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Auditor Performance, Implicit Guarantees, and the Valuation of Legal Liability

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Liability exposure is now such a major concern for auditors that any discussion of equilibrium audit fee structures needs to take account of the expected costs of liability exposure. We develop a model of audit litigation risk and then proceed to apply insights gained from the application of finance theory to show how liability exposure is related to guarantee provision. Given auditors wish to incorporate the expected cost of guarantees when planning and pricing services, we apply contingent claims analysis to derive valuation equations for expected (litigation) costs. We then proceed to consider wider regulatory issues. Whereas the above analysis assumes stable legal liability rules, we subsequently consider the auditor incentive effects of parametric variation in the rules. Since the quality of audits is unobservable, we consider how the rules could be set so as to ensure auditors are not tempted to provide a low degree of care and collude with management. To illustrate the applicability of this approach we also consider whether recent proposals to reform auditor liability to a proportional basis will always provide appropriate incentives. © 1997 by John Wiley & Sons, Ltd. Int. J. Audit. 1(1), 13–30 (1997)

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SUMMARY

Recent evidence suggests that auditors can face significant litigation costs even when they have been most diligent. For instance, when a company enters a state of financial stress, frequently one of the first groups to be criticised is the auditors. Sometimes, this is irrespective of the merits of the case. When pricing the provision of audit services in such environments, it is critical that the auditor incorporate some adjustment (valuation) to take into account the risk of costly litigation. In order to aid auditors to perform this complex valuation task, this research develops a theoretical model which provides guidelines for how an auditor could determine the necessary valuations.

As a by-product of this valuation research, an analysis is conducted which shows how an auditor...
may be influenced by a move to a proportional liability standard. It is shown that proportional liability may not provide a sufficient deterrent against some auditors allowing themselves to be influenced by management when auditing financial disclosures.

**INTRODUCTION**

The expansion of auditor liability over the past two decades has resulted in an unprecedented crisis of prodigious proportions for the public accounting profession. (Hill et al., 1993, p. 13).

There has recently been much lobbying by the auditing profession for changes in the legal liability rules that govern the provision of audit services. For instance, in part as a result of lobbying by the profession, the US legislature recently passed the Private Securities Litigation Reform Act of 1995 which makes provisions for a scheme of proportionate liability for non-fraudulent defendants. Pressures for reform exist not just in the US, but also, for instance, in Australia and the UK as evidenced by the recent Law Commission report for the DTI on a Feasibility Investigation of Joint and Several Liability (1996). In the UK, the profession has presented a vigorous case for a general reduction in liability exposure. However, some commentators have argued that it is not clear that the profession has presented well-grounded guidelines for proposing by how much liability should be reduced. Indeed, the above Law Commission report comes firmly down against proposals to introduce proportionate liability along the US lines, instead suggesting an alternative proposal based upon a client negotiated cap on liability.

Given uncertainty over the optimal form of legal liability regime for auditors, we suggest that a necessary precursor to such an analysis is the development of a clear understanding of the costs and benefits associated with audit provision. This is required because, without a clear understanding of the overall costs and benefits of providing audit services, under alternative liability regimes, it is not possible to appraise effectively the efficacy of individual liability regimes. When auditors claim that liability levels are 'too high' we propose that it is important to recognize that the validity of such claims tacitly assumes that a cost for audit risk is factored into the overall evaluation process. However, it is not clear that audit firms have always in the past carried out a risk return analysis of clients and hence the central objective of this research is to develop (contingent claims) valuation equations for the potential litigation component of audit risk. We propose this is necessary because, without pricing this important component of audit risk, it seems appropriate to view auditors claims about litigation levels being 'too high' as largely unsubstantiated and at best ad hoc.

We focus upon litigation risk because it has become a significant cost associated with the provision of audits. For instance, claims in the US, UK and Australia against auditors are now so large that Epstein and Spalding (1993, p. 1) go as far as to question the continued existence of the auditing profession by asking whether 'As juries award multimillion dollar recoveries to plaintiffs "injured" by the work of accountants, will the profession shrink out of sight?' In this respect it is argued that one cannot simply conclude that only 'guilty' auditors are found against in court or choose to settle out of court (Arthur Andersen et al., 1992). In fact there is much concern that in the case of securities litigation, the merits of the case do not necessarily matter (Alexander, 1991). Thus, even the most fastidious of auditors needs to consider their potential exposure to future litigation costs. One reason why securities-related litigation poses such problems arises because of volatile movements in stock prices. For instance, an empirical study in the US has shown that 'most lawsuits are filed after large single-day stock price declines' (Jones and Weingram, 1996, p. 37).

The differential volatility in stock prices between client firms creates special problems for the evaluation of expected litigation costs since it becomes apparent that it is undesirable to assume there is some constant probability p (across clients) which characterizes the probability of litigation arising.

8 See Arthur Andersen et al. (1992) and the special issues of *Journal of Economics & Management Strategy, Fall* (1993), *Journal of Accounting Research* (1994) and Critical Perspectives on Accounting (1994) dedicated to these issues.

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Valuing these expected costs when underlying stock price movements are stochastic. Thus, one of our primary research objectives is to introduce a methodology for valuing the expected costs of auditor litigation in such stochastic environments. Here the established finance literature on contingent claims analysis is most helpful in providing the basic analytical tools, however, it is important to note that the institutional setting giving rise to the possibility of auditor litigation (and hence cost) is considerably different from the typical type of cost and benefit streams that result for the standard derivative securities analysed in the literature. Thus, given the special nature of our application, in the following section we present a detailed discussion of the special modelling assumptions required to characterize and value expected auditor litigation costs.

Having noted that difficulties in estimating the expected cost of litigation arise because of the stochastic movement in stock prices, we also note that further complications are caused by the fact that the performance of auditors is difficult to determine (indeed 'audit'), given the partially unobservable nature of their chosen degree of care; that is, audit litigation risk is not just a function of the client (and the stock price) but also dependent upon the behaviour of the auditor. If audits are to be of value to users, then users need to be confident that the auditors do not collude with management to bias financial reports and that, simultaneously, auditors apply a sufficient degree of care. If the auditor condones, or does not discover biases introduced by management, the firm is more likely unexpectedly (to the shareholders) to fall into financial distress. Thus, we need to consider the interplay between strategic auditor behaviour, firm financial distress and legal liability. These factors are related because auditors recognize that if they collude with management and provide a low degree of care, this increases the likelihood of legal action against them, because of unexpected financial distress of the company. However, a countervailing factor is that such collusive behaviour increases the chance of reappointment and the possibility to earn future rents. Thus, given auditors face such a trade-off and given the unobservable nature of their actions, if shareholders are to value audits, they need to be confident that legal liability levels are sufficiently high to deter collusive, low degree of care behaviour. Hence, legal liability has an important positive role to play since the level at which legal liability is set, is an important commitment device for the continued functioning of (and confidence in) the audit industry.

In the next section, we model stochastic litigation risk in order to understand explicitly how legal liability considerations influence auditors' behaviour. We apply contingent claims analysis to the valuation of this risk. In the section after we model auditor preferences and introduce the possibility that the auditor may act strategically when choosing independence and degree of care levels. In the first sub-section we establish why the auditor may choose to adopt a less than complete level of independence from management. As we can view legal liability as a choice variable for regulators, we then show how legal liability can be chosen optimally to create incentives for auditors such that they find it in their own self-interest to maintain full independence from management. In the next sub-section we extend our auditor strategic behaviour model also to consider auditors' choice over degree of care considerations. In the section after we appraise recent calls for the introduction of proportional liability for auditors. We show why proportional liability may not always provide sufficient incentives for auditors. Finally, we present concluding comments.

APPLICATION OF CONTINGENT CLAIMS ANALYSIS TO IMPLICIT AUDITOR GUARANTEES

Much of the present public disaffection with the efficacy of audits follows from the dramatic financial distress that some companies encounter, shortly after an audit has been performed that gives the company a clean bill of health. For instance, Power (1994, p. 41) has commented that 'the cost of audit in relation to its claimed benefits, particularly in the fields of financial audit and quality assurance arrangements, is being questioned in the light of its potential impact upon competitiveness'. In the model which we develop below we focus upon how a significant stock price decline may trigger auditor litigation. This modelling assumption has the support of a number of empirical studies. For instance, Alexander (1991, p. 513) looking at litigation in class action suits following initial public offerings provides empirical evidence that 'strongly suggest that suits alleging securities violations were filed whenever the stock price declined sufficiently.' In addition, from a lawyers' perspective, Lerach (1985) identi-

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See Dye (1991) and Acemoglu and Gietzmann (1993) for a more detailed discussion of these issues.
fies stock price declines as a frequent trigger for litigation. In fact, Coffee (1988) argues that stock price declines are used by plaintiffs' lawyers to locate potentially meritorious suits and this is given empirical support by the work of Jones and Weingram (1996). This arises because for many courts the basis of assessing damages is functionally dependent on the stock price decline since purchase and hence (self-interested) lawyers rewarded on a contingent fee basis recognize that their fee may be monotonically increasing in the absolute magnitude of the decline.

It is important here to note that a significant decline in stock price may subsequently result in bankruptcy in only a restricted number of cases and hence we make no assumption that bankruptcy is a necessary trigger. Indeed, Carcello and Palmrose (1994, p. 27) provide support for our modelling assumption by finding that in those restricted cases that did result in bankruptcy, 'litigation preceded bankruptcy in most cases.' Other supporting empirical work includes Kellog (1984) and that of St. Pierre and Anderson (1984, p. 256) who in their study of US lawsuits against public accountants argue that 'The initiation of error search by potential plaintiffs was motivated by signals emitted from the client company and situational characteristics of the client and the client's industry. Signals which motivated error search primarily related to negative financial information concerning bankruptcy or significant client losses.'

In addition to this empirical support for the modelling assumption, it has also recently been used in the related theoretical research of Narayan (1994, p. 40) who argues 'it is optimal for lawyers to sue when the share price falls below a threshold level, if the audited financial statements released earlier disclosed favourable news.' Thus, in summary, in the model that follows we assume that a significant stock price decline increases auditors' expectations that litigious action will follow. This is not to presuppose guilt, but merely to identify a common litigative trigger. In this respect, Liggio (1975) stated that 'the new wave of litigation that has hit the accounting profession clearly proceeds on the assumption that the auditor is the guarantor of the accuracy of a company's financial statements'. However, Liggio (1975) goes on to argue that this notion of the auditor as a guarantor is flawed because 'The plaintiffs and their attorneys simply refuse to acknowledge the judgement-making process that is integral to the auditing function'. Although it is clearly correct for Liggio (1975) to highlight the judgemental nature involved in audit attestation, we believe that this does not provide a completely convincing defence for auditors. Motivated by agency theory considerations, we investigate how auditor behaviour influences the probability distribution of stock price decline and firm financial distress, rather than imposing a simple dichotomous model in which auditors can either perfectly, or not at all, detect imminent distress.

Typically, the benefits of audits are appraised in terms of the provision by an auditor of direct investigative acts to detect fraud or accounting misstatement. However, it should also be noted that shareholders have little direct information confirming whether or not auditors actually provide such services or at what level. At issue is the question of who audits the auditor's work. Since shareholders typically find it most difficult to assess the performance of an auditor, given the partially unobservable nature of auditor tasks, the shareholders look for statistics which may be informative about auditor performance. As our earlier discussion of the empirical literature makes clear, one such (imperfect) statistic is the observed failure of a company which previously published a positive audit report. On observing unexpected financial distress, the shareholders infer that this increases the likelihood that the auditor did not perform audit duties diligently and hence may consider mounting litigation against an auditor. Thus, shareholders act as if the auditors gave an implicit guarantee that the company would not fall into financial distress shortly after producing a positive audit report. We stress that the guarantee is implicit, since we recognize that auditors would prefer shareholders not to act as if one was in place and they give no such explicit undertaking.

The point we are making here is that given limited observability of auditor behaviour, shareholders use unexpected financial distress as a trigger for litigation whether or not the auditors like it or offer one. That is, it is the fact that unexpected financial distress may trigger litigation that prompts us to identify an implicit guarantee. We make no assumption that an explicit guarantee is offered.

Explicit evidence for the existence of this implicit guarantee has been hard to establish empirically because of the lack of data to test the hypothesis that investors act as if auditors provide such a guarantee (insurance). However, the recent bankruptcy of Leventhal and Howarth allowed Menon and Williams (1994) explicitly to test to see if investors acted in this fashion. As a result they
were able to provide 'empirical support, showing that investors assign a value to the right to recover investment losses from the auditor' (Menon and Williams, 1994, p. 327), thus giving clear empirical support for the existence of the implicit guarantee (insurance) function. Other studies such as Wallace (1987), Chow et al. (1988), Schwartz and Menon (1985) provide additional empirical support for the view that auditors act as implicit guarantors of financial statements.

Given its existence there are at least two possible ways an implicit guarantee may become institutionalized in the auditing setting. One way is to argue that even though auditors do not explicitly give (or desire) such an implicit guarantee, either verbally or in writing, it is de facto in existence as the above empirical evidence, showing how litigation is triggered by stock price decline, establishes. Alternatively, we may argue that auditors explicitly recognize the nature of the guarantee, provided it is subject to limitations which arise from the judgemental nature of auditing. This latter case illustrates that a theory of guarantee provision associated with an audit, must therefore allow for imperfect controllability by the auditor of the guaranteed process.

In order to understand how the implicit guarantee influences auditor behaviour, we need to evaluate the risk associated with its provision. Indeed without understanding the nature and the cost of the risk we cannot discuss in an internally consistent fashion whether there is a litigation crisis because we need to evaluate the relationship between audit fees, direct audit costs, and the cost of risk. In this respect Merton (1977b, p. 3) has demonstrated that there exists 'an isomorphic correspondence between loan guarantees and common stock put options'. The details of the auditors' guarantee, which we specify, differ (as explained below) from those of a loan guarantee. However, such differences do not affect the general principle. Therefore, once we have established the special institutional setting faced by auditors, we can appeal to the well established theory of contingent claims analysis as the basic modelling device for the risk associated with a guarantee. In this respect Dye (1993) was the first to model an auditor guarantee explicitly as an option. However, in his model he does not also model the underlying stock price process and hence the contingent claims valuation process is not explicitly related to the underlying assets of the firm and thus client specific risk is not considered. Let us now develop the required risk valuation relationships for our problem.

Black and Scholes (1973), and Merton (1973) introduced the modern approach to the valuation of contingent claims. Among other things, they recognized formally that almost all corporate liabilities can be viewed as a combination of options. Merton (1974, 1977a) developed a general theory for the valuation of any contingent claim based upon the dynamics of the assets of the firm.

To simplify our analysis we shall consider an all-equity financed firm. Using the now standard aforementioned continuous time finance approach, we model the auditor guarantee process as follows. We use the standard frictionless market assumptions (see for instance Merton, 1974) and the assumption that the dynamics of the asset value of the firm can be described by the stochastic differential equation

$$dV = \mu V dt + \sigma V dz$$

(1)

where

- $t = 0$ = publication date of most recent financial statements;
- $V(t) = V = \text{market value of the firm at time } t$;
- $\mu = \text{instantaneous rate of return on the firm per unit time period}$;
- $\sigma = \text{instantaneous standard deviation of the return on the firm per unit time}$;
- $dz = \text{a standard increment of a Wiener process}$.

This means that the value of the firm is instantaneously log-normally distributed (it should be noted that $V$ is often said to be a geometric Brownian motion).

We refer the interested reader to a text such as Hull (1993) for an extended discussion of the various forms of contingent claims such as European put options that are often analysed. However, these standard forms are not directly helpful to us because the special institutional form which the

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3 We shall assume the insurance that the auditor chooses to hold is sold at a fair price taking account of the risk faced by the individual auditor (partnership). Thus, typically the auditor will not fully insure against all potential losses (they choose a limit on the insurance). Even though an auditor may be insured against losses for a fixed time period, this does not remove their incentive to prevent such losses since once losses are incurred, this may have a dynamic effect upon the way future insurance premiums are determined. In addition, the occurrence of a loss may also have a reputational damaging effect and thus we assume that, in any period, an auditor prefers less claims to be made against them.

4 See Selby et al. (1988) for the case of incorporation of debt financing. This extension does not materially change our analysis.
auditor litigation process takes requires us to define a non-standard contingent claim. Specifically, we are interested here in the type of contingent claim 'with more complicated pay-offs than the standard European or American calls and puts ... sometimes referred to as exotic options' Hull (1993, p. 414). This arises because the trigger which gives rise to auditor litigation is a significant fall in stock price. That is, the payment of the contingent claim (guarantee (insurance)) arises only when the stock price falls below some given value and hence the specific class of exotic options that we are interested in are sometimes referred to as barrier options. Thus, in general, we assume that if the value of the firm falls to a level \( L(t) \), the auditor is sued for legal damages of \( a \). In this section, in order to concentrate upon our contingent claims analysis, we shall assume that \( a \) is a fixed parameter. However, we will relax this assumption in later sections to investigate the effect upon incentives of a parametric variation in the liability level. We now consider carefully what would be a reasonable assumption to make about the nature of this trigger \( L(t) \). One possibility would be to assume that given the initial value of the firm \( V(0) \) and a favourable audited financial statement, if the market value of the firm falls by more than a fixed proportion \( \lambda \) of \( V(0) \), this will act as a trigger. This would be subject to some time limit \( T \) beyond which it would not be reasonable to rely on historic financial statements, i.e. \( t \in [0, T] \). Thus, we let

\[
L(0) = l = \lambda V(0), \quad 0 < \lambda < 1
\]  

(2)

More generally, if the audited financial statements have reported an instantaneous rate of return of \( \mu > 0 \), then we may want to argue that the litigation barrier should rise through time, though not at a faster rate than \( \mu \). Since the historic financial statements do not incorporate information arising after \( t = 0 \), it therefore follows that it is assumed that the statements should be relied on less as \( t \) increases to \( T \). This assumption embodies the notion that auditors will not be held responsible for events long after the publication of annual financial statements. We make this assumption so as not to be implicitly supporting a view that auditors can have perfect foresight, and also as a mean of providing recognition that auditors are attesting the validity of historical data. In making this assumption it is, however, important to note that this assumption is not necessary for the subsequent results on the efficacy of proportional liability which we derive. Thus, if we denote the growth rate of the litigation barrier as \( \gamma \), then we are at first suggesting \( 0 < \gamma \leq \mu \).

One functional form\(^5\) for describing the litigation barrier path which aids mathematical tractability is

\[ L(t) = le^{\gamma t} \]  

(3)

If we wish to emphasize even more strongly that the audited statements should be relied on less and less, then we can relax the assumption that \( \gamma \) is non-negative. We illustrate these possibilities in Fig. 1, which illustrates how different assumptions

\(^5\)See Black and Cox (1976) for a detailed discussion of barrier specification.

Figure 1. Shape of the schedules.
about $\gamma$ influence the shape of the litigation barrier path.

To determine the value of the implicit guarantee, we shall work within the framework of Black and Cox's (1976) analysis of bonds with safety covenants, but reinterpreting their results within our context. It is important to realize that we are therefore assuming inter alia, that appropriate hedge portfolios consisting of the assets and the guarantee can be formed. Although this is a strong assumption, we could work in an explicit general equilibrium environment, following Rubinstein (1978) and Brennan (1979), and rederive our results. The principal difference between our approach and that of Rubinstein (1978) and Brennan (1979) is the assumptions relating to whether one needs to be in equilibrium and have complete markets, or to be simply consistent with equilibrium, not requiring complete markets, but assuming that one has the opportunity to span states of nature dynamically.

Let us now introduce some further notation. We assume a flat term structure, with instantaneous riskless interest rate, $r$. We further define variables $y$ and $\tilde{y}(t)$ as follows

$$y = \frac{L(T)}{V(0)}$$

$$\tilde{y}(t) = \frac{L(t)}{V(t)}$$

The terms of the guarantee are that the auditor will pay the shareholders an indemnity payment $\alpha$, if shortly after confirming that the state of the firm is good, the firm then suffers financial distress. Clearly the guarantee is of value to the shareholders and, in turn, imposes a cost on the auditor. To evaluate this cost formally we proceed as follows. If for all time, $t$, prior to the terminal date, $T$, the value of the net assets always exceeds the value of the litigation barrier, then $\tilde{y}(t) < 1$. On the other hand, if the value of the net assets equals the value of the litigation barrier, at time $t^*$, say, then $\tilde{y}(t^*) = 1$.

We define the indicator function as follows.

**Definition of the Indicator Function**

$A$ is an event corresponding to the barrier being reached and $A^c$ is its complement. Thus we define the indicator function on the event space $\Omega$, $I_A$: $\Omega \rightarrow \mathbb{R}$ where

$$I_A(w) = \begin{cases} 1 & \text{if } w \in A \\ 0 & \text{if } w \in A^c \end{cases}$$

and can represent the auditor's payments as

$$- \max(\alpha I_A(w), 0)$$

Hence, if $G(T)$ is the actual payment made by the auditor due to the guarantee, when the length of time ($t \in [0, T]$) until no reliance is placed on the historical financial reports is $T$, then

$$G(T) = - \max(\alpha I_A(w), 0)$$

where

$$I_A(w) = \begin{cases} 1 & \text{if } \tilde{y}(t) = 1 \\ 0 & \text{if } \tilde{y}(t) < 1 \end{cases}$$

It is important to note that when $\tilde{y}(t) = 1$, the process stops and thus for $\tilde{y}(t^*) = 1$, $t^*$ is the first passage time to the litigation barrier. As $t^*$ is unknown, and is in fact a random variable, to determine the current value of the expected guarantee, we must evaluate

$$G(x, r, T) = x \int_0^T e^{-rt} p_T(t) dt$$

where $p_T(t)$ is the first passage time probability density function (p.d.f.) of reaching the litigation barrier up to $T$ time periods after commencement. Given our assumed firm value diffusion process in (1) and the path of the litigation barrier, this probability density function takes the form of the Inverse Gaussian density function (see for instance Cox and Miller, 1965, and note that we put $\mu = (r - \gamma - 0.5 \sigma^2)$). In Appendix A we show how to evaluate the first passage time probability distribution $\int_0^T p_T(t) dt$ in terms of the parameters of the model. This identification of this functional relationship is required for actual empirical work and in the worked example which we shall subsequently present. However, given the somewhat cumbersome form of this functional expression we shall use some summary notation when we are merely rearranging equations which use the expression but do not require manipulation of individual component terms. Thus, we shall denote the distribution summarily as

$$\Delta(T) = \int_0^T p_T(t) dt$$

In principle, we could now empirically estimate the cost of audit guarantees given actual data on litigation settlements. We could then compare this to results derived from application of the incentive model presented in the following section. There we define a level of sufficient liability (guarantee cost) which would provide auditors with incentives to maintain independence and provide appropriate care when conducting audits. Given the claim that, at present, auditors' liability exposure is too high, application of the theoretical incentive model
would allow us to be more precise about what we mean by too high. If the actual guarantee cost was greater than the sufficient (theoretical) guarantee cost, then we could conclude that auditors' liability was indeed too high in relation to what is needed to ensure adequate incentives. Unfortunately we were unable to collect systematic data on actual litigation settlements given its highly confidential nature. However, since at a later date, others may be successful in collecting such data, we believe it is valuable to report on these new test procedures for establishing whether auditors' liability is indeed too high. Moreover, the approach we develop here allows us to consider whether if litigation settlements were assessed in the courts on a proportional basis, would this provide sufficient incentives for auditors? After developing our model of auditor incentives in the next section, we turn to this specific issue in the following section.

STRATEGIC AUDITOR BEHAVIOUR

In the following sub-sections we first consider the case of the auditor having only one level of effort to provide but where information concerning the financial state may be communicated strategically. In the sub-section following we then relax the assumption concerning effort and allow the auditor a wider choice set.

Auditor Independence

We now follow, in spirit, the pioneering approach of Holmstrom and Milgrom (1987), which explicitly applied incentive theory in a stochastic setting. Thus, we allow the specification of the stochastic firm valuation process to be conditioned on agent strategic choice. In particular, we assume that the auditor controls the reporting of the instantaneous rate of return at time $t=0$.

Following Hart and Moore (1990), we shall assume that the manager is an absolute 'empire builder' in the sense that the manager always prefers to report good financial statements rather than bad ones. This arises because we assume improvements in reports increase their ability to raise additional finance and build the manager's empire (scope of influence) and when this is combined with limited managerial liability, the manager always prefers to report good (see Gietzmann and Sen, 1996), for an extended discussion of this point). In this setting, the primary role of the auditor is to act on behalf of shareholders' interests to ensure that when the state of the world (instantaneous rate of return at time $t=0$) is bad, either management report it to be bad, or the auditor exposes management's incorrect reporting strategy. In our model the good and bad discrete dichotomy is captured by two possible instantaneous rates of return for the firm where:

$\mu_g =$ good return, 
$\mu_b =$ bad return.

In addition, we assume that the auditor has imperfect audit technology in the following sense:

$b =$ actual probability that the expected return is bad ($= \mu_b$);

$\beta =$ probability that the auditor (A) finds out the expected return is bad

$\beta < b$  \hspace{1cm} (10)

We shall further assume that the litigation barrier only applies after the auditor has reported that the instantaneous rate of return at $t=0$ was good. Thus, the shareholders are only entitled to compensation if they were misled by the auditor's favourable report $\mu_g$.

Let us now consider the preferences of the auditor and question why an auditor may not act purely in the shareholders' interests. Let 

$F =$ fixed audit fee received net of investigative auditing costs, i.e. the fee for the contingent claim$^6$; 

$X =$ present value of future rents earned from re-employment as the firm's auditor (not observable by outsiders); 

$e =$ auditor's effort; 

$\psi(e) =$ disutility of degree of care effort.

Dye (1991, proposition 2, p. 359), proposes a similar incentive model, in which an auditor faces a trade-off between future rents (which Dye calls quasi-rents) resulting from collusion versus litiga-$^6$ In general one could assume that auditors may be paid by a fixed fee plus some report contingent and whistleblowing fees as well as being subject to liability penalties. However, in a multiple auditor task model, Acemoglu (1995) has demonstrated that the use of report contingent and whistleblowing fees creates a problem because rewarding the auditor for disagreeing with management decreases the incentive for the auditor to perform tasks which both management and shareholders want carried out, but which only management can observe. In this model environment he shows how the optimal contract is to offer the auditor a flat fee plus the threat of legal liability if found guilty of negligence, with no report contingent or whistleblowing fees being paid. Since this also agrees with what we commonly observe in practice we shall make the same assumption.
tion costs. He establishes that 'there is no equilibrium in which auditors receive zero quasi-rents when these quasi-rents are not observable to outsiders'. Thus, we assume \( X > 0 \). Furthermore, we shall assume that auditors preferences \( U(\mu, \rho) \) are separable in money income \( m \) (giving utility \( U(m) \)) and effort, thus \( U(m, e) = U(m) - \psi(e) \). Since managers are 'empire builders', we shall assume that they put pressure on the auditor to report that the instantaneous rate of return at \( t = 0 \) was \( \mu_B \), by threatening them that they will lose the audit if they report \( \mu_B \). We assume therefore that the auditor knows that they will lose \( X \) (future audit and consultancy work) if they report \( \mu_B \). Thus, we now have the possibility that the auditor will report \( \mu_E \) knowing full well that \( \mu_B \) is the truth. Let:

\[
\mu^O = \text{the instantaneous rate of return at } t = 0 \text{ that the auditor observes (O) where } O \in \{g, b\}.
\]

\[
\mu^R = \text{the instantaneous rate of return at } t = 0 \text{ that the auditor reports (R) where } R \in \{g, b\}.
\]

In order to be able to sign-off the audited financial statements the auditor needs to form a judgement concerning the instantaneous rate of return at \( t = 0 \). Let us now consider what policy instrument can be applied to ensure auditors act independently of management. Shareholders want to be confident that auditors will not have an incentive to act collusively by reporting \( \mu_E \) even though it is believed to\(^7\) be \( \mu_B \). From (31) derived in Appendix A, it is clear that because the guarantee is a cost to the auditor, if \( \alpha \) is sufficiently high, auditors will not choose to act collusively. Thus, we shall assume that regulators use the level of legal liability to which auditors are subject to, as an incentive policy instrument. We require that \( \alpha \) be set, so as to ensure auditor independence.

Let \( \Pi(\mu^R | \mu^O) = \) the auditors' expected utility when the auditor reports \( \mu^R \) having observed \( \mu^O \) (reports and observations are either good or bad respectively). Thus, we have

\[
\Pi(\mu^R | \mu^O) = U(F+X) - \psi(e) > 0
\]

the auditor reporting good when the underlying state is good, receives the fee and future rent possibilities after supplying effort to perform the audit. In this section we assume that a constant level of effort (degree of care) is supplied. We will delay introducing variable effort (degree of care) considerations until the following sub-section. For the bad state realization either:

\[
\Pi(\mu^B | \mu^O) = U(F) - \psi(e)
\]

if the auditor reports bad when the underlying state is bad and is discontinued as the auditor\(^8\), or

\[
\Pi(\mu^R | \mu^O) = (1 - \Delta(T))U(F + X) - \Delta(T)U(F - \alpha) - \psi(e)
\]

if the auditor reports good when the underlying state is bad and hence receives the possibility of earning future rents provided the firm does not suffer financial distress (which recalling (9), with probability \( (1 - \Delta(T)) \)), but also needs to recognize there is a probability \( \Delta(T) \) that legal action will result because the firm suffers financial distress. Our discussion of the determinants of \( \Delta(T) \) in Appendix A, illustrates how our analysis differs from that of Dye (1993). Dye (1993, p. 893) derives an auditor's expected liability by assuming that an 'audited firm fails' (which occurs with probability \( 1 - p \)) and the auditor does not qualify his report (which occurs with probability \( 1 - q \) if the audit is of quality \( q \)). In contrast to Dye's exogenously imposed \( 1 - p \) assumption, we exploit the results of contingent claims analysis to derive endogenously \( \Delta(T) \) (which replaces \( 1 - p \)) from an explicit formulation of the dynamics of the asset value of the firm. An advantage of this more general finance theory approach is illustrated later, where for instance, we are able to relate requirements concerning legal liability to variations in the instantaneous standard deviation of the return on the firm.

It is clear by comparing (12) and (13), that if \( \alpha \) is sufficiently high, then the auditor will not be tempted to report good when the underlying state is bad since the auditor does not want to risk high liability. Let us now formally establish what we mean by sufficiently high. For incentive compatibility we require

\[
\Pi(\mu^B | \mu^O) \leq \Pi(\mu^B | \mu^O)
\]

Thus, a regulator would be concerned with identifying the minimum level of legal liability \( \alpha^* \) which would always ensure satisfaction of (14). We demonstrate in Appendix B, in principle, how such an \( \alpha^* \) could be determined. However, for our purposes we characterize (in Appendix B) a lower bound \( \alpha^* \) which \( \alpha \leq \alpha^* \). This lower bound is most useful because it enables us in the following section

\(^7\)We assume that auditors and shareholders bear their own litigation costs or that auditors may go bankrupt in the case of a large ruling against them. Thus, shareholders will not be indifferent between incentive compatible auditor behaviour and collusive behaviour with liability transfers.

\(^8\)Another assumption giving rise to the same model predictions does not require auditors always to be replaced but just that they receive no rents when they choose not to collude with management.
to relate the incentive properties of actual litigation levels to $g$ in a quite general fashion. In the Appendix we show that:

$$g = \frac{1}{R} \left( 1 - \left( 1 + \frac{2R(1 - \Delta(T))}{\Delta(T)} \left( X - \frac{1}{2} X^2 R \right) \right)^{1/2} \right)$$

(15)

where $R$ is the Arrow-Pratt measure of absolute risk aversion. Thus, the lower bound on liability depends on the relative magnitudes of the auditor's attitude towards risk, potential rents and the first passage time probability of financial distress. In order to develop further understanding of this relationship it is appropriate to consider a rearrangement of (15) and apply (31) which gives

$$(1 + R g \Delta(T))^2 = 1 + \frac{2R(1 - \Delta(T))}{\Delta(T)} \left( X - \frac{1}{2} X^2 R \right)$$

(16)

In the case when the auditor is risk neutral ($R = 0$), we can directly develop a particularly clear interpretation because (16) simplifies to

$$g \Delta(T) = (1 - \Delta(T)) X$$

(17)

that is, a lower bound on liability is the condition that expected liability equals expected rents. Equation (16) has more terms because of complexities introduced by the effect of risk aversity.

Thus, if regulators propose that the law of the land imposes less liability than $g$ we know that an incentive exists for auditors not to maintain independence. Let us now consider how the introduction of degree of care considerations for the auditor affects the determination of such a lower bound.

**Choice over Degree of Care and Independence**

In the above section we assumed that the auditor can only provide a single degree of care (effort). We shall now extend the model to allow for a dichotomous choice, over levels of degree of care. The significance of this extension is that it now allows us to model implicit auditor collusion in which the auditor consciously chooses to provide a low degree of care in full knowledge that this increases the probability of not finding management misstatements and hence enjoying the continued benefits of re-employment. Whether or not the auditor uses a high degree of care, there is always the chance that the auditor actually believes $\mu_S$ is the state of the world but was mistaken because of the imperfect audit technology. We shall assume that the auditor's effort is productive and thus, using superscripts to denote high ($h$) and low ($l$) effort, we shall assume

$$b > b_h > b_L$$

(18)

That is, for the higher level of effort the auditor is more likely to estimate the state of the world correctly.

This assumption embodies the principle that the auditor using low effort is more likely to report the state of the world is $\mu_L$ (since $b_h - b_L > 0$) even though the simple independence requirement (15) may be satisfied. Thus, we wish to explore by what order of magnitude the legal liability should be adjusted, to ensure incentives both for the maintenance of independence and appropriate provision of degree of care (effort). We proceed by assuming that (15) is satisfied, so we can exclude consideration of the auditor reporting $\mu_S$ even though they believe it to be $\mu_L$ for a given choice of effort level.

This means we do not need to distinguish between $\mu^R$ and $\mu^S$ (reports will correspond to observations) since with (15) holding, $\mu^R = \mu_S$. What we are concerned with here is whether degree of care motivation considerations require more stringent requirements on liability levels. Now let $\Pi(\mu^R; \mu^S)$ – the auditors pay-off when the auditor truthfully reports $\mu^R$ under $\alpha$ liability determined by (15) and the actual state of the world is $\mu^S$.

We require the expected pay-off from a high degree of care to exceed that resulting from the choice of a low degree of care, which implies that

$$(1 - b) \Pi^h(\mu^R; \mu^S) + (b - b_h) \Pi^h(\mu^S; \mu^R) + b_h \Pi^h(\mu^S; \mu^L) > (1 - b) \Pi^l(\mu^R; \mu^S) + (b - b_h) \Pi^l(\mu^S; \mu^R) + b_h \Pi^l(\mu^S; \mu^L)$$

(19)

The first term (on each side of the inequality) corresponds to the event that the state of the world is good and the auditor observes this. The second term corresponds to the event that the auditor believes the state of the world to be bad when in fact it is bad. The final term covers the event that the auditor correctly assesses the state to be bad.

Now if we let $\delta b = b_h - b_L > 0$ then we show in Appendix C that this implies that (19) is equivalent to the requirement that

$$\Pi^h(\mu^R; \mu^S) > \Pi^l(\mu^R; \mu^S) + \frac{[\psi^R - \psi^L]}{\delta b} > 0$$

(20)

Since $[\psi^R - \psi^L]/\delta b > 0$, comparison of (20) with (14), establishes that in order to ensure that auditors provide a high degree of care we require an even higher level of legal liability than required to resolve
the simple collusion problem the previous subsection. Thus, if we establish lack of incentive compatibility for low effort then it will also hold for higher levels of effort\(^9\).

**PROPORTIONAL LIABILITY AND AUDITOR INCENTIVES**

Given the above analysis of the determinants of a legal liability level \(\alpha\), which ensures auditor incentive compatibility, we can now apply this to the formal policy issues at hand. In particular, there have been calls for the liability that auditors are subjected by the courts to be restricted to proportional liability. Recently, Narayanan (1994) produced a model for which moving to a proportionate liability regime is predicted to increase audit quality (auditor effort). However, as Gigler (1994, p. 63) points out, 'The ultimate concern of regulators is the incidence of fraud or misstatement, and one cause of such failures is arguably a lack of auditor effort. However, if something else like management effort of management's incentive to be honest can substitute for auditor effort (as may be the case with comparative negligence), comparing negligence regimes on the basis of auditor effort alone misses the regulators' true objective.' In this respect, our model considers the application of proportional liability in a more general setting than that of Narayanan (1994) since we consider the strategic interplay between auditor effort, rents and management incentives to be honest and hence are closer in spirit to Gigler's (1994) notion of wider regulators' objectives. Similarly, in an experimental setting, Dopuch et al. (1994) provide support for the use of proportional liability but do so with an experimental game that does not allow verifiers to earn rents from sellers and hence they also do not test the wider regulators' objectives. For our model let us now consider how to model changes in the regulatory regime in which the form of \(\alpha\) (auditor liability) becomes a regulatory choice variable.

In order to be able to address this issue we now need to consider, in general terms, how plaintiffs' losses are assessed by the courts. Rather than propose a unitary method of assessment of loss, we will model the assessment in a flexible manner such that by varying choice parameters one can determine the effect of a range of different assessment procedures.

\(^9\)This proof implicitly assumes a strict liability regime for auditors. The analysis could also be extended to a due care auditor negligence regime by linking the liability level \(\alpha\) to the level of effort provided.

![Figure 2. Calculation of loss at time \(t^*\).](image)

A natural modelling assumption would be to assume that the loss is the difference between the expectation of the firm's value if it had evolved according to the instantaneous rate of return at \(t = 0\), and the firm's liquidation barrier value as illustrated in Fig. 2.

Let \(\kappa\) be the proportion of the loss that the courts attribute to the auditor. In general, \(\kappa\) could be assessed by the courts to be any value within the closed interval \(\kappa \in [0, 1]\), that is \(\kappa\) would be case specific. Our concern is to question whether such liability assessment is robust over a wide range of case settings and thus sufficient to ensure incentive compatibility for all \(\kappa\). Thus, if we can show that with \(\kappa = 1\), a liability rule is not incentive compatible, then it will not be incentive compatible for all smaller \(\kappa\). Similarly, the issue of the existence of multiple defendants is likely to be case specific. However, if we maintain the above assumption of \(\kappa = 1\), which gives an upper bound on the liability an auditor would face, we can still evaluate a rule, if it is not incentive compatible for this upper bound.

Proceeding along these lines we define the expected cost of legal damages to the auditor to be

\[
a = D(T) = \int_0^T e^{-\lambda t} [V(0)e^{\gamma t} - \lambda e^\gamma p_T(t)] dt
\]

(21)

Recalling how \(l\) is defined allows us to rewrite this in the more convenient form

\[
a = D(T) = V(0) \int_0^T p_T(t) dt - \lambda V(0) \int_0^T e^{-\lambda t} p_T(t) dt
\]

(22)
Evaluation of $D(T)$ is straightforward using the results of Appendix A where we choose $\beta = 0$ for the first integral and $\beta = (\gamma - \rho)$ for the second.

In principle then we could compare $D(T)$ directly with incentive compatible $\beta$. However, here we need only present a counter example which shows that $D(T)$ cannot always be relied upon to provide sufficient incentives.

Since $\lambda = 0$, an upper bound on legal damages is

$$\tilde{D}(T) = V(0) \int_0^T p\tau(t) dt = V(0) \Delta(T)$$

Thus by (15) if

$$V(0) \Delta(T) < - \frac{1}{R} \left( 1 - \left( 1 + \frac{2R(1-\Delta)}{\Delta} \right) \left( X - \frac{1}{2} X^2 R \right) \right)^{1/2}$$

(24)

legal damages will not provide an auditor with sufficient incentives. As an example of where this inequality is easy to analyse recall (17). Thus we have:

$$V(0) \Delta(T) < X \left( \frac{1-\Delta(T)}{\Delta(T)} \right)$$

(25)

In such a situation, potential legal damages will not provide sufficient incentives to ensure auditors maintain independence (incentive compatibility will not hold).

To summarize, if (25) is satisfied, proportionate liability is not sufficient to provide auditors with the required incentives. Liability levels are required to be higher. We provide a more detailed discussion of the intuition behind this result later. This follows the presentation next of a simple numerical example which illustrates our analysis and demonstrates the plausibility of (25) being satisfied.

**Critical Rents**

When applying the above analysis, we need to determine the expected present value of future rents $X$. Clearly such figures are not publicly available and are difficult to estimate because audit personnel charge-out rates are full costs, which incorporate overheads and required partner returns, in such a way that notionally no explicit rent is earned on audits.\(^{10}\)

Rather than present a theory of such rent determination, we need only consider the minimum level of rents, which would result in proportional

\(^{10}\) For a detailed discussion of equilibrium rents, see the earlier referred to work of Dye (1991).

liability not providing sufficient incentives to auditors. After determining analytically the form of the minimum rent levels, we show via a simple numerical example that these rents need only be relatively small.

By rearranging terms we can rewrite (25) as

$$X > V_0 \left( \frac{\Delta(T)^2}{1 - \Delta(T)} \right)$$

Thus, at the critical value $X_c$

$$X_c = V_0 \left( \frac{\Delta(T)^2}{1 - \Delta(T)} \right)$$

(27)

liability is just sufficient. This critical value is a useful construct since it is a function of the underlying asset valuation parameters and hence for a given choice of parameter(s), we can determine the form of the $X_c$ function. We can then determine for which side of the function, (25) is always satisfied.

To illustrate the form of this functional relationship via a numerical example assume $T = 1; \tau = 0.1; \lambda = 0.5; \gamma = 0.05; V_0 = 100000000$; and we wish to consider how the critical rent $X_c$ varies with the instantaneous variance of return (for the firm per unit time) $\sigma^2 \in [0.1, 0.3]$.

To illustrate the implied order of magnitudes involved for this numerical example if $\sigma^2 = 0.1; X_{c, \text{R}} = 9193.59$ and it means that if rents are greater than one tenth of one percent of the firm’s value, proportional liability provides insufficient incentives for the auditor.\(^{11}\) Figure 3 also demonstrates that the auditor requires larger rents the more volatile the audit client’s market value since increasing volatility increases the chance of the barrier being reached.

**The Nature of Sufficient Liability**

The above analysis clearly demonstrates that proportional liability will not, in general, provide auditors with sufficient incentives. How can we then relate this result to the present debate in which it is claimed that legal reform is required, because auditors are being sued for damages way beyond

\(^{11}\) Figure 3 demonstrates that an interesting area for further research may be to consider whether auditors’ liability could be assessed in part on the basis of the ex ante reported risk associated with a given audit. Estimating the risks associated with the conducting of an audit is an integral part of audit practices (see for instance Wallace, 1991, for further details). Thus it would be interesting to consider the implications of requiring the auditor to communicate the adjudged estimate of audit risk.
what is reasonable to assume they are responsible for? It does superficially appear that, at present, auditors are subject to excessive liability exposure. However, a change in the liability regime under which auditors become assessed on a proportionate basis may not be in the public interest because auditors would not have sufficient incentives to signal commitment to independence and a high degree of care.

One of the central purposes of this paper is to recognize the dual role that auditors' legal liability exposure performs. First, to ensure that plaintiffs are reimbursed for the proportionate damages auditors cause. Secondly, to ensure that auditors have sufficient performance incentives. Given this dual role, at issue is whether liability rules which satisfy the requirements of the first role also satisfy the requirements of the second role. Using contingent claims analysis, we have demonstrated that in a wide range of circumstances (where \( 26 \) is satisfied), liability levels which ensure satisfaction of the auditor commitment requirement (role two) need to be greater than those determined to satisfy proportionate liability. Put simply, liability levels should be set both to make good damages to plaintiffs and discourage bad auditor behaviour; proportionate liability may only satisfy the former or perhaps even neither. In this respect it is interesting to note that in a somewhat similar critical tone over the possible introduction of proportionate liability, the Law Commission's report on a Feasibility Investigation of Joint and Several Liability (1996, p. 16) concludes that:

The joint and several liability principle rests on standard, well accepted, principles of causation that present a formidable hurdle for plaintiffs. In order to be jointly and severally liable for loss, each defendant must have been causally responsible for the whole of that loss. ... As a matter of causation, ... joint and several liability follows from each defendant being 100 per cent responsible for the whole of P's loss.

It is important to note that the auditing profession makes claims that it self-regulates its members so as to ensure that bad behaviour is discouraged, at least in part. Clearly, the extent to which this is realized, will determine limits on the need for liability to be greater than proportionate to blame. A research topic of some importance therefore, is work on comparing the efficacy of auditor performance induced by professional internal regulatory controls, as compared with that induced by high liability exposure (over and above that required to satisfy proportionate liability).

We do not develop this issue here, but do note that recent research has called into question the ability of the accounting profession to regulate itself adequately\(^{12}\). For instance, using the Savings and Loan crisis as an example, Merino and Kenny (1994, p. 181) recognize 'the importance that auditors have placed on judgement as the critical attribute justifying their professional status, and ... ask if they exercised professional judgement when auditing routine everyday accounting techniques in the S & L industry'. They carefully document how changes in the late 1970s and early 1980s resulted in the previously low risk thrift industry, becoming significantly more risky. Thus, they argue (Merino and Kenny, 1994, p. 184) that 'As such, the changes should have been a signal to auditors of the need for closer scrutiny of management and more stringent audit procedures. Apparently, the AICPA did not recognize the signal as the S & L Audit Guide was not updated between 1979 and 1985'. In conclusion (Merino and Kenny, 1994, p. 189) they argue that 'we think auditors failed in terms of what they have consistently maintained is the central attribute justifying professional status, which is the ability to use their experience and expertise to determine what is appropriate in a particular circumstance.'

**CONCLUSIONS**

Auditors have become increasingly concerned about the risk of litigation associated with the provision of audit services. We have developed a

\(^{12}\)For a discussion on these issues from the perspective of a lawyer see Freedman (1993).
model of how litigation is triggered and have applied contingent claims analysis to determine the expected cost of litigation. The model is of potential value to auditors since it can be utilized to analyse the expected cost of audit litigation for existing and potential clients, once the risk characteristics of the clients have been estimated. This would enhance auditors’ ability to price audit services effectively and identify clients that are good candidates for adoption or continuance. The model may also be of interest to regulators concerned with reforming auditors’ present legal liability exposure. In order to appraise various reform proposals, it is critical that regulators are able to identify how reduced liability could influence auditors’ incentives. When considering proposals to reduce liability, regulators should consider whether such revisions still motivate auditors to conclude that the option of colluding with management is less desirable than maintaining independence.

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APPENDIX A
Given our assumed firm value diffusion process and the path of the litigation barrier the probability density function takes the form of the Inverse Gaussian density function and so if we put 
\[ \mu = (r - \gamma - \frac{1}{2} \sigma^2) \] we have

\[
G(s, r, T) = \alpha \int_0^T e^{-\alpha t} \frac{-ln y}{\sigma \sqrt{2 \pi t}} \exp \left( \frac{-ln y - [(r - \gamma - \frac{1}{2} \sigma^2) t]^2}{2 \sigma^2 t} \right) dt
\]

(28)

Our strategy will be to evaluate this integral to give

\[
G(s, r, T) = \alpha \sqrt{\frac{2 \pi}{y^2}} \exp \left( \frac{-ln y - [(r - \gamma - \frac{1}{2} \sigma^2)]^2}{2 \sigma^2 t} \right)
\]

Our strategy will be to evaluate this integral to give

\[
G(s, r, T) = \alpha \sqrt{\frac{2 \pi}{y^2}} \exp \left( \frac{-ln y - [(r - \gamma - \frac{1}{2} \sigma^2)]^2}{2 \sigma^2 t} \right)
\]

(29)

By choosing

\[ \Delta(r, T) = y^{-\frac{1}{2}} \left[ N \left( \frac{ln y - \gamma \sigma^2 T}{\sigma \sqrt{T}} \right) + y^2 N \left( \frac{ln y + \gamma \sigma^2 T}{\sigma \sqrt{T}} \right) \right] \]

(30)

so we can express \( G(s, r, T) \) as

\[ G(s, r, T) = \alpha \Delta(r, T) \]

(31)

where

\[ \delta = \left( r - \gamma - \frac{\sigma^2}{2} \right) + 2r \sigma^2 \]

\[ \eta = \frac{(\delta - 2r \sigma^2)^{1/2}}{\sigma^2} \]

\[ \zeta = \frac{\sqrt{\delta}}{\sigma^2} \]

\( N(.) \) denotes the standard normal cumulative distribution function.

We wish to evaluate (28). However anticipating later analysis we instead evaluate

\[ I = \int_0^T \frac{e^{-\beta t} - \ln y}{\sigma \sqrt{2 \pi t}} \exp \left( \frac{-ln y - [r - \gamma - \frac{1}{2} \sigma^2] t^2}{2 \sigma^2 t} \right) dt \]

(32)

Evaluating this integral will allow us to evaluate (28) by simply requiring us to choose \( \beta = r \). In later analysis we will also be free to choose \( \beta = r - \gamma \). Rearranging terms gives

\[ \frac{\sigma \sqrt{2 \pi}}{\ln y} I = \int_0^T \frac{e^{-\beta t} - \ln y - [r - \gamma - \frac{1}{2} \sigma^2] t^2}{2 \sigma^2 t} dt \]

(33)

Consider now the argument of the exponential function. We have

\[ \beta t + \frac{\ln y - [r - \gamma - \frac{1}{2} \sigma^2] t^2}{2 \sigma^2 t} \]

\[ = \frac{1}{2 \sigma^2 t} \left( 2 \beta \sigma^2 t^2 + \left( \ln y - \left[ r - \gamma - \frac{1}{2} \sigma^2 \right] t \right)^2 \right) \]

\[ = \frac{1}{2 \sigma^2 t} \left( \ln y^2 + \left[ r - \gamma - \frac{1}{2} \sigma^2 \right]^2 + 2 \beta \sigma^2 \right) t \]

(34)

\[ \frac{\ln y}{\sigma^2} \left[ r - \gamma - \frac{1}{2} \sigma^2 \right] \]

(35)

Define

\[ \delta = \left[ r - \gamma - \frac{1}{2} \sigma^2 \right]^2 + 2 \beta \sigma^2 \]

(36)
Thus we can now rewrite (35) as

$$\frac{1}{2\sigma^2 t} (\ln y)^2 + \delta t^2) - \ln \left[ y \left( \frac{\tau - \gamma - \frac{1}{2} \sigma^2}{\sigma^2} \right)^2 \right] = \frac{\delta}{\sigma}$$

(37)

Now we have \( \delta - 2 \beta \sigma^2 = (\tau - \gamma - \frac{1}{2} \sigma^2)^2 \) thus (37) is equivalent to

$$\frac{1}{2\sigma^2 t} (\ln y)^2 + \delta t^2) - \ln \left[ y \left( \frac{\delta - 2 \beta \sigma^2}{\sigma^2} \right)^{1/2} \right]$$

(38)

Define

$$\sigma^2 \theta = \tau - \gamma - \frac{1}{2} \sigma^2$$

(39)

and thus

$$\sigma^2 (\theta - 1) = \tau - \gamma - \frac{1}{2} \sigma^2$$

(40)

Hence

$$\delta - 2 \beta \sigma^2 = \sigma^4 (\theta - 1)^2$$

(41)

now define

$$\left( \frac{\delta - 2 \beta \sigma^2}{\sigma^4} \right)^{1/2} = \eta$$

(42)

and

$$\eta = \frac{\sigma^2}{\sigma^4} (\theta - 1)$$

(43)

then

$$\eta = \frac{\delta - 2 \beta \sigma^2}{\sigma^4} (\theta - 1)$$

(44)

Thus (38) is

$$\frac{1}{2\sigma^2 t} (\ln y)^2 + \delta t^2) - \ln y^\eta$$

(45)

Negating (45) and exponentiating we have

$$y^\eta \exp \left( -\frac{1}{2\sigma^2 t} (\ln y)^2 + \delta t^2 \right)$$

(46)

thus we can rewrite (33) as

$$-\frac{\sigma^2}{\ln y} I = \int_0^\tau y^{\tau - 3/2} \exp \left( -\frac{1}{2\sigma^2 t} [(\ln y - t\sqrt{\delta})^2 + 2 \ln yt\sqrt{\delta}] \right) dt$$

(47)

and hence completing the square we have

$$-\frac{\sigma^2}{\ln y} I = y^{-\sigma/\sqrt{\delta}} \int_0^\tau t^{3/2} \exp \left( -\frac{(\ln y - t\sqrt{\delta})^2}{2\sigma^2 t} \right) dt$$

(48)

Hence with

$$\zeta = \frac{\sqrt{\delta}}{\sigma^2}$$

(49)

$$I = -\frac{\gamma^{1-\gamma} \ln y}{\sigma^2} \int_0^\tau t^{3/2} \exp \left( -\frac{(\ln y - t\sqrt{\delta})^2}{2\sigma^2 t} \right) dt$$

(50)

Now since the normal distribution with zero mean and unit variance has density function \( N(x) \equiv 1/\sqrt{2\pi e^{-x^2/2}} \) we can rewrite (50) as

$$I = -\frac{\gamma^{1-\gamma} \ln y}{\sigma} \int_0^\tau t^{3/2} \left( \frac{\ln y - t\sqrt{\delta}}{\sigma} \right) dt$$

(51)

From Cox and Miller (1965), for example, we can immediately recognize that \( I \) is the time integrand of the first passage probability density function of an arithmetic Brownian motion to an absorption point at ln y. We can thus immediately evaluate (51) by referring to the derivation of the first passage time p.d.f. Hence

$$I = y^{\gamma-\gamma} \left( N \left( \frac{\ln y - t\sqrt{\delta}}{\sigma^2/\tau} \right) - \exp \left( \frac{2\sqrt{\delta} \ln y}{\sigma^2/\tau} \right) \right)$$

(52)

thus rearranging terms and recognizing that \( N(-x) = 1 - N(x) \) and \( N(x) = (2\pi)^{-1/2} \int_\infty^\infty e^{-x^2/2} dt \),

gives

$$I = y^{\gamma-\gamma} \left[ N \left( \frac{\ln y - c\sigma^2 T}{\sigma^2/\tau} \right) + y^{2\gamma} N \left( \frac{\ln y + c\sigma^2 T}{\sigma^2/\tau} \right) \right]$$

(53)

as required.

Note that \( \Delta(t, T) \) denotes the discounted first passage time probability distribution, \( \int_0^T e^{-\rho T} \tau(t) dt \). In later analysis when we use the notation \( \Delta(T) \) we will be referring to the (undiscounted) first passage time probability distribution \( \int_0^T \tau(t) dt \) with obvious distinction between the notation \( \Delta(t, T) \) and \( \Delta(T) \).

APPENDIX B

For incentive compatibility we require

$$\Pi(\mu^b | \mu^b) \leq \Pi(\mu^b | \mu^b)$$

(54)

where

$$\Pi(\mu^b | \mu^b) = (1 - \Delta(T))U(F + X) + \Delta(T)U(F - x) - \phi(\epsilon)$$

(55)

and

$$\Pi(\mu^b | \mu^b) = U(F) - \phi(\epsilon)$$

(56)
In the following analysis we will consider the $\alpha$ which results in (54) holding with strict equality. Let us now consider the Taylor series second-order expansions about $F$ of the $U(F + X)$ and $U(F - \alpha)$ terms which appear in (55)

$$U(F + X) = U(F) + XU'(F) + \frac{1}{2} X^2 U''(F^*) \quad F^* \in [F, F + X]$$

(57)

$$U(F - \alpha) = U(F) - \alpha U'(F) + \frac{1}{2} \alpha^2 U''(F^*) \quad F^* \in [F - \alpha, F]$$

(58)

It will be useful to note that for (57), since $U(F + X) - U(F) > 0$ we also have $XU'(F) + \frac{1}{2} X^2 U''(F^*) > 0$. Thus we can rewrite the strict form of (54) as

$$\frac{1 - \Delta(T)}{\Delta(T)} \left( XU'(F) + \frac{1}{2} X^2 U''(F^*) \right) = \alpha U'(F) - \frac{1}{2} X^2 U''(F^*)$$

(59)

Let

$$K = \frac{1 - \Delta(T)}{\Delta(T)} \left( XU'(F) + \frac{1}{2} X^2 U''(F^*) \right) > 0$$

(60)

Thus the critical $\alpha$ satisfies the quadratic equation

$$\frac{1}{2} U''(F^*) \alpha^2 - U'(F) \alpha + K = 0$$

(61)

which implies that

$$\alpha = \frac{U'(F)}{U''(F^*)} \pm \left( \frac{(U'(F))^2 - 2U''(F^*)K}{U''(F^*)} \right)^{1/2}$$

(62)

making the standard assumptions that $U'(.) > 0$ and $U''(.) < 0$ and excluding any negative root of $\alpha$, we thus require that

$$\alpha = \frac{U'(F)}{U''(F^*)} \left( 1 - \frac{2U''(F^*)K}{(U'(F))^2} \right)^{1/2} > 0$$

(63)

(64)

At present this expression seems to be a somewhat complex mix of terms. However once we recognize that we are searching for a lower bound on $\alpha$ and that the Arrow-Pratt measure of absolute risk aversion is $R = -U''(F)/U'(F)$ we can considerably simplify the expression. Noting that

$$\frac{U'(F)}{U''(F^*)} \geq \frac{U'(F)}{U'(F)}$$

(65)

gives us

$$\alpha \geq -\frac{1}{R} \left( 1 - \left( 1 + \frac{2K}{U'(F)} \right)^{1/2} \right)$$

(66)

and similarly recognizing that

$$\frac{U''(F^*)}{U'(F)} \geq \frac{U''(F)}{U'(F)}$$

(67)

gives us the following lower bound

$$\alpha \geq -\frac{1}{R} \left( 1 + \frac{2R(1 - \Delta(T))}{\Delta(T)} \left( X - \frac{1}{2} X^2 R \right)^{1/2} \right)$$

(68)

thus we are able to write a lower bound on $\alpha$ which depends on three primitive variables; the first passage time probability, rents and risk attitude.

APPENDIX C

We require

$$(1 - b)\Pi^b(\mu^b; \mu^b) + (b - b^1)\Pi^b(\mu^b; \mu^b) + \Pi^1(\mu^b; \mu^b) > 0$$

(69)

let us now consider term by term comparisons for the expanded terms

$$(1 - b)[\Pi^b(\mu^b; \mu^b) - \Pi^1(\mu^b; \mu^b)] = (1 - b)[-\psi(\mu^b) + \psi(\mu^b)] < 0$$

(70)

since by definition $\psi^b > \psi^i$

$$(b - b^i)[\Pi^b(\mu^b; \mu^b) - \Pi^1(\mu^b; \mu^b)] - \delta b \Pi^b(\mu^b; \mu^b) = (b - b^i)[-\psi(\mu^b) + \psi(\mu^b)] - \delta b \Pi^b(\mu^b; \mu^b) < 0$$

(71)

and finally

$$b^i(\Pi^b(\mu^b; \mu^b) - \Pi^1(\mu^b; \mu^b))$$

(72)

$$+ \delta b \Pi^b(\mu^b; \mu^b) = b^i[-\psi(\mu^b) + \psi(\mu^b)] + \delta b \Pi^b(\mu^b; \mu^b)$$

If the auditor is to prefer to provide high effort (i.e. (69) holds) this implies that we must have

$$\delta b \Pi^b(\mu^b; \mu^b) > (1 - b)[\psi(\mu^b) - \psi(\mu^b)] + (b - b^i)[-\psi(\mu^b) + \psi(\mu^b)]$$

(73)
rerranging terms gives
\[
\delta b [\Pi^b (\mu^b; \mu^b) - \Pi^b (\mu^b; \mu^b)] > [1 - b] + (b - b^e_s) + b^e_s \left[ \psi (e^b) - \psi (e^s) \right]
\]
(74)
which is equivalent to requiring
\[
\Pi^b (\mu^b; \mu^b) > \Pi^b (\mu^b; \mu^b) + \frac{[\psi (e^b) - \psi (e^s)]}{\delta b}
\]
(75)
Thus since clearly \( [\psi (e^b) - \psi (e^s)] / \delta b > 0 \), by inspection of (14) we note that in order to induce the auditor to self-select a high degree of care, we require liability to be higher than as determined by (14) at equality.

REFERENCES


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