Enhanced Choice-Based Network
Revenue Management

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Independent versus Dependent Demand

**Independent Demand**
- class availability  
  - Y  
  - K  
  - B  
  - Q
- booking classes
- customer segments

**Dependent Demand**
- class availability  
  - Y  
  - K  
  - B  
  - Q
- booking classes
- customer segments
Dimensions of Customer Choice

_Lufthansa_

- **Flight Options**
- **Price**
- **Passenger Details**
- **Payment**
- **Booking Summary**

Please select your departing and returning flights, then review the price before booking.

**Lowest Price**

_Flight Price 1) -
1 adult
£ 249.58 2)_

**Duration**

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>Flight</th>
<th>Duration</th>
<th>Economy Basic</th>
<th>Economy Flex</th>
<th>Business Flex</th>
</tr>
</thead>
<tbody>
<tr>
<td>06:25 London, Heathrow</td>
<td>Frank.furt</td>
<td>LH921</td>
<td>1h35</td>
<td>£ 176.58</td>
<td></td>
<td></td>
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<tr>
<td>07:15 London, Heathrow</td>
<td>Frank.furt</td>
<td>LH923</td>
<td>1h35</td>
<td>£ 197.58</td>
<td>£ 302.58</td>
<td></td>
</tr>
<tr>
<td>09:10 London, Heathrow</td>
<td>Frank.furt</td>
<td>LH901</td>
<td>1h35</td>
<td>£ 249.58</td>
<td>£ 302.58</td>
<td>£ 356.58</td>
</tr>
</tbody>
</table>

**Currency Converter**

- Rebooking: EUR 50
- Not included
- Mileage Credit:* 750 miles
- Allowed
- Mileage Upgrade:** not applicable
- Allowed
- Fee
- Fee
Importance of Choice Models in Mixed Fare Environments

Motivation

- Restricted fares: Use e.g. mandatory Saturday night stay to segment the market.
- Unrestricted fares: Price is the only differentiating feature.
  ⇒ customers only buy at the lowest open fare
- Trend in recent year towards simplified fare structures in some markets
- RM model need to handle mix of restricted & unrestricted fares, as well as customer choice behaviour

Example

Fiig, Isler, Hopperstad and Belobaba (2010)
“Optimization of mixed fare structures: Theory and applications”. *JRPM*

Meissner and Strauss (2010)
“Pricing Structure Optimization in mixed restricted/unrestricted fare environments”. *JRPM*
How to model customer choice?

Choice Models

- Reservation price models
- Random utility models (e.g. Multinomial Logit)
**Choice Modelling for Fixed Itinerary**

**Example**

Lufthansa Systems’ Choice Model

- Buy-down forecasts
- Nested product sets: allows to pre-compute choice probabilities

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**Product Clusters**

- C
- D
- Y
- B
- M
- H
- G
- T

**Fare**

- $699
- $419
- $399
- $379
- $250
- $219
- $149
- $99

**Demand Graph**

- Direction of buy-down
- 100% buy-down
- 4 Buy-down potential
- 8 Demand

Nodes and edges indicate the structure and flow of the choice model.
Disjoint Consideration Sets

In sufficiently fenced markets:
Customer segments consider disjoint sets of products for purchase.
If strong segmentation not possible:
Customer segments consider **overlapping** sets of products for purchase.
Multinomial Logit

Parameter estimation requires only past capacity availability, revenue accounting and flight schedule data.

**Multinomial Logit Choice Model**

1. Define preference vector for each segment
2. Probability that a customer of a segment purchases product $j$ subject to which alternatives are offered

\[
\text{Probability} = \frac{\text{preference for product } j}{\text{sum of preferences for considered available alternatives}}
\]

G. Vulcano, G. van Ryzin and W. Chaar (2010)
“Choice-Based Revenue Management: An Empirical Study of Estimation and Optimization”. MSOM.
**Model Context and Purpose**

**Airline Modelling Framework**

- Multiple products, sold over finite booking time horizon
- Flight network with fixed capacities
- Unused capacity becomes worthless at departure $T$
- Customer choice model: MNL with overlapping consideration sets

**Goal**

Policy stating which set of products to offer given time and remaining inventory.
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Goal
Policy stating which set of products to offer given time and remaining inventory.
General Approach

Given time period $t$ and vector of available inventory $x$, which set $S^*$ to offer?

$$S^* = \arg\max_{\text{feasible } S} E \left[ \text{revenue} - \text{opp.cost}(t, x) | \text{offer set } S \right]$$

How to get opportunity cost? Dynamic Programming formulation:

$$v_t(x) = \max_{\text{feasible } S} \sum_{j \in S} \lambda P_j(S) \left[ r_j - (v_{t+1}(x) - v_{t+1}(x - A_j)) \right] + v_{t+1}(x), \forall t, x.$$  

Boundary condition: $v_{T+1}(x) = 0$ for all $x$
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**Choice-Based Deterministic Linear Program (CDLP)**

- $R(S)$ — expected revenue from offering $S$
- $Q_i(S)$ — inventory consumption on resource $i$
- $w(S)$ — number of time periods that set $S$ is offered

\[
\begin{align*}
\max & \quad \sum_{S \subseteq J} \lambda R(S) w(S) & \text{Expected revenue} \\
\text{s.t.} & \quad \sum_{S \subseteq J} \lambda w(S) \tilde{Q}(S) \leq \tilde{c} & \text{Capacity constraints} \\
\sum_{S \subseteq J} w(S) = T & \quad \text{Finite time horizon} \\
0 \leq w(S), \forall S \subseteq J. & \end{align*}
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Gallego et al. (2004)  

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“On the choice-based linear programming model for network revenue management”. MSOM.
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Problems with CDLP

- $2^n$ variables ($n$: number of products)
- Column generation NP-hard for overlapping segments (Miranda Bront et al. 2009, OR)
- Only guaranteed upper bound once optimal solution has been reached
- Time-consuming
Numerical Example

Joint work with K. Talluri and J. Meissner:

<table>
<thead>
<tr>
<th>Spokes</th>
<th>Legs</th>
<th>OD pairs</th>
<th>Itineraries</th>
<th>Products</th>
<th>Segments</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>36</td>
<td>90</td>
<td>172</td>
<td>860</td>
<td>360</td>
</tr>
<tr>
<td>16</td>
<td>68</td>
<td>306</td>
<td>596</td>
<td>2980</td>
<td>1224</td>
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New Approach Outperforms CDLP
Forced CDLP to stop after maximum of 10 hours.
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How Does It Work?

Idea

- Each customer segment considers only a few products
- Therefore, CDLP can be solved for a single segment
  → Segment-based Deterministic Concave Program (SDCP), Talluri (2010)
- Add constraints to enforce consistency between segment-based decisions
  → SDCP with “product constraints”

Talluri (2010)

### Segment-based Deterministic Concave Program

\[
\begin{align*}
\text{max} & \quad \sum_l \sum_{s_l} \lambda_l R_l(s_l) w^l(s_l) \\
\sum_l \tilde{y}^l & \leq \bar{c} \\
\sum_{s_l} \lambda_l \bar{Q}_l(s_l) w^l(s_l) & \leq \tilde{y}^l & \forall l \\
\sum_{s_l} w^l(s_l) & = T & \forall l \\
w^l(s_l) & \geq 0 \\
\tilde{y}_l & \leq \lambda_l T \bar{t} \\
\tilde{y}_l & \geq 0
\end{align*}
\]

- **max total revenue**
- **network capacity constraints**
- **segment capacity constraints**
- **time constraints**
- **duration of offering \( S_l \)**
- **segment demand constraint**
- **capacity allocation to segment \( l \)**
Product Cuts

Example

Two segments with $C^l = \{1, 2\}$ and $C^k = \{2\}$.

Suppose the SDCP solution for segments $l$ and $k$ is:

- Need to coordinate interaction on overlap $C^l \cap C^k = \{2\}$
- Add constraint $w^l(\{2\}) + w^l(\{1, 2\}) = w^k(\{2\})$
- Solve SDCP with new constraints.
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- Solve SDCP with new constraints.
SDCP with Product Cuts

\[ \text{Solve} \quad \text{SDCP} \]
\[ \text{s.t.} \quad \sum_{S_l \supseteq S_{lk}} w^l(S_l) = \sum_{S_k \supseteq S_{lk}} w^k(S_k) \quad \forall S_{lk} \subseteq C_l \cap C_k, \forall \{l, k\} \]

Results

- Provides upper bound on optimal expected revenue
- Same objective as CDLP in all test scenarios where CDLP tractable
- Sufficient to consider constraints for \(|S_{lk}| \leq 2\)
- Dramatic run time reductions (10 min vs. 10 hours)
***Dynamic Programming Decomposition***

Solve for all flight legs $i$:

\[ v^i_t(x_i) = \max_{\text{feasible } S} \sum_{j \in S} P_j(S) \left( \frac{A_{ij} \pi_j r_j}{\sum_k A_{kj} \pi_k} - (v^i_{t+1}(x_i) - v^i_{t+1}(x_i - 1)) \right) \]

\[ + v^i_{t+1}(x_i) \quad \forall t, x_i, \]

\[ v^i_{T+1}(x_i) = 0 \quad \text{for all } x_i, \]

\[ v^i_t(0) = 0 \quad \text{for all } t. \]

- Runs sufficiently quick
- Good performance with iterations and resolving

Approximate opportunity cost with

\[ \sum_{i \in A_j} (v^i_{t+1}(x_i) - v^i_{t+1}(x_i - 1)) \] to obtain policy
Further Potential for Improvement

Joint with T. Winter and P. Kemmer at Lufthansa Systems:

Idea

- Dynamic Programs are already solved in parallel
- Exploit knowledge on $v_t^i$ as soon as it becomes available

$\Rightarrow$ can obtain time-dependent marginal capacity values (almost) for free!
Dynamic Marginal Capacity Value Candidates

In time $t$, suppose we have solved $v_i^t$ for all legs $i$ and $t \in \{t + 1, \ldots, T\}$.

$$\pi_{t+1}^i := \frac{\sum_{h=1}^{c_i} (v_{t+1}^i(h) - v_{t+1}^i(h-1))}{c_i}$$

Increased average revenues in simulation studies up to 1%. 
Summary

- Choice-based demand model important but complex
- Difficult optimisation if products are considered by more than one segment
- But sufficiently powerful tools now available
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- E-Mail: arne.strauss@wbs.ac.uk

“An Enhanced Concave Program Relaxation for Choice Network Revenue Management”.

“Dynamic Simultaneous Fare Proration for Large-Scale Network Revenue Management”