Volatility Options: Hedging Effectiveness, Pricing, and Model Error*

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Abstract

Motivated by the growing literature on volatility options and their imminent introduction in major exchanges, this paper addresses two issues. First, we examine whether volatility options are superior to standard options in terms of hedging volatility risk. Second, we investigate the comparative pricing and hedging performance of various volatility option pricing models in the presence of model error. Monte Carlo simulations within a stochastic volatility setup are employed to address these questions. Alternative dynamic hedging schemes are compared, and various option-pricing models are considered. The results have important implications for the use of volatility options as hedging instruments, and for the robustness of the volatility option pricing models.

JEL Classification: G11, G12, G13.

Keywords: Hedging Effectiveness, Model Error, Monte Carlo Simulation, Stochastic Volatility, Volatility risk, Volatility Options.

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I. Introduction

The main sources of risk that an investor faces are price and volatility risk (vega risk). Price risk is the investor's exposure to changes in the asset price. Volatility risk is the exposure to changes in volatility. The latter type of risk has been responsible for the collapse of major financial institutions in the past fifteen years (e.g. Barings Bank, Long Term Capital Management). To date, the hedging of volatility risk has been carried out by using the exchange traded standard futures and plain-vanilla options. However, these instruments are designed so as to deal with price risk, primarily. A natural candidate to hedge volatility risk is volatility options. These are instruments whose payoff depends explicitly on some measure of volatility.

The growing literature on volatility options has emerged after the 1987 crash. Brenner and Galai (1989, 1993) first suggested options written on a volatility index that would serve as the underlying asset. Towards this end, Whaley (1993) constructed VIX (currently termed VXO), a volatility index based on the S&P 100 option's implied volatilities traded in the Chicago Board of Exchange (CBOE). Ever since, other implied volatility indices have also been developed (e.g., VDAX in Germany, VXN in CBOE, VX1 and VX6 in France) and the properties of some of them have been studied (see e.g., Fleming et *al*. 1995, Moraux et *al*. 1999, Whaley 2000, Blair et *al*. 2001, Corrado and Miller 2003, and Simon 2003). Various models to price volatility options written on the instantaneous volatility have also been developed (see e.g., Whaley 1993, Grünbichler and Longstaff 1996, and Detemple and Osakwe 2000). These models differ in the specification of the assumed stochastic process, and the assumptions made about the volatility risk premium. In 2003, CBOE adopted a new methodology to calculate the implied volatility index, and it announced the immediate introduction of volatility options in an organized exchange.

However, to the best of our knowledge, the hedging effectiveness of volatility options compared to that of plain-vanilla options has not yet been studied. Jiang and Oomen (2001) have examined the hedging performance only of volatility futures versus standard options; we comment further on the relevance of their study to ours in the concluding section of the paper. This may be surprising given that one of the main arguments for introducing volatility options is based on their use as hedging instruments[1]. Furthermore, the comparative hedging and pricing performance of the existing volatility option pricing models in the presence of model error has attracted very little attention[2]; Daouk and Guo (2004) have focused on the pricing side and they have investigated the impact of model error to the performance of only one (Grünbichler and Longstaff 1996) of the developed volatility option pricing models.

This paper makes two contributions to the volatility options literature by exploring these two issues, respectively. First, it compares the hedging performance of volatility versus standard European options. Second, it answers the following question: "Assuming that we know the true data generating process of the underlying asset price and of volatility, what is the impact of using a mis-specified process on the hedging and pricing performance of the volatility option pricing models under scrutiny?" Understanding the hedging performance of volatility options, as well as the comparative pricing performance of various volatility option

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[1] Volatility options can also be used to speculate on the fluctuations of volatility. Interestingly, Dupire (1993), Derman et *al*. (1997), and Britten-Jones and Neuberger (2000) have shown that volatility trading/hedging can also be performed indirectly by using static positions in standard European calls. For a review of the volatility trading/hedging techniques, see also Carr and Madan (1998). However, transaction costs may hamper the implementation of such strategies.

[2]Crouhy et *al*. (1998) define as model error either the mis-specification of the model, and/or the parameter mis-estimation within any given model, and/or the incorrect implementation of any given model. The existing studies on the impact of model error to the hedging effectiveness use as a target option either a standard European option (see e.g., Galai 1983, Figlewski 1989, and Carr and Wu 2002) or various exotic options (see e.g., Hull and Suo 2002).

pricing models will facilitate the introduction of volatility options in organized exchanges, and their use by investors[3].

To address our research questions, Monte Carlo (MC) simulations under a stochastic volatility setup are employed. MC simulation has been used in the literature extensively to investigate the pricing and hedging performance of various models, as well as the impact of model error (see e.g., Hull and White 1987, Figlewski 1989, Jiang and Oomen 2001, Carr and Wu 2002, Daouk and Guo 2004). This is because it enables the selection of the data generating process, and the control of the values of its parameters. Comparative analysis for various parameter values is also possible. Moreover, in our case the use of MC simulation is dictated by the lack of data on volatility options; volatility options are not traded yet. Alternative methods such as historical simulation (Green and Figlewski 1999), or calibration of the pricing model to market data (see e.g., Backshi et *al*. 1997, Dumas et *al*. 1998, and Hull and Suo 2002) that have been used to answer similar questions cannot be followed. Following Hull and Suo (2002), the stochastic volatility setup has been adopted as the true data generating process. This is a legitimate assumption since there is broad empirical evidence that volatility is stochastic. Moreover, this setup is preferred to a more complex one that also includes other sources of risk, e.g., jumps and stochastic interest rates. Backshi et *al*.

[3] Surprisingly, the trading of volatility derivatives in exchanges has not yet been instituted. The only attempt to introduce contracts on volatility in an organized market was undertaken by the German Exchange in 1997; that was a volatility future (VOLAX) on the German implied volatility index VDAX. However, the trading of VOLAX ceased in 1998. An anecdotal explanation that is offered by practitioners for the failure of VOLAX, as well as for the delay in introducing volatility options, is that market makers are neither familiar with the models that have been developed to price volatility futures and options, nor with their use for hedging purposes. In accordance with this claim, Whaley (1998) also states "In summary, I believe that volatility derivatives are a viable exchange-traded product…I also believe that the contracts have not been successful largely because potential market makers have not stepped forward. The reason is fear."

(1997) examine such a general model and they conclude "…taking stochastic volatility into account is of first-order importance in improving upon the BS formula" (pp. 2042-2043).

We assume that a short position in a standard European stock call option is to be hedged (target option). A natural way to hedge both price and volatility risk is to hedge the target option with the stock and another option (delta-vega strategy, see e.g., Hull and White 1987 and Scott 1991). Two alternative dynamic delta-vega neutral hedging schemes are considered. These differ on the type of option employed as the hedging instrument; standard and volatility European call options are used as option hedging instruments, respectively. Next, the stock and volatility price series are jointly generated. We introduce the hedging error by rebalancing the position in the hedging instrument discretely. Then, the performance of the two hedging strategies is assessed on the simulated data across different strikes and maturities of the target option, and for various values of correlation between the underlying asset and its volatility. Two different rebalancing frequencies (daily and weekly) are used. This will enable us to investigate the effect of the contract specifications, correlation, and the rebalancing frequency on the hedging effectiveness of the two strategies.

To implement the two hedging schemes, the hedge ratios are calculated by employing separately the Black-Scholes (BS, 1973) and Heston (1993) models in the case of (target and instrument) standard options. In the case of volatility options, Whaley (1993), Grünbichler and Longstaff (1997), and Detemple and Osakwe (2000) models are used. The use of various option pricing models will shed light on the robustness of the volatility option pricing models for hedging and pricing purposes in the presence of model error.

In terms of the hedging effectiveness, we find that volatility options are not superior hedging vehicles to standard European options in the context of the examined hedging schemes. However, the difference in the performance of the two hedging schemes depends on the characteristics of the target option, and on whether a roll-over strategy in volatility options is carried out. In terms of the effect of the model error, we find that the seemingly worst mis-specified volatility option-pricing model can be reliably used for pricing and hedging purposes.

The remainder of the paper is structured as follows. In the next Section, the employed option pricing models are described. In Section 3, the simulation setup and the measure of hedging effectiveness are introduced. Section 4 presents and discusses the results on the hedging performance of the two hedging strategies. Section 5 presents and discusses the results on the robustness of the volatility option pricing models for hedging and pricing purposes in the presence of model error. The last Section provides a brief summary, the implications of the results are addressed, and topics for future research are suggested.

II. Description of the Models

In this section, the Black-Scholes (1973), Heston (1993), Whaley (1993), Grünbichler and Longstaff (1996), and Detemple and Osakwe (2000) option pricing models that will be used in the remainder of the paper are briefly described. The first two models are used for the pricing/hedging of standard European options. The remaining three are used for the pricing/hedging of volatility options. Only the European call pricing formulae are provided for the purposes of our analysis.

A. Standard Options: Black-Scholes (1973)

Let the stock price S_t of a non-dividend paying stock follow a Geometric Brownian Motion Process (GBMP), i.e.:

$$dS_t = rS_t dt + VS_t dW_t, \quad t \in [0, +\infty), \tag{1}$$

where V is the volatility of the asset price, and W_t is a Wiener process under the risk-adjusted measure Q. The Black and Scholes (BS) price $C(S_t, K, T)$ of a European call option is given by:

$$C(S_t, K, T) = S_t N\left(d_1\right) - e^{-r(T-t)} K N\left(d_2\right) \tag{2}$$

where *T-t* is the time to maturity, *K* is the strike price, *N(·)* is the cumulative standard normal probability distribution function, and

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + (r + \frac{1}{2}V^2)(T-t)}{V\sqrt{(T-t)}},\ d_2 = d_1 - V\sqrt{T-t}. \tag{3}$$

B. Standard Options: Heston (1993)

Heston (1993) assumes that the stock price follows a GBMP and the variance υ_t evolves as a Mean Reverting Square Root Process (MRSRP), i.e.:

$$dS_t = rS_t dt + \sqrt{\upsilon_t} S_t dW_{1t} \tag{4}$$

$$d\upsilon_t = \beta\left(m - \upsilon_t\right)dt + \sigma\sqrt{\upsilon_t}\, dW_{2t} \tag{5}$$

where W_{1t} and W_{2t} are two correlated Wiener processes under the risk-adjusted measure *Q* and σ is the volatility of the variance. The parameters β and *m* are the rate of mean reversion and the long-run mean of volatility, respectively, that incorporate the market price of volatility risk *h*. Heston assumes that *h* is proportional to the current level of variance, i.e. $h = \varsigma\upsilon$, where $\zeta \in [0, +\infty)$ is a constant parameter. Therefore, under the risk-adjusted measure *Q*,

$$\beta = \lambda + \zeta, \text{ and}$$

$$m = \frac{\lambda\mu}{\lambda + \zeta},$$

where λ and μ are the rate of mean reversion and the long-run mean of the equivalent MRSRP under the physical probability measure *P*, respectively. Then, the price at time *t* of a European call option is given by:

$$C(S_t, \upsilon, t) = S_t P_1 - Ke^{-r(T-t)} P_2, \tag{6}$$

where P_j (j=1,2) is defined in Heston (1993, page 331, equations 16, 17, and 18).

C. Volatility Options: Whaley (1993)

Whaley uses the Black's (1976) futures model to price volatility options written on an implied volatility index; he considers volatility futures options with a zero cost-of-carry. Hence, he assumes implicitly that the volatility index is a traded asset that follows a GBMP. The value $C(V_t, K, T)$ of a European volatility call option on the volatility index at time t is given by:

$$C(V_t, K, T) = e^{-r(T-t)}\left[V_t N\left(d_1\right) - KN\left(d_2\right)\right], \tag{7}$$

where V_t is the value of the volatility index at time t, and

$$d_1 = \frac{\ln\left(\frac{V_t}{K}\right) + 0.5\sigma^2\left(T-t\right)}{\sigma\sqrt{\left(T-t\right)}} \; and \; d_2 = d_1 - \sigma\sqrt{T-t}. \tag{8}$$

where σ is the volatility of the volatility index returns.

D. Volatility Options: Grünbichler and Longstaff (1996)

Grünbichler and Longstaff (GL) derive a closed form expression to price European volatility options. The underlying asset is the instantaneous volatility of the returns of a stock index. They regard volatility as a non-tradable asset; the market price of volatility risk has to be introduced. Following Heston (1993) they assume that the expected volatility risk premium h is proportional to the current level of volatility. GL model the volatility (and not the variance as in Heston's case) as a MRSRP, i.e.:

$$dV_t = \beta\left(m - V_t\right)dt + \sigma\sqrt{V_t}dW_t, \qquad t \in \left[0, +\infty\right) \tag{9}$$

where W_t is the Brownian motion under the risk-adjusted probability measure Q. Then, the value of a European volatility call option $C(V_t, K, T)$ at time t is given by:

$$\begin{aligned} C(V_t, K, T) = {} & e^{-r(T-t)} \times e^{-\beta(T-t)} \times V_t \times X\left(\gamma K; \nu + 4, \theta\right) + \\ & e^{-r(T-t)} \times m \times \left(1 - e^{-\beta(T-t)}\right) \times X\left(\gamma K; \nu + 2, \theta\right) - \\ & e^{-r(T-t)} \times K \times X\left(\gamma K; \nu, \theta\right) \end{aligned} \tag{10}$$

where $X(\cdot)$ is the cumulative distribution function of the complementary non-central chi-squared distribution $\chi_v^2(\theta)$, with v degrees of freedom and non-centrality parameter θ (see Chapter 28 of Johnson and Kotz 1970). The parameters γ, θ, and v are defined in Grünbichler and Longstaff (1996, page 989, equation 6).

E. Volatility Options: Detemple and Osakwe (2000)

Detemple and Osakwe (DO) provide an analytic pricing formulae for European volatility options assuming that the market price of volatility risk is zero, and that volatility follows a Mean Reverting Logarithmic process (MRLP), i.e.:

$$d\ln V_t = \lambda\left(\mu - \ln V_t\right)dt + \sigma dW_t, \tag{11}$$

Then, the value of a European volatility call option is given by:

$$C(V_t, K, T) = e^{-r(T-t)}\left[V_t^{\phi_T} e^{\mu(1-\phi_T)+\frac{1}{2}a_T^2} N\left(d_T + a_T\right) - KN\left(d_T\right)\right], \tag{12}$$

where φ_T, α_T, and d_T are defined in Detemple and Osakwe (2000, page 25).

III. Assessing the Hedging Effectiveness

We investigate whether the hedging performance of volatility options is superior to that of standard plain vanilla options in a stochastic volatility environment. Towards this end, the performance of various hedging strategies is assessed under a *stochastic volatility* simulation setup.

A. Generating the Data: The Simulation Setup

A joint Monte Carlo simulation of the stock price and volatility generates the data. We assume that the asset price S follows a GBMP, and the volatility V follows a MRSRP (i.e. equations 1 and 9). The MRSRP has been widely used to model the stochastic evolution of volatility over time (see among others, Hull and White 1988, Heston 1993, Ball and Roma

1994, Grünbichler and Longstaff 1996, and Psychoyios et *al*. 2003 for a review on the continuous time stochastic volatility processes). This is because it is consistent with the empirical evidence that volatility follows a mean-reverting process (see e.g., Scott 1987, Merville and Pieptea 1989, and Sheikh 1993), and it precludes volatility from taking negative values.

Furthermore, a zero volatility risk premium is assumed, i.e. $\zeta=0$. Hence, equations (1) and (9) are re-written as

$$\frac{dS_t}{S_t} = rdt + V_t dW_{1,t}, \tag{13}$$

$$dV_t = \lambda\left(\mu - V_t\right)dt + \sigma\sqrt{V_t}\,dW_{2,t}, \quad t \in \left[0, +\infty\right). \tag{14}$$

The Brownian motions in the two processes are assumed to be correlated with instantaneous correlation coefficient ρ, i.e.

$$Cor(dW_{1,t}, dW_{2,t}) = \rho \tag{15}$$

The assumption of a zero volatility risk premium is necessary so as to include Whaley (1993) and DO in the list of models to be compared subsequently. On the other hand, this assumption is in contrast with the empirical evidence that suggests that the volatility risk premium is non-zero and time varying (see e.g., Backshi and Kappadia 2003). However, the assumption of a zero volatility risk premium does not impose any limitations on our subsequent analysis. We comment further on this in Section 4.4. Moreover, such an assumption is not unrealistic within our context. Whaley (1993) argues that risk neutral valuation is valid given that the (implied) volatility index can be replicated by forming an option trading strategy in line with the steps necessary to construct the index.

Finally, the non-zero correlation between the asset price and volatility is also empirically documented. Typically it has been found to be negative; this has been termed as leverage effect (see e.g., Figlewski and Wang 2000, for a detailed review and an empirical analysis).

B. The Hedging Scenario and the Hedging Error Metric

Assume that a financial institution sells a standard European call option with τ_1 days to maturity (target portfolio). Accepting the stochastic nature of volatility, the aim is to delta-vega hedge the target portfolio with an instrument portfolio that is composed of the underlying asset and either a standard option, or a volatility option. To make this concrete, let T_t and I_t be the prices of the target and instrument portfolios at time t, respectively. Then, the price P_t of the portfolio formed by the target and the instrument portfolio (extended portfolio) is given by

$$P_t = T_t + I_t = T_t + n_{1,t} S_t + n_{2,t} i_t \tag{16}$$

where i_t and S_t are the prices of the instrument option and the stock price at time t, respectively. Note that the weights n_1 and n_2 are functions of time t, since the delta-vega neutrality requires (continuous) rebalancing. In particular, a continuous delta-vega neutral hedge requires the following equations to hold $\forall t$

$$\frac{\partial P_t}{\partial V} = \frac{\partial T_t}{\partial V} + n_{2,t} \frac{\partial i_t}{\partial V} = 0 \tag{17}$$

and

$$\frac{\partial P_t}{\partial S} = \frac{\partial T_t}{\partial S} + n_{1,t} + n_{2,t} \frac{\partial i_t}{dS} = 0 \tag{18}$$

Equations (17) and (18) express the conditions for vega and delta neutrality, respectively.

However, in practice the presence of transaction costs does not allow continuous rebalancing; this introduces a hedging error. Assuming a rebalancing frequency Δt, the price of the extended portfolio at time $t+\Delta t$ is given by

$$P_{t+\Delta t} = T_{t+\Delta t} + I_{t+\Delta t} = T_{t+\Delta t} + n_{1,t} S_{t+\Delta t} + n_{2,t} i_{t+\Delta t} \tag{19}$$

Notice that the weights in equation (19) are still functions of time t because the hedge has not been adjusted since time t (see also Boyle and Emanuel 1980 for a similar setup). Assuming that there are no arbitrage opportunities, a riskless extended portfolio (i.e. a perfect hedge is achieved) should earn the risk-free rate, i.e. $\Delta P_t = r\ P_t\ \Delta t$. Following Figlewski (1989) and Scott (1991), the hedging error $HE(t+\Delta t)$ at time $t+\Delta t$ is defined as

$$\begin{aligned} HE(t+\Delta t) = \Delta P_t - rP_t\Delta t = (T_{t+\Delta t} - T_t) + n_{1,\mathrm{t}}(S_{t+\Delta t} - S_t) + \\ n_{2,\mathrm{t}}(i_{t+\Delta t} - i_t)\text{-}r[\mathrm{T_t}+\mathrm{n_{1,t}}S_t+\mathrm{n_{2,t}}i_t]\Delta t \end{aligned} \tag{20}$$

HE(t) is calculated and recorded for each date *t* and for each hedging scheme. A commonly used metric to assess the hedging effectiveness is the Total Dollar Hedging Error (TDHE, see Figlewski 1989, and Backshi et *al*. 1997). The TDHE is defined as:

$$TDHE = \sum_{l=1}^{M} HE\left(l\Delta t\right)\times\left(1+r\Delta t\right)^{M-l} \tag{21}$$

where M is the number of rebalancing dates. Therefore, the TDHE is the sum of hedging errors across the rebalancing dates compounded up to the expiry date (aggregate hedging error). Consequently, the hedging scheme with the lowest *TDHE* should be chosen among alternative competing schemes.

However, within a simulation setup the calculated hedging error is conditional on the generated asset and volatility paths. To eliminate this dependence, we calculate $HE(t+\Delta t, k)$ by running K joint MC simulation runs of the asset and volatility prices ($k=1,2,\ldots,K$). Then, the *TDHE* is calculated for each simulation run. Finally, the *unconditional TDHE* (*UTDHE*) is calculated as the average of the absolute values of the *TDHE* across the K simulation runs, i.e.:

$$UTDHE(\Delta t) = \frac{1}{K}\sum_{k=1}^{K}\left|TDHE(k)\right| = \frac{1}{K}\sum_{k=1}^{K}\left|\sum_{l=1}^{M} HE(l\Delta t,k)\times\left(1+r\Delta t\right)^{M-l}\right| \tag{22}$$

The absolute values are used so as to avoid offsetting the positive with the negative signed TDHEs and hence distorting the magnitude of the chosen measure of hedging error. The UTDHE can be interpreted as a measure of the dispersion of the distribution of the TDHEs. The variability of the hedging error has been commonly used in the literature as a metric to assess the hedging performance (see e.g., Hull and White 1987, Figlewski 1989, Green and Figlewski 1999). The most effective hedging scheme is the one with the lowest UTDHE; the idea is analogous to the approach followed to calculate a standard optimal hedge ratio, i.e. to minimize the variance in the changes of the price of an extended portfolio.

C. Hedging Schemes and Option Pricing Models

The described simulation setup is applied to two different hedging schemes:

Hedging Scheme 1 (HS1): An instrument portfolio consisting of n_1 shares, and n_2 at-the-money standard European call options with τ_2 days to maturity ($\tau_1<\tau_2$), and

Hedging Scheme 2, (HS2): An instrument portfolio consisting of n_1 shares, and n_2 at-the-money European volatility call options with τ_3 days to maturity ($\tau_3 \leq \tau_1 < \tau_2$). Hence, a roll-over strategy with volatility options may be considered.

Within the above two schemes, five models are employed to calculate the hedge ratios (equations 17 and 18). These are: Black-Scholes (BS, 1973), Heston (He, 1993), Whaley (1993), Grünbichler and Longstaff (GL, 1996), and Detemple and Osakwe (DO, 2000) models[4] [5]. The observed prices of the target and the instrument options are assumed to be generated by the Heston (1993) and Grünbichler and Longstaff (1996) models in the cases of the plain-vanilla and volatility options, respectively. These are the models that are closest to the assumed data generating stock/volatility processes. Then, the resulting hedging error is calculated (equation 20). Hence, the results will be a joint test of the hedging scheme and of the model employed. In total, the hedging performance of eight combinations is compared. These are the following:

[4]The delta-vega neutral strategy is inconsistent with the BS assumption of constant volatility. However, it is widely used in practice by practitioners so as to accommodate the non-constancy of volatility within the BS model.

[5] There exist no closed-form formulae to calculate the GL hedge ratios; these are calculated numerically. The hedge ratios in the other models are calculated by using the formulae that are provided in the respective papers. The simulated stock price/ instantaneous volatility are used as inputs.

HS 1, BS-BS: In Hedging Scheme 1, BS model is used to calculate the Greeks of the target and of the instrument portfolio.

HS 1, He-He: In Hedging Scheme 1, Heston model is used to calculate the Greeks of the target and of the instrument portfolio.

HS 2, BS-DO: In Hedging Scheme 2, BS and DO models are used to calculate the Greeks of the target and of the instrument portfolio, respectively.

HS 2, He-DO: In Hedging Scheme 2, Heston and DO models are used to calculate the Greeks of the target and of the instrument portfolio, respectively.

HS 2, BS-GL: In Hedging Scheme 2, BS and GL models are used to calculate the Greeks of the target and of the instrument portfolio, respectively.

HS 2, He-GL: In Hedging Scheme 2, Heston and GL models are used to calculate the Greeks of the target and of the instrument portfolio, respectively.

HS 2, BS-W: In Hedging Scheme 2, BS and Whaley models are used to calculate the Greeks of the target and of the instrument portfolio, respectively.

HS 2, He-W: In Hedging Scheme 2, Heston and Whaley models are used to calculate the Greeks of the target and of the instrument portfolio, respectively[6].

D. The Implementation

The joint MC simulations in equations (13) and (14) are performed by adopting the Grünbichler and Longstaff (1996) parameters values $\mu=0.15$ and $\lambda=4$. The initial asset price

[6] Heston (1993) models the stochastic evolution of the variance. In our simulation framework, the volatility is simulated. This raises two issues. First, to implement Heston's model that requires the variance as an input the simulated volatility has to be squared. Second, Heston defines vega as the first derivative of the option price with respect to the variance. In our case, vega is defined as the first derivative of the option price with respect to volatility. Hence, the chain rule is applied to calculate Heston's vega using the latter definition, i.e.:

$$\frac{dC}{dV} = \frac{dC}{dVar} \times \frac{dVar}{dV} = \frac{dC}{dVar} \times \frac{d\left(V^2\right)}{dV} = 2V\frac{dC}{dVar}$$

where C is the call option price, and V, Var denote the volatility and the variance, respectively.

and the initial volatility are set equal to 100 and 0.1, respectively, r=5%, σ^2=0.133, and τ_2=100. 1,000 simulation runs are performed to calculate the UTDHE.

Equation (22) shows that the *UTDHE* is a function of the rebalancing frequency Δt. Therefore, the hedging effectiveness of each one of the eight combinations is examined by calculating the *UTDHE* for two different rebalancing frequencies: daily and every five days.

Under the assumption that volatility follows a mean reverting process, the hedging effectiveness of the volatility option decreases as the time to maturity increases (see Grünbichler and Longstaff 1996, and Detemple and Osawke 2000). Therefore, short maturity volatility options should be preferred as hedging vehicles. To test this hypothesis, we considered two different hedging strategies within HS2. A roll-over in volatility options strategy with N short maturity τ_3 volatility options ($\tau_3 << \tau_1$), so as $N \cdot \tau_3 = \tau_1$, and a no-roll-over strategy with only one volatility option with τ_1 time-to-maturity.

Moreover, the hedging performance of the instrument options may depend on the strike price and the time-to-maturity of the target option (see e.g., Hull and White 1987, and Backshi et *al.* 1997). Hence, regarding the strike price, three target options are hedged separately: an in-the-money (ITM), an at-the-money (ATM), and an out-of-the-money (OTM) call option (K=90, 100, and 110, respectively). For each strike price, target options of short, intermediate, and long maturities are studied separately (τ_1=20, 40, and 80, respectively). In the case of the roll-over strategy, the time-to-maturity of the volatility option is τ_3=30, and the roll-over is performed ten days prior to it's expiry date. Obviously, in the case of the short maturity target option, there is no roll-over. In the intermediate and the long maturity, there are one and three roll-over points, respectively.

Finally, it may be the case that the hedging performance also depends on the value of correlation between the asset price and the volatility. For example, Nandi (1998) finds that the Heston (1993) model performs better than the BS model in terms of hedging only in the case where the correlation is non-zero. Hence, for each strike price and time-to-maturity, the joint MC simulations in equations (13) and (14) are performed for different values of ρ=-0.5, 0, +0.5. The results are not sensitive to the choice of the correlation value. Therefore, they are reported only for the case of ρ=-0.5 that is consistent with the empirically documented leverage effect. Cholesky decomposition is applied to create the correlated random numbers for the purposes of MC simulation.

IV. Hedging Effectiveness: Results and Discussion

In this Section, the hedging effectiveness of volatility versus plain vanilla options is examined. Table 1 shows the UTDHE for HS1 and HS2 across different rebalancing frequencies, various times-to-maturity, and moneyness levels of the target option. The results are also reported for the roll-over versus the no roll-over of the volatility option case; in the short maturity no roll-over is performed.

(INSERT TABLE 1 HERE)

We can see that in general, the hedging performance of HS1 is superior to that of HS2, i.e. the distribution of the TDHE has a smaller dispersion under HS1 than HS2. HS1 outperforms HS2 in both the roll-over and the no roll-over of the volatility option case. In particular, HS1 He-He has the lowest UTDHE for all maturities and moneyness levels.

Next, the results on the hedging effectiveness are discussed by focusing on the change in the difference in the hedging performance of the two hedging schemes across each one of the following four criteria: the rebalancing frequency, the application of roll-over versus no-roll-over, the time-to-maturity of the target option, and the strike price of the target option. The analysis is ceteris paribus.

A. Hedging Effectiveness: Rebalancing Frequency and Roll-Over Effect

Table 1 shows that the difference between HS1 and HS2 increases in the case of the weekly rebalancing. In the short and intermediate maturities and for all moneyness levels, the hedging error is less under daily than weekly rebalancing. Interestingly, for some combinations in the case of the long maturity target options, the hedging error is slightly greater under daily than weekly rebalancing; the result may seem counterintuitive. Boyle and Emanuel (1980) have showed that the hedging error decreases as the rebalancing frequency increases. However, their study focuses on the effect of the rebalancing frequency on the *delta* hedging error under the Black-Scholes assumptions. To the best of our knowledge, there is no similar study in the context of delta-vega hedging strategies in a stochastic volatility setup. Intuitively, our findings may be explained as follows. The hedging error includes the delta and vega hedging errors among other sources of error. Given the assumed mean-reverting nature of volatility, the vega hedging error term is expected to be smaller under weekly rebalancing in the case of the long-maturity target option. This is because there is a greater probability for the long-run mean of volatility to be reached, and hence volatility does not vary any longer.

Regarding the roll-over effect, Table 1 shows that HS1 dominates HS2 in both the no-roll-over and the roll-over strategies. Interestingly, in the roll-over case, the difference between HS1 and HS2 has decreased, in general. This finding is expected; long-dated volatility options should not be used to hedge volatility since their prices are less sensitive to changes in volatility than the short-term ones (see Grünbichler and Longstaff 1996, and Detemple and Osakwe 2000). The result implies that the hedging effectiveness of volatility options increases by following a roll-over strategy with short maturity volatility options.

B. Hedging Effectiveness: Maturity and Strike Effect

To compare the hedging effectiveness across various maturities and strikes, the UTDHE calculated from equation (22) needs to be standardized. The choice of the numeraire to be used for the standardisation depends on the type of risk to be hedged. In the context of comparing alternative delta hedging strategies, the resulting hedging error is usually divided by the initial premium of the target option (see e.g., Figlewski and Green 1999, Carr and Wu 2002, and Bakshi and Kappadia 2003). For the purposes of our analysis, we choose the numeraire to be the Black-Scholes vega of the target option at the initiation date of the hedging strategy. Hence, the standardised hedging errors can be compared across various target options that have the same exposure to volatility risk[7]. Table 2 shows the vega-standardised UTDHE for HS1 and HS2 across different rebalancing frequencies, and various times-to-maturity and moneyness levels of the target option. The results are also reported for the roll-over versus the no roll-over of the volatility option case.

(INSERT TABLE 2 HERE)

HS1 dominates HS2 for all maturities. The effect of the time-to-maturity of the target option on the standardized UTDHE depends on the moneyness of the target option. In particular, the standardized UTDHE increases in the cases of ITM and OTM target options. On the other hand, the standardized UTDHE decreases in the case of ATM options. The results hold for both HS1 and HS2. The effect of the time-to-maturity of the target option on the *difference* between the hedging performance of HS1 and HS2 depends on the moneyness of the target option, as well. In general, as the time-to-maturity increases, the difference in the performance of the two hedging schemes decreases for ITM and OTM target options, and it increases for ATM target options. The results hold for both the roll-over and the no roll-over strategies.

Finally, as far as the strike price of the target option is concerned, HS1 dominates HS2 for all moneyness levels. We can see that the difference in the hedging performance of

[7] Given that our analysis focuses on hedging vega risk, it is not appropriate to standardise the hedging error by using the option premium as a numeraire. This is because the vega hedging error is expected to increase monotonically with the vega. However, the relationship between the vega hedging error and the premium is not monotonic. On the other hand, in the context of delta hedging, the delta hedging error is monotonically related to both the option premium and the delta; either of them can be used as numeraire for the purposes of comparing delta hedging errors across various target positions.

the two schemes depends on the moneyness level. The difference is maximised for ITM target options, and it is minimised for ATM target options. Regarding the performance of each individual scheme, both HS1 and HS2 perform best for ATM target options and worse for ITM target options. Focusing on HS2, we can see that it performs best when ATM options are to be hedged; these happen to be the most liquid instruments in the derivatives exchanges. The results hold for both the roll-over and the no-roll-over strategies.

C. Assessing the Differences in the Hedging Effectiveness

So far, we have commented on the differences between the UTDHE across the various hedging schemes. The UTDHE measures the dispersion of the distribution of the TDHE. To get a feel of whether the TDHE differs across the hedging schemes under scrutiny, we construct 95% confidence intervals from the distribution of the TDHE. The intersection of any two of the constructed 95% confidence intervals may suggest that the corresponding values of the TDHE are not significantly different in a statistical sense[8]. Tables 3 and 4 show the UTDHE and (within the brackets) the lower and upper bounds of the 95% confidence intervals of the TDHE in the case of daily and weekly rebalancing, respectively. Results are reported for the roll-over and no-roll-over case.

(INSERT TABLES 3 AND 4 HERE)

The confidence intervals "test" suggests that the difference between HS1 and HS2 becomes stronger as the time-to-maturity of the target option increases; this holds regardless of the rebalancing frequency. Interestingly, this relationship is more evident in the no-roll-

[8] The standard statistical methods that assess the statistical significance of two means (e.g. the one sample *t*-test or the independent-samples *t* test) cannot be used in our case because they are based on the assumption that the population follows a normal distribution. However, application of Jarque-Bera tests showed that the TDHE is not normally distributed.

over strategy; these findings are in accordance with the discussion in Section IV.B where the differences between HS1 and HS2 are reported to be greater for the no-roll-over case.

In particular, we can see that the differences in the TDHE between HS1 and HS2 are not significant for the short maturity options; this holds for all moneyness levels. In the case of the intermediate maturity options, the results depend on the moneyness level. More specifically, in the case of ITM options the differences between HS1 and HS2 are not significant. In the case of ATM options, the differences between HS1 He-He and HS2 are significant. On the other hand, the differences between the HS1 BS-BS and HS2 are not significant. Finally, considering OTM options, the differences are in general significant; some exceptions occur comparing the differences of HS1 BS-BS with HS2 He-GL and HS2 He-DO. In the case of the long maturity options, the differences between HS1 and HS2 are significant for ATM and OTM options.

D. The Effect of a Non-Zero Volatility Risk Premium

At this point, we should examine whether our results are robust to the assumption of a zero volatility risk premium. The existence of a non-negative volatility risk premium may affect the hedging effectiveness of volatility options (provided that the respective model admits the existence of a volatility risk premium). For instance, Henderson et *al*. (2003) showed that in a quite general stochastic volatility setting, the prices of standard calls and puts are a decreasing function of the market price of risk of volatility. There may be an analogous result to be proved for the case of volatility options. Therefore, we investigated the comparative performance of combinations that admit the existence of a non-zero volatility risk premium, i.e. HS1 He-He versus HS2 He-GL. To this end, the same analysis was performed by choosing $\zeta=-1$. This choice is consistent with the average estimate of ζ over the period 1987-1992 in Guo (1998); he estimated the implicit volatility risk premium from Heston's model using currency option prices. Hence, his results are directly applicable to implementing

Heston and GL models in our setup. We found that HS1 still outperforms HS2. Hence, the previously reported results are robust to the choice of a zero value for the volatility risk premium. This is in line with the results in Daouk and Guo (2004) who also found that the effect of model error on the pricing performance of the GL model is not affected by the choice of the volatility risk premium.

V. Volatility Options and the Effect of Model Error

In this Section, the focus is on HS2, and the effect of model error on the hedging and pricing performance of the various volatility option pricing models is studied. The He, GL models are used as benchmarks. This is because among the option models under scrutiny, they assume a volatility process (MRSRP) that is closest to our assumed (possibly correlated stock and volatility) data generating processes.

A. Model Error and the Hedging Performance

Given that the He-GL is the true combination, the model error may arise from any of the three following alternative scenarios: (a) the hedger uses the true model to calculate the hedge ratio of the volatility option, and a false model to calculate the hedge ratio of the target option. In this case, we compare He-GL with BS-GL. He-GL outperforms BS-GL only in the intermediate maturity roll-over case, irrespectively of the moneyness level. The out performance of this combination also appears in the OTM intermediate and long maturity options for both the roll-over and no-roll-over case. (b) The hedger uses the false model to calculate the hedge ratio of the volatility option and the true model to calculate the hedge ratio of the target option. In this case, we compare He-DO and He-W with He-GL. He-GL is superior to the He-W in almost all cases. On the other hand, the performance of the He-GL relative to He-DO depends on whether roll-over is performed. In the case of the no-roll-over strategy, He-DO performs best, while in the case of the roll-over strategy He-GL performs best. (c) The hedger uses the false model to calculate the hedge ratios of both the volatility and the target option. In this case, we compare BS-DO, and BS-W with He-GL. In the case of the no roll-over strategy, BS-DO, and BS-W perform better than He-GL. In the case of the roll-over strategy, He-GL performs, in general, better than BS-DO, and BS-W[9]

Overall, Table 1 shows that the combinations that use simpler models such as BS and Whaley's seem to perform equally well with the benchmark combination He-GL. This evidence is corroborated by the "confidence intervals tests" presented in Tables 3 and 4 where the differences in the hedging error across the various models appear to be statistically insignificant. These findings have an important implication: there is no need to resort to "complex" volatility option pricing models in order to hedge a target option with a volatility option. It suffices to use Whaley's model that is based on "simpler" assumptions. This result is analogous to the empirical findings of Scott (1991) and Backshi et *al*. (1997) who compared the delta-vega hedging performance of BS versus more complex models (stochastic volatility and jump models, respectively). In a broader study, Dumas et *al*. (1998) reached similar results and they state, "Simpler is better".

B. Model Error and the Pricing Performance

The effect of model error on the pricing performance of the various volatility option pricing models is examined by evaluating the pricing performance of the DO and Whaley models relative to the "true" GL model. The square root of the unconditional mean square pricing error is used as a measure of the pricing error (true GL price minus the price of each one of the other two models). This is calculated as follows.

For each model and for any given simulated volatility path, the square pricing errors are calculated at each rebalancing point. Next, the mean square pricing error is calculated across the rebalancing points. This exercise is repeated for each one of the 1,000 volatility

[9] Some exceptions occur in the case of the OTM target options.

simulation runs. Finally, the unconditional mean square pricing error is obtained as the average of the 1,000 mean square pricing errors. Three different correlation values between the stock price and volatility (ρ=-0.5, 0, +0.5) are used to perform the joint MC simulation. The pricing performance of each model is assessed for a volatility option with three separate moneyness levels (ITM, ATM, OTM), and three separate maturities (short, intermediate, long).

Table 5 shows the square root unconditional mean square pricing error of the DO and Whaley models for different maturities and moneyness levels across the three correlation values. We can see that the pricing performance of the Whaley model is superior to that of the DO. This holds regardless of the option moneyness/maturity, as well as of the correlation values. Despite the fact that the DO model relies on a volatility process that is closer to the "real" one, it performs worse than Whaley's model that is based on a seemingly more unrealistic process. This is in accordance with the results from the analysis of the hedging performance where we found that "the simpler is better".

(INSERT TABLE 5 HERE)

Focusing on the performance of each individual model, regarding the moneyness dimension, we can see that Whaley's model performs better for OTM options; the DO model performs equally worse for all moneyness levels. Regarding the maturity dimension, Whaley's model performs better for short maturity options. On the other hand, the DO model performs better for long maturity options.

VI. Conclusions

This paper makes two contributions to the rapidly evolving volatility options literature by addressing the following two questions: (a) Are volatility options superior to standard options in terms of hedging volatility risk?, and (b) Are the volatility option pricing models robust for hedging and pricing purposes in the presence of model risk?

To this end, a joint Monte Carlo simulation of the stock price and volatility in a stochastic volatility setup has been employed. First, two alternative dynamic delta-vega with discrete rebalancing hedging schemes were constructed to assess the hedging performance of plain vanilla options versus volatility options. A short standard European call is assumed to be the option to be hedged (target option). A standard European call and a European volatility call option are the alternative hedging instruments. Black Scholes (1973), Heston (1993), Whaley (1993), Grünbichler and Longstaff (1996), and Detemple and Osakwe (2000) models have been used to hedge the standard and volatility options. Then, the robustness of the hedging and pricing effectiveness of the volatility option pricing models in the presence of model error was investigated. The Grünbichler-Longstaff and Heston models were assumed to be the "true" models; this is consistent with the choice of the volatility data generating process used to simulate the data. Our two research questions have been examined for various expiry dates and strikes of the target option, as well as for alternative correlation values between the stock price and volatility, and for different rebalancing frequencies. Roll-over and no-roll over strategies in the volatility option were also considered.

In terms of the hedging effectiveness, we found that the hedging scheme that uses volatility options as hedging instruments is not superior to the one that uses standard options. In terms of the impact of model error to the hedging performance, combinations that use simpler models such as Black-Scholes and Whaley's seem to perform equally well with the benchmark combination (Grünbichler-Longstaff and Heston). Regarding the impact of model error to the pricing performance, Whaley's model performs better than Detemple-Osakwe's model.

This study has at least five important implications for the use of volatility options and their pricing models. First, volatility options are not better hedging vehicles than plain-vanilla options for the purposes of hedging standard options. This finding extends the conclusions in Jiang and Oomen (2001) who examined the hedging effectiveness of volatility futures versus plain-vanilla options; they found that the latter perform better than the former. However, this does not invalidate the imminent introduction of volatility options (and volatility futures) in various exchanges. Volatility options may be proved to be very useful for volatility trading and for hedging other types of options, e.g., exotic options. The liquidity and the transaction costs will be critical factors for the success of this emerging new market, as well. Second, in the case that an investor chooses the volatility options as hedging instruments, these should be used to hedge at-the-money and out-of-the-money rather than in-the-money target options. This feature may encourage the use of volatility options for hedging purposes given that most of the options trading activity is concentrated on at-the-money options. Third, the hedging performance of volatility options increases as their rebalancing frequency increases. Fourth, the roll-over strategy with volatility options should be preferred since it performs better than the no-roll-over strategy. Finally, despite the fact that Whaley's model is the worst mis-specified model within our simulation framework, it can be reliably used to hedge standard options with volatility options, and to price volatility options. This is in accordance with the results from previous studies in the model error (standard options) literature that found that increasing the complexity of the option pricing model does not necessarily improve it's pricing and hedging performance (see e.g., Backshi et *al*. 1997, Dumas et *al*. 1998).

The simulation setup presented creates three strands for future research. First, the ability of volatility options to hedge exotic options (e.g., barrier options) satisfactorily should be explored. Second, the hedging effectiveness of options written on alternative measures of volatility should be investigated. For example, Brenner et al. (2001) suggest the introduction of options on straddles. Finally, it may be worth studying the hedging effectiveness of volatility options and the impact of model error for alternative data generating processes (see e.g., Carr and Wu 2002, and Daouk and Guo 2004). In the interests of brevity, these extensions are best left for future research.

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TABLE 1

Unconditional Total Dollar Hedging Error

			Short Maturity		Intermediate Maturity				Long Maturity			
			no roll-over		***roll-over***		***no roll-over***		***roll-over***		***no roll-over***	
			Daily	Weekly	Daily	Weekly	Daily	Weekly	Daily	Weekly	Daily	Weekly
ITM	**HS1**	BS-BS	0.142	0.141	0.249	0.290	0.249	0.290	0.504	0.544	0.504	0.544
		He-He	0.121	0.122	0.134	0.139	0.134	0.139	0.231	0.203	0.231	0.203
	HS2	BS-DO	0.143	0.144	0.310	0.357	0.323	0.357	1.273	1.349	1.316	1.368
		He-DO	0.145	0.144	0.332	0.380	0.334	0.366	1.400	1.428	1.380	1.403
		BS-GL	0.142	0.145	0.309	0.348	0.322	0.356	1.188	1.305	1.316	1.359
		He-GL	0.146	0.145	0.270	0.331	0.344	0.372	1.191	1.281	1.412	1.421
		BS-W	0.142	0.146	0.309	0.346	0.320	0.356	1.180	1.298	1.325	1.348
		He-W	0.149	0.151	0.278	0.334	0.361	0.382	1.255	1.314	1.456	1.453
ATM	**HS1**	BS-BS	0.487	0.561	0.777	0.789	0.777	0.789	0.696	0.685	0.696	0.685
		He-He	0.187	0.225	0.170	0.214	0.170	0.214	0.140	0.133	0.140	0.133
	HS2	BS-DO	0.734	0.828	1.493	1.527	1.475	1.511	2.943	2.896	2.913	2.885
		He-DO	0.737	0.823	1.519	1.529	1.506	1.527	3.061	3.020	2.969	2.930
		BS-GL	0.738	0.831	1.394	1.451	1.490	1.523	2.689	2.690	2.960	2.923
		He-GL	0.752	0.822	1.276	1.354	1.551	1.564	2.711	2.735	3.062	3.023
		BS-W	0.741	0.834	1.416	1.465	1.514	1.542	2.730	2.705	3.022	2.985
		He-W	0.762	0.830	1.385	1.408	1.603	1.611	2.865	2.835	3.150	3.130
OTM	**HS1**	BS-BS	0.162	0.162	0.590	0.591	0.590	0.591	0.998	0.934	0.998	0.934
		He-He	0.132	0.135	0.134	0.135	0.134	0.135	0.142	0.140	0.142	0.140
	HS2	BS-DO	0.163	0.162	0.734	0.735	0.725	0.727	2.750	2.707	2.807	2.777
		He-DO	0.167	0.164	0.703	0.706	0.672	0.679	2.614	2.547	2.711	2.624
		BS-GL	0.163	0.162	0.718	0.719	0.727	0.727	2.612	2.585	2.831	2.802
		He-GL	0.168	0.164	0.643	0.643	0.690	0.692	2.350	2.316	2.761	2.677
		BS-W	0.163	0.162	0.719	0.720	0.731	0.731	2.665	2.616	2.856	2.834
		He-W	0.167	0.165	0.700	0.710	0.707	0.707	2.529	2.438	2.790	2.732

Unconditional Total Dollar Hedging Error (UTDHE) for Hedging Scheme 1 (HS1) and Hedging Scheme 2 (HS2): The results are reported separately for three different maturity dates and three different moneyness levels of the option to be hedged (target option). Two rebalancing frequencies (daily and weekly) have also been examined. The correlation ρ between the underlying index price and volatility is assumed to be $\rho = -0.5$. A roll-over and a no-roll-over in volatility options strategy have been considered. In the case of the short maturity target option, both the target and the volatility option have the same time-to-maturity and therefore no-roll-over is performed.

TABLE 2

The vega standardized Unconditional Total Dollar Hedging Error

			Short Maturity		Intermediate Maturity				Long Maturity			
			no roll-over		**roll-over**		**no roll-over**		**roll-over**		**no roll-over**	
			Daily	Weekly	Daily	Weekly	Daily	Weekly	Daily	Weekly	Daily	Weekly
ITM	HS1	BS-BS	41.224	40.992	1.614	1.875	1.614	1.875	0.434	0.469	0.434	0.469
		He-He	35.186	35.418	0.868	0.900	0.868	0.900	0.199	0.175	0.199	0.175
	HS2	BS-DO	41.369	41.863	2.008	2.311	2.094	2.310	1.098	1.164	1.135	1.180
		He-DO	41.950	41.892	2.150	2.458	2.162	2.368	1.208	1.232	1.190	1.211
		BS-GL	41.340	42.095	1.998	2.254	2.085	2.307	1.025	1.126	1.135	1.172
		He-GL	42.385	42.153	1.747	2.142	2.228	2.409	1.028	1.106	1.218	1.226
		BS-W	41.253	42.501	1.999	2.240	2.069	2.302	1.018	1.120	1.143	1.163
		He-W	43.198	43.721	1.796	2.162	2.336	2.470	1.083	1.134	1.256	1.254
ATM	HS1	BS-BS	0.045	0.052	0.053	0.054	0.053	0.054	0.036	0.036	0.036	0.036
		He-He	0.017	0.021	0.012	0.015	0.012	0.015	0.007	0.007	0.007	0.007
	HS2	BS-DO	0.068	0.077	0.102	0.104	0.100	0.103	0.154	0.151	0.152	0.151
		He-DO	0.068	0.076	0.103	0.104	0.103	0.104	0.160	0.158	0.155	0.153
		BS-GL	0.068	0.077	0.095	0.099	0.102	0.104	0.140	0.140	0.154	0.153
		He-GL	0.070	0.076	0.087	0.092	0.106	0.107	0.141	0.143	0.160	0.158
		BS-W	0.069	0.077	0.096	0.100	0.103	0.105	0.143	0.141	0.158	0.156
		He-W	0.071	0.077	0.094	0.096	0.109	0.110	0.150	0.148	0.164	0.163
OTM	HS1	BS-BS	1.763	1.758	0.270	0.271	0.270	0.271	0.084	0.078	0.084	0.078
		He-He	1.436	1.467	0.061	0.062	0.061	0.062	0.012	0.012	0.012	0.012
	HS2	BS-DO	1.772	1.767	0.336	0.337	0.332	0.333	0.231	0.227	0.236	0.233
		He-DO	1.822	1.783	0.322	0.324	0.308	0.311	0.219	0.214	0.227	0.220
		BS-GL	1.772	1.767	0.329	0.329	0.333	0.333	0.219	0.217	0.238	0.235
		He-GL	1.824	1.788	0.294	0.294	0.316	0.317	0.197	0.194	0.232	0.225
		BS-W	1.771	1.766	0.329	0.330	0.335	0.335	0.224	0.219	0.240	0.238
		He-W	1.822	1.795	0.321	0.325	0.324	0.324	0.212	0.205	0.234	0.229

The vega standardized Unconditional Total Dollar Hedging Error (UTDHE) for Hedging Scheme 1 (HS1) and Hedging Scheme 2 (HS2): The roll-over strategy case. The vega of the target option has been calculated using the Black-Scholes (1973) model. The results are reported separately for three different maturity dates and three different moneyness levels of the option to be hedged (target option). Two rebalancing frequencies (daily and weekly) have also been examined. The correlation ρ between the underlying index price and volatility is assumed to be $\rho = -0.5$. A roll-over and a no-roll-over in volatility options strategy have been considered. In the case of the short maturity target option, both the target and the volatility option have the same time-to-maturity and therefore no-roll-over is performed.

TABLE 3

Unconditional Total Dollar Hedging Error and the corresponding confidence intervals

			Short Maturity	Intermediate Maturity		Long Maturity	
			no roll-over	*roll-over*	*no roll-over*	*roll-over*	*no roll-over*
ITM	**HS1**	BS-BS	0.142 [0.020 , 0.256]	0.249 [0.030 , 0.513]	0.249 [0.030 , 0.513]	0.504 [0.088 , 0.955]	0.504 [0.088 , 0.955]
		He-He	0.121 [0.009 , 0.236]	0.134 [0.017 , 0.272]	0.134 [0.017 , 0.272]	0.231 [0.014 , 0.652]	0.231 [0.014 , 0.652]
	HS2	BS-DO	0.143 [0.019 , 0.257]	0.310 [0.073 , 0.568]	0.323 [0.077 , 0.577]	1.273 [0.663 , 1.681]	1.316 [0.703 , 1.692]
		He-DO	0.145 [0.021 , 0.259]	0.332 [0.023 , 0.717]	0.334 [0.023 , 0.638]	1.400 [0.709 , 2.435]	1.380 [0.703 , 2.273]
		BS-GL	0.142 [0.020 , 0.257]	0.309 [0.063 , 0.566]	0.322 [0.074 , 0.577]	1.188 [0.458 , 1.667]	1.316 [0.763 , 1.689]
		He-GL	0.146 [0.018 , 0.262]	0.270 [0.015 , 0.611]	0.344 [0.033 , 0.660]	1.191 [0.508 , 2.019]	1.412 [0.752 , 2.347]
		BS-W	0.142 [0.021 , 0.257]	0.309 [0.072 , 0.567]	0.320 [0.065 , 0.576]	1.180 [0.434 , 1.656]	1.325 [0.869 , 1.686]
		He-W	0.149 [0.019 , 0.267]	0.278 [0.015 , 0.676]	0.361 [0.022 , 0.777]	1.255 [0.512 , 2.183]	1.456 [0.774 , 2.564]
ATM	**HS1**	BS-BS	0.487 [0.076 , 0.879]	0.777 [0.345 , 1.346]	0.777 [0.345 , 1.346]	0.696 [0.219 , 1.147]	0.696 [0.219 , 1.147]
		He-He	0.187 [0.022 , 0.417]	0.170 [0.012 , 0.379]	0.170 [0.012 , 0.379]	0.140 [0.013 , 0.321]	0.140 [0.013 , 0.321]
	HS2	BS-DO	0.734 [0.186 , 1.208]	1.493 [0.689 , 2.126]	1.475 [0.826 , 2.149]	2.943 [1.815 , 3.779]	2.913 [1.735 , 3.808]
		He-DO	0.737 [0.163 , 1.319]	1.519 [0.552 , 2.351]	1.506 [0.756 , 2.298]	3.061 [1.532 , 4.247]	2.969 [1.861 , 4.177]
		BS-GL	0.738 [0.207 , 1.203]	1.394 [0.528 , 2.065]	1.490 [0.877 , 2.146]	2.689 [1.399 , 3.737]	2.960 [2.012 , 3.756]
		He-GL	0.752 [0.203 , 1.313]	1.276 [0.292 , 2.140]	1.551 [0.830 , 2.295]	2.711 [1.418 , 3.892]	3.062 [2.074 , 4.145]
		BS-W	0.741 [0.196 , 1.195]	1.416 [0.652 , 2.045]	1.514 [0.999 , 2.119]	2.730 [1.467 , 3.737]	3.022 [2.337 , 3.709]
		He-W	0.762 [0.201 , 1.292]	1.385 [0.444 , 2.243]	1.603 [1.004 , 2.314]	2.865 [1.525 , 4.174]	3.150 [2.112 , 4.070]
OTM	**HS1**	BS-BS	0.162 [0.015 , 0.314]	0.590 [0.300 , 0.914]	0.590 [0.300 , 0.914]	0.998 [0.318 , 1.714]	0.998 [0.318 , 1.714]
		He-He	0.132 [0.021 , 0.248]	0.134 [0.013 , 0.257]	0.134 [0.013 , 0.257]	0.142 [0.017 , 0.302]	0.142 [0.017 , 0.302]
	HS2	BS-DO	0.163 [0.013 , 0.310]	0.734 [0.461 , 1.035]	0.725 [0.468 , 1.018]	2.750 [1.932 , 3.535]	2.807 [1.983 , 3.448]
		He-DO	0.167 [0.017 , 0.335]	0.703 [0.119 , 1.483]	0.672 [0.120 , 1.466]	2.614 [0.946 , 4.482]	2.711 [1.190 , 4.562]
		BS-GL	0.163 [0.013 , 0.310]	0.718 [0.453 , 1.016]	0.727 [0.478 , 1.016]	2.612 [1.629 , 3.418]	2.831 [2.096 , 3.368]
		He-GL	0.168 [0.015 , 0.334]	0.643 [0.147 , 1.370]	0.690 [0.108 , 1.440]	2.350 [0.693 , 4.157]	2.761 [1.251 , 4.385]
		BS-W	0.163 [0.013 , 0.310]	0.719 [0.462 , 1.014]	0.731 [0.489 , 1.011]	2.665 [1.759 , 3.529]	2.856 [2.266 , 3.347]
		He-W	0.167 [0.018 , 0.327]	0.700 [0.185 , 1.421]	0.707 [0.180 , 1.441]	2.529 [0.824 , 4.508]	2.790 [1.357 , 4.280]

Unconditional Total Dollar Hedging Error (UTDHE) and the corresponding lower and upper confidence interval bounds (within brackets) for Hedging Scheme 1 (HS1) and Hedging Scheme 2 (HS2): Roll-over and no roll-over strategies with **daily** rebalancing. The results are reported separately for three different maturity dates and three different moneyness levels of the option to be hedged (target option). The correlation ρ between the underlying index price and volatility is assumed to be ρ=*-0.5*. In the case of the short maturity target option, both the target and the volatility option have the same time-to-maturity and therefore no-roll-over is performed.

TABLE 4

Unconditional Total Dollar Hedging Error and the corresponding confidence intervals

			Short Maturity	Intermediate Maturity		Long Maturity	
			no roll-over	*roll-over*	*no roll-over*	*roll-over*	*no roll-over*
ITM	HS1	BS-BS	0.141 [0.021 , 0.258]	0.290 [0.028 , 0.538]	0.290 [0.028 , 0.538]	0.544 [0.105 , 0.941]	0.544 [0.105 , 0.941]
		He-He	0.122 [0.040 , 0.195]	0.139 [0.021 , 0.289]	0.139 [0.021 , 0.289]	0.203 [0.090 , 0.356]	0.203 [0.090 , 0.356]
	HS2	BS-DO	0.144 [0.019 , 0.259]	0.357 [0.100 , 0.583]	0.357 [0.081 , 0.582]	1.349 [0.668 , 1.715]	1.368 [0.798 , 1.733]
		He-DO	0.144 [0.020 , 0.262]	0.380 [0.028 , 0.895]	0.366 [0.051 , 0.693]	1.428 [0.572 , 2.443]	1.403 [0.652 , 2.546]
		BS-GL	0.145 [0.019 , 0.259]	0.348 [0.083 , 0.579]	0.356 [0.084 , 0.582]	1.305 [0.598 , 1.698]	1.359 [0.716 , 1.732]
		He-GL	0.145 [0.020 , 0.265]	0.331 [0.019 , 0.812]	0.372 [0.042 , 0.755]	1.281 [0.416 , 2.153]	1.421 [0.625 , 2.554]
		BS-W	0.146 [0.019 , 0.256]	0.346 [0.082 , 0.578]	0.356 [0.095 , 0.582]	1.298 [0.556 , 1.699]	1.348 [0.769 , 1.706]
		He-W	0.151 [0.018 , 0.274]	0.334 [0.016 , 0.834]	0.382 [0.039 , 0.789]	1.314 [0.467 , 2.204]	1.453 [0.673 , 2.760]
ATM	HS1	BS-BS	0.561 [0.071 , 1.244]	0.789 [0.175 , 1.515]	0.789 [0.175 , 1.515]	0.685 [0.075 , 1.237]	0.685 [0.075 , 1.237]
		He-He	0.225 [0.054 , 0.540]	0.214 [0.033 , 0.533]	0.214 [0.033 , 0.533]	0.133 [0.009 , 0.309]	0.133 [0.009 , 0.309]
	HS2	BS-DO	0.828 [0.068 , 1.589]	1.527 [0.345 , 2.592]	1.511 [0.410 , 2.398]	2.896 [0.956 , 4.181]	2.885 [0.910 , 4.045]
		He-DO	0.823 [0.083 , 1.647]	1.529 [0.218 , 2.737]	1.527 [0.351 , 2.609]	3.020 [1.184 , 4.656]	2.930 [0.986 , 4.398]
		BS-GL	0.831 [0.097 , 1.601]	1.451 [0.322 , 2.481]	1.523 [0.378 , 2.378]	2.690 [0.633 , 4.029]	2.923 [1.213 , 4.017]
		He-GL	0.822 [0.068 , 1.531]	1.354 [0.143 , 2.493]	1.564 [0.454 , 2.592]	2.735 [1.001 , 4.280]	3.023 [1.453 , 4.324]
		BS-W	0.834 [0.103 , 1.601]	1.465 [0.361 , 2.503]	1.542 [0.513 , 2.435]	2.705 [0.550 , 4.267]	2.985 [1.493 , 3.956]
		He-W	0.830 [0.116 , 0.310]	1.408 [0.129 , 2.539]	1.611 [0.588 , 2.602]	2.835 [0.776 , 4.427]	3.130 [1.907 , 4.337]
OTM	HS1	BS-BS	0.162 [0.016 , 0.308]	0.591 [0.247 , 0.905]	0.591 [0.247 , 0.905]	0.934 [0.258 , 1.649]	0.934 [0.258 , 1.649]
		He-He	0.135 [0.012 , 0.312]	0.135 [0.030 , 0.300]	0.135 [0.030 , 0.300]	0.140 [0.009 , 0.294]	0.140 [0.009 , 0.294]
	HS2	BS-DO	0.162 [0.015 , 0.313]	0.735 [0.370 , 1.038]	0.727 [0.428 , 1.026]	2.707 [1.563 , 3.541]	2.777 [2.075 , 4.061]
		He-DO	0.164 [0.012 , 0.344]	0.706 [0.069 , 1.467]	0.679 [0.050 , 1.379]	2.547 [0.738 , 4.937]	2.624 [0.862 , 4.737]
		BS-GL	0.162 [0.015 , 0.313]	0.719 [0.368 , 1.028]	0.727 [0.439 , 1.027]	2.585 [1.328 , 3.518]	2.802 [1.927 , 3.614]
		He-GL	0.164 [0.005 , 0.057]	0.643 [0.062 , 1.338]	0.692 [0.066 , 1.395]	2.316 [0.413 , 4.964]	2.677 [1.056 , 4.641]
		BS-W	0.162 [0.015 , 0.313]	0.720 [0.341 , 1.022]	0.731 [0.468 , 1.025]	2.616 [1.450 , 3.602]	2.834 [2.001 , 3.531]
		He-W	0.165 [0.007 , 0.055]	0.710 [0.095 , 1.432]	0.707 [0.120 , 1.398]	2.529 [0.391 , 5.143]	2.790 [1.208 , 4.590]

Unconditional Total Dollar Hedging Error (UTDHE) and the corresponding lower and upper confidence interval bounds (within brackets) for Hedging Scheme 1 (HS1) and Hedging Scheme 2 (HS2): Roll-over and no roll-over strategies with **weekly** rebalancing. The results are reported separately for three different maturity dates and three different moneyness levels of the option to be hedged (target option). The correlation ρ between the underlying index price and volatility is assumed to be ρ=*-0.5*. In the case of the short maturity target option, both the target and the volatility option have the same time-to-maturity and therefore no-roll-over is performed.

TABLE 5

Volatility Option Pricing Models: Square Root Unconditional Mean Squared Pricing Errors

		Short Maturity			Intermediate Maturity			Long Maturity		
		ρ= -0.5	**ρ = 0**	**ρ = +0.5**	**ρ = -0.5**	**ρ = 0**	**ρ = +0.5**	**ρ = -0.5**	**ρ = 0**	**ρ = +0.5**
ITM	DO	0.3842	0.3842	0.3843	0.2683	0.2686	0.2680	0.1863	0.1866	0.1868
	W	0.0080	0.0080	0.0080	0.0124	0.0128	0.0125	0.0151	0.0149	0.0154
ATM	DO	0.3873	0.3872	0.3874	0.2709	0.2707	0.2708	0.1874	0.1877	0.1887
	W	0.0075	0.0074	0.0074	0.0114	0.0114	0.0114	0.0110	0.0111	0.0114
OTM	DO	0.3890	0.3890	0.3891	0.2726	0.2730	0.2727	0.1887	0.1886	0.1886
	W	0.0043	0.0043	0.0042	0.0078	0.0079	0.0078	0.0065	0.0066	0.0064

The Effect of Model Error on the Pricing Performance of the Volatility Option Pricing Models: Square Root Unconditional Mean Squared Pricing Errors of Detemple-Osakwe (DO, 2000) and Whaley (W, 1993) models. The results are reported separately for three different maturity dates, three different moneyness levels of the volatility option to be priced, and three different values of correlation between the underlying index price and volatility. $\frac{dS_t}{S_t} = rdt + V_t dW_{1,t}$, and $dV_t = \lambda\left(\mu - V_t\right)dt + \sigma\sqrt{V_t}dW_{2,t}$ are assumed to be the data generating processes for the stock price and the volatility, respectively, where $Cor(dW_{1,t}, dW_{2,t}) = \rho$.