A Model of UK Libor as a Jump-Diffusion Process

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Abstract

This paper presents a jump-diffusion model of the UK sterling interbank short rate ('libor'). We are able to decompose libor into the sum of a mean-reverting continuous part, and a jump process. The latter is identified with the clearing bank base rate. In addition, including a transition function allows libor to 'anticipate' base rate movements.

If this decomposition is a reasonable description of the process followed by libor then models that do not take this behaviour into account may be mis-specified. In particular the models and the methodology used by Chan, Karolyi, Longstaff and Sanders (1992) in their empirical study of American short rates may not be applicable to UK libor.

Implications are drawn for the valuation of derivative securities on libor. The valuation of the base rate process. Long maturity options reflect the conditional distribution of the base rate process. Long maturity options reflect the conditional distribution of the base rate process. Short term options are influenced by the probability of single base rate jumps within the horizon of their time to maturity.

1. Introduction

In a recent paper Chan, Karolyi, Longstaff and Sanders (1992) presented an empirical comparison of interest rate models using American data. All of the models they compared were continuous diffusion models. In this paper we present an empirical study of the behaviour of UK 3 month libor, ('libor') and we draw some implications for the valuation of derivative securities on libor. We conclude that libor may be represented as a sum of a continuous part and a jump part. The basis for the decomposition is the observation that UK libor and UK base rate levels are closely related. Indeed, jumps in libor are frequently associated with jumps in the base rate. This is more than just an empirical accident; since 1981 the UK government has exerted close control over the level of UK interest rates, in particular 3 month libor . By intervening in the money markets, the Bank of England has attempted to prevent interest rates straying outside 'acceptable' bands. Hence libor is at least partially constrained to keep near to base rate levels.

This implies that the models investigated by Chan, Karolyi, Longstaff and Sanders (1992) may be inappropriate as models of UK libor, since none of their set of models incorporates a jump component. This has serious implications for pricing derivative securities based on libor. Libor is multimodal; so that models of derivatives that fail to reflect this property will possess systematic pricing biases. The purpose of this paper is to model UK 3 month libor as a jump-diffusion process. In the next section a summary of previous works on jump-diffusion models is presented. Section three provides an empirical investigation to decompose the UK libor into a continuous part and a jump component. In the fourth section we note implications for pricing derivatives on libor. Finally, conclusions are given.

2. Previous works on jump diffusion models.

Interest rate models often assume that the underlying interest rate process is a diffusion process. For instance, the short rate models of Vasicek (1977) and Cox, Ingersoll and Ross (1985b) model the short rate as a Markovian diffusion process. However, often motivated by empirical evidence, many authors have investigated models that allow for the presence of jumps in the underlying process. Merton (1976) conducted an investigation into the effects on the Black-Scholes (1973) option pricing formula of incorporating jumps into the stock process. To obtain workable formulae he had to assume that stock price jumps were uncorrelated with the market and were thus unpriced. This assumption may hold if jumps are small and firm specific, but evidently fails to hold when jumps occur simultaneously across a market. The most notorious recent example of this happening is the 1987 crash.

Merton considered several cases, including one where jumps in stock prices were log-normally distributed. In this case his general formula takes a particularity simple form. Merton also noted that in general it is impossible to hedge an option on a stock that has a jump-diffusion process with a portfolio containing only the stock and riskless borrowing.

Jump processes had also been considered by Cox and Ross (1976) in a survey of option pricing models. Cox, Ross and Rubinstein (1979) approximated diffusion stock processes as the continuous limit of a binomial jump process. This results in the binomial method of option pricing. An earlier paper by Brennan and Schwartz (1978) had highlighted the parallels between certain types of finite difference methods used to solve the Black-Scholes differential equation, and multinomial approximations to the diffusion process of the underlying stock. Press (1967) considered a jump model of stock price movements.

A number of authors, including Beckers (1981), Ball & Torous (1985), and Powell (1989), have attempted to estimate parameters for option pricing models based on Merton's paper. These attempts have usually centred on methods of moments, where the theoretical relationships between a stock's moments and the parameters of the jump-diffusion model are derived. The sample moments are then found and used in the relevant option pricing formula.

The results of these tests have been mixed. Allowing the underlying stock to jump has an effect on the price of an option written on that stock, but empirically the effect is not large. Furthermore, tests often assume, in line with Merton's formula, that jumps in stock prices are log-normally distributed. This assumption, although natural, needs justification.

In the late seventies Cox, Ingersoll and Ross, (1985a,1985b), developed an equilibrium model of economies that included derivative securities. Their model specified endogenously the processes followed by assets and derivatives, and allowed the price of risk of non-traded assets to be consistently determined. They demonstrated as a special case a formula for pricing options on interest rates. They produced an interest rate process, in which the interest rate and its associated price of risk are guaranteed to be arbitrage free.

The model of Cox, Ingersoll and Ross did not incorporate jumps. Jarrow & Rosenfeld (1984) set up an asset pricing model that did incorporate jumps. Although not a full equilibrium model, they developed a CAPM type pricing model and showed how asset jump risk could be priced in the market.

A general equilibrium model which allows for jump risk was published by Ahn & Thompson in 1988. Their model is a straightforward extension of Cox, Ingersoll and Ross (1985a, 1985b) where the underlying state variables and the production processes may have a jump component. In their paper they restate the main results of Cox, Ingersoll and Ross, emphasising the effect of the presence of jumps. They conclude with an example of an interest rate process analogous to that presented in Cox, Ingersoll and Ross, and indicate how an approximate option pricing formula may be found. Recently Bates (1991) has analysed the 1987 crash using an option pricing model based on a general equilibrium framework, including jumps, analogous to Ahn and Thompson.

Meanwhile Oldfield, Rogalski & Jarrow (1977) had examined jump diffusion-processes for stocks which there could be autocorrelation between successive jumps. Various authors including Jones (1984), Page and Sanders (1986) and Jarrow and Madan (1991) have investigated option pricing on processes that have jump components.

3. Empirical Investigation of UK Libor

Figure 1 is a plot of daily UK 3 month libor from November 1988 to October 1990, with the level of UK base rates superimposed upon it. This plot is remarkable for several reasons. The first is that it naively resembles a jump-diffusion process. Libor remains relatively static, but from time to time moves to a new level. This behaviour is reflected in the frequency histogram for libor, shown in figure 2. Peaks occur at, for example, 10%, 11%, 12%, 13%, 14%, 15%, etc. In periods between shifts libor appears to possess a relatively small volatility. The second feature is that the shifts occur close to the dates of changes in base rates. This in itself is not surprising because some control is exerted over libor to encourage it to remain near to the level of prevailing base rate. Since 1981 the government has regarded control of interest rates as mechanism for control of the economy, and has actively exercised this control. If this is to be a persistent feature of libor, then there may be an empirical justification to decompose libor into a base rate component, that controls the overall level of libor, and a continuous part that represents the usual daily fluctuations of libor. If it is the case that the main contributor to large movements in libor has been the discrete base rate component, then studies that attempt to analyse libor as a continuous process with no jump component may be mis-specified. Moreover, model estimates derived from such studies may have to be regarded as suspect unless it can be shown that the methodology employed is consistent with the empirically observed behaviour.

We shall decompose libor into a base rate component, minus an offset, f, equal to the base rate minus libor. We shall split the offset, f, into three parts. Firstly, a mean-reverting part that reverts to zero. This part represents the steady state continuous part of libor in between jumps. Secondly, a jump part that jumps at the same time as the base rate jumps, which takes values close to zero. This part represents the difference, if any, between the base rate level and the level about which libor actually reverts. A null hypothesis is that this part is identically zero. Thirdly, it is necessary to include a 'transition' part. When a base rate jump occurs libor often jumps with the base rate. However, in a significant number of cases, a change in libor may anticipate or lag the base rate change, in that libor may move part of the way towards the new base rate before or after the base rate change takes place. The transition part is included as an attempt to model this effect.

We shall write;

$$L=b+f, \text{ or}$$

$$L=b+g+\beta+T,$$

where:

L is libor, b is the base rate process, f is the offset process, g is the continuous part of the offset, β is the correction to the mean, and T is the transition process.

We shall expect that β jumps only when b jumps, and that T is non-zero only near jumps.

The data set used in the empirical analysis of libor are daily rates for 3 month Libor, and for the London clearing bank base rate, from 2 January 1975 to 11 Nov 1991, inclusive.[3] Shown in figure 3 is a plot of the offset series f. A casual inspection of this plot indicates that the offset series appears to exhibit mean reversion, and that the mean is close to, or equals, zero. The first third of the plot appears to be distinctly more volatile than the remaining two thirds. This change

[3] The data was obtained from Datastream

seems to correspond to the enforcement of interest controls that took place in the early 80's. We shall restrict some of our analysis to this latter more settled period.

Initially, the series for libor, base rates, and the offset series were separately tested for the presence of unit roots. If units roots are detected then the series is not stationary and cannot be a mean-reverting Ornstein-Uhlenbeck process. Non-stationary variables present difficulties since the standard distributions do not apply.

The Augmented Dickey-Fuller (ADF) test [see Dickey and Fuller (1979) and (1981)] was performed on the data for the period November 1980 to November 1991. In the ADF test, the first difference of the variable is regressed on a time trend, Tr, the first lag of the variable and on the lagged first differences of the variable:

$$\Delta Y_t = a_0 + a_1\, Tr + a_2\, Y_{t-1} + \sum_{i=1}^{p} a_i\, \Delta Y_{t-i} + u_t$$

where p is chosen so that the residual series (u_t) is white noise. The results are presented in table 1.

Table 1. ADF test

Variables	**lags**	t_{a_0} [a]	t_{a_1} [a]	t_{a_2} [b]
Base rate (b)	0	1.708	0.6479	-1.997
libor (l)	2	1.737	0.4383	-2.029
Offset (f)	3	-4.883	4.1350	-8.799
(Δb)	1	-0.487	0.2833	-52.455
(Δl)	1	-0.516	0.2668	-38.243

[a] The critical value for the t-statistic is 1.96 at the 5% significance level.
[b] The critical value for the ADF test is -3.4140 at the 5% significance level

The null hypothesis of the presence of a unit root in the base rate and libor series is not rejected at the 5% significance level. However, when both series are differenced the ADF test reflects no presence of unit roots. Thus, the base rate and libor series are integrated of order one. In contrast, the offset series f was not found to have a unit root, showing that the offset is a stationary process.

In order to test for cointegration between two variables X_t and Y_t, the Augmented Dickey-Fuller test for cointegration should be performed [see Engle and Granger (1987)]. This test consists of estimating the cointegration regression through the following static equation:

$$Y_t = a_0 + a_1 X_t + u_t,$$

under the null hypothesis that the residuals u_t are integrated of order one or non-cointegration. The results of doing this for the base rate and libor series are presented in table 2.

Table 2. Cointegrating regressions

Dependent variable	Independent variable	ADF test
Base rate	libor	-8.1460
libor	base rate	-8.1753

The critical value for the ADF test at the 5% of significance level is -3.3398.

The null hypothesis that libor and base rates are not cointegrated was rejected at the 5% level. In other words, for the period analysed libor and base rate are cointegrated; they tend to change stochastically together and do not drift apart. This result implies that there exists a long-run or equilibrium relationship between the base rate and libor. We thus feel some confidence in decomposing libor into the base rate plus a stationary offset process.

3.1. The Base Rate Component

From the series of base rates between 1975 and 1991 shown in figure 1 various observations may be made. Firstly, large scale fluctuations have occurred over the period, with rates first declining to 6% before rising to 17%. The series exhibits autocorrelation, both before and after differencing. A downward change is move likely to be followed by another downward change, and an up more likely to be followed by an up. When the base rate takes on mid-values it seems to move more rapidly. More time seems to be spent at the extremes, once the base rate gets there, than at mid range.

The base rate process appears to be Poisson, but with autocorrelation. The distribution of time intervals between jumps in the base rate is consistent with an exponential distribution with parameter $\lambda = 7$. This implies that the base rate process may be Poisson, with intensity 7, so that on average 7 jumps may be observed over a one year period.

Table 4 shows the average waiting time at each base rate level. The base rate has spent less time at mid-range levels. If the average time between jumps at a base rate b is l_b, then the jump intensity at that level may be estimated as $\lambda_b = 1/l_b$. λ_b appears to be higher at mid-range values of b.

Downward jumps are most frequently -0.5, whereas upwards jumps are likely to be larger (see figure 4).

3.2. Analysis of the Offset Series *f*

We have seen that the offset series *f* is a stationary process, taking values close to zero for the period analysed. To model the offset it is natural to investigate if a mean-reverting process sufficiently captures its behaviour. Thus, we hypothesize that the offset series *f* mean reverts to a long-run level, γ :

(1) $$df = \alpha(\gamma - f)dt + \sigma dz$$

where:

α is the speed of mean reversion parameter, which should be positive in order to ensure that f is stationary,

$\alpha(\gamma - f)dt$ is the instantaneous drift in the offset process and

σdz is the stochastic part of the offset component, where σ is the volatility of *f*.

We first test whether or not f may be modelled by equation (1) over the period November 1980 to November 1991.

Expressing equation (1), in discrete terms, we can express it in the following form:

(2) $$\Delta f_t = c + d\, f_{t-1} + \varepsilon_t$$

where:

$$c = \alpha \gamma \Delta t,$$

$$d = -\alpha \Delta t,$$

$1/\Delta t$ is the number of observations in unit time, in our case, $\Delta_t = 1/260$, and

ε_t is a random noise, which should be normally distributed, serially uncorrelated and with constant variance.

To look for classical mean-reversion, the first differences $\Delta f_t = f_t - f_{t-1}$ of the offset process were regressed against the values of the offset, f_{t-1} as in equation (2). If f follows an Ornstein-Uhlenbeck process, then from the regression above we can obtain estimates of α , γ and σ .

The estimate of σ was calculated as:

$$\sigma = \sqrt{\sum_{t=1}^{N} \varepsilon_t^{2} \Big/ (N-1)}$$

where:

N is the total number of observations in the period analysed

and the annualized standard deviation is:

$$\sigma_A = \sigma \big/ \sqrt{\Delta t}$$

We estimate the regression equation (2) for the entire data series. The results are given in table 3:

Table 3. Annualized parameters of the Ornstein-Uhlenbeck process for *f*

Depend. variable	Independ. variable	α [a]	γ	σ	Wald test [b]
Δf_t	f_{t-1}	14.005 (7.060)	-0.00157	0.02564	6.841

[a] The value in parenthesis is the t-ratio. The critical value for α is 1.96 at the 5% significance level.

[b] The Wald test permits the significance of the parameter γ to be tested. Note that γ = -c/d is a non-linear restriction from equation (2). The Wald test allows us to test the null hypothesis of nonsignificance of γ, i.e. $\gamma = 0$. The critical value for *b* is 3.84 at the 5% significance level.

From the regression (2) we conclude that the offset series is a stable process, since α= 14.005 is positive and significant. This is a high mean reversion speed parameter. The parameter γ, although it is near zero, in fact is significantly different from zero. In other words, the estimated long-run level in the offset part of libor cannot be taken to be zero. Diagnostic tests (e.g. serial correlation, normality and heteroscedasticity) were performed. The tests show that the residuals of regression equation (2) are not normally distributed and non-homoscedastic. So it is not possible to model the offset series for the entire period as an Ornstein-Uhlenbeck process mean reverting to zero. A casual investigation revealed that the offset series *f* occasionally exhibited large fluctuations for short periods at times when base rate changed. It was hypothesized that special behaviour takes place near base rate jumps resulting in a process for *f* that is too unstable to be modelled by (1) over the entire period.

In order to investigate this the data was divided into segments. A segment was defined as a period when no base rate changes occur; that is, a sequence of daily libor in between two successive base rate changes There are 88 segments in the period November 1980 to November 1991. Each segment was examined to discover if the behaviour of libor within the segment is consistent with the decomposition posited before. For instance, libor should have no large jumps within a segment.

In the next section we regress equation (2) for each of the individual segments.

3.2.1 Continuous part of the offset series g and the β correction to the mean

We analyse the behaviour of the offset series in between base rate jumps, ignoring the transition part in each segment. The division of the offset series into segments allows us to estimate g and β from equation (2). The estimated value of γ is taken as the estimate of β on each segment. The process for g is then:

$$dg = -\alpha g dt + \sigma dz,$$

where α and σ are estimated through regression (2) on each segment. Since each segment is associated with a particular base rate level this procedure permits a b-dependence of σ to be tested.

In analysing the offset process on the segments we have filtered the data in two ways.

Firstly, short segments have been ignored. If there were less than 14 days between base rate changes, the data for that period was excluded from the sample. This was done on several grounds; a short period between base rate changes may indicate high volatility in the economy, so interest rates during these periods may display spurious behaviour. Furthermore, a short period is more likely to be dominated by transition effects. More prosaically, a longer sample was required to calculate the volatility of the residual part over the segment. Out of the 88 segments in the data set, 40 were excluded by this length criterion.

Secondly, in order to compensate at least in part for any effects associated with transitions, the first two and last two days of each segment were excluded from the data set. We are concerned to remove, as far as possible, any spurious effects due to transitions. An inspection of the data reveals that immediately around base rate changes there are likely to be larger movements in libor than in periods away from base rate changes. Removing the initial and final two days data from the sample removed much of this excess volatility.

The regression equation (2) was estimated for 48 segments out of the 88, for the period November 1980-November 1991. Estimates for α, β, σ and their Wald statistics were obtained for each segment. The estimate α is negative only twice and is never significantly non-positive. It is significantly positive on almost half of 48 segments considered. The β parameter is significantly different from zero in 31 segments.

Diagnostic tests were also performed on each segment to test for normality and for serial correlation. Out of 48 segments, 15 failed the normality test and 2 failed the test for the absence of serial correlation.[4]

These tests provide some evidence that an Ornstein-Uhlenbeck process restricted to individual segments can fit the data, once

i) short segments are ignored

ii) days immediately adjacent to base rate changes are removed

iii) parameters are estimated separately on each segment

Some analysis was performed on the behaviour of σ over the set of segments. An ADF test rejected the hypothesis of the presence of a unit root in this series of σ at the 5% significance level, implying that the σ series is stationary.

The analysis also enables the standard deviation estimate σ to be compared to the base rate level it stems from. There has been much speculation on the relationship between the level of interest rates and the volatility of interest rates. In our decomposition, we are able to address this question directly. Figure 5 is a plot of σ_b against b, where σ_b is the sample estimate calculated over all accepted segments at base rate level b. The values are given in table 4. There seems to be a relationship between σ_b and b, in that σ_b may be larger at higher b. The general level of interest rates may effect the volatility of the offset process. Regressing

[4] Detailed statistics available from the authors upon request

against b we find $\sigma_b = 0.0085 + 0.0073\ b + e$. Neither coefficient is significant at the 5% level.

Table 4. Volatility and Occupancy at each base rate level.

(Volatility for 1980-1991, occupancy for 1975-1991)

Base Rate	Average * Volatility	Average Occupancy (days)
6	_	31
6.5	_	74
7	_	24
7.5	_	18
8	0.0145	20
8.5	0.0097	35
8.75	_	1
9	0.0133	26
9.125	0.0241	10
9.5	0.0148	32
10.0	0.0157	37
10.5	0.0164	29
11.0	0.0171	37
11.5	0.0107	29
12.0	0.0174	37
12.5	0.0208	29
12.75	_	1
13	0.0148	36
14	0.0237	64
14.5	0.0270	36
15	0.0100	140
15.5	_	18
16	_	56
17	_	165

* No volatility shown if no eligible segments appeared at the corresponding base rate level.

3.2.2. Transitions in libor when Base Rates Change

We have seen that by examining periods during which no base rate change occurs, excluding values near to the times of base rate changes, a reasonable description of the offset process is in terms of an Ornstein-Uhlenbeck process. In order to arrive at a fuller description of libor we now investigate the transitions to libor that take place when base rate changes occur. In the period January 1975 to November 1991, 125 base rates changes took place. We selected for investigation those base rate changes which were not within 20 days, either before or after, of another base rate change. This selects a sample of 'isolated' base rate changes, with 40 days of libor data for each: 20 days preceding the jump and 19 days following. 63 base rate changes satisfied this criterion. For each jump, the average of the first five days of the 40 day data series was used as an estimate of the level of libor in the period immediately preceding the jump. Deviations away from this level as a proportion of the size of the base rate jump were calculated, and averaged over the 63 jumps.

Set $L_{i,t}$, $i = 1,...,63$, $t = -20,...,+19$, to be the value of libor for the i'th base rate jump and day t relative to the date of the base rate jump, and define $a_i = \frac{1}{5} \sum_{t=-20}^{-16} L_{i,t}$ to be the prior level of libor. Suppose that at t = 0 the base rate jumps from b_i to b_i', with jump size $\Delta_i = b_i' - b_i$.

Then the proportional deviation for the i'th jump on day t is:

$$D_{i,t} = \frac{L_{i,t} - a_i}{\Delta_i}, \quad i = 1,...,63, \quad t = -15,...,+19$$

$D_{i,t}$ represents the proportion of the base rate jump that libor has moved above the level a_i, on day t. Define $D_t = \frac{1}{63} \sum_{i=1}^{63} D_{i,t}$ to be the average $D_{i,t}$ across every segment. D_t represents the average movement in libor on day t, away from its previous average value, as a proportion of the size of the base rate jump.

The resulting 'average' transition D_t is shown in figure 6a. By day 0, the day of the jump, libor has 'anticipated' the jump. Libor starts to move some 6 or 7 days before the jump. After the jump, average libor continues to move in the direction of the jump, reaching 1.5 times the base rate increment 10 days after the base rate change. There is a wide variation in the behaviour of libor during a jump transition period, so that the proportional libor deviations are not significantly different from zero on any day.

Of the 63 jumps, 43 are downward and 20 are upward. It was hypothesized that libor may behave differently when base rates jump down to when they jump up. The average proportional deviations restricted to positive jumps are shown in figure 6b and those for negative jumps in figure 6c. From figure 6b it can be seen that on average libor at first follows slightly behind positive jumps, but then by day 12 libor has on average increased by an amount equal to the gain due to the base rate change. This contrasts to the cases where the jump size is negative. Then on average libor slightly over anticipated the fall in base rate and then overshoots by an amount equal to half the change due to the base rate change. Once again, care must be taken with interpreting this average behaviour since the deviations are not significantly different from zero.

D_t represents an empirically determined estimate of a transition function T. In the absence of a theoretical model to account for the form of T we take the analysis no further at this stage.

4. Implications for Option Pricing

We have modelled libor as a jump-diffusion process. The jump component reverts towards a central value and the diffusion component reverts to the value of the jump component. Including a transition function, this captures some of the essential features of libor. This form of process is dissimilar to those frequently used to price derivative securities on libor.

One of the most heavily traded derivative instruments on libor are short sterling options. Normally thought of as futures options they may nevertheless be analysed as libor options. Since they are subject to LIFFE-style margining it is never optimal to exercise them prior to maturity, and at maturity the price of the futures contract on which they are written converges to 100-libor. A short sterling call option on a short sterling futures contract may be analysed as a put on libor itself. Other derivative products containing interest rate options include caps and floors - used to manage interest rate risk on floating rate borrowing - captions, swaptions, etc.

To value any of these instruments a correct specification of the underlying interest rate process is essential. We have restricted our analysis to 3 month libor so our comments and analysis are of greatest relevance to options on 3 month libor. Examples of such options are caplets (the individual options making up an interest rate cap) and short sterling options.

It is well known (e.g. Cox and Ross (1976)) that derivatives may be priced as if investors are risk neutral. This means is that the value at time 0 of a call on a share, for instance, is given by:

$$C = e^{-r_0 T} E^* [\max(0, S_T - X)]$$

where:

r_0 is the riskless rate,

S_T is the value of the stock price at the terminal time T,

X is the exercise price, and

E^* is the risk adjusted expectations operator

The option value equals the discounted expected risk neutral payoff to the option. The expected value of the payoff to the option is calculated as if the share evolved according to the process:

$$\frac{dS}{S} = r dt + \sigma dz$$

Interest rates themselves are not directly traded so this procedure cannot be applied directly to interest rates. Instead the market price of interest rate risk must be estimated. For instance, in the Vasicek (1977) interest rate model the prices of derivatives on an instantaneous interest rate r obey the partial differential equation:

$$\frac{1}{2}\sigma^2 P_{rr} + (\mu - \eta\sigma) P_r - rP + P_t = 0$$

Here, r is assumed to follow the stochastic process:

$$dr = \mu\, dt + \sigma\, dz$$

where:

$\mu = \alpha(\beta - r)$ is the drift of r and η is the price of risk for L.

η is an additional price of risk parameter that must be estimated before the pricing equation for P can be solved.

We use libor L as a surrogate for the short rate r, so that option values are contingent upon L. We qualitatively compare option pricing using our jump-diffusion process;

$$L = b + g + \beta + T$$

to that obtained using a single factor diffusion model, such as the Vasicek model for short interest rate.

Firstly, note that if one attempts to fit a Vasicek type model to libor, care needs to be taken in estimating the volatility of the interest rate process.

An estimate of the Vasicek volatility, σ , based on historical time series data of libor may give poor estimates. For instance, a 60 day historical estimate will underestimate σ if no jumps have taken place in its sampling period. Using $\lambda = 7$, one might expect on average approximately 1.6 jumps to occur in a 60 day period. If two or more jumps occur in the sampling then the estimate may be too high. For long dated options a long sampling period seems appropriate. For options with only a short time to maturity the appropriate volatility is determined by (i) the volatility of the mean-reverting component g, σ_g, and (ii) the probability and likely size of a jump occurring in the period to maturity. As the time to maturity goes to zero the probability of a jump goes to zero, so σ_g becomes the appropriate volatility.

Secondly, the dynamic of the process g, may have a relatively small impact on option values, except for options with only a short period to run before maturity. Jumps are generally of greater magnitude than the size of movements caused by g. For options with a long time to run before maturity the future distribution of libor is largely determined by the parameters of the jump process, since the process g can account for only relatively small perturbations. For options whose time to maturity is on the same scale as $l = 1/\lambda$ the probability of individual jumps becomes crucial; if a jump occurs libor will take values close to the new base rate level, otherwise it takes values close to its current value. In practice, short sterling futures contracts are available to hedge part of the jump risk. We argue that the value of a short sterling futures contract is largely determined by the expected behaviour of the jump component to libor.

Thirdly, an examination of table 4 shows that the average time libor spends at a base rate level, conditional on having reached that level, is surprisingly constant (except at higher base rate levels where libor has historically stayed longer). This is consistent with the observation that the jump component b has Poisson jump frequency. This behaviour is very different to that observed with a Vasicek type model. Compared with an interest rate obeying a Vasicek type model (using a large-sample volatility estimate) libor spends more time at levels away from its mean values. The implication for option pricing is that, when libor is greater than its historical mean values, a Vasicek type model will underprice calls and overprice puts, since libor spends more time at these levels than predicted by a Vasicek type model. Conversely, when libor is lower than its historical mean values, a Vasicek type model will overprice calls and underprice puts, since libor will remain lower for longer than a Vasicek type model expects.

5. Conclusions

We have established that libor may be decomposed into a base rate component minus an offset process. The continuous part of the offset process mean reverts to a level close to zero, except close to base rate changes. It appears that libor may not always jump at the time of a base rate change, but may instead move between levels over a period of several days before to several days after the base rate change. Variations of behaviour occur depending on whether the base rate movement is up or down.

The full model for libor is:

$$L = b + g + \beta + T,$$

where:

b is the base rate jump process

g is the continuous part of the offset process, mean reverting to 0,

β is a jump process that jumps simultaneously with b and takes values close to zero; it represents a correction to the reversion level of L.

T is a transition function that is zero everywhere except near base rate jumps.

This leads to option prices systematically different from those of models that assume a pure mean-reverting process for libor. The jump component b is crucial for correctly pricing interest rate options on libor. Among the implications of the interest rate process described above is that the density of libor is not unimodel. Rather, its density peaks around levels corresponding to base rate levels. This effect may be very significant for short dated options. Also, since the jump component has Poisson jump frequency, libor spends significantly longer away from its mean than a Vasicek type process, leading to future pricing biases.

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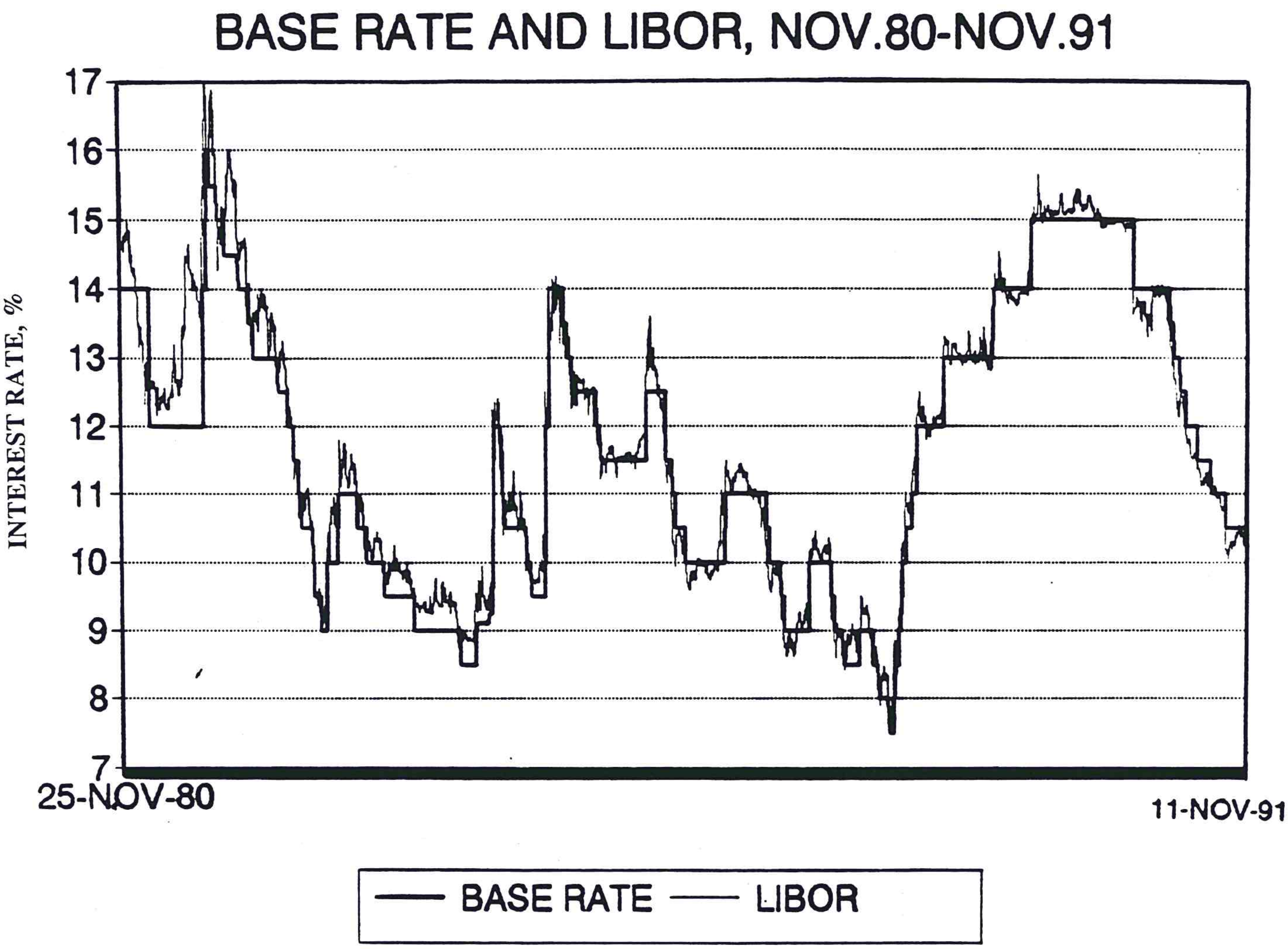
Figure 1



BASE RATE AND LIBOR, NOV.80-NOV.91
17
16
15
14
13
12
11
10
9
8
7
INTEREST RATE, %
25-NOV-80
11-NOV-91
BASE RATE
LIBOR

Figure 2

FREQUENCY HISTOGRAM FOR LIBOR, JAN. 75-NOV. 91

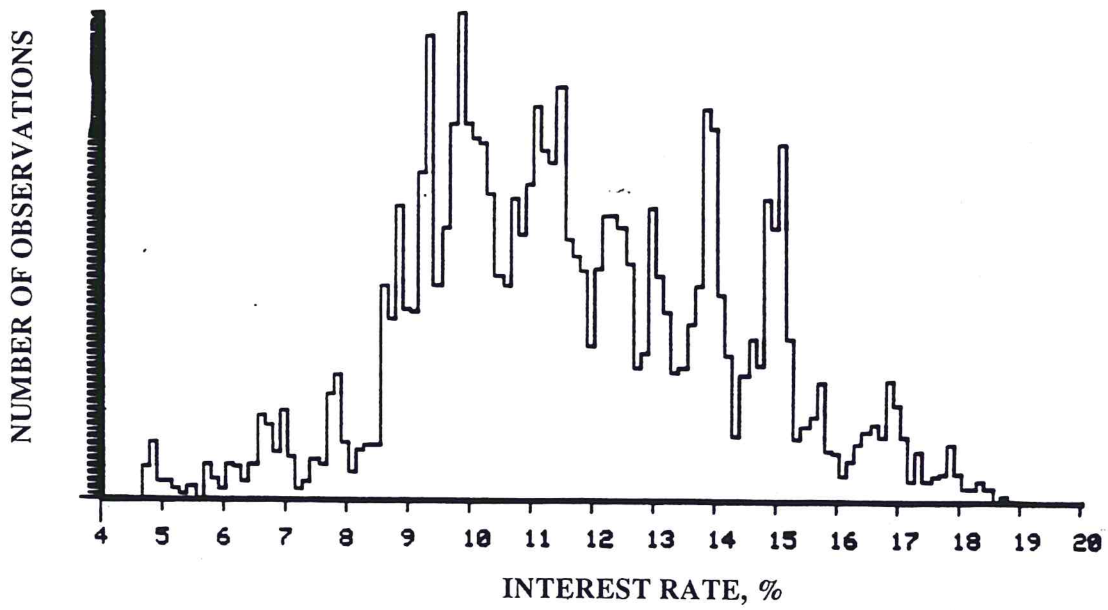
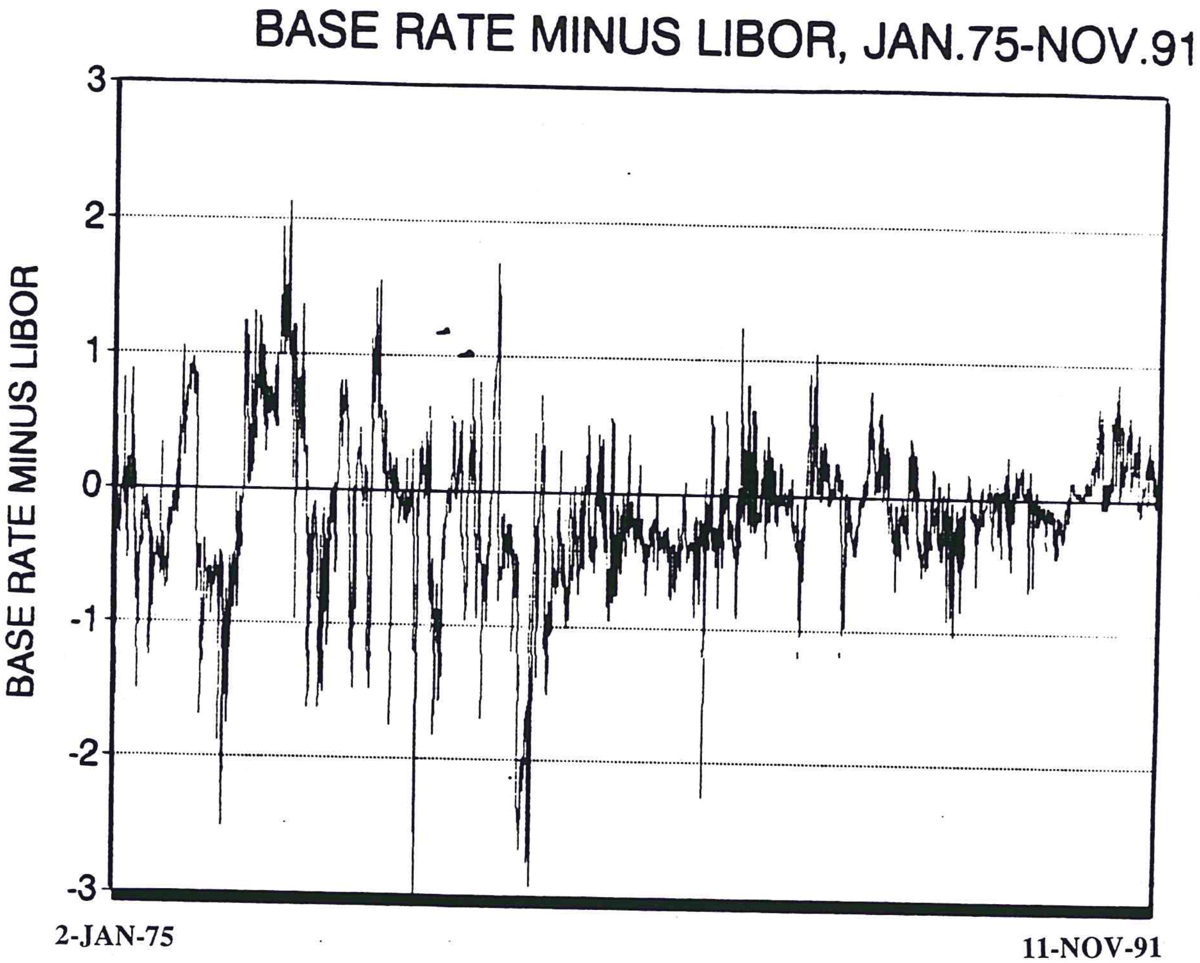


Figure 3



BASE RATE MINUS LIBOR, JAN.75-NOV.91
BASE RATE MINUS LIBOR
3
2
1
0
-1
-2
-3
2-JAN-75
11-NOV-91

Figure 4

DISTRIBUTION OF BASE RATE JUMP SIZES

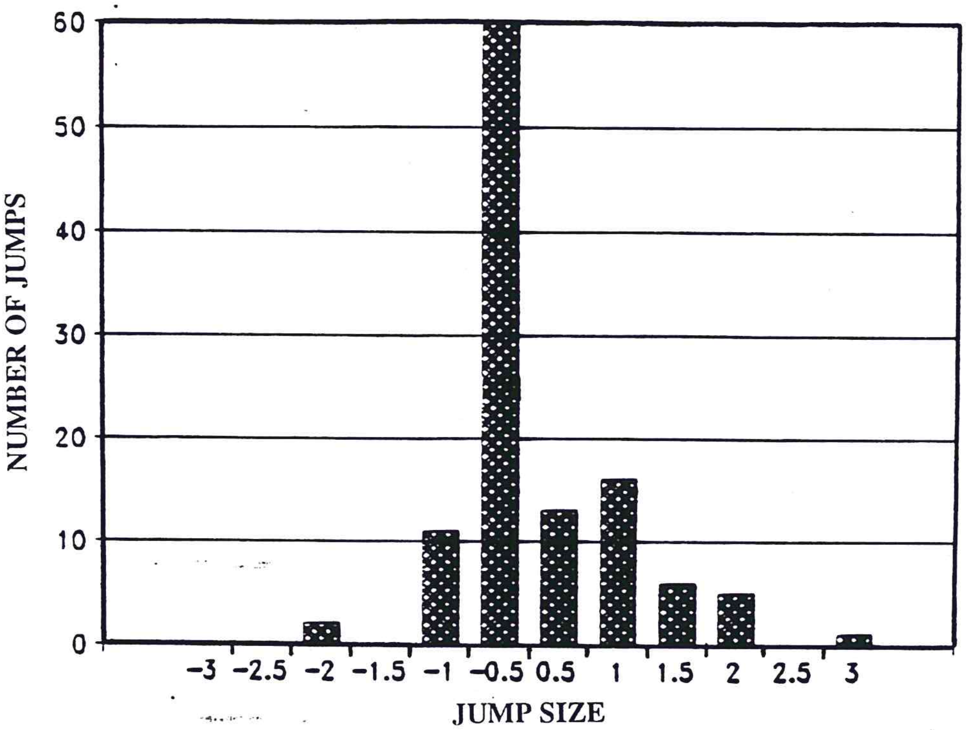
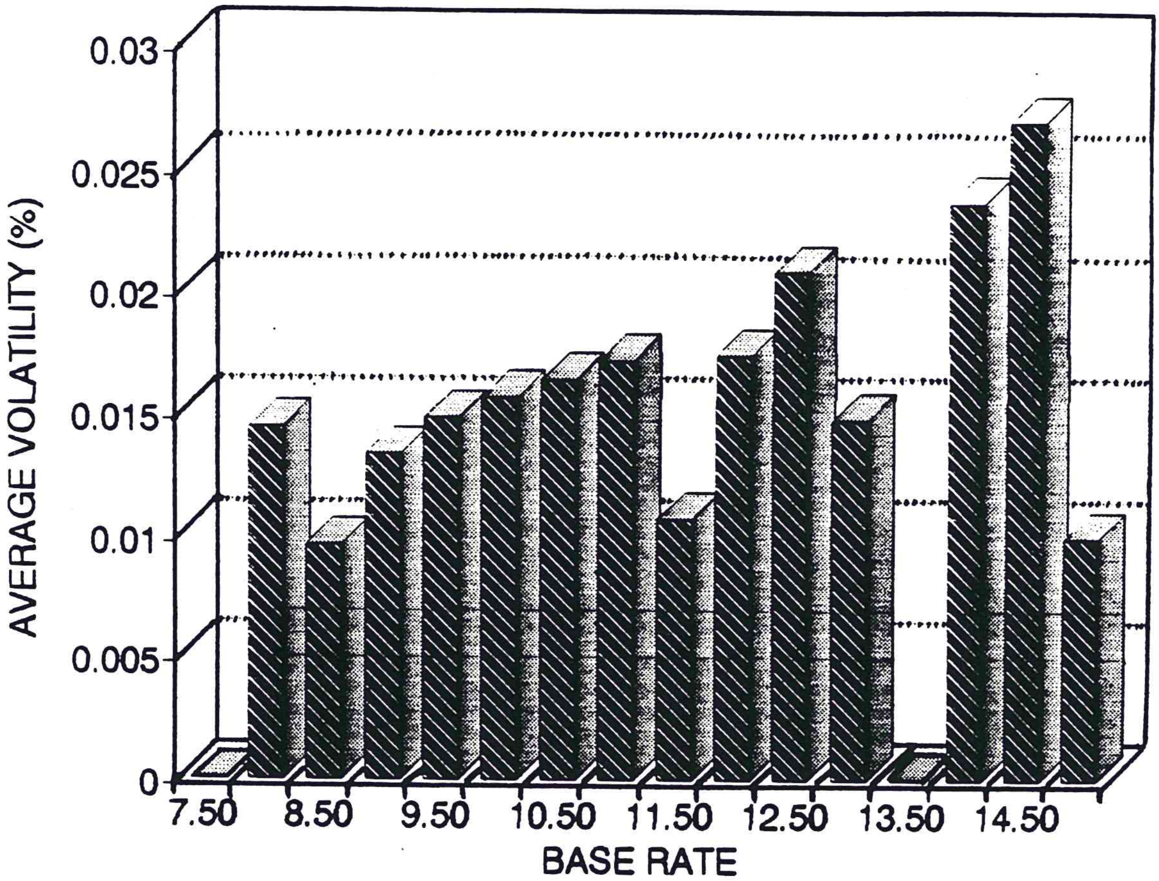


Figure 5

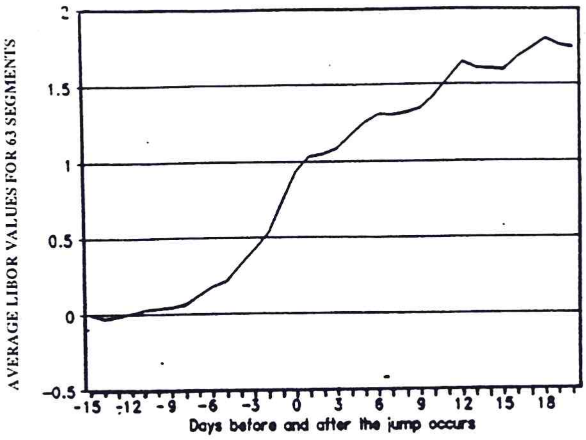
AVERAGE VOLATILITY OF THE OFFSET SERIES AGAINST THE BASE RATE *



*Excludes the volatility for the single occurrence of a segment at base rate 9.125. This had length 10 days and a volatility of 0.024.

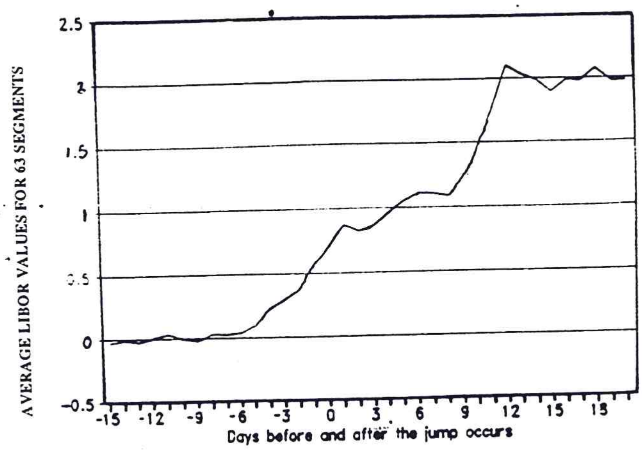
Figure 6a

AVERAGE MOVEMENT IN LIBOR NEAR BASE RATE CHANGES



b

AVERAGE MOVEMENT IN LIBOR NEAR POSITIVE BASE RATE CHANGES



c

AVERAGE MOVEMENT IN LIBOR NEAR NEGATIVE BASE RATE CHANGES