Effect of executive share ownership and private hedging on executive stock option exercise and values.*

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November 2008

JEL: G13, J33

Keywords: executive stock options, valuation, exercise threshold, optimal partial hedging, ownership, retention on exercise

*Thanks to Peter Corvi, Gordon Gemmill, Vicky Henderson, Stewart Hodges, and especially Silvia Rossetto, and participants at the EFMA meeting 2006 and at seminars at ISMA Centre, University of Reading and Warwick Business School for valuable feedback and suggestions. All errors remain my own.

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Abstract

Executives can substantially increase the subjective value of their stock options by hedging optimally with an imperfectly correlated traded asset, even if trades are costly, because of the reduction in their cost of unhedged risk. Through the effect on their exercise strategy, this also increases the options' cost to shareholders. Subjective values, exercise thresholds and shareholders' objective values thus depend on unhedgeable (idiosyncratic) as well as total risk. All differ from their perfect market equivalents more for larger unhedgeable risk, grant sizes and executive ownership levels. If shares are retained on exercise, unhedgeable risk increases optimal exercise thresholds; if sold, thresholds decrease. Objective values always decrease with unhedgeable risk.

1 Introduction

Executive stock options (ESOs) are a widely used and important component of executive compensation.[1] Recently Accounting Standards Boards around the world have adopted standards which require the calculation of the 'fair value' of an ESO to the company, taking account of the effects of expected early exercise.[2] The widely documented empirical finding[3] that ESOs are often exercised well before maturity reduces the cost recognised in the accounts but also highlights the need for models of executive valuation of ESOs which predict such early exercise based on features specific to ESOs.

In this paper we present a continuous-time utility-based model which takes account of a risk-averse executive's overall portfolio of holdings in both the firm's stock and outside tradeable wealth and incorporates both restrictions on the executive's ability to trade his company's stock and also the costs of adjusting the composition of the tradeable portion of his portfolio. We also consider the effects of planned future changes in executive ownership by considering both retention of shares acquired on exercise of the ESOs as well as immediate sale on exercise.

We use this framework to examine the benefit to the executive of trading opti-

[1]Frydman & Saks (2007) report around 80% of top executives in their sample of large US firms are granted options each year, around 90% held options over 1995-2005 and that the median value of option grants reached 37% of total compensation in the late 1990s. Boyd *et al.* (2007) note "practically all firms in the S&P/ASX300 index have ESO plans", and Clarkson *et al* (2005) find option grants represent $10 - 21\%$ of annual total remuneration between 2000-2005 for their sample of large Australian firms.

[2]See International Financial Reporting Standard 2 Appendix B (paragraph B9), American Financial Accounting Standard Board's SFAS 123(R) and Australian Accounting Standard AASB2.

[3]See *e.g.* Bettis, Bizjak & Lemmon (2005), Boyd, Brown & Szimayer (2007).

mally in his non-firm liquid wealth in order to partially hedge the option position when such trades are costly (relative to not hedging). We find such hedging is always worthwhile for the executive and the benefits can be substantial. Thus, particularly for those executives for whom hedging is most beneficial, we predict that both option early exercise thresholds (chosen by the executive) and hence, the cost of option grants to the shareholders should reflect the executive's optimal partial hedging and so vary with unhedgeable (idiosyncratic)[4] risk as well as the executive's exposure to this risk through the size of his option grants and his current and future ownership levels. The executive's discount on the value of the option due to his risk-aversion and the illiquidity of his holding, resulting from non-tradeability and costs, is transmitted to give a smaller reduction in the cost to the shareholders of the same grant, even though they do not face the same imperfections.

A utility-based framework is necessary for the executive's valuation of the ESOs because of the restrictions on an executive's ability to trade their own company's stock, which we detail in section 2.1. These effectively rule out the dynamic hedging using the underlying asset on which the Black-Scholes world or 'perfect market' option values are based and leave the executive exposed to his firm's stock price risk. This risk reduces the value the risk-averse executive places on the option grant[5] (effectively producing an illiquidity discount) and generally causes him to exercise and sell shares earlier than would be optimal in a perfect market setting.[6]

[4]If the executive uses the market to hedge.

[5]See *e.g.* Lambert, Larcker & Verrechia (1991) for the general principle and Henderson (2005) for an application to European options.

[6]See for example Hall & Murphy (2000, 2002), Detemple & Sundaresan (1999), Leung & Sircar (2007), Carpenter, Stanton & Wallace (2007) and Grasselli & Henderson (2008).

The executive's early exercise threshold is sub-optimal for an investor who is risk-neutral or who can trade freely in the firm stock. Thus the cost to the shareholders, who value the option in a perfect market setting taking into account the executive's early exercise threshold (the objective value) is also lower than the perfect market value. We find larger costs of unhedged risk faced by the executive generally result in both early exercise thresholds and objective values which differ by more from their perfect market equivalents. Thus the determinants of the executive's overall illiquidity discount also affect exercise thresholds and objective values.

The executive's exposure to firm-specific risk is not generally limited to a single option grant[7] but also arises from his holdings of company stock[8] and potentially other option grants. Our framework summarises this additional exposure as the executive's ownership of firm stock and finds this ownership level has a significant effect, consistently lowering both executive option values and shareholder costs of the option grants[9] and also increasing the benefits of hedging optimally. The ownership level affects the executive's option value because the cost of unhedged risk to a risk-averse executive increases non-linearly with the level of risk. Hence an overall illiquidity discount must be calculated for all of the executive's holdings which expose him to his firm's stock price risk. This also means the value he places on a new grant of options additional to his existing shareholding is its *marginal* value

[7]Existing utility-based models of ESO valuation which incorporate dynamic hedging (*e.g.* Henderson (2005), Leung & Sircar (2007), Carpenter, Staunton & Wallace (2007)) have considered only a single grant of options with sale on exercise and no additional holdings of stock, as well as ignoring the costs associated with such dynamic hedging.

[8]The empirical evidence on the levels and changes in executive ownership of shares in the firms they manage is summarised in section 2.2.

[9]See also Lambert, Larcker & Verrechia (1991), Hall & Murphy (2000, 2002) and Cai & Vijh (2005).

reflecting the *additional* risk resulting from the option grant. The additional risk depends not only on the size of the grant itself but also on the executive's existing unhedged risk exposure through his ownership of firm stock before and after exercise, *i.e.* whether and for how long he retains the shares acquired on exercise.[10]

The level of unhedged risk also depends on the executive's hedging choices. Although he is unable to hedge using the firm's stock, we assume he can adjust the composition of his tradeable wealth between the risk-free asset and a risky hedging asset,which is imperfectly correlated with his firm's stock. Such trading reduces the level of unhedged risk, which increases both the subjective per-option value to the executive and the per-option cost to the shareholders. However, the optimal trading strategy in the absence of costs involves trading continuously in order to minimise the unhedged risk, *i.e.* eliminate all potentially hedgeable risk, and in the presence of trading costs this is infeasible.

Transaction costs modify the hedging strategy to introduce a no-transaction band limiting the level of 'unhedgedness' or additional potentially hedgeable risk. The optimal levels of potentially hedgeable risk and the costs in following the hedging strategy also reduce the executive's subjective option value and hence affect exercise thresholds and shareholder cost. So with transaction costs, the benefits of hedging are reduced (relative to the costless case). For a given total risk level, increasing the proportion of potentially hedgeable risk decreases the cost of unhedgeable risk but increases the costs of hedging optimally. However we find it is still worthwhile for the executive to hedge optimally (the benefits outweigh the

[10] Aboody *et al* (2006) find almost half of executives exercising stock options in 1996-2003 retain shares acquired on exercise.

costs), and the net benefit of hedging still increases as the level of unhedgeable risk decreases. Since it is worthwhile incurring costs to reduce potentially hedgeable risk with optimal hedging, the executive's overall illiquidity discount or cost of unhedged risk is reduced, so option values increase.

We document the firm and executive characteristics which increase the benefit of hedging for the executive: larger holdings of stock and options, longer times to vesting and longer holding periods of company stock, lower idiosyncratic risk (if hedging with the market) and lower dividend yields.

Increased levels of unhedged risk and exposure size always reduce subjective and objective option values. However, the effect on exercise thresholds varies depending on whether shares acquired on exercise are retained or sold immediately. Generally executives would prefer to sell shares acquired on exercise of options in order to reduce the overall illiquidity discount on their holdings of firm-related wealth.[11] This sale reduces the executive's overall exposure to unhedged risk, increasing the attractiveness of exercising and thus decreasing the optimal exercise thresholds below their perfect market equivalents, by more the larger the option grant, the higher the executive's shareholdings and the greater the unhedgeable risk. However, if the executive expects to (have to) retain the shares acquired on option exercise, perhaps as part of a movement towards an explicit or implicit ownership target, exercising locks in a higher exposure to unhedged risk. The unhedged risk effect reduces the payoff on exercise to the executive, but also increases the exercise threshold above

[11] Our model shows that executives' subjective option values are greater when the shares acquired on exercise will be sold rather than retained, and that the difference in subjective values under sale and retention policies increases with executives' existing shareholdings, consistent with the findings of Ofek & Yermack (2000).

the perfect market threshold, again by more the larger the option grant, the higher the executive's shareholdings, the greater the unhedgeable risk and the longer the executive continues to hold the shares after exercise. In both cases the exercise thresholds differ more from the optimal perfect market threhsold, so the shareholder cost per option, which takes the executive's exercise strategy into account, decreases.

Since it makes sense for executives to hedge privately using a partially correlated traded asset, we expect exercise thresholds and shareholder costs, particularly for firms and executives with the greatest potential benefit from hedging, to depend on idiosyncratic (unhedgeable) risk as well as total risk and also to vary with the executive's holdings of (exposure to) firm stock and options, and whether shares are retained or sold on exercise. Empirical studies of the determinants of exercise thresholds for executive stock options[12] have generally considered only total firm risk and not included executive's holdings as a potential determinant. So further empirical investigation of executives' early exercise thresholds taking private hedging and the executive's exposure to firm stock price risk is merited. Moreover, since the relationship between exercise thresholds and shareholder cost depends on the length of the holding period after exercise, care must be taken in estimating shareholder cost based on calibrations to early exercise data.

The rest of the paper is organised as follows. Section 2 reviews related literature, section 3 explains the model, section 4 considers its results and their implications, and section 5 summarises and concludes.

[12]See Bettis, Bizjak & Lemmon (2005), Boyd, Brown & Szimayer (2007), Armstrong, Jagolinzer & Larcker (2006).

2 Related literature

2.1 Trading restrictions for executives

In practice there are limits on the ability of executives to trade freely in their firms' shares. Executives can face formal restrictions imposed by their firms, by law and by regulators and are also subject to informal pressures resulting from market reactions to insider sales. Some firms issue restricted stock to their executives (Cai & Vijh (2007)), which may not be sold before the end of an initial vesting period. In addition firms may require or encourage minimum ownership levels (see section 2.2) and may also place limitations on the timing of trades by executives in their own firms' shares.[13] As insiders, executives face legal restrictions on short-selling.[14] In the US, SEC insider trading regulations limit the timing and volume of sales by executives of their existing holdings of company stock.[15] Executives also need to take into account the likely price impact of stock sales.

This suggests executives are able to trade their company stock only infrequently. In particular the dynamic hedging which would be required to hedge their option grants using company stock is not possible,[16] particularly for executives with low

[13] Bettis, Coles & Lemmon (2000) find that 78% of US firms also have blackout periods, during which trading by insiders is prohibited, and over 72% have 'trading windows' explicitly defined relative to earnings announcement dates, which generally occur only once a quarter.

[14] In the US, insiders are prohibited by Section 16-c of the Securities Exchange Act from selling short the shares of their firm (Carpenter (1998)).

[15] SEC Rule 144 specifies the minimum holding period and maximum rate of sale for 'control stock' by 'affiliates', which includes stock acquired on exercise of ESOs (Kahl, Liu & Longstaff (2003)).

[16] Derivative transactions such as zero-cost collars and equity swaps, which can be used by executives to reduce their firm-specific exposure without selling shares or changing ownership reported in proxy

ownership for whom it would involve short selling.

2.2 Executive own firm share ownership

Whilst the mean proportional stock ownership by CEOs of large US firms is slightly over 2% (Khan *et al* (2005), Dharwadkar *et al* (2008)), the distribution is highly skewed. von Lilienfeld & Ruenzl (2007) report that around 80% of CEOs in S&P500 firms over 1994-2003 have zero ownership of restricted and unrestricted stock, around $7-10\%$ own more than 5% and around 2% own 20% or more.

Ofek & Yermack (2000) find 46% of their sample of executives granted options or restricted stock own less than the number of shares underlying the new grant. They also find these 'low prior ownership' executives do not adjust their holdings as a result of the grant. In contrast, executives with higher prior ownership tend to sell shares after option and restricted stock grants, and the propensity to sell (average proportion sold) increases with prior ownership. Similarly, they find that whilst overall executives tend to sell immediately most shares acquired on exercise of options, the proportion sold also increases with prior ownership for executives with high prior ownership (they do not include 'low ownership' executives in this analysis). Aboody *et al* (2006) find in almost half stock option exercises filed with the SEC over 1996-2003 no shares were sold within 30 days and 72.5% of these

statements, are not restricted in the US but are required to be reported to the SEC, although the exact format is not specified. Bettis, Bizjak & Lemmon (2001) report that only 89 zero cost collars and equity swap transactions were filed with the SEC over 1996-98 and that these had long maturities and reduced effective ownership by on average 25%. Hall & Murphy (2000) suggest there is no evidence that these are widespread. In any case their effect would be like a partial static hedge which reduces but does not eliminate the executive's firm specific exposure.

shares were retained for at least a year, but do not examine the relationship with executive's ownership levels.

These findings are consistent with executives adjusting their holdings of firm stock in response to new option grants, but only once they have reached a certain ownership level. This 'target' ownership level may be determined by the executive but may be influenced formally or informally by the firm. Core & Larcker (2002) consider a sample of firms in 1991-95 which adopt "target ownership plans" for CEOs. They find the median target level is $4\times$ base salary achievable within five years and that 27% have penalties for non-compliance. Nevertheless they find 38% of firms have CEOs and 84% of firms have a top executive with holdings below the minimum. Cai & Vijh (2007) find for 457 firms involved in acquisitions in 1993-2001 who issued proxy statements, 21% had stated minimum ownership requirements of on average between 4 and $5\times$ base salary, together with penalties for not meeting these in time, a further 40%+ had 'expectations of substantial ownership' and around 50% had plans facilitating share purchases for executives.

This suggests executives with low holdings may not sell shares when granted new options and may retain shares acquired on exercise of options in order to move towards a target level. Executives with higher holdings who may be above the target level are likely to sell shares acquired on exercise of options and may adjust their holdings to reduce their overall exposure to firm risk when granted new options.

2.3 Utility-based subjective option valuation

The restrictions faced by executives and the resulting firm-specific risk they are forced to bear has motivated the evaluation of ESOs from the executive's viewpoint using utility-based frameworks.[17] Lambert, Larcker & Verrecchia (1991) and Hall & Murphy (2000, 2002) use a certainty equivalent framework to value ESOs assuming the executive's external wealth is invested in fixed, non-optimised proportions in the risk-free asset and company stock. Similarly Carpenter (1998) sets wealth proportions invested in the risk-free asset and company stock to the optimal ones in the absence of options. Cai & Vijh (2005) allow a single choice of proportions of wealth invested in the risk-free asset and the market for a given proportion of wealth invested in company stock. None allow the possibility of optimal dynamic hedging using a partially correlated trading asset (*e.g.* the market) and ignore restrictions on executives trading own company stock which would in practice limit their ability to maintain an exogenously given proportion of their overall wealth in company stock.

Henderson (2005) and Leung & Sircar (2007), in a CARA framework, and Carpenter, Stanton & Wallace (2008) using power utility value a single grant of identical ESOs on a standalone basis, allowing optimal investment of the executive's outside wealth in the risk-free asset and a partially correlated trading asset.[18] All consider only a single grant of options with no prior stock or option holdings by the executive and assume all shares acquired on exercise are sold immediately. They do not consider the effect of grant size on per-option subjective values or shareholder cost. In addition all dynamic trading is assumed to be costless[19] whereas in practice transactions incur costs (*e.g.* bid-ask spreads).

A large literature has modeled the impact of the costs of dynamic hedging on

[17]Kahl, Liu & Longstaff (2003) consider the valuation of restricted stock in a similar framework.

[18]See also Detemple & Sundaresan (1999) and Ingersoll (2006).

[19]Carpenter, Stanton & Wallace (2008) allow constraints on liquid portfolio weights.

the optimal hedging strategy and value of financial options.[20] Most of these papers assume the asset used for hedging is the same as that underlying the derivative contract (the hedged asset). Recently Whalley (2008) presents a model for the total value of a portfolio of European options with cash payoff where hedged and hedging assets differ. We base our analysis on this model, extending it to find the marginal value of American style options with delayed vesting and both settlement either in cash or by acquisition of non-tradeable stock.

2.4 Objective valuation and early exercise thresholds

Carpenter (1998) compared the ability of a power-utility based binomial model with random potential exercise and forfeiture and an American model with exogenous random exercise and forfeiture to match the exercise behaviour for her sample of 40 stock option grants and found the extended American model performed as well as the utility-based model. However Bettis *et al* (2005) performed a similar comparison with an adjusted-maturity American model for a sample of 141,120 option exercises at 3966 US firms over 1996-2002 and found the utility-based model gave markedly better forecasts of exercise behaviour.

A few studies have investigated the empirical determinants of early exercise behaviour directly (Bettis *et al* (2005), Boyd *et al* (2007), Armstrong *et al* (2006)). and consistently found higher total volatility is associated with earlier exercise of stock options at lower levels of moneyness, consistent with a role for risk-aversion. Higher dividend yield is also associated with earlier exercise (Bettis *et al*, Boyd

[20]This was first studied by Hodges & Neuberger (1989). See also Davis, Panas & Zariphopoulou (1993), Whalley & Wilmott (1997), Andersen & Damgaard (1999) and Zakamouline (2003).

et al), as predicted by perfect-market option valuation models. CEOs, directors or higher level employees were also consistently found to exercise options closer to maturity than lower level employees. However none of these empirical tests considered idiosyncratic volatility, the size of the option grant, the ownership level and the sale/retention policy, which the model in this paper predicts will be significant determinants of the executive's early exercise strategy.

3 The model

3.1 Setup

We consider an executive who receives a grant of options on the stock of the firm he manages. The option grant gives him the right to buy n shares, each with a strike price K. The options have maturity T and time to vest T_V.

We summarise the executive's exposure to the firm's stock by assuming he has a holding of N shares in the firm which he will continue to hold until time $T_G \geq T$. This is a simplification of a typical executive's holdings which may consist of both stocks and options with varying maturity, vesting dates and restriction schedules but captures the principal level effect (the magnitude of the sensitivity of the value of the executive's holdings to stock price movements, or its Delta), which we find has a significant influence on subjective and objective option values in our model. The empirical evidence outlined in section 2.2 is consistent with executive ownership levels remaining constant once they have reached a certain formal or informal target level. We thus initially assume N remains constant over the life of the option, that options are exercised as a block[21] and all shares acquired on exercise are sold immediately. Later we will consider the situation of an executive who retains shares acquired on option exercise so his ownership level increases from N to $N + n$. This would be relevant for an executive with a shareholding below his target level in the process of building up his ownership stake.

The firm's stock price follows geometric Brownian motion

$$dV = (r + \xi\eta - q)V dt + \eta V dZ \tag{1}$$

where $\xi = \frac{\mu_V - r}{\eta}$ is the Sharpe ratio for the stock and q is its dividend yield. We abstract from leverage and dilution considerations, which should not affect our main results.

We assume the executive cannot trade in the shares of the firm to hedge his option exposure, but may trade in the riskless asset and also in an imperfectly correlated risky *hedging asset*, M, where

$$dM = (r + \lambda\sigma)M dt + \sigma M dZ_M \tag{2}$$

with $dZdZ_M = \rho dt$ and λ the hedging asset's Sharpe ratio. Trades in the hedging asset incur a cost proportional to the value traded of $k(M, dy) = kM|dy|$, where dy is the number of hedging assets traded. Transaction costs reduce the amount held in the riskless asset, B, so

$$dB = rB - Mdy - k(M, dy).$$

[21]Simultaneous exercise is not the optimal exercise policy if executives are unconstrained and exercise is costless (see Jain & Subramanian (2003), Leung & Sircar (2007) and Grasselli & Henderson (2008)) but may correspond reasonably well to exercise decisions in practice given restrictions on executive share sales outlined in section 2.1. Grasselli & Henderson (2008) show if exercise is costly, exercise of multiple options simultaneously becomes optimal again.

The executive is risk-averse and maximises his utility of terminal wealth (at T_G) which consists of his liquid wealth, net of costs of liquidating his position, given by $W = B + yM - k(M, y)$, where y is the number of the hedging asset held by the executive, plus the value of his holding in company stock.

Let $J(V, M, B, t, y; N, n) \equiv J(V, W, t; N, n)$ represent the optimised subjective utility at the earlier time t of an executive with a holding of N shares and n options. Once he no longer holds the option ($n = 0$, after exercise or the maturity date), he chooses his investment policy for his liquid wealth taking into account his non-tradeable holding of company stock and the transaction costs in adjusting his portfolio of tradeable wealth, to maximise this utility of terminal wealth.

$$J(V, W, t; N, 0) = \sup_{dy_u, t \leq u \leq T_G} E_t[U(W_{T_G} + NV_{T_G})] \tag{3}$$

Whilst he still holds the option, the executive has another choice at each point in time: whether or not to exercise the option. He still maximises his utility of terminal wealth net of transaction costs by his choice of exercise and liquid portfolio composition choices, and takes into account his exposure to the company stock price both before and after exercise.

If he immediately sells all shares acquired on exercise, so his holding of company stock remains at N, then in determining his exercise decision, the executive compares the maximised utility of exercising and investing the additional liquid wealth (including the cash payoff from the option exercise and sale) optimally until T_G, $J(V, W + n\Lambda(V), t; N, 0)$, where $\Lambda(V) = \max(V - K, 0)$ is the cash payoff to a single option, with the subjective utility of continuing to hold the option grant, $J(V, W, t; N, n)$. If he retains the shares acquired on exercise, then the comparison is between continuing to hold the option, with subjective utility $J(V, W, t; N, n)$, and the subjective utility of exercising, paying n times the strike price, nK, and holding $N+n$ non-tradeable shares, *i.e.* $J(V, W-nK, t; N+n, 0)$.

As is standard in the literature following Hodges & Neuberger (1989), the value of the option to the executive, E, is given by its *reservation* or *utility indifference* value, at which the executive is indifferent between holding the option grant or instead holding E in cash and no option grant. Where necessary we distinguish between the per-option values when shares are sold on exercise and when they are retained on exercise by writing $\bar{E}^{(n|N)}$ for the per-option reservation value of a grant of n options with sale on exercise and an additional shareholding of N, and $\hat{E}^{(n|N)}$ for the per-option reservation value with retention on exercise and $\bar{J}$, $\hat{J}$ for the associated subjective utilities including the option grant. So $\bar{E}^{(n|N)}$ and $\hat{E}^{(n|N)}$ are defined by

$$\begin{aligned} J(V, W+\bar{E}^{(n|N)}, t; N, 0) &= \bar{J}(V, W, t; N, n) \qquad (4) \\ &= \sup_{\tau} \sup_{dy_u, t\le u\le\tau} E_t[J(V_\tau, W_\tau + n\Lambda(V_\tau), \tau; N, 0)] \\ J(V, W+\hat{E}^{(n|N)}, t; N, 0) &= \hat{J}(V, W, t; N, n) \qquad (5) \\ &= \sup_{\tau} \sup_{dy_u, t\le u\le\tau} E_t[J(V_\tau, W_\tau - nK, \tau; N+n, 0)] \end{aligned}$$

where $J(V, W, t; N, 0)$ is given by the solution to (3). We also need the per-stock reservation value to the executive of a stock holding of N shares, which we denote $G^{(N)}$, which is given by $J(V, W+G^{(N)}, t; 0, 0) = J(V, W, t; N, 0)$.

In general (3), (4) and (5) are five-dimensional free boundary problems that are computationally intensive to solve. However Whalley (2008) recently investigated the optimal hedging strategy and reservation value of a European derivative on a non-traded underlying asset in a similar setup. By assuming an exponential utility function[22] $U(x) = -\frac{1}{\gamma}e^{-\gamma x}$ and approximating using asymptotic expansions assuming transaction costs k are small, she showed the dimension and complexity of the problem can be reduced so per-option values can be represented as the sum of solutions of two dimensional differential equations.

Proposition 1 *(Whalley (2008))*

1. *The value of a European derivative portfolio, F, on a non-traded security, V, with payoff $\Lambda_F(V)$ at maturity T_F hedged using a partially correlated hedging asset, M, with small transaction costs proportional to the value traded, $k \ll 1$, to a risk-averse investor with exponential utility with coefficient of absolute risk aversion γ can be approximated by*

$$F \approx FP + FB + FF \tag{6}$$

with $FP = O(1)$, $FB = O(k^{\frac{2}{3}})$ and $FP = O(k)$ where

   (a) *FP is the value in the absence of transaction costs of the partially hedged option and satisfies*

$$\mathcal{L}_{BS}(FP) - \hat{\gamma}(t)\frac{\eta^2(1-\rho^2)}{2}V^2(FP_V)^2 = 0 \tag{7}$$

subject to $FP(V, T_F) = \Lambda_F(V)$ where $\hat{\gamma}(t) = \gamma e^{r(T_F - t)}$ and $\mathcal{L}_{BS}(FP)$ is the Black-Scholes differential operator:

$$\mathcal{L}_{BS}(FP) \equiv FP_t + (r-q)VFP_V - rFP + \frac{\eta^2}{2}V^2FP_{VV}$$

[22]Most utility-based models of transaction costs use exponential utility. Andersen & Damgaard (1999) compare option values from more computationally intensive power- and other HARA utility functions with exponential utility and find that, for the same initial level of absolute risk aversion, option values are not very sensitive to the choice of utility function.

(b) *FB represents the leading order effect of transaction costs and the risk associated with the dynamic hedging strategy on the derivative portfolio value over its life and satisfies*

$$\mathcal{L}_{BS}(FB) - \hat{\gamma}(t)\eta^2(1-\rho^2)V^2 FP_V FB_V \quad = \quad \hat{\gamma}(t)\frac{\sigma^2}{2}\left(X_F^{*^2} - X_0^{*^2}\right) \quad (8)$$

subject to $FB(V, T_F) = 0$ where $X_F^*(V,t) = \left(\frac{3k}{\hat{\gamma}\sigma^2}\right)^{\frac{1}{3}} |\mathcal{V}(FP)|^{\frac{1}{3}}$ with

$$\begin{aligned} \mathcal{V}(FP) \quad &= \quad \frac{\sigma^2}{2}\left(x_0 - \beta V FP_V + \beta^2 V(FP_V + VFP_{VV})\right)^2 \\ &\quad + \frac{\eta^2(1-\rho^2)}{2}\beta^2 V^2 (FP_V + VFP_{VV})^2, \end{aligned} \quad (9)$$

$\beta = \frac{\rho\eta}{\sigma}$ is the sensitivity of the firm's stock returns to those of the hedging asset, and $X_0^* = \left(\frac{3k\lambda^2}{2\hat{\gamma}^3\sigma^2}\right)^{\frac{1}{3}}$.

(c) *FF represents the leading order effect on the derivative portfolio of the transaction costs incurred to unwind the hedge and satisfies*

$$\mathcal{L}_{BS}(FF) - \hat{\gamma}(t)\eta^2(1-\rho^2)V^2 FP_V FF_V \quad = \quad 0 \quad (10)$$

subject to $FF(V, T_F) = -k\frac{|\rho|\eta}{\sigma} V \frac{\partial \Lambda_F}{\partial V}$.

2. *The optimal hedging strategy is to hold a number of the hedging asset, y, such that the value of the hedging asset held, $x = My$, lies in a no transaction band*

$$x_0^*(t) - \beta V(FP_V + FB_V) - X_F^* \leq x \leq x_0^*(t) - \beta V(FP_V + FB_V) + X_F^*$$

so the centre of the band, to leading order, is $x_F^* \equiv x_0^*(t) - \beta V(FP_V + FB_V)$, where $FP_V + FB_V$ represents the leading order Delta of the derivative position and $x_0(t) = \frac{\lambda}{\hat{\gamma}(t)\sigma}$ is the centre of the band in the absence of the derivative position, and X_F^* defined above is the leading order semibandwidth.

Subjective values (F) are strictly less than their equivalent perfect market values in the presence of either unhedgeable risk or transaction costs ($FP < F^{BS}$, $FB, FF < 0$). The main factors driving the imperfect hedging discount of the subjective value from the perfect market value are terms relating to the level of unhedgeable risk and the optimal level of additional potentially hedgeable risk in the equations for FP and FB respectively. The additional terms $-\hat{\gamma}(t)\frac{\eta^2(1-\rho^2)}{2}V^2FP_V^2$ and $-\frac{\hat{\gamma}\sigma^2}{2}(X_F^{*2} - X_0^{*2})$ in (7) and (8) respectively are < 0,[23] and thus by a comparison argument, reduce option values, by more, the longer the executive holds the option so the unhedgeable and potentially hedgeable risks continue to be borne.

In the absence of costs the ideal holding in the hedging asset due to the non-tradeable derivative is an amount $x_F^* = -\beta VFP_V$ or equivalently a number $y_F^* = \frac{x_F^*}{M}$. In the presence of unhedgeable risk *and* costs, the optimal hedging strategy is to transact only when the potentially hedgeable risk becomes too large, *i.e.* when the holding in the risky hedging asset moves too far from the ideal holding. This strategy places a limit on the variance of changes in the executive's incremental portfolio due to exposure to his company stock $\Pi = F + yM$.[24]

$$d\Pi = VF_V\left(\frac{dV}{V}\right) + My_F^*\left(\frac{dM}{M}\right) + M(y - y_F^*)\left(\frac{dM}{M}\right) + \dots$$

$$\begin{aligned} Var[d\Pi] &= \eta^2V^2F_V^2dt + \sigma^2(-\beta VF_V)^2dt + \sigma^2M^2(y-y_F^*)^2dt - 2\rho\sigma\eta\beta V^2F_V^2dt \\ &\quad + 2\sigma^2\beta VF_VM(y-y_F^*)dt - 2\rho\sigma\eta VF_VM(y-y_F^*)dt \\ &= dt[\eta^2(1-\rho^2)V^2F_V^2 + \sigma^2M^2(y-y_F^*)^2] \qquad (11) \end{aligned}$$

The first term in (11) represents the portion of the risk associated with the exec-

[23]Providing $|X_F^*| > |X_0^*|$ which is the case for the portfolios we consider.

[24]This is a generalisation of the hedging strategy when the stock itself is tradeable of holding a number $-F_V$, which completely eliminates the risk associated with the option position.

utive's stock and option holdings which is unhedgeable and must be borne. The second represents the additional potentially hedgeable risk which the executive can control by his choice of hedging strategy or y.

If hedging is costless, the executive chooses $y = y_F^*$ to completely eliminate all potentially hedgeable risk so $Var[d\Pi] = \eta^2(1-\rho^2)V^2F_V^2dt$. The remaining unhedgeable risk still reduces his subjective value of his position and is reflected in the additional term in the differential equation for the leading order portfolio value, multiplied by the executive's risk aversion, γ: $-\hat{\gamma}(t)\frac{\eta^2(1-\rho^2)}{2}V^2FP_V^2$.

With costly hedging the executive trades off the cost of bearing additional potentially hedgeable risk with the transaction costs involved in trading to reduce risk. The optimal strategy places a limit $X_F^*(V,t)$ on the 'unhedgedness' or difference between the value of the actual holding in the hedging asset, My, and My_F^*. So $Var[d\Pi] \leq \eta^2(1-\rho^2)V^2F_V^2dt + \sigma^2X_F^{*^2}dt$. The combined effects on subjective option values of the additional unhedged risk and costs associated with the hedging strategy are reflected in the term $\frac{\hat{\gamma}\sigma^2}{2}(X_F^{*^2} - X_0^{*^2})$ on the right-hand side of (8).

The optimal bandwidth X_F^* increases with factors which increase the effect of transaction costs (for a given bandwidth) more than that of unhedged risk, *e.g.* the transaction cost level, k, and the expected number of trades. The latter depends on the variability of the unhedgedness $y - y_F^*$ and, since y_F^* depends on $-\beta\frac{V}{M}FP_V$, on the portfolio's Delta and Gamma and on $|\beta|$ or the absolute level of correlation. There are additional asymmetric effects, so the bandwidth is higher for hedging assets with negative correlation with the firm's stock,[25] however in our numerical simulations the asymmetric effect is relatively small and bandwidth and costs

[25]For more details see Whalley (2008).

generally increase with $|\rho|$.

We adapt the analysis in Proposition 1 to the current setting. The main difference arises because we consider exercise at times before maturity. We thus use a horizon-unbiased form of discounted exponential utility,[26] which effectively chooses the discount rate to reflect the assumption that the executive invests his liquid wealth optimally, and has no effect on the differential equations. Details of the differences are given in the Appendix. Applying Proposition 1 directly, we have the reservation value of the holdings of restricted stock $G^{(N)}$:

Corollary 1 *The value per share of a holding of N shares to an executive who is unable to trade them until T_G can be approximated by:*

$$G^{(N)} = GP^{(N)} + GB^{(N)} + GF^{(N)}$$

where

1. *$GP^{(N)}$ satisfies*

$$\mathcal{L}_{BS}(GP^{(N)}) + qV - N\hat{\gamma}(t)\frac{\eta^2(1-\rho^2)}{2}V^2(GP_V^{(N)})^2 = 0$$

with $\hat{\gamma} = \gamma e^{r(T_G - t)}$ subject to $GP^{(N)}(V, T_G) = V$,

2. *$GB^{(N)}$ satisfies*

$$\mathcal{L}_{BS}(GB^{(N)}) - N\hat{\gamma}(t)\eta^2(1-\rho^2)V^2 GP_V^{(N)} GB_V^{(N)} = \frac{\hat{\gamma}(t)}{N}\frac{\sigma^2}{2}\left(X^{*2}_{(N,0)} - X^{*2}_0\right)$$

subject to $GB^{(N)}(V, T_G) = 0$ where $X^*_{(N,0)(}(V,t) = \left(\frac{3k}{\hat{\gamma}\sigma^2}\right)^{\frac{1}{3}} |\mathcal{V}(N\, GP^{(N)})|^{\frac{1}{3}}$

3. *$GF^{(N)}$ satisfies*

$$\mathcal{L}_{BS}(GF^{(N)}) - N\hat{\gamma}(t)\eta^2(1-\rho^2)V^2 GP_V^{(N)} GF_V^{(N)} = 0$$

[26] This was developed independently by Oberman & Zariphopoulou (2003) for finite maturity and Henderson (2007) for perpetual options. See Henderson & Hobson (2007) for further discussion.

subject to $GF^{(N)}(V,T_G) = -k\frac{|\rho|\eta}{\sigma}V$.

The optimal hedging strategy is to hold a number of the hedging asset, y*, such that the value of the hedging asset held,* $x = My$*, lies in a no transaction band* $x^*_{(N,0)} - X^*_{(N,0)} \leq x \leq x^*_{(N,0)} - X^*_{(N,0)}$ *where* $x^*_{(N,0)} = x^*_0(t) - N\beta V(GP^{(N)}_V + GB^{(N)}_V)$ *is the centre of the band and* $X^*_{(N,0)}$ *is the semibandwidth defined above.*

Given the subjective value of a holding of restricted stock, we can also determine the equations governing the marginal value of n additional options

Proposition 2 *The value per option of a grant of options on* n *shares with strike price* K *to an executive who holds* N *non-tradeable shares, who hedges optimally using a partially correlated hedging asset with associated transaction costs can be approximated by:*

$$E^{(n|N)} = EP^{(n|N)} + EB^{(n|N)} + EF^{(n|N)} \tag{12}$$

where

1. $EP^{(n|N)}$ *satisfies*

$$\mathcal{L}_{BS}(EP^{(n|N)}) - \hat{\gamma}(t)\frac{\eta^2(1-\rho^2)}{2}V^2 EP^{(n|N)}_V(2N\, GP^{(N)}_V + n\, EP^{(n|N)}_V) = 0 \tag{13}$$

2. $EB^{(n|N)}$ *satisfies*

$$\begin{aligned} \mathcal{L}_{BS}(EB^{(n|N)}) - \hat{\gamma}(t)\eta^2(1-\rho^2)V^2 N(EP^{(n|N)}_V GB^{(N)}_V + GP^{(N)}_V EB^{(n|N)}_V) & \qquad (14) \\ -\hat{\gamma}(t)\eta^2(1-\rho^2)V^2 n EP^{(n|N)}_V EB^{(n|N)}_V - \frac{\hat{\gamma}(t)}{n}\frac{\sigma^2}{2}\left(X^{*2}_{(N,n)} - X^{*2}_{(N,0)}\right) &= 0 \end{aligned}$$

where $X^*_{(N,n)}(V,t) = \left(\frac{3k}{\hat{\gamma}\sigma^2}\right)^{\frac{1}{3}} |\mathcal{V}(N\, GP^{(N)} + n\, EP^{(n|N)})|^{\frac{1}{3}}$.

3. $EF^{(n|N)}$ *satisfies*

$$\begin{aligned} \mathcal{L}_{BS}(EF^{(n|N)}) - \hat{\gamma}(t)\eta^2(1-\rho^2)V^2 n EP^{(n|N)}_V EF^{(n|N)}_V & \qquad (15) \\ -\hat{\gamma}(t)\eta^2(1-\rho^2)V^2 N(EP^{(n|N)}_V GF^{(N)}_V + GP^{(N)}_V EF^{(n|N)}_V) &= 0 \end{aligned}$$

The executive's optimal hedging strategy is to hold a number of the hedging asset, y, such that the value of the hedging asset held, $x = My$, lies in a no transaction band $x^*_{(N,n)} - X^*_{(N,n)} \leq x \leq x^*_{(N,n)} - X^*_{(N,n)}$ where $x^*_{(N,n)} = x^*_0(t) - \beta V(N(GP^{(N)}_V + GB^{(N)}_V) + n(EP^{(n|N)}_V + EB^{(n|N)}_V))$ is the centre of the band and $X^*_{(N,n)}$ is the semiband-width defined above.

If the executive sells the shares associated with the option immediately on exercise $E \equiv \bar{E}$, $EP \equiv \bar{EP}$ etc. and (13)-(15) must be solved subject to final conditions $\bar{EP}^{(n|N)}(V,T) = \Lambda(V) = \max(V - K, 0)$, $\bar{EB}^{(n|N)}(V,T) = 0$, $\bar{EF}^{(n|N)}(V,T) = -k\frac{|\rho|\eta}{\sigma}VI_{V\geq K}$ where $I_{V\geq K} = 1$ if $V \geq K$ and 0 otherwise, and early exercise condition once the options have vested

$$\bar{E}^{(n|N)} \geq \Lambda(V) - k\frac{|\rho|\eta}{\sigma}VI_{V\geq K} \quad \forall T_V \leq t \leq T.$$

If the executive retains the shares on exercise of the option $E \equiv \hat{E}$ etc. and (13)-(15) must be solved subject to final conditions

$$\hat{EP}^{(n|N)}(V,T) = G^{(N+n)}(V,T) - K + \frac{N}{n}(G^{(N+n)}(V,T) - G^{(N)}(V,T)),$$

$\hat{EB}^{(n|N)}(V,T) = 0$, and $\hat{EF}^{(n|N)}(V,T) = 0$, and early exercise condition once the options have vested

$$\hat{E}^{(n|N)} \geq G^{(N+n)} - K + \frac{N}{n}(G^{(N+n)} - G^{(N)}) \quad \forall T_V \leq t \leq T.$$

Given the executive's optimal early exercise threshold from the calculation of E, we can find the cost of the grant to the shareholders.

Proposition 3 *The cost to the shareholders (objective value) per option of the same grant is given by $\bar{S}^{(n|N)}$ (or equivalently $\hat{S}^{(n|N)}$ if the executive retains the shares acquired on exercise) which satisfies $\mathcal{L}_{BS}(S^{(n|N)}) = 0$ subject to $\bar{S}^{(n|N)}(V,t) =$*

$\Lambda(V)$ if $V \geq \bar{V}^*_{(n|N)}(t)$ or $\hat{S}^{(n|N)}(V,t) = \Lambda(V)$ if $V \geq \hat{V}^*_{(n|N)}(t)$ where $\bar{V}^*_{(n|N)}(t)$, $\hat{V}^*_{(n|N)}(t)$ are the optimal early exercise thresholds associated with the executive's subjective option values $\bar{E}^{(n|N)}$ and $\hat{E}^{(n|N)}$ respectively.

Insert Figure 1 here

The centre of the hedging band at inception for a six-year option grant with two years to vesting for an executive who already owns an equivalent number of shares in the firm is shown in the top left-hand graph in Figure 1 plotted against moneyness and correlation.[27] The location of the band moves rapidly away from the perfect market value (the left-hand edge of the graph) as the correlation decreases from 1, primarily due to the linear effect of changes in β. The effect of unhedgeable risk in reducing subjective sensitivity (Delta) can however also be seen in the inverted S-shape with respect to ρ. The width of the band, shown in the top right-hand graph in Figure 1, generally increases with absolute correlation. However, the cost of unhedgeable risk decreases as absolute correlation increases and overall this effect dominates, so, as shown in the bottom left-hand graph, subjective option values ($\bar{E}$) decrease as unhedgeable risk increases. The resulting effect on early exercise thresholds once the options have vested for cash exercise is shown in the bottom right-hand graph in Figure 1 for different times to maturity and correlations and has the same U-shape with respect to correlation as subjective option values. Objective values have a similar but smaller U-shape with respect to correlation.

Since utility-based pricing is nonlinear, the marginal per-option value of the

[27] Other parameter values are given in the caption. Most ESOs in the US have maturity 10 years and vesting periods 2-4 years. In Australia ESOs typically have maturities 5-6 years and vest after 2-3 years (Boyd *et al* (2007)).

grant to the executive depends not only on its size but also on the magnitude of the executive's existing stockholdings. This can be seen in the cost of unhedgeable risk term in the differential equation for the leading order option value,[28] $-\frac{\hat{\gamma}}{2}\eta_e^2 V^2 \left(N\, GP_V^{(N)} + n\, EP_V^{(n|N)}\right) EP_V^{(n|N)}$, which increases with both n and N,[29] (and with the option's moneyness), and also for the hedging strategy cost term in (14) of $-\frac{\hat{\gamma}(t)}{n}\frac{\sigma^2}{2}(X^{*2}_{(N,n)} - X^{*2}_{(N,0)})$, which increases with n and N but at a lower rate.[30] Hence unhedgeable risk becomes relatively more important in determining subjective values as the executive's exposure to his own company stock rises.

Insert Figure 2 here

Combining the leading order cost of unhedgeable risk term with other terms involving $V EP_V$ we see the cost of unhedgeable risk effect can be viewed as an additional dividend yield of $\frac{\hat{\gamma}}{2}\eta_e^2 V \left(N\, GP_V^{(N)} + n\, EP_V^{(n|N)}\right)$ which increases with moneyness. The graphs in Figure 2 compare the subjective option values at inception against correlation for different combinations of dividend yield and $\nu = n\gamma$ with $N = 0$ and show that larger grants or higher risk aversion (larger $\nu = n\gamma$) reduce subjective option values, by more, the greater the unhedgeable risk (the lower $|\rho|$). Comparing the two also shows the impact of ν increases with moneyness.[31]

[28]The cost of unhedgeable risk terms in the equations for EB and EF have similar forms.

[29]Note since $GP_V^{(N)} < 1$ and $EP_V^{(n|N)} < E_V^{BS}$ decrease with n and N, the rate of increase decreases with both N and n. Furthermore, the size of the existing shareholding has much greater influence than the size of the new grant, since each additional stock held increases the cost of unhedgeable risk term by $2 \times GP_V^{(N)}$ whereas each additional option only increases it by $1 \times EP_V^{(n|N)}$.

[30]This term also depends on the portfolio Gamma and so has greater relevance close to the money.

[31]In the right-hand graph for at-the-money options the option value with $\nu = 1, q = 0, \rho = 0$ is close to the value for lower $\nu = 0.25$ and higher dividend yield ($q = 0.02$), whereas for in-the-money options with $V/K = 2$, it is closer to that for $\nu = 0.25$ and $q = 0.04$.

Insert Figure 3 here

If the executive sells the shares acquired on exercise, the reduction in subjective option value due to the cost of unhedgeable risk and the cost of the hedging strategy reduce the early exercise threshold relative to its perfect market equivalent. The left-hand graph in Figure 3 plots the subjective and objective value on vesting for sale on exercise, (dotted and bold lines respectively) and, for comparison, the perfect market value (solid line). Note since exercise is not optimal from the shareholders' perspective, the slope of $\bar{S}$ at $\bar{V}^*$ does not match the payoff's and the objective value differs from the equivalent value with optimal exercise (the solid line) by more, the closer the firm's share price is to the executive's exercise threshold.

If the executive retains the shares acquired on exercise, shown in the right-hand graph in Figure 3 for the same parameter values, the value of the payoff to the executive is lower and concave. Since exercising locks in his exposure to firm stock price risk, he 'delays' exercising until the stock price is higher. This does not increase the objective value however. Since the exercise threshold is higher than optimal under perfect market conditions the objective option value is still lower than its perfect market equivalent. The objective value thus falls below the payoff when close to the exercise threshold, although the effect is small.

So objective option values, early exercise thresholds and subjective option values all differ from perfect market values because of the costs of bearing unhedged risk due to restrictions and costs of trading. We now investigate the implications of this for executives and shareholders by considering the benefits of hedging to the executive and the effect of this, and the executive's holdings of company stock, on the shareholder cost which should be reflected in the accounts.

4 Analysis

4.1 Effects of hedging on option values

If the executive does not hedge, he does not incur additional transaction costs but he bears the full exposure to his firm's stock price risk. His subjective option value is $EP^{(n|0)}(V,t;\rho=0)$ and hence solves (13) with $\rho=0$ s.t. $EP(V,t) \geq \max(V-K,0)\ \forall t \geq T_V$ (cash settlement). In the graphs of option values against correlation, the unhedged value corresponds to $\rho = 0$.

Hedging reduces the level and hence cost of unhedgeable risk, by more, the greater the absolute correlation between firm stock and hedging asset returns. So if hedging were costless, it would be optimal for executives to hedge maximally, using the hedging asset with the highest absolute correlation with their firm's stock returns. In practice, however, hedging incurs transaction costs in following the optimal hedging strategy and the effect of these costs generally increases with the level of potentially hedgeable risk, or absolute correlation. In the presence of costs the optimal hedging strategy lies between the extremes of no and maximal hedging. It limits potentially hedgeable risk by imposing a hedging bandwidth. The costs in maintaining the hedging strategy and the residual potentially hedgeable risk reduce the subjective option value but, since the choice of bandwidth is optimal for the executive, by less than leaving the risk unhedged. So some hedging, limiting the risk, is always worthwhile for an executive if a partially correlated tradeable asset exists. This is reflected in the bottom lines in the graphs in Figure 4, which show at-the-money subjective per-option values at inception as a proportion of the perfect market option value *vs* correlation, and which all have a minimum at $\rho = 0$.

Insert Figure 4 here

The impact of unhedged risk and costs on the executive's subjective option value translates into an impact on the executive's optimal exercise threshold once the options have vested and hence to a muted effect on the objective per-option value or shareholder cost. The top lines in the graphs in Figure 4 show that objective values as a proportion of perfect market values also decrease with the level of unhedged risk, but to a lesser extent than the equivalent subjective value (the lower lines of the same style on the same graph).

The top left graph in Figure 4 shows the effect of different transaction cost levels. The higher the proportional cost k, the larger the bandwidth (which increases as $k^{\frac{1}{3}}$) so the greater the residual potentially hedgeable risk and the effect on subjective option values (which increases as $k^{\frac{2}{3}}$). Changes in transaction costs have a greater effect on subjective option values the larger the potentially hedgeable risk (higher $|\rho|$), but have little effect on objective values. The volatility of the hedging asset, σ, also affects the impact of the bandwidth hedging strategy on subjective option values, decreasing the hedging bandwidth but, for the parameter values we consider, making very little difference to the effect of hedging costs on either subjective or objective option values. Increases in the volatility of the firm's stock price, η, increase unhedgeable risk for a given correlation so, as shown in the bottom left graph in Figure 4, proportionate subjective and objective values decrease.

Increases in the firm's dividend yield decrease both subjective and objective option values and also early exercise thresholds, as in the perfect market case, because the options are not dividend-protected. However, as shown in the top right graph in Figure 4, the proportional value (relative to the equivalent perfect market value with the same dividend yield) of both subjective and objective ESO values are higher, the larger the dividend yield. Recall the cost of unhedgeable risk can be thought of as an additional 'dividend' borne by executives which increases with moneyness. In a perfect market setting, the magnitude of the sensitivity of call option values to the dividend yield decreases as the dividend yield increases. So the higher the actual dividend yield, the smaller the effect of the additional cost of unhedged risk effect on subjective option values and, through the exercise threshold, also on objective option values.

Changes in the vesting period have opposite effects on proportional subjective and objective option values at inception as shown in the bottom right graph in Figure 4. During the vesting period executives cannot eliminate the unhedgeable risk associated with their option position by exercising. The cost of unhedgeable risk effect thus decreases subjective option values more, the longer the vesting period. Although the executive's exercise threshold increases with time to maturity, it still always differs from the perfect market exercise threshold, so the objective value is lower than the perfect market value, by more, the longer the time *after* vesting. For fixed maturity, proportional objective values thus increase with time to vesting.

Insert Figure 5 here

Figure 5 shows the effect of the executive's exposure to his firm's stock price movements through the effective size of the option grant, $\nu = n\gamma$ and his existing stock-holdings, $\alpha = N\gamma$. Top graphs show subjective values of at-the-money options at inception as a proportion of the perfect market value, middle graphs show proportional objective values and bottom graphs show exercise thresholds on vesting, all *vs* correlation. Left-hand graphs show the case where shares acquired on exercise are sold and right-hand graphs where they are retained until T_G.

Increasing either grant size or ownership level increases the magnitude of the cost of unhedgeable risk term and decreases per-option subjective values, so unhedged ($\rho = 0$) subjective option values decrease significantly as either the grant size or the executive's shareholding[32] increase, though at a decreasing rate.[33] The effect of hedging costs also increases with effective grant sizes and executive shareholdings, but at a lower rate, since the executive optimally adjusts the limits on the potentially hedgeable risk (the bandwidth per option decreases with ν or α), whereas no such adjustment for the unhedgeable risk is possible. So the effects of unhedgeable risk not only decrease proportionate subjective option values more as ν or α increase, but also become relatively more important (than transaction costs) in determining these values, *i.e.* the relative benefit of hedging for the executive increases.

Insert Figure 6 here

Comparison of the right- and left-hand graphs in Figure 5 shows the cost of unhedgeable risk effect is also much greater (subjective option values are much lower) when shares are retained on exercise than when they are sold immediately. The top left graph in Figure 6 shows subjective values are also significantly affected by the length of time the executive expects to (have to) retain the shares (the retention period T_G), but only if his shareholding increases as a result of exercising

[32]Since the exposure from holding a share, $GP_V^{(N)}$, is generally closer to 1 and so larger than the exposure from an option, $EP_V^{(n|N)}$, the level of executive shareholding generally has greater effect on subjective option values than the grant size.

[33]This can be seen by comparing the top, middle and bottom lines in the option graphs. The more the effects of unhedgeable risk reduce subjective option values closer to zero, the less the scope for further decreases at the same rate.

the option. The subjective payoff on exercise is lower under retention than sale, and decreases with the retention period, so under retention, subjective option values are lower for the same level of exposure (ν, α, ρ). The benefits of hedging are thus greater for an executive who retains rather than sells shares acquired on exercise and increase with the length of the retention period. Increases in the effective sizes of the option grant or executive shareholding (ν or α), or the retention period, T_G, also decrease proportionate objective option values.

4.2 Implications for executives

The analysis in section 4.1 has shown it is always worthwhile for executives to hedge optimally using a partially correlated hedging asset. The benefit of hedging is greater for more risk-averse executives with larger shareholdings or option grants where the options have long times to vesting, when the executive retains shares on exercise and for firms which have low dividend yields.

An executive may have a number of potential tradeable assets he could use to hedge his option position (*e.g.* the market index or a basket of stocks from the same industry). Overall the main characteristics affecting the executive's selection of a hedging asset are a choice between higher absolute correlation with firm stock returns, equivalent to lower unhedgeable risk, and lower transaction costs (*e.g.* trading in a market index is likely to have lower transaction costs but may also have lower correlation than a basket of stocks from the same industry). The exact outcome (*i.e.* which combination gives the higher subjective option value) depends on a number of factors and should be determined for each case; however minimising unhedgeable risk becomes increasingly important for more risk-averse executives with larger holdings of firm stock, larger option grants with longer times to vesting and for firms with lower dividend yield stocks.

The incentive for an executive to hedge can be large, particularly for executives with large existing shareholdings and large option grants where the firm has a low dividend yield. For example, hedging with a traded asset with $\rho = \beta = 0.8$ and transaction costs of 1% increases per-option subjective value at inception for an executive with risk aversion $\times$ existing holding of $\alpha = 1$ with a grant of six-year maturity options with two years to vesting with an effective grant size of $\nu = 1$ from 0.111 to 0.165, a proportionate increase of 49%.

Apart from hedging with a partially correlated traded asset, another way for executives to reduce the cost of unhedgeable risk associated with their overall portfolio is to reduce their holdings of firm shares. This increases the value of the executive's option compensation (as well as removing the illiquidity discount on the shares sold). Returning to the example above, if the executive sold his existing stock-holding, the value of the hedged option grant would increase by 31%, and if he held twice his existing stock-holding, the option grant would be worth 19% less to him. So executives with higher levels of share holdings in the firms they manage can reduce the overall illiquidity discount associated with their exposure to firm risk more by selling shares, the larger these holdings are and the larger their exposure through options granted to them. This is consistent with Ofek & Yermack (2000)'s findings that executives with higher prior ownership had higher propensity to sell shares on exercise or on grant. Alternatively, entering into a derivative contract such as those described in Bettis, Bizjak & Lemmon (2001) will also have greater effect in reducing the illiquidity discount, the larger their holdings of stock and options.

4.3 Implications for shareholders

Given the benefit obtained by the executive from hedging,shareholder cost calculations should take account of this hedging and the resulting increase in objective option values this implies, particularly when benefits to exe3cutives are large. The effect of hedging on shareholder cost is less pronounced than the effect on executive values but can still represent a significant difference. In the example above, hedging by the executive increases the equivalent objective value from 0.217 to 0.238, a proportional increase of 10% (representing 84% and 93% of the equivalent perfect market value respectively, showing perfect market valuation still gives values which are too high). The impact on shareholder value is higher for firms with lower dividend yields, for larger grants of options with shorter times to vesting and for executives with larger holdings if firm stock.

Comparing the middle graphs of Figure 5 shows the magnitude of the effect of executive hedging differs considerably depending on the retention policy of the executive. The example above was for sale on exercise (which corresponds to the left-hand graph), where the effects can be considerable. In contrast, if the executive retains the shares acquired on exercise until the maturity of the option grant (shown in the right-hand graph), the effect on the objective value is negligible, even though the exercise threshold increases significantly with unhedgeable risk, grant size and original ownership level.[34] However, if the executive retains the shares on exercise, the effect on the objective value is very sensitive to the executive's retention period,

[34]The bottom right-hand graph shows that, under retention, the exercise threshold depends primarily on the exposure after exercise (the thresholds for *e.g.* $\nu = 2, \alpha = 0$ and $\nu = 1, \alpha = 1$ are close). The dependency on the exposure after exercise carries over to the objective values.

as shown in the top right graph in Figure 6. Retaining shares beyond the maturity of the option decreases subjective payoff values, particularly closer to maturity, so, as shown in the bottom graphs in Figure 6, exercise thresholds increase further from the perfect market threshold, the longer the retention period. Whilst the effect of a higher than optimal (in a perfect market setting) exercise threshold on objective values is small when the difference in exercise thresholds is small, it increases rapidly as the difference between the thresholds increases, particularly for shorter times to maturity. Thus objective values are much lower and more sensitive to the level of unhedgeable risk ($|\rho|$) and the size of the executive's exposure (ν, α), the longer the retention period. This is relevant for executives at the beginning of their career with the firm who may be building up their ownership stake and expect to retain shares for a considerable period.

Thus the shareholder's objective option value is both lower than the perfect market value and is sensitive to unhedgeable risk and the executive's exposure through his option and stock holdings both for executives with ownership above their target level who sell shares acquired on exercise and also for new executives building up their ownership stake who retain shares acquired on exercise with expectations of holding for a considerable period. However the relationship between shareholder value and exercise threshold is more complex when retention of shares by the executive on exercise is allowed. With sale on exercise there is a significant positive relationship between exercise thresholds and shareholder cost; if shares are retained the relationship has the opposite sign and its magnitude depends significantly on the executive's retention period. The shareholder cost of an option grant is sometimes implied from a calibration of early exercise threshold data. In such calculations care should be taken to identify features such as the likely retention period which have a significant impact on this relationship.

4.4 Exercise thresholds

Given the potentially significant benefit to executives from hedging using a partially correlated traded asset, exercise thresholds should vary with unhedgeable risk and the executive's exposure to this risk. The bottom graphs in Figures 5 and 6 show how the exercise thresholds vary with unhedgeable risk (correlation), grant sizes, ownership levels, sale or retention on exercise decisions, retention periods and time to exercise. The graphs show executives' optimal early exercise threshold are significantly influenced by the level of unhedgeable risk (idiosyncratic risk if the executive uses the market to hedge), the sizes of the option grant and the executive's existing shareholding, whether shares are retained or sold on exercise and, if retained, on the retention period. The effect of retention rather than sale on exercise is important because it reverses the sign of the effects of unhedgeable risk, grant size and size of existing shareholding on exercise thresholds from negative to positive. This is particularly relevant given Aboody *et al* (2006)'s finding that nearly half ESO exercises are not accompanied by the immediate sale of acquired shares.

Prior studies of the determinants of early exercise thresholds (Bettis *et al* (2005), Boyd *et al* (2007), Armstrong *et al* (2006)) have only considered total firm stock volatility rather than the idiosyncratic component and have not incorporated the size of the option grant or executive shareholdings[35] as potential determinants of

[35] Another consistent empirical finding is that higher level executives are likely to exercise later than lower level employees. The model would suggest such an effect for any directors who retained shares on

early exercise activity. Nevertheless, they find significant negative effects of total volatility on the exercise threshold, lending support to the idea that risk-aversion plays a role in executive decisions on ESO exercise. Thus more empirical work is merited in this area.

5 Conclusion

The model for ESO valuation in this paper incorporates the executive's costs of hedging in a partially correlated tradeable risky asset, his holdings of firm stock, the restrictions on trading these and both retention or sale of shares acquired on option exercise.

We show optimal hedging in a tradeable asset is worthwhile for the executive, even if imperfectly correlated and costly to trade, since it reduces the risk he has to bear due to his exposure to the firm's stock price. The executive's subjective option value, his optimal exercise strategy and hence also the shareholders' objective value of the option grant thus all depend on the level of unhedgeable risk (idiosyncratic risk if the executive uses the market to hedge), as well as the executive's exposure level (the size of the option grant and his ownership level), and the length of his exposure (time to vesting, time to maturity, sale/retention policy on exercise and time to the executive's sale of his ownership stake). However the relative importance of these factors and the relationship between the exercise strategy and the shareholder cost can differ substantially depending on the executive's sale or retention policy, which is relevant given Aboody *et al*'s finding that almost 50% of shares are retained for at least 30 days after exercise. The exercise threshold is *higher* than perfect market

exercise, although this is likely to be only a partial explanation of this finding.

threshold and *increases* with unhedgeable risk and executive exposure if shares are retained on exercise. We also show the incentive to sell shares on exercise or on grant is greater for higher ownership (and higher grant) executives, consistent with Ofek & Yermack's empirical findings.

The model implies that empirical tests of early exercise thresholds for ESOs as well as models for the shareholder cost of ESOs should take account of unhedgeable (idiosyncratic) risk, option grant size, executive's additional holdings of firm related wealth and whether shares are retained or sold on exercise. Additionally, if using the results of such calibrations to find the shareholder cost for accounting purposes, calculation of the shareholder cost should take account of particular characteristics of the grant[36] rather than using sample average values.

Although the model in this paper has made various simplifying assumptions about the option grant and the executive's portfolio of firm-specific and non firm-specific wealth, the sizes of the grant and existing exposure and the sale/retention decision are still likely to be major influences on the value of incremental option grants in models which incorporate more realistic and hence complex executive portfolios, which we leave for further work. More generally, recognising that the nonlinearity of utility-based valuations means that incremental values and decisions are not independent of executive's existing holdings may have wider applicability.

[36] *E.g.* grant size and terms, the executive's existing and predicted future ownership levels and the idiosyncratic risk associated with the firm's stock price.

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6 Appendix

Corollary 1: horizon unbiased utility functions

Whalley (2008) considers European-style derivatives and maximises expected utility of wealth at the maturity of the derivative. We consider option exercise before maturity and allow the terminal date, T_G to differ from the option maturity, T. Between the exercise and terminal dates, the executive is assumed to invest optimally, so on exercise his subjective utility incorporates this. Hence $J(V, W, t; 0, 0) = \sup_{dy_s, t \leq s \leq T_G} E_t[U(W_{T_G})]$ is given by the Merton solution $J = -\frac{1}{\gamma} e^{-\frac{\lambda^2}{2}(T_G - t)} e^{-\gamma e^{r(T_G - t)} W}$. Effectively we maximise the value on exercise of expected discounted utility of terminal wealth, with the discount rate reflecting the executive's optimal investment over $(T_V, T_G]$. See Oberman & Zariphopoulou (2003) and Henderson & Hobson (2007) for further discussion.

This has no effect on European option values, since these are given by the difference in certainty equivalent values of positions with and without the option and the effect is the same for both certainty equivalent values. (In the notation of Whalley (2008), both h_0^w and h_0^{wo} include a term $\frac{\lambda^2(T-t)}{2\hat{\gamma}}$ which is removed by discounting. The leading order effect on the option value, $H_0 = h_0^w - h_0^{wo} \equiv FP$ is unaffected.) Otherwise the analysis is as in Whalley (2008). Corollary 1 corresponds to Proposition 2 and Section 5.5 in Whalley (2008).

Proposition 2

Let the reservation value of holding N shares and n options be given by $H^{(n,N)} = NG^{(N)} + nE^{(n|N)}$. In the continuation region, $H^{(n,N)}$ can be approximated by $H^{(n,N)} \approx HP^{(n,N)} + HB^{(n,N)} + HF^{(n,N)}$, where $HP^{(n,N)}$ satisfies (7), $HB^{(n,N)}$ satisfies (8) and $HF^{(n,N)}$ satisfies (10). Substituting $H^{(n,N)} = NG^{(N)} + nE^{(n|N)}$

and subtracting we find $EP^{(n|N)} = \frac{HP^{(n,N)} - NGP^{(N)}}{n}$ satisfies

$$\mathcal{L}_{BS}(EP) - \frac{\hat{\gamma}(t)}{n}\frac{\eta^2(1-\rho^2)}{2}V^2\left((N\,GP_V^{(N)} + n\,EP_V^{(n|N)})^2 - (N\,GP_V^{(N)})^2\right) = 0$$

Similarly, $EB^{(n|N)} = \frac{HB^{(n,N)} - N\,GB^{(N)}}{n}$ and $EF^{(n|N)} = \frac{HF^{(n,N)} - N\,GF^{(N)}}{n}$ satisfy

$$\mathcal{L}_{BS}(EB^{(n|N)}) - \frac{\hat{\gamma}(t)}{n}\eta^2(1-\rho^2)V^2\left((N\,GP_V^{(N)} + n\,EP_V^{(n|N)})(N\,GB_V^{(N)} + n\,EB_V^{(n|N)})\right.$$
$$\left. -(N\,GP_V^{(N)})(N\,GB_V^{(N)})\right) = \frac{\hat{\gamma}(t)}{n}\frac{\sigma^2}{2}\left(X^{*2}_{(N,n)} - X^{*2}_{(N,0)}\right),$$

$$\mathcal{L}_{BS}(EF^{(n|N)}) - \frac{\hat{\gamma}(t)}{n}\eta^2(1-\rho^2)V^2\left((N\,GP_V^{(N)} + n\,EP_V^{(n|N)})(N\,GF_V^{(N)} + n\,EF_V^{(n|N)})\right.$$
$$\left. -(N\,GP_V^{(N)})(N\,GF_V^{(N)})\right) = 0$$

which after expansion and cancellation of the terms in $GP^{(N)2}_V$, $GP_V^{(N)}GB_V^{(N)}$ and $GP_V^{(N)}GF_V^{(N)}$ give (13), (14) and (15) respectively.

Consider initially sale on exercise so $E^{(n|N)} \equiv \bar{E}^{(n|N)}$. After exercise, the reservation value of holding N shares is $NG^{(N)}$ with $G^{(N)}$ given by Corollary 1, so on exercise, $H^{(n,N)}$ has value $NG^{(N)} + n\Lambda(V) - kn|\beta|V$. Before exercise $H^{(n,N)} > NG^{(N)} + n\Lambda(V) - kn|\beta|V$. Substituting $H^{(n,N)} = NG^{(N)} + n\bar{E}^{(n|N)}$, we have $\bar{E}^{(n|N)} \geq \Lambda(V) - k|\beta|V$. The optimal exercise threshold $\bar{V}^*_{(n|N)}(t)$ is defined for $T_V \leq t \leq T$ by $\bar{V}^*_{(n|N)}(t) \equiv \inf\{V_t|\bar{E}^{(n|N)}(V_t,t) \leq \Lambda(V_t) - k|\beta|V_t.\}$

For retention on exercise, so $E^{(n|N)} \equiv \hat{E}^{(n|N)}$, analysis of the differential equations and value before exercise is identical. On exercise, $H^{(n,N)} = (N+n)G^{(N+n)} - nK$, so $H^{(n,N)} \geq (N+n)G^{(N+n)} - nK$ and .

$$\bar{E}^{(n|N)} = \frac{H^{(n,N)} - NG^{(N)}}{n} \geq \frac{(N+n)}{n}G^{(N+n)}(V,t) - \frac{N}{n}G^{(N)}(V,t) - K.$$

The optimal exercise threshold $\hat{V}^*_{(n|N)}(t)$ is defined for $T_V \leq t \leq T$ by $\hat{V}^*_{(n|N)}(t) \equiv \inf\{V_t|\hat{E}^{(n|N)}(V_t,t) \leq \frac{(N+n)}{n}G^{(N+n)}(V_t,t) - \frac{N}{n}G^{(N)}(V_t,t) - K\}$.

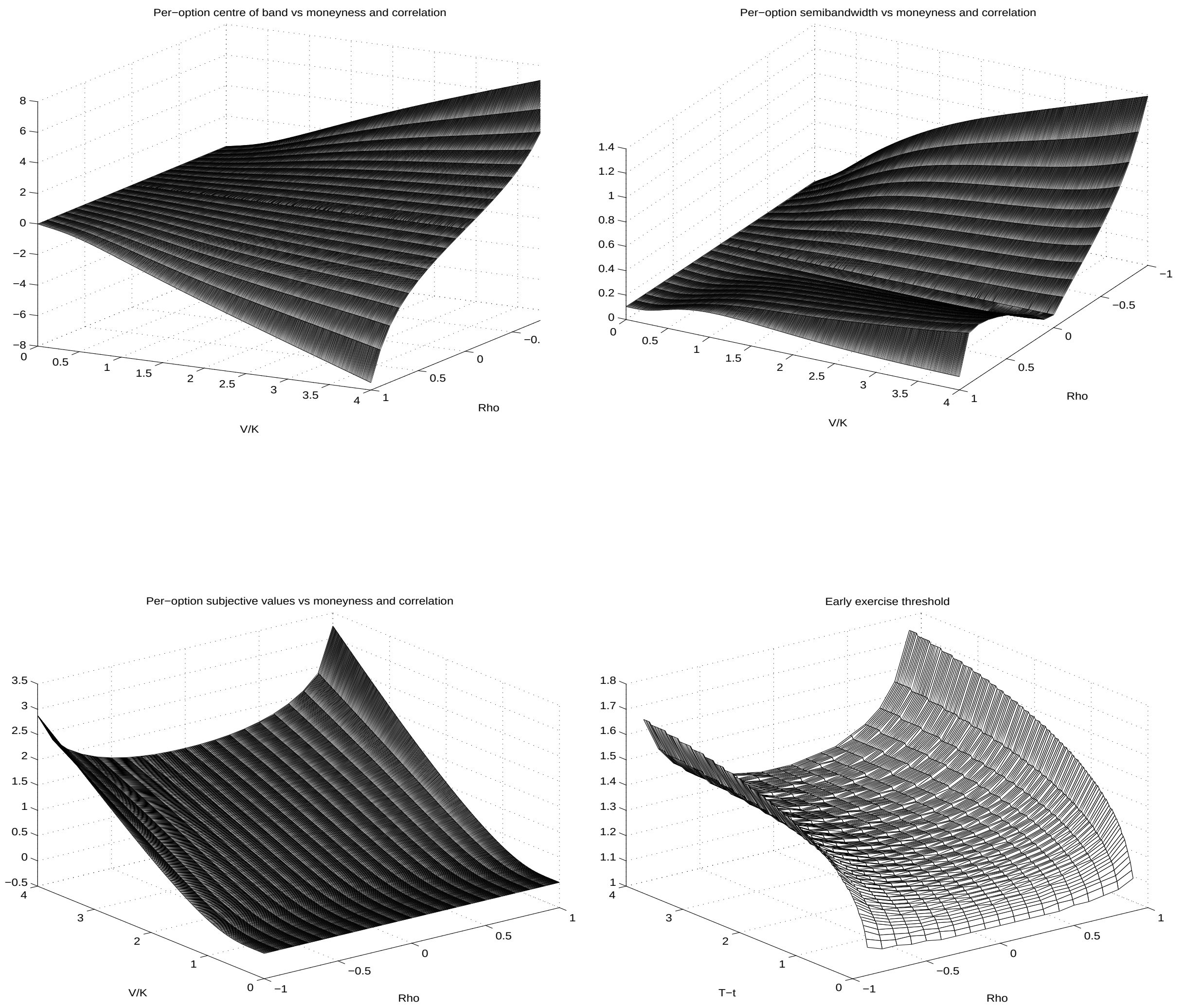


Figure 1: Leading order centre of no-transaction band per option in the absence of transaction costs, $x^*_{(N,n)}/n$ (top left graph), leading order semibandwidth per option, $X^*_{(N,n)}/n$ (top right graph), and per-option subjective value, $\bar{E}$ (bottom left graph), all at inception plotted against moneyness, V/K, and correlation between hedged and hedging assets, ρ, for a grant of n options with sale of shares on exercise. Bottom right graph: exercise threshold, $\bar{V}$ for the same option grant *vs* time to maturity and correlation. Other parameter values where not specified: $T_G = T = 6$, $T_V = 2$, $\eta = 0.25$, $\sigma = 0.25$, $r = 0.04$, $q = 0.02$, $\lambda = 0.1$, $K = 1$, $\nu = n\gamma = 1$, $\alpha = N\gamma = 1$ and $k = 0.01$. Note the direction of the axes differs between graphs.

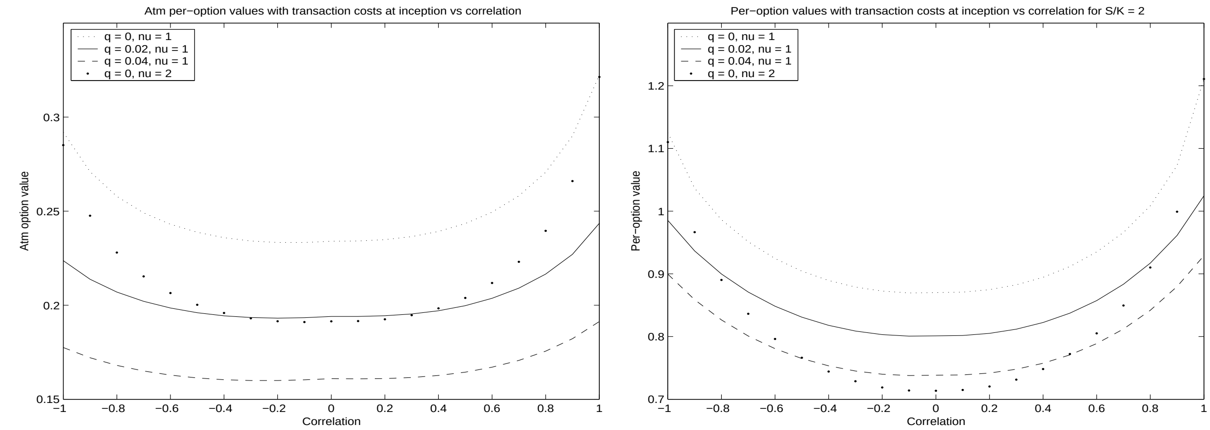
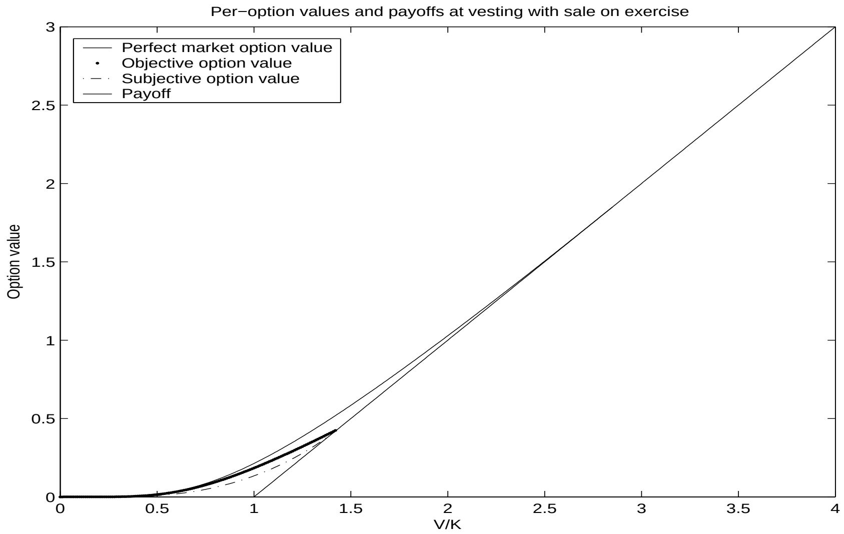


Figure 2: Top graphs: Subjective per-option values $\bar{E}$ at grant for at-the-money $V/K = 1$ (left hand graph) and in-the-money $V/K = 2$ (right hand graph) stock prices *vs* correlation between hedged and hedging assets, ρ for different dividend yields and effective sizes of option grant $\nu = n \times \gamma$ for an executive with no additional ownership of firm stock ($\alpha = N\gamma = 0$) and sale of shares on exercise. Bottom graph: Exercise threshold on vesting $\bar{V}(T_V)$ *vs* correlation for the same parameter combinations. Other parameter values: $T_G = T = 6$, $T_V = 2$, $\eta = 0.25$, $\sigma = 0.25$, $r = 0.04$, $q = 0.02$, $\lambda = 0.1$, $K = 1$, and $k_p = 0.01$.



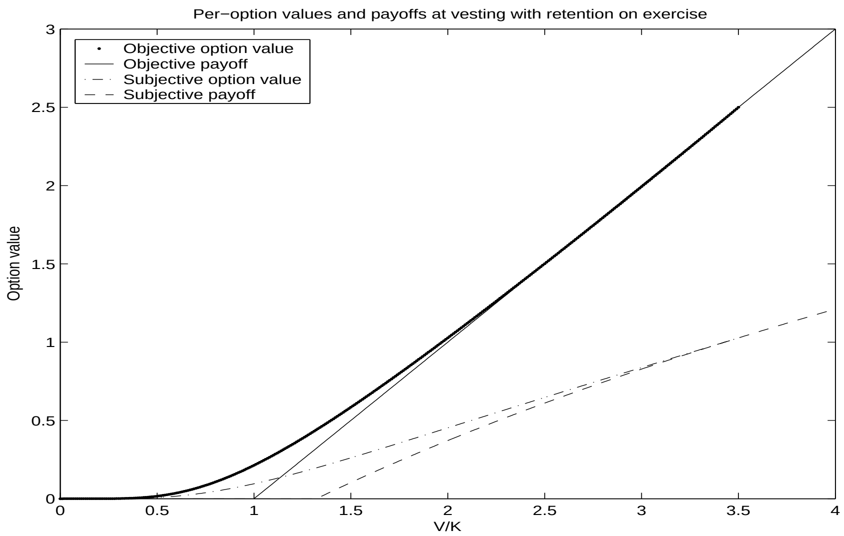


Figure 3: Per-option payoffs and values on vesting *vs* stock price for sale of shares on exercise (top graph) and retention of shares on exercise (bottom graph). Bold lines represent objective shareholder option value, solid line represents objective payoff, lower dash-dotted line is subjective executive value. In top graph, top solid line is perfect market value and in bottom graph, dashed line is subjective payoff value. Parameter values: $T = 6$, $T_V = 2$, $\eta = 0.25$, $\sigma = 0.25$, $r = 0.04$, $q = 0.02$, $\lambda = 0.1$, $K = 1$, $\nu = n\gamma = 1$, $\alpha = N\gamma = 1$, $\rho = 0.6$ and $k = 0.01$.

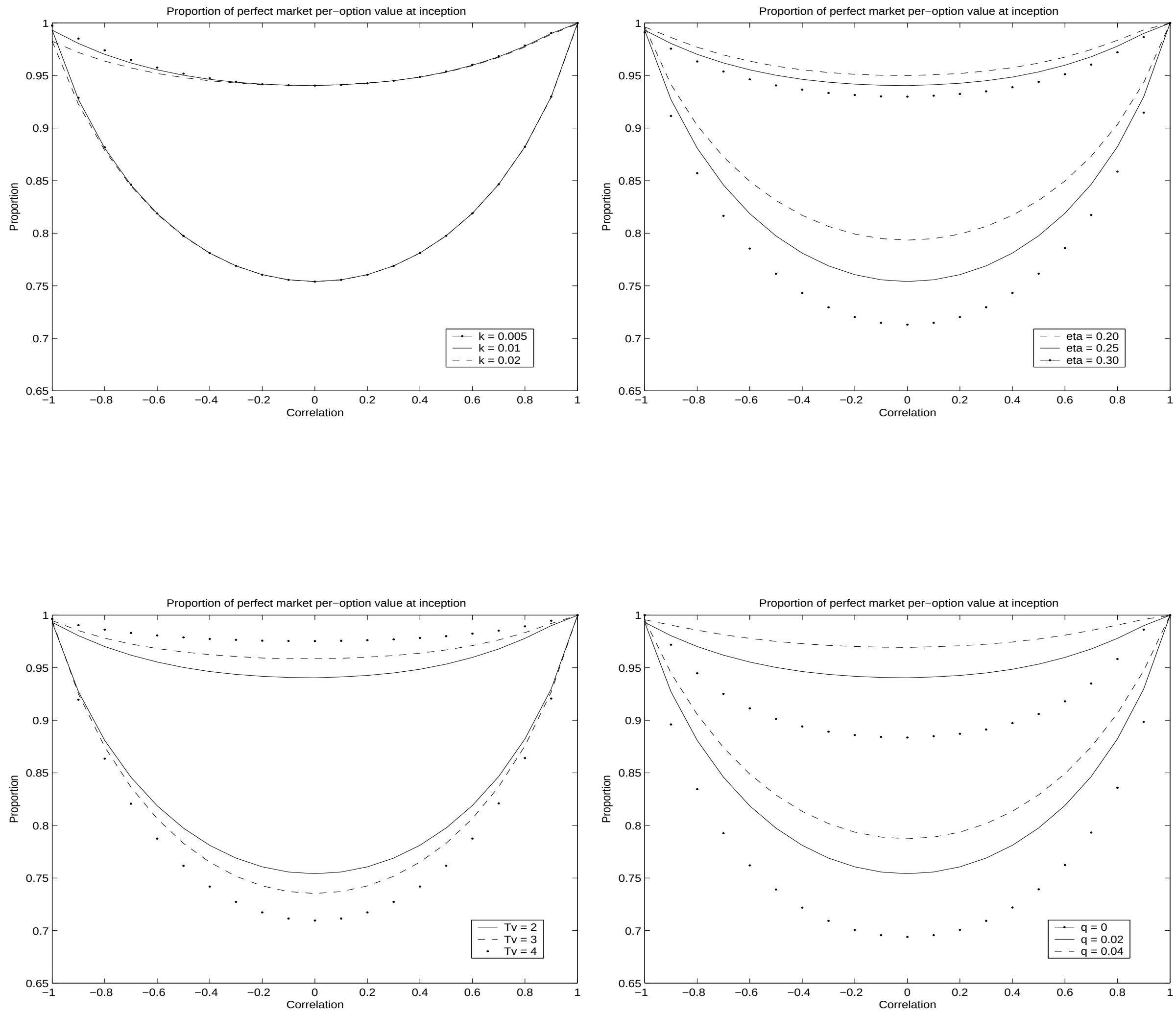


Figure 4: Subjective and objective at-the-money option values at inception as a proportion of the perfect market option value *vs* correlation ρ for different levels of transaction costs, k, (top left graph), dividend yields, q, (top right graph), times to vesting, T_V, (bottom left graph) and firm stock price volatility, η (bottom right graph). Parameter values where not specified: $T_G = T = 6$, $T_V = 2$, $\eta = 0.25$, $\sigma = \eta$, $r = 0.04$, $q = 0.02$, $\lambda = 0.1$, $K = 1$, $\nu = n\gamma = 1$, $\alpha = N\gamma = 0$ and $k = 0.01$.

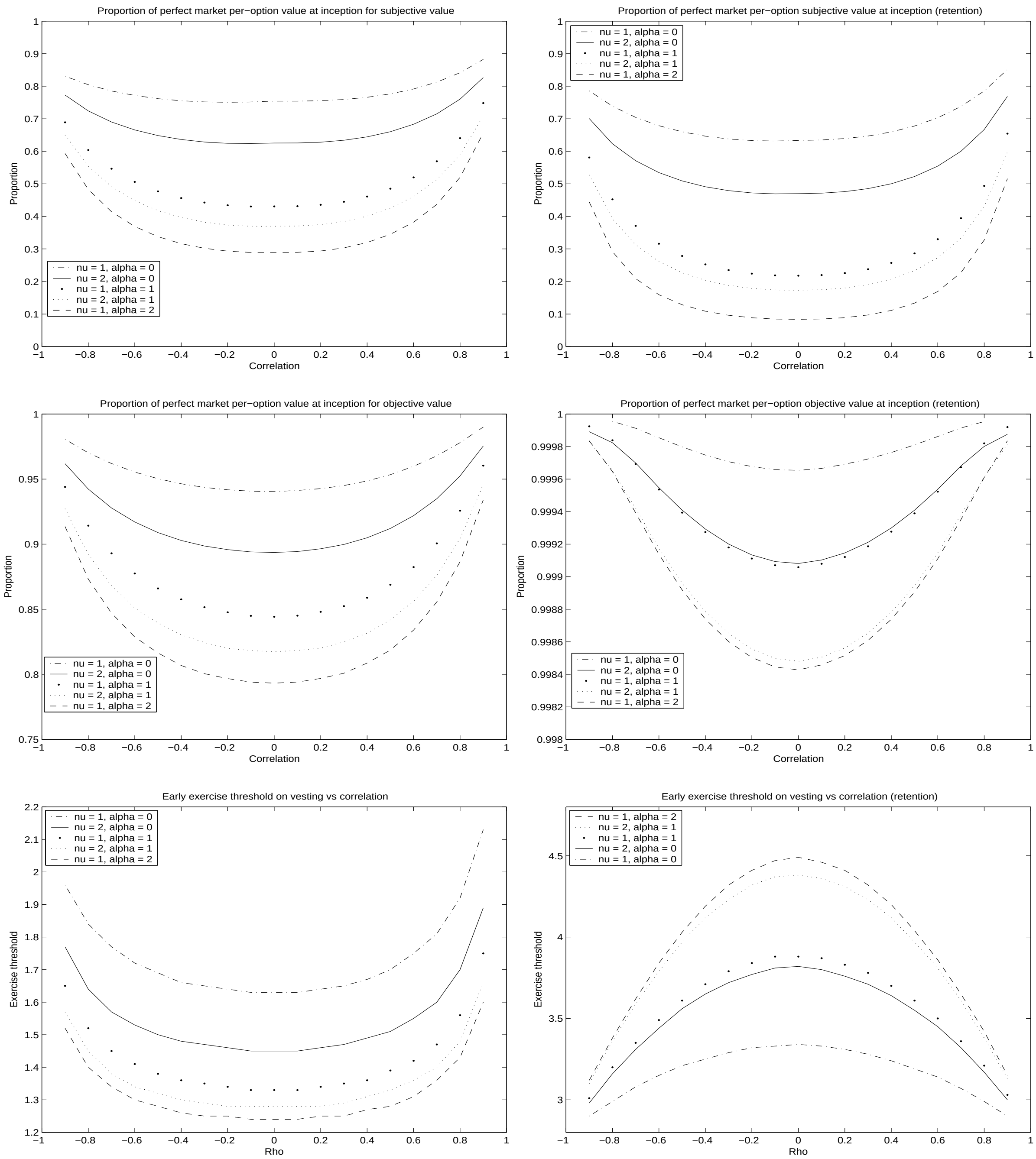


Figure 5: Top graphs: Subjective at-the-money option values at inception as a proportion of the perfect market option value *vs* correlation for different sizes of option grants $\nu = n\gamma$ and existing stock-holdings $\alpha = N\gamma$ for sale on exercise (top left graph) and retention on exercise (top right graph). Middle graphs: Objective at-the-money option values at inception *vs* correlation for the same parameter combinations. Bottom graphs: Exercise threshold on vesting *vs* correlation for the same parameter combinations. Parameter values where not specified: $T_G = T = 6$, $T_V = 2$, $\eta = 0.25$, $\sigma = 0.25$, $r = 0.04$, $q = 0.02$, $\lambda = 0.1$, $K = 1$, and $k = 0.01$.

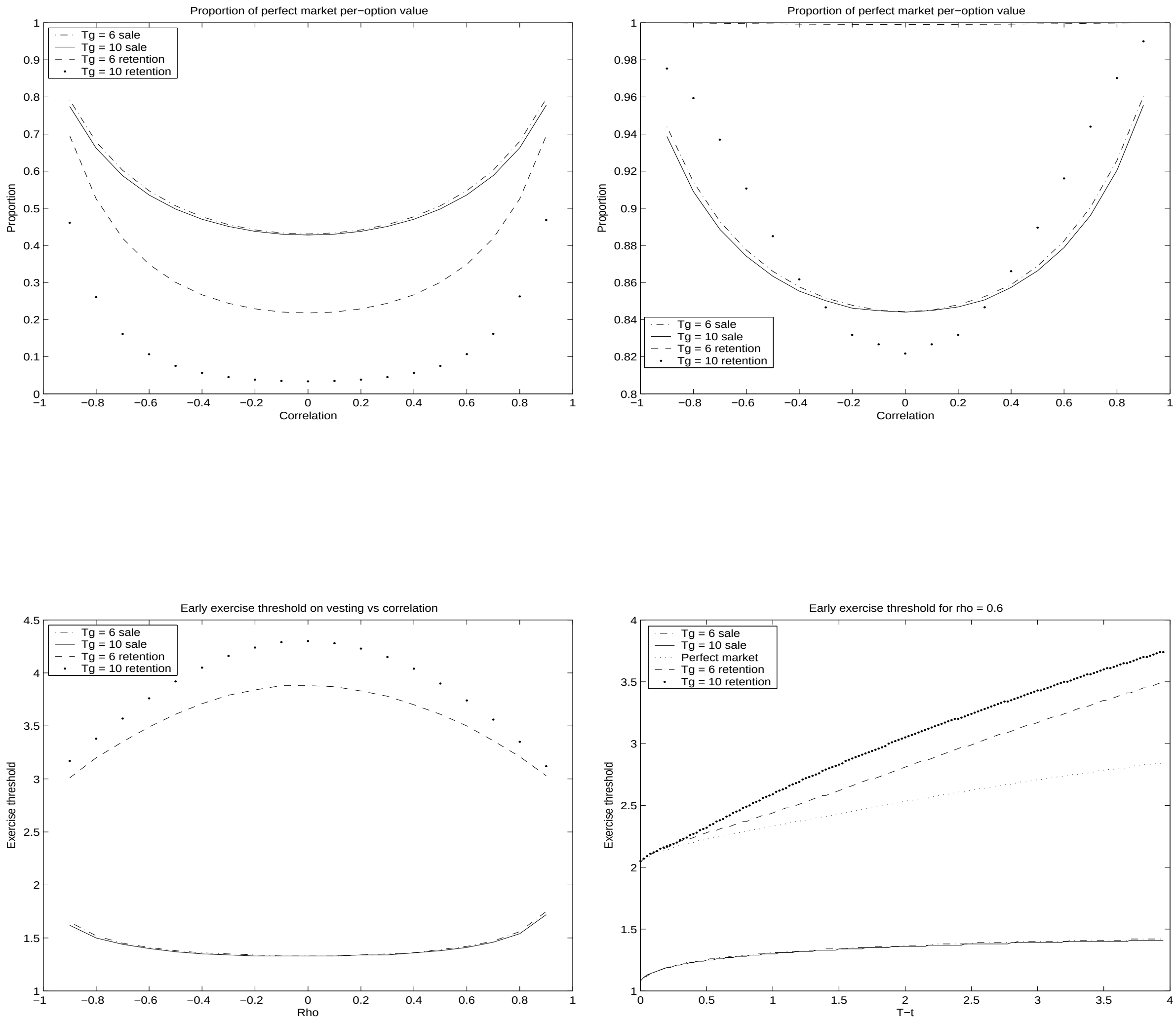


Figure 6: Top graphs: Subjective (left) and objective (right) at-the-money option values at inception as a proportion of the perfect market option value *vs* correlation for different holding periods T_G of firm stock for sale and retention on exercise. Bottom left graph: Exercise thresholds on vesting *vs* correlation for different holding periods T_G and sale/retention decision combinations. Bottom right graph: Exercise thresholds *vs* time to maturity for different holding periods T_G and sale/retention decision combinations. Parameter values where not specified: $T = 6$, $T_V = 2$, $\eta = 0.25$, $\sigma = 0.25$, $r = 0.04$, $q = 0.02$, $\lambda = 0.1$, $K = 1$, $\nu = n\gamma = 1$, $\alpha = N\gamma = 1$ and $k_p = 0.01$.