The Identification of Price Jumps: 
Stock Market Indices During the Crisis

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Abstract

We performed an extensive simulation study to compare the relative performance of many price-jump indicators with respect to false positive and false negative probabilities. We simulated twenty different time series specifications with different intraday noise volatility patterns and price-jump specifications. The double McNemar non-parametric test (McNemar, 1947) has been applied on constructed artificial time series to compare fourteen different price-jump indicators that are widely used in the literature. The results suggest large differences in terms of performance among the indicators, but we identified the best-performing indicators. In the case of false positive probability, the best-performing price-jump indicator is based on thresholding with respect to centiles. In the case of false negative probability, the best indicator is based on bipower variation. Then, we applied all price jump indicators to estimate the price jumps for nine stock market indices using 5-minute data for years from 2007 to 2010. Results show that the price jump indicators tend to group in clusters with close numbers of estimated price jumps per month. However, the overlap of the identified price jumps inside a cluster is rather low, which confirms their different real performance.

Keywords: Price jumps; price-jump indicators; non-parametric testing; Monte Carlo simulations; financial econometrics; stock market indices; cluster.

JEL: C14, C58, F37, G15, G17

1. Motivation and relevant literature

Discontinuities in price evolution have been recognized as an essential part of the price time series generated on financial markets. Many studies, from the seminal work of Merton (1976) to Andersen et al. (2002), demonstrate that continuous-time models have to incorporate the discontinuous component known in the literature as price jumps. The presence of price jumps has serious consequences for financial risk management and pricing. Nyberg and Wilhelmsson (2009) discuss the importance of including event risk as recommended by the Basel II accord, which suggests employing a VAR model with a continuous component and price jumps representing event risks. Andersen et al. (2007) conclude that most of the standard approaches in the financial literature on pricing assets assume a continuous price path. Since this assumption

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is clearly violated in most cases the results tend to be heavily biased. Before a price jump can be accounted for in an estimation stage, it first has to be identified. Surprisingly, the literature up to now does not offer a consensus on how to identify price jumps properly. Jumps are identified with various techniques that yield different results. While there exist quite extensive use of jump-diffusion processes to model interactions in financial data, including Monte Carlo simulations, a broader simulation study comparing discrete identification of price jumps is still missing. The value-added of this paper is that we perform extensive simulations and detailed non-parametric study that employs a wide variety of price-jump indicators to identify the superior techniques. Specifically, we have employed the double McNemar (1947) test and compared the fourteen different price-jump indicators most frequently used in the literature employing simulated time series.

Researchers agree on the presence of price jumps, but they disagree about the source. One branch of the literature considers new information as a primary source of price jumps (see e.g. Merton, 1976; Lee and Mykland, 2008; Lahaye et al., 2011). Macroeconomic news announcements were shown to affect stock prices on both developed as well as emerging markets (Hanousek et al., 2009; Hanousek and Kocenda, 2011). Joulin et al. (2008); Bouchaud et al. (2006) conclude, on the other hand, that price jumps are usually caused by a local lack of liquidity on the market. They also claim that news announcements have a negligible effect on the origin of price jumps. The behavioral finance literature provides other explanations for price jumps. Schiller (2000) claims that price jumps are caused by market participants who themselves create an environment that tends to cause extreme reactions and thus price jumps. Finally, price jumps can be viewed as a manifestation of Black Swans, as discussed by Taleb (2007), where the jumps are rather caused by complex systemic interactions that cannot be easily tracked down. In this view, the best way to understand jumps is to be well aware of them and be ready to react to them properly, instead of trying to forecast them.

The key role price jumps play in financial engineering triggered interest in the financial econometrics literature, especially how to identify price jumps. Several different approaches evolved over the recent years. Generally, we can identify in the literature four groups of econometric price-jump indicators.

The first group is represented by the work of Aït-Sahalia and Jacod (2009a); Aït-Sahalia et al. (2009); Aït-Sahalia and Jacod (2009b) using proper statistical methods to derive and analyze the jump statistics based on different analytic models. The indicators have well-defined analytic properties; however, they do not identify price jumps one by one but rather measure the intensity of price jumps for a given period. These methods are more suitable to assess the jumpiness of ultra-high-frequency data, even though they were also employed in previous studies for high-frequency time series.

The second group of price-jump indicators comprises indicators based on bipower variation and is promoted in a series of papers: Barndorff-Nielsen and Shephard (2004, 2006); Barndorff-Nielsen et al. (2006, 2008). The method is based on two distinct measures of overall volatility, where the first one takes into account the entire price time movement while the second one ignores the contribution of the model-dependent price-jump component. The papers above also illustrate a broad range of application. This method was further improved by Lee and Mykland

\footnote{For illustration, Jarrow and Rosenfeld (1984); Nietert (2001); Pan (2002) study pricing in the presence of jumps and all of them confirm the presence of the jump risk premium. Pricing with jumps using continuous-time diffusion equations was studied by Brodilie and Jain (2008), where the authors consider the pricing of volatility and variance swaps. They conclude that the pricing of swaps significantly differs when jumps are taken into account, thus one cannot appropriately price the risk connected with jumps while ignoring the jumps (See also Kubilius and Platen, 2002; Ogawa, 2008). Continuous processes has been used to model simultaneously price stock options and credit default swaps and find a significant presence of the interplay between credit and market risks (E.g., Duffie et al., 2000; Liu et al., 2003; Johannes, 2004; Kiessler and Tempone, 2011, among others).}
(2008), who develop a statistics for the exact identification of moments when particular price jumps occur and employ it for high-frequency stock returns. However, the main disadvantage of bipower variation-based methods lie in the sensitivity of the intraday volatility patterns, which leads to a high rate of jump mis-identification.

The third group is represented by a test developed by Jiang and Oomen (2008). This test relies on the difference between the swap variance and the realized variance. The authors claim that their test is better than the one based on bipower variation since it amplifies the discontinuities to a larger extent, as they show with a comparative analysis using Monte Carlo simulation. The amplification of discontinuities tends, according to the authors, to suppress the effects of intraday volatility patterns.

Finally, the fourth group of price jump-indicator techniques has its roots in Econophysics. Such techniques are based on the scaling properties of time series known in physics, see e.g. Mantegna and Stanley (2000). Gopikrishnan et al. (1999); Eryigit et al. (2009); Jiang et al. (2009); Joulin et al. (2008) take normalized price time series—the normalization differs across these papers—and define the scaling properties of the tails of the distributions. Then, the scaling index enables them to define the jumpiness of the market purely based on how much of the weight lies in the tails and how this weight is distributed.

As mentioned above, there is still no clear consensus in the literature on how to identify price jumps properly. Bajgrovicz and Scaillet (2009) treat the problem of the spurious identification of price jumps by adaptive thresholds in the testing statistics. The problem with most of the price-jump indicators lies in what model they are built upon. This study illustrates the need for robustness when dealing with price jumps.

The question of the intraday patterns of overall volatility mentioned above is deeply studied in the literature. The work of Wood et al. (1985) documents a U-shaped pattern in the intraday equity volatility. Bollerslev et al. (2008) confirm this effect more robustly. In addition, Novotný (2010) employed many price-jump indicators and studied the difference in price-jump properties during the recent financial crisis using stocks from the NYSE.

Still, to our best knowledge, the literature lacks a deep non-parametric study based on a wide variety of price jump indicators. The literature suggests that identification techniques vary a lot, therefore direct comparison of different papers is not easy. We have focused on this gap in the research and perform a detailed Monte Carlo simulation study to compare price-jump indicators. We have compared price-jump indicators with respect to the false positive and false negative probabilities. We have simulated twenty different kinds of time series with various intraday noise volatilities and different price-jump specifications. Using these simulated time series, we have employed the double McNemar test and compared the fourteen different price-jump indicators most frequently used in the literature.

Our analysis revealed significant differences among the indicators. It was very often the case that one type of indicator clearly dominated the others with respect to the given criterion. Namely, the comparison with respect to the false positive probability was significantly dominated by the indicator based on thresholding with respect to centiles. On the other hand, the comparison with respect to the false negative probability yielded results in which the bipower variation-based indicator dominated. The differences in indicators is very often significant at the highest significance level, which further supports the initial suspicion that the results obtained using different price-jump indicators are not comparable.

In the next step, we have applied the same battery of price jump indicators to nine different stock market indices at 5-minutes frequency for period spanning from 2007 to 2010. The price jump indicators tend to cluster in groups, where number of estimated price jumps per month in the same cluster are very close. This suggests the similarity between the price jump indicators. In reality, the similarity is just seeming since overlap between identified price jumps in the same cluster is very low. Further, we have tested the stability of the estimated numbers of
price jumps over time, with a particular focus on the recent financial crisis. The price jump estimates remain the same over time for every cluster, the time series formed by the number of price jumps per month were stationary. This gives the intuition that recent crisis did not change the particular price jump properties of financial time series.

2. Price-Jump Indicators

We employ a set of price-jump indicators divided into four groups as outlined in the introduction. These indicators are widely used in the empirical literature but the results of two or more indicators are rarely compared with respect to a single string of data. Hence, the results derived in different papers are hard to compare. In our study we perform a non-parametric comparison of the set of price-jump indicators whose details are outlined in section 4. In this section we first introduce the four groups of price-jump indicators. The technical details of the indicators are further elaborated in Appendix A.

2.1. Group 1: Aït-Sahalia

The first class of indicators goes in a similar direction as Aït-Sahalia and Jacod (2009a,b). The price process is assumed to be decomposed into the Gaussian component—corresponding to normal (white) noise—and the non-homogenous Poisson component—corresponding to price jumps. Under certain assumptions it holds that whenever a significant price jump appears, the price increment is dominated by the non-homogenous Poisson component. On the other hand, when the price movements are governed solely by Gaussian noise, the average and/or maximum magnitudes of such increments can be estimated (at a given confidence level). Therefore, one can invert such an argument and set a threshold value that will effectively distinguish the two components.

In practical cases, however, the proper threshold values require knowledge of what should be estimated. Thus we employ an empirical approach and set the threshold by calculating certain threshold levels, or certain percentiles, of the distribution of returns observed over the entire sample. In addition, the financial time series often have intraday volatility patterns, i.e., the average absolute returns systematically differ over the trading day. To reflect this phenomenon, we further divide every trading day into several blocks and calculate percentiles over these blocks separately.

2.2. Group 2: Bipower Variation

The second group is based on bipower variation, as in Barndorff-Nielsen and Shephard (2004, 2006). Specifically, it is based on the difference between the two measures of variation: realized variation and bipower variation. Assuming the price generating process can be decomposed into two components—regular white noise and price jumps—the realized variation measures the variation in the prices coming from both the white noise and the price jumps, while the bipower variation measures the variation coming from the white noise only. This measure can be applied in two different ways.

The first approach, proposed by Barndorff-Nielsen and Shephard (2004, 2006) and further elaborated by Huang and Tauchen (2005), involves the construction of a statistics whose purpose is to determine the presence of price jumps over a given time window, i.e., to test the hypothesis that a given time window contains price jumps at all. This statistics, known as the Max-Adjusted statistics, can be thus employed to identify the exact moment when a price jump occurs. Namely, we fix the length of the testing window and for every time step we test a given window ending at that time step for the presence of price jumps. Then, we say that a price jump occurs at that moment if the window ending at that moment contains a price jump while the window ending at the preceding time step does not.
A problem occurs for consecutive price jumps. If two price jumps are separated by an interval shorter than the given window, the second price jump cannot be identified. Hence, we modify the technique in such a way that after we identify a price jump, we replace it with an average calculated over a moving window of the same length. Since these observations by definition do not contain a price jump, their average also excludes price jumps.

The second approach, constructed by Lee and Mykland (2008), also employs bipower variation. However, it is by definition constructed as a statistics to identify price jumps and the moments when they occurred. The statistics compares the current price movement with the bipower variation calculated over a moving window with a given number of preceding observations, excluding the current one.

2.3. Group 3: Jiang-Oomen Statistics

Jiang and Oomen, 2008 proposed another statistics to test for the presence of price jumps over a certain time window based on Swap Variance. It is claimed that this test amplifies the contribution of price jumps to a larger extent than bipower variation indicators and thus are less sensitive to intraday volatility patterns. Since the Jiang and Oomen statistics is constructed as a test statistics for a certain time window, the price-jump indicator is analogous to the one following the Berneroff-Nielsen and Shephard method: For every time step, we define a moving time window of a given length ending at the time step and test for the presence of price jumps over the window. Then, we identify a certain moment as the one when the price jump occurred if the window ending at the current time step contains a price jump and the one ending at the previous time step does not. In addition, we define an analogous improved indicator, which involves replacing the identified price jumps with moving averages and thus allows for identification of consecutive price jumps.

2.4. Group 4: Statistical Finance

The last group of identification techniques come from the field of statistical finance, as it is called by Bouchaud (2002), although it is also known as Econophysics. This group of indicators relies on the scaling properties of price movements. We employ the price-jump index defined by Joulin et al. (2008).

The price index is defined as the absolute returns normalized with respect to the $L_1$ variance, i.e., the variance defined as an average of absolute returns over a certain moving window. The price-jump index has, as the literature confirms (Joulin et al., 2008), certain scaling properties of the tail part of its distribution. Thus, we define price jumps as those returns for which the price-jump index exceeds a certain empirically determined threshold.

3. Test to compare the performance of the different price-jump indicators

Here we introduce the procedure to compare the performance of the different price-jump indicators. The procedure itself is based on the double McNemar (1947) test, which is a non-parametric method used on nominal data. The intuition behind employing this method lies in the fact that, based on extensive simulations, we want to compare the price-jump indicators with each other rather than study their finite sample properties. Hence, the comparison will be based on the prediction accuracy of the indicators. This means that price-jump indicator $A$ dominates indicator $B$ if $A$ has a significantly better accuracy of jump prediction. This strategy leads to a test procedure to compare the proportions of correctly and incorrectly predicted jumps. The main idea for this approach is natural: if the price-jump indicators are not different in terms of the accuracy of the prediction, it is hard to judge whether one indicator dominates the other in other ways (for binary models see Hanousek, 2000).
Table 1: Pair-wise comparison of the prediction accuracy of two jump indicators.

<table>
<thead>
<tr>
<th>$I_1$</th>
<th>$I_2$</th>
<th>$\sum$</th>
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<tbody>
<tr>
<td>+</td>
<td>$n_{11}$</td>
<td>$n_{12}$</td>
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<tr>
<td>-</td>
<td>$n_{21}$</td>
<td>$n_{22}$</td>
</tr>
<tr>
<td>$\sum$</td>
<td>$n_{\ast1}$</td>
<td>$n_{\ast2}$</td>
</tr>
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</table>

Since the jump process $\{Y_t\}$ could be understood as a binary process $(0 - 1)$, 1 being associated with a jump, studying the prediction accuracy would lead to the following binary outcomes with the probabilities:

\[ p_{11} \equiv \Pr(1|1) = \Pr(\hat{Y} = 1|Y = 1): \text{prob. of correct prediction when } Y = 1; \]
\[ p_{22} \equiv \Pr(0|0) = \Pr(\hat{Y} = 0|Y = 0): \text{prob. of correct prediction when } Y = 0; \]
\[ p_{12} \equiv \Pr(1|0) = \Pr(\hat{Y} = 1|Y = 0): \text{prob. of wrong prediction when } Y = 0; \]
\[ p_{21} \equiv \Pr(0|1) = \Pr(\hat{Y} = 0|Y = 1): \text{prob. of wrong prediction when } Y = 1, \]

where $\hat{Y}$ stands for the estimated price jump.

In diagnostics terminology the above probabilities are usually called **sensitivity** ($p_{11}$), **selectivity** ($p_{22}$), **false positive** ($p_{12}$) and **false negative** ($p_{21}$) probabilities. It is clear that in different situations we might prefer different treatments of misclassification by giving to $\Pr(1|0)$ and $\Pr(0|1)$ different weights. For the sake of simplicity let us consider the case when both misclassifications have equal weights, i.e., we concentrate on the standard case in which the probability of correct prediction is maximized and where

\[ \Pr(\text{correct prediction}) = \Pr(0|0) + \Pr(1|1) = p_{11} + p_{22}. \]

Using a complementary approach, one can minimize the probability of incorrect prediction:

\[ \Pr(\text{incorrect prediction}) = \Pr(0|1) + \Pr(1|0) = p_{21} + p_{12}. \]

The above approach could be used to search for and study the “best” price-jump indicator in terms of the accuracy of prediction. However, the optimization procedures conducted on simulated data (potentially) might not capture the relationship, as well as the difference (in prediction) between the relationships between the studied indicators, and therefore it is better to use this approach for a pair-wise comparison of jump indicators.

In terms of any pair-wise comparison/test we can assume the following. We have the available outcomes of price-jump prediction given by two jump indicators denoted as $I_1$ and $I_2$. The combination of their outcomes in terms of the accuracy of price-jump prediction can be summarized by Table 1.

$n$ is the total number of simulated returns, $n_{11}$ denotes the number of cases when both indicators correctly identify a price jump. $n_{12}$ is the number of cases when the $I_2$ correctly identifies a price jump and $I_1$ does not. $n_{21}$ is the number of cases when the $I_1$ correctly identifies a price jump and $I_2$ does not. Finally, $n_{22}$ denotes the number of cases when both indicators do not correctly identify a price jump. In other words, Table 1 is a contingency table, summarizing the outcomes of the two binary variables $I_1$ and $I_2$ using the accuracy of prediction as the additional classification dimension. Therefore we adopt standard notation for contingency tables and a dot used in a subscript indicates the corresponding marginal distribution, for example, $n_{\ast1}$ stands for the number of price jumps correctly identified by $I_1$.

The statistical inference of whether jump indicator $I_1$ dominates $I_2$ in prediction accuracy can assessed by testing the null hypothesis $H_0: n_{1\ast} = n_{\ast1}$, or equivalently $H_0: n_{12} = n_{21}$. This approach directly leads to the well-known McNemar (1947) test, whose underlying test statistics
is \( \chi_1^2 = \frac{(n_{12} - n_{21})^2}{(n_{12} + n_{21})} \) and is distributed asymptotically as \( \chi_1^2 (n_{12} + n_{21} \geq 8) \). For smaller values of \( n_{12} + n_{21} \) we can construct an exact test using probabilities in multinomial distribution.\(^2\)

In Table 1 we can set various criteria for prediction accuracy, for example the classification used in the table. The example above used the approach where misclassifications \( \Pr(1|0) \) and \( \Pr(0|1) \) have the same weight in selecting price-jump indicators. We compare the correct identification of the price jump only with the incorrect identification in which we combine both types of misclassification. Since in reality those misclassifications have different impacts we test and compare the price-jump indicators using only false negative and false positive classifications. This means that we can treat misclassification only when an indicator predicts a jump but there was no jump (false positive) or when the indicator does not predict a jump, but we observe a jump (false negative). If we minimize the false positive criterion, the winning indicator would identify fewer returns as false price jumps and would potentially miss some true price jumps. A similar logic is valid for the false negative criterion.

The testing procedure in the simulation framework is applied as follows:

- In the first step, we simulate 100 trading days and compare the indicators pair-wise.\(^3\) As the prediction criteria, we use:
  - the number of correctly identified price jumps and
  - the number of falsely identified price jumps.

We conduct the McNemar-type test described above and we count it if indicator \( I_1 \) dominates \( I_2 \) (90%, 95% and 99% significance levels).

- In the second step, we repeat each simulation 100 times. The results from the test procedure (the first step) are used as the input for the second step. The second test is again the McNemar-type test, where we compare the number of cases when one indicator dominates the other at a given confidence level. For both tests, we use three confidence levels—90%, 95% and 99%.

To summarize, first we use the test for a given (simulated) window of trading days to analyze if one jump indicator dominates the other in terms of the accuracy of the prediction of the price jump. The second step analyzes the results of repeated simulation using the same time window.

4. Data Generation of Artificial Time Series

The goal of this part is to compare the price-jump indicators to find the one that performs best. For that purpose, we perform an extensive simulation study with simulated data. We simulate the price of a virtual asset during a trading day: every trading day lasts seven trading hours or 420 trading minutes. The price time series is simulated at a 1-minute frequency as a discrete process generally defined using the Euler scheme: \( p_t - p_{t-1} = F_t \), where \( p_t \) is a log-price, or \( p_t \equiv \log(F_t) \) with \( F_t \) being a price of the asset, and \( F_t \) is the time-dependent log-price generator. Generally, the drift is insignificant for high-frequency data.

\(^2\)It is also recommended to conduct an exact test if 20% of \( n \cdot n_{i1} \cdot n_{j0} \) is less than 5, or if any of \( n \cdot n_{i1} \cdot n_{j0} \) is smaller than one (see for example Gibbons, 1985).

\(^3\)We actually simulate 105 trading days and then cut off the first five trading days.
4.1. Normal Price Movements

The most intuitive log-price generation process uses an iid normal distribution

\[ p_t - p_{t-1} = \sigma Z_t, \]

where \( Z_t = Z \sim N(0, 1) \) and \( \sigma \) is a constant. This is the first intraday volatility pattern we employ.

4.2. Intraday Volatility Patterns

The flat intraday volatility pattern is, however, not close to real data. Therefore, we mimic the well-known U-shaped volatility pattern, which says that log-return time series show a significant increase in volatility at the beginning and end of the trading day. We implement three different intraday volatility patterns. The purpose is to test the behavior of the indicators under these intraday volatility patterns as well as to compare them over the broadest possible range of situations. The four different specifications for intraday volatility patterns further serve as a testing ground for a proper understanding of price-jump indicators.

4.2.1. Step function I

The second intraday volatility pattern is based on the assumption that volatility undergoes a two-regime switching process, where one regime is at the beginning and end of the trading day, while the other regime is at the middle of the trading day. Namely, we assume a log-price generating process given as

\[ p_t - p_{t-1} = \sigma Z_t, \]

where \( p_t \) being the log-price, and the volatility \( \sigma_t \) governs the two-regime process and is defined as

\[
\sigma_t = \begin{cases} 
\sigma_{high} & t \in [0, 1/4 \cdot \text{Day}) \\
\sigma_{low} & t \in [1/4 \cdot \text{Day}, 3/4 \cdot \text{Day}) \\
\sigma_{high} & t \in [3/4 \cdot \text{Day}, \text{Day}] 
\end{cases}
\]

where \( \sigma_{low} < \sigma_{high} \) and \( \alpha \) and \( \beta \) are parameters governing periods with different volatility regimes. Compared to the previous case, there is an artificial "jump" in volatility at the moment when the volatility changes from \( \sigma_{low} \) to \( \sigma_{high} \).

4.2.2. Step function II

The third intraday volatility pattern is an extension of the previous one. We employ a four-level volatility regime to mimic the U-shaped volatility smile in a more subtle way. Such a definition also partially gets rid of the artificial jumps at the corners where volatility regimes change. Namely, we assume the log-price generating process is given as

\[ p_t - p_{t-1} = \sigma Z_t, \]

with

\[
\sigma_t = \begin{cases} 
3\sigma_{high} & t \in [0, \alpha \cdot \text{Day}) \\
2\sigma_{high} & t \in [\alpha \cdot \text{Day}, \beta \cdot \text{Day}) \\
1\sigma_{high} & t \in [\beta \cdot \text{Day}, \gamma \cdot \text{Day}) \\
\sigma_{low} & t \in [\gamma \cdot \text{Day}, \delta \cdot \text{Day}) \\
1\sigma_{high} & t \in [\delta \cdot \text{Day}, \varepsilon \cdot \text{Day}) \\
2\sigma_{high} & t \in [\varepsilon \cdot \text{Day}, \phi \cdot \text{Day}) \\
3\sigma_{high} & t \in [\phi \cdot \text{Day}, \text{Day}] 
\end{cases}
\]
Note: The figures depict the spread of returns over an artificial trading day for four different specifications.

where $\sigma_{\text{low}} < \sigma_{\text{high}}$ and parameters $\alpha$, $\beta$, $\gamma$, $\delta$, $\varepsilon$ and $\phi$ define periods with different volatility regimes. In this case, the volatility pattern is smoother and mimics the empirical patterns better.

4.2.3. Linear-like smooth smile

The fourth volatility pattern mimics the U-shaped volatility smile more closely. We use three linear functions, which ensure a smooth transition in volatility between different parts of the trading day. Namely, we assume the log-price generating process is given as

$$p_t - p_{t-1} = \sigma_t Z_t,$$

with

$$\sigma_t = \begin{cases} 
3\sigma_{\text{high}} - \frac{(3\sigma_{\text{high}} - \sigma_{\text{low}})}{\alpha \cdot \text{Day}} (t) & t \in [0, \alpha \cdot \text{Day}) \\
\sigma_{\text{low}} & t \in [\alpha \cdot \text{Day}, \beta \cdot \text{Day}) \\
\sigma_{\text{low}} + \frac{(3\sigma_{\text{high}} - \sigma_{\text{low}})}{(1-\beta) \cdot \text{Day}} (t - 285) & t \in [\beta \cdot \text{Day}, \text{Day}] 
\end{cases}$$

and $\sigma_{\text{low}} < \sigma_{\text{high}}$. The parameters define periods with different volatility. $3\sigma_{\text{high}}$ was chosen to be able to compare this pattern with the previous one.

4.3. Volatility Specifications

We employ the four different intraday volatility patterns defined above with the parameters as follows. Their meaning is further depicted in Figure 1, where we show the spread of returns over an artificial trading day for four different specifications.
Volatility Pattern A: The first type of intraday volatility pattern consists of a basic homogeneous iid normal process, namely

\[ p_t - p_{t-1} = \sigma Z_t, \]

where we use \( \sigma = 0.0004 \), which corresponds to the values observed in the real data (used in the literature and based on the annual realized volatility).

Volatility Pattern B: The second intraday volatility pattern is given as

\[ p_t - p_{t-1} = \sigma Z_t, \]

with

\[ \sigma_t = \begin{cases} 
\sigma_{\text{high}} & t \in [0, \text{Day}/4) \\
\sigma_{\text{low}} & t \in [\text{Day}/4, 3\text{Day}/4) \\
\sigma_{\text{high}} & t \in [3\text{Day}/4, \text{Day}] 
\end{cases}, \]

using the values \( \sigma_{\text{low}} = 0.0001 \) and \( \sigma_{\text{high}} = 0.0004 \).

Volatility Pattern C: The third intraday volatility pattern is defined as

\[ p_t - p_{t-1} = \sigma Z_t, \]

with volatility defined as

\[ \sigma_t = \begin{cases} 
3\sigma_{\text{high}} & t \in [0, 45\text{min}) \\
2\sigma_{\text{high}} & t \in [45\text{min}, 90\text{min}) \\
1\sigma_{\text{high}} & t \in [90\text{min}, 135\text{min}) \\
\sigma_{\text{low}} & t \in [135\text{min}, 285\text{min}) \\
1\sigma_{\text{high}} & t \in [285\text{min}, 330\text{min}) \\
2\sigma_{\text{high}} & t \in [330\text{min}, 375\text{min}) \\
3\sigma_{\text{high}} & t \in [375\text{min}, 420\text{min}] 
\end{cases}. \]

The 45-minute step corresponds approximately to \( \text{Day}/9 \), thus the trading day has three periods of approximately the same duration: the first at the beginning of the day with decreasing volatility, the second at the middle of the day with increasing volatility and the third at the end of the day with increasing volatility. We use \( \sigma_{\text{low}} = 0.0001 \) and \( \sigma_{\text{high}} = 0.0002 \).

Volatility Pattern D: This pattern prevents a possible critique that could emerge in the previous cases: whenever we change the volatility regime, we introduce an artificial jump of average size \( \sigma_{\text{high}} \). This can have a negative effect on the performance of some indicators; therefore we make the transition smoother. Thus, volatility is defined as

\[ \sigma_t = \begin{cases} 
3\sigma_{\text{high}} - \frac{(3\sigma_{\text{high}} - \sigma_{\text{low}})}{135\text{min}} (t) & t \in [0, 135\text{min}) \\
\sigma_{\text{low}} & t \in [135\text{min}, 285\text{min}) \\
\sigma_{\text{low}} + \frac{(3\sigma_{\text{high}} - \sigma_{\text{low}})}{135\text{min}} (t - 285) & t \in [285\text{min}, 420\text{min}] 
\end{cases}, \]

where \( \sigma_{\text{low}} < \sigma_{\text{high}} \). We use \( \sigma_{\text{low}} = 0.0001 \) and \( \sigma_{\text{high}} = 0.0002 \).
4.4. Price-Jump Specification

This study focuses on price jumps, so we extend the price movements defined above with non-normal price jumps. The Euler scheme for price evolution with price jumps is defined as $p_t - p_{t-1} = f_t = \sigma_t Z_t + J \cdot j_t$, where $\sigma_t Z_t$ is the term defined above and $J \cdot j_t$ is the term generating price jumps. We conveniently define $j_t$ as a Poisson process with a rate of price-jump arrival $\lambda_j$:

$$j_t = \begin{cases} 0 & p_j, \\ 1 & 1 - p_j, \end{cases}$$

where $p_j = e^{-\lambda_j}$ and parameter $J$ govern the size of the jumps. For single-size price jumps, $J = \pm J_{\text{param}}$, where both signs have the same probability of occurring. In the most sophisticated cases, parameter $J$ can have a value from any statistical distribution.

Due to the independence of increments, the probability to observe $n$ jumps at a time step is given as

$$\Pr(\text{N of Jumps} = n) = \frac{e^{-\lambda} (\lambda)^n}{n!}.$$ 

By definition, we assume that only one price jump per time step can occur and thus we define first the probability that no price jump will occur as $\Pr(\text{N of Jumps} = 0) = e^{-\lambda}$ and the probability that one price jump will occur as a complement value $\Pr(\text{N of Jumps} = 1) = 1 - e^{-\lambda}$.

We employ five different specifications of price jumps. These five specifications are combined with the four different groups of indicators. Thus we will have twenty different price time series (excluding four different time series without price jumps). 4

**Price Jumps 1–3**: The first three price-jump specifications have the same rate of arrival and a constant size of jump $J = \pm \text{const}$. Both signs occur with the same probability. We employ combinations:

- Price Jump 1: $J = 5\sigma_{\text{jump}}$ and $\lambda = 5/N_{\text{Day}}$.
- Price Jump 2: $J = 7\sigma_{\text{jump}}$ and $\lambda = 5/N_{\text{Day}}$.
- Price Jump 3: $J = 9\sigma_{\text{jump}}$ and $\lambda = 5/N_{\text{Day}}$.

The parameter $\sigma_{\text{jump}} = 0.0004$ and $N_{\text{Day}}$ means the number of minutes per trading day.

**Price Jumps 4–5**: The next two price-jump specifications use a uniform distribution to select the size of price jumps. We select price jumps from a given distribution $J \propto \pm U(a, b)$, with $0 < a < b$, and both signs occur with the same probability. We use the following specifications:

- Price jump 4: $J \propto \pm U(5\sigma_{\text{jump}}, 9\sigma_{\text{jump}})$ and $\lambda_j = 5/N_{\text{Day}}$.
- Price jump 4: $J \propto \pm U(5\sigma_{\text{jump}}, 9\sigma_{\text{jump}})$ and $\lambda_j = 15/N_{\text{Day}}$.

The parameter for volatility is chosen as $\sigma_{\text{jump}} = 0.0004$ and $N_{\text{Day}}$ was defined above.

5. Comparison Strategy

The goal of the simulation procedure is to compare price-jump indicators with each other, understand their properties and select the most appropriate indicator for real data.

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4 An alternative approach to estimate price jumps is to assume that the error term follows a given stochastic distribution and combine it with a non-homogeneous Poisson term describing a jump. Then, Score Method of Moments or Simulated Method of Moments could be employed to estimate the parameters of the model. See for example Jiang and Oomen, 2007, who estimate an affine jump diffusion model for a series of returns from the S&P 500 index. However, this approach relies on the proper specification of the underlying model, including the intraday volatility pattern as well as the distribution. Thus, this approach is not appropriate for our analysis.
5.1. Price Jump Indicators

We employ the following extensive list of price-jump indicators that are defined in the Appendix A:

1. Centiles as defined in A1.1: The price jump is identified as those returns below the 0.5th centile or above the 99.5th centile. Centiles are calculated for the entire sample.
2. Block-centiles as defined in A1.2: The price jump is identified as those returns below the 0.5th centile or above the 99.5th centile. Every trading day is divided into 15-minute blocks and centiles are calculated for every block separately for the entire sample.
3. \( Z_{RJTP} \), as defined in A2.1 with a 99% confidence interval (CI) and length of moving window \( n = 60 \).
4. \( Z_{RJTP} \), as defined in A2.1 with a 99% CI and \( n = 120 \).
5. Improved \( Z_{RJTP} \), as defined in A2.2 with a 99% CI and \( n = 60 \).
6. Improved \( Z_{RJTP} \), as defined in A2.2 with a 99% CI and \( n = 120 \).
7. \( \xi \)-statistics as defined in A2.3 with a 99% CI and \( n = 60 \).
8. \( \xi \)-statistics as defined in A2.3 with a 99% CI and \( n = 120 \).
9. \( JO_{Ratio} \) as defined in A3.1 with a 99% CI and \( n = 60 \).
10. \( JO_{Ratio} \) as defined in A3.1 with a 99% CI and \( n = 120 \).
11. Improved \( JO_{Ratio} \) as defined in A3.2 with a 99% CI and \( n = 60 \).
12. Improved \( JO_{Ratio} \) as defined in A3.2 with a 99% CI and \( n = 120 \).
13. The price jump index as defined in A4.1: The price jump is identified as those returns with \( pji > 4 \) and \( n = 120 \).
14. The price jump index as defined in A4.1: The price jump is identified as those returns with \( pji > 4 \) and \( n = 420 \).

5.2. Artificial Time Series

We employ a Monte Carlo simulation technique to simulate an artificial time series with price jumps. We simulate all the combinations of four different intraday volatility patterns (specified in section 4.3) and five different price-jump specifications (specified in section 4.4), thus there are 20 different time series in total. Every trading day is sampled at a one-minute frequency, starting at 9:01 and ending at 16:00; seven hours in total, which gives 420 trading minutes per trading day. We further match the end of the trading day with the beginning of the next trading day and thus produce continuous time series.

We simulate 105 trading days and define price-jump indicators. Then, we cut off the first five days, which serve to settle down the simulation as well as produce the necessary observations for the moving windows. In addition, the Jiang-Oomen statistics-based indicators require absolute levels. For that purpose, we set an initial value \( p_0 = 100 \) and produce price levels instead of returns.

5.3. Relative Comparison of Price Jump Indicators

In the last step we perform an extensive comparison of the performance of the different price-jump indicators. We follow the methodology outlined in section 3 based on the McNemar (McNemar, 1947) test.

6. Comparison Strategy – Results

We compared 14 different price-jump indicators with respect to false positive and false negative probabilities. We do not present all the details of the comparisons here; all tables are available in Hanousek et al. (2011) in Appendix B, or upon a request. In Table 2 we present a summary of our results: the number of cases when a given price-jump indicator dominates
Table 2: Summary of the analysis based on false positive and false negative probabilities – number of dominances.

<table>
<thead>
<tr>
<th>Indicator</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<td>0</td>
<td>0</td>
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<tr>
<td>False negative</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>15</td>
<td>0</td>
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</table>

the other indicators with respect to both false positive and false negative probabilities. Several times there were two indicators that were dominating the other indicators. In such a case, we counted both indicators as dominating the given simulated specification.

In the case of the false positive probability—the false identification of non-jump cases—the best indicator seems to be the indicator No. 1 based on centiles. This indicator dominated others the most often. In addition, there are other indicators, which performs in some specifications well, namely No. 2—the one based on block-centiles—and Nos. 7 and 8 based on the $\xi$-statistics with 99% CI and $n = 60$ or $n = 120$, respectively.

The other case, false negative probability—jumps that occur are not identified—shows that the best performing statistics is the indicator No. 8, the $\xi$-statistics with 99% CI and $n = 120$. In addition, the analysis shows that even the version with time window $n = 60$ performs well since these two statistics are in many cases statistically indistinguishable.

The analysis further reveals that the performance of price-jump indicators is not homogeneously distributed among all the indicators but rather their performance is dominated by a few best indicators. This can be further seen in the results, where most of the time when one indicator dominates another, it dominates it at the highest significance level.

7. Economic Implications: Stock Market Indices

In the following section, we relate the findings of the Monte Carlo study to the real stock market data. Namely, we employ 5-minute prices of nine stock market indices from countries all over the world including both mature as well as emerging markets. We employ all price jump indicators and study the results the indicators provide using the clustering analysis.

7.1. Data

We employ data of nine stock market indices at 5-minute frequency spanning the period from January 2007 to December 2010.\(^5\) We use following stock market indices from following countries: Japan (NKY index), Germany (DAX index), France (CAC index), the United Kingdom (UKX), the Czech Republic (PX index), Poland (WIG index), Hungary (BUX index) and USA (SPX—the S&P 500 index, and INDU—the Dow Jones Industrial Average index). Data comes from Datastream and Bloomberg.

The presented set is formed by a wide range of different stock market indices including both mature and emerging markets, markets with different market micro-structure and levels of regulations, and even stock exchanges with and without lunch time break. Therefore, the data set represents a variety of different time series, which allows us to provide a robust answer.

The data in our study are processed as follows: For every stock market index, we take a last price available for every fifth minute. We consider prices from the regular trading hours only.\(^6\) Then, we construct for every time step time series of log-returns given as $r_t = \log (P_t/P_{t-1})$.

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\(^5\)We take also data for December 2006, as will be explained further.

\(^6\)We mention this since many stock exchanges have pre-opening trading period as well as trading after the closing hours, or the final auction. We do not consider such activity in our analysis.
At the first time step, log-returns are not defined as they are not available whenever there is a break in the trading period. This discards all discontinuities due to the non-trading activities. Then, we make a continuous time series of log-returns by connecting consecutive trading days.

7.2. Price Jump Indicators

We employ fourteen price jump indicators as they are described above to estimate number of price jumps per every calendar month in the sample. There are two further clarifications at place: First, for centile-based price jump indicators—price jump indicators No. 1 and 2—we need a certain period to estimate the centiles. We take calendar quarter as a period over which we estimate threshold centiles. Therefore, every three months share the same level of centiles which we use to construct the thresholds for price jumps.

Second, the remaining price jump indicators require a certain history to get an appropriate price jump statistics. This poses a question how to consider the initial moments of every month. For that purpose, when we estimate the price jump statistics at the beginning of every month, we take a certain part of the previous month to be able to identify price jump even at the very first moments of the month. This also require to start our sample even before January 2007 and thus we have to employ data from December 2006, as was mentioned above.

Provided price jump indicators allow us to obtain 14 different estimates of the number of the price jumps per month. The simulation analysis suggests that these numbers will differ across the price jump indicators. We therefore analyze how the estimated numbers based on the real time series differ from each other using the clustering analysis.

7.3. Clustering analysis

Clustering analysis allows us to split the price jump indicators into clusters based on the similarity of the estimated number of price jumps per month they provide. There are various methods how form the clusters, see for example survey of Gordon (1987) or work of Milligan and Cooper (1985). We have implemented the algorithm based on the average link clustering as it coded in Stata. The average-link-based clustering method determines the closest groups by the average proximity of observations in the groups. Therefore, the clusters indeed provide the groups of price jump indicators, which for gives for every month the most similar estimates of the number of price jumps.

The total number of clusters is an exogenous parameter, which can be further determined using either the algorithmic procedure or the expected behavior of the sample. In our study, we have studied several number of clusters and examined the pseudo F-index of such clustering. After the analysis to cluster into 2-14 groups, we have decided for 5 clusters. Such a clustering was robust in the sense that cluster No. 1 and 2, see below, were stable and formed of the same price jump indicators and provided a feasible F-index. In the following, we provide results of the clustering analysis.

8. Clustering of the Price Jump Indicators: Stock Market Indices

In this section, we provide results of the clustering analysis for the stock market indices. For this analysis, we have effectively mapped the entire data-set onto a data-set, which contains for every stock market index and price jump indicator time series of estimated number of price

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7Namely, there are big price changes over-night, which emerged because of the trading activity in different markets. For example, the price at the European stock markets changes over-night due to the trading in the North America and Asia. In this study, we are interested in the price jumps coming from the trading activity only.

8In particular, it is the sub-command average linkage.

9See Caliński and Harabasz (1974) for a definition of the pseudo F-index.
Table 3: Clustering of the price jump indicators.

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<tr>
<th>No.</th>
<th>NKY</th>
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</table>

Note: We have used average-link clustering algorithm and formed 5 clusters. Clusters are sorted according to the mean of the number of price jumps per month they provide. Bold entries are price jump indicators optimal with respect to the False Positive, or Type I error (No. 1), and False Negative, or Type II error (No. 8) probability, respectively. GC stands for the global cluster common for all the stock market indices.

jumps per month. We have further sorted clusters according to the mean of number of price jumps they provide. Namely, cluster No. 1 gives on average less number of price jumps per month than clusters No. 2 to 5 etc.

8.1. Global clustering

First, we use the clustering algorithm to form clusters of price jump indicators for every stock market index and the entire time series. Table 3 provides an overview of the clustering analysis by stock market indices. We perform the analysis by stock market indices as well as one clustering for all stock market indices together. Therefore, we can distinguish composition of clusters on the level of individual indices and its difference to the global clustering.

The five estimated clusters can be characterized as follows (number of brackets denotes number of price jump indicators in a particular cluster):

*Cluster 1 (8): Type II-optimal like indicators*. All indicators expect those below. Nos. 7, 8, 9, 10, 11, 12, 13 and 14.

*Cluster 2 (2): Type I-optimal like indicators – centiles based*. Price jump indicators based on the extreme returns thresholded using the centiles (Nos. 1 and 2).

*Cluster 3 (2): Higher-moments indicators*. This cluster contains price jump indicators based on the $Z_{R,TP}$ statistics with both time windows (Nos. 3 and 4).


First, the analysis clearly reveals an existence of a big cluster No. 1, which comprises of more than a half of the price jump indicators used in the study. These clusters seems to behave
Table 4: Overlap of price jumps in cluster No. 1.

<table>
<thead>
<tr>
<th>No.</th>
<th>NKY</th>
<th>CAC</th>
<th>DAX</th>
<th>UKX</th>
<th>BUX</th>
<th>PX</th>
<th>WIG</th>
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<th>SPX</th>
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<td>8</td>
<td>298</td>
<td>469</td>
<td>549</td>
<td>380</td>
<td>516</td>
<td>999</td>
<td>674</td>
<td>401</td>
<td>419</td>
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<td>257/381</td>
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<td>767/500</td>
<td>577/328</td>
<td>933/481</td>
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<td>1,038/615</td>
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<td>865/320</td>
<td>578/173</td>
<td>562/159</td>
</tr>
</tbody>
</table>

Note: Table contains number of price jumps identified by the price jump indicator No. 8 and then number of price jumps identified by other price jump indicators from the same cluster followed by the number of overlapping price jumps, i.e., those identified jointly by the price jump indicator and the optimal one—No. 8.

in the same way as the one optimal with respect to False Negative probability. This first finding of this analysis suggests that there is a stable sub-set of price jump indicators, which seemingly behave as optimal. They provide estimated number of price jumps close to the number of given by the optimal price jump indicator.

We further focus our attention on the cluster No. 1 and investigate how much the identified price jumps overlap each other using different price jump indicators from this particular cluster. Table 4 contains the statistics of the overlapped price jump indicators for cluster No. 1.\(^\text{10}\)

We present number of identified price jumps by the optimal indicator—No. 8—and then number of identified price jumps by other price jump indicators followed by the number of price jump jointly identified together with the optimal price jump indicator. The results clearly suggest that overlap between the price jump indicators in the same cluster is not so big, in very few cases it exceeds 25%. Therefore, the price jump indicators in the same cluster tend to provide the false accuracy.

In addition, the results clearly suggests that PX index (the Prague Stock Exchange) deviates from the sample. This suggests a different price generating process, which is in agreement with recent findings of Hanousek and Novotný (2012), where the author shows the presence of the “PX Puzzle” a reverse scaling behavior of this stock index with respect to the other regional—both from the emerging and mature markets—stock market indices.

8.2. Stability of Clusters over Time

In the next step, we answer the question whether the clusters are composed from the same price jump indicators over time. For that purpose, we perform the clustering analysis by years. Table 5 contains estimation of cluster by years. The result suggests a stability of the clustering over years. In particular, we focus our attention on two years, which were economically the most distinct: year 2007 and 2009. Year 2007 can be, having in hand our data sample, considered as period of financial stability. Year 2009, on the other hand, is the year when financial and economic crisis emerged in its full strength globally.

We perform a Stuart-Maxwell test statistic for symmetry test of the contingency table 6, which describes the number of cases when price jump indicators remain in the same cluster—the diagonal—and number of cases when price jump indicators changed cluster during the financial crisis—off-diagonal terms. The Stuart-Maxwell test is in this case asymptotically equal to \(\chi^2\) and, in particular, equal to 4.41, giving to rise \(p\)-value of 0.22, i.e., we cannot reject the null

\(^{10}\)Numbers for other clusters are available upon request.
Table 5: Clusters (5) | separation of price jump indicators using averages and pseudo F index. By years.

<table>
<thead>
<tr>
<th>No.</th>
<th>NKY</th>
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<td>1111</td>
<td>1111</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: GC stands for the global cluster common for all the stock market indices.

Table 6: Contingency table for clustering over years.

<table>
<thead>
<tr>
<th></th>
<th>07/09</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<th>5</th>
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<tr>
<td>1</td>
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<td>0</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Note: Entries in the table denotes number of cases when price jump indicators remain in the same cluster—the diagonal—and number of cases when price jump indicators changed cluster during the financial crisis—off-diagonal terms.

hypothesis that the table is symmetric. We therefore see that size of the clusters remained the same and despite the mild migration among them the clusters remain stable.

8.3. Stability of Clusters with Respect to the Stock Market Indices

In the previous parts, we have mentioned stability of the clustering analysis over different stock market indices—except the PX index and its possible relation to the PX Puzzle. To confirm such a stability, Table 7 is a contingency table capturing number of cases, when clustering on an individual level deviates from the global clustering. Further, we report Stuart-Maxwell test statistic for symmetry test of this contingency table, which is again asymptotically equal to $\chi^2$ and have a value 4.09, giving rise to $p$-value of 0.39, i.e., we cannot reject the null hypothesis that table is symmetric. Table thus suggests that global clustering is in agreement with individual clustering and cannot be claimed as an average of the individual level.

8.4. Price Jumps in Time

Next, we analyze the evolution of the estimated number of price jumps per month over the time horizon for every cluster and every stock market index. To accomplish that, Table 8 contains augmented Dickey-Fuller test statistics for monthly estimated numbers of price for every cluster and every stock market index. The null hypothesis states that the time series with numbers of price jumps per month contains a unit root and the alternative states that time series is stationary. The McKinnon $p$-value is in all the cases $p < 0.001$, which means that
Table 7: Stability of clustering with respect to individual stock market indices.

<table>
<thead>
<tr>
<th>Global/Individual</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>66</td>
<td>5</td>
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<td>0</td>
<td>0</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

Note: Entries in the table denote number of cases when price jump indicators remain in the same cluster for both the individual and global level—the diagonal—and number of cases when the clusters changed—off-diagonal terms.

Table 8: Augmented Dickey-Fuller: estimated number of price jumps per month.

<table>
<thead>
<tr>
<th></th>
<th>NKY</th>
<th>CAC</th>
<th>DAX</th>
<th>UKX</th>
<th>BUX</th>
<th>PX</th>
<th>WIG</th>
<th>INDU</th>
<th>SPX</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 4</td>
<td>-4.286</td>
<td>-6.607</td>
<td>-4.886</td>
<td>-5.129</td>
<td>-6.463</td>
<td>-6.323</td>
<td>-7.618</td>
<td>-5.822</td>
<td>-6.091</td>
</tr>
<tr>
<td>No. 5</td>
<td>-5.432</td>
<td>-6.610</td>
<td>-5.446</td>
<td>-5.425</td>
<td>-6.189</td>
<td>-6.475</td>
<td>-6.983</td>
<td>-5.831</td>
<td>-6.466</td>
</tr>
</tbody>
</table>

Note: Table contains augmented Dickey-Fuller test statistics for monthly estimated numbers of price for every cluster and every stock market index. The null hypothesis states that the time series with numbers of price jumps per month contains a unit root and the alternative states that time series is stationary. McKinnon $p$-values are in all the cases $< 0.001$.

at very high significance level, we can reject the null hypothesis and conclude that time series are stationary.

The results suggests that number of price jumps per month for every cluster and every stock market index is stable at very high significance level. This means that even the financial crisis did not cause the increase in the rate of price jumps and this result is robust since it was confirmed by all price jump indicators.\(^{11}\)

Still, there is a question, whether the difference between clusters remains stable over time. For that purpose, we repeat the augmented Dickey-Fuller test but with difference between estimated number of price jumps using different clusters. Table 9 contains augmented Dickey-Fuller test statistics for differences in monthly estimated numbers of price between clusters for every stock market index. First column on the left denotes the clusters for which we do a difference. The null hypothesis states that the time series with differences in the number of price jumps per month contains a unit root and the alternative states that time series is stationary. Since $p$-value is for all the entries $p < 0.001$, we can reject the null hypothesis and accept the alternative that time series are stationary. This means that the difference between the estimated number of price jumps using different clusters is stable over time. The consequence of this finding is such that any stationarity related conclusions using different price jump indicators are consistent no matter what price jump indicator is one using and thus allows us to perform meta-analysis over the existing literature.

For the sake of clarity, Figure 2 contains the estimated number of price jumps for every stock market index and every cluster by months. The figures complete the previous analysis.

\(^{11}\)It is also in agreement with Hanousek and Novotný (2012); Novotný (2010).
Table 9: Augmented Dickey-Fuller: differences in estimated number of price jumps per month.

<table>
<thead>
<tr>
<th></th>
<th>NKY</th>
<th>CAC</th>
<th>DAX</th>
<th>UKX</th>
<th>BUX</th>
<th>PX</th>
<th>WIG</th>
<th>INDU</th>
<th>SPX</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 1/4</td>
<td>-4.276</td>
<td>-7.064</td>
<td>-5.928</td>
<td>-5.320</td>
<td>-6.408</td>
<td>-6.444</td>
<td>-6.860</td>
<td>-6.116</td>
<td>-6.423</td>
</tr>
<tr>
<td>No. 1/5</td>
<td>-5.378</td>
<td>-6.677</td>
<td>-5.668</td>
<td>-5.374</td>
<td>-6.629</td>
<td>-6.461</td>
<td>-6.803</td>
<td>-5.865</td>
<td>-6.602</td>
</tr>
<tr>
<td>No. 4/5</td>
<td>-6.797</td>
<td>-6.630</td>
<td>-5.790</td>
<td>-6.951</td>
<td>-7.861</td>
<td>-7.038</td>
<td>-6.650</td>
<td>-5.998</td>
<td>-6.910</td>
</tr>
</tbody>
</table>

Note: Table contains augmented Dickey-Fuller test statistics for differences in monthly estimated numbers of price between clusters for every stock market index. First column on the left denotes the clusters for which we do a difference. The null hypothesis states that the time series with differences in the number of price jumps per month contains a unit root and the alternative states that time series is stationary. McKinnon p-values are in all the cases < 0.001.

Figure 2: Cluster analysis 5c over months.

Note: The figures depict the number of price jumps per month using different clusters. The notation is as follows: solid line for cluster No. 1, dash line for cluster No. 2 , dash line for cluster No. 3, dash-dot line for cluster No. 4, and short-dash line for cluster No. 5.
9. Conclusion

We performed an extensive simulation study to compare the relative performance of a broad class of price-jump indicators with respect to false positive and false negative probabilities. We simulated twenty different time-series specifications with different intraday noise volatility patterns and price-jump specifications and using these artificial time series, we employed the double McNemar test and compared fourteen different price-jump indicators that are widely used in the literature. We compared them with respect to false positive and false negative probabilities. The results suggest large differences among the indicators in terms of their performance. In the case of false positive probability, the best-performing price-jump indicator is the one based on thresholding with respect to centiles. In the case of false negative probability, the best indicator is the one based on bipower variation. Significant differences among the indicators further confirms the fact that any meta-analysis based on different price-jump indicators is not possible since the indicators tend to perform in very different ways. The price jump indicators applied on nine stock market indices from all over the world including both mature and emerging markets still supports the difference between the price jump indicators. The price jump indicators tend to group into clusters according to the number of price jumps per month they estimate. However, the actual overlap between the estimated price jumps is very low. In addition, the estimated numbers of price jumps do not change over time, which suggests that the recent financial crisis did not change the overall jumpiness of the markets.

10. Acknowledgment

Jan Hanousek gratefully acknowledges financial support from the GAČR grant (403/11/0020). Evžen Kočenda and Jan Novotný acknowledge financial support of the GACR grant P40212G097. Jan Novotný is recently a Marie Curie Postdoctoral Fellows at Manchester Business School. The research leading to these results has received funding from the European Community’s Seventh Framework Programme FP7-PEOPLE-ITN-2008 under grant agreement number PITN-GA-2009-237984 (project name: RISK). The usual disclaimer applies.


Appendix A: Price-Jump Indicators – The Details

In this section, we provide technical details for all of the price-jump indicators we tested.

A1 Group 1: Ait-Sahalia

This class of indicator assumes that the underlying price increment process is given as \( \Delta S = \sigma \Delta X + \Delta J \), where the log-price increment means \( \Delta S = S_t - S_{t-1} \), where we assume that we observe the realization of prices in equidistant time steps \( \Delta t \), i.e., \( \Delta S \) denotes a price change over the time interval \( \Delta t \). In this definition, \( X \) corresponds to the Brownian motion and \( J \) is a \( \beta \)-stable process. The increments of the two components can be expressed as \( \Delta X = (\Delta t)^{1/2} X_1 \) and \( \Delta J = (\Delta t)^{1/\beta} J_1 \), where the equalities are equalities in distribution. In this specification, \( X \) corresponds to the Brownian motion and \( J \) is a \( \beta \)-stable process.

The different magnitudes in the two components can be used to discriminate between the noise components and the big price jumps coming solely from the \( J \)-process.\(^{12}\) The big price jumps cause \( \Delta S = \Delta J \) (in distribution), while in the presence of no big price jumps, which is most of the time, \( \Delta S = \sigma (\Delta t)^{1/2} X_1 \). Therefore, we can, for a given \( \Delta t \), choose a threshold value equal to \( \alpha (\Delta t)^\gamma \), with \( \alpha > 0 \) and \( \gamma \in (0, 1/2) \), such that if \( \Delta S > \alpha (\Delta t)^\gamma \) then \( \Delta S \) is at a given moment dominated by \( J \) with a certain probability.

This argument can be inverted. Assuming that we know the rate of the arrival of big jumps, we can easily construct a threshold based on the centile value. Therefore we will use centiles as a threshold to discriminate price jumps from the noise. Using centiles, however, can produce biased results due to the intraday volatility patterns. The intraday volatility pattern means that \( \sigma = \sigma (t) \). In addition, the \( J \)-process can also be different either across different phases of the trading day or across different trading days. To account for the former, we divide every trading day into several trading blocks and assume that inside every trading block the price process is constant no matter the trading day. In this case, we can apply the same logic block by block. Namely, we calculate the centiles for the same block over different trading days and threshold price jumps for every trading block separately.

A1.1 Global Centiles

We define price jumps as those returns that are higher/lower than a given upper/lower centile. Centiles are calculated based on the observation of the entire sample, where we use the 99.5-th centile as the upper threshold and the 0.5-th centile as the lower threshold.

A1.2 Centiles over Block-Windows

To compensate for intraday volatility, we divide every trading day into several 15 minute-long blocks. Then, we apply the procedure defined above for every trading block separately, i.e., we calculate the upper/lower threshold for every trading block independently and then define the price jumps as those price movements that are higher/lower than the corresponding threshold values.

A2 Group 2: Bipower Variation

The two different measures for variation, as defined by Barndorff-Nielsen and Shephard (2004, 2006) are Realized Variation defined as \( RV_t = \sum_{i=t-n+3}^{t} \sum_{j=t-n+2}^{i} r_{ij}^2 \) and Bipower Variation defined as \( BV_t = \mu_2^{-2} (n-1) \sum_{i=t}^{n} \sum_{j=t-n+3}^{i} |r_{ij}|^{2\alpha} \), with \( \mu_2 = E ([|Z|^{n}) \) for \( Z \sim N (0, 1) \), or generally \( \mu_\alpha = 2^{\alpha/2} \Gamma (\frac{\alpha+1}{2}) / \sqrt{\pi} \). In the following, \( r_{ij} = p_{ij} - p_{i-1} \) and \( p_{i} = \log (P_{i}) \), with \( P_{i} \) being the price of the asset (denominated in currency units).

\(^{12}\)The \( J \)-process contributes to a large amount of small price jumps; however, we want to focus on big price jumps only. The goal is not to completely determine the properties of the \( J \)-process but rather to determine how to discriminate extreme price movements.
A2.1 The Max-adjusted Statistics

The difference between the two variations is the key ingredients; however, one also needs to estimate the conditional standard deviation $\int \sigma^2$. There are at least two possible ways to estimate this: Andersen et al. (2007) introduced tripower quarticity:

$$TP_j = n\mu^{-3}_{n} \left( \frac{n-1}{n-3} \right) \sum_{i=j-n+4}^{j} |r_i| \left| |r_{i-1}| \right| \left| |r_{i-2}| \right|^4$$

to measure the conditional standard deviation, while Barndorff-Nielsen and Shephard (2004, 2006) used Quadpower Quarticity:

$$QP_j = n\mu^{-4}_{n} \left( \frac{n-1}{n-4} \right) \sum_{i=j-n+5}^{j} |r_i| \left| |r_{i-1}| \right| \left| |r_{i-2}| \right| \left| |r_{i-3}| \right|$$

Barndorff-Nielsen and Shephard (2004, 2006) then proposed several different asymptotically equal statistics to estimate the presence of price jumps.

According to Huang and Tauchen (2005), the best statistics is $Z_{RJ,TP}$ defined as:

$$Z_{RJ,TP} = \frac{RJ}{\sqrt{\left( \frac{\pi}{2} \right)^2 + \pi - 5} \left( \frac{1}{n} \right) \max \left( 1, \frac{TP}{BV^2} \right)}$$

with $RJ = (RV_j - BV_j)/RV_j$. The null hypothesis states that there is no jump in a given period. If the statistics exceeds the critical value $\Phi^{-1}(\alpha)$, then we reject the null hypothesis of no price jump at confidence level $\alpha$.

Realized Variation and Bipower Variation are forward-looking, however, we need a backward-looking specification re-defined as:

$$RV_j = \sum_{i=j-n+2}^{j} r_i^2,$$

$$BV_j = \mu^{-2}_{n} \left( \frac{n-1}{n-2} \right) \sum_{i=j-n+3}^{j} |r_i| |r_{i-1}|.$$

The statistics thus refer to a window of length $n$ ending at time step $j$. Thus, observing a significant jump at time step $j$ means that somewhere in the window of length $n$ ending at time step $j$ was at least one significant jump. Thus, the change between periods with no price jump and periods with a price jump can serve as an indicator for the moments when jumps happened the first time. This also assumes that the average time between two jumps will be much larger than the window used in this statistics. On the other hand, a very short time window skews the results with a small-sample bias. Since we assume more than one price jump per day, we employ $n = 60$ and $n = 120$.

The indicator for a price jump is defined as follows: price jumps are those prices for which $Z_{t-1} \leq \Phi^{-1}(\alpha)$ and $Z_t > \Phi^{-1}(\alpha)$. By definition, the indicator cannot distinguish two consecutive steps, otherwise we would have to work with the absolute levels of the statistics.

A2.2 Max-adjusted Statistics: Improved Identification Method

The improvement works as described in the main section, namely returns identified as price jumps are replaced by the average value calculated over the same length as was used for identification. The replaced value is clearly not a price jump, otherwise the price jump would not be identified as a price jump. Therefore, we define a pair of improved indicators based on the above-defined Max-adjusted statistics with $n = 60$ and $n = 120$. 

\[ \text{25} \]
A2.3 Lee-Mykland

The statistics of Lee and Mykland (2008) is based on bipower variation and is given as $\mathcal{L}(i) = \frac{\hat{r}_n}{\hat{\sigma}(i)}$, with $\hat{\sigma}^2(i) = \frac{1}{n-2} \sum_{j=i-n+2}^{i-1} |r_i| |r_{i-1}|$. Then

$$\frac{\max_{1 \leq i \leq n} |\mathcal{L}(i)| - C_n}{S_n} \to \xi,$$

where $\xi$ has a cumulative distribution function $P(\xi \leq x) = \exp(-e^{-x})$, and the constants are given as $C_n = \frac{(2 \log n)^{1/2}}{c} \frac{\log x + \log(\log n)}{2(2 \log n)^{1/2}}$, $S_n = \frac{1}{c(2 \log n)^{1/2}}$, and $c = \sqrt{2}/\sqrt{\pi}$. Whenever the $\xi$-statistics exceeds the critical value $\xi_{CV}$, we reject the null hypothesis of no price jump at time $t_i$.

Lee and Mykland recommends $n_{15-min} = 156$ and $n_{5-min} = 270$. In our analysis, we use $n = 60$ and $n = 120$.

A3 Group 3: Jiang-Oomen Statistics

The Jiang and Oomen (2008) statistics is based on Swap Variance defined as

$$SwV = 2 \sum_{i=2}^{n} (R_i - r_i),$$

where $R_i = \frac{P_i - P_{i-1}}{P_i}$, with $P_i$ being the price of the asset as introduced above. The authors claim that employing swap variance further amplifies the contribution coming from price jumps and thus makes the estimator less sensitive to intraday variation.

A3.1 Jiang-Oomen Statistics-based Price-Jump Indicator

The Jiang-Oomen statistics is defined as

$$JO_{Ratio} = \frac{n \cdot BV}{\sqrt{\Omega_{SwV}}} \left(1 - \frac{RV}{SwV}\right),$$

where the Realized Variation $RV$ and the Bipower Variation $BV$ are defined as above. The statistics is asymptotically equal to $z \sim N(0,1)$ and tests the null hypothesis that a given window does not contain any price jump. The indicator for a price jump is defined as those price movements for which $JO_{i-1} \leq \Phi^{-1}(\alpha)$ and $JO_i > \Phi^{-1}(\alpha)$. The same comments as for the Max-adjusted statistics apply. We use two price-jump indicators with $n = 60$ and $n = 120$.

A3.2 Jiang-Oomen Statistics: Improved Identification Method

We use the same improvement technique as in section A2.1 and define two improved indicators based on the Jiang-Oomen statistics with $n = 60$ and $n = 120$.

A4 Group 4: Statistical Finance

The scaling properties of returns can be studied using different techniques (see Mantegna and Stanley, 2000, and references therein), where we employ the price-jump index, as defined by Joulin et al. (2008), for this study.

A4.1 Price-Jump Index

The price-jump index is defined as

$$pji = \frac{1}{n} \sum_{j=i-n+1}^{i} \frac{|r_j|}{\sum_{j=i-n+1}^{i} |r_j|},$$

where $n$ governs the length of the moving window over which we normalize the absolute returns at a given time moment. The empirical observations suggest (Joulin et al., 2008) that the
scaling properties behave as $\Pr (pji > s) \sim s^{-\alpha}$, therefore we define a price jump as those price returns where the price-jump index exceeds a given threshold $\hat{s}$. In our analysis, we choose $\hat{s} = 4$ and $n = 120$ and $n = 420$. 