Term Structure Modelling Under Alternative Official Regimes

Simon Babbs
First National Bank of Chicago and
University of Warwick
Coventry

and

Nick Webber
The University of Warwick

October 1995

Acknowledgements: The authors wish to thank Jorgen Nielsen for a number of helpful contributions.

Financial Options Research Centre
Warwick Business School
University of Warwick
Coventry
CV4 7AL
Phone: (01203) 524118

FORC Preprint: 95/61

Term Structure Modelling Under Alternative Official Regimes

Simon Babbs

and

Nick Webber[1]

Abstract

Monetary authorities exercise control of domestic short term interest rates. We argue that models of the term structure of interest rates must take into account the consequences of this control if they are to capture important empirical features of interest rate dynamics. In particular, previous one or two factor models of the term structure may not adequately describe the short rate process. Interest rate regimes in the United Kingdom, the United States, France and Germany are described. A common characteristic is the presence of 'non-effective' official interest rates used for signalling purposes. We construct a term structure model, consistent with general equilibrium, that captures, in idealised form, some important features of these regimes. The paper concludes with an illustrative example of a regime in which the short rate is constrained to lie within an officially controlled corridor.

---

1 This work was carried out while Nick Webber was a First National Bank of Chicago Visiting Academic.

1. Introduction

A recent paper by Babbs and Webber (Babbs and Webber (94)) presents the first attempt, in the modern finance literature on the term structure of interest rates, to develop a theory in which the rate setting behaviour of the monetary authorities plays an explicit role. Monetary authorities within a currency's domestic market attempt to control the level of short term interest rates in that currency. Very few papers attempt to take this into account[2].

Focusing on the behaviour of the United Kingdom authorities over recent decades, Babbs and Webber were motivated to develop a theoretical framework in which the short term interest rate, r, follows a pure jump process, with fixed jump sizes, and with stochastic intensities influenced by a diffusion state variable, x. Simulations of an illustrative model, constructed within this framework, mimicked successfully various empirical regularities of UK interest rates. Diffusion models of the term structure such as Duffie and Kan (93), Heath, Jarrow and Morton (92), or extended Vasicek models due to several authors, cannot satisfactorily capture the observed jump-diffusion behaviour of short rates in the UK regime. Jump-diffusion extensions, such as Shirakawa (91), Ahn and Thompson (88), or Bjork, Kabanov and Runnaldier (95) do not attempt to take rate setting behaviour into account.

As Babbs and Webber pointed out, while the existence of rate setting behaviour by the monetary authorities is common to many economies, the precise form of this behaviour differs substantially from country to country, necessitating generalisations of Babbs and Webber's theoretical framework.

In this paper we present a generalisation to multiple jump and diffusion state variables, and in which the short rate, r, is a specified function of the state variables, rather than as in Babbs and Webber where r is directly equal to a particular state variable following a pure jump process.

Such a generalisation could, no doubt, be put to many uses. In the present paper we motivate the generalisation by a desire to take account of the existence of additional officially set "discount" and "Lombard" rates (however termed in particular countries) which, as discussed below, enable the monetary authorities to signal to the markets their intentions concerning the future levels of the short rate. In practice these rates provide "soft" floors and ceilings for short term interest rates.

We provide simplified specialisations of this framework to provide stylised representations of the rate setting behaviour of the monetary authorities in the United States, France and Germany. The

2 A step in this direction was taken by Balduzzi Bertola and Foresi (93). See Babbs and Webber (94) for a fuller discussion.

original Babbs and Webber framework for the UK emerges as a special case. The focus in the present paper is upon signalling aspects of discount and Lombard rates. It does not deal explicitly with other official actions, such as repo auctions.

The results of this paper are of relevance to the management of interest rate risk, and the pricing of interest rate derivatives. The results are of interest in the manner in which changes to boundaries may influence the shape of the term structure, in other words, the effect of signalling behaviour upon the term structure. Although the exact mechanisms will vary from economy to economy, a change to an official rate may cause the term structure to alter, even if the short rate remains unchanged.

The plan of this paper is as follows. In section two we describe the main features of the short term interest rate regimes in several currencies. Drawing from these examples, in section three we construct a model that captures in mathematical form important characteristics of the regimes. In section four we show how idealised versions each of the regimes discussed in section two might be incorporated within the framework. Section five concludes with a numerical illustration.

2. Monetary Authorities and Interest Rate Regimes

We briefly examine the regimes imposed in several economies. Except where otherwise noted, the descriptions are based upon those given by Schnadt (94). The descriptions are by necessity a summary only, outlining only those features that we believe are most pertinent to term structure modelling. Furthermore, the descriptions in this paper are based upon the regimes in existence in the period covered by Schnadt. As monetary authorities adapt to changing conditions, the mechanisms and character of their control may alter, potentially leading to different short rate behaviour. Where regimes have altered, the descriptions given in this paper need to be modified accordingly.

Monetary authorities have several different roles in an economy. These may be undertaken by a single agency, or by several. Sometimes a role may not be the explicit responsibility of a specific agency or group of agencies. Monetary authorities exist because of the need to service the economic and regulatory requirements of the domestic banking system, and to service the needs of the domestic government. Governments undertake programmes of expenditure and raise revenue to finance their activities. This may result in significant fluctuations in the demand for short term funds. The monetary authorities seek to manage the quantity of these funds. Regulatory structures may explicitly exist to control the levels of bank reserves. In any case banks will attempt to maintain a minimum level of liquid assets to enable them to satisfy short term demand. This leads to a role for the monetary authorities to act as a 'lender of last resort' when banking illiquidity threatens the integrity of the domestic banking system. A price extracted for this service is often a regulatory requirement that banks should maintain operational deposits of liquid assets with the monetary authorities, above a minimum percentage of their capital base. These reserve levels are frequently calculated on an averaging basis that may have some degree of complexity. Since reserves may be non-interest bearing, and in any case represent an opportunity cost, banks are motivated to keep reserve levels to a minimum, and reserve management may be a significant aspect of day-to-day banking activity.

Because levels of short term liquidity varies exogenously, the aggregate level of reserves is subject to exogenous pressure. The sole net supplier, or purchaser, of marginal reserves within an economy is the monetary authorities. The level of interest rates at which marginal liquidity is supplied, or removed, from the banking system is set at the discretion of the monetary authorities. This provides the mechanism by which the authorities are able to set the level of short term interest rates as part of a macro economic management policy. This freedom to set rates is however constrained. If rates are set at levels that result in liquidity flows that are predominately into, or conversely, predominately out of, the monetary authorities, one party or the other to the transaction will eventually be unable to sustain the flows.

The mechanisms by which the roles sketched above are implemented varies from economy to economy. In practice a monetary authority, on a day-to-day basis, must forecast the net liquidity requirements of the domestic banking system. Based on its forecasts it will then make funds available to the market, or absorb funds in the event of a surplus. In the examples described below funds are frequently managed by authorities' open market operations in the sale or purchase of repos, or 'eligible bills'. Supply of funds is often accomplished by a tendering system. Purchase of funds may be accomplished at a published fixed rate. The frequency and volume of intervention varies widely from economy to economy. In the UK there may be several auctions per day. In Germany auctions are held once a week, although there may be additional daily fine tuning through, for instance, switching public sector deposits between the Bundesbank and the commercial banks.

Any residual imbalance between reserves supply and demand, after the authorities' intervention, is met on the interbank market. Overnight interbank rates may fluctuate considerably in response to short term day-to-day residual liquidity requirements. Overnight and short term interbank rates therefore reflect the degree and direction of the monetary authorities residual forecasting and supply errors. These effects are purely short range and may not be expected to effect market expectations of future levels of rates. We argue that overnight interbank rates do not reflect the underlying levels of the short term interest rate, and that the short rate is better represented by the official interest rates set by the monetary authorities in supplying money to the market. We refer to the effective rate at which the authorities supply or remove funds as the intervention rate. The overnight interbank rate typically fluctuates about the level of the intervention rate.

Even though the intervention rate may, in some regimes, be a two week rate, and may not be continuously available, nevertheless the intervention rate effectively sets the level of short term interest rates. Market expectations of the prospective levels of the intervention rate are widely discussed both in practitioner publications and the popular press, and it may be supposed that these expectations are impounded into the spot rate curve. In terms of term structure formation, the intervention rate behaves as if it were the short rate. It does not seem unreasonable to identify the short rate with the intervention rate.

In addition to rate setting behaviour, authorities may consider that part of their role is to smooth interest rate variability by signalling in advance to the markets their future intentions in interest rate transactions. This allows the authorities to more effectively control rates beyond simple direct rate setting.

Signalling may be facilitated by publishing interest rates at which alternative levels of intervention might take place. Typically, these rates are floors or ceilings to the rate at which the authorities may immediately intervene in the markets. A change to such a rate is a signal of the authorities' future intentions, even though that rate may not be one at which a large volume of transactions is normally made. For instance in Germany banks do not normally wish to borrow at the Lombard rate because it is expensive, and they do not borrow significantly at the cheap Discount rate because strict quotas are applied to its use. The Bundesbank may therefore alter the Discount rate or the Lombard rate at will without necessarily altering the value of the short rate in the DM money market. It uses changes in these two rates to signal to the market its intentions concerning future levels of the short rate. For instance, if the Discount rate is raised, this may be interpreted as a signal that the authorities do not expect, or will not allow, rates to fall below the new level in the near future. Similarly, the Lombard rate acts as a ceiling to the short rate, with analogous signalling.

Signalling by changing floor and ceiling rates will affect the term structure, even through the intervention rate itself may not have changed, because the market's expectations of future levels of the short rate will have been altered. We contend that these signalling aspects should be modelled to more fully account for changes in the term structure at short maturities.

In the examples below, floor and ceiling rates typically remain fixed for extended periods, being changed from time to time by discrete amounts. These rates may thus be modelled as jump processes. The floor and ceiling rates may coincide, forcing the short rate to be a jump process.

We now briefly discuss in turn the economies of the United Kingdom, Germany, France and the United States to see how these features are implemented in specific economies.

United Kingdom

Since the early 1980s the banking system has been purposefully kept short of funds. The Bank of England must inject money into the system on a daily basis. This is done by the purchase of eligible bills. Each morning the Bank estimates the quantity of market assistance required. It publishes this estimate, together with an invitation to eligible institutions to tender for purchase. The Bank accepts tenders above a minimum rate, called the stop rate, in each of a number of maturity bands. Quantities tendered for may be scaled down if the total exceeds the banks forecast of cash requirements. The Band of shortest maturity is called band one. It covers bills maturing in between one and fourteen days, although the greatest number of transactions are for bills of 3 or 4 days maturity. The Bank may offer further assistance through the day if demand in excess of its forecast becomes significant. Since the banks must obtain residual funds through the interbank market, overnight and very short term interbank rates are especially sensitive to residual cash requirements resulting from the inevitable mismatch between the level of Bank of England assistance and market requirements.

In Babbs and Webber it was argued that the major determinant on money market interest rates was the rate at which the Bank of England offers assistance to the market, the band one stop rate. This remains fixed from day to day over periods of weeks or months. When it is changed it jumps to a new level. Babbs and Webber argued that UK interest rates could be modelled by requiring the short rate to follow a pure jump process.

A plot of three month sterling libor is shown in figure 1, taken from Babbs and Webber (94). The plot also shows the value of UK base rate, which closely follows the band one stop rate. It is clear from the graph that the values of libor and the base rate are closely linked.

Germany

Banks are required to maintain high average reserves, relative to countries such as France or America. Since reserves are non–interest bearing, there is a motivation to keep reserves as low as possible to achieve an optimum use of funds. If reserves are kept too low, so that a bank fails to meet its reserves requirement, it may be required to borrow from the Bundesbank at a penal interest rate in order to increase its reserves to the required level. Funds may be obtained from the interbank market or from the Bundesbank. Cheap funding may be obtained from the Bundesbank by borrowing at the Discount Rate. However, there is a quota imposed on how much may be borrowed at this rate, and the level of this quota has been declining. More expensive funding may be obtained at the Lombard Rate. This is set at a level one percent or so above the Discount Rate, but funds are relatively freely available. The main method by which the Bundesbank makes funds available to banks is by operations in the repo market, although other forms for 'fine–tuning' are available. It holds weekly auctions of repos, predominantly of 2 week maturity. It does so in one of two ways, each way leading to qualitatively different behaviour of the short rate. The Bundesbank invites tenders from the banks on either on a fixed rate basis, 'mengentender', where it announces a rate and allows banks to tender on a volume basis only, or on a floating basis, 'zinstender', where it allows the market to tender both for volumes and for rates. Tenders above a minimum rate, chosen by the Bundesbank, are accepted.

In the first regime the level of the repo rate is determined by the Bundesbank. The repo rate is thus a third official rate, lying between the Discount Rate and the Lombard Rate. In the second regime, the level of the repo rate is effectively determined by the demand for funds in the market. The Bundesbank may switch between regimes at its discretion, and speculation about its intentions is an important topic in the DM money markets. Trading the overnight interbank rate, the Tagesgeld rate, is reported by Schnadt to be driven mainly by reserve management considerations, rather than by trading considerations. It is not a volatile rate, although towards the end of a reserves maintenance period there may be circumstances when it might trade outside the Discount rate – Lombard rate interval, because of its short maturity compared to the other two rates.

Figure 2, taken from Schnadt, shows the Tagesgeld rate, the Lombard rate and the Discount rate for the period 1979 to 1989. The Tagesgeld rate, in later years, has usually stayed close to the centre of the official interval, although it has reached the boundaries of the interval and even breached them.

France

Average reserve requirements in France are low to the point that day to day fluctuations in banks' cash positions due to trading requirements may significantly interfere with the banks' average reserves position. To maintain the benefits of low reserve requirements while avoiding negative balances presents special cash management problems. This means that marginal funding requirements, normally met through the interbank market, cause the overnight interbank rate to be very sensitive to reserves pressure. The Banque de France allows banks to borrow from it by inviting tenders for 7 day repos. Auctions are held twice a week. It accepts bids above a minimum rate, uniformly at this rate, the 'taux des pensions appel d'offres'. Bids are scaled so that the volume matches the Banque de France's forecast of demand. Additional funding may be obtained on the banks' initiative. They are able to obtain funds at a fixed premium above the taux des pensions appel d'offres, the 'taux des pensions de 5 a 10 jour'. The premium has been around 0.75%. The taux des pensions appel d'offres and the taux des pensions de 5 a 10 jour form a 'soft floor' and a 'soft ceiling' respectively for the overnight rate. They are not hard barriers because they are for relatively long maturities. The Banque de France also 'fine tunes' with overnight repos.

Figure 3, taken from Schnadt, shows the taux des pensions appel d'offres, the taux des pensions de 5 a 10 jour, and the overnight interbank rate in 1991 and 1992. The premium between the taux des pensions appel d'offres and the taux des pensions de 5 a 10 jour has been between 0.75% and 1% in this period. On only one occasion did the two rates not change simultaneously.

At the beginning of 1992 a change in regime took place, after which greater control was exerted on the overnight rate to lie close to the centre of the official corridor. After the change the overnight rate has stayed roughly centred in the band. Operations by the Banque de France ensure that the overnight rate stays, usually, within the corridor.

United States

Banks must maintain reserves over a two week averaging period. There is a very active interbank market. Most funds are obtained this way. The Fed permits banks to borrow at an official discount rate, but this form of funding is limited and discouraged by non–price mechanisms. The Fed sets a 'fed funds target rate' and by active open market operations attempts to keep the fed fund overnight rate close to the target rate.

The main supply of liquidity is via overnight repos. Tenders are invited and bids are accepted down to a minimum 'stop–out rate', or until the target volume is achieved. Borrowing at the discount rate is discouraged. The fed funds rate is normally greater than the discount rate. This is not absolute, however; the fed funds rate may fall below the discount rate.

Figure 4, taken from Schnadt, shows the fed funds rate and the Discount rate from 1985 to 1992.

3. Mathematical Formulation.

We wish to model key features of the interest rate regimes that have been described in section 2. In a general formulation the term structure will need to be a function of several state variables. The discussion above has mentioned informally potential factors such as 'reserves pressure', the intervention repo rate, the prospective intervention rate, etc. Variables such as 'reserves pressure' may well exhibit a cyclical pattern, becoming particularly pronounced towards the end of a reserves maintenance period. This effect may have practical significance, because of its possible implications for interest rate behaviour, and could be explicitly modelled. We shall ignore it in this paper.

We have also seen the need to include jump variables into the model, representing, for instance, potential floor and ceiling rates. In the examples we saw above, neither the intervention rate nor the floor or ceiling rates need be as short as overnight. In Germany the intervention rate is normally a two week rate, while in France the ceiling rate is approximately a one week rate. This leads to the existence of 'soft' floors or ceilings. In this paper we assume that all official rates are instantaneous rates. Thus we cannot explicitly account for the existence of 'soft' floors or ceilings. We can do so implicitly, however, by allowing bounds to be breached because of 'liquidity factors'. Where an official short rate, realised through the use of overnight repos, for instance, is constrained to admit bounds, an overnight interbank rate may not be constrained to lie within these bounds. This is discussed more fully in section 4.

We allow all coefficient functions and intensities to be functions of the state variables, the official rates, and time. We suppose that there are N diffusion state variables in the economy,

$X_i, \ i = 1, \ldots, N.$

In addition we suppose there are M state variables in the economy following pure jump processes,

$Y_j, \ j = 1, \ldots, M.$

In this paper we restrict to jump processes each of whose jump sizes belong to a known finite set. This is not a severe restriction since we shall use the jump variables to model official interest rates such as the Lombard or discount rate. These rates typically jump in a small number of multiples of a minimum jump size, such as 0.25%.

We allow the short rate r to be a function of the state variables. Babbs and Webber took M = 1 and identified the short rate with $Y = Y_1$. Other one or two factor models of the term structure may identify r with a diffusion state variable. For instance, both Fong and Vasicek (91)(92) and Brennan and Schwartz (79) identify one of their two state variables with the short rate. The second state variable is taken to be the short rate volatility for Fong and Vasicek and the yield on a perpetual coupon bond for Brennan and Schwartz. Longstaff and Schwartz (91) set up a model in terms of two state variables representing asset returns related and unrelated to volatility, respectively. In their model the short rate is expressible as a linear combination of the two state variables. Longstaff and Schwartz are able to invert their model to express the underlying two state variables in terms of the short rate and short rate volatility. Longstaff and Schwartz are a special case of the general equilibrium framework of Cox, Ingersoll and Ross (85a)(85b). In that framework the short rate may be expressed as a function of a set of underlying state variables. r is given by

$$r = \frac{1'.\Omega^{-1}.\alpha - 1}{1'.\Omega^{-1}.1}$$

where α is a vector of returns to a set of production processes and Ω is the covariance matrix of returns to the set of production processes. Both α and Ω are functions of the set of underlying state variables. It may be seen that in this framework the short rate r may be an extremely complex, and in general non-linear, function of the state variables. Ahn and Thompson generalise Cox, Ingersoll and

Ross by admitting jump processes with in their specification. It can be shown that with non-stochastic jump magnitudes the short rate is given by

$$r = \frac{1'.\Omega^{-1}.\alpha - 1}{1'.\Omega^{-1}.1} + \frac{1'.\Omega^{-1}.\delta}{1'.\Omega^{-1}.1},$$

where δ is a vector whose components are related to the jump magnitudes and jump intensities.

In our general framework, rather than on the one hand requiring r to be identified as a state variable, or on the other requiring r to have a particular functional dependence upon the state variables determined within an explicit general equilibrium framework, we allow r to be a relatively arbitrary function of the state variables.

In our context the short rate may either be an official rate controlled by the authorities, or it may be a rate partially determined by the market. In the former case the functional form of r represents the authorities' response to market pressure as described by the state variables. In the latter case the market's beliefs about the pattern of the authorities' future behaviour influences the demand for funds of various maturities.

The process for X_i will be written

$$dX_i = a_i(X_1,\ldots,X_N,\ Y_1,\ldots,Y_M,\ t).dt + b_i(X_1,\ldots,X_N,\ Y_1,\ldots,Y_M,\ t).dz_i,$$

where a_i and b_i are functions of the X_i and Y_j, z_i is a standard Wiener process, and $dz_i.dz_j = \rho_{ij}.dt$. Each Y_j will be represented as a sum of counting processes

$$Y_j(t) = Y_j(0) + \sum_{k=1}^{N_j} c_{jk}.N_{jk}(t),$$

where c_{jk}, $k = 1,\ldots, N_j$ are constants, and each N_{jk} is a counting process. c_{jk} is the size of the jump to Y_j that occurs when N_{jk} jumps. Each Y_j has a finite number N_j of allowable jump sizes. Each N_{jk} has an associated jump intensity, λ_{jk}. We allow each intensity to depend upon the full set of state variables and jump processes, and current time,

$$\lambda_{jk} = \lambda_{jk}(X_1,\ldots,X_N,\ Y_1,\ldots,Y_M,\ t).$$

We write

$$B(t,T) = B(t,T,\ X_1,\ldots,X_N,\ Y_1,\ldots,Y_M)$$

for the value at time t of a pure discount bond yielding 1 at time T. Within this framework, under a number of mild restrictions, we are able to price B(t,T) by solving a set of partial differential difference equations.

We wish to allow for the possibility of more than one variable jumping simultaneously. For instance, a monetary authority may wish to change both a ceiling rate and a floor rate at the same time. We incorporate this into the framework as follows. For $j = 1, \ldots, M$ define $c_{j0} = 0$, and set

$$V = \left(\prod_{j=1}^{M} \{0,\ldots,N_j\}\right) \backslash (0,\ldots,0).$$

For $v = (v_1,\ldots,v_M) \in V$, each v_j defines the jump size associated with Y_j.

$$Y_j \rightarrow Y_{v,j} = Y_j + c_{j,vj}, \qquad j = 1,\ldots, M.$$

If $v_j = 0$, then since $c_{j,vj} = 0$, the variable Y_j does not jump. V denotes the full set of possible jumps that may occur simultaneously to the set $\{ Y_1,\ldots,Y_M \}$.

Given $v \in V$, we define the new value of the bond B(t,T) after the set of jumps v has occurred to be

$$J_v(B) = B(t,T,\ X_1,\ldots,X_N,\ Y_{v,1},\ldots,Y_{v,M}),$$

where $Y_{v,j}$ is defined as before.

With this notation we then obtain the following theorem:

Theorem: $B(t,T) = B(t,T,\ X_1,\ldots,X_N,\ Y_1,\ldots,Y_M)$ obeys the following partial differential difference equation:

$$\tfrac{1}{2}\sum_{i=1}^{N} b_i^2 \frac{\partial^2 B}{\partial X_i^2} + \sum_{i,j=1}^{N} \rho_{ij}.b_i.b_j.\frac{\partial^2 B}{\partial X_i \partial X_j} + \sum_{i=1}^{N} a_i^*.\frac{\partial B}{\partial X_i} + \frac{\partial B}{\partial t} +$$

$$\sum_{v \in V} \lambda_v^*.[\, J_v(B) - B\,] = r.B,$$

where $a_i^* = a_i - \theta_i.b_i$,

ρ_{ij} is the instantaneous correlation between z_i and z_j,

θ_i, $i = 1,\ldots,N$,

are (relatively arbitrary) price of risk functions, and

λ_v^*, $v \in V$,

are risk adjusted jump intensities.

This theorem is a simple extension of theorem 6.2 in Babbs and Webber (94). Its proof is omitted.

Note that there is one equation for every possible set of values that the variables $\{ Y_1, \ldots, Y_M \}$ may take.

The mild restrictions mentioned above refer chiefly to issues of limiting behaviour and differentiability. For the purposes of the present paper none of them are crucial.

The importance of the theorem is that we may choose the functions θ_i and λ^*_v essentially freely, while being guaranteed, under the conditions of the theorem, that we have not introduced the possibility of arbitrage. The theorem shows how bonds may be valued as solutions to a set of differential equations. If jumps in two or more variables cannot occur simultaneously the differential equation simplifies to

$$\tfrac{1}{2}\sum_{i=1}^{N} b_i^2 \frac{\partial^2 B}{\partial X_i^2} + \sum_{i,j=1}^{N} \rho_{ij}.b_i.b_j.\frac{\partial^2 B}{\partial X_i \partial X_j} + \sum_{i=1}^{N} a_i^*.\frac{\partial B}{\partial X_i} + \frac{\partial B}{\partial t} +$$

$$\sum_{j=1}^{M} \sum_{k=1}^{N_j} \lambda^*_{jk}.[\, B(t, \ldots, Y_j + c_{jk}, \ldots) - B(t, \ldots, Y_j, \ldots)\,]$$

$$= r.B,$$

where $\lambda^*_{jk} = (1 - \theta_{jk}).\lambda_{jk}$,

and θ_{jk}, $j = 1, \ldots, M$, $k = 1, \ldots, N_j$, are price of risk functions.

When there is one diffusion variable and one jump variable, $N = M = 1$. We reduce to the result of Babbs and Webber (94). The differential equation in this case is:

$$\tfrac{1}{2}.b^2.\frac{\partial^2 B}{\partial X^2} + a^*.\frac{\partial B}{\partial X} + \frac{\partial B}{\partial t} + \sum_{k=1}^{N} \lambda^*_k.[\, B(t, X, Y + c_k) - B(t, X, Y)\,] = r.B,$$

where $a^* = a - \theta_0.b$,

$\lambda^*_k = (1 - \theta_k).\lambda_k$,

and θ_0, and

θ_k, $k = 1, \ldots, N$,

are the price of risk functions. In Babbs and Webber r was identified with Y.

Explicit solutions to these equations are unavailable in general, but they may be solved using numerical methods. Note that if only a limited number of jump sizes are permitted by the model, introducing additional jump variables is far less expensive computationally than introducing additional diffusion variables. Thus, from the point of view of solving the differential equations, we have only a limited reluctance to increase the number of jump variables.

4. Application to Selected Regimes

We illustrate how the system developed above might be used to model interest rates in idealisations of the regimes outlined in section 2. Initially we set up a general model for which the illustrative regimes may be considered as special cases.

We shall take $N = 1$. As in Babbs and Webber (94), the sole diffusion state variable will be the 'prospective' level of the intervention rate within the economy: X_1 follows a mean reverting process, similar to an interest rate. Jump intensities are set so that the short rate tends to jump to the vicinity of X_1. Thus X_1 represents a likely level of the short rate in the near future. We discuss below the possibility of introducing a second diffusion state variable to model a money market liquidity factor representing the residual demand for funds in the economy remaining after official intervention.

We identify $D = Y_1$ with an official floor, and $L = Y_2$ with an official ceiling, where this exists. The notation D and L is adopted because of the use of the term 'discount rate' for the floor rate, and because in the German system the ceiling rate is called the Lombard rate. In an economy where the short rate is bounded by official action we allow the short rate and its intensities to be functions of X_1, D and L. The functional form of r, and the structure of the jump intensities for D and L, will ensure that r remains bounded by D and L.

We do not exclude the case where the short rate is an official rate following a third jump process. We then set $Y_3 = R$, and

$$r = r(X_1, X_2, D, L, R) \equiv R.$$

In this case we may wish to constrain the process for R to ensure that r cannot leave the interval [D,L].

The dynamics of the model are contained in the specification of the processes of X_1, D, L, and any other state variables.

Outside of the scope of our discussion is the effect of changes to the monetary regime. For instance, in periods of intense pressure the Banque de France may temporarily suspend the ceiling rate, allowing it to fix the short rate at arbitrarily high levels using overnight repos. However, by leaving the floor rate unchanged, it signals that its underlying stance remains unaltered.

Similarly, the Bundesbank adopts one of two possible regimes, fixed or floating, and may switch between them. In this paper we consider each regime separately. A full treatment would require the possibility of regime switches to be explicitly considered and incorporated within the model.

The Prospective Intervention Rate.

The prospective interest rate, as described above, represents the market's view on anticipated levels of interest rates. The prospective rate reflects the market's beliefs of the level of the intervention rate in the near future. Alternatively, the prospective rate may be viewed as a quantification of the notion frequently expressed by market practitioners that 'interest rates are too low' or 'too high'. It also may be interpreted as the level the short rate would be expected to have, were it not constrained by official action.

We require the prospective rate to be a diffusion process. It is reasonable to write the process for X_1 as a standard mean reverting process. However, we explicitly allow the reversion level of X_1 to depend upon D and L. This permits feed back between the controlled levels of D and L, and the state of the economy as epitomised by X_1. Feed back may be posited to take two forms. Firstly the long range effects on the economy caused by shifts in levels of rates. Secondly, and perhaps more importantly in the short term, the effect that changes in D and L have upon market expectations of future interest rate levels. This signalling affect is widely recognised as being important by practitioners.

For illustrative purposes we adopt a process for X_1 of the following form:

$$dX_1 = a.(b.\mu + c.D + d.L + e.r - X_1).dt + \sigma_1.X_1.dz_1,$$

$$a,\ b,\ c,\ d,\ \sigma_1 > 0,$$

$$b + c + d + e = 1.$$

X_1 reverts to a level given by a weighted average of D, L, r, and constant μ. An increase in either D or L causes X_1 to revert to a higher level. A decrease in D or L causes X_1 to revert around a lower level. Adjusting the weightings allows the strength of a signal to be varied to suit a particular regime. r is included to represent the feedback between the level of r in the economy and the process followed by the state variable denoting the state of the economy. e may be positive of negative depending on the nature of the feedback. The form of the volatility term is not central to this paper.

The Liquidity Factor.

The overnight rate may be modelled by including a second diffusion factor X_2. X_2 may be taken to represent either the actual liquidity pressure within the economy, or it may simply be taken to be a noise term added on to the official short rate to give the overnight rate. The latter route has the advantages of simplicity. The former may allow a deeper analysis into the formation of the overnight rate.

If the liquidity factor is represented purely as an error term we could write

$$dX_2 = -a_2.X_2.dt + \sigma_2.dz_2,$$

and set

$$r_{overnight} = r + X_2.$$

$r_{overnight}$ could then be used in the differential equation instead of r. However, this may not lead to further explanatory power. We anticipate that the reversion rate a_2 is high, and that the volatility σ_2 is comparatively high, compared to estimations of the drift and volatility of three month money market rates, for instance.

We discuss a liquidity factor for generality only. In practice we do not expect this factor to significantly influence the term structure beyond the extremely short end, although it could be relevant to the valuation of derivatives on repos or overnight rates. Although an official short rate may be constrained to lie with in the interval [D,L], an overnight rate may not be so constrained. On occasion, overnight rates in both Germany and France have been observed at values outside the interval, and for the dollar, the fed funds rate has fluctuated below the floor level defined by the discount rate. For sterling, the overnight rate is not a pure jump process. This effect, where relevant, should be modelled.

The Floor and Ceiling rates, D and L

We consider D and L together, although L may be absent in some economies, notably the US. All of the economies considered in section 2 maintain an official interest rate at levels less than the official short rate. The floor rate, D, and the ceiling rate L, where present, are pure jump processes, but their jump intensities will be influenced by the levels of the state variables in the economy. For instance, we prohibit jumps by D that would leave it at a value greater than that of the short rate r. Jumps are not precluded, however, if a functional dependence of r on D takes r to a level in excess of D when a jump occurs. Nor is D allowed to jump so that D > L, when a ceiling level L exists. Similar observations apply to L. The form of the intensity functions are highly dependent upon the particular interest regime. See Babbs and Webber (95) for a discussion in the UK case.

Write λ_{Dj} for the jump intensity functions of D, $j = 1,\ldots,N_j$, and write λ_{Lj} for the jump intensity functions of L. We shall consider the functions λ_{Dj}, but similar arguments apply to λ_{Lj}.

One may suppose that differing policies could result in several alternative forms of intensities λ_{Dj}. It seems reasonable to express λ_{Dj} in terms of the offset between D and each of X_1, r, and L. L would be included to reflect the possibility that D and L tend on average to have a fairly constant spread, that is, L - D approximately constant. One may set up the jump intensities to reflect this. A model that includes a liquidity factor X_2 might allow λ_{Dj} to depend on X_2. For instance, if an overnight rate $r + X_2$ were persistently below D, a monetary authority might be unable to maintain control over the official short rate. If might then be forced to reduce D. The market's expectation of a change in D, reflected in this model by the specification of the intensities for D, would legitimately depend upon X_2 as well as X_1.

In the appendix we illustrate how several differing policy stances by the monetary authorities could be modelled. These policies are:

i) A 'smoothing' policy.
The authorities wish to allow the short rate to generally follow the rate X_1,
but wish to reduce the volatility.

ii) A 'static' policy.
The authorities wish to keep the short rate within a static band,
now and again permitting the band to shift when it can no longer be sustained.

iii) A 'nudging' policy.

The authorities wish the short rate to tend in the long term to some level μ_2.

They encourage moves towards μ_2, and discourage moves away from μ_2.

These three possibilities are included to show the range of behaviour that might plausibly be modelled within this context.

We now briefly consider each of the economies described in section 2 with a view to incorporating them into the context of the general model described above. Since our interest is on the yield curve beyond very short maturities, in the illustrations below we prefer to exclude a 'liquidity factor' state variable, X_2. However we shall mention this factor where it may shed light upon the behaviour of rates of interest to us.

We assume that all rates are instantaneous, and that they are continuously available. This implies that all floor and ceiling rates are 'hard', since they have the same maturity as the short rate.

United Kingdom

Take $N = M = 1$; one diffusion variable and one pure jump variable. Set $r = Y_1 = Y$. We identify the short rate with the jump variable. In Babbs and Webber the diffusion variable X was taken to be a 'prospective' interest rate. Jump intensities reflect market expectations of jumps occurring as functions of the difference in level between the prospective level of interest rates and the current value of the short rate.

France

Take D to be the taux des pensions appel d'offres. We have seen that the taux des pensions de 5 a 10 jour tends to remain at a relatively fixed premium above D. It is therefore not an unreasonable approximation to suppose that the ceiling rate L is a fixed amount above D. As a consequence only one jump variable, D, is required. The diffusion variable X is taken to be a prospective level of interest rates.

We identify the overnight rate with the short rate, and set

$r = \max(D, \min(X, L))$.

If we were to allow the presence of a significant liquidity factor, X_2, following a process mean reverting to zero, one might set r to be

$$r = \max(D, \min(X + X_2, L)).$$

Prior to the regime shift in the French market towards the end of 1991 the overnight rate was very volatile. With a lquidity factor this could be modelled by attributing most of the volatility in the overnight rate to the liquidity factor. The regime shift is then consistent with a reduction in the volatility of the liquidity factor.

Germany

D and L provide hard boundaries, so that the short rate is constrained to lie between them. We identify the short rate with the intervention rate.

In the floating rate regime the intervention rate may be modelled as the diffusion variable, $X = X_1$. In the fixed rate tender regime, the short rate is effectively a third pure jump process, R. In either case the short rate may be set equal to the middle of D, L, and the intervention rate;

$$r = \max(D, \min(X, L)), \text{ in a floating regime,}$$

$$r = \max(D, \min(R, L)), \text{ in a fixed regime.}$$

United States

Two alternative models may be considered for the fed funds rate.

The first ignores the discount rate and models the fed funds rate as a pure jump process with liquidity factor noise. There is a single jump variable, the fed funds target rate, R, and two diffusion processes, the prospective rate, X_1, and the liquidity factor, X_2. The liquidity factor may be dropped if one is not interested rates of very short maturities. In this case we are reduced to the situation of Babbs and Webber.

The second takes the discount rate into account as a signalling factor. Then there is an additional jump variable, D. Depending on the regime operated by the fed, D may sometimes be considered as a hard lower bound on the short rate. At other times the regime in operation may permit the short rate to fall below D.

5. Illustrative Example

As an illustration of the model we solve numerically the partial differential difference equation in a case where the short rate is constrained to lie within an official corridor. In practice, the floor and ceiling rates tend to maintain an approximately constant interval between them. For simplicity, in this illustration, we presume that D and L always jump simultaneously and by the same amounts. In effect there is a single jump variable, which we take to be D. We set

$$L = D + 0.01$$

to maintain a corridor of constant width 1%. This form of regime is similar to the French and German regimes. Although in neither case is the assumption exact, nevertheless we believe it to be a reasonable approximation, particularly for the French case.

We take a single diffusion state variable, $X = X_1$, representing a prospective interest rate. The process for X is taken to be

$$dX = 0.1 \times (\tfrac{1}{2}.(0.085 + D) - X).dt + 0.015.dz.$$

Feedback between the official rate D and the state variable X is incorporated by allowing X to revert to a weighted average of D and a long term level of 8.5%.

The short rate is set equal to X capped by D and L.

$$r = \max(D, \min(X, L)).$$

This corresponds to a situation where the authorities allow the short rate to be determined by market forces while it lies within the official interval. When the short rate reaches the boundary of its permitted band the authorities intervene to prevent it moving outside the band. This is an extreme case. In practise the authorities may decide to intervene before the short rate reaches the edges of the band. In France and Germany the authorities may attempt to maintain the short rate near the centre of the band.

We allow D to jump by four possible magnitudes; ±1% and ±½%. These sizes are large compared to values typically seen in France and Germany, but they suffice for this illustrative example. The jump intensities are taken to be functions of $X - \tfrac{1}{2}.(D+L)$, that is, functions of the offset between X and the centre of the corridor. The functional form of the intensities λ_j are shown in figure 5. These are set up so that the jump which is most likely to occur is the one that takes X to being closest to the centre of the corridor. The parameter h determines the expected number of jumps per year. it was given a value corresponding to a small number, 2 or 3, jumps a year per year on average.

The PDDEs were solved for various initial values of X and D. The results are presented in figures 6, 7 and 8, where the spot rates derived from the computed values of pure discount bonds are shown. For these figures the prices of risk were set to zero. Other term structures, with non–zero values for prices of risk, were computed. The results were qualitatively similar to those in the figures presented here, which may be taken to be representative.

The short rate is constrained to lie inside the official corridor, and the asymptotic values of the term structure at the long end are determined by the value of the long run reversion level, 8.5%. Medium term levels of spot rates are determined by the initial value of X. X denotes a prospective short rate, so the numerical results informally bear out a version of an expectations hypothesis that equates spot rates to the expected future levels of the short rate.

Figure 6 shows a number of term structures for an initial corridor of 8% – 9%, and various initial values of X, ranging from 7% to 10%. The long run value of 8.5% lies within the initial range of the corridor. For these values the term structures are relatively flat.

The effect of jumps in the value of the corridor may be seen in figure 7. Each term structure in figure 7 has the same initial value of X, but is constrained by a different initial corridor. The bottom term structure belongs to an interval of 7% – 8%. The top term structure to an interval 9.5% – 10.5%. The three term structures with the same short rate value of 8.5% lie in the intervals 7.5% – 8.5%, 8% – 8.5%, and 8.5% – 9.5%. Jumps from one of these intervals to another does not cause the short rate to jump, but nevertheless cause the term structures as a whole to shift. Jumps in D that leave the value of r unchanged will cause the term structure to change shape. This corresponds to observed behaviour in the markets, where changes in D are interpreted as signals about future intentions. We have built the market's interpretation into the formulation of the illustrative model. The effects observed in this illustrative example are not great, but they are real. Taking other choices of parameters leads to more acute behaviour.

A special effect may be seen at the short end when the short rate lies close to D or L. For example, if r has a value just less than L when X is larger than L, then the term structure may dip at very short maturities before rising towards X. This is illustrated in figure 8, which shows a number of term structures for a corridor of 8% – 9%. X ranges between 8.75% and 9.15%. A dip can be observed even when X lies 0.15% above L. The size of the dip may exceed 10 basis points. This surprising effect may be interpreted in terms of the likely future value of the short rate. r is capped by L. Because only a few jumps per year are anticipated in the short run r is unlikely to be permitted to exceed the current value of L. Hence the expected future value of r, in the short run, may less than its current value. In the medium term a jump is increasingly likely to have occurred. Here the expected future value of r may rise to the vicinity of X.

6. Conclusion

We have seen how within several important interest rate regimes the short rate may be modelled as a process bounded by officially controlled levels. Changes to these levels pass signals to the markets concerning anticipated future interest rate levels. Although the details vary from regime to regime all of the examples given here fit into a common pattern. We have seen how very short term money market rates are sensitive to 'liquidity' or 'forecast error' pressure. In particular it seems appropriate to use as a surrogate for the short rate not an overnight money market rate, but the intervention level of the monetary authorities within the economy. An overnight rate can then be modelled as fluctuations away from the official short rate. We have seen how changes to a boundary may act as a signal to the market and cause a change to the shape of the term structure, even through the monetary authorities may not have altered the level of the official short rate. We obtain qualitatively different term structure behaviour from this model to that seen in other formulations. We expect our formulation to have implications for hedging and valuing derivative securities based on short maturity interest rates.

References

[AT] **Ahn & Thompson**, Journal of Finance, 431(88), 155–174.
Jump–Diffusion Processes and the Term Structure of Interest Rates

[BW94] **Babbs and Webber**, Forc working paper, 94/49, (94)
A Theory of the Term Structure with an Official Short Rate

[BBF93] **Balduzzi, Bertola and Foresi**,
National Bureau of Economic Resesarch, working paper, 4347, (93)
A Model of Target Changes and the Term Structure of Interest Rates

[BKR] **Bjork, Kabanov & Runnaldier**, Stockholm School of Economics, Working paper, (95)
Bond Markets when Prices are Driven by a General Marked Point Process

[BrS] **Brennan & Schwartz**, Journal of Banking and Finance, 3(79), 133–155.
A Continuous Time Approach to the Pricing of Bonds

[CIRa] **Cox, Ingersoll & Ross**, Econometrica, 53(85),363-384
An intertemporal general equilibrium model of asset prices.

[CIRb] **Cox, Ingersoll & Ross**, Econometrica, 53(85),385-407
A theory of the term structure of interest rates.

[DK] **Duffie and Kan**, GBS Stanford University working paper, 1993.
A Yield Factor Model of Interest rates

[FV91] **Fong and Vasicek**, Journal of Portfolio Management, Summer, 1991, 41–46.
Fixed Income Volatility Management

[FV92] **Fong and Vasicek**, Gifford Fong Associates Working Paper, 1992.
Interest Rate Volatilty as a Stochastic Factor

[HJM] **Heath, Jarrow & Morton**, Econometrica, 60(92), 77–105
Bond Pricing and the Term Structure of Interest Rates:
A New Methodology for Contingent Claims Evaluation.

[LS] **Longstaff & Schwartz**, Journal of Finance, 47(91), 1259-1282
Interest Rate Volatility and the Term Structure:
A Two Factor General Equilibrium Model.

[Sc] **Schnadt**, The City Research Project,
Subject report VII, Paper I, Corporation of London, (94)
The Domestic Money Markets of the UK, France, Germany and the US.

[Sh] **Shirakawa**, Mathematical Finance, 1(91), 77–94
Interest Rate Option Pricing with Poisson–Gaussian Forward rate curve Processes

Appendix

In this appendix we illustrate how three different policy stances might be modelled. The three policies are:

i) A 'smoothing' policy.
   The authorities allow the short rate to follow X_1 but with reduced volatility.

ii) A 'static' policy.
   The authorities wish to keep the short rate within a static band,
   shifting the band when it can no longer be sustained.

iii) A 'nudging' policy.
   The authorities wish the short rate to tend in the long term to some level μ_2.

These different policies might be reflected in the model as follows. It is assumed that the markets are aware of the nature of the policy stance adopted by the authorities, and that signals are interpreted correctly. Authorities may attempt to 'defend' boundaries, that is, attempt to prevent the short rate from crossing them, with various degrees of vigour. Although an authority may always defend a boundary, we have previously noted that there is a cost to doing so. An authority may not be able to sustain a boundary. We assume that if this is the case then the boundary is shifted, permitting the authorities the opportunity to allow the short rate to cross the previous boundary level. We assume that the effect of a defence of a boundary is to dampen the volatility of r as x approaches the boundary, D say. We interpret this as follows. The volatility of r as a function of that of X_1, in the absence of jumps, is given by

$$\sigma_r = \frac{\partial r}{\partial X_1} \cdot \sigma_{X_1}.$$

If we assume that $\frac{\partial r}{\partial X_1}$ is positive, and intervention reduces the volatility of r as r approaches D, then

$\frac{\partial r}{\partial X_1}$ must decrease as r decreases towards D, that is,

$$\frac{\partial^2 r}{\partial X_1^2} > 0.$$

Similarly, as r approaches a ceiling, L, we have $\frac{\partial r}{\partial X_1}$ decreases as r increases towards L, that is,

$$\frac{\partial^2 r}{\partial X_1^2} < 0.$$

We assume that the greater the intervention by the authorities, that is, the stronger the defence of the boundary, the greater the dampening effect. With no attempt to defend, r might depend on D, L and X_1 as

$$r = \max(D, \min(X_1, L)).$$

If both D and L were equally defended, r might take the form

$$r = X_1 + D - L + c(X_1, D, \sigma) - c(X_1, L, \sigma),$$

where

$$c(X_1, D, \sigma) = X_1.N(d_1) - D.N(d_2)$$

$$d_1 = \tfrac{1}{\sigma}.\ln \tfrac{X_1}{D}, \quad d_2 = d_1 - \sigma,$$

and N(d) is the cumulative normal density function.

This functional form has been chosen for illustrative purposes only. The value of the parameter σ reflects the amount of the dampening caused by the defence of D and L. When $\sigma \rightarrow 0$, r tends to the function

$$r = \max(D, \min(X_1, L)).$$

As σ increases, r exhibits greater and greater dampening as it approaches D or L. The formula may be modified to allow differing degrees of dampening as r approaches D and L. In this case there will be two parameters, σ_D and σ_L, determining the dampening at D and L respectively.

The more strongly defended a boundary is, the less likely the authorities are to shift the boundary when approached by r.

i) A Smoothing Policy

The authorities may defend D and L only lightly, being prepared to shift them if X_1 remains away from defensible levels. Shifts in D and L are not seen as strong signals. Set

σ_D and σ_L small:

Little dampening near D or L.

$dX_1 = a.(b.\mu + c.D + d.L + e.r - X_1).dt + \sigma_{X_1}.X_1.dz$, with c and d small

Little feedback between D and L, and the process for X_1.

$\lambda_{Dj} = 0$ for jumps up, unless i) r is close to L,

or ii) r is further away from D than some threshold

λ_{Dj} = is small for jumps down, unless r approaches D

When r approaches close to D, jumps down become increasingly certain.

r is dampened only slightly by the actions of the authorities to defend the official rates. However, when a jump to D or L occurs r will jump, if only slightly.

ii) A Static Policy

The authorities may defend D and L strongly, being prepared to shift them only if X_1 approaches close to D or L. Shifts in D and L are seen as strong signals. Set

σ_D and σ_L large:

Significant dampening near D and L.

$dX_1 = a.(b.\mu + c.D + d.L + e.r - X_1).dt + \sigma_{X_1}.X_1.dz_1$, with c and d large

Significant feedback between D and L, and the process for X_1.

$\lambda_{Dj} = 0$ for jumps up, unless i) r is close to L,

or ii) r is further away from D than some threshold

λ_{Dj} = is small for jumps down, unless r approaches close to D.

When r approaches sufficiently close to D, jumps down become increasingly certain.

When a jump occurs both L and D jump to place r in roughly mid-band.

r is dampened considerably by the actions of the monetary authorities. When a jump occurs the largest reasonable jump is made. The rate structure is re-aligned.

iii) A Nudging Policy

We suppose that the desired level of r, μ_2, is outside the current band,

$\mu_2 \notin [D,L]$.

If the short rate is moving towards the desired direction when it reaches a boundary (the 'near' boundary) that boundary will not be defended. If the boundary in the direction away from the desired level (the 'far' boundary) is approached the authorities will defend it to some extent. Shifts in the far boundary are seen as strong signals, shifts in the near boundary as weak signals. (We do not consider the form of action the authorities might take if μ_2 lies within the current band). Set

σ_{far} large, σ_{near} small:

Significant dampening at the far boundary, little at the near.

$dX_1 = a.(b.\mu + c.D + d.L + e.r - X_1).dt + \sigma_{X_1}..X_1.dz_1$,

with c and d functions of r, D and L so that

c or d large for the far boundary, small for the near boundary.

Signalling is significant when r is close to the far boundary,

not so important when r is close to the far boundary.

$\lambda_{far,j} = 0$ for jumps towards,

unless i) r is close to L,

or ii) r is further away from D than some threshold

$\lambda_{far,j}$ is small for jumps away, unless r approaches close to the far boundary.

When a jump to the far boundary occurs,

the near boundary may jump by a similar amount.

$\lambda_{near,j} = 0$ for jumps away, unless r is close to the far boundary

$\lambda_{near,j}$ is small for jumps towards, unless r approaches to the near boundary.

When a jump by the near boundary occurs,

the far boundary may jump by a similar amount.

Figure 1

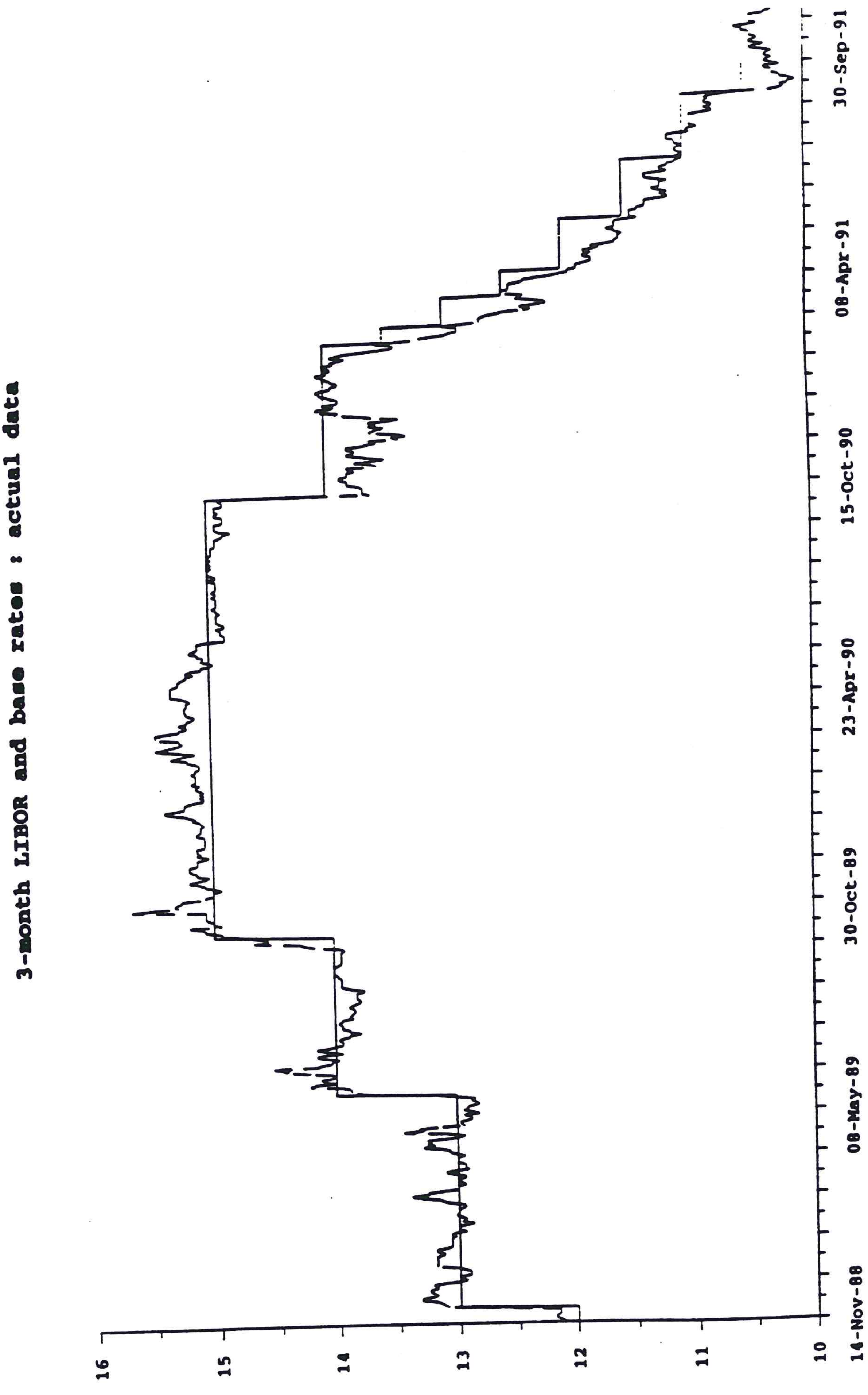
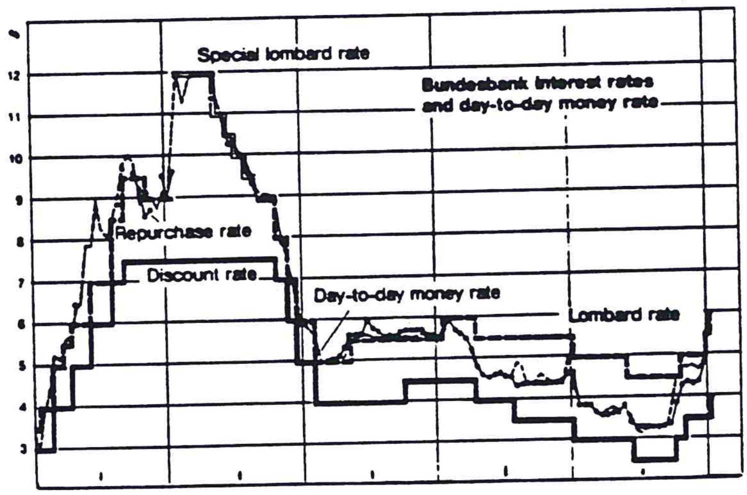

3-month LIBOR and base rates : actual data
16
15
14
13
12
11
10
14-Nov-88
08-May-89
30-Oct-89
23-Apr-90
15-Oct-90
08-Apr-91
30-Sep-91

Figure 2

(Taken from Schnadt (94))

German Day to Day Interbank Interest Rate 1979 - 1989

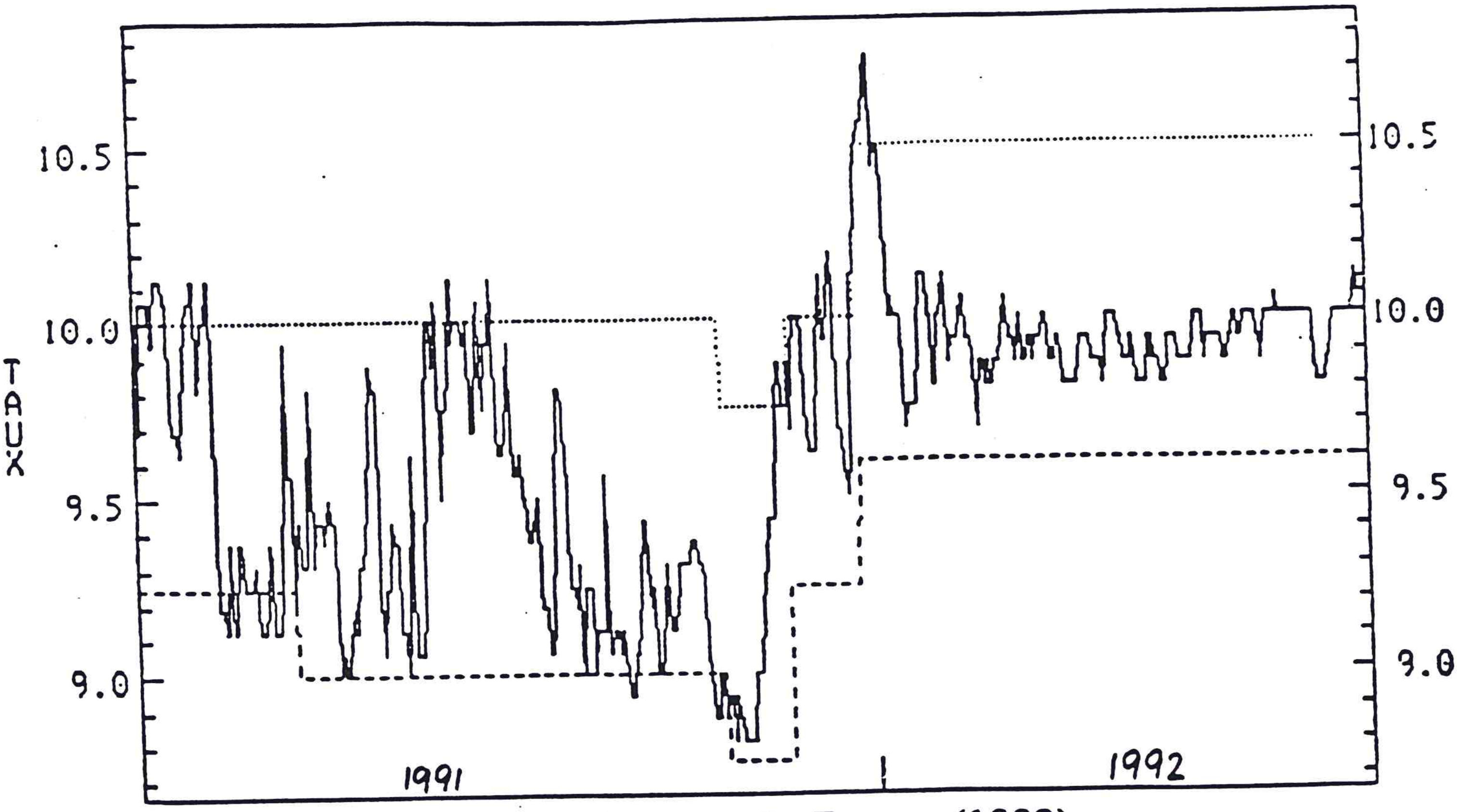


Source: Deutsche Bundesbank

Figure 3

(Taken from Schnadt (94))

French Interbank Rates 1991 - 1992

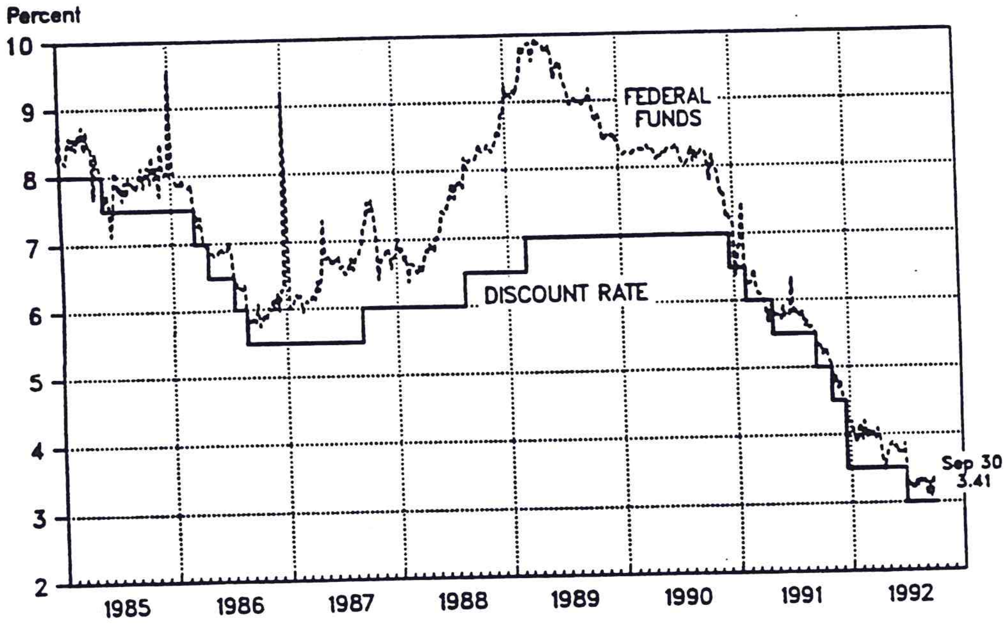


Source: Banque de France (1992)

Figure 4

(Taken from Schnadt (94))

The Fed Funds Rate and Discount Rate 1985 - 1992



Source: Federal Reserve

Figure 5

λ a function of X - D

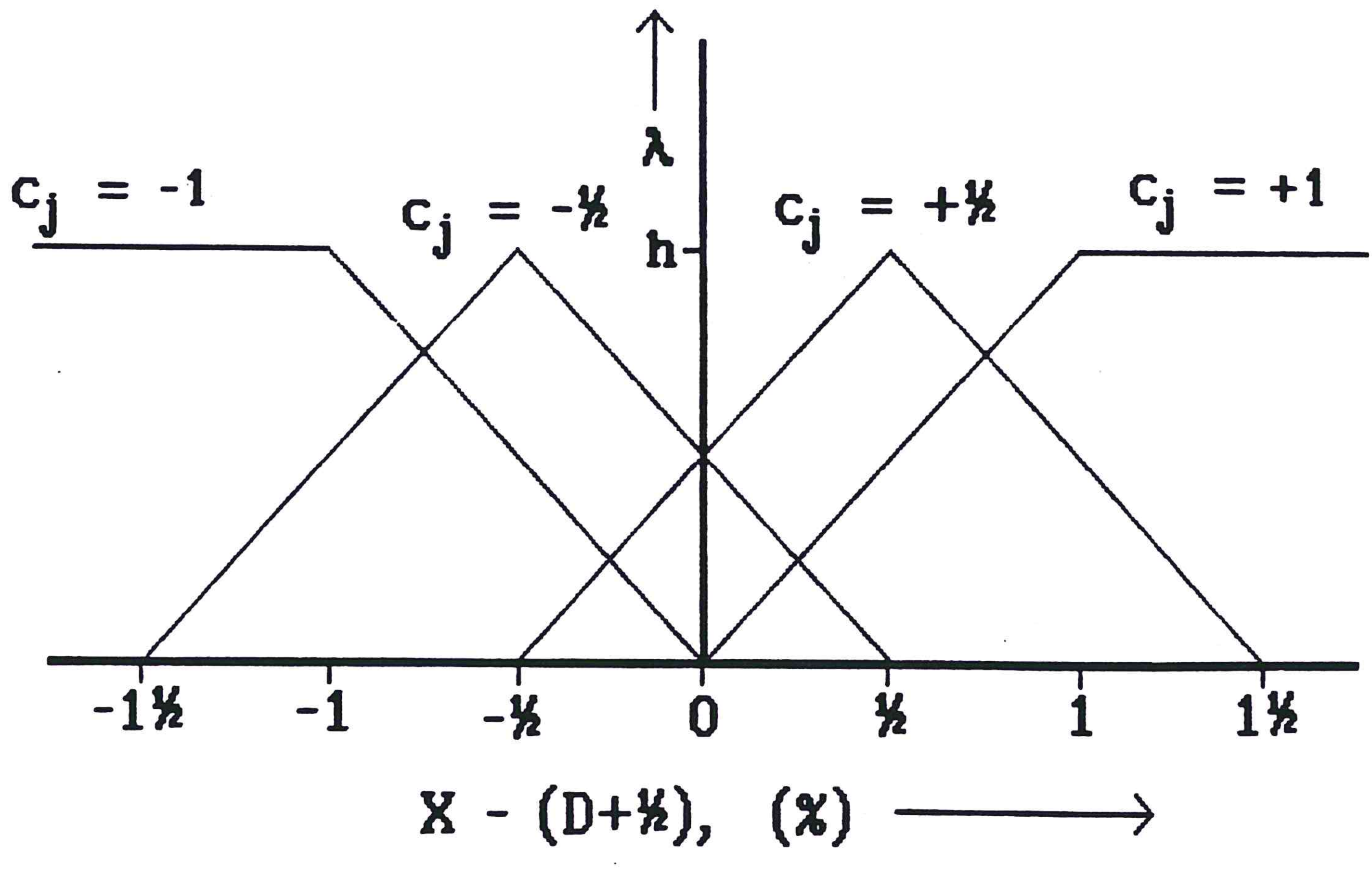


Figure 6

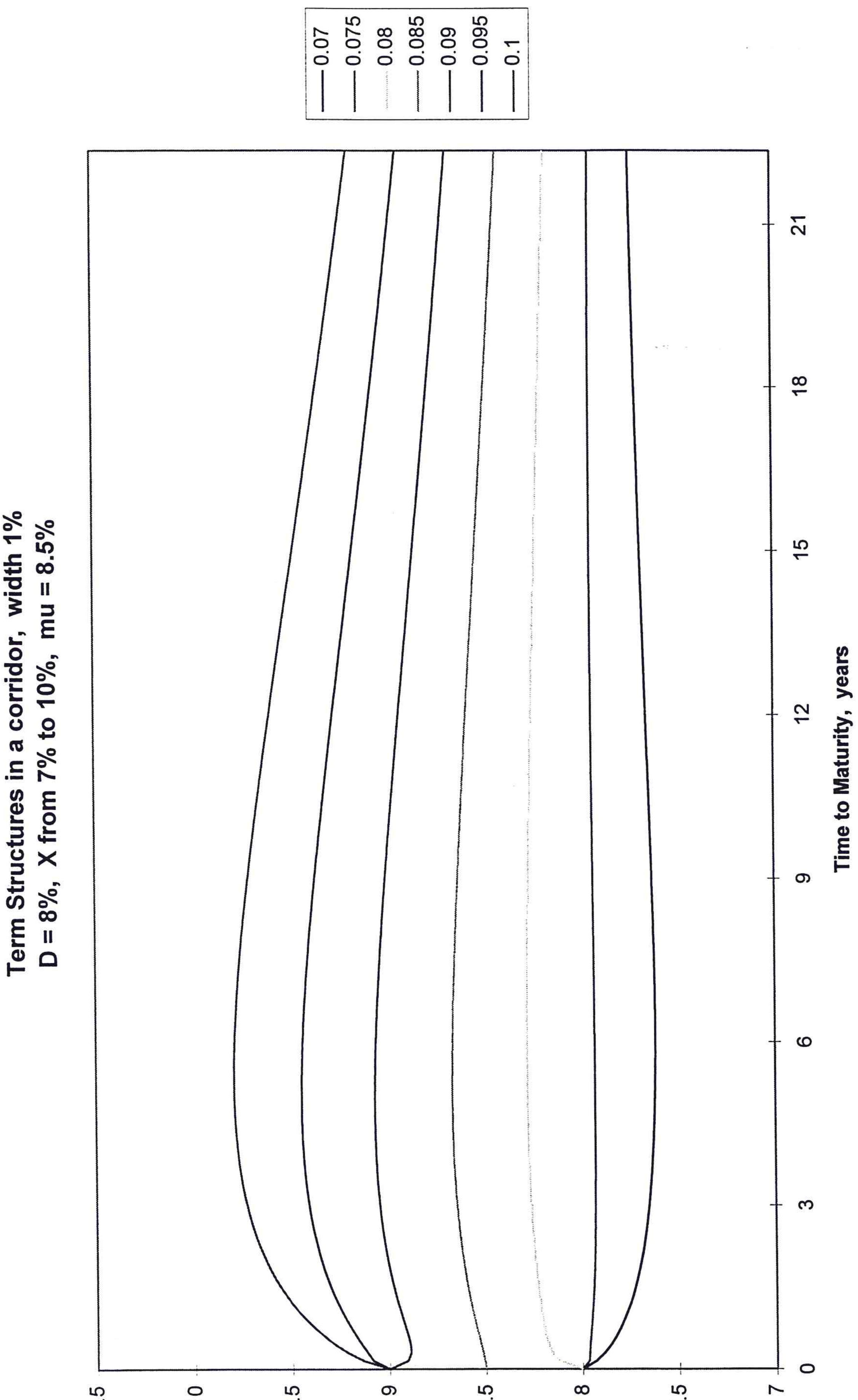
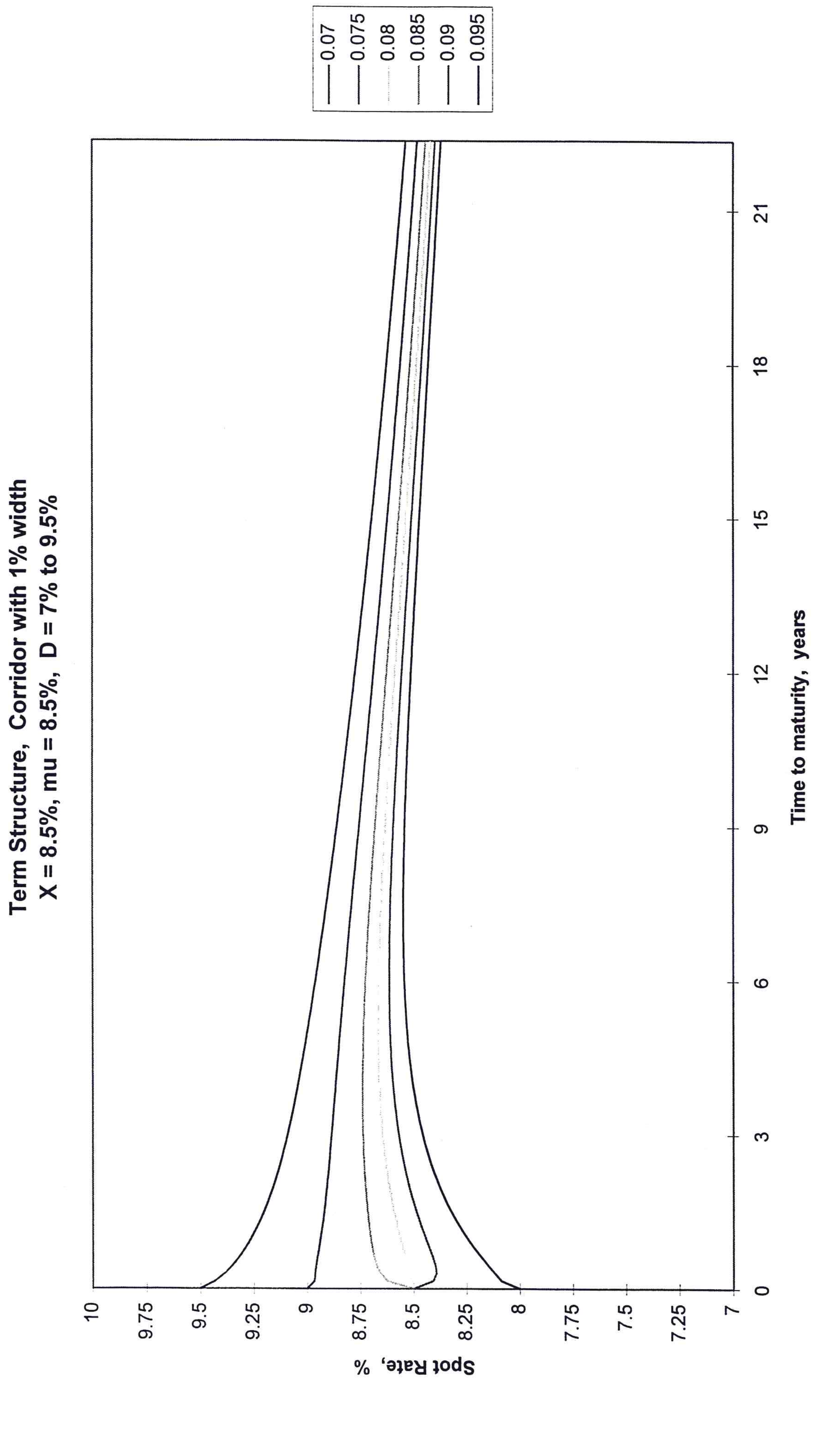
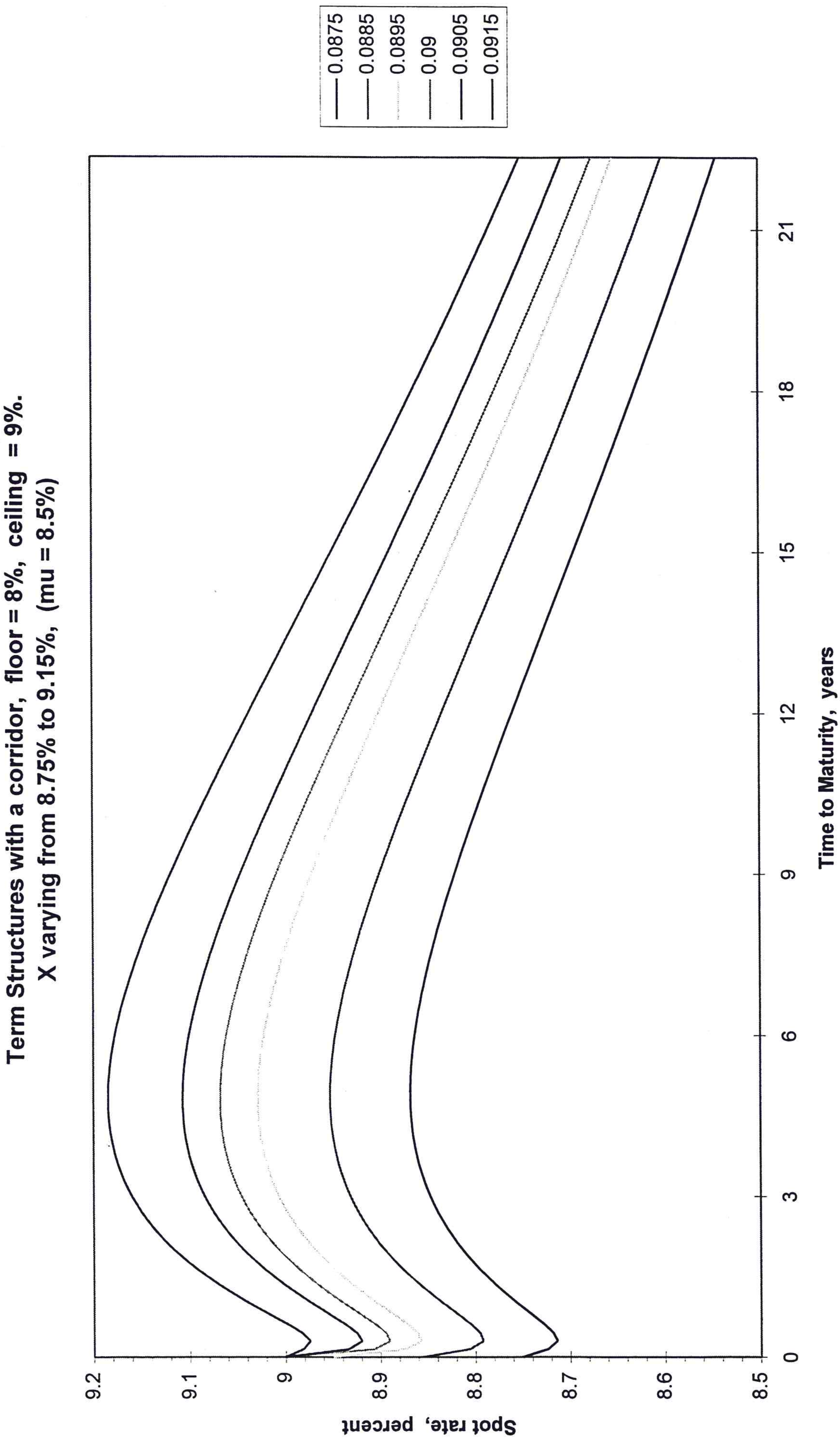

Term Structures in a corridor, width 1%
D = 8%, X from 7% to 10%, mu = 8.5%
Spot rate, percent
Time to Maturity, years
10.5
10
9.5
9
8.5
8
7.5
7
0
3
6
9
12
15
18
21
0.07
0.075
0.08
0.085
0.09
0.095
0.1

Figure 7



Term Structure, Corridor with 1% width
X = 8.5%, mu = 8.5%, D = 7% to 9.5%
0.07
0.075
0.08
0.085
0.09
0.095
Spot Rate, %
10
9.75
9.5
9.25
9
8.75
8.5
8.25
8
7.75
7.5
7.25
7
0
3
6
9
12
15
18
21
Time to maturity, years

Figure 8



Term Structures with a corridor, floor = 8%, ceiling = 9%.
X varying from 8.75% to 9.15%, (mu = 8.5%)
Spot rate, percent
9.2
9.1
9
8.9
8.8
8.7
8.6
8.5
0
3
6
9
12
15
18
21
Time to Maturity, years
0.0875
0.0885
0.0895
0.09
0.0905
0.0915