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Abstract

Analytical formulae exist for continuously fixed barrier options under the standard Black-Scholes assumptions and this implies a hedging strategy equivalent to the Black-Scholes delta hedging strategy for standard options. However, the key assumptions of geometric Brownian motion, continuous trading and no transactions costs do not hold in reality. In real markets jumps in asset prices occur, volatility changes randomly, the hedging strategy can only be implemented in discrete time and transaction costs are incurred each time the hedge is rebalanced. For barrier options reasonable risk reduction implies such frequent rebalancing of the hedge, particularly near the barrier, that the transaction costs make this impractical. Recently, the construction of static portfolios of standard options which replicate the value of a barrier option within a binomial or trinomial tree have been proposed. However, this approach has problems; the tree cannot fully reflect the jumps and stochastic volatility, they do not take into account transaction costs and most importantly they do not allow for adjusting the portfolio. The requirement to adjust the replicating portfolio implies a trade-off between the risk reduction and the transaction costs incurred. In this paper we propose a Monte Carlo simulation based myopic optimisation model to control transaction costs and we compare the performance of this model against naive Black-Scholes delta and delta-gamma hedging, and static replicating portfolios in the presence of jumps or stochastic volatility.

1. Introduction

Analytical formulae exist for continuously fixed barrier options under the standard Black-Scholes assumptions which also implies a hedging strategy equivalent to the Black-Scholes delta hedging strategy for standard options. However, this relies critically on the assumptions of geometric Brownian motion, continuous trading and no transactions costs. In reality asset prices have jumps, volatility changes randomly, the hedging strategy can only be implemented in discrete time and transaction costs are incurred each time the hedge is rebalanced. For standard options this leads to Black-Scholes delta hedging and higher order hedges such as delta-gamma hedging being sub-optimal and dominated by optimal strategies which take account of the market imperfections (Hodges and Neuberger (1989), Hodges and Clewlow (1993), Clewlow and Hodges (1994)). For barrier options reasonable risk reduction implies such frequent rebalancing of the hedge, particularly near the barrier, that transaction costs make the Black-Scholes hedging approach impractical. Recently, the construction of static portfolios of standard options which replicate the value of a barrier option within a binomial or trinomial tree have been proposed (Dupire(1994), Derman *et al* (1994)). However, these approaches have problems; the tree cannot fully reflect the jumps and stochastic volatility, they do not take into account transaction costs and they do not allow for adjusting the portfolio which is essential in the presence of stochastic volatility and jumps. The requirement to adjust the replicating portfolio implies a trade-off between the risk reduction and the transaction costs incurred. In this paper we propose a Monte Carlo based myopic optimisation model to control transaction costs and we compare the performance of this model against naive Black-Scholes delta and delta-gamma hedges, and static replicating portfolios in the presence of jumps or stochastic volatility.

The motivation for this work stems firstly from an earlier empirical analysis (Hodges, Clewlow *et al* (1993)) of hedging CME options on the S&P500 futures. This study confirmed that gamma hedging provided a significant risk reduction compared to delta hedging, and did so even when rebalancing was implemented daily. The reason why the improvement is so marked appears to be because real asset processes involve jumps and stochastic volatility. However, the study also showed that this kind of standard gamma hedging involved high turnover of options positions - which with transactions costs would make the hedge prohibitively costly.

We therefore have the following problem: What is the best way to manage gamma when options are costly to transact and we have jumps and/or stochastic volatility?

We have some insights into this problem from the literature on optimal delta hedging under transactions costs. The gamma hedging problem, though, is very much harder since, unlike delta, there are many different candidate instruments that could be used to modify gamma. We would have many state variables to solve for using a dynamic programming formulation (as, for example, in Hodges and Neuberger (1989) and Hodges and Clewlow (1993)), though it could of course be written in that framework in a formal sense. Clewlow and Hodges (1994) used a simulation approach to search for optimal methods of hedging based on heuristics which are consistent with myopic policies. We extend that approach here to the situation where an investment bank has written and needs to hedge barrier options using market traded standard options.

The structure of the paper is as follows. In the following section we review some of the previous theoretical and empirical work on hedging which is relevant to the problem studied here. Section 3 describes our methodology for implementing almost optimal delta-gamma hedges in the presence of jumps and/or stochastic volatility and transaction costs. We then describe our simulation analysis of alternative hedging approaches under a variety of assumptions. The final section draws some conclusions and sketches some ideas for further work in this area.

2. A Review of Related Work

In this section we review the relevant empirical and theoretical literature on hedging. Most of the work on hedging has been focused on the hedging of standard European options. Although there are quite a few theoretical studies of hedging errors (for example, Boyle and Emmanuel (1980)), and there are others which employ Monte Carlo simulation to study hedging effectiveness (e.g. Figlewski (1989)), there are few published empirical studies of hedging effectiveness. The earliest study is the pre-CBOE work of Black and Scholes (1992).

Other studies of interest are the Hull and White (1987) paper on hedging FX options, Dolbear (1992) on hedging lookback options, and Kat (1994) on hedging lookback and Asian options.

Hodges *et al* (1993) describe the results of using delta, delta-gamma, and delta-vega hedges to hedge the CME S&P 500 index futures option contracts. They hedged ten non-overlapping contracts between 1985 and 1992, and report on both the terminal replication errors and on the daily replication errors from marking to market. The main results are reproduced in Table 1. It is evident that delta hedging brings about a dramatic reduction in the risk exposure, and that delta-gamma hedging roughly halves again the standard deviation of the replication error. The vega hedge is particularly naive, as it is mismatched on expiry dates but pays no attention to the way in which long term volatilities move less than short term ones. Table 2 reproduces the levels of turnover involved from the various hedges. Note that daily rebalancing of the gamma or vega hedges implies prohibitively high turnover, both for the option contract and also for the delta adjustment made through the futures contract.

There are very few papers that we are aware of which study the problem of hedging exotic options under realistic market conditions. Dolbear (1992) used a simulation approach to compare the performance of delta hedging and hedges based on straddles for exchange rate lookback options under stochastic volatility and interest rates. Both methods used simple filters to control the level of transaction costs. The straddle based method was found to perform significantly better than the delta hedge for realistic levels of transaction costs in terms of cost/risk trade-off. Kat (1994) uses a sophisticated stochastic simulation model to study the performance of delta hedging of lookback and Asian options on S&P 500 index. A set of heuristic filter based rules are used to control the level of transaction costs; rebalancing after a certain number of days, rebalancing after the index has moved by a certain percentage and rebalancing when the delta is in error by a certain percentage. The main conclusions were that rebalancing every fixed number of days provided the best rule, and volatility misestimation and computing hedge ratios based on Black-Scholes assumptions was an important source of hedging error standard deviation.

The prohibitively high turnover found by Hodges *et al* (1993) leads us to consider work on optimal hedging under transactions costs and its implications for the current problem. The work in this area has so far only considered delta hedging. Leland (1985) provided important early insights into this problem. Hodges and Neuberger (1989) and Hodges and Clewlow (1993) (building on earlier work by Davis and Norman (1990)), formulated the optimal delta hedging problem under transactions costs as a problem in stochastic optimal control. This has a natural dynamic programming structure (i.e. similar to the binomial method) where we work backwards from the expiry date. By choosing an exponential utility function they reduced the problem to one with just two state variables, asset price and portfolio delta, and they were able to provide numerical solutions. Other related work on this problem includes papers by Bensaid *et al* (1992), Davis *et al* (1991, 1993), Edirisinghe *et al* (1993), and by Wilmott (1993).

The nature of the optimal solutions to this problem is that with proportional transactions costs (i.e. no fixed component) the optimal hedging strategy is to transact only as much as is necessary to maintain the asset holding within a region which can be computed. This region takes the form of a band around the preferred delta, which itself differs from the Black-Scholes delta (and indeed the Black-Scholes delta can even lie outside the optimal region). When we think of the gamma hedging problem, the difficulty of a computable solution is immediately apparent. Although the problem could be formally stated in a dynamic programming framework, at best we will end up with state variables for the holdings in every single available option, and this cannot be computationally tractable.

Recently a static hedge approach for barrier options has been proposed independently by Derman *et al* (1994) and Dupire (1994). Their approach is firstly to build a binomial tree (or trinomial in the case of Dupire) for the asset such that the market prices of standard options are returned. This tree then embeds the risk neutral distribution of the asset at various horizons and thus to some degree the fat tailedness due to jumps and stochastic volatility. Barrier options can then be priced consistently with the market prices of standard options within the world represented by the tree. Furthermore, Derman *et al* (1994) describe how the tree can be used to create a portfolio of standard options which replicates the payoff of a barrier option. The idea is to choose a portfolio of standard options which match the value of the barrier option at expiry and at the barrier. That is the portfolio satisfies the boundary conditions for the barrier option in the tree and thus the value of the portfolio inside the boundaries will be equal to the barrier option. Within the model world represented by the tree this is a perfect hedge - there were no transaction costs in setting it up and it replicates the barrier option in all states of the world. If the barrier is attained the portfolio is sold which results in no cashflow since it's value is zero. Note that this must be done exactly at the barrier since the portfolio does not replicate the barrier option outside the barrier. In practice there are many difficulties with this approach. The methodology which Derman describes involves working backwards from the maturity date of the barrier option using options with maturities at date $(i+1)dt$ and strike prices outside the barrier to match the value at the barrier at date idt where dt is the time step in the tree. The maturity and strike must satisfy these conditions so that the option doesn't interfere with the match which has been achieved up to date $(i+1)dt$. Now if we want to replicate under a realistic behaviour for the asset then we must construct the tree with a small time step. This immediately gives us the problem both for constructing the tree and the hedge of requiring options with maturities corresponding to every date in the tree. Furthermore; the tree can only capture a rather limited range of non geometric Brownian motion behaviours, this methodology does not take account of the transaction costs involved in constructing the replicating portfolio, and the methodology does not naturally allow for rebalancing of the hedge. In particular if volatility changes the hedge could become badly out of line. If we are going to use standard options to hedge barrier options then the natural way is as a dynamic gamma or vega hedge which takes account of the transaction costs involved in trading the standard options. Clewlow and Hodges (1994) described a methodology for delta-gamma hedging under jumps and stochastic volatility and transaction costs. We adopt and extend this approach to barrier options.

3. Myopically Optimal Gamma Hedges

In this section we describe our methodology for computing myopically optimal gamma hedges in the presence of jumps and/or stochastic volatility and transaction costs. Firstly we discuss the issue of moneyness as it applies to gamma hedges, and the notion of speed which provides a local measure of the robustness of a gamma hedge.

A delta hedge is simple in the sense that there is a natural underlying instrument to use. A gamma hedge is more complex in that to adjust gamma we have to buy or sell some kind of option, and depending on what strike price it has we will obtain a different gamma profile.

The gamma of an option peaks at spot prices close to the present value of the strike price. It is therefore usually advisable to gamma hedge using an at the money option, for if we neutralise gamma by buying or selling options a long way from the money, our gamma hedge is likely to be very sensitive to changes in the underlying asset price (see Figure 1).

A lesser known "greek" called speed (see Garman (1992)) measures the change in gamma with respect to the underlying, and in constructing gamma hedges (and choosing the strike price as well as the option amounts) we will want to try to ensure that we manage speed as well as gamma and delta.

We therefore assume that the cost of being mishedged is a quadratic function of the gamma and speed of the portfolio and at each date we minimise the sum of this quadratic function and the transaction costs incurred in adjusting the portfolio. Formally we solve the following problem

$$\begin{aligned} &\text{Minimise } y'Cy + d'y + e'|x| \\ &\text{subject to } y = y_0 + Gx \end{aligned}$$

where C and d are matrix and vector of coefficients of the quadratic function, e is the vector of transaction costs on the available standard options per unit of option transacted, y_0 is the vector of the current gamma and speed sensitivity of the portfolio, G is the matrix of gamma and speed of the available standard options, and x is the vector of adjustments to make in our holdings of the standard options for which we want to solve. We further simplify this by assuming

$$C = \begin{bmatrix} c_g & 0 \\ 0 & c_s \end{bmatrix}, \; d = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

where c_g and c_s are parameters which determine the relative of importance of gamma and speed relative to the transaction costs.

In order to obtain optimal values for c_g and c_s we simulate this hedging strategy and search for values which maximise expected utility of terminal wealth w_T. We assume a negative exponential utility function

$$U(w_T) = -e^{-\lambda w_T}$$

with a coefficient of risk aversion λ of 1.

4. The Structure of the Simulations

The simulations run delta and delta-gamma hedging portfolios under three different assumptions for the nature of "the world" and under a variety of methods of implementing the hedges.

The three "worlds" are:

a Black-Scholes world, where the asset follows geometric Brownian motion,
a Stochastic Volatility world, based on the Hull and White (1988) model,
a Jump-Diffusion world, based on the Merton (1976) model, and where the proportional jump is to a conditionally normal distribution with fixed standard deviation.

In each of these worlds we have a fairly low level of costs for transacting in the underlying (or future) at 0.5% but a much higher transaction cost for buying or selling options to adjust the gamma exposure. The latter cost is a fixed spread chosen so that it is 1.0% of the premium on a three month at the money option. We have chosen initially to simply hedge a single one year written up-and-out call option, at-the-money (initial asset price of 100) with a barrier at 120. We gamma hedge using options with an expiry of three months from our starting date. The hedges are rebalanced dynamically, and we look at the hedge performance at the end of an eleven month period. This is to avoid the problems associated with the gamma and speed changing rapidly near to expiry. We view this as a separate problem. We also rollover the hedge options into the next maturity when they reach one month to expiry for similar reasons.

For the Black-Scholes world we also examine the benefit of optimising the parameters c_g and c_s within the sub-periods between the rollovers of the hedge options.

Each of our worlds has a risk free interest rate of 5% and gives the underlying asset a volatility of either 0.20 or reverting to 0.20. In the jump diffusion world the intensity parameter is 5 (i.e. we expect 5 jumps a year) and it jumps to a distribution with a standard deviation of 0.05. The volatility of the diffusion is chosen so that the overall annualised standard deviation is 0.20. In the stochastic volatility model (which takes the CIR square root form for the variance) we have a mean reversion parameter of 1, and a volatility parameter of 0.30 as the volatility of volatility. Table 3 summarises the distributional properties of the asset returns we obtain under these parameter values. Notice that the excess kurtosis under the jump-diffusion model falls off rapidly and has disappeared by the one month horizon. In contrast, for the stochastic volatility model the kurtosis builds up gradually as we look at longer horizons. If we combine the two models we obtain values typical for equity indices for example.

Within each of our three worlds we explore the performance of a number of hedging strategies; naive delta and delta-gamma strategies implemented with a range of rebalancing periods from one day up to 25 days, static portfolios with from 10 up to 100 maturities, and our myopically optimised delta-gamma strategies.

All the strategies use Black-Scholes models for computing prices and sensitivities.

5. Results of the Simulations

Figures 2 to 4 show the results of our simulations in each of the three "worlds". In each case we have plotted the expected level of transactions cost against the standard deviation of the replication error. Our myopic optimisation approach has been run on a with daily rebalancing and we have then computed its subsequent performance on an out of sample basis.

Given our low level of transactions cost of changing delta, the delta hedging approach produces less replication variance as we rebalance more frequently, and the expected cost of this only increases fairly slowly, up to rebalancing every four days. At this point the increasing transaction costs variance begins to dominate the reduction in replication variance. As expected, it performs less well once the Black-Scholes assumptions are violated by jumps, or to a lesser extent by stochastic volatility.

The naive gamma hedging strategy is completely dominated over this length of hedging period (over a three month period it performs more comparably for rebalancing periods of approximately 20 days) because of the high level of transactions costs it incurs. Note that the reason it peforms so badly is that the transactions costs worsen the overall replication variance as well as increasing the expected cost.

The static hedges perform quite well although they are dominated by naive Black-Scholes hedging and our myopic strategy particularly in the jump-diffusion world. Interestingly, it seems hard to dominate naive Black-Scholes in the stochastic volatility world. This is probably because the stochastic volatility world is not that different to Black-Scholes for short horizons. We think a jump-diffusion model for volatility would be a more severe and realistic test.

Our simulations have enabled us to calibrate reasonable and robust parameter values for the myopic optimisation policy. This policy gives a significant advantage over the other strategies under either jumps or stochastic volatility, particularly in the robustness of the performance across the different worlds.

Finally, Figure 5 and 6 illustrates the nature of the myopic strategies we obtain. Figure 5 illustrates how revisions are made to the portfolio in gamma-speed space. It will be noted that they correspond to a kind of control region strategy, but where the exact location of the optimal region varies through time depending on the opportunities available in the option market. Figure 6 shows the actual adjustments the strategy makes to holdings in the standard options through time for some typical simulation runs. Note how the strategy, particularly in the early period, makes quite infrequent discrete adjustments to the holdings when good opportunities present themselves.

6. Conclusions

We have described a simulation methodology which allows the construction of almost optimal dynamic delta-gamma hedges for barrier options under realistic market conditions such as jumps, stochastic volatility and transaction costs. Our simulations have confirmed that gamma hedging is particularly useful in realistic situations with stochastic volatility or jumps, but can be dangerous if applied in a naive manner. Hodges, Clewlow *et al* (1993) and Hodges and Clewlow (1994) found that the choice of volatility input and model were not important when delta-gamma hedging long dated standard options. But for barrier options the choice of model and/or the estimation/forecast of volatility does appear to be important (Kat (1994) also found this to be the case for lookback and Asian options). This is probably due to the increased gamma sensitivity of exotic options.

Further work remains to be done. Investigation of the benefit of more accurate option pricing models and volatility forecasts. More realistic costs structures so that the strategy chooses from the complete set of available options of all maturities as well as strikes. This should provide a better prescription for rolling over the hedge options. The added benefit of optimising the delta hedging aswell should be explored. Finally, more refined prescriptions for conserving transactions costs, and also to extend the analysis to a world with both jumps in asset prices and stochastic jump-diffusion volatility would be interesting.

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Table 1

Summary of Hedging Results: $100 Initial Premium

Contract Date	Expiry Hedging Errors Under: No Hedge	Delta	D-Gamma	D-Kappa
8509	-81.8	11.2	-35.4	-39.9
8606	-929.5	-28.3	16.9	15.1
8703	-77.8	-16.0	6.4	20.8
8712	104.8	-66.9	-5.6	64.6
8809	75.8	52.6	14.7	-49.2
8906	-141.3	32.5	2.5	-70.9
9003	104.9	-1.7	17.0	132.6
9012	104.9	4.6	4.5	-24.4
9112	-64.9	21.8	-20.0	-45.3
9209	31.8	35.0	-5.1	-65.8
Mean	-87.3	4.5	-0.4	-6.2
SD	294.0	33.2	16.0	61.7
SD from daily errors:				
underlying	207.0	28.1	14.7	87.9
volatility	58.5	58.5	45.1	34.3
Total	202.3	52.4	36.1	89.5

Table 2

Average Turnover

Futures Turnover (#'s of contracts)

Volatility	Delta	D-Gamma	D-Kappa
Implied	3.1	6.5	17.0
Constant	3.2	3.6	18.2

Options Turnover (#'s of contracts)

	D-Gamma	D-Kappa
Implied	25.1	38.3
Constant	20.5	39.6

Table 3

Distributional Properties of Simulated Asset Returns

Black-Scholes World

Horizon	SD (%)	Skewness	Excess Kurtosis
1d	1.25	0.04	0.05
1w	2.76	0.06	0.10
1m	5.80	0.19	0.48
1y	23.16	0.37	0.09

Jump Diffusion World

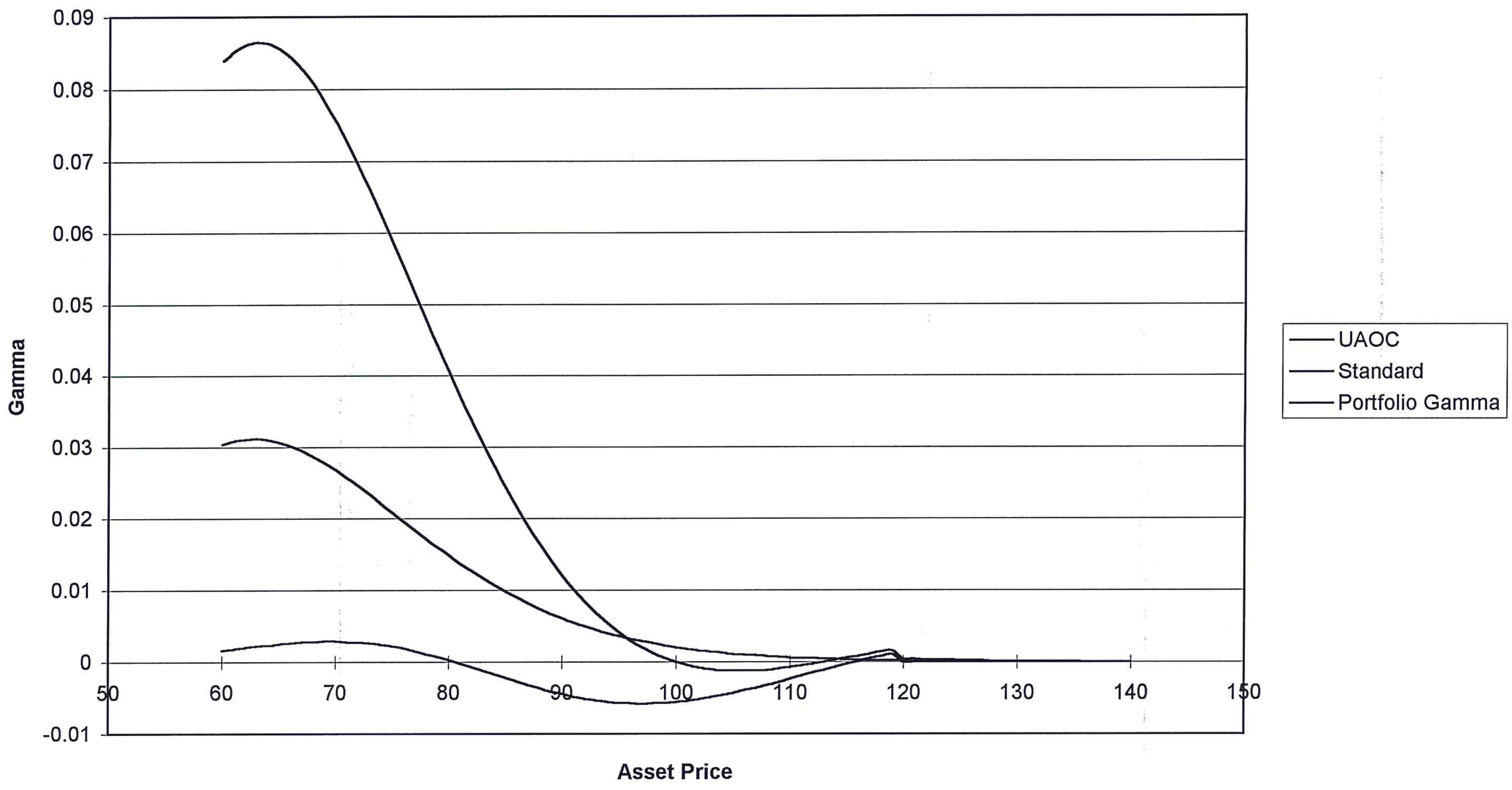
Horizon	SD (%)	Skewness	Excess Kurtosis
1d	1.42	0.15	8.06
1w	3.15	0.11	1.42
1m	6.61	0.14	0.28
1y	26.92	0.34	0.49

Stochastic Volatility World

Horizon	SD (%)	Skewness	Excess Kurtosis
1d	1.23	0.00089	1.6086
1w	2.79	0.0129	1.1273
1m	5.74	0.0426	2.3044
1y	22.43	2.0852	6.1318

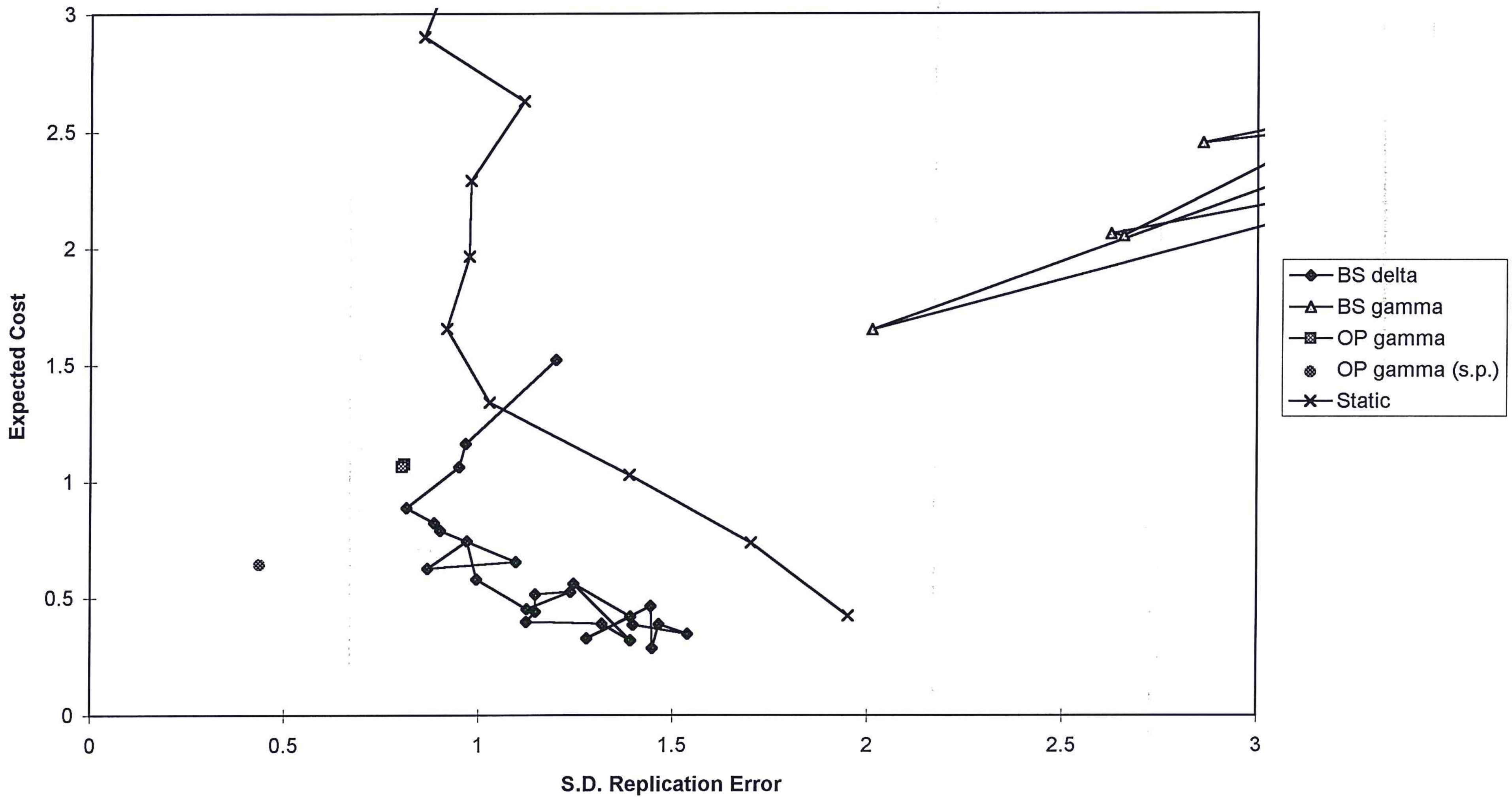
SD = Standard Deviation, 1d = 1 day, 1w = 1 week, 1m = 1 month, 1y = 1 year.

Figure 1: Gamma Hedging an Up-And-Out Call Option (Barrier=120, Strike=100) with a Standard Call Option (Strike=70)



HS7-GHR1.XLS, BS E v SD plot

Figure 2: Hedging in a Black-Scholes World



HS7-GHR1.XLS, SV E v SD plot

Figure 3: Hedging in a Stochastic Volatility World

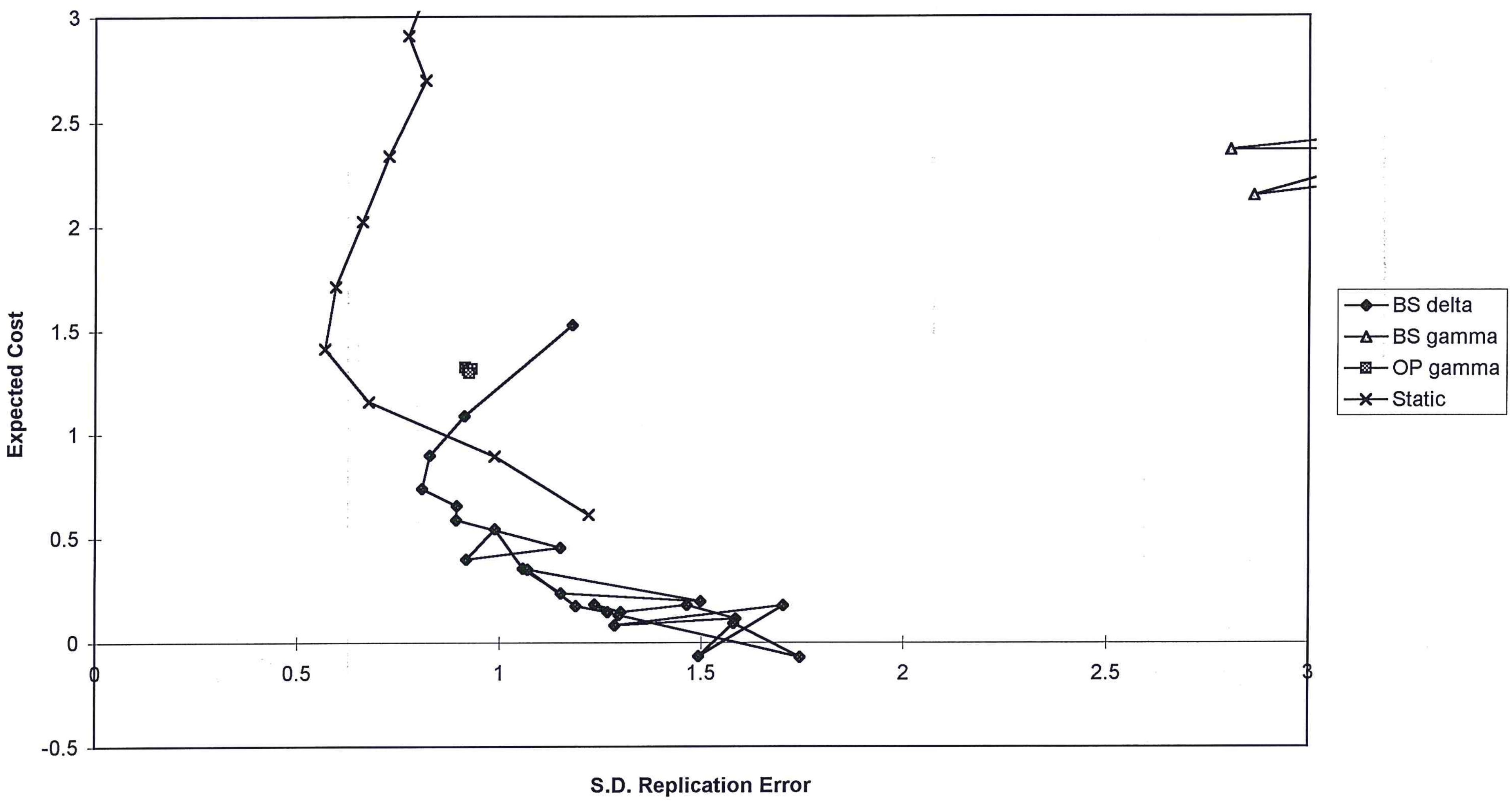


Figure 4: Hedging in a Jump Diffusion World

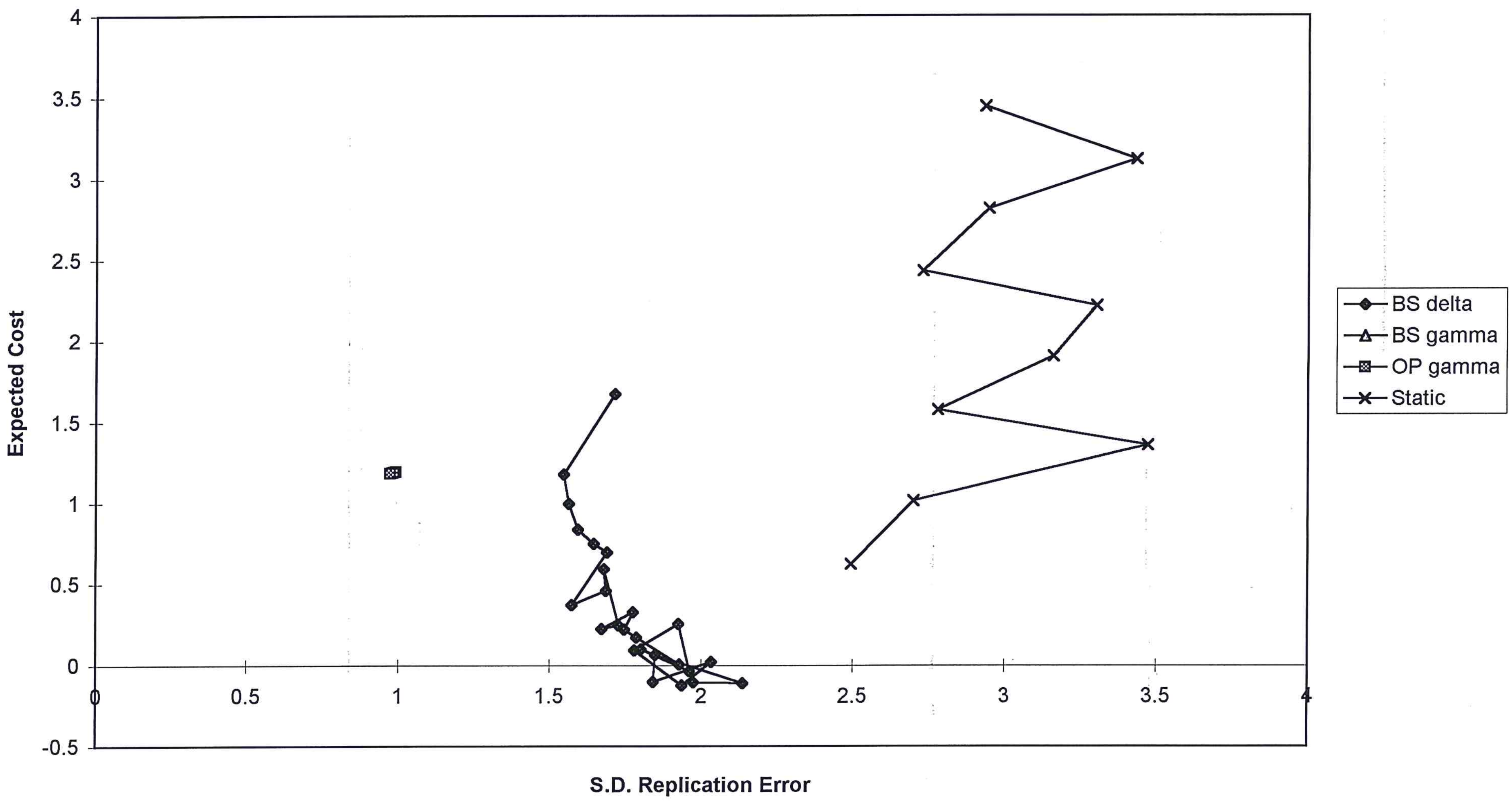


Figure 5: Example Myopic Strategy Gamma-Speed Rebalances

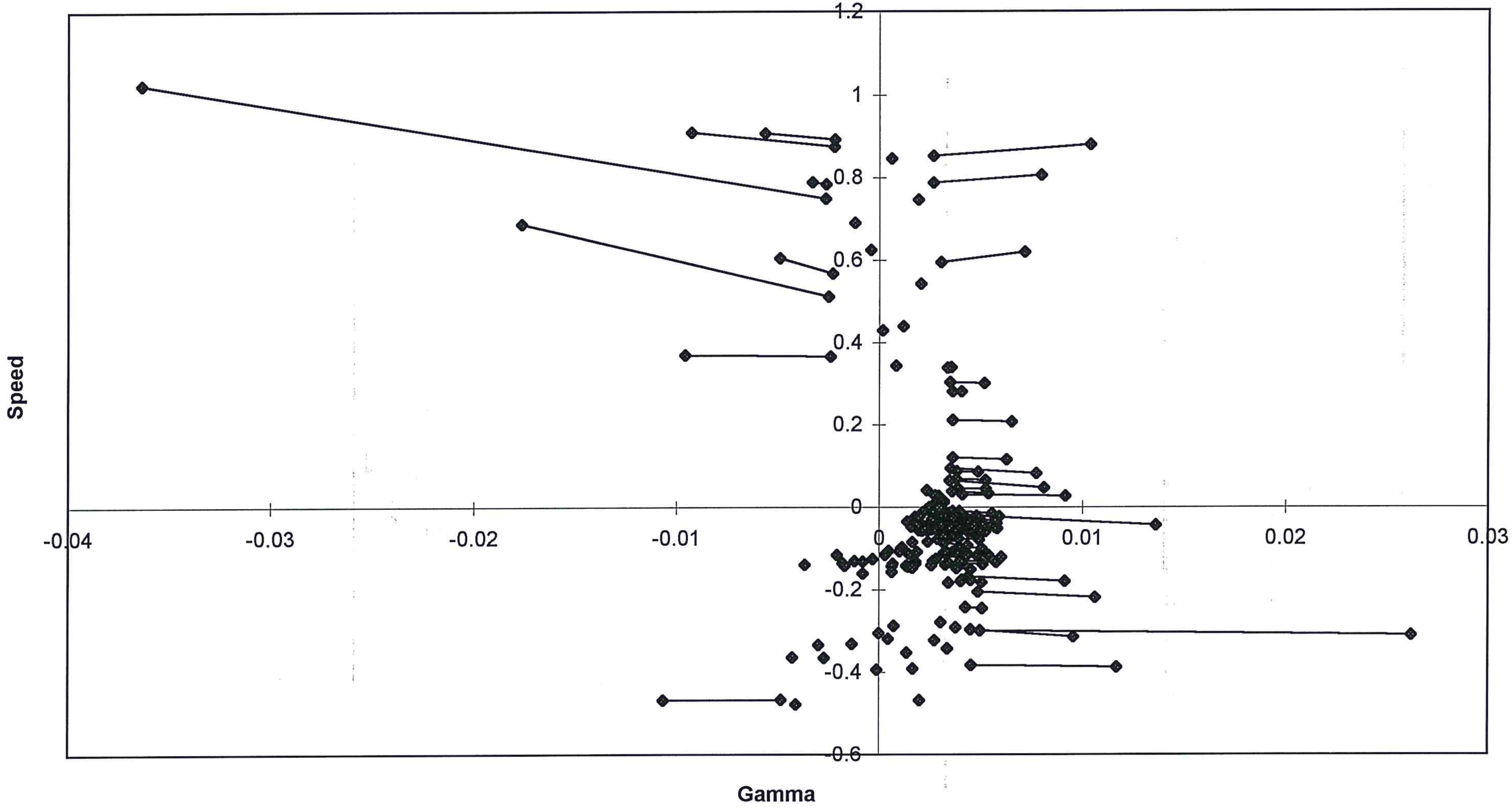


Figure 6: Example of Myopic Strategy Holdings
(Hx(K) is holding in shortest maturity call of strike K)