

FINANCIAL OPTIONS RESEARCH CENTRE

University of Warwick

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**Russell Grimwood
and
Stewart Hodges**

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*Financial Options Research Centre
Warwick Business School
University of Warwick
Coventry
CV4 7AL
Phone: (0)24 76 524118*

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Russell Grimwood †
Stewart Hodges

The Financial Options Research Centre
Warwick Business School
University of Warwick
Coventry CV4 7AL
United Kingdom

Phone: +44 (0)24 7652 2701, Fax: +44 (0)24 7652 4167

Email Russell Grimwood : `forcrg@wbs.warwick.ac.uk`
Email Stewart Hodges : `forcsh@wbs.warwick.ac.uk`

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Abstract

This work investigates the different languages used to characterize contingent claim payoffs and cash flows. The mathematical operators used to mark up these payoffs and cash flows are formalized. A translation is provided between equivalent colloquial (jargon), legal and computational terms used. This part of the work should therefore be useful for those learning about derivatives who are unclear on the legal and mathematical terminology.

The tensions and inadequacies inherent in using different languages to describe contingent claims are illustrated and thus the desirability of a legally binding machine readable language becomes clear. The progress towards a coherent computer language for characterization of contracts (and their improved classification in terms of information monitoring) is reviewed.

JEL classification: G13

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1 Introduction

1.1 The Treachery of Language

This work is concerned with the nature of language and the translation or mapping between languages. That is, a language's finite vocabulary confines one's ability of expression and that translation between different languages has abundant scope for error. This study is limited to the languages that are used to characterize contingent claim contracts¹, nevertheless there is still ample opportunity to err.

A financial contract is described by a language, this (formal) language has a syntax and a type system. The choice of type system is a trade-off between expressiveness and the complexity of implementation. Within the finance industry there exist, along side each other, at least four languages for the characterization of contingent claims. The first is the colloquial language (jargon) which is used typically in the front office by traders to verbally describe a deal. The second is the written language of mathematics which is used by researchers and quantitative analysts to rigorously describe contingent claim cash-flows. The third is the written legal language used by lawyers in confirmation documents. Finally, the fourth language is the computer readable code used by IT professionals and computer scientists for the automatic processing of contracts, their valuation and risk management. When a single organization, such as a bank, has different departments and different groups of professionals describing the same product with different languages then there will be tensions and the scope for error when mapping between these languages is clearly acute. It is therefore desirable research objective to develop an accurate and consistent treatment for the description of contingent claims across front, middle and back offices².

1.2 The Anatomy of Trade Confirmation

A trade is defined as an agreement between two or more parties to exchange some financial instrument(s) and to convey whatever information is necessary in order to carry out that exchange.

The process of trade confirmation initially involves the trade being verbally agreed by the two parties over the phone, a confirmation document is then drawn up. The document itself usually starts with an International Swap and Derivatives Association (ISDA) Master Agreement. This is a contract widely used by industry participants, it has established international contractual standards governing privately negotiated derivatives transactions that reduce legal

¹All securities, except possibly government bonds without call features, are contingent claims of one kind or another.

²Front office: Is the area or function which relates to trading, investing, or sales activities for a financial firm. Orders start here, flow through the middle office, if any, and get processed by the back office. Middle office: Is the area or function which relates to risk management. This area measures, monitors and occasionally pro-actively manages the firm's exposure. Back office: Refers to the area or function which relates to the processing, record keeping, and other operational aspects of transactions for financial firms.

uncertainty and allow for reduction of credit risk through netting of contractual obligations. The contract is then modified and checked by each party, faxed back and forth and finally when both parties are satisfied with its content it is signed. The whole process of trade confirmation can take several days. Here then there is clearly a mapping process going on between the verbal description of the trade i.e. the jargon language and the legal language used in the confirmation agreement. The inadequacies of this mapping mean the counter parties have to repeatedly amend the legal document until they are satisfied it is an accurate representation of the trade they initially characterized verbally.

1.3 Description Standards

The reason the facsimile has been adopted as the medium for contract confirmation is that, as the derivatives industry has grown, contingent claim contracts have come to be described by instrument specific proprietary processing systems. As the trade processing systems of different banks cannot therefore communicate with one another electronically the back office task of trade confirmation and settlement has become extremely inefficient and manually intensive, resulting in considerable fax, phone and paper transfer between counter-parties. The desire to reduce these inefficiencies and move towards Internet based confirmation has created a demand in the industry for the standardization of the description of contingent claim contracts, see for example the functional programming work of S. Jones, J. Eber and J. Seward (2000) [32] and the Financial Products Markup Language of J.P. Morgan and PricewaterhouseCoopers (1999) [29]. Internet based confirmation offers the possibility of reducing confirmation time and unit costs and therefore increasing the volume of trades.

Here then is a problem of the legal language representation of the contract being mapped onto a myriad of different computer based trade processing systems with the contract in some machine readable language which is not as yet standardized.

1.4 Motivation

The motivation for this work is threefold; firstly to formalize the mathematical operators (Section 3) which are involved in the mark up of cash flows and payoffs, to provide a guide to equivalent colloquial (Section 4) and legal (Section 5) terms; secondly to highlight throughout the different sections the tensions that exist between these different languages and thirdly to suggest future research into a machine readable legally binding language and to review the work currently in progress on the consistent treatment and automation of contingent claims (Section 6). An example of the different languages used to characterize contingent claims is given in Section 2. Conclusions are drawn in Section 7.

2 Contingent Claim Languages: An Example

This section exemplifies the four contingent claim languages with respect to a type of barrier option.

2.1 Colloquial

“Long a European up-and-out barrier call option with a strike price of X , a maturity of T and a barrier at H ”. In plain English this means: one has purchased an option which at maturity (the end of its life) gives one the right, but not the obligation, to purchase the underlying asset minus the strike price, if and only if the barrier has not been reached, otherwise it expires worthless.

2.2 Mathematical

$\max(S_T - X, 0)I_{S_t < H}$ where S_T is the underlying asset price at maturity T , X is the strike price and $I_{S_t < H}$ is the condition indicator which is true (has a value of 1) when $\sup(S_t) < H$ otherwise it is false (has a value of 0). H is the barrier level and $t \in [0, T]$.

2.3 Legal

The following tract is an example of a hypothetical confirmation document and ISDA Master Agreement:

The purpose of this letter agreement (this “Confirmation”) is to confirm the terms and conditions of the Transaction entered into between us on the Trade Date specified below (the “Transaction”). This Confirmation constitutes a “Confirmation” as referred to in the ISDA Master Agreement specified below.

The definitions and provisions contained within the 1996 ISDA Equity Derivatives Definitions (the “Equity Definitions”) as published by the International Swap and Derivatives Association, Inc., are incorporated into this Confirmation. In the event of any inconsistency between the Equity Definitions and this Confirmation, this Confirmation will govern.

1. This Confirmation supplements, forms part of, and is subject to, the ISDA Master Agreement dated as of 8th June 1994 (the “Agreement”), between X and Y as amended and supplemented from time to time, between you and us. All provisions contained in the Agreement govern this Confirmation except as expressly modified below:
2. The terms of the particular Transaction to which this Confirmation relates are as follows:
Trade Date: day month year amended day month year

Effective Date: day month year subject to adjustment in accordance with the Following Business Day Convention
Maturity Date: day month year subject to adjustment in accordance with the Following Business Day Convention
Termination Date: day month year subject to adjustment in accordance with the Following Business Day Convention
Index: name of index
Exchange: name of exchange

Equity Amounts payable by:
Equity Amount Payer: X
Equity Notional Amount: currency and amount
Equity Amount Payer Payment Date: day month year subject to adjustment in accordance with the Following Business Day Convention
Type of Return: Price Return
The Equity Amount Payer will pay:
 $(1 - A) \times \max(S - X, 0) \times \text{Equity Notional Amount}$
where:
 $A = 1$ if the closing level of the Index from and including the Effective Date to and including the Maturity Date is at a level above level.
 $A = 0$ if the closing level of the Index from and including the Effective Date to and including the Maturity Date never closes above level.
 S means, the closing level of the Index on the Maturity Date
Averaging Date Market Disruption: Postponement

Fixed Amount payable by:
Fixed Amount Payer: Y
Fixed Amount: currency and amount
Fixed Amount Payment Date: day month year
Business Days: London
Calculation Agent: X or Y

3. Account Details: branch, account number, sort code
Counter-party Account Details: To be advised

4. Broker: Z

5. Documentation:
Please confirm that the foregoing correctly confirms the terms of our agreement by executing the copy of this confirmation enclosed for that purpose and returning it to
Fax Number: number

Telex Number: number
Telephone number for confirmation queries: number

2.4 Machine Readable (Computer Algorithm)

Ideally the description of a financial contract should be kept independent of the evaluation model. However, in most banks this is not the case, there is simply a large library of algorithms of which perhaps only one will price a certain contingent claim. The algorithm explicitly requires the barrier option parameters (the strike level X , the barrier level H and the maturity T), ancillary information (valuation date, settlement date of premium and settlement date of payoff) and the modeling assumptions (the underlying asset price S , the volatility σ , the interest rates r and any dividends q or foreign interest rates r_f) for example a routine may be called to price our example contingent claim using `euro_up_and_out_call(S,X,T,H,r, σ)`. The model itself is implicit (for example a Black Scholes world) and is coded into the algorithm (different models and algorithms require different input parameters.) The pricing algorithm is usually coded in Visual Basic (for speed of implementation), C and C++ (where run time speed is a priority) or Java (for Internet applications). If a new instrument is developed or different modeling assumptions are explicitly made then a new algorithm must be coded.

2.5 Information Capture

The four different languages have arisen to capture information about the contingent claim contract for specific applications. The colloquial language is a medium for the rapid oral description of the contract terms between traders. The mathematical language is more precise in that it avoids the subjective interpretation of words and is useful for quantitative analysts. The legal language is a medium for resolving disputes in the courts and therefore gives written definitions for the contract terms and attempts to make provisions for future events. The description of the contract pricing algorithm is a machine readable medium.

Each language serves its particular application but has limitations. The colloquial language often has colorful terms but these can be imprecise and subjective. The legal language is very verbose and in the experience of the author is not always complete enough to reproduce all the contract cash flows. The mathematical language is not accessible to those without a mathematical background and also often lacks ancillary information such as bank account numbers and addresses. The contract pricing algorithm is not in a human readable form so is not accessible to professionals with a non-computing background. The fact that interpretation is sometimes subjective or that information may be missing results in the difficulty of mapping between the different languages. This is where the tension between the different languages has its source and is the motivation

for a developing a machine readable, legally binding language which is also relatively readable for humans. The ideal is to translate the informal jargon into a formal mathematical syntax and to represent legal contingent claim documents using this syntax in such a way that enough pricing and ancillary³ information is communicated with the back office in order for it to carry out the necessary confirmation and settlement processes for the contract⁴, see Appendix A.

3 Mathematical Characterization of Contingent Claims

3.1 Mathematical Operations

To catalogue and treat each individual contingent claim as an elemental object would be to ignore the common underlying mathematical building blocks of many of these contracts. It would also mean that one would be ill equipped to deal with the inevitable demand for a new type of contract which can often be decomposed into these smaller mathematical components. We will define an algebra of quantities from which we can compose our contingent claim contracts:

Quantities

$D, S, X, Y, P \in \mathcal{A}$ where \mathcal{A} is an algebra.
 $\mathfrak{R} \in \mathcal{A} \Rightarrow X + Y, X - Y, X \times Y, X \div Y$ and $X^Y \in \mathcal{A}$.
 $\max\{X, Y, \dots, Z\} \in \mathcal{A}$
 $\min\{X, Y, \dots, Z\} \in \mathcal{A}$
 $[X]^+ = \max\{X, 0\} \in \mathcal{A}$
 $\sum_i n_i S_i \in \mathcal{A}$
 $\prod_i n_i S_i \in \mathcal{A}$

Events

$\sup\{S_{t_i}\}$ the supremum of S_{t_i} where $t_i \in \{t_1, t_2, \dots, t_N\}$.

$\inf\{S_{t_i}\}$ the infimum of S_{t_i} where $t_i \in \{t_1, t_2, \dots, t_N\}$.

Indicators

I is the condition indicator which is equal to 1 when the condition is true and 0 when it is false.

³The ancillary information includes the trade ID, the product description, counter-party identification data (name and address), settlement instructions (dates and account details), fees and terms, market data (e.g. implied volatility) and pricing parameters (e.g. the level of the FTSE100 at close of trading).

⁴There needs to be sufficient information for the back office to monitor business events (new trades, exercise events and unwinds), to validate trades, to match trades and to perform prompt settlement to minimize the cost of carry charges associated with trade failures.

3.2 Mathematical Payoffs

Throughout we will use the sign convention that cash flows to the holder of the contingent claim are positive and cash flows to the issuer of the contingent claim are negative.

3.2.1 European Option

Call $D_T = S_T - X$. Pay $[D_T]^+$ at T .

Put $D_T = X - S_T$. Pay $[D_T]^+$ at T .

3.2.2 American Option

$$D_t = [f(S, X)]^+.$$

The option pays D_t at a maximum of one date $t \in [0, T]$ chosen by the holder.

American call $D_t = [S_t - X]^+$ where the holder chooses $t \in [0, T]$.

American put $D_t = [X - S_t]^+$ where the holder chooses $t \in [0, T]$.

3.2.3 Bermudan Option

$$D_t = [f(S, X)]^+.$$

The option pays D_t at a maximum of one date t chosen by the holder where $t = \{t_1, t_2, \dots, t_N\}$, $t_1 \geq 0$ and $t_N \leq T$.

American call $D_t = [S_t - X]^+$ where the holder chooses t .

American put $D_t = [X - S_t]^+$ where the holder chooses t .

3.2.4 Asian

$\left\{ \begin{array}{l} \text{Geometric} \\ \text{Arithmetic} \end{array} \right\}$ average $\left\{ \begin{array}{l} \text{Price} \\ \text{Strike} \end{array} \right\}$ $\left\{ \begin{array}{l} \text{Call} \\ \text{Put} \end{array} \right\}$ option.

Arithmetic average of Y is $\bar{Y} = \frac{1}{N} \sum_{i=1}^N Y_{t_i}$ where $t_i = t_1, t_2, \dots, t_N$.

Geometric average of Y is $\bar{Y} = \prod_{i=1}^N (Y_{t_i})^{\frac{1}{N}}$

Averaging Price $S_T = \bar{Y}$.

Averaging Strike $X = \bar{Y}$.

The option pays D_T at time T where:

Call $D_T = [S_T - X]^+$.

Put $D_T = [X - S_T]^+$.

Partial Asian $\Rightarrow t_1 \geq 0$ and or $t_N \leq T$.

Asian Tail $\Rightarrow t_1 \geq 0$.

3.2.5 Barrier Option

$\left\{ \begin{array}{l} \text{Up} \\ \text{Down} \end{array} \right\}$ and $\left\{ \begin{array}{l} \text{In} \\ \text{Out} \end{array} \right\}$ barrier $\left\{ \begin{array}{l} \text{Call} \\ \text{Put} \end{array} \right\}$ option.

Up $\Rightarrow S_0 < H$ and $M_{Up} = \sup\{S_t\}$ where H is the barrier level and $t = t_1, t_2, \dots, t_N$.

Down $\Rightarrow S_0 > H$ and $M_{Down} = \inf\{S_t\}$.

In $\Rightarrow 1_{M_{Up} \geq H}$ or $1_{M_{Down} \leq H}$.

Out $\Rightarrow 1_{M_{Up} \leq H}$ or $1_{M_{Down} \geq H}$.

The option pays D_T at time T where:

Call $D_T = [S_T - X]^+ \cdot 1_{Condition}$

Put $D_T = [X - S_T]^+ \cdot 1_{Condition}$

If the option is knocked-out at time t then there can be provision in the contract for a predetermined cash rebate amount R . This is typically payed out at time t rather than at maturity T .

Partial Barrier $\Rightarrow t_1 \geq 0$ and or $t_N \leq T$.

3.2.6 Binary

$\left\{ \begin{array}{l} \text{Cash-or-Nothing} \\ \text{Asset-or-Nothing} \\ \text{General Gap} \\ \text{Pay-later (contingent premium)} \end{array} \right\} \left\{ \begin{array}{l} \text{Call} \\ \text{Put} \end{array} \right\} \text{ option.}$

Cash-or-nothing \Rightarrow the holder receives $Q \cdot 1_{Condition}$ at T where Q is a cash payoff not the underlying asset.

Asset-or-nothing \Rightarrow the holder receives $S_T \cdot 1_{Condition}$ at T where S is the underlying asset.

General Gap \Rightarrow the holder receives $(S_T - Q) \cdot 1_{Condition}$ at T .

Pay-later call (contingent premium) \Rightarrow the holder receives $(S_T - X - Q) \cdot 1_{Condition}$ at T where $Q > 0$ is set such that the option has zero initial value.

Pay-later put (contingent premium) \Rightarrow the holder receives $(X - S_T - Q) \cdot 1_{Condition}$ at T where $Q > 0$ is set such that the option has zero initial value.

The *Condition* is:

Call $\Rightarrow 1_{S_T \geq X}$.

Put $\Rightarrow 1_{S_T \leq X}$.

Range binaries (cash-or-nothing options) $D_T = Q$ if and only if $X_1 \leq S_T \leq X_2$.

Range binaries (asset-or-nothing options, supershare) $D_T = S_T/X_1$ if and only if $X_1 \leq S_T \leq X_2$.

3.2.7 Chooser

$\left\{ \begin{array}{l} \text{Regular (Simple)} \\ \text{Complex} \end{array} \right\} \text{ chooser } \left\{ \begin{array}{l} \text{Buyer's Choice} \\ \text{Dealer's Choice} \end{array} \right\} \text{ option.}$

$C_t(X, T)$ is the price at time t of a European call with strike price X and maturity T i.e. $C_t(X, T) = E_t(\max(S_T - X, 0))$ where E_t is the expectation at time t .

$P_t(X, T)$ is the price at time t of a European put with strike price X and maturity T i.e. $P_t(X, T) = E_t(\max(X - S_T, 0))$ where E_t is the expectation at time

t .

Regular (Simple) $Y = C_t(X, T)$ and $Z = P_t(X, T)$.

Complex $Y = C_t(X_1, T_1)$ and $Z = P_t(X_2, T_2)$ where $X_1 \neq X_2$ and $T_1 \neq T_2$.

At time t the buyer or the dealer chooses whether they want a call or a put with the above strike and maturity. Optimally (but they are not obligated to choose optimally) this is determined by:

Buyer's choice $D_t = \max[Y, Z; t]$ where $t \leq T_n$.

Dealer's choice $D_t = \min[Y, Z; t]$.

At time T the following payoff occurs:

Call $D_T = [S_T - X]^+$

Put $D_T = [X - S_T]^+$

3.2.8 Compound

$\left\{ \begin{array}{c} \text{Call} \\ \text{Put} \end{array} \right\}$ on a $\left\{ \begin{array}{c} \text{Call} \\ \text{Put} \end{array} \right\}$ option.

Call $Z = C_t(X_2, T_2)$ where $t \leq T_1 \leq T_2$.

Put $Z = P_t(X_2, T_2)$.

At time T_1 the holder of the compound option can choose to pay strike price X_1 and receive option Z . Optimally (but they are not obligated to choose optimally) this is determined by:

Call $\max(Z - X_1; T_1, 0)$.

Put $\max(X_1 - Z; T_1, 0)$.

At time T_2 the option pays D_{T_2} .

Call $D_{T_2} = [S_{T_2} - X]^+$.

Put $D_{T_2} = [X - S_{T_2}]^+$.

3.2.9 Currency Translated

$\left\{ \begin{array}{c} \text{Foreign Equity} \\ \text{Domestic Equity} \end{array} \right\}$ $\left\{ \begin{array}{c} \text{Call} \\ \text{Put} \end{array} \right\}$ struck in $\left\{ \begin{array}{c} \text{Foreign Currency} \\ \text{Domestic Currency} \end{array} \right\}$.

Foreign Equity $Y = K_T \times S_T$ where K_T is the foreign exchange rate at time T .

Domestic Equity $Y = S_T$.

Struck in *Foreign Currency* $Z = K_T \times X$.

Struck in *Domestic Currency* $Z = X$.

The option pays D_T at T where:

Call $D_T = [Y - Z]^+$.

Put $D_T = [Z - Y]^+$.

$\left\{ \begin{array}{c} \text{Fixed Exchange Rate Foreign Equity (Quanto)} \\ \text{Equity Linked Foreign Exchange} \end{array} \right\}$ on a $\left\{ \begin{array}{c} \text{Call} \\ \text{Put} \end{array} \right\}$ option.

Fixed exchange rate foreign equity call (Quanto) $D_T = [K_0 \times S_T - K_0 \times X]^+$.

Fixed exchange rate foreign equity put (Quanto) $D_T = [K_0 \times X - K_0 \times S_T]^+$.

Equity linked foreign exchange call $D_T = S_T \times [K_T - X]^+$.

Equity linked foreign exchange put $D_T = S_T \times [X - K_T]^+$.

3.2.10 Dual Date

$$\left\{ \begin{array}{l} \textit{Shout} \\ \textit{Holder Extendible} \\ \textit{Writer Extendible} \end{array} \right\} \left\{ \begin{array}{l} \textit{Call} \\ \textit{Put} \end{array} \right\} \text{ option}$$

The holder of a shout option can shout at time t and lock in the intrinsic value of the option and at time T receive the maximum of either the payoff of the European option or the intrinsic value:

$$\textit{Shout Call } D_T = \max(S_T - S_t, 0) + (S_t - X) \text{ where } t \in [0, T].$$

$$\textit{Shout Put } D_T = \max(S_t - S_T, 0) + (X - S_t).$$

At time t the holder of an extendible option can choose to extend the life of the option from t to T by paying an additional premium A where $t \leq T$. One can also change the strike price of the new option from X_1 to X_2 . Optimally (but they are not obligated to choose optimally) this is determined by:

$$\textit{Holder extendible call } \max(S_t - X_1, C(X_2, T_2) - A).$$

$$\textit{Holder extendible put } \max(X_1 - S_t, P(X_2, T_2) - A).$$

$$\textit{Writer extendible call } \min(S_t - X_1, C(X_2, T_2) - A).$$

$$\textit{Writer extendible put } \min(X_1 - S_t, P(X_2, T_2) - A).$$

At time T the payoff D_T is:

$$\textit{Call } D_T = [S_T - X]^+.$$

$$\textit{Put } D_T = [X - S_T]^+.$$

3.2.11 Exchange

$$\left\{ \begin{array}{l} \textit{Exchange one asset for another} \\ \textit{Better of (out-performance)} \\ \textit{Worse of (under-performance)} \end{array} \right\} \text{ option.}$$

$$\left\{ \begin{array}{l} \textit{Ratio} \\ \textit{Product} \end{array} \right\} \left\{ \begin{array}{l} \textit{Call} \\ \textit{Put} \end{array} \right\} \text{ option.}$$

At time T the payoff D_T is:

$$\textit{Exchange } D_T = \max(S_T^2 - S_T^1, 0).$$

$$\textit{Out Performance } D_T = \max(S_T^1, S_T^2).$$

$$\textit{Under Performance } D_T = \min(S_T^1, S_T^2).$$

$$\textit{Ratio } Z = \frac{S_T^1}{S_T^2}.$$

$$\textit{Product } Z = S_T^1 \times S_T^2.$$

$$\textit{Call } D_T = [Z - X]^+.$$

$$\textit{Put } D_T = [X - Z]^+.$$

3.2.12 Forward

Employee Stock Option $D_T = [S_T - \alpha S_t]^+$ where $\alpha > 0$ and $t < T$.

3.2.13 Lookback

$$\left\{ \begin{array}{l} \textit{Fixed Strike} \\ \textit{Floating Strike} \\ \textit{Straddle} \\ \textit{Ladder} \\ \textit{Do Nothing} \\ \textit{Extreme Spread} \\ \textit{Reverse Extreme Spread} \\ \textit{Partial Time Floating Strike} \\ \textit{Partial Time Fixed Strike} \end{array} \right\} \left\{ \begin{array}{l} \textit{Call} \\ \textit{Put} \end{array} \right\} \text{ option.}$$

At time T the payoff D_T is:

Maximum $Z = \sup\{S_t\}$ where $t \in [0, T]$.

Minimum $Z = \inf\{S_t\}$ where $t \in [0, T]$.

Fixed Strike $S_T = Z$

Floating Strike $X = Z$

Call $D_T = [S_T - X]^+$

Put $D_T = [X - S_T]^+$

Maximum $Y = \sup\{S_t\}$ where $t \in [0, T]$.

Minimum $Z = \inf\{S_t\}$ where $t \in [0, T]$.

Straddle Call (range) $D_T = [(Y - Z) - X]^+$.

Straddle Put $D_T = [X - (Y - Z)]^+$.

Ladder call $D_T = [\max(\min(S_{t_1}, Y), \min(S_{t_2}, Y), \min(S_{t_3}, Y), \dots) - X]^+$ where $S_1 < S_2 < S_3 < \dots$

Do-Nothing Call $D_T = [(\alpha S)/(Y - Z + K) - X]^+$ where K is a predetermined payoff.

$V = \sup\{S_t\}$ where $t \in [0, t]$.

$W = \inf\{S_t\}$ where $t \in [0, t]$.

$Y = \sup\{S_t\}$ where $t \in [t, T]$.

$Z = \inf\{S_t\}$ where $t \in [t, T]$.

Extreme Spread Call $D_T = [Y - V]^+$.

Extreme Spread Put $D_T = [W - Z]^+$.

Reverse Extreme Spread Call $D_T = [Y - W]^+$.

Reverse Extreme Spread Put $D_T = [V - Z]^+$.

Partial-Time Floating Strike Call $D_T = [S_T - \alpha W, 0]^+$ where $\alpha > 1$.

Partial-Time Floating Strike Put $D_T = [\alpha V - S_T, 0]^+$ where $0 \leq \alpha \leq 1$.

Partial-Time Fixed Strike Call $D_T = [Y - S_T]^+$.

Partial-Time Fixed Strike Put $D_T = [S_T - Z]^+$.

3.2.14 Power

$$\left\{ \begin{array}{l} \textit{Power} \\ \textit{Self Quanto (Turbo)} \end{array} \right\} \left\{ \begin{array}{l} \textit{Call} \\ \textit{Put} \end{array} \right\} \text{ option.}$$

At time T the payoff D_T is:

Power Call $D_T = [S^Y - X]^+$.

Power Put $D_T = [X - S^Y]^+$.

Self Quanto Call $D_T = S \times [S - X]^+$.

Self Quanto Put $D_T = S \times [X - S]^+$.

3.2.15 Rainbow

The payoffs here are for 2-color rainbow options but they can be generalized to N-colors.

$\left\{ \begin{array}{l} \text{Call} \\ \text{Put} \end{array} \right\}$ on the $\left\{ \begin{array}{l} \text{Maximum of Two Risky Assets} \\ \text{Minimum of Two Risky Assets} \end{array} \right\}$.

Maximum of two risky assets $Y = \max(S^1, S^2)$.

Minimum of two risky assets $Y = \min(S^1, S^2)$.

At time T the holder receives the payoff D_T :

Call $D_T = [Y - X]^+$.

Put $D_T = [X - Y]^+$.

Spread Call $D_T = [(S_T^1 - S_T^2) - X]^+$.

Spread Put $D_T = [X - (S_T^2 - S_T^1)]^+$.

Portfolio call (two color basket) $D_T = [(\alpha_1 S_T^1 + \alpha_2 S_T^2) - X]^+$ where α is a portfolio weighting.

Portfolio put (two color basket) $D_T = [X - (\alpha_1 S_T^1 + \alpha_2 S_T^2)]^+$.

Dual Strike Call $D_T = \max(S_T^1 - X_1, S_T^2 - X_2, 0)$ where $X_1 \neq X_2$.

Dual Strike Put $D_T = \max(X_1 - S_T^1, X_2 - S_T^2, 0)$.

Dual Asset Chooser $\left\{ \begin{array}{l} \text{Call} \\ \text{Put} \end{array} \right\} \left\{ \begin{array}{l} \text{Call} \\ \text{Put} \end{array} \right\} \left\{ \begin{array}{l} \text{Buyer's} \\ \text{Dealer's} \end{array} \right\}$ Choice.

At time t the buyer or dealer has the choice between various call and put combinations with the same maturity T but different underlying assets S^1 and S^2 , respectively. Optimally (but they are not obligated to choose optimally) this is determined by D_t :

Call $Y = C(S_t^1, T)$.

Put $Y = P(S_t^1, T)$.

Call $Z = C(S_t^2, T)$.

Put $Z = P(S_t^2, T)$.

Buyer's Choice $D_t = \max(Y, Z; t)$.

Dealer's Choice $D_t = \min(Y, Z; t)$.

At time T the holder receives the payoff D_T :

$D_T = [S_T - X]^+$.

$D_T = [X - S_T]^+$.

Outside Barrier $\left\{ \begin{array}{l} \text{Up} \\ \text{Down} \end{array} \right\}$ and $\left\{ \begin{array}{l} \text{In} \\ \text{Out} \end{array} \right\} \left\{ \begin{array}{l} \text{Call} \\ \text{Put} \end{array} \right\}$ option (barrier determined by second asset).

Up $\Rightarrow S_0^2 < H$ and $M_{Up} = \sup\{S_t^2\}$.

Down $\Rightarrow S_0^2 > H$ and $M_{Down} = \inf\{S_t^2\}$.

In $\Rightarrow 1_{M_{Up} \geq H}$ or $1_{M_{Down} \leq H}$.

Out $\Rightarrow 1_{M_{Up} \leq H}$ or $1_{M_{Down} \geq H}$.

The option pays D_T at time T where:

Call $D_T = [S_T^1 - X]^+ \cdot 1_{Condition}$

Put $D_T = [X - S_T^1]^+ \cdot 1_{Condition}$

Dual Asset Double Barrier $\left\{ \begin{array}{c} Up \\ Down \end{array} \right\}$ and $\left\{ \begin{array}{c} In \\ Out \end{array} \right\}$ $\left\{ \begin{array}{c} Call \\ Put \end{array} \right\}$ on the $\left\{ \begin{array}{c} Maximum \\ Minimum \end{array} \right\}$

Maximum $Y = \max(S_T^1, S_T^2)$.

Minimum $Y = \min(S_T^1, S_T^2)$.

Up $\Rightarrow S_0^1 < H_1, S_0^2 < H_2, M_{Up}^1 = \sup\{S_t^1\}$ and $M_{Up}^2 = \sup\{S_t^2\}$.

Down $\Rightarrow S_0^1 > H_1, S_0^2 > H_2, M_{Down}^1 = \inf\{S_t^1\}$ and $M_{Down}^2 = \inf\{S_t^2\}$.

In $\Rightarrow 1_{M_{Up}^1 \geq H_1, M_{Up}^2 \geq H_2}$ or $1_{M_{Down}^1 \leq H_1, M_{Down}^2 \leq H_2}$.

Out $\Rightarrow 1_{M_{Up}^1 \leq H_1, M_{Up}^2 \leq H_2}$ or $1_{M_{Down}^1 \geq H_1, M_{Down}^2 \geq H_2}$.

The option pays D_T at time T where:

Call $D_T = [Y - X]^+ \cdot 1_{Condition}$

Put $D_T = [X - Y]^+ \cdot 1_{Condition}$

Asset-or-nothing call (Dual-Asset Binary Option) $D_T = 0$ if $S_T^2 \leq X$ or $D_T = S_T^1$ if $S_T^2 > X$.

Asset-or-nothing put $D_T = 0$ if $S_T^2 \geq X$ or $D_T = S_T^1$ if $S_T^2 < X$.

Dual Asset Lookback $\left\{ \begin{array}{c} Call \\ Put \end{array} \right\}$ on the $\left\{ \begin{array}{c} Maximum \\ Minimum \end{array} \right\}$.

Maximum $Y = \sup\{S_t^2\}, Z = \sup\{S_t^1\}$ where $t \in [0, T]$.

Minimum $Y = \inf\{S_t^2\}, Z = \inf\{S_t^1\}$ where $t \in [0, T]$.

The option pays D_T at time T where:

Call $D_T = [(Y - Z) - X]^+$.

Put $D_T = [X - (Y - Z)]^+$.

$Y = \sup\{S_t^2\}$ where $t \in [0, T]$.

$Z = \inf\{S_t^1\}$ where $t \in [0, T]$.

Semi-Dual Asset Lookback Call $D_T = [Y - X]^+$.

Semi-Dual Asset Lookback Put $D_T = [X - Z]^+$.

3.2.16 Forward Rate Agreements

$D_{T_1} = -P$ where P is the principal amount.

$D_{T_2} = P[1 + R_{fixed}(T_2 - T_1)]$ where R_{fixed} is some predefined fixed interest rate agreed at time t and $t \leq T_1 \leq T_2$.

3.2.17 Vanilla Interest Rate Swap

$D_{t_i} = -\sum_{i=1}^N c_{fixed}$ where c_{fixed} are the coupon cash-flows derived from the fixed rate R_{fixed} on some notional principal P and $t = t_1, t_2, \dots, t_N$.

$D_{t_i} = \sum_{i=1}^N c_{floating}$ where $c_{floating}$ are the coupon cash-flows derived from the floating rate $R_{floating}$ on some notional principal P .

3.2.18 Interest Rate Caps, Floors and Collars

Cap $D_{t_i} = \sum_{i=1}^N [R_{t_i} - R_c]^+$ for $t_i = t_1, t_2, \dots, t_N$ where R_{t_i} is the floating interest rate and R_c is the fixed cap rate.

Floor $D_{t_i} = \sum_{i=1}^N [R_f - R_{t_i}]^+$ for $t_i = t_1, t_2, \dots, t_N$ where R_f is the fixed floor rate.

Collar $D_{t_i} = \sum_{i=1}^N ([R_{t_i} - R_c]^+ + [R_f - R_{t_i}]^+)$ where $t_i = t_1, t_2, \dots, t_N$ and $R_c > R_f$.

3.2.19 Swaptions

Call swaption or payer swaption (the holder has the choice to become the fixed rate R_{fixed} payer) $D_T = [R_{fixed} - R_X]^+$ where R_X is the strike rate.

Put swaption or receiver swaption (the holder has the choice to become the floating rate $R_{floating}$ payer) $D_T = [R_X - R_{floating}]^+$ where R_X is the strike rate.

3.2.20 Convertible Bonds

The holder of the convertible bond has the choice to convert the bond and receive equity of the value $D_t = n_t S_t$ where S_t is the current stock price of the company, n_t is the current conversion price and $t \in [0, T]$.

If the bond has a call provision the issuer has the choice to call the bond back at a price of P_{t_j} to terminate the deal at t_{j+m} where m is typically 30 days and $0 \leq j \leq T$.

If the bond is neither called nor converted then the holder will receive a series of coupons c_{t_i} and the principal amount P_T at maturity T i.e. $D_{t_i} = \sum_{i=1}^N c_{t_i} + P_T$. More complicated convertible bonds can have put provisions (which allow the holder to put the bond back to the issuer at a certain price P_{t_j}) and re-fix clauses which allow the conversion price n_j to be adjusted during the life of the bond.

3.2.21 Swing

The holder of a swing (take-or-pay) option has the choice at each date t_i where $t_i = t_1, t_2, \dots, t_N$ to take a maximum or minimum amount of a commodity:

$D_{t_i} = \sum_{i=1}^N \max((S_{t_i} - X)V_{max}, (S_{t_i} - X)V_{min} : k)$ where $k = 0, \dots, M_{max}$, V_{max} is the maximum amount of the commodity that can be taken and V_{min}

is the minimum amount of the commodity that can be taken per swing. When the maximum amount of the commodity is taken the number of swings k is incremented by 1 up to a maximum of M_{max} . After which no more gas can be taken. There is often a minimum quantity of gas that can be taken over the entire life of the contract T below which a fine is incurred:

$D_{t_i} = \max((S_{t_i} - X)V_{max}, -c(V_{max} - V_{min})(M_{min} - k)^+)$ where c is the unit penalty multiplied by the volume short fall.

4 Colloquial Language (Jargon)

The derivatives industry has amassed a huge colloquial language, littered with jargon, to describe contingent claim contracts. For example American, Asian, barrier, Bermudan, binary, cap, call, chooser, compound, convertible, dual-date, exchange, European, floor, lookback, passport, power, put, rainbow, Russian, swaption and swing, just to name but a few! This section catalogues and defines the commonly used colloquial language. Many of the following definitions of contingent claims are based on those of Rubinstein and Reiner (1992) [45], Lyden (1996) [35], Haug (1997) [22] and Hull (2000) [28].

4.1 Class

Nearly all contingent claims can be categorized as either calls or puts.

Call Option The right but not the obligation to buy an asset at a certain price and date.

Put Option The right but not the obligation to sell an asset at a certain price and date.

4.2 Style

The style of a contingent claim determines when in time a cash flow(s) will be made, if any are due.

American Option A contingent claim that can be exercised by the holder at any time during its life.

European Option A contingent claim that can only be exercised by the holder at the end of its life.

Bermudan (Atlantic, Restricted Exercise) A contingent claim that can be exercised by the holder at a time chosen from a discrete set of dates.

Russian (Perpetual) A contingent claim that can be exercised by the holder at any time however, there is no maturity date. Anecdotal evidence suggests these are not used by practitioners and are of academic interest only.

4.3 Payoffs That Only Depend On Functions

The payoff depends on a mathematical function which is independent of events and choices.

Asian (Average) Contingent claims whose payoff depends not only on the price at expiration of the underlying asset but also on the average (arithmetic [48], [10] or geometric [33]) price experienced by the underlying asset

during at least some portion of the life of the option. Asian options are especially popular in the currency and commodity markets⁵.

Arithmetic Arithmetic method of averaging.

Geometric Geometric method of averaging i.e. the n th root of the product of n numbers.

Average price The payoff is a function of the final asset price and the fixed strike price.

Average strike The payoff is a function of the final asset price and a strike price given by the average asset price.

Partial The period over which the average of the underlying asset price is recorded is only a fraction of the life of the option. An **Asian Tail** contingent claim has the averaging period confined to the final part of the option's life.

Currency Translated (Guaranteed Exchange Rate or Quanto Options)

Currency contingent claims [11], [38], [12] have the underlying asset or striking price denominated in a foreign currency at a random or prespecified exchange rate. At expiry the foreign asset is translated into the domestic currency. **Quanto** a contingent claim where the payoff is defined by variables associated with one currency but is paid in another currency.

Deferred Payment (Boston, Break Forward) a contingent claim where the premium is deferred until the end of the claims life.

Forward Rate Agreement an agreement that at a certain interest rate will apply to a certain principal amount for a certain time period in the future.

Forward Start Options (Deferred Start and Ratchet Options) Forward start contingent claims [41] are paid for in the present but are only received at a prespecified future date. A series of forward start options is called a **Ratchet Option (Moving Strike Option or Cliquet Option)**.

Lookback (High-Low Options, Ladder Options) Lookback contingent claims [20], [15], [9], [24] have a payoff that depends not only on the price at expiration of the underlying asset but also on the minimum or maximum price experienced by the underlying asset during at least some portion of the life of the option. **Ladder options** are lookback options which are discretely sampled in the asset price rather than in time.

Fixed Strike The payoff is a function of the fixed strike price and the extremum underlying asset price.

⁵An average is less volatile than the underlying asset itself and will lower the price of an average rate option compared with a similar standard option. If the option is based on an average, an attempt to manipulate the asset price just before expiration will normally have little or no effect on the option's value. Asian options should therefore be of particular interest in markets for thinly traded assets.

Floating Strike The payoff is a function of the underlying asset price at maturity and the extremum underlying asset price.

Partial The period where the extremum value of the underlying asset is observed is only a fraction of the life of the option.

Packages Contingent claims that consist or a portfolio of standard call, put and forward contracts together with the underlying asset. Packages are often structured by traders so that they have zero initial cost.

Passport This is a contingent claim on a trading account, it grants the holder the right to engage in a short / long trading strategy of his / her own choice, while obligating the writer to cover any net losses on the strategy.

Power A power option has a payoff which is an exponent of the underlying asset.

Swap An agreement to exchange cash flows in the future according to a prearranged formula.

Accrual An interest rate swap where interest on one side accrues only when a certain condition is met.

Amortizing A swap where the notional principal decreases in a predetermined way as time passes.

Commodity A swap where cash flows depend on the price of a commodity.

Constant Maturity A swap where a swap rate is exchanged for either a fixed or a floating rate on each payment date.

Constant Maturity Treasury Swap A swap where the yield on a Treasury bond is exchanged for either a fixed rate or a floating rate on each payment date.

Credit Default An instrument that gives the holder the right to sell a bond for its face value in the event of a default by the issuer.

Currency A swap where interest and principal in one currency are exchanged for interest and principal in another currency.

Deferred / Forward An agreement to enter into a swap at some time in the future. Also called a forward swap.

Differential A swap where a floating rate in one currency is exchanged for a floating rate in another currency and both rates are applied to the same principal.

Equity A swap where the return on an equity portfolio is exchanged for either a fixed or floating rate of interest.

Extendable A swap whose life can be extended at the option of the holder.

Indexed Amortizing / Principal A swap where the principal declines over time. The reduction in the principal on a payment date depends on the level of interest rates.

Interest Rate An exchange of a fixed rate of interest on a certain notional principal for a floating rate of interest on the same notional principal.

Putable A swap where one side has the right to terminate early.

Step-up A swap where the principal increases over time in a predetermined way.

Total Return A swap of the return on one portfolio of assets for the return on another portfolio of assets.

4.4 Payoffs That Depend on Fixed Events

The payoff is a function which is contingent on a fixed event or events occurring for example a barrier being reached.

Barrier (Knock-In, Knock-Out, Vulnerable and Parisian Options) Contingent claims whose payoff depends not only on the price at expiration of the underlying asset but also on whether or not the underlying asset has previously reached some pre-determined barrier price. Barrier options [39], [1], [17], [30], [25], [23], [21], [7] have become extremely popular and are certainly the most popular class of exotic options. They are traded on some exchanges as well as in the OTC market. **Parisian** options are barrier options for which the barrier is only triggered if the underlying asset has spent a predetermined time beyond the barrier.

Up The barrier is above the initial underlying asset price.

Down The barrier is below the initial underlying asset price.

In The option pays the holder contingent on the barrier being reached.

Out The option pays the holder nothing at maturity or an immediate **rebate** contingent on the barrier being reached.

Double There is an upper and a lower barrier with respect to the initial underlying asset price.

Partial (protected) The position of the barrier is time dependent and may disappear for specified periods.

Roll up/down Vanilla call or put contracts which become barrier options if the underlying asset reaches a predefined level.

Binary All or Nothing, Contingent Premium, Corridor, Digital and Switch Options) Contingent claims with binary and discontinuous payoff patterns. Binary options [39], [26] are popular in the OTC market for hedging and speculation. They are also important to financial engineers as building blocks for constructing more complex option products.

Cash-or-nothing If the payoff condition is triggered by the value of the underlying asset the holder receives a predetermined cash amount at maturity and zero otherwise.

Asset-or-nothing If the payoff condition is triggered by the value of the underlying asset the holder receives the underlying asset at maturity and zero otherwise.

Gap If the payoff condition is triggered by the underlying asset the holder receives the cash difference between the underlying asset minus a strike price.

Pay Later (Contingent Premium) These are like asset-or-nothing options however the payoff is adjusted so that the option is worth nothing at the time it is purchased.

Range If the underlying asset lies within an upper and a lower range at maturity then the holder receives either a payoff of a predetermined cash amount at maturity and zero otherwise for a cash-or-nothing option or an amount based on the value of the underlying asset and zero otherwise for an asset-or-nothing option.

Interest Rate Cap A contingent claim which provides a payoff when a specified interest rate is above a certain level. The interest rate is a floating rate that is reset periodically. **Caplet** is one component of an interest rate cap. **Auto-cap** is an interest rate option analogous to swing options in the commodity markets and describes an option where the holder has the right to choose up to a maximum number of caplets from a larger series of caplets with different maturities.

Interest Rate Collar (a floor-ceiling agreement) A contingent claim that is a combination of an interest rate cap and an interest rate floor.

Interest Rate Floor A contingent claim which provides a payoff when a specified interest rate is below a certain level. The interest rate is a floating rate that is reset periodically. **Floorlet** is one component of an interest rate floor.

4.5 Payoffs That Depend on Choices

The payoff is a function which is contingent on a choice being made at some time for example choosing whether to extend the maturity of a cliquet.

Chooser (as you like it options) Contingent claims which are paid for in the present but which at some prespecified future date are chosen to be either a put or a call.

Regular (Simple) A simple chooser option [43] has the same maturity and strike for the put and the call.

Complex A complex chooser option [43] can have different maturities and strikes for the put and the call.

Compound Compound options [18], [27], [42] have underlying assets that are themselves options. Compound options have two strike prices and two exercise dates. On the first exercise date the holder of the option is entitled to pay the first strike price and receive the underlying option. The underlying option then gives the holder a conventional option on the underlying asset for exercise at the second strike price and exercise date. The compound option will only be exercised on the first exercise date if the value of the underlying option on that date is greater than the first strike price.

Convertible A corporate bond that can be converted into a predetermined amount of the company's equity at certain times during its life at the holder's choice. Convertible bonds can have additional clauses which make them more complex:

A call provision gives the issuer the choice to call the bond back at a predetermined price.

A put provision gives the holder of the bond the choice to put the bond back to the issuer at a predetermined price.

A re-fix clause allows the conversion price of the bond to be adjusted during the life of the bond. The clause is normally contingent on the price of the underlying equity reaching a predetermined level for a certain period of time.

Exchange (Better / Worse of Two Assets, Margrabe, Out-performance, Options on Max / Min, Spread Options) Exchange options [36], [49], [3], [8] give the option to exchange one asset for another. **Spread options** payoff the difference between two market variables.

Extendible Maturity Extendible maturity contingent claims [34] can be exercised at their maturity date but they allow their holder to extend the life of the option by paying an additional premium to the writer. The strike price of the option can be adjusted at the time of the extension.

Shout Shout options combine features of look-back, ladder and ratchet options. The holder of a shout option can contact the writer of the option at one point in its life and lock in (*shout*) the intrinsic value of the option (the difference between the exercise price of the option and the spot price at the time the shout was made).

Swaption An option to enter into an interest rate swap where a specified fixed rate is exchanged for floating.

Swing (Take or Pay) A contract in the gas or electricity industry where the buyer agrees to purchase fixed price gas (or electricity) up to a maximum volume (capacity) in a given period but is subject to penalty payments if the taken volume (capacity) falls below an agreed minimum. **Swing** is the ability to change the volume purchased.

4.6 Miscellaneous Terms

Exotic Option Contingent claims with non-standard payoffs.

Rainbow (Basket Options, Multiple Underlying Assets, State Variables) Options whose payoff depends on the value of two or more underlying assets [47], [31], [5], [6], [44], [40]. A **basket** option's payoff depends on the aggregate value of a specified *basket* of financial assets, rather than on the value of the individual assets.

Vanilla Option Contingent claims with standard call and put payoffs i.e., without any special properties attached. They include packages, European and American [19], [2] options on stocks (with [37] and without dividends), indices, futures [4], [46], currencies (with and without foreign interest rates [16]) and commodities (with and without storage costs and convenience yield).

5 Legal Language

Masters Agreements and trade confirmation contracts are written by lawyers in a hybrid of plain English and limited mathematics. These documents are intended to carefully define all the important terms, dates (premium payment, commencement, termination, settlement) and required market data which cash-flows may be contingent upon, such as the level of an index. Moreover, unlike the mathematical, colloquial and computational descriptions the legal contracts contain clauses which attempt to provide for unforeseen events. For example, what if a valuation date is not a business date or if a counter-party defaults. The following examples are provided to give a flavor of the legal language:

5.1 Master Agreement

5.1.1 Example Definitions

An ISDA Master Agreement for a particular generic family of contingent claim (for example, a Financial Futures Agreement) often has a definitions section as its first clause, this will describe terms used in the subsequent clauses. The confirmation document then relies on these definitions and clauses but adds the detail which actually describes the particular deal. So the Master Agreement will define what is meant by an Index or Commencement Date and the confirmation document will specify the particular exchange and the actual date.

Act means the Insurance and Companies Act 1982 (as amended from time to time).

Affected Party has the meaning set out in Clause X.

Business Day means a day (other than a Saturday and Sunday) when banks are open for business (including dealings in foreign exchange and foreign currency deposits) in London.

Commencement Date means date month year or, if such is not a Business Day, the next following date which is a Business Day.

Exchange means the exchange or exchanges set out as such in the relevant Financial Futures Confirmation.

Expiration Valuation Date means, subject to Clause X, each Business Day which falls within the period commencing on the First Expiration Valuation Date and ending on the Final Expiration Valuation Date.

Expiration Valuation Time means each of the times set out as such in the Financial Futures Confirmation.

Futures and Options Exchange means the exchange or exchanges set out as such in the Financial Futures Confirmation.

Index means, subject to Clause X, the index set out in the Financial Futures Confirmation.

Initial Valuation Date(s) means, subject to Clause X, the date or dates set out as such in the Financial Futures Confirmation or, if any such date is not a Business Day, the immediately following Index Business Day provided that if any such date is so adjusted all subsequent Initial Valuation Dates shall be adjusted accordingly.

Initial Valuation Times means each of the times set out as such in the Financial Futures Confirmation.

Non-affected Party has the meaning set out in Clause X.

Sponsor means the index sponsor set out as such in the relevant Financial Futures Confirmation.

Valuation Date means each and any of the Initial Valuation Dates and the Expiration Valuation Dates.

Valuation Times means each of the Initial Valuation Times and the Expiration Valuation Times.

5.1.2 Example Clauses

After the definitions are further clauses which govern procedures such as which jurisdiction will govern the agreement and the procedure to be adopted in the occurrence of a possible event such as a required index level being unavailable for observation.

Governing Law This Agreement will be governed by, and construed in accordance with, English law

Market Disruption Event Means the occurrence or existence, on any Valuation Date during the one-half hour period that ends at any Valuation Time on such a Valuation Date, of any suspension of or limitation imposed on trading (by reason of movements in price exceeding limits permitted by the relevant exchange or otherwise) on (i) the Exchange in securities that comprise 20 per cent. or more of the level of the Index or (ii) the Futures and Options Exchange in futures and/or options contracts on the Index if, in the determination of the Bank, such suspension or limitation is material. For the purposes of determining whether a Market Disruption Event exists at any time, if trading in a security included in the Index is, in the determination of the Bank, materially suspended or materially limited at that time, then the relevant percentage contribution of that security to the level of the Index shall be based on a comparison of (i) the portion of the level of the Index attributed to that security relative to (ii) the overall level of the Index, in each case immediately before the suspension or limitation.

Adjustment to the Index If the index is (i) not calculated and announced by the Sponsor but is calculated and announced by a successor sponsor acceptable to the Bank or (ii) replaced by a successor index using the same or a substantially similar formula for and method of calculation as used in the calculation of the Index, then the Index will be deemed to be the index so calculated and announced by that successor sponsor or that successor index, as the case may be.

Early Termination upon Change in the Law If as a result of any change in law (including, without limitation, the Act), regulation or requirements applicable to the Investor (the "Affected Party") or in the application or interpretation of any such law, regulation or requirements, the Investor would cease to be able lawfully to remain party to this Agreement in compliance with all laws, regulations and requirements applicable to the Investor, the Investor shall be entitled, by written notice to the Bank (the "Non-affected Party") to terminate this Agreement on the date specified in the notice, which date shall not be earlier than the date upon which the notice is effective.

5.2 Confirmation Agreement (Financial Futures Agreement)

5.2.1 Example Details

Finally, the Confirmation Agreement gives the detailed trade particulars which pertain to the definitions in the Master Agreement.

Exchange The International Stock Exchange of the United Kingdom and Republic of Ireland.

Futures and Options Exchange London International Financial Futures and Options Exchange Limited.

Initial Valuation Date 7th June 1996.

Initial Valuation Time 11:00am and 3:00pm.

First Expiration Valuation Date 7th December 2000.

Final Expiration Valuation Date 6th June 2001.

Expiration Valuation Time The close of business on the Exchange.

Index FTSE100.

Sponsor The Exchange.

6 Computer Readable Language

The goal when designing a computer language which can characterize contingent claim contracts is to have enough flexibility to characterize all existing and future contracts⁶ and yet preclude contracts which are non-sensical. By non-sensical we mean contracts which are meaningless, impossible or completely unfair to one party. A meaningless contract would be for example, a contract with a payoff function including $\sqrt{-1}$ or that pays S_{t_2} at t_1 where $t_2 > t_1$. An impossible contract would for example, have a payoff of the order or in excess of the United States Gross National Product. An unfair contract would be completely one way and would never pay-out in any state of the world. In this Section we examine various languages for representing contingent claim contracts and consider their particular advantages and disadvantages.

⁶The nature of contingent claims contracts is such that we can always invent new types of contracts. For instance, given any new contract we can simply invent another by writing a new contract contingent on the original contract, and so on, *ad infinitum*. This does, of course, have rapidly diminishing returns and adds little value hence there would be few buyers for such contracts. In theory the head of trading at any bank or securities house should limit a trader from writing contingent claims which cannot be priced or that are too complicated to understand. However, Kurt Gödel's first incompleteness theorem (1931) [14] proves that all logical systems of any complexity are, by definition, incomplete; each of them contains, at any given time, more true statements than it can possibly prove according to its own defining set of rules. That is to say within the system, there exist certain clear-cut statements that can neither be proved or disproved. Thus within a trading system it is impossible to prove whether or not one can invent an arbitrary contingent claim which cannot be priced.

6.1 FinXML

FinXML is an XML⁷ application for the financial industry - specifically, it was developed by the FinXML Consortium to provide a universal standard for data interchange within the capital markets. FinXML version 1.0 was released in July 1999 and registered users can download it for free from the web site of the FinXML Consortium. FinXML 1.0 includes Document Type Definitions (DTDs)⁸ supporting a set of vocabularies and schemas to cover the various aspects of capital markets transactions. It is designed to be used to conduct e-commerce over the Internet, as well as within a financial institutions for transaction processing and risk management purposes.

FinXML allows capital markets-related vocabularies to be defined, and for applications based on them to be developed. Current vocabularies defined as part of FinXML support a broad set of elements and attributes that represent financial transactions, reference data, market data, payments, settlements and confirmations. FinXML supports a wide variety of financial products including interest rate, foreign exchange and commodity derivatives, bonds, money markets, loans and deposits, and exchange traded futures and options.

According to the online journal *Wall Street & Technology*, since the FinXML specification became available over 1,000 downloads from every major financial institution have taken place.

6.2 Financial Products Markup Language

FpML, or Financial Product Markup Language, was also introduced in June 1999, and was developed through a collaboration among JP Morgan, a major player in the capital markets, and consultants from PriceWaterhouseCoopers. Like FinXML, FpML is an XML specification intended to function as a protocol for Internet-based electronic dealing and information sharing of financial derivatives. Unlike FinXML, however, FpML at the moment can handle only interest rate derivative products and foreign exchange (FX) products⁹. FinXML already incorporates in its DTDs the ability to handle these types of transactions as well as the broader range of capital markets products, such as futures and options, loans and deposits, and bonds.

Like FinXML, an FpML Consortium has been formed with members including JP Morgan, PriceWaterhouseCoopers, IBM, Sybase, several major financial

⁷XML, the eXtensible Markup Language, is a metalanguage, based on Standard Generalized Markup Language (SGML). XML allows for the development of markup languages for specific needs and purposes. These resulting languages are applications of XML, allowing for the transmission of data among various users (primarily institutional ones) without regard to platform or operating system - XML is universally portable. XML became a recommendation (i.e., standard) of the World Wide Web consortium in February of 1998.

⁸The DTD allows the author to formally define the document's schema through a definition of the document's entities and elements, the elements' attributes and contents, and the elements' relationships to other elements.

⁹It aims ultimately, to allow for the electronic integration of a range of services, from electronic trading and confirmations to portfolio specification for risk analysis for all categories of over-the-counter (OTC) derivatives.

institutions, and Integral Corp., developer of FinXML. As both FinXML and FpML have been developed for the same purpose, they run the risk of becoming competing specifications. This would create new problems of inter-system compatibility, compounding those that already exist among financial institutions running legacy systems. Therefore, Integral has joined the FpML Consortium with the declared aim of working towards the interoperability of both standards. Nevertheless, both FinXML and FpML will continue to exist as de facto separate standards and only the passage of time will reveal if both systems will work seamlessly with one other over the Internet.

6.3 Mathematical Markup Language

The World Wide Web Consortium says of MathML (the Mathematical Markup Language):

MathML is intended to facilitate the use and re-use of mathematical and scientific content on the Web, and for other applications such as computer algebra systems, print typesetting, and voice synthesis. MathML can be used to encode both the presentation of mathematical notation for high-quality visual display, and mathematical content, for applications where the semantics plays more of a key role such as scientific software or voice synthesis.

As we have seen in Section 3 mathematics can be used to mark up contingent claim payoffs. Therefore it would seem logical that MathML could be adapted as the standard to characterize contingent claim contracts. The subset of operators could be rationalized as for example, integration and complex numbers are superfluous to our requirements for marking up contingent claim cash flows and payoffs. One would also need to include the ability to characterize the ancillary contract details (i.e. counter-party address, fax number, phone number and banking details).

6.4 Functional Programming Languages

Meta languages¹⁰ such as ML (Meta Language) and HOL (High Order Language) are functional programming languages (as opposed to imperative languages like C, C++ and Java). They allow for theorem proving i.e. the computer can look at an algebraic conjecture and the theorem prover can check its validity. For example,

```
even(x)=true
even(x+increment+increment)=even(x)
```

this kind of theorem proving is not possible with a conventional imperative programming language. Thus a contingent claim contract characterized by a meta

¹⁰A meta language is a language used to create other languages.

language can be verified to be consistent and bug free unlike a legal contract or normal computer program. Jean Marc Eber (1999) [13] at Société Général has been working on the logical representation of contingent claim contracts using functional programming languages for a number of years. He aims to rewrite CAML (a French functional programming language which is derived from ML) for this purpose. His motivation for writing a new language is so that it will be well adapted to the application domain, the semantics will be simplified, the complexity class of the language will be reduced and it will therefore compile faster¹¹. However, in moving away from the ML language he cannot use its standard tools. Simulation must be used to check for software and hardware bugs¹² rather than theorem proving. Thus the language can only be checked to have produced consistent results via simulation. Eber's ambition to design and implement a formalized language is more ambitious than FpML and FinML in that it aims to be consistently bug free and the specification encompasses the pricing models as well as the contract characterization. Jean Marc Eber has also been working on a similar project with S. Jones and J. Seward (2000) [32] using another functional programming language called Haskell. However, as the authors admit, the language cannot yet describe many contracts and a variety of its promised improvements over existing technologies have yet to be implemented.

7 Conclusion

This paper has three goals: firstly to teach those learning about derivatives the colloquial, mathematical and legal terms associated with contingent claim contracts, secondly to highlight the inadequacies of the existing paradigm and thirdly to review the work currently in progress on the consistent treatment and automation of contingent claims.

The work formalizes the mathematical operations which are utilized in the marking up of contingent claim cash flows and payoffs. It provides a lexicographical guide to equivalent colloquial and legal forms.

We describe how financial institutions have traditionally used several languages to characterize contingent claims. These different languages have developed to serve different functions and do not necessarily capture the same information. This has introduced the difficulty of consistent characterization when mapping between these languages. We have argued for the development of a machine readable, legally binding language which is also relatively decipherable for humans.

The progress towards a coherent computer language for the characterization of contingent claims has been reviewed. At the time of writing there are several competing languages, those based on XML and those based on functional programming languages.

¹¹Imperative languages such as C are often faster than functional programming languages.

¹²Even if the contracts and the functional programming language are bug free there may still be bugs in the compiler and the computer processor.

References

- [1] S. Barle and N. Cakici. Growing a smiling tree. *Risk*, 8:76–81, October 1995.
- [2] G. Barone-Adesi and R. Whaley. Efficient analytic approximations of American option values. *The Journal of Finance*, 42:301–320, June 1987.
- [3] P. Bjerksund and G. Stensland. American exchange options and a put-call transformation: A note. *Journal of Business Finance and Accounting*, 20(5):761–764, 1993.
- [4] F. Black. The pricing of commodity contracts. *Journal of Financial Economics*, 3:167–179, 1976.
- [5] P. Boyle, J. Evnine, and S. Gibbs. Numerical evaluation of multivariate contingent claims. *The Review of Financial Studies*, 2:241–250, 1989.
- [6] P. Boyle and Y. Tse. An algorithm for computing values of options on the maximum and minimum of several assets. *Journal of Financial and Quantitative Analysis*, 25:215–227, 1990.
- [7] M. Broadie, P. Glasserman, and S. Kou. A continuity correction for discrete barrier options. Columbia Business School, April 1996.
- [8] P. Carr. The valuation of sequential exchange opportunities. *Journal of Finance*, 43:1235–1256, 1988.
- [9] A. Conze and Viswanathan. Path dependent options: The case of lookback options. *Journal of Finance*, 46:1893–1907, 1991.
- [10] M. Curran. Beyond average intelligence. *Risk Magazine*, 5(10), 1992.
- [11] E. Derman, P. Karasinski, and J. Wecker. Understanding guaranteed exchange-rate contracts in foreign stock investments. Goldman Sachs, June 1990.
- [12] A. Dravid, M. Richardson, and T. Sun. Pricing foreign index contingent claims: An application to nikkei index warrants. *Journal of Derivatives*, 1(1):33–51, 1993.
- [13] J. Eber. What can finance learn from computer science? Financial Options Research Centre Twelfth Annual Conference, July 1999.
- [14] S. Feferman, editor. *Kurt Gödel Collected Works*, volume 1. Oxford University Press, 1994. Publications 1929-1936.
- [15] M. Garman. Reflection in tranquility. *Risk Magazine*, 2(3), 1989.
- [16] M. Garman and S. Kohlhagen. Foreign currency option values. *Journal of International Money and Finance*, 2:231–237, 1983.

- [17] H. Geman and M. Yor. Pricing and hedging double-barrier options: A probabilistic approach. *Mathematical Finance*, 6(4):365–378, 1996.
- [18] R. Geske. The valuation of compound options. *Journal of Financial Economics*, 7:63–81, March 1979.
- [19] R. Geske and H. Johnson. The american put option valued analytically. *The Journal of Finance*, 39(5):1511–1523, December 1984.
- [20] B. Goldman, H. Sosin, and M. Gatto. Path dependent options: Buy at the low, sell at the high. *Journal of Finance*, 34(5):1111–1127, 1979.
- [21] I. Hart and M. Ross. Striking continuity. *Risk Magazine*, 7(6), 1994.
- [22] E. Haug. *The Complete Guide to Option Pricing Formulas*. McGraw-Hill, 1997.
- [23] R. Heynen and H. Kat. Crossing barriers. *Risk Magazine*, 1994a.
- [24] R. Heynen and H. Kat. Crossing barriers. *Risk Magazine*, 7(11), 1994b.
- [25] R. Heynen and H. Kat. Partial barrier options. *Journal of Financial Engineering*, 3:253–274, 1994c.
- [26] R. Heynen and H. Kat. Brick by brick. *Risk Magazine*, 9(6), 1996.
- [27] S. Hodges and M. Selby. On the evaluation of compound options. *Management Science*, 33(3):347–355, 1987.
- [28] J. Hull. *Options, Futures, and Other Derivative Securities*. Prentice-Hall, fourth edition, 2000.
- [29] T. Ibikunle, K. Jackson, A. Kramer, C. McDonald, R. Parmar, B. Sieber, W. Ali, P. Lee, and M. Panjwani. Financial products markup language. Technical Report 1.0 b2, J.P. Morgan and PricewaterhouseCoopers, December 1999.
- [30] M. Ikeda and N. Kunitomo. Pricing options with curved boundaries. *Mathematical Finance*, 2:275–298, 1992.
- [31] H. Johnson. Options on the maximum and minimum of several assets. *Journal of Financial and Quantitative Analysis*, 22(3):277–283, 1987.
- [32] S. Jones, J. Eber, and J. Seward. Composing contracts: an adventure in financial engineering. Submitted to the International Conference on Functional Programming, 2000, March 2000.
- [33] A. Kemna and A. Vorst. A pricing method for options based on average asset values. *Journal of Banking and Finance*, 14:113–129, 1990.
- [34] F. Longstaff. Pricing options with extendible maturities: Analysis and applications. *Journal of Finance*, 45(3):935–957, 1990.

- [35] S. Lyden. Reference check: A bibliography of exotic options models. *The Journal of Derivatives*, pages 79–91, Fall 1996.
- [36] W. Margrabe. The value of an option to exchange one asset for another. *The Journal of Finance*, 33(1):177–186, March 1978.
- [37] R. Merton. Theory of rational option pricing. *Bell Journal of Economics and Management Science*, 4:141–183, 1973.
- [38] E. Reiner. Quanto mechanics. *Risk Magazine*, 5:59–63, 1992.
- [39] E. Reiner and M. Rubinstein. Breaking down the barriers. *Risk Magazine*, 4(8), 1991.
- [40] D. Rich and D. Chance. An alternative approach to the pricing of options on multiple assets. *Journal of Financial Engineering*, 2(3):271–285, 1993.
- [41] M. Rubinstein. Pay now, choose later. *Risk Magazine*, 1990.
- [42] M. Rubinstein. Double trouble. *Risk Magazine*, 5(1), 1991a.
- [43] M. Rubinstein. Options for the undecided. *Risk Magazine*, 4(4), 1991c.
- [44] M. Rubinstein. Somewhere over the rainbow. *Risk Magazine*, 4(10), 1991d.
- [45] M. Rubinstein and E. Reiner. Exotic options. A compilation of papers on exotic options which have appeared as separate articles in RISK.
- [46] S. Schaefer and E. Schwartz. Time dependent variance and the pricing of bond options. *Journal of Finance*, 42:1113–1128, 1987.
- [47] R. Stulz. Options on the minimum and the maximum of two risky assets. *Journal of Financial Economics*, 10:161–185, 1982.
- [48] S. Turnbull and L. Wakeman. A quick algorithm for pricing european average options. *Journal of Financial and Quantitative Analysis*, 26:377–389, 1991.
- [49] P. Zhang. Correlation digital options. *Journal of Financial Engineering*, 4:75–96, 1995.

Table 1: Back office monitoring requirements.

Information Flows	Example
Prices at T and cash flows at T .	European call
Prices at t_1, t_2, \dots, t_n and cash flows at T .	European lookback
Prices at t_1, t_2, \dots, t_n may prompt cash flows at t_1, t_2, \dots, t_n .	European barrier
Prices at t_1, t_2, \dots, t_n counter-party may exercise.	Short American option or the holder of a convertible bond with a call provision.
Prices at t_1, t_2, \dots, t_n one must decide when to exercise	Long American option or the holder of a convertible bond with a put provision.

A High Level Classification of Contracts (Information and Monitoring Requirements)

A high level classification of contracts is outlined in Table 1 indicating their information (in this case observable prices) and monitoring (time) requirements. The back office of a bank has to be aware of contract specifications so that the relevant cash flows can be executed in a timely fashion to avoid interest penalties on cash flows (and to verify interest owed on counter party cash flow failures). Moreover, some contracts have associated choices (free boundary problems), the optimal execution of which necessitates the monitoring of market information. A knowledge and monitoring of information requirements allows the back office to foresee any cash flows resulting from a counter-party exercising a right against the bank. The corollary of this is that the back office will be able to optimally exercise a contract against a counter-party.