psi}} \frac{Y_t^{1-\frac{1}{\epsilon}}}{\omega_{N+1,t}} \\
&\quad + \beta\left(1-\frac{1}{\psi}\right)(W_t - C_t)^{1-\frac{1}{\psi}}\,\zeta_t^{\frac{1}{\theta}-1} E_t \phi_{t+1}^{1-\gamma} R_{p,t+1}^{-\gamma} R_{N+1,t+1},
\end{aligned}
\tag{A.14}
$$

where ϱ_t is the Lagrange multiplier on the constraint that all the weights sum up to one, $\sum_{i=1}^{N+1} \omega_{it} = 1$.

Let us rewrite these constraints equivalently using the definition of the consumption return and the discount factor:

$$
\begin{aligned}
\varrho_t &= (1-\beta)(1-\alpha)\left(1-\frac{1}{\psi}\right)(W_t - C_t)\, V_t^{\frac{1}{\epsilon}-\frac{1}{\psi}} C_t^{-\frac{1}{\psi}} E_t M_{t+1} R_{i,t+1}, \\
\varrho_t &= (1-\beta)\left(1-\frac{1}{\psi}\right) V_t^{\frac{1}{\epsilon}-\frac{1}{\psi}} C_t^{-\frac{1}{\psi}} \left(\alpha C_t^{\frac{1}{\epsilon}} \frac{Y_t^{1-\frac{1}{\epsilon}}}{\omega_{N+1,t}} + (1-\alpha)(W_t - C_t) E_t M_{t+1} R_{N+1,t+1}\right).
\end{aligned}
\tag{A.15}
$$

Let us multiply each of the first constraint for asset i by its weight ω_{it} and add the second constraint for the durable good holding multiplied by its share $\omega_{N+1,t}$. Using the definition of the portfolio return in (A.3) and constraint (A.4), we obtain the following equation for the multiplier ϱ_t :

$$
\varrho_t = (1-\beta)(1-\alpha)\left(1-\frac{1}{\psi}\right)(W_t - C_t)\, V_t^{\frac{1}{\epsilon}-\frac{1}{\psi}} C_t^{-\frac{1}{\psi}}. \tag{A.16}
$$

Comparing it to the first-order conditions in Equations A.15, it immediately follows that M_{t+1} is a valid discount factor, so that

$$
E_t M_{t+1} R_{i,t+1} = 1, \tag{A.17}
$$

for any financial asset i. In particular, this Euler condition for the durable good $i = N+1$ gives the characterization of the user cost in Equation (8).

B Model Solution

The log-linearization parameter for the consumption asset κ_1 satisfies the following recursive equation:

$$\begin{aligned}\log \kappa_1 = \log \beta &+ \left(1 - \frac{1}{\psi}\right) \left((1-\chi) i_c + \chi(i_a + i_s)\right)' \mu \\ &+ \frac{1}{2}\theta \left(1 - \frac{1}{\psi}\right)^2 \left((1-\chi) i_c + \chi(i_a + i_s)\right)' \Sigma_g \Sigma_g' \left((1-\chi) i_c + \chi(i_a + i_s)\right) \\ &+ \frac{1}{2}\theta \kappa_1^2 A_x' \Sigma_x \Sigma_x' A_x.\end{aligned} \tag{B.1}$$

The discount factor parameters are given by

$$m_0 = \theta \log \delta + (1-\theta) \log \kappa_1 - \lambda_g' \mu. \tag{B.2}$$

The nominal discount factor parameters satisfy

$$m_0^{\$} = m_0 - i_\pi \mu_g, \quad m_x^{\$} = m_x - F' i_\pi, \quad \lambda_g^{\$} = \lambda_g + i_\pi, \tag{B.3}$$

where $i_\pi = \begin{bmatrix} 0 & 1 \end{bmatrix}'$.

The solution for real bond price loadings are given by,

$$\begin{aligned} B_{0,n} &= B_{0,n-1} - m_0 - \frac{1}{2}\lambda_g' \Sigma_g \Sigma_g' \lambda_g - \frac{1}{2}(\lambda_x + B_{x,n-1})' \Sigma_x \Sigma_x' (\lambda_x + B_{x,n-1}), \\ B_{x,n} &= \Pi' B_{x,n-1} - m_x, \end{aligned} \tag{B.4}$$

and similar for nominal bonds using the parameters of the nominal discount factor in Equation (B.3).

C Change of Numeraire

In our model, the inflation rate of durable good is endogenous. Before jumping to the derivation of real yield under different numeraires, we need to derive the model implied durable good inflation rate.

C.1 Model Implied Durable Good Inflation Rate

By definition, the nominal user cost Q_t is

$$Q_t = P_t^d - (1-\delta) E_t[M_{t+1} P_{t+1}^d]$$

Denote $F_t = \frac{Q_t}{P_t^d}$ and rewrite the equation above as

$$F_t = 1 - (1-\delta)E_t[M_{t+1}\frac{Q_{t+1}}{Q_t}\frac{F_t}{F_{t+1}}]$$

With the notations above, the model implied durable good inflation is:

$$\ln\frac{P_{t+1}^d}{P_t^d} = \ln\frac{Q_{t+1}}{Q_t} - \ln\frac{F_{t+1}}{F_t}$$

In our model, the log-SDF takes the form of:

$$m_{t+1} = m_0 + m_x^{'} x_t - (\lambda_g^{'}\Sigma_g + \lambda_x^{'}\Sigma_x)\eta_{t+1}$$

The cointegration residual is defined as:

$$res_t = const + c_t - s_t + \epsilon p_t^c - \epsilon q_t$$

where c_t is the log level non-durable consumption, s_t is the log level durable consumption, p_t^c is the log level of non-durable good purchase price and q_t is the log level durable good user cost. Taking first order difference on both side, we can obtan:

$$\Delta res_{t+1} = (i_c' - i_s^{'} + \epsilon i_\pi^{'})g_{t+1} - \epsilon \ln\frac{Q_{t+1}}{Q_t}$$

And also notice that $\Delta res_{t+1} = (1-\epsilon)\Delta a_{t+1} = (1-\epsilon)i_a^{'} g_{t+1}$, so

$$\begin{aligned} \ln\frac{Q_{t+1}}{Q_t} &= \frac{1}{\epsilon}[i_c' - i_s^{'} + \epsilon i_\pi^{'} - (1-\epsilon)i_a^{'}]g_{t+1} \\ &= h_q^{'} g_{t+1} \end{aligned}$$

where the constant $h_q^{'} = \frac{1}{\epsilon}[i_c' - i_s^{'} + \epsilon i_\pi^{'} - (1-\epsilon)i_a^{'}]$.

Now let's define $s_t = \ln\{(1-\delta)E_t[M_{t+1}\frac{Q_{t+1}}{Q_t}\frac{F_t}{F_{t+1}}]\}$ and log-linearize the function F_t:

$$\ln F_t = \ln(1-e^{s_t}) \approx \ln(1-e^{\overline{s}}) - \frac{e^{\overline{s}}}{1-e^{\overline{s}}}(s_t - \overline{s})$$

We conjecture that $\ln(F_t) = f_0 + f_x^{'} x_t$, then

$$\begin{aligned} E_t[M_{t+1}\frac{Q_{t+1}}{Q_t}\frac{F_t}{F_{t+1}}] &= E_t[\exp(m_{t+1} + g_{q,t+1} - f_x(x_{t+1} - x_t))] \\ &= \exp\{[m_0 + h_q^{'}\mu_g + \frac{1}{2}(\lambda_g^{'} - h_q^{'})\Sigma_g\Sigma_g^{'}(\lambda_g - h_q) + \frac{1}{2}(\lambda_x^{'} + f_x^{'})\Sigma_x\Sigma_x^{'}(\lambda_x + f_x)] \\ &\quad +[m_x^{'} + h_q^{'} - f_x^{'}(\Pi - I)]x_t\} \end{aligned}$$

We also obtain s_t as follows:

$$\begin{aligned} s_t &= \ln\{(1-\delta)E_t[M_{t+1}\frac{Q_{t+1}}{Q_t}\frac{F_t}{F_{t+1}}]\} \\ &= \ln(1-\delta) + [m_0 + h_q^{'}\mu_g + \frac{1}{2}(\lambda_g^{'} - h_q^{'})\Sigma_g\Sigma_g^{'}(\lambda_g - h_q) + \frac{1}{2}(\lambda_x^{'} + f_x^{'})\Sigma_x\Sigma_x^{'}(\lambda_x + f_x)] \\ &\quad +[m_x^{'} + h_q^{'} - f_x^{'}(\Pi - I)]x_t \\ &= \ln(1-\delta) + s_0 + s_x' x_t \end{aligned}$$

where

$$\begin{aligned} s_0 &= m_0 + h_q^{'}\mu_g + \frac{1}{2}(\lambda_g^{'} - h_q^{'})\Sigma_g\Sigma_g^{'}(\lambda_g - h_q) + \frac{1}{2}(\lambda_x^{'} + f_x^{'})\Sigma_x\Sigma_x^{'}(\lambda_x + f_x) \\ s_x^{'} &= m_x^{'} + h_q^{'} - f_x^{'}(\Pi - I) \end{aligned}$$

Now substitute s_t into $\ln F_t$ and match the coefficients:

$$f_0 + f_x^{'} x_t = \ln(1 - e^{\overline{s}}) - \frac{e^{\overline{s}}}{1 - e^{\overline{s}}}[\ln(1-\delta) + s_0 + s_x' x_t - \overline{s}]$$

Matching the coefficient of x_t :

$$f_x = (\Pi' - e^{-\overline{s}} I)^{-1}(m_x + h_q)$$

Now mathcing the constant term:

$$f_0 = \ln(1 - e^{\overline{s}})$$

As a result, the model implied durable good inflation is

$$\begin{aligned} \ln\frac{P_{t+1}^d}{P_t^d} &= \ln\frac{Q_{t+1}}{Q_t} - f_x^{'}(x_{t+1} - x_t) \\ &= h_q^{'} g_{t+1} - f_x^{'}(x_{t+1} - x_t) \end{aligned}$$

C.2 Durable Good Purchase Price as Numeraire

Let m_{t+1} denote the real log-SDF under the numeraire of non-durable good price. If we change numeraire to durable good purchase price, the new log-SDF we shall use to calculate real yield is now equal to:

$$m^{p^d}_{t+1} = m_{t+1} - (\ln \frac{P^d_{t+1}}{P^d_t} - \pi_{t+1})$$

where π_{t+1} is the non-durable good inflation rate.

Now substitute in the formula of durable good inflation rate we obtained above:

$$\begin{aligned} m^{p^d}_{t+1} &= [m_0 - (h^{'}_q - i^{'}_\pi)\mu_g] + [m^{'}_x - h^{'}_q + i^{'}_\pi + f^{'}_x(\Pi - I)]x_t \\ &\quad -[(\lambda^{'}_g + h^{'}_q - i^{'}_\pi)\Sigma_g + (\lambda^{'}_x - f^{'}_x)\Sigma_x]\eta_{t+1} \end{aligned}$$

Applying this new log-SDF in the set of difference equations to calculate the real yields for different maturities.

C.3 Ideal Price Index as Numeraire

The ideal price index P^{ipx}_t is defined through the consumption bundle:

$$\frac{P^{ipx}_t}{P^c_t} = [(1-\alpha)^\epsilon + \alpha^\epsilon A^{\epsilon-1}_t (\frac{Q_t}{P^c_t})^{1-\epsilon}]^{\frac{1}{1-\epsilon}}$$

So the inflation rate of the ideal price index $\ln(P^{ipx}_{t+1}/P^{ipx}_t)$ satisfies:

$$\begin{aligned} \ln(\frac{P^{ipx}_{t+1}}{P^{ipx}_t}) - \pi_{t+1} &= \frac{1}{1-\epsilon} \ln\{\frac{(1-\alpha)^\epsilon + \alpha^\epsilon A^{\epsilon-1}_{t+1}(\frac{Q_{t+1}}{P^c_{t+1}})^{1-\epsilon}}{(1-\alpha)^\epsilon + \alpha^\epsilon A^{\epsilon-1}_t(\frac{Q_t}{P^c_t})^{1-\epsilon}}\} \\ &\approx \frac{1}{1-\epsilon} \frac{\alpha^\epsilon e^{(1-\epsilon)\overline{q}}}{(1-\alpha)^\epsilon + \alpha^\epsilon e^{(1-\epsilon)\overline{q}}}[(\epsilon - 1)i^{'}_a + (1-\epsilon)(h^{'}_q - i^{'}_\pi)]g_{t+1} \\ &= h^{'}_{ipx} g_{t+1} \end{aligned}$$

where the constant $h^{'}_{ipx} = \frac{1}{1-\epsilon}\frac{\alpha^\epsilon e^{(1-\epsilon)\overline{q}}}{(1-\alpha)^\epsilon + \alpha^\epsilon e^{(1-\epsilon)\overline{q}}}$.

Now if we change numeraire to the ideal price index, the new log-SDF we shall use to calculate the real yield is:

$$
\begin{aligned}
m_{t+1}^{ipx} &= m_{t+1} - [\ln(\frac{P_{t+1}^{ipx}}{P_t^{ipx}}) - \pi_{t+1}] \\
&= [m_0 - h_{ipx}^{'}\mu_g] + [m_x^{'} - h_{ipx}^{'}]x_t - [(\lambda_g^{'} + h_{ipx}^{'})\Sigma_g + \lambda_x^{'}\Sigma_x]\eta_{t+1}
\end{aligned}
$$

Applying this new log-SDF in the set of difference equations to calculate the real yields for different maturities.

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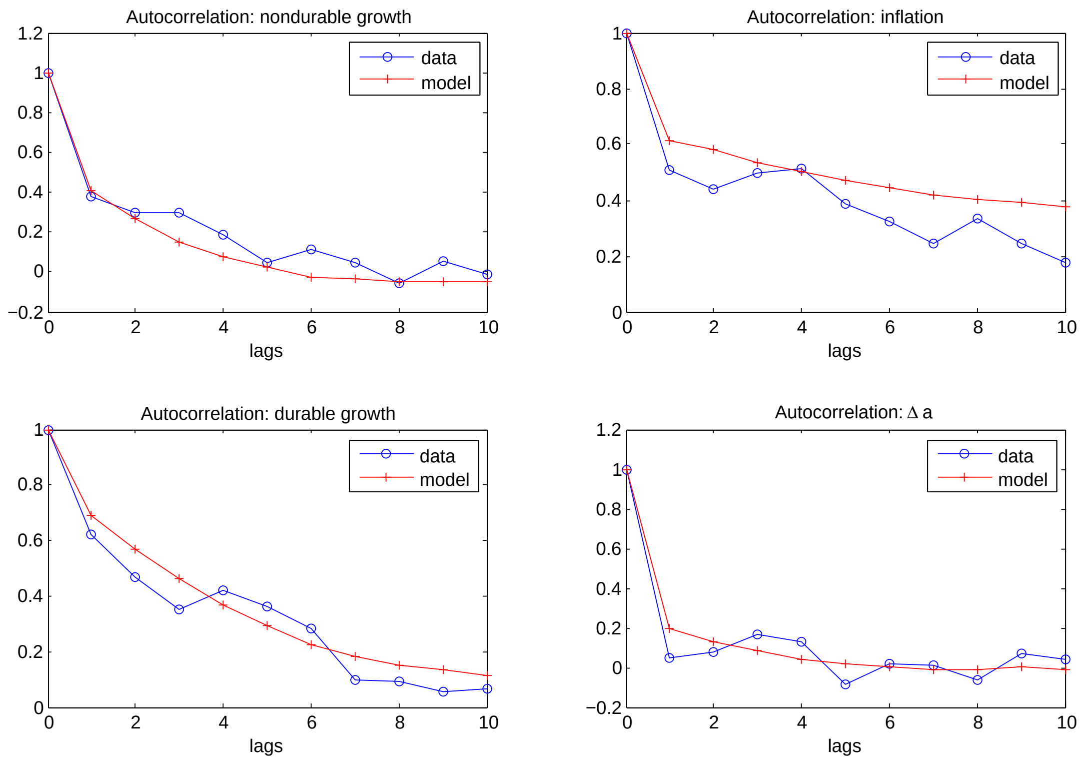

Figure 1: Autocorrelation functions of non-durable consumption growth, durable growth, inflation rate and growth of estimated preference shock. Quarterly observations from 1965Q1 to 2009Q4.

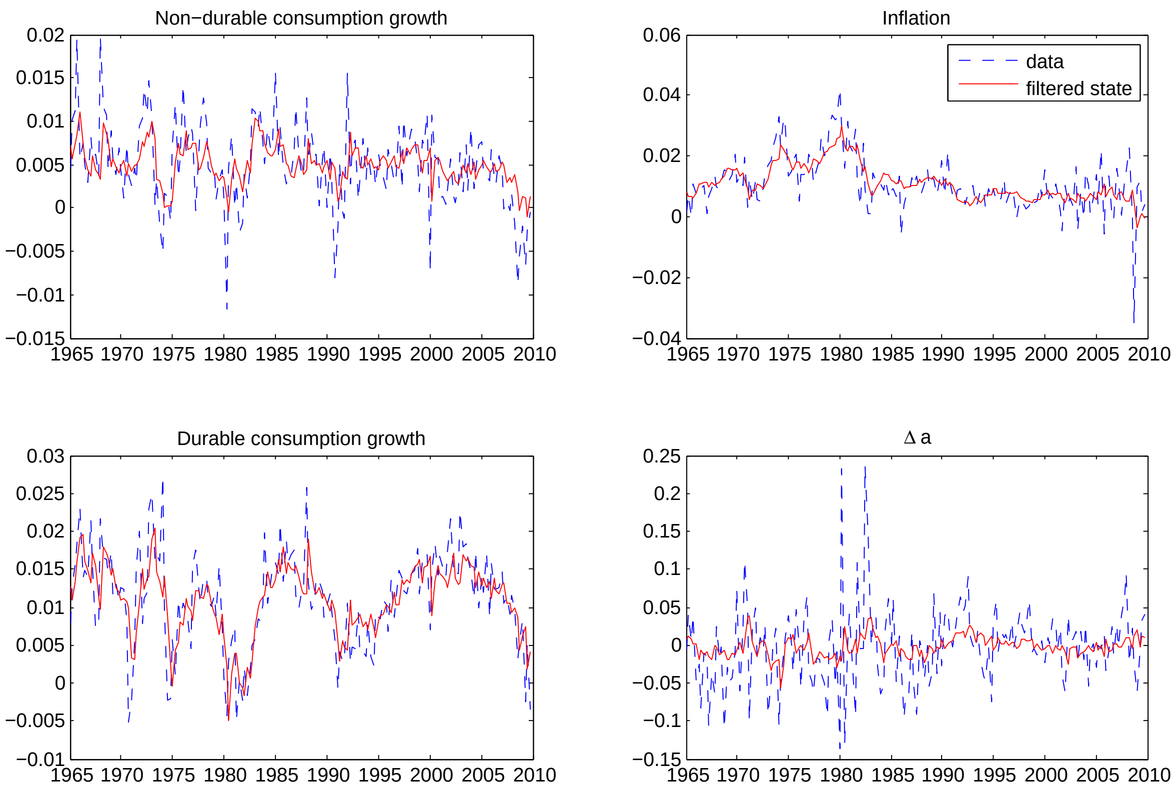

Figure 2: Realized and expected nondurable consumption and durable consumption growth rates, inflation rate and change in preference shock.

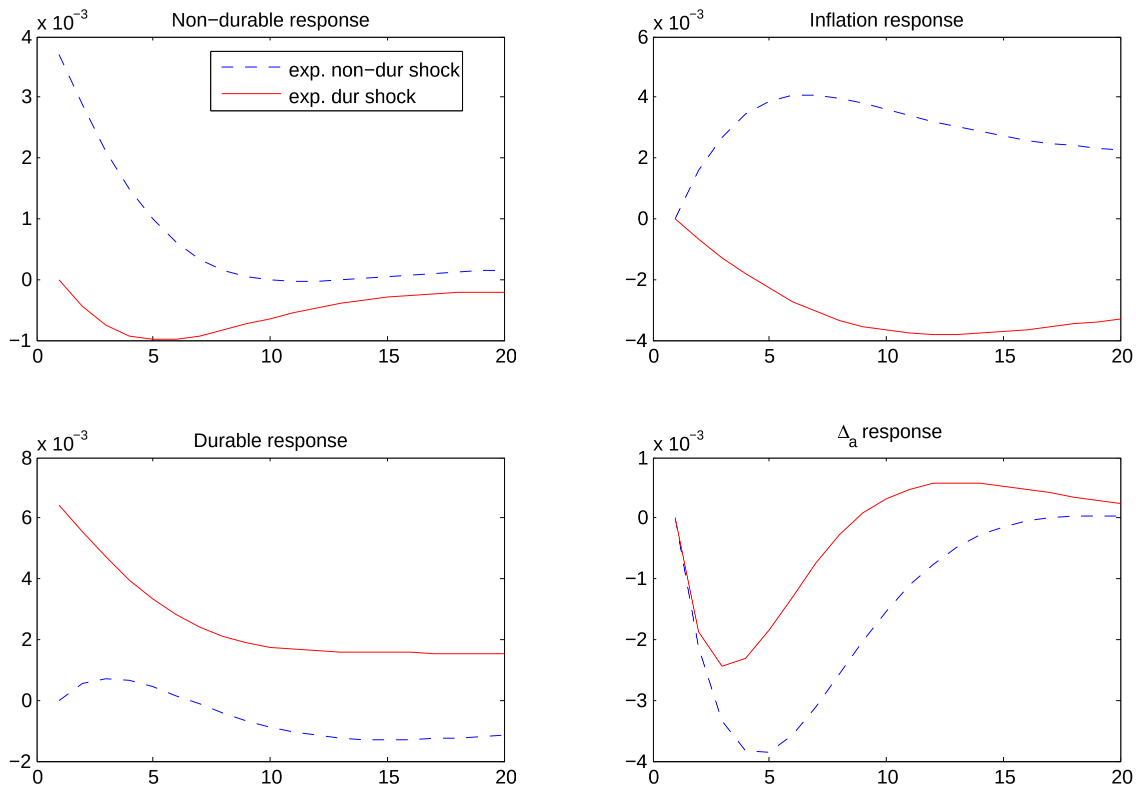

Figure 3: Impulse response functions for shocks to expected non-durable consumption (dashed blue) and expected durable consumption (solid red), respectively.

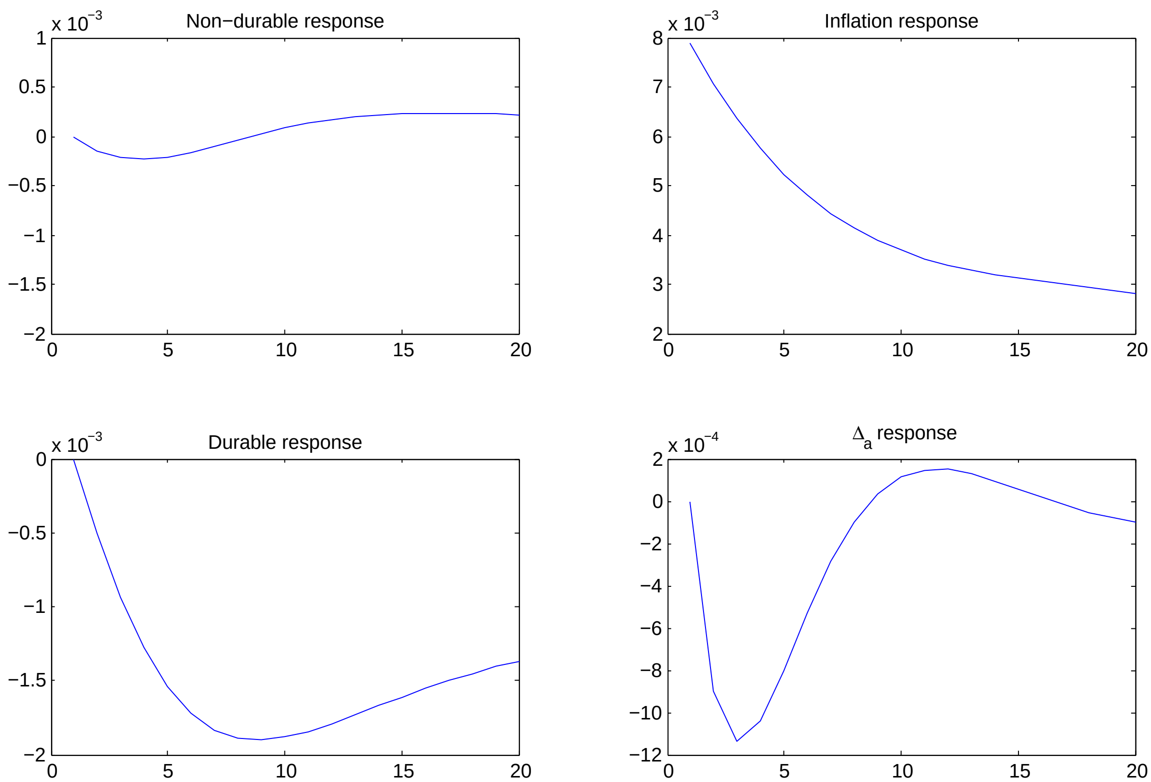

Figure 4: Impulse response functions for a shock to expected inflation.

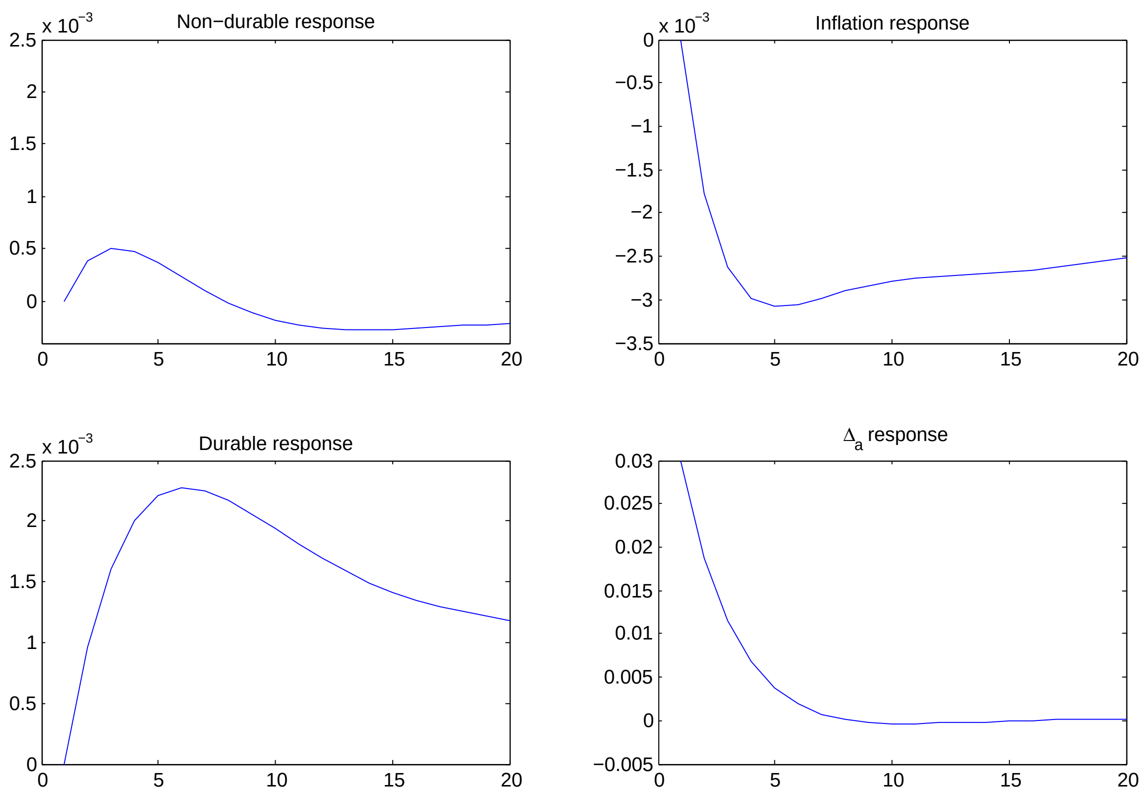

Figure 5: Impulse response functions for a shock to expectation of relative preference of durable consumption.

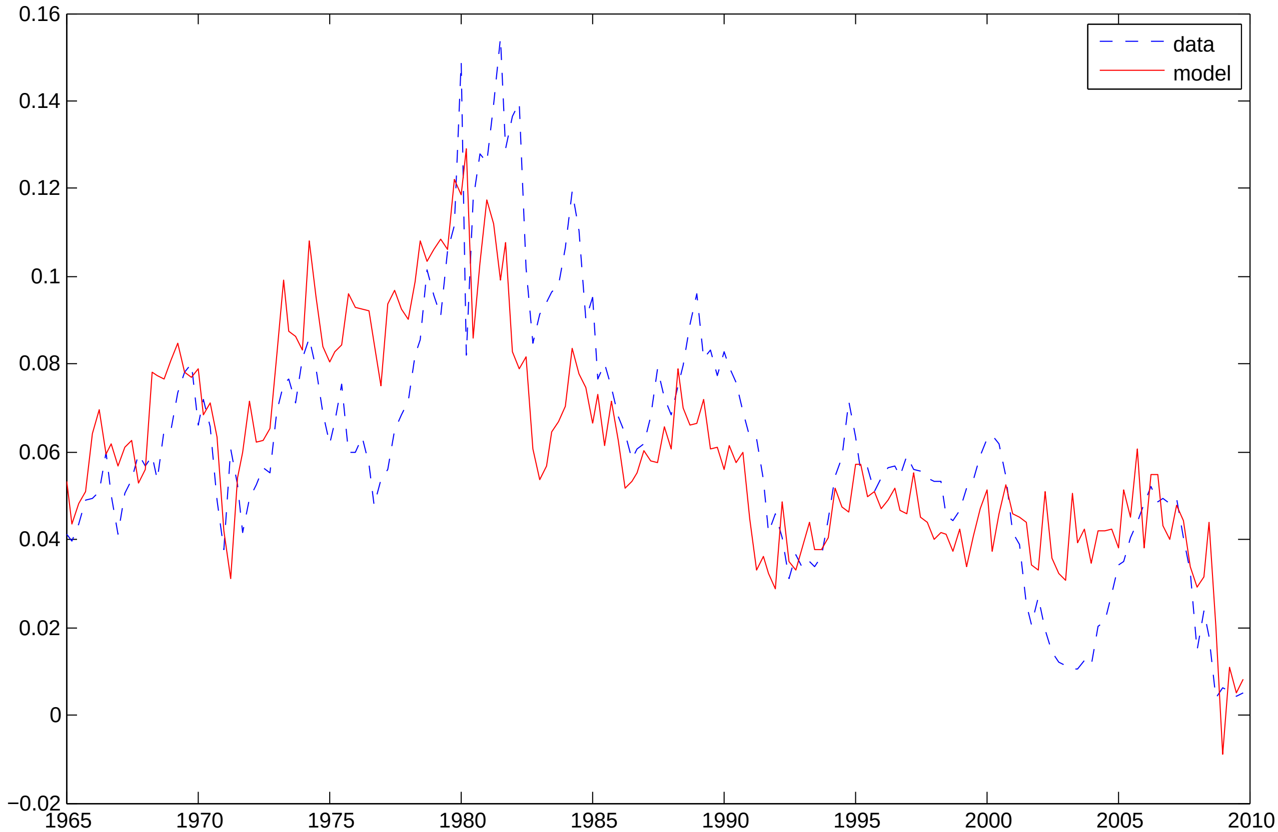

Figure 6: One year nominal zero coupon yields. The plot shows the data and the theoretical model prediction computed from estimated parameters, Kalman filtered state-variables, and preference parameters $\psi = 2$, and $\gamma = 17$.

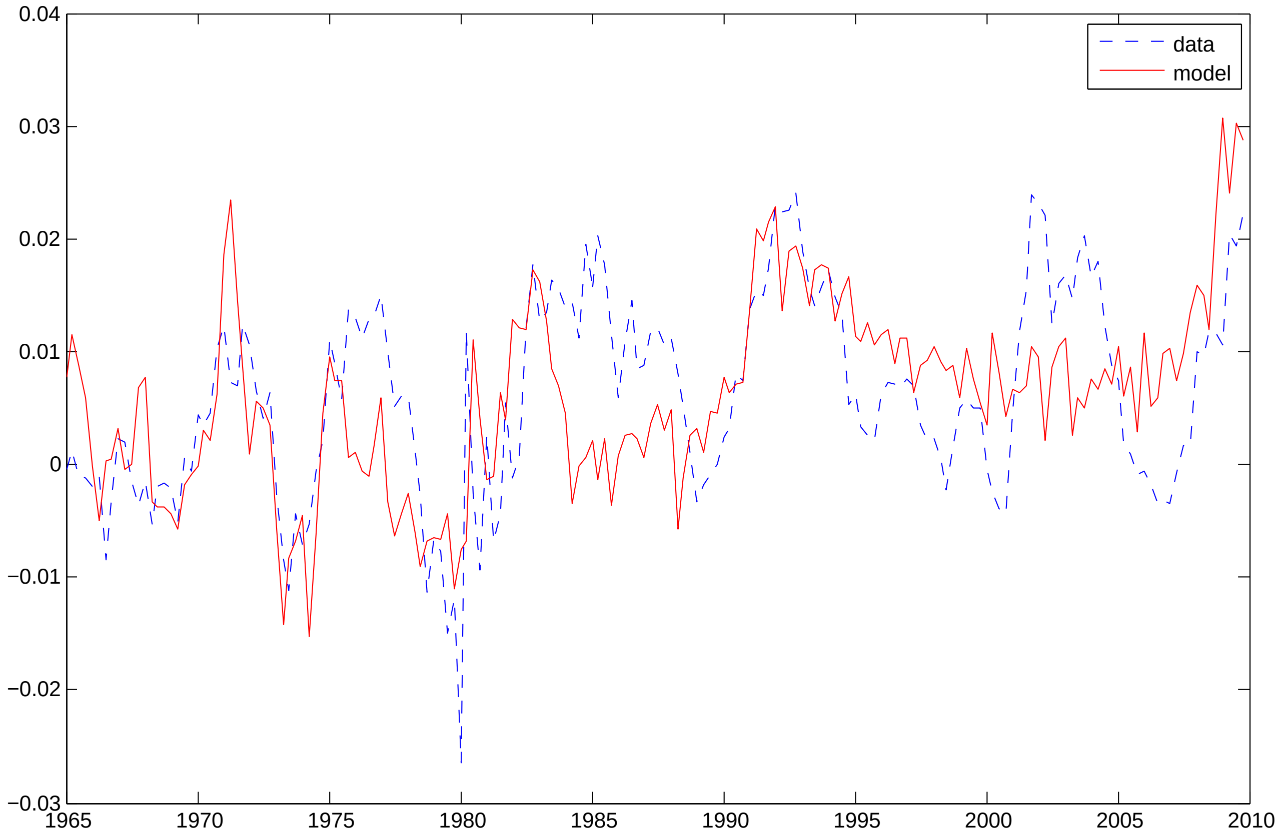

Figure 7: Term structure nominal spread (5yr vs. 1yr). The plot shows the data and the theoretical model prediction computed from estimated parameters, Kalman filtered state-variables, and preference parameters $\psi = 2$, and $\gamma = 17$.

Table 1: Descriptive Statistics

	Δc_t	$\Delta \pi_t$	Δs_t	Δq_t
Mean	0.0051	0.0105	0.0109	-0.0015
Std. Dev.	0.0049	0.0089	0.0061	0.0516
Autocorr.	0.3766	0.5110	0.6299	0.0485

Descriptive statistics for real non-durable consumption growth (Δc_t), inflation $\Delta \pi_t$, durable consumption, Δs_t, and user cost Δq_t.

Table 2: Consumption Growth Predictability: Interest Rate

			Non-durable		
Lag	1	2	3	4	5
Est.	−0.1005	−0.0764	−0.0343	0.0052	0.0283
St. err.	(0.0332)	(0.0261)	(0.0218)	(0.0185)	(0.0171)
R^2	0.0528	0.0493	0.0149	0.0005	0.0163
			Durable		
Lag	1	2	3	4	5
Est.	−0.2952	−0.3003	−0.2639	−0.2085	−0.1555
St. Err.	(0.0503)	(0.0434)	(0.0392)	(0.0363)	(0.0341)
R^2	0.1732	0.2252	0.2163	0.1670	0.1124

OLS regressions of future cumulative average consumption growth on one year nominal yield.

Table 3: Consumption Growth Predictability: Inflation and Cointegration Residual

A: Future consumption growth on inflation

Non-durable

Lag	1	2	3	4	5
Est.	−0.6396	−0.4956	−0.3325	−0.1901	−0.0975
St. err.	(0.1032)	(0.0814)	(0.0691)	(0.0606)	(0.0575)
R^2	0.1892	0.1837	0.1233	0.0565	0.0171

Durable

Lag	1	2	3	4	5
Est.	−0.9104	−1.1402	−1.1365	−1.0157	−0.8627
St. Err.	(0.1719)	(0.1401)	(0.1195)	(0.1078)	(0.1013)
R^2	0.1455	0.2869	0.3545	0.3502	0.3057

B: Future consumption growth on the cointegration residual

Non-durable

Lag	1	2	3	4	5
Est.	0.1267	0.1150	0.0973	0.0783	0.0657
St.err	(0.0190)	(0.0142)	(0.0115)	(0.0099)	(0.0095)
R^2	0.2132	0.2842	0.3034	0.2754	0.2234

Durable

Lags	1	2	3	4	5
Est.	0.2118	0.2338	0.2341	0.2164	0.1901
St.err	(0.0305)	(0.0250)	(0.0209)	(0.0184)	(0.0172)
R^2	0.2262	0.3466	0.4320	0.4569	0.4263

OLS regressions of future cumulative average consumption growth on inflation and the cointegrating residual $res_t = const - (s_t - c_t) - \epsilon q_t$ where s_t is log level of durable consumption stock, c_t is log non-durable consumption and q_t is log real user cost of the durable good.

Table 4: Parameter Estimates

	Π				
	Δc_t	$\Delta \pi_t$	Δs_t	Δa_t	
Δc_{t+1}	0.7718	-0.0187	-0.0714	0.0130	
$\Delta \pi_{t+1}$	0.4243	0.8939	-0.1053	-0.0593	
Δs_{t+1}	0.1477	-0.0637	0.8632	0.0322	
Δa_{t+1}	-0.5756	-0.1138	-0.2899	0.6294	
	diag$(\Sigma_g) \times 1000$		$\Sigma_x \times 1000$		
		Δc_t	$\Delta \pi_t$	Δs_t	Δa_t
Δc_{t+1}	3.2820	2.4194			
$\Delta \pi_{t+1}$	6.0247	-0.8818	0.6024		
Δs_{t+1}	2.9463	1.6764	2.1077	1.1982	
Δa_{t+1}	46.0393	0.4422	-16.8733	-10.6347	9.3478

Parameter estimates for the Full model specification: $g_t = \mu_g + F x_t + \Sigma_\eta \eta_t$, $\eta_t \sim N(0, I), x_t = \Pi x_{t-1} + \Sigma_x u_t,\ \ u_t \sim N(0, I)$ Parameters are estimated by MLE using Kalman filtering. Quarterly observations of non-durable consumption, durable consumption, inflation rate and durable user cost from 1965Q1 to 2009Q4.

Table 5: Model-Implied Nominal Yield Curves

Term Structure : Mean

Maturity	1 yr	2 yr	3 yr	4 yr	5 yr
Data	6.10	6.31	6.48	6.62	6.71
Full	6.10	6.32	6.46	6.58	6.70
No-dur	6.13	5.98	5.83	5.71	5.61
PW1	38.15	35.99	34.92	34.51	34.36
PW2	8.62	8.59	8.57	8.56	8.54
NPW1	37.33	36.34	35.96	35.87	35.86
NPW2	8.44	8.40	8.38	8.36	8.34

Term Structure : Std. Dev.

Maturity	1 yr	2 yr	3 yr	4 yr	5 yr
Data	2.91	2.85	2.76	2.69	2.62
Full	2.41	2.17	2.01	1.90	1.82
No-dur	2.19	2.01	1.88	1.78	1.70
PW1	11.74	7.21	4.85	3.59	2.87
PW2	2.23	1.99	1.87	1.79	1.72
NPW1	9.71	6.25	4.74	4.06	3.71
NPW2	2.40	2.14	2.01	1.92	1.84

Term Structure : Autocorrelation

Maturity	1 yr	2 yr	3 yr	4 yr	5 yr
Data	0.927	0.938	0.944	0.948	0.953
Full	0.899	0.912	0.926	0.934	0.938
No-dur	0.921	0.925	0.935	0.941	0.944
PW1	0.691	0.706	0.723	0.741	0.760
PW2	0.887	0.927	0.946	0.953	0.955
NPW1	0.721	0.795	0.868	0.911	0.931
NPW2	0.881	0.920	0.940	0.948	0.951

Mean, standard deviation and autocorrelation of our models vs. the data over the sample period (1965-2009). The "Full" model is the model with utility defined over both durable and non-durable goods and EZ preferences with $\gamma = 17$ and $\psi = 2$. Constrained models are "Non-dur" (only utility from non-durable goods, EZ preferences $\gamma = 17$ and $\psi = 2$), "PW1" (power-utility with $\gamma = 17$), "PW2" (power utility with $\gamma = 2$), "NPW1" (power utility of non-durable goods only, $\gamma = 17$), and "NPW2" (power utility of non-durable goods only, $\gamma = 2$).

Table 6: **Time Series Fit of Nominal Yield and Spread**

A: One year nominal yield

	Data	Model					
		Full	Non-dur	PW1	PW2	NPW1	NPW2
Mean	6.10	6.10	6.13	38.15	8.62	37.33	8.44
Stdev	2.91	2.41	2.19	11.74	2.23	9.71	2.40
MSE		1.92	1.94	34.34	3.19	32.67	3.05

B: Five year nominal spread

	Data	Model					
		Full	Non-dur	PW1	PW2	NPW1	NPW2
Mean	0.60	0.60	−0.51	−3.80	−0.07	−1.47	−0.10
Stdev	0.86	0.82	0.67	9.46	0.96	7.84	1.01
MSE		0.84	1.37	10.68	1.37	8.25	1.37

Yield curve implications of our models vs the sample period (1965-2009). The "Full" model is the model with utility defined over both durable and non-durable goods and EZ preferences with $\gamma = 17$ and $\psi = 2$. Constrained models are "Non-dur" (only utility from non-durable goods, EZ preferences $\gamma = 17$ and $\psi = 2$), "PW1" (power-utility with $\gamma = 17$), "PW2" (power utility with $\gamma = 2$), "NPW1" (power utility of non-durable goods only, $\gamma = 17$), and "NPW2" (power utility of non-durable goods only, $\gamma = 2$).

Table 7: Term Structure Factor Loadings on State-Variables

	X_c	X_π	X_s	X_a
Data	3.477	3.766	−1.288	0.116
	(1.112)	(0.446)	(0.745)	(0.250)
Full	2.312	4.000	−0.407	−0.407
No-dur	1.905	4.000	0.000	0.000
PW1	59.153	4.000	8.846	8.846
PW2	7.554	4.000	0.446	0.446
NPW1	67.999	4.000	0.000	0.000
NPW2	8.000	4.000	0.000	0.000

Loadings of a 1-quarter nominal yield on state variables predicted by our models vs. the loadings obtained from regression in data over the sample period (1965-2009). The "Full" model is the model with utility defined over both durable and non-durable goods and EZ preferences with $\gamma = 17$ and $\psi = 2$. Constrained models are "Non-dur" (only utility from non-durable goods, EZ preferences $\gamma = 17$ and $\psi = 2$), "PW1" (power-utility with $\gamma = 17$), "PW2" (power utility with $\gamma = 2$), "NPW1" (power utility of non-durable goods only, $\gamma = 17$), and "NPW2" (power utility of non-durable goods only, $\gamma = 2$). X_c, X_π, X_s and X_a are Kalman filtered state variables.

Table 8: Real Yield Implication

	3 mo	1 yr	2 yr	3 yr	4 yr	5 yr
Full	1.49	1.62	1.66	1.67	1.68	1.69
No-dur	1.83	1.77	1.73	1.72	1.71	1.71
PW1	36.06	34.37	31.88	30.53	29.91	29.61
PW2	4.34	4.33	4.31	4.30	4.29	4.29
NPW1	34.15	33.37	32.30	31.84	31.72	31.71
NPW2	4.15	4.14	4.13	4.12	4.12	4.12

Term structure of average real yields predicted by our models. The "Full" model is the model with utility defined over both durable and non-durable goods and EZ preferences with $\gamma = 17$ and $\psi = 2$. Constrained models are "Non-dur" (only utility from non-durable goods, EZ preferences $\gamma = 17$ and $\psi = 2$), "PW1" (power-utility with $\gamma = 17$), "PW2" (power utility with $\gamma = 2$), "NPW1" (power utility of non-durable goods only, $\gamma = 17$), and "NPW2" (power utility of non-durable goods only, $\gamma = 2$).

Table 9: Real Yields: Change of Numeraire

	3 mo	1 yr	2 yr	3 yr	4 yr	5 yr
Non-dur Price	1.49	1.62	1.66	1.67	1.68	1.69
Ideal Price Index	1.44	2.34	2.78	2.94	3.04	3.12
Dur Pur Price	0.67	0.92	0.97	1.02	1.08	1.14

Term structure of real yields under diffrent numeraires. The “Non-dur Price” row reports the real yields when the traditional non-durable good purchase price is taken as numeraire; the “Ideal Price Index” row reports the real yields when the price of consumption bundle is used as numeraire, and “Dur Pur Price” reports the real yields when the durable good purchase price is used as numeraire.).

Table 10: Inflation Premium

	1 yr	2 yr	3 yr	4 yr	5 yr
Full	0.120	0.303	0.439	0.551	0.656
No-dur	0.013	−0.099	−0.232	−0.349	−0.448
PW1	0.058	0.198	0.338	0.460	0.569
PW2	−0.007	−0.034	−0.056	−0.074	−0.092
NPW1	0.008	−0.026	−0.061	−0.093	−0.129
NPW2	−0.010	−0.045	−0.076	−0.102	−0.127

Inflation premium predicted by our models. The "Full" model is the model with utility defined over both durable and non-durable goods and EZ preferences with $\gamma = 17$ and $\psi = 2$. Constrained models are "Non-dur" (only utility from non-durable goods, EZ preferences $\gamma = 17$ and $\psi = 2$), "PW1" (power-utility with $\gamma = 17$), "PW2" (power utility with $\gamma = 2$), "NPW1" (power utility of non-durable goods only, $\gamma = 17$), and "NPW2" (power utility of non-durable goods only, $\gamma = 2$).