Smoothing, Nonsynchronous Appraisal and Cross-Sectional Aggregation in Real Estate Price Indices

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Abstract

In this paper three econometric issues related to private-equity return indices, such as real estate indices, are explored (smoothing, nonsynchronous appraisal, and cross-sectional aggregation). Under certain assumptions, it is found that index returns based on appraisals follow an ARFIMA(0,d,1) process, where the long memory parameter (d) explains the level of smoothing and the MA parameter explains the nonsynchronous appraisal problem. The empirical results show that: 1) the level of smoothing in appraisal based real estate indices is far less than assumed in many academic studies; 2) nonsynchronous appraisal problem exists and becomes a more serious problem for higher frequency returns; and, 3) the level of volatility of real estate securities is higher than that recovered from an appraisal based index by around 50 percent. We interpret this difference as the level of noise in stock markets.

Key Words: Smoothing, Nonsynchronous Appraisal, Long Memory

JEL Codes: C4, G12.

1 Introduction

The reduction in global interest rates and the poor performance of stock markets in recent years has seen investors turn to less liquid assets in order to maintain yield or support carry trades. Attention has increasingly focused on asset classes such as commercial real estate, emerging market bonds and hedge funds. However one difficulty for investors in these markets is that readily observable measures of performance based on transactions are not generally available. As an example, indices based on infrequent estimates of market price (appraisals) are widely used to measure performance in the commercial real estate sector in contrast to the equity or bond markets, where minute by minute transactions data can be used to compile real-time performance measures. When current transaction prices for an asset can not be readily observed it is necessary to infer market values from the evidence available. In many developed markets there are established procedures for appraising or valuing assets from limited transaction information.

A number of well known problems exist with use of these appraisal or valuation based indices, perhaps the most well known problem is that of smoothing. Smoothing helps explain why such indices exhibit an extreme downward bias in the volatility and thus movements in the series appear excessively flat or 'smooth'. This topic has been extensively investigated in many studies in real estate finance (*inter alia* Barkham and Geltner 1994, 1995; Geltner, MacGregor and Schwann, 2003; Cho, Kawaguchi and Shilling, 2003; Bond and Hwang, 2003). However, to our knowledge none of the previous studies in this area explain how the smoothing level found in an appraisal-based index is related to the smoothing levels of individual assets. Moreover, the variability of appraisal time points of an individual asset and how this may distort the underlying return process has not yet been investigated.

In this paper we investigate three econometric problems associated with appraisal-based performance indices. In addition to the problem of smoothing discussed above, we also identify nonsynchronous appraisal and cross-sectional aggregation as potential problems for appraisal-based indices. Our approach has application in many areas of finance as it may help to understand other non-transaction based indices such as hedge fund, consumer confidence, business survey, or market sentiment indices. All of these indices suffer similar econometric problems to those investigated in this paper though the effects of each of these issues may differ.

The first issue we explore is the smoothing problem associated with the valuation of individual assets which form an appraisal-based index. According to the efficient market hypothesis, decision makers fully update asset prices at the arrival of new information. However, as argued in behavioral finance (see for instance Mullainathan and Thaler, 2001), the bounded rationality of human beings could deter a full updating of the market price.[1] For example when an appraisal-based real estate index is compared with its equivalent securitized (public) price index, the former is noticeably smoother than the latter with much lower volatility.[2]

The second issue we discuss is the nonsynchronous appraisal on individual assets. In financial markets many assets trade infrequently, with few assets trading in such a way that their price processes can be observed continuously. For example, a price reported for a given calendar time interval such as day, week or month may not reflect the true underlying price; the reported price may be the price traded before the reporting time. In this we argue a

[1] Smoothing is closely related with underreaction in behavioural finance. In finance evidence of underreactions has been reported in many studies such as stock prices (Jegadeesh and Titman, 1993), divesting firms (Cusatis et al. 1993), stock splits (Desai and Jain, 1997), open-market share repurchases (Ikenberry et al. 1995).

[2] Some examples of appraisal-based real estate indices are the National Council of Real Estate Investment Fiduciaries (NCREIF) index in the US or the Investment Property Databank (IPD) index in the UK.

similar process may occur with the timing and recording of market appraisals. This problem of nonsynchronous trading (or appraisal) is prevalent in most financial and economic data, though the seriousness of the problem varies.

The third econometric problem we investigate is cross-sectional aggregation. An index is constructed by cross-sectionally aggregating its constituents. When individual asset returns do not follow a specific time series process, aggregation is a simple task. However, cross-sectionally aggregated processes in the presence of smoothing do not necessarily have the same characteristics as the constituent assets.

In this study we investigate the effects of these three econometric problems on modelling asset returns under the assumption that asset returns are log-normally distributed. Using a simple exponential smoothing process, we first show that asset returns follow an AR(1) process in the presence of smoothing. The smoothing creates positive autocorrelation and lowers the volatility of the returns. Our study also shows that when the true asset returns are infinitely divisible, the reported returns follows an MA(1) process in the presence of nonsynchronous appraisal. It is found that the MA parameter is negative and the returns have a larger variance than the true returns. The magnitudes of the negative MA parameter and the variance of the returns increase with the Sharpe ratio and with the variability of nonsynchronous appraisals. When the nonsynchronous appraisal process is modelled by a negative exponential distribution, these results become more clear. The combined effects of nonsynchronous appraisals and smoothing appear as an ARMA(1,1) process, where the AR parameter explains the level of smoothing and the MA parameter explains the level of nonsynchronous appraisal. Finally, we show that an appraisal-based index, which is obtained by cross-sectionally aggregating individual assets, is far more persistent than the average smoothing level of individual assets. Analytically appraisal-based indices can be modelled by an ARFIMA(0,d,1) process under the assumption that individual AR parameters follow the Beta distribution.[3] The long memory parameter d approximates the average level of individual AR parameters better than conventional short memory processes such as an AR(1) or ARMA(1,1) process.

Using the results, we investigate appraisal-based and securitized real estate indices in the UK and US. We find that conventional methods to determine the level of smoothing in an index suggest a much higher level of smoothing than is the case. Estimates from an ARFIMA(0,d,1) model show that the smoothing levels of calculated for UK and US commercial real estate indices are 0.856 and 0.572 respectively. These estimates are much smaller than the values suggested by other previous studies. We also find evidence of nonsynchronous appraisal, which is more prominent in the monthly UK index than in the quarterly US index.

Finally, as an interesting outcome of the methods discussed in this paper we are able to compare the volatility of the same asset traded in both the public and private markets. In doing this we can obtain an estimate of the noise associated with stock markets. By comparing the volatility of returns calculated from the securitized real estate index with that from the adjusted appraisal-based index we find that the securitized index return volatility is higher than the adjusted appraisal-based volatility by around 50 percent. We interpret the difference between the two as an estimate of the noise in equity markets. In other words 30 to 35 percent of monthly and quarterly stock returns are noise. These numbers look significantly larger than French and Roll (1986) who suggest 4 to 12 percent of daily returns come from misspricing.

[3]Simulation results presented in the paper show that even if AR parameters are normally distributed, the ARFIMA model works well.

2 An Econometric Investigation of Appraisal-based Performance Measures

In this section we analytically investigate three econometric issues related to appraisal-based indices; these are (1) smoothing, (2) discrepancies between the time an appraisal is conducted and when the information is recorded (that is nonsynchronous appraisal), and (3) the process by which appraisals of the individual assets are cross-sectionally aggregated to form an index. For the data generating process of the individual asset returns we use a widely accepted model in finance; a mean plus noise process;

$$r_{it} = \mu_i + \sigma_i \varepsilon_{it} \tag{1}$$

where r_{it} is the log-return of asset i at time t, $\varepsilon_{it} \overset{iid}{\sim} N(0,1)$, and μ_i and σ_i are the expected return and standard deviation of the log-returns per unit time respectively. The log-normality is equivalent to a geometric Brownian motion in continuous time, which we need to examine the nonsynchronous appraisal problem.

A The Effects of Smoothing on Asset Returns

There is substantial empirical evidence of momentum in finance. Momentum occurs because of investors' underreaction to information; in other words, a degree of smoothing takes place in the formation of an estimate of value. The problem of smoothing is a particularly important issue when widely used data are based on non-transaction price estimates such as appraisals. For example, in real estate markets where appraisal-based indices play an important role, many different models have been proposed to reverse appraisers' smoothing behavior in order to recover market prices (see for example, Geltner, MacGregor and Schwann, 2003 for a survey on appraisal smoothing).

When appraisers underreact to information, they combine past and current information during the process of extracting price signals from noise, and tend to lag the true innovations. A simple but widely used smoothing model assumes that past information affects current price with an exponentially decreasing weight. When smoothing is present, the innovation at time t, ε_{it}, is not fully reflected at time t, but over time with an exponential rate. When the rate is ϕ_{si}, the smoothed return process for asset i, r_{sit}, is

$$r_{sit} = \mu_i + (1 - \phi_{si})\sigma_i \varepsilon_{sit}, \tag{2}$$

where

$$\varepsilon_{sit} = \phi_{si}\varepsilon_{sit-1} + \varepsilon_{it}.$$

In this model, ϕ_{si} is an AR parameter for the level of smoothing, where $0 \leq \phi_{si} < 1$. Note that $1 - \phi_{si}$ in (2) is necessary to make the sum of the weights on past innovations one so that asset returns do not under or over reflect the innovations in the long run. The smoothed process in (2) can be written as

$$\begin{aligned} r_{sit} - \mu_i &= \phi_{si}(r_{sit-1} - \mu_i) + \sigma_{si}\varepsilon_{it} \qquad (3) \\ &= \phi_{si}(r_{sit-1} - \mu_i) + (1 - \phi_{si})(r_{it} - \mu_i), \end{aligned}$$

where $\sigma_{si} = (1 - \phi_{si})\sigma_i$. When $\phi_{si} = 0$, there is no smoothing and the return process in (3) is the same as the data generating process in equation (1). On the other hand, as ϕ_{si} becomes larger, the relative weight on the current information (ε_{it}) decreases and the past information (ε_{it-1}, ε_{it-2}, ...) becomes more important in the return process.

The variance and autocorrelation of the smoothed return process are

$$\begin{aligned} Var(r_{sit}) &= \frac{(1-\phi_{si})}{1+\phi_{si}}\sigma_i^2 \\ Cor(r_{sit}, r_{sit-\tau}) &= \phi_{si}^{\tau} \text{ for } \tau = 1, 2, ... \end{aligned} \quad (4)$$

The variance of smoothed returns decreases by $\frac{1-\phi_{si}}{1+\phi_{si}}$ times and thus is less volatile than the true process; i.e., $Var(r_{sit}) < Var(r_{it})$ for $0 \leq \phi_{si} < 1$. However, the expected return (μ_i) remains unchanged by the smoothing procedure.

When there are two asset returns for the same underlying asset, one smoothed and the other unsmoothed, then the level of smoothing can be obtained by equating the smoothed asset's return volatility to the unsmoothed asset's return volatility, or simply comparing the AR parameters of the two asset returns. However, it is not easy to find asset returns free from smoothing but with the same underlying asset. Returns from securitized markets could be used as unsmoothed return series, but it is not clear if securitized returns can provide us with the true volatility since stock markets are well documented to be noisy. In addition, as will be explained later, the AR parameter from index returns whose constituents are smoothed should not be used to find an appropriate smoothing level of the constituents since there is a significant difference between the smoothing level of the index and the smoothing level of its constituents.

Although the level of smoothing, ϕ_{si}, is an empirical matter, some studies such as Chatfield (1978) and Makridakis et al. (1982) suggest a range of 0.3 to 1 as a plausible candidate in general. In real estate markets, Brown and Matysiak (1998) show that for 30 randomly-selected individual properties among the Investment Property Databank (IPD) index constituents, the values of ϕ_{si} range from 0.08 to 0.95.[4]

B The Effects of Nonsynchronous Appraisal on Asset Returns

The next econometric issue we discuss is the nonsynchronous appraisal problem of individual assets. When an appraiser evaluates an asset whose returns follow the mean plus noise model in (1), but the appraisal time points change over time, we face a problem that the appraisal-based price does not reflect the price at the time when it is reported. Nonsynchronous appraisal is different from smoothing as the former is about the time difference between appraisal and reporting, while the latter is about how much new information is reflected in the price when it is updated.

There are two sources of nonsynchronous appraisal. The first one arises when assets are not valued at the specific time required. For example, a consumer confidence index or market sentiment index is not constructed by individual evaluations which are carried out at the same time. Another example is in the construction of a commercial real estate index, such at the UK IPD monthly index, which states in their index rules that "the values recorded and included in the computation [of the index] will be as at the end of each month - or no more than ten working days prior to that date".[5] This implies that the reported estimates of price at the end of each month may be the estimates made up to ten working days previously.

The second source, which could be more significant, comes from the way appraisers use information. For example, let us consider the interest rate, an important factor in determining the discount rate which is used to estimate a market value of an asset based on its cash

[4]Brown and Matysiak (1998) assume that the smoothing parameter changes over time. This range comes from 23 time-varying and 7 constant smoothing parameters reported in their study. The 'smoothing parameter' used in other real estate studies is equivalent to $1 - \phi_{si}$.

[5]See "UK Monthly index - Rules for Construction, Computation and Review" of Investment Property Databank Ltd. We expect similar practice for the National Council of Real Estate Investment Fiduciaries (NCREIF), which is a US equivalent appraisal based index to the UK IPD index.

flows. Unless a significant change in interest rates is observed, appraisers might treat small daily changes in the interest rate as uninformative or simply noise. This heuristic approach could create a large difference between the time points when asset prices are reported and when the information appraisers use becomes available. The latter may be far earlier than the former, and thus reported appraisals may reflect past information well before the time the appraisal is official reported. This could be a particularly significant problem with the appraisal of commercial real estate values as often only one appraiser (or firm of appraisers) provides a price estimate and as this is done without knowing other appraisers price estimates. It is hard to expect that information is fully updated as in securitized markets where prices are determined by the interplay of a large number of market participants.

The discrepancy between the appraisal time point and the time point when the asset should be appraised (we call this 'reporting time' from now on, which is usually the end of the unit time) creates the econometric problem of 'errors in variables'. The effects of the nonsynchronous trading (or appraisal) problem have been studied since its importance was first recognized by Fisher (1966). Many studies have investigated the effects on asset pricing (systematic risk) (see for example, Scholes and Williams, 1977; and Dimson, 1979). Recent studies such as Lo and Mackinlay (1990) investigate the effects of nonsynchronous trading on spurious autocorrelations.[6] Most of these studies investigate nonsynchronous trading in a multiperiod framework; i.e., infrequent trading over multiple periods.

To investigate the effects of the nonsynchronous appraisal problem, we follow the method of Scholes and Williams (1977). Let a nonsynchronous variable, s_{it}, represent the time difference between appraisal and reporting such that $0 \leq s_{it} \leq 1$. This implicitly assumes

[6] See Chapter 3 of Campbell, Lo, and Mackinlay (1997) for a summary of studies on nonsynchronous trading.

that appraisers use new information after the last appraisal. When $s_{it} = 0\ \forall t$, there is no nonsynchronous appraisal problem for asset i, and thus the reported returns represent the true process in (1). On the other hand when $s_{it} = 1$, there is no price updating between the unit time interval $t-1$ and t, and the last price estimate is the information used to evaulate the current price. In our study s_{it} is not allowed to be larger than 1 because assets are appraised once in a unit time. Since s_{it} can take any value from 0 to 1, the nonsynchronous appraisal problem we investigate in our study requires underlying asset returns to be infinitely divisible (or a geometric Brownian motion).

Let r_{nit} be the discrete time returns calculated with reported prices at time t. To examine the effects of nonsynchronous appraisal on the return process, we derive the mean, variance, and autocovariance of reported returns, and then investigate what process they follow.

Theorem 1 *When asset returns follow a mean plus noise process (an infinitely divisible process), but are appraised with a non-negative nonsynchronous variable s_{it} which is iid and not correlated with ε_{it}, then the mean, variance, and autocovariance of the appraisal based returns, r_{nit}, are*

$$\begin{aligned} E[r_{nit}] &= E[1 - s_{it} + s_{it-1}]\mu_i \\ Var(r_{nit}) &= \sigma_i^2 E(1 - s_{it} + s_{it-1}) + \mu_i^2 Var(1 - s_{it} + s_{it-1}), \\ Cov(r_{nit}, r_{nit-\tau}) &= \begin{cases} \mu_i^2 Cov(1 - s_{it} + s_{it-1}, 1 - s_{it-1} + s_{it-2}) \text{ for } \tau = 1, \\ 0 \text{ for } \tau > 1. \end{cases} \end{aligned} \tag{5}$$

Proof. See the Appendix. ■

The result suggests that when the true process follows a random walk, the mean and variance of appraisal based returns are not the same as those of the true process because of the nonsynchronous appraisal problem (which depends on s_{it}). Only when s_{it} is constant, is the above statistic the same as those of the true process in (1). Even if there is a difference between appraisal and reporting times, as far as the difference is constant over time, the nonsynchronous problem does not create 'errors in variables'. Thus the variability of s_{it} matters, not the average value of s_{it}.

Theorem 1 shows that the return process in the presence of nonsynchronous appraisal is autocorrelated only with lag 1. Therefore the underlying process (random walk) becomes a moving average process with lag 1 (MA(1)). However without an appropriate assumption on the nonsynchronous variable, s_{it}, it is not easy (*delete* possible) to further analyze the effects of nonsynchronous appraisal on the asset return process.

a Modelling Nonsynchronous Appraisal

The statistics in Theorem 1 can be simplified with some additional assumptions on the nonsynchronous variable. In finance information arrival is usually assumed to follow the Poisson distribution (for example, Easley, Hvidkjaer, and O'Hara, 2002). Then the nonsynchronous variable, which measures time difference between any two (Poisson distributed) information arrivals, follows the negative exponential distribution.[7]

When the nonsynchronous variable s_{it} follows a negative exponential distribution with mean and variance $\lambda_i(= E(s_{it}))$ and λ_i^2 respectively, the probability density function of s_{it} is[8]

$$f(s_{it}) = \frac{1}{\lambda_i} \exp(-\frac{s_{it}}{\lambda_i}). \tag{6}$$

[7] In some cases the arrival time is modelled by the Weibull distribution which inlcudes the negative exponential distribution as a special case. See for instance Dufour and Engle (2000).

[8] Usually the negative exponential distribution function is expressed in terms of the number of appraisals during a unit time period. However, since we are interested in the time difference between appraisal and reporting, we use the distribution function as in equation (6).

Remark 2 *When the nonsynchronous variable s_{it} is iid negative exponentially distributed, the mean, variance, and autocovariance of the reported returns are*

$$
\begin{aligned}
E[r_{nit}] &= \mu_i \qquad (7)\\
Var(r_{nit}) &= \sigma_i^2 + 2\mu_i^2\lambda_i^2, \\
Cov(r_{nit}, r_{nit-\tau}) &= \begin{cases} -\mu_i^2\lambda_i^2 \ for\ \tau = 1, \\ 0 \ for\ \tau > 1. \end{cases}
\end{aligned}
$$

Proof. See the Appendix. ■

Although the expected return is the same as the true expected return, the variance and autocovariance of reported returns are not. This is similar to the results of Lo and Mackinlay (1990) in a multiperiod setting, though in our case the nonsynchronous variable is assumed to be less than or equal to one. The variance increases in the presence of the nonsynchronous appraisal because of the additional term $2\mu_i^2\lambda_i^2$. The more volatile the appraisal variables, the larger the variance of the reported returns. The negative autocovariance also becomes larger when the variability of s_{it} is larger.

The autocorrelation function of the reported returns is

$$
Cor(r_{nit}, r_{nit-\tau}) = \begin{cases} -\frac{1}{2+v_i^{-2}\lambda_i^{-2}} \text{ for } \tau = 1, \\ 0 \text{ for } \tau > 1, \end{cases}, \qquad (8)
$$

where v_i is the Sharpe ratio calculated for total returns rather than excess returns ($v_i = \frac{\mu_i}{\sigma_i}$). For a given positive Sharpe ratio we have a negative first order autocorrelation. When the variability of s_{it} is small, the autocorrelation will be close to zero while for a large variability of s_{it} the autocorrelation will approach -0.5, suggesting $-0.5 < Cor(r_{nit}, r_{nit-1}) < 0$.

Autocorrelations with lags larger than one, on the other hand, are all zero, a direct result of assuming that there is one appraisal in a unit of time.[9] Therefore, the reported returns follow a moving average (MA) process in discrete time even if the true asset price follows the mean plus noise process.

The MA(1) process which shows the variance and covariance structure in (7) is[10]

$$r_{nit} - \mu_i = \sigma_{ni}\varepsilon_{it} + \theta_i \sigma_{ni}\varepsilon_{it-1}, \tag{9}$$

where

$$\theta_i = -\frac{2v_i^2\lambda_i^2}{1 + 2v_i^2\lambda_i^2 + \sqrt{1 + 4v_i^2\lambda_i^2}}, \tag{10}$$

$$\sigma_{ni} = \sigma_i \sqrt{\frac{1}{2}\left[1 + 2v_i^2\lambda_i^2 + \sqrt{1 + 4v_i^2\lambda_i^2}\right]}, \tag{11}$$

and

$$Var(r_{nit}) = \sigma_i^2(1 + 2v_i^2\lambda_i^2). \tag{12}$$

The MA(1) process depends on two parameters, v_i^2 and λ_i^2. The importance of the nonsynchronous appraisal problem will be answered with empirical tests in the next section. However, it is interesting to see how the MA parameter and the volatility of the returns are affected by the volatility of the nonsynchronous variable (λ_i^2) as well as the Sharpe ratio (v_i^2).

We calculate the values of θ_i and the volatility ratios ($\sqrt{\frac{Var(r_{nit})}{\sigma_i^2}} = \sqrt{1 + 2v_i^2\lambda_i^2}$) in

[9]The result suggests that when we allow the nonsynchronous variable to be larger than 1, we need to consider higher lags for autocorrelation. Then we have similar results as those of Lo and Mackinlay (1990) where assets are allowed not to be traded (appraised) for multiperiod and thus the reported process becomes persistent over multiperiod.

[10]We only consider the invertible MA process.

equations (10) and (12) for various values of λ_i^2 and v_i^2. When the annual Sharpe ratio is less than one, the MA parameter and the volatility of the MA process increase little by the nonsynchronous appraisal. Only when a large Sharpe ratio is combined with a large volatility of the nonsynchronous variable, does the MA coefficient becomes large. In some cases where the standard deviation of the nonsynchronous variable is 0.7 (or $Var(s_{it}) = \lambda_i^2 = 0.49$) and the annual Sharpe ratio is 3, then we have $\theta_i = -0.222$ and the standard deviation of returns increases by 31.7 percent for the monthly case. The quarterly cases show that the effects of the nonsynchronous appraisal are even larger; $\theta_i = -0.399$ and the standard deviation of returns increases by 79 percent. The difference between the monthly and quarterly cases comes from the higher Sharpe ratios of the quarterly case. The details of these results are available from the authors upon request.

The effect of nonsynchronous appraisal on volatility is the opposite to that of smoothing. Smoothing reduces volatility and increases persistence while nonsynchronous appraisal increases volatility. However, the effects of nonsynchronous trading do not seem to be more serious than those of smoothing, since we do not expect λ_i^2 and v_i^2 to be excessively large.

C The Combined Effects of Smoothing and Nonsynchronous Appraisal on Asset Returns

The true mean plus noise process becomes an AR(1) process when there is smoothing or a MA(1) process when the nonsynchronous appraisal problem exists. In practice, these two effects are likely to exist together. To find out the form of the process in the presence of the two econometric problems, we need to note that 1) the exponential smoothing can be modelled by making error terms follow an AR process as in (2), and 2) nonsynchronous appraisal can be modelled by making the error terms follow a MA(1) process as in (9).

These results suggest that we have an ARMA(1,1) process because of the smoothing and nonsynchronous appraisal. When the error term in the MA(1) process in (9) follows an AR(1) process, we have

$$\begin{aligned} r_{cit} - \mu_i &= \sigma_{ni}(1+\theta_i L)(1-\phi_{si})\varepsilon_{sit}, \\ \varepsilon_{sit} &= \phi_{si}\varepsilon_{sit-1} + \varepsilon_{it}, \end{aligned}$$

where r_{cit} represents asset return at time t in the presence of the smoothing and nonsynchronous appraisal and L is the lag operator. This gives us

$$r_{cit} - \mu_i = \phi_{si}(r_{cit-1} - \mu_i) + \theta_i \sigma_{ci}\varepsilon_{it-1} + \sigma_{ci}\varepsilon_{it}, \tag{13}$$

where $\sigma_{ci} = (1-\phi_{si})\sigma_{\eta i}$ and $\varepsilon_{it} \overset{iid}{\sim} N(0,1)$ as in (1).

Note that the AR and MA parameters are the same as those in the smoothed process of (3) and in the nonsynchronously appraised process of (10) respectively. However, the volatility (σ_{ci}) of the error term in the smoothed and nonsynchronously appraised process is a function of both smoothing and nonsynchronous appraisal;

$$\sigma_{ci} = \sigma_i(1-\phi_{si})\sqrt{\frac{1}{2}\left[1 + 2v_i^2\lambda_i^2 + \sqrt{1+4v_i^2\lambda_i^2}\right]}, \tag{14}$$

which can be obtained using $\sigma_{\eta i}$ in (11). The variance of r_{cit} is

$$Var(r_{cit}) = \frac{1+\theta_i^2 + 2\theta_i\phi_{si}}{1-\phi_{si}^2}\sigma_{ci}^2, \tag{15}$$

and comparing the variance of true return process in (1) with that we have after the smoothing and nonsynchronous appraisal in (15) gives

$$\frac{Var(r_{cit})}{\sigma_i^2} = \frac{1}{2}\frac{(1+\theta_i^2+2\theta_i\phi_{si})(1-\phi_{si})}{1+\phi_{si}}\left[1+2v_i^2\lambda_i^2+\sqrt{1+4v_i^2\lambda_i^2}\right], \qquad (16)$$

which is a nonlinear function of $v_i^2\lambda_i^2$ (or θ_i) and ϕ_{si}.

Figure 1 shows the ratios of the two variances for various values of θ_i and ϕ_{si}. We choose $-0.301 \leq \theta_i \leq 0$ (or $0 \leq v_i^2\lambda_i^2 \leq 0.653$) and $0 \leq \phi_{si} \leq 0.9$. Figure 1 shows that in most cases $Var(r_{cit})$ becomes smaller than σ_i^2, in particular for $\theta_i = 0$ or large values of ϕ_{si}. The ratio is more sensitive to the level of smoothing than the nonsynchronous appraisal, suggesting that the main component that affects the volatility ratio is the smoothing level. When the level of smoothing is high, e.g., the AR parameter is 0.8 or 0.9, the ratio becomes less than 0.15. The ratio is not sensitive to different values of θ_i especially when ϕ_{si} is close to 1. However, when the MA parameter becomes large, i.e., $\theta_i \leq -0.15$, the nonsynchronous appraisal increases volatility significantly.

Another point that arises from the nonsynchronous appraisal is that when we calculate the smoothing level of appraisal-based property returns, we should not just consider the autocorrelation with lag 1. Because of a negative MA parameter, the autocorrelation with lag 1 is likely to be less than it should be. A proper way to calculate the persistence level is to consider higher lags.

In real estate, for the smoothing levels ($0.08 < \phi_{si} < 0.95$) suggested by Brown and Matysiak (1998) and the annual Sharpe ratio less than 3, the variance ratio ranges from 0.03 to 1.4. This suggests that the true volatility (standard deviation) is at least 0.85 ($= 1.4^{-1/2}$) times or up to 5.77 ($= 0.03^{-1/2}$) times of the volatility of the smoothed reported monthly returns.

D The Effects of Cross-sectional Aggregation of Smoothed and Nonsynchronously Appraised Individual Asset Returns

In this section we investigate what happens when N individual assets are cross-sectionally aggregated to construct an index. For instance, the IPD index consists of thousands of individual UK properties, each of which is appraised once at the end of a month. The investigation in this section also provides an analytical explanation for why the smoothing levels of individual properties are significantly lower than those of an index.

Our analysis so far showed that a simple mean plus noise model becomes an ARMA(1,1) process after smoothing and nonsynchronous appraisal. With N such assets in the market, an index is constructed by aggregating these N ARMA(1,1) processes. Applying the result of Lippi and Forni (1990), we have an ARMA(p, q) process for the index where $p \leq N$ and $q \leq N$.[11] However, it cannot be an realistic option to allow thousands of AR and MA components for index returns.

To investigate the effects of cross-sectional aggregation in detail, we decompose the error term in (1) into two components; an idiosyncratic error, η_{it}, and a market-wide common factor, ϵ_t. Let these two components be independent of each other and over time; $E(\eta_{it}\epsilon_t) = 0$, $\eta_{it} \sim iid(0, \sigma_\eta^2)$, and $\epsilon_t \sim iid(0, \sigma_\epsilon^2)$ such that

$$\begin{aligned} \sigma_i \varepsilon_{it} &= \beta_i \epsilon_t + \eta_{it}, \text{ and} \\ \sigma_i^2 &= \beta_i^2 \sigma_\epsilon^2 + \sigma_\eta^2, \end{aligned}$$

where β_i is the coefficient which represents asset i's sensitivity to the factor ϵ_t. The de-

[11] In general, Lippi and Forni (1990) show that when ARMA(m_1,n_1) and ARMA(m_2,n_2) are aggregated, we have ARMA(m,n) where $m \leq m_1 + m_2$ and $n \leq \max(m_1 + n_2, m_2 + n_1)$.

composition of the error term into an idiosyncratic error and a market-wide common factor can be better understood with linear factor models. An idiosyncratic error is asset specific whereas market-wide common factors include macroeconomic factors, regions or other persistent features. One factor is assumed in our study for simplicity. We do not need a specific assumption on β_i, but for any asset which reacts positively to any positive news, we expect $\beta_i > 0$ and thus assume $E[\beta_i] > 0$ without loss of generality. Then we have

$$r_{cit} - \mu_i = \phi_{si}(r_{cit-1} - \mu_i) + \theta_i \frac{\sigma_{ci}}{\sigma_i}(\beta_i \epsilon_{t-1} + \eta_{it-1}) + \frac{\sigma_{ci}}{\sigma_i}(\beta_i \epsilon_t + \eta_{it}). \tag{17}$$

The process obtained by cross-sectionally aggregating N ARMA(1,1) return processes is

$$r_{mt} - \mu_m = \sum_{\tau=0}^{\infty} E_c \left[\phi_{si}^{\tau}(\theta_i \epsilon^*_{it-1-\tau} + \epsilon^*_{it-\tau}) \right] \tag{18}$$

as $N \to \infty$, where

$$\begin{aligned} \epsilon^*_{it-\tau} &= \frac{\sigma_{ci}}{\sigma_i}\beta_i \epsilon_{t-\tau} \\ &= (1 - \phi_{si})\sqrt{\frac{1}{2}\left[1 + 2v_i^2\lambda_i^2 + \sqrt{1 + 4v_i^2\lambda_i^2}\right]}\beta_i \epsilon_{t-\tau}, \end{aligned}$$

and $E_c(.)$ represents cross-sectional expectation. See the Appendix for proof.

The smoothing level of an index and the average smoothing level of its constituents are not the same and Gourieroux and Monfort (1997) show the following relationship between them.

Remark 3 *For $0 < \phi_{si} < 1$ $\forall i$, the aggregated process, r_{mt}, is more smooth than the average smoothing level of the individual assets.*

Proof. See page 444, Gourieroux and Monfort (1997) for proof. ∎

Remark 2 shows that any survey or appraisal based index which is constructed by cross-sectionally aggregating individual evaluations which suffer smoothing problem could result in a higher level of smoothing than the average smoothing level suggests. This explains why there is a significant difference in the smoothing levels reported by Brown and Matysiak (1998) between the 30 individual properties and their portfolio returns.

Although Remark 2 shows that the aggregated series is more smooth than the average level of individual series, further analytical analysis is not possible without additional assumptions on ϕ_{si}. In the next subsection we use Monte Carlo simulations to show how much the smoothing level of the aggregated series is affected by cross-sectional aggregation.

One simple analytical solution for the process in (18) is to assume the Beta distribution for ϕ_{si}. Using the same method as in Granger (1980), when ϕ_{si} is generated by the Beta distribution, we have a long memory (fractionally integrated) process from the cross-sectional aggregation of AR(1) processes.[12] More generally, we have an autoregressive fractionally integrated moving average (ARFIMA) (0,d,q) process by aggregating ARMA(1,q) processes.

Theorem 4 *Consider the following ARMA(1,q) process of* x_{it}*;*

$$x_{it} = \psi_i x_{it-1} + \theta_{iq}\xi_{t-q} + \theta_{iq-1}\xi_{t-q+1} +, ..., +\xi_t, \;\; i = 1, 2, ..., N$$

where ψ_i *and* θ_{iq} *are independent for all* i *and* q*, and* $\xi_t \sim iid(0,1)$*. When* ψ_i *follows* $Beta(d, 1-d)$ *distribution with* $0 < \psi_i < 1$ $\forall i$*, the cross-sectionally aggregated process*

[12] Studies on cross-sectionally aggregated variables can be found in econometrics literature. Zaffaroni (2004) uses a more general semiparametric distribution to show how cross-sectional aggregation can lead to long memory. He also suggests some conditions in addition to Granger (1980), which should be satisfied for cross-sectionally aggregated AR process to follow a long memory process.

follows an ARFIMA(0,d,q) process;[13]

$$(1-L)^d E_c(x_{it}) = E_c(\theta_{iq})\xi_{t-q} + E_c(\theta_{iq-1})\xi_{t-q+1} +, ..., +\xi_t. \quad (19)$$

Proof. See Appendix for proof. ■

When we apply the above result to equation (18), we have the following ARFIMA(0,d,1) process;

$$(1-L)^d(r_{mt} - \mu_m) = \theta\epsilon^*_{t-1} + \epsilon^*_t, \quad (20)$$

where $\theta = E_c(\theta_i)$, $d = E_c(\phi_{si})$ and $\epsilon^*_t = E_c[\epsilon^*_{it}]$.

Therefore, with some further assumptions an index return process whose constituents suffer the smoothing and nonsynchronous appraisal problems follows a long memory process whose properties are summarized by an autocorrelation function with a hyperbolic decay rate. When the individual AR parameters follow the Beta distribution, we can directly estimate the average smoothing level and its variance from the ARFIMA(0,d,1) process since $d = E_c(\phi_{si})$ and $Var_c(\phi_{si}) = \frac{d(1-d)}{2}$. Therefore estimating d for an index return series is an alternative way of obtaining the average smoothing level of individual processes. In addition, in our setting the MA parameter still explains the average level of the nonsynchronous appraisals. Therefore we can investigate the effects of nonsynchronous appraisal separately from those of smoothing for any appraisal based index.

a Monte Carlo Simulations

We showed that when individual ARMA(1,1) processes with positive AR parameters are cross-sectionally aggregated, the aggregated process becomes far more persistent. In this

[13] To be more strict, ψ_i should be large with large variance for a long memory process. See Zaffaroni (2004).

section we use Monte Carlo simulations to examine the increase in persistence for an aggregated time series. In addition, we also evaluate whether or not long memory processes are appropriate for the aggregated process. In the simulations we compare the normal distribution with the Beta distribution for AR parameters. Alternative distributions could be used, but the normal distribution is by far the most common distribution.

We first generate 1000 series each of which follows a mean zero ARMA(1,1) process. AR parameters are assumed to follow either the normal or Beta distribution, while MA parameters are drawn from the normal distribution with standard deviation of 0.1.[14] We allow several different standard deviations for the AR parameters (σ_ϕ), i.e., 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, but to save space we only report the cases of 0.15 and 0.25. When AR parameters are assumed to be normal, some parameters are outside the range of $0 \leq \phi_{si} < 1$. Since smoothing implies a positive value of ϕ_{si} and Remark 2 and Theorem 2 hold when AR parameters lie between 0 and 1, we truncate the AR parameter values that are less than 0 to 0 and larger than 1 to 1.[15] Likewise positive MA parameters are truncated to 0. We generate 200 observations for each series, the number of which is similar to that of the UK monthly appraisal based index returns we use in the next section. These 1000 individual ARMA(1,1) series are then cross-sectionally aggregated to construct one single series which has 200 observations. Then we estimate ARMA(1,1) and ARFIMA(0,d,1) models for the aggregated series.[16] The generating and estimating procedure is repeated 1000 times and

[14] We also used different standard deviations for MA parameters, but the results are not significantly different from those in Table 1.

[15] When the average levels of AR parameters are close to 0 or 1, there are many cases that individual AR parameters lie outside the range between 0 and 1. Thus the average level of AR parameters after the truncation is slightly higher when the average level of AR parameters is close to 0 while it is slightly lower when the average level of AR parameters is close to 1.

[16] There are several issues in estimating ARFIMA models. When sample sizes are more than 150 and $d>0$, there is little difference between the time domain and frequency domain ML estimates. Both of these estimates perform well. See Cheung and Diebold (1994) for example. Thus to reduce calculation time we use frequency domain ML estimation as in Hwang (2000). For a recent study that compares various estimation methods for ARFIMA(0,d,1) models, see Nielsen and Frederiksen (2004). The purpose of our study is neither

the results are summarized in Table 1.

We first discuss the estimates of ARMA(1,1) models for the aggregated series. As expected when ARMA(1,1) models are used for the aggregated series, the AR estimates are significantly larger than the average level of individual AR parameters. For example when the average level of AR parameters is 0.595, the estimates of AR parameters for the aggregated series are over 0.95 with σ_ϕ=0.25 and 0.83 with σ_ϕ=0.15 when AR parameters are normally distributed, and 0.95 when they are Beta distributed. In general the cases of the Beta distribution show far more persistence in aggregated level. The second result is that estimated MA parameters are significantly downward biased. For example when $\theta_i \sim N(-0.1, 0.1^2)$ and $\phi_{si} \sim N(0.6, 0.25^2)$, the average value of $\widehat{\theta}_i$s is -0.395. These biases tend to decrease with the level of AR parameters, but they are still large for large AR parameters.

Therefore, the large significant upward tendency in the AR estimates and downward biases in the MA estimates clearly show the inadequate consequences of using misspecified ARMA(1,1) models for an aggregated series. The level of smoothing of individual assets estimated from an appraisal based index is significantly inflated with these short memory processes. However, when the dispersion of individual AR parameters (variance of ϕ_{si}s) is close to zero, the difference between the smoothing level measured with ARMA(1,1) models and the average smoothing level of individual AR parameters becomes smaller (not reported). The high level of smoothing of aggregated series comes from a large disperse of individual AR parameters, which confirms the results of Zaffaroni (2004).

We next estimate long memory models, ARFIMA(0,d,1), for the aggregated series. Table 1 shows that the upward tendency of $\widehat{d}$ of ARFIMA(0,d,1) models is not as large as that of

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to introduce unbiased estimator nor to compare different estimation methods, and thus we do not discuss about these econometric issues further in this study.

AR estimates of ARMA(1,1) models. In fact when ϕ_{si}s are small or the standard deviation of ϕ_{si}s is small, $\widehat{d}$s tend to be less than the average levels of ϕ_{si}s. However, the differences are relatively smaller than those of ARMA(1,1) models. On the other hand with ϕ_{si}s from the Beta distribution, $\widehat{d}$s of ARFIMA(0,d,1) models are significantly upward biased. Although our arguments for long memory processes is based on the Beta distribution for ϕ_{si}s, the simulation results show that $\widehat{d}$s are seriously larger than the average level of ϕ_{si}s.

The estimates of the MA parameters show significant upward bias when ϕ_{si}s are drawn from the normal distribution. The bias is more severe when σ_ϕ is small (Panel B). However, when ϕ_{si}s are drawn from the Beta distribution, we observe negative biases in the estimates of MA parameters though the biases become trivial for large ϕ_{si}s.

The simulation results support that the aggregated series becomes by far more persistent and ARMA(1,1) models are not appropriate for the aggregated series. The results also suggest that it is highly likely that we overestimate the smoothing of an appraisal based index returns when using an ARMA(1,1) model for appraisal based index returns. Although ARFIMA(0,d,1) models are based on the rather unintuitive assumption that AR parameters follow the Beta distribution, the simulation results support ARFIMA(0,d,1) models, even more strongly when AR parameters follow the normal distribution. For the purpose of filtering out the average level of AR and MA parameters from an aggregated series, the overall performance of ARFIMA(0,d,1) models are better than ARMA(1,1) models.

It is important to note that the simulation results depend on the probability density function assumed for the AR parameters. For the two distributions we used for the individual AR parameters, we find that the estimates of d could provide useful information for the average level of individual AR parameters, in particular when ϕ_{si}s are dispersed and large as in Panel A. The question is what is the level of dispersion of individual AR parameters in practice. For the real estate properties Brown and Matysiak's (1998) study reports that individual ϕ_{si}s are widely dispersed from 0.08 to 0.95. Therefore, without further information on the distribution on individual AR parameters we use ARFIMA(0,d,1) models for the aggregated series with $\sigma_\phi = 0.25$ as in Panel A.

3 An Application to the Real Estate Index Returns

We apply the analytical results in the previous section to investigate appraisal based real estate indices in the UK (IPD) and US (NCREIF), and the difference between these indices and their equivalent private market indices such as FTSE Real Estate and NAREIT indices respectively.

A Data

We use the IPD index and its equivalent FTSE Real Estate (FTR from now on) index for a total number of 192 monthly log-returns from January 1988 to December 2003. The FTSE All-Share (FTA from now on) index is also used for comparison purpose. Table 2A reports some basic properties of these three index log-returns. As expected, the IPD index returns display a far smaller volatility than the FTR index returns. One other clear difference is that the IPD index returns are highly persistent; autocorrelation of the IPD index returns decays very slowly. We examine if the IPD returns have a unit root using an augmented Dickey-Fuller test. The result rejects a unit root for the IPD returns at the usual 5% significance level, but not at the 1% significance level. Therefore the test does not provide a decisive conclusion as in FTR or FTA index returns.

The properties of the FTR and FTA returns have similar characteristics; high volatility, negative skewness, and little autocorrelation. These basic statistics suggest that the FTR is related more to the FTA than to the IPD. Direct comparison between the three indices can be found in the estimated cross-correlation coefficients. The FTR and FTA are highly correlated, while the IPD is far less correlated with these two securitized indices. Although the IPD and FTR indices represent the same underlying real estate, the two show little similarity.

Direct comparison between FTR and IPD however has some problems. Firms in the public markets are leveraged and liable to corporate tax while the appraisal based IPD index is not. We use a similar method to Geltner (1993), Fisher, Geltner and Webb (1994), and Barkham and Geltner (1995) to obtain an leverage-free and tax-free index from the FTR. From the weighted average cost of capital, the return on the unlevered and untaxed property, r_{umt}, can be obtained with

$$r_{umt} = \frac{E_t}{V_t}\frac{r_{lmt}}{1-t_c} + \frac{D_t}{V_t}r_{Dt}, \tag{21}$$

where r_{lmt} is the return on the levered and taxed firms at time t, r_{Dt} is the return on debt at time t, t_c is the corporate tax rate, D_t and E_t are total debt and shareholders' equity respectively, and $V_t = D_t + E_t$. Note that the returns in the equity market need to be 'untaxed' by dividing r_{lmt} by $1 - t_c$. Using total debt for D_t, market value of equities for E_t, the UK corporate bond yield for r_{Dt}, the FTR index returns for r_{lmt}, and assuming 30 percent corporate tax, we recover unlevered untaxed property index returns and report the statistical properties of these returns in Table 2A. The unlevered untaxed property index (from now on we call it FTRU) returns show little difference from those of the FTR returns, except for a few interesting differences; 1) the volatility of the FTRU decreases, and 2) the average return of the FTRU is close to that of the IPD index. Figure 2A shows that although the average return of the FTRU is similar to that of the IPD, the level of the FTRU at the end of 2003 is slightly lower than the IPD because of the high volatility of the FTRU returns. However, there are still significant differences in the volatility and correlation between FTRU and IPD. Thus leverage and tax do not explain the difference between the appraisal based index and the security market index.

For the US real estate market, we use 104 quarterly log-returns of NCREIF, NAREIT, unlevered NAREIT, and CRSP value weighted index returns for the period of the first quarter of 1978 to the fourth quarter of 2003. The basic statistical properties in Table 2B show similar attributes as those of the UK cases. The securitized index returns are closer to normality than the appraisal based index returns. This tendency towards normality of NAREIT can be explained by the central limit theorem for the low frequency (quarterly) returns. Other characteristics - the high persistence and the possibility of nonstationarity of the NCREIF index returns, and a higher correlation between NAREIT and CRSP index returns - are similar to those of the UK market. Augmented Dickey-Fuller tests show that the NCREIF index returns are non-stationary at conventional 5% significance level.

We calculate the unlevered NAREIT index returns using equation (21) with $t_c = 0$ (there is no tax on REITs in the US under certain tax regulations). We use the US corporate bond yield (Moodys AAA) for r_{Dt}, market value of equities for E_t, and the total debt for D_t. Again the unlevered NAREIT index (from now on we call it NAREITU) returns show a significant drop in volatility. The average return of the NAREITU is also smaller than the NAREIT, suggesting that leverage increases average returns as well as volatility in the US market. These results in the UK and US markets are in line with the results Barkham and Geltner (1995).

Figures 2A and 2B shows that appraisal based index returns are smoother and less volatile than the equity market index returns. In the US market, it is the NCREIF index that performs worse than the NAREIT index while in the UK it is the FTR index that performs worse than the IPD index. In the US market, the decreased volatility and average return of the NAREITU result in a large cumulative difference in the NAREIT and NAREITU at the end of our sample period.

B Evidence of Smoothing and Nonsynchronous Appraisal

a Estimation Methods

We estimate AR(1), ARMA(1,1) and ARFIMA$(0, d, 1)$ models for the securitized and appraisal based index returns. The popular AR(1) and ARMA(1,1) models are used to compare their results with those of ARFIMA(0,d,1) models. We use the exact time domain maximum likelihood (ML) estimation method of Sowell (1992).[17]

Estimating ARMA models is straightforward whereas estimating ARFIMA models is not so simple. The confusion between short and long memory processes is well documented in empirical tests, especially in small samples, and most econometric tests are not powerful enough to differentiate these two in finite samples. For example, Agiakloglou, Newbold and Wohar (1993) and Agiakloglou and Newbold (1993) show that it is difficult to detect the existence of long memory in the presence of AR or MA processes. Therefore unless a

[17] We use Ox version 3.30 (see Doornik, 2002) and the ARFIMA package version 1.00 (Doornik and Ooms, 1999) for the exact time domain ML estimates. Because of the possible nonstationarity of the appraisal based index returns (see Table 2), the appraisal based index returns are differenced again for the estimation of ARFIMA models.

well specified model is used for ARFIMA models, inserting short and long memory parameters could cause a serious bias in long memory parameters. From this reason we focus on ARFIMA$(0, d, 1)$ models rather than including arbitrary numbers of AR and MA lags.

Another important characteristic of appraisal based index returns such as IPD and NCREIF is that they show seasonal patterns. Using autocorrelations of residuals from the estimated ARFIMA$(0, d, 1)$ models and other model selection criteria such as ML, AIC and BIC, we include AR(3) and AR(4) for the IPD and NCREIF respectively. There are seasonal movements every three months (or quarterly patterns) for the monthly IPD while for the quarterly NCREIF we have seasonal movements every four quarters (or annual patterns).

b Securitized Returns

Panel A of Tables 3A and 3B show the estimates of AR(1), ARMA(1,1), and ARFIMA(0,d,0) models for the FTR and the NAREIT returns. We find no evidence of long memory in these securitized index returns. In addition, the estimated AR and MA parameters are not significant. When individual equity returns in securitized markets follow an AR(1) process because of momentum, the cross-sectionally aggregated returns (index returns) will follow an ARMA(p,q) process or a long memory process. No evidence of a short or long memory suggests that individual equities' AR parameters are not significantly different from zero on average or could be positive or negative because of momentum or contrarian effects.

For the securitized index returns we select the mean plus noise model, and conclude that smoothing and nonsynchronous appraisal do not exist in securitized markets. These results are not different for those in the FTRU and NAREITU in Panel B of Tables 3A and 3B.

c Level of Smoothing

Panel C of Tables 3A and 3B show that our theoretical ARFIMA(0,d,1) models are selected rather than the widely used AR(1) or ARMA(1,1) models (short memory processes) for the appraisal based index returns. In addition, as reported in the simulations in the previous section, the estimates of AR(1), ARMA(1,1) and ARFIMA(0,d,1) models for the appraisal based index returns show that AR parameters are much higher than long memory parameters in both UK and US markets. The large AR parameters are not different from those reported in previous studies such as Barkham and Geltner (1994).

However, the average level of smoothing approximated by the estimates of the long memory parameter is 0.572 for the NCREIF index and 0.856 for the IPD index. The differences between $\widehat{\phi}$ of the ARMA(1,1) model and $\widehat{d}$ are at least 0.36 for the quarterly NCREIF returns and 0.09 for the monthly IPD returns, suggesting that the index returns are far more persistent than the average level of smoothing of individual properties. Therefore, studies such as Barkham and Geltner (1994) which implicitly assume that the smoothing level estimated from an appraisal based index represents the average smoothing level of individual properties, lead to a serious upward bias in the average smoothing level of individual properties.

The smoothing level of the monthly IPD returns (0.856) is higher than that of the quarterly NCREIF returns (0.572). This could be attributed to the difference between the two countries. However, this can also be interpreted that smoothing may be less serious in lower frequency data (i.e., quarterly returns). In general temporal aggregation does not affect the long memory parameter (see Hwang, 2000). Therefore the reason why we observe a smaller long memory parameter for low frequency data is that the long memory parameter in our study represents the average level of AR parameters which decrease with frequency of returns.

In terms of stationarity of appraisal based index returns, we are not conclusive. The estimates of d show that the IPD returns are nonstationary (since $\widehat{d}$ is larger than 0.5 at the 5% significance level) while we do not reject stationarity for the NCREIF returns ($\widehat{d}$ is not significantly different from 0.5 at the 5% significance level). These are opposite results to those of the Dickey-Fuller test in Table 2.

When individual AR parameters are large and dispersed, there are many cases where individual ϕ_{si}s are close to 1. For example, for the cases in Panel A of Table 1 the mass near one increases with the average level of AR parameters. Therefore for the IPD index, where d is much larger than that of the NCREIF index, has many individual properties where the appraisals are not updated with new information as quickly as the NCREIF, resulting in 'stale appraisal'.[18]

d Nonsynchronous Appraisal

From the estimates of ARFIMA(0,d,1) models we find that the MA estimates are negative, i.e., -0.193 and -0.111 in the UK and US markets, though neither of them are significant. However, the simulation results in Panel A of Table 1 indicate that these estimates are positively biased by 0.05 to 0.1, and the insignificance of the MA estimates could be affected by the upward bias in the MA estimates. Thus the real values are likely to be around -0.25 and -0.2 on average for the IPD and NCREIF index returns. On the other hand, the MA estimates are large, negative and significant in the ARMA(1,1) models. However, the simulation results in Table 1 show that these are significantly negatively biased. In fact using the simulation results and the MA estimates from ARMA(1,1) models we arrive at

[18]The argument for the existence of stale appraisal is more clear with the Beta distribution. The result relating to the Beta distribution can be obtained from authors upon request.

similar levels of the MA estimates as those suggested above.

The large negative MA estimates suggest that the nonsynchronous variable (s_{it}) is highly volatile. In particular for a given annual Sharpe ratio and volatility of the nonsynchronous variable we expect larger MA parameters for the quarterly index returns. However, it is the monthly index which suffers greater nonsynchronous appraisal problems when the country difference is disregarded.

This is because a time difference between the appraisal time point (or information arrival time point) and the time point when the asset should be appraised becomes smaller for lower frequency. For example, the value of the nonsynchronous variable for 10 days difference in a month is 1/3 while the same 10 days in a quarter is only 1/9 when we assume that there are 30 days in a month. Therefore *ceteris paribus*, the standard deviation of the nonsynchronous variable for quarterly data will be $\lambda/3$, where λ is the standard deviation of nonsynchronous variable for monthly data. Because of the nonlinear relationship between the MA coefficient and the nonsynchronous variable in equation (10), we cannot directly compare the UK and US nonsynchronous effects. However it is clear that the monthly index suffers more nonsynchronous appraisal problems.

4 How Noisy Are Stock Markets?

Our analytical results make it possible to investigate an interesting issue in finance; the level of noise in stock markets. Many studies in real estate investigate the smoothing level of appraisal-based real estate index by matching the appraisal-based index return volatility to the securitized market index return volatility. However this method is appropriate only when the securitized market index return volatility represents the true volatility. It is now commonly accepted that financial asset prices are noisy and deviate from fundamentals. For example Black (1986) emphasizes the role of noise in financial markets, while De Long at el. (1990) show that equilibrium prices are influenced by noise traders. However, very few suggest how much noise involved in asset returns. Note that calculating the absolute level of volatility for individual asset returns is not possible in our model because of the idiosyncratic error and β_i, which are unknown.

A Unsmoothed Appraisal Based Index Return Volatilities with AR(1) Models

We first calculate the level of volatility of real estate index returns by applying the most widely used unsmoothing method in the real estate finance literature. This method entails applying an AR(1) filter to the appraisal-based index disregarding the nonsynchronous appraisal problem and the cross-sectional aggregation effects. The obtained AR parameters are then used to unsmooth the appraisal based index returns by applying the estimated AR model to the series and then taking the residuals (or the scaled residuals) to represent the true return process.

Using a over-bar for an average value calculated from the estimates in Table 2, the variance ratios ($\overline{\frac{Var(r_{cit})}{Var(r_{it})}}$) for the NCREIF and the IPD indices are 0.194 and 0.055 respectively (see equation (4)). The unsmoothed standard deviations are calculated as 2.27 (=$0.194^{-1/2}$) and 4.24 (=$0.055^{-1/2}$) times larger than those of the NCREIF and IPD index returns respectively. Therefore we have standard deviations of 3.76 for the NCREIF and 3.53 for the IPD, which are significantly larger than the volatilities of the NCREIF and IPD index returns reported in Table 2. Figures 2A and 2B show that the unsmoothed indices with AR(1) models are far more volatile than the equivalent appraisal based indices. However these reconstructed standard deviations are likely to be significantly upward biased since the AR parameters are seriously upward biased from the cross-sectional aggregation.

B Unsmoothed Appraisal Based Index Return Volatilities with ARFIMA(0,d,1) Models

We next calculate volatility considering the nonsynchronous appraisal and cross-sectional aggregation problems. Using the estimates of the ARFIMA(0,d,1) model in equations (14) and (16), we calculate the average ratio of the smoothed nonsynchronously appraised return volatility to the true volatility.

Using a over-bar for an average value calculated from the estimates in Table 2, we have $\overline{\frac{Var(r_{cit})}{Var(r_{it})}} = 0.345$ and $\overline{\frac{\sigma_{ci}}{\sigma_i}} = 0.535$ for the NAREIT index, since $\widehat{d} = 0.572$ (or $\overline{\phi}_{si} = 0.572$) and $\overline{v_i^2 \lambda_i^2} = 0.311$ from $\widehat{\theta} = -0.2$ (see equation (10)). On the other hand, for the IPD index, we have $\overline{\frac{Var(r_{cit})}{Var(r_{it})}} = 0.088$ and $\overline{\frac{\sigma_{ci}}{\sigma_i}} = 0.192$ using $\widehat{d} = 0.856$ (or $\overline{\phi}_{si} = 0.856$) and $\overline{v_i^2 \lambda_i^2} = 0.443$ from $\widehat{\theta} = -0.25$. These results suggest that for the individual property returns the true standard deviations are on average 1.70 (=$0.345^{-1/2}$) and 3.37 (=$0.088^{-1/2}$) times larger than the standard deviations we obtain with appraisal based individual returns for the NCREIF and the IPD respectively.

Applying these multipliers to the standard deviation of appraisal based index returns in Table 2, we have standard deviations of 2.82 for the NCREIF returns and 2.80 for the IPD returns. The reconstructed standard deviation of the NCREIF returns is 42.7 percent of the NAREIT returns. For the IPD the reconstructed standard deviation of returns is 50.7 percent of the standard deviation of the FTR returns. Compared with the unlevered and untaxed securitized index return volatilities, the reconstructed standard deviation of the NCREIF returns is 71.1 percent of the standard deviation of the NAREITU returns, while the reconstructed standard deviation of the IPD returns is 64.8 percent of the standard deviation of the FTRU returns.

In these two cases the differences between volatilities from appraisal based and securitized index returns are around 30% to 35% for quarterly and monthly returns. Under the assumption that the reconstructed returns from an appraisal based index are not likely to be exposed to noise traders, the difference could be interpreted as the noise level in stock markets. Disregarding the difference between countries we could say that low frequency returns have less noise than higher frequency returns; high frequency returns such as daily returns are expected to include more noise than low frequency returns such as annual returns. This is because noise in high frequency returns is expected to cancel out each other when temporally aggregated in low frequency returns. Therefore we expect that the proportion of noise in daily returns is much higher than 35 percent of the FTRU while that in annual returns is lower than 30 percent of the NAREITU.

Our results suggest a higher level of noise than those suggested by French and Roll (1986) who investigate the level of noise by comparing volatilities during trading and nontrading hours. They find 4 to 12 percent of mispricing from daily returns. The difference could come from our assumption that the volatility calculated with ARFIMA models is appropriate to filter out smoothing and nonsynchronous appraisal problems. However considering the series of studies on excess volatility in financial markets by Shiller (1981, 2000, 2003) and others, we believe that the level of noise calculated in this paper is not different from the actual level.

5 Conclusion

In this study we have investigated the effects of three problems of an appraised-based performance index; smoothing, nonsynchronous appraisal and cross-sectional aggregation. In the presence of these econometric problems, it is found that an index comprised of the performance of individual assets is far more persistent than the average level of smoothing in individual assets. Using simulations we show that an ARFIMA(0,d,1) process is a good analytical representation of a cross-sectionally aggregated process, where the long memory parameter d explains average level of smoothing of individual constituents and the MA parameter explains the level of nonsynchronous appraisal.

We have applied our analytical results to UK and US appraisal-based real estate index returns and then compared the results with their equivalent stock market index returns. Several interesting empirical results are derived from the analytical models developed in this paper. The level of smoothing of appraisals (downward bias in volatility) is far less than assumed in many academic studies. In addition, when the assets also trade in public markets, the securitized market volatility is much higher than that of the true underlying process. The empirical results also find evidence of nonsynchronous appraisal, which is a more serious problem for indices constructed using higher frequency returns, such as the monthly IPD index in the UK.

In our study we applied our model to real estate index returns. However, the results of this study also have application to other non-transaction based indices used in macroeconomics and finance (such as consumer confidence measures). Any time series index based on a survey, expert's opinion or non-transaction based estimate of price has potentially similar econometric problems to those discussed in this study. Application of these results to a broader range of performance measures may provide an interesting area for future study.

Appendix

Proof of Theorem 1

The length of the interval for the reported return at time t is $1 - s_{it} + s_{it-1}$. The expected return and variance conditional on the nonsynchronous variable is

$$\begin{aligned} E(r_{nit}|s_{it}) &= \mu_i(1 - s_{it} + s_{it-1}), \\ Var(r_{nit}|s_{it}) &= (1 - s_{it} + s_{it-1})\sigma_i^2, \\ Cov(r_{nit}, r_{nit-\tau}|s_{it}) &= 0 \text{ for all } \tau > 0, \end{aligned}$$

since

$$\begin{aligned} Cov(r_{nit}, r_{nit-\tau}|s_{it}) &= E[(r_{nit} - E(r_{nit}|s_{it}))(r_{nit-\tau} - E(r_{nit-\tau}|s_{it}))] \\ &= E[(\sqrt{1 - s_{it} + s_{it-1}}\sigma_i\varepsilon_{it})(\sqrt{1 - s_{it-\tau} + s_{it-\tau-1}}\sigma_i\varepsilon_{it-\tau}))] \\ &= \sigma_i^2\sqrt{1 - s_{it} + s_{it-1}}\sqrt{1 - s_{it-\tau} + s_{it-\tau-1}}E(\varepsilon_{it}\varepsilon_{it-\tau}) \\ &= 0. \end{aligned}$$

Since we assume the nonsynchronous variable, s_{it}, is neither autocorrelated nor correlated with ε_{it}, the unconditional variance of r_{nit} is

$$\begin{aligned} Var(r_{nit}) &= E[Var(r_{nit}|s_{it})] + Var(E[r_{nit}|s_{it}]) \\ &= \sigma_i^2 E(1 - s_{it} + s_{it-1}) + \mu_i^2 Var(1 - s_{it} + s_{it-1}). \end{aligned}$$

Unconditional autocovariance with lag 1 is

$$
\begin{aligned}
Cov(r_{nit}, r_{nit-1}) &= E[Cov(r_{nit}, r_{nit-1}|s_{it})] + Cov[E(r_{nit}|s_{it}), E(r_{nit-1}|s_{it})] \\
&= \mu_i^2 Cov(1 - s_{it} + s_{it-1}, 1 - s_{it-1} + s_{it-2}).
\end{aligned}
$$

Using similar method, autocovariance with lag $\tau > 1$, is zero.

Proof of Remark 1

The results can be obtained with

$$
\begin{aligned}
Var(1 - s_{it} + s_{it-1}) &= [Var(s_{it}) + Var(s_{it-1})] \\
&= 2\lambda_i^2, \\
Cov(1 - s_{it} + s_{it-1}, 1 - s_{it-1} + s_{it-2}) &= -Var(s_{it-1}) \\
&= -\lambda_i^2,
\end{aligned}
$$

since s_{it} is *iid* negative exponentially distributed with mean λ_i and variance λ_i^2.

Proof of Equation (18)

Using (10) and (14), we can rewrite equation (17) as

$$
r_{cit} - \mu_i = \sum_{\tau=0}^{\infty} \phi_{si}^{\tau} \frac{\sigma_{ci}}{\sigma_i} \left[\theta_i(\beta_i \epsilon_{t-\tau-1} + \eta_{it-\tau-1}) + (\beta_i \epsilon_{t-\tau} + \eta_{it-\tau}) \right]. \tag{22}
$$

Index returns are equivalent to cross-sectionally aggregated N ARMA(1,1) return processes.

$$\begin{aligned} r_{mt} - \mu_m &= \frac{1}{N}\sum_{i=1}^{N}(r_{cit} - \mu_i) \qquad (23) \\ &= \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{\tau=0}^{\infty}\phi_{si}^{\tau}\frac{\sigma_{ci}}{\sigma_i}\left[\theta_i(\beta_i\epsilon_{t-\tau-1} + \eta_{it-\tau-1}) + (\beta_i\epsilon_{t-\tau} + \eta_{it-\tau})\right]\right], \end{aligned}$$

where r_{mt} and μ_m are the aggregated smoothed reported index return at time t and its expected return respectively. As N increases, the idiosyncratic errors can be removed with a large number of assets,

$$\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\eta_{it-\tau} = 0 \text{ for all } \tau.$$

This argument is based on the assumption that $\theta_i\frac{\sigma_{ci}}{\sigma_i}$ is not related to η_{it} for all t. We also assume that β_i is not related with $\frac{\sigma_{ci}}{\sigma_i}$ since asset i's sensitivity to the factor ϵ_t is not expected to be related with smoothing or nonsynchronous appraisal. Therefore

$$r_{mt} - \mu_m = \sum_{\tau=0}^{\infty}E_c\left[\phi_{si}^{\tau}(\theta_i\epsilon_{it-1-\tau}^{*} + \epsilon_{it-\tau}^{*})\right]$$

where $\epsilon_{it-\tau}^{*} = \frac{\sigma_{ci}}{\sigma_i}\beta_i\epsilon_{t-\tau}$ and $E_c(.)$ represents cross-sectional expectation.

Proof of Theorem 2

Consider the following stationary ARMA(1,q) process;

$$x_{it} = \psi_i x_{it-1} + (\theta_{iq}\xi_{t-q} + \theta_{iq-1}\xi_{t-q+1}+, ..., +\xi_t),$$

where $\xi_t \sim iid(0,1)$, ψ_i and θ_{iq} are independent for all i and q. Therefore, we have

$$E_c(x_{it}) = \sum_{\tau=0}^{\infty} E_c(\psi_i^\tau)\left[E_c(\theta_{iq})\xi_{t-\tau-q} + E_c(\theta_{iq-1})\xi_{t-\tau-q+1}+, ..., +\xi_{t-\tau}\right].$$

When ψ_i follows $Beta(d, 1-d)$ distribution with $0 < \psi_i < 1$ $\forall i$, whose probability density function is

$$f(\psi_i) = \frac{1}{\Gamma(d)\Gamma(1-d)}\psi_i^{d-1}(1-\psi_i)^{-d},$$

where $d > 0$, $1-d > 0$, and $\Gamma(.)$ is the Gamma function. Then, the cross-sectional expectation of ψ_i^τ, $E_c[\psi_i^\tau]$, is

$$\begin{aligned} E_c[\psi_i^\tau] &= \int_{-\infty}^{\infty} \psi_i^\tau \frac{1}{\Gamma(d)\Gamma(1-d)}\psi_i^{d-1}(1-\psi_i)^{-d} d\psi \\ &= \frac{1}{\Gamma(d)}\frac{\Gamma(\tau+d)}{\Gamma(\tau+1)}\int_{-\infty}^{\infty} \frac{\Gamma(\tau+1)}{\Gamma(\tau+d)\Gamma(1-d)}\psi_i^{\tau+d-1}(1-\psi_i)^{-d} d\psi \\ &= \frac{1}{\Gamma(d)}\frac{\Gamma(\tau+d)}{\Gamma(\tau+1)}, \end{aligned} \tag{24}$$

since $\int_{-\infty}^{\infty} \frac{\Gamma(\tau+1)}{\Gamma(\tau+d)\Gamma(1-d)}\psi_i^{\tau+d-1}(1-\psi_i)^{-d} d\psi = 1$. Therefore, we have

$$E_c(x_{it}) = \sum_{\tau=0}^{\infty} \frac{1}{\Gamma(d)}\frac{\Gamma(\tau+d)}{\Gamma(\tau+1)}\left[E_c(\theta_{iq})\xi_{t-\tau-q}+, ..., +\xi_{t-\tau}\right].$$

Note that $\frac{1}{\Gamma(d)}\frac{\Gamma(\tau+d)}{\Gamma(\tau+1)}$ is the τth moving average coefficient of fractionally integrated process. Therefore, we have

$$(1-L)^d E_c(x_{it}) = E_c(\theta_{iq})\xi_{t-q} + E_c(\theta_{iq-1})\xi_{t-q+1}+, ..., +\xi_t,$$

where L is the lag operator.

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Table 1 Monte Carlo Simulations for the Effects of Cross-sectional Aggregation on Persistence

We first generate 1000 ARMA(1,1) processes using the parameter values in the column of the 'data generating process'. AR parameters are generated from normal distribution with standard deviations of 0.25 (Panel A) and 0.15 (Panel B) or with beta distribution (Panel C). MA parameters are generated from normal distribution with standard deviation of 0.1. Any AR parameters less than 0 are truncated to 0 and any AR parameters larger than 1 are truncated to 1. MA parameters are also truncated such that all MA parameters are negative. 200 observations are generated for each ARMA(1,1) process, and then these 1000 ARMA(1,1) series are cross-sectionally aggregated to construct one single time series. The results are based on 1000 iterations. STD represents standard deviation of estimates.

A. When AR and MA Parameters are Normally Distributed: Standard Deviation of AR Parameter is 0.25

	Data Generating Process		Average Values of Truncated AR and MA Parameters*		ML Estimates of ARMA(1,1) Model		ML Estimates of ARFIMA(0,d,1) Model	
	AR	MA	AR from Normal Distribution with Standard Deviation of 0.25	MA from Normal Distribution with Standard Deviation of 0.1	AR	MA	*d*	MA
Estimates	0.400	-0.100	0.405	-0.108	0.911	-0.580	0.383	-0.009
STD					0.111	0.162	0.132	0.141
Estimates	0.400	-0.200	0.405	-0.201	0.932	-0.693	0.374	-0.092
STD					0.105	0.149	0.147	0.158
Estimates	0.400	-0.300	0.405	-0.300	0.953	-0.794	0.376	-0.192
STD					0.099	0.130	0.194	0.205
Estimates	0.500	-0.100	0.500	-0.108	0.944	-0.496	0.507	-0.022
STD					0.071	0.120	0.147	0.152
Estimates	0.500	-0.200	0.500	-0.201	0.954	-0.602	0.501	-0.106
STD					0.064	0.112	0.161	0.168
Estimates	0.500	-0.300	0.500	-0.300	0.965	-0.704	0.500	-0.204
STD					0.056	0.098	0.196	0.200
Estimates	0.600	-0.100	0.595	-0.108	0.957	-0.395	0.619	-0.028
STD					0.058	0.102	0.137	0.142
Estimates	0.600	-0.200	0.595	-0.201	0.962	-0.497	0.618	-0.117
STD					0.055	0.098	0.163	0.169
Estimates	0.600	-0.300	0.595	-0.300	0.969	-0.601	0.619	-0.215
STD					0.052	0.091	0.196	0.200
Estimates	0.700	-0.100	0.686	-0.108	0.975	-0.310	0.722	-0.038
STD					0.038	0.086	0.130	0.139
Estimates	0.700	-0.200	0.686	-0.201	0.977	-0.408	0.719	-0.123
STD					0.036	0.085	0.149	0.158
Estimates	0.700	-0.300	0.686	-0.300	0.980	-0.511	0.725	-0.225
STD					0.034	0.082	0.188	0.197
Estimates	0.800	-0.100	0.770	-0.108	0.983	-0.237	0.808	-0.046
STD					0.029	0.079	0.125	0.135
Estimates	0.800	-0.200	0.770	-0.201	0.984	-0.330	0.808	-0.131
STD					0.028	0.080	0.147	0.158
Estimates	0.800	-0.300	0.770	-0.300	0.985	-0.430	0.814	-0.230
STD					0.027	0.079	0.182	0.190
Estimates	0.900	-0.100	0.842	-0.108	0.988	-0.181	0.874	-0.053
STD					0.020	0.076	0.124	0.139
Estimates	0.900	-0.200	0.842	-0.201	0.989	-0.270	0.872	-0.132
STD					0.019	0.078	0.136	0.152
Estimates	0.900	-0.300	0.842	-0.300	0.990	-0.367	0.875	-0.227
STD					0.018	0.079	0.166	0.180

B. When AR and MA Parameters are Normally Distributed: Standard Deviation of AR Parameter is 0.15

	Data Generating Process		Average Values of Truncated AR and MA Parameters*		ML Estimates of ARMA(1,1) Model		ML Estimates of ARFIMA(0,d,1) Model	
	AR	MA	AR from Normal Distribution with Standard Deviation of 0.15	MA from Normal Distribution with Standard Deviation of 0.1	AR	MA	*d*	MA
Estimates	0.400	-0.100	0.400	-0.108	0.551	-0.243	0.175	0.132
STD					0.227	0.254	0.094	0.102
Estimates	0.400	-0.200	0.400	-0.201	0.573	-0.359	0.145	0.071
STD					0.292	0.321	0.098	0.109
Estimates	0.400	-0.300	0.400	-0.300	0.589	-0.468	0.108	0.010
STD					0.398	0.417	0.105	0.118
Estimates	0.500	-0.100	0.500	-0.108	0.682	-0.264	0.281	0.138
STD					0.160	0.193	0.094	0.102
Estimates	0.500	-0.200	0.500	-0.201	0.704	-0.384	0.250	0.078
STD					0.187	0.227	0.098	0.108
Estimates	0.500	-0.300	0.500	-0.300	0.733	-0.517	0.212	0.018
STD					0.230	0.271	0.104	0.117
Estimates	0.600	-0.100	0.600	-0.108	0.832	-0.296	0.421	0.115
STD					0.112	0.144	0.098	0.105
Estimates	0.600	-0.200	0.600	-0.201	0.847	-0.413	0.393	0.053
STD					0.116	0.156	0.103	0.112
Estimates	0.600	-0.300	0.600	-0.300	0.865	-0.539	0.358	-0.010
STD					0.124	0.168	0.112	0.124
Estimates	0.700	-0.100	0.699	-0.108	0.929	-0.279	0.586	0.068
STD					0.066	0.101	0.107	0.112
Estimates	0.700	-0.200	0.699	-0.201	0.934	-0.385	0.565	-0.001
STD					0.065	0.103	0.117	0.124
Estimates	0.700	-0.300	0.699	-0.300	0.941	-0.499	0.538	-0.072
STD					0.064	0.105	0.130	0.139
Estimates	0.800	-0.100	0.794	-0.108	0.968	-0.217	0.741	0.016
STD					0.039	0.081	0.111	0.120
Estimates	0.800	-0.200	0.794	-0.201	0.969	-0.315	0.727	-0.057
STD					0.038	0.082	0.124	0.134
Estimates	0.800	-0.300	0.794	-0.300	0.971	-0.419	0.710	-0.136
STD					0.037	0.081	0.139	0.149
Estimates	0.900	-0.100	0.877	-0.108	0.986	-0.158	0.863	-0.027
STD					0.022	0.073	0.114	0.127
Estimates	0.900	-0.200	0.877	-0.201	0.986	-0.248	0.857	-0.103
STD					0.022	0.075	0.128	0.143
Estimates	0.900	-0.300	0.877	-0.300	0.986	-0.346	0.850	-0.188
STD					0.022	0.076	0.151	0.164

C. When AR Parameters are beta-distributed and MA Parameters are Normally Distributed

	Data Generating Process		Average Values of Truncated AR and MA Parameters*		ML Estimates of ARMA(1,1) Model		ML Estimates of ARFIMA(0,d,1) Model	
	AR	MA	AR from Beta Distribution	MA from Normal Distribution with Standard Deviation of 0.1	AR	MA	*d*	MA
Estimates	0.400	-0.100	0.400	-0.108	0.967	-0.572	0.653	-0.228
STD					0.042	0.086	0.191	0.203
Estimates	0.400	-0.200	0.400	-0.201	0.973	-0.652	0.670	-0.332
STD					0.037	0.076	0.205	0.209
Estimates	0.400	-0.300	0.400	-0.300	0.979	-0.729	0.673	-0.426
STD					0.031	0.066	0.201	0.194
Estimates	0.500	-0.100	0.500	-0.108	0.972	-0.477	0.724	-0.202
STD					0.034	0.080	0.171	0.185
Estimates	0.500	-0.200	0.500	-0.201	0.976	-0.562	0.737	-0.301
STD					0.031	0.074	0.183	0.191
Estimates	0.500	-0.300	0.500	-0.300	0.980	-0.646	0.740	-0.397
STD					0.028	0.067	0.187	0.185
Estimates	0.600	-0.100	0.600	-0.108	0.978	-0.386	0.797	-0.183
STD					0.028	0.076	0.156	0.172
Estimates	0.600	-0.200	0.600	-0.201	0.980	-0.474	0.815	-0.286
STD					0.026	0.073	0.181	0.193
Estimates	0.600	-0.300	0.600	-0.300	0.983	-0.563	0.823	-0.386
STD					0.024	0.069	0.192	0.195
Estimates	0.700	-0.100	0.700	-0.108	0.983	-0.301	0.852	-0.151
STD					0.025	0.076	0.154	0.170
Estimates	0.700	-0.200	0.700	-0.201	0.985	-0.390	0.862	-0.246
STD					0.024	0.075	0.171	0.186
Estimates	0.700	-0.300	0.700	-0.300	0.986	-0.483	0.871	-0.346
STD					0.023	0.073	0.187	0.196
Estimates	0.800	-0.100	0.800	-0.108	0.989	-0.222	0.907	-0.121
STD					0.017	0.074	0.137	0.156
Estimates	0.800	-0.200	0.800	-0.201	0.990	-0.309	0.916	-0.211
STD					0.017	0.075	0.159	0.176
Estimates	0.800	-0.300	0.800	-0.300	0.991	-0.403	0.921	-0.305
STD					0.017	0.076	0.173	0.186
Estimates	0.900	-0.100	0.900	-0.108	0.993	-0.146	0.964	-0.103
STD					0.015	0.068	0.136	0.155
Estimates	0.900	-0.200	0.900	-0.201	0.993	-0.229	0.969	-0.184
STD					0.015	0.073	0.149	0.170
Estimates	0.900	-0.300	0.900	-0.300	0.993	-0.320	0.976	-0.278
STD					0.015	0.077	0.169	0.188

Table 2A Properties of IPD and FTSE Real Estate Index Monthly Log-Returns

The basic statistical properties in the table are obtained using 192 monthly log-returns from January 1988 to December 2003. Bold numbers represent significance at 5% level. Monthly Sharpe ratios are calculated with total returns rather than excess returns, and thus slightly higher than the values found in other studies during similar sample period. Annual Sharpe ratios are calculated by multiplying square-root of 12 to the monthly Sharpe ratios. We use the Augmented Dickey-Fuller test to examine if log-return series are unit root. The number of AR lags is chosen using Bayesian information criteria. The probability values of the augmented Dickey-Fuller test statistics is MacKinnon (1996) one-sided probability values.

	IPD Index Returns	FTSE Real Estate Index Returns	Unlevered Untaxed FTSE Real Estate Index Returns	FTSE All Share Index Returns
Mean	0.797	0.569	0.810	0.799
STD	0.831	5.516	4.320	4.357
Skewness	**0.433**	**-0.489**	**-0.408**	**-0.396**
Kurtosis	**4.152**	3.035	2.952	3.462
Jarque-Bera	**16.634**	**7.669**	5.344	**6.728**
Sharpe Ratio (Monthly)	0.959	0.103	0.187	0.183
Sharpe Ratio (Annual)	3.321	0.358	0.649	0.635
Autocorrelation				
1	**0.897**	0.133	0.099	0.052
2	**0.851**	0.012	0.019	-0.116
3	**0.817**	0.023	0.016	-0.053
4	**0.731**	0.077	0.079	0.049
5	**0.652**	0.062	0.044	0.022
6	**0.578**	0.024	0.027	0.039
7	**0.486**	-0.051	-0.073	-0.059
8	**0.411**	0.002	0.013	0.031
9	**0.329**	0.005	-0.013	0.045
10	**0.264**	-0.071	-0.043	0.046
Augmented Dickey-Fuller Test Statistic	-3.021 (3 Lags)	-12.033 (No Lag)	-12.475 (No Lag)	-13.078 (No Lag)
Probability	0.035	0.000	0.000	0.000
Correlation				
IPD Index Returns	1.000			
FTSE Real Estate Index Returns	0.037	1.000		
Unleveraged Untaxed FTSE Real Estate Index Returns	0.035	**0.988**	1.000	
FTSE All Share Index Returns	-0.014	**0.625**	**0.610**	1.000

Table 2B Properties of NCREIF and NAREIT Quarterly Log-Returns

The basic statistical properties in the table are obtained using 104 quarterly log-returns from the first quarter of 1978 to the fourth quarter of 2003. During 2003 CRSP index returns are not available and we use the MSCI Investible 2500 index which explains 98% of the US equity market. Bold numbers represent significance at 5% level. Quarterly Sharpe ratios are calculated with total returns rather than excess returns, and thus slightly higher than the values found in other studies during similar sample period. Annual Sharpe ratios are calculated by multiplying square-root of 4 to the quarterly Sharpe ratios. We use the Augmented Dickey-Fuller test to examine if the log-return series are unit root. The number of AR lags is chosen with Bayesian information criteria. The probability values of the augmented Dickey-Fuller test statistics is MacKinnon (1996) one-sided probability values.

	NCREIF	NAREIT	Unlevered NAREIT Index Returns	CRSP
Mean	2.233	3.318	2.816	3.523
STD	1.656	6.597	3.963	8.581
Skewness	**-1.081**	-0.105	-0.098	**-0.479**
Kurtosis	**7.286**	3.337	3.716	3.368
Jarque-Bera	**99.872**	0.683	2.387	**4.561**
Sharpe Ratio (Quarterly)	1.348	0.503	0.711	0.411
Sharpe Ratio (Annual)	2.696	1.006	1.421	0.821
Autocorrelation				
1	**0.680**	0.061	0.065	-0.037
2	**0.682**	0.021	0.034	0.000
3	**0.598**	-0.080	-0.047	-0.029
4	**0.714**	0.006	0.036	-0.059
5	**0.468**	-0.051	-0.020	-0.069
6	**0.423**	0.004	0.003	0.001
7	**0.381**	-0.120	-0.104	-0.111
8	**0.436**	0.073	0.086	0.145
9	**0.250**	-0.061	-0.030	0.049
10	**0.237**	-0.110	-0.114	0.057
Augmented Dickey-Fuller Test Statistic	-2.193 (4 Lags)	-9.407 (0 Lag)	-9.402 (0 Lag)	-10.407 (0 Lag)
Probability	0.2101	0.000	0.000	0.000
Cross-Correlation				
NCREIF	1.000			
NAREIT	-0.013	1.000		
Unlevered NAREIT Index Returns	-0.003	**0.991**	1.000	
CRSP	-0.040	**0.593**	**0.582**	1.000

Table 3A Estimates of ARFIMA(*p,d,q*) Models for IPD and FTSE Real Estate Index Monthly Log-Returns

The results are obtained using 192 monthly log-returns from January 1988 to December 2002. The sample averege return is taken out from the log-returns and are not estimated in the ARFIMA models. AR1 and AR3 represent the autoregressive parameters with lags 1 and 3, and MA1 represents the moving average parameter with lag 1. AIC and BIS are Akaike and Bayesian information criteria respectively. Estimates are obtained with the exact maximum likelihood module provided by Ox version 3.30 (see Doornik, 2002) and the Arfima package version 1.00 (Doornik and Ooms, 1999). Bold estimates show significance at 5% level.

A. Estimation Results of FTSE Real Estate Index Monthly Returns

Models	Parameters	Estimates	Standard Deviation	Maximum Likelihood Value	AIC	BIC
ARFIMA(0,d,0)	Long Memory Parameter (d)	0.089	(0.064)	-598.781	1201.562	1208.077
	Standard Deviation of Error Term	5.472				
AR(1)	AR1	0.133	(0.072)	-598.083	**1200.165**	**1206.680**
	Standard Deviation of Error Term	5.452				
	AR1	0.099	(0.614)			
ARMA(1,1)	MA1	0.035	(0.619)	**-598.079**	1202.159	1211.931
	Standard Deviation of Error Term	5.452				

B. Estimation Results of Unlevered and Untaxed FTSE Real Estate Index Monthly Returns

Models	Parameters	Estimates	Standard Deviation	Maximum Likelihood Value	AIC	BIC
AR(1)	AR1	0.099	(0.072)	-551.917	**1107.835**	**1114.350**
	Standard Deviation of Error Term	4.287				
	AR1	0.617	(0.468)			
ARMA(1,1)	MA1	-0.535	(0.501)	**-551.852**	1109.703	1119.476
	Standard Deviation of Error Term	4.286				

C. Estimation Results of IPD Index Monthly Returns

Models	Parameters	Estimates	Standard Deviation	Maximum Likelihood Value	AIC	BIC
AR(1)	AR1	**0.895**	(0.031)	-79.276	162.552	169.067
	Standard Deviation of Error Term	0.364				
	AR1	0.948	(0.023)			
ARMA(1,1)	MA1	-0.279	(0.065)	-72.143	150.285	165.315
	Standard Deviation of Error Term	0.351				
	Long Memory Parameter (d)	**0.856**	(0.153)			
ARFIMA(0,d,1)	MA1	-0.193	(0.161)	**-66.622**	**141.244**	**159.532**
	AR3	**0.306**	(0.085)			
	Standard Deviation of Error Term	0.343				

Table 3B Estimation Results of ARFIMA(*p,d,q*) Models for NCREIF and NAREIT Quarterly Log-Returns

The results are obtained using 104 quarterly log-returns from the first quarter of 1978 to the third quarter of 2003. The sample averege return is taken out from the log-returns and is not estimated in the ARFIMA models. AR1 and AR4 represent the autoregressive parameters with lags 1 and 4, and MA1 represents the moving average parameter with lag 1. AIC and BIS are Akaike and Bayesian information criteria respectively. Estimates are obtained with the exact maximum likelihood module provided by Ox version 3.30 (see Doornik, 2002) and the ARFIMA package version 1.00 (Doornik and Ooms, 1999). Bold estimates show significance at 5% level.

A. Estimation Results of NAREIT Index Quarterly Returns

Models	Parameters	Estimates	Standard Deviation	Maximum Likelihood Value	AIC	BIC
ARFIMA(0,d,0)	Long Memory Parameter (d)	0.005	(0.089)	-343.281	690.562	695.850
	Standard Deviation of Error Term	6.566				
AR(1)	AR1	0.061	(0.098)	-343.088	**690.175**	**695.464**
	Standard Deviation of Error Term	6.553				
	AR1	0.141	(0.834)			
ARMA(1,1)	MA1	-0.079	(0.835)	**-343.083**	692.166	700.100
	Standard Deviation of Error Term	6.553				

B. Estimation Results of Unlevered NAREIT Index Quarterly Returns

Models	Parameters	Estimates	Standard Deviation	Maximum Likelihood Value	AIC	BIC
AR(1)	AR1	0.064	(0.098)	-290.061	**584.123**	**589.412**
	Standard Deviation of Error Term	3.936				
	AR1	0.258	(0.980)			
ARMA(1,1)	MA1	-0.193	(0.993)	**-290.042**	586.085	594.018
	Standard Deviation of Error Term	3.935				

C. Estimation Results of NCREIF Index Quarterly Returns

Models	Parameters	Estimates	Standard Deviation	Maximum Likelihood Value	AIC	BIC
AR(1)	AR1	**0.675**	(0.071)	-167.441	338.882	344.171
	Standard Deviation of Error Term	1.207				
	AR1	**0.934**	(0.038)			
ARMA(1,1)	MA1	**-0.549**	(0.082)	-156.868	316.740	327.669
	Standard Deviation of Error Term	1.088				
	Long Memory Parameter (d)	**0.572**	(0.160)			
ARFIMA(0,d,1)	MA1	-0.111	(0.160)	**-139.745**	**287.490**	**293.423**
	AR4	**0.556**	(0.084)			
	Standard Deviation of Error Term	0.931				

Figure 1 The Ratio of the Variance of the True Mean Plus Noise Returns to That of the Smoothed Reported Returns When the Variance of Nonsynchronous Variable is 0.49

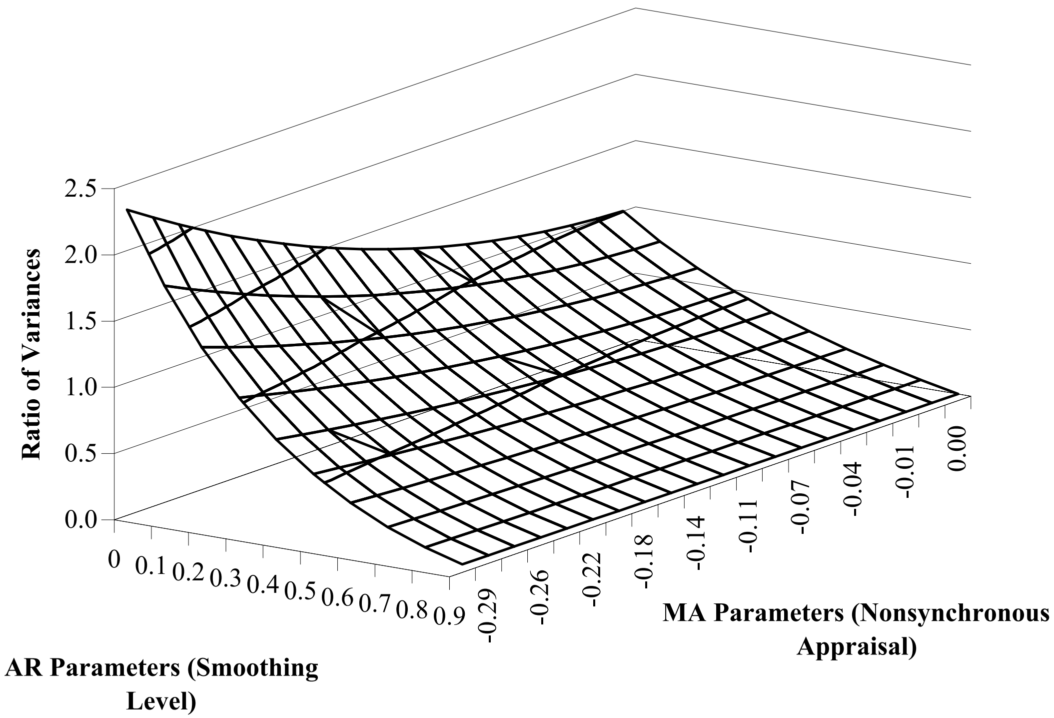

Figure 2A UK Commercial Real Estate Indices
(End of 1987=100)

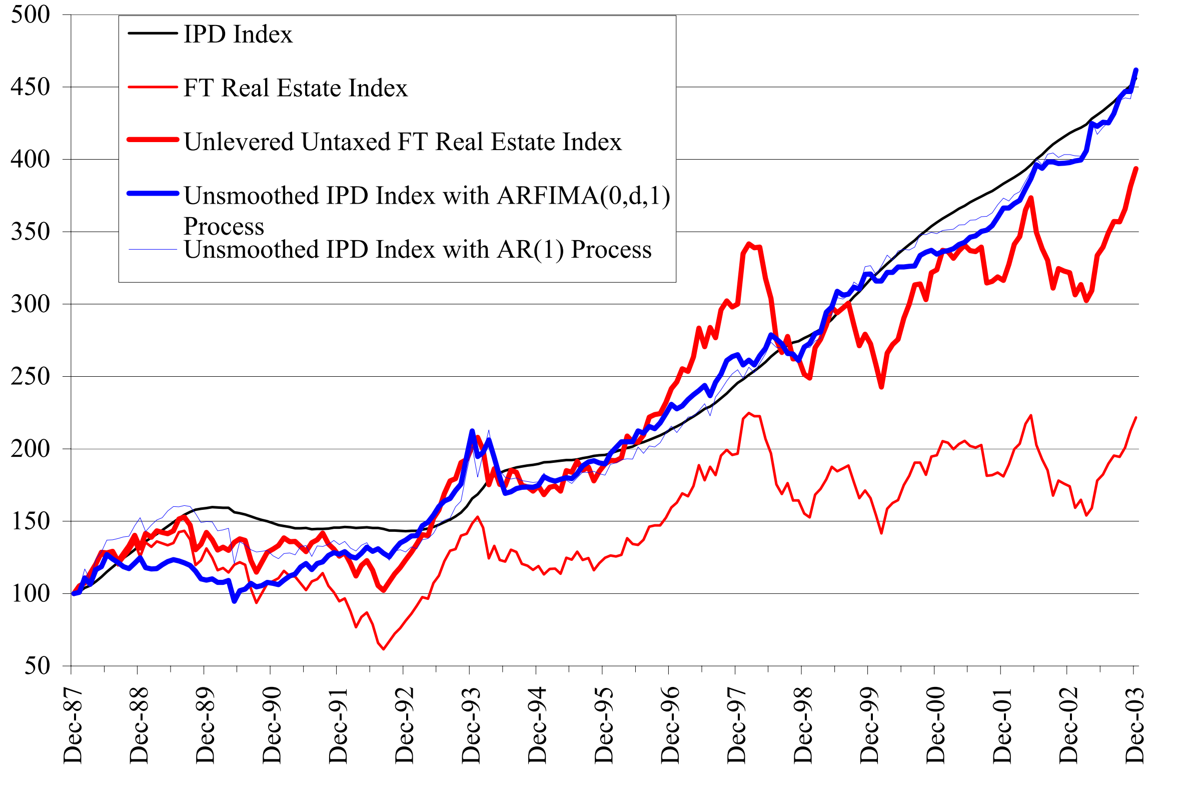

Figure 2B US Commercial Real Estate Log-Indices
(End of 1977=100)